Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.2/48-1.1.3.2-f

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**132**]. This is test number [48].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (132)	0.00 (0)
Mathematica	100.00 (132)	0.00 (0)
Maple	97.73 (129)	2.27 (3)
Fricas	93.18 (123)	6.82 (9)
Reduce	93.18 (123)	6.82 (9)
Mupad	88.64 (117)	11.36 (15)
Giac	86.36 (114)	13.64 (18)
Sympy	84.09 (111)	15.91 (21)
Maxima	59.85 (79)	40.15 (53)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

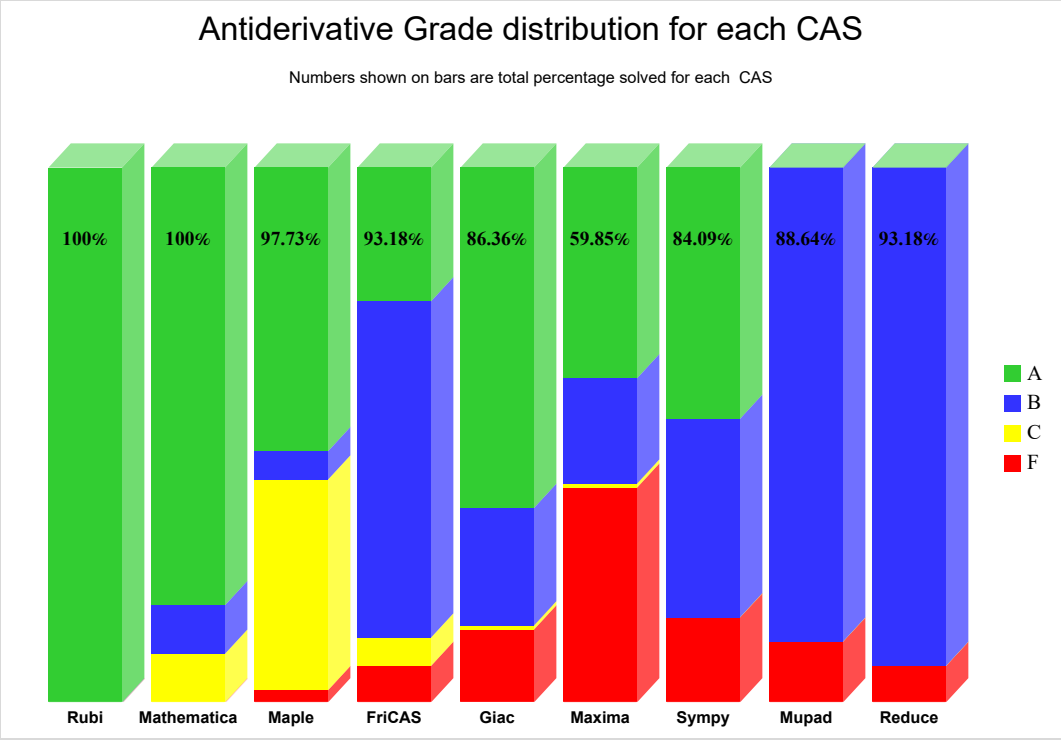
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

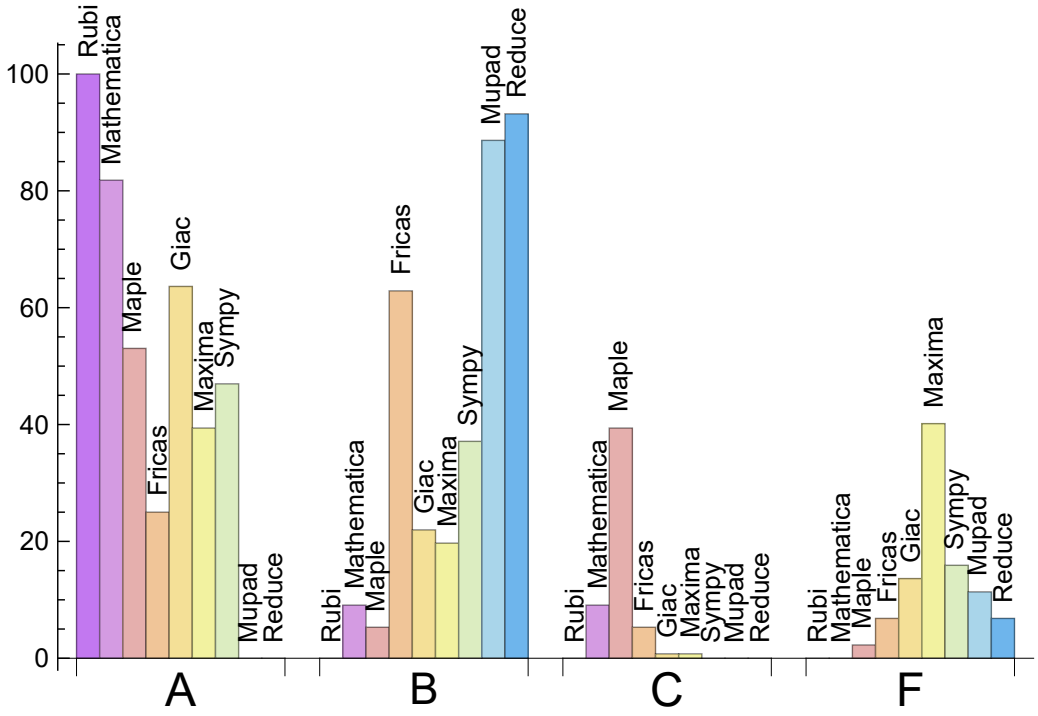
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	81.818	9.091	9.091	0.000
Giac	63.636	21.970	0.758	13.636
Maple	53.030	5.303	39.394	2.273
Sympy	46.970	37.121	0.000	15.909
Maxima	39.394	19.697	0.758	40.152
Fricas	25.000	62.879	5.303	6.818
Mupad	0.000	88.636	0.000	11.364
Reduce	0.000	93.182	0.000	6.818

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	3	100.00	0.00	0.00
Fricas	9	88.89	11.11	0.00
Reduce	9	100.00	0.00	0.00
Mupad	15	0.00	100.00	0.00
Giac	18	77.78	0.00	22.22
Sympy	21	61.90	38.10	0.00
Maxima	53	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Giac	0.13
Reduce	0.24
Fricas	0.25
Rubi	0.39
Mathematica	0.58
Mupad	0.66
Maple	1.00
Sympy	2.76

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	82.52	1.73	28.00	1.08
Rubi	95.95	0.96	50.00	0.98
Mathematica	115.54	1.31	64.00	0.95
Giac	146.75	2.43	117.50	1.27
Sympy	159.24	3.05	83.00	1.38
Mupad	182.73	2.18	131.00	1.71
Maple	233.26	1.91	74.00	0.96
Reduce	543.31	4.46	126.00	2.42
Fricas	655.72	4.61	156.00	3.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

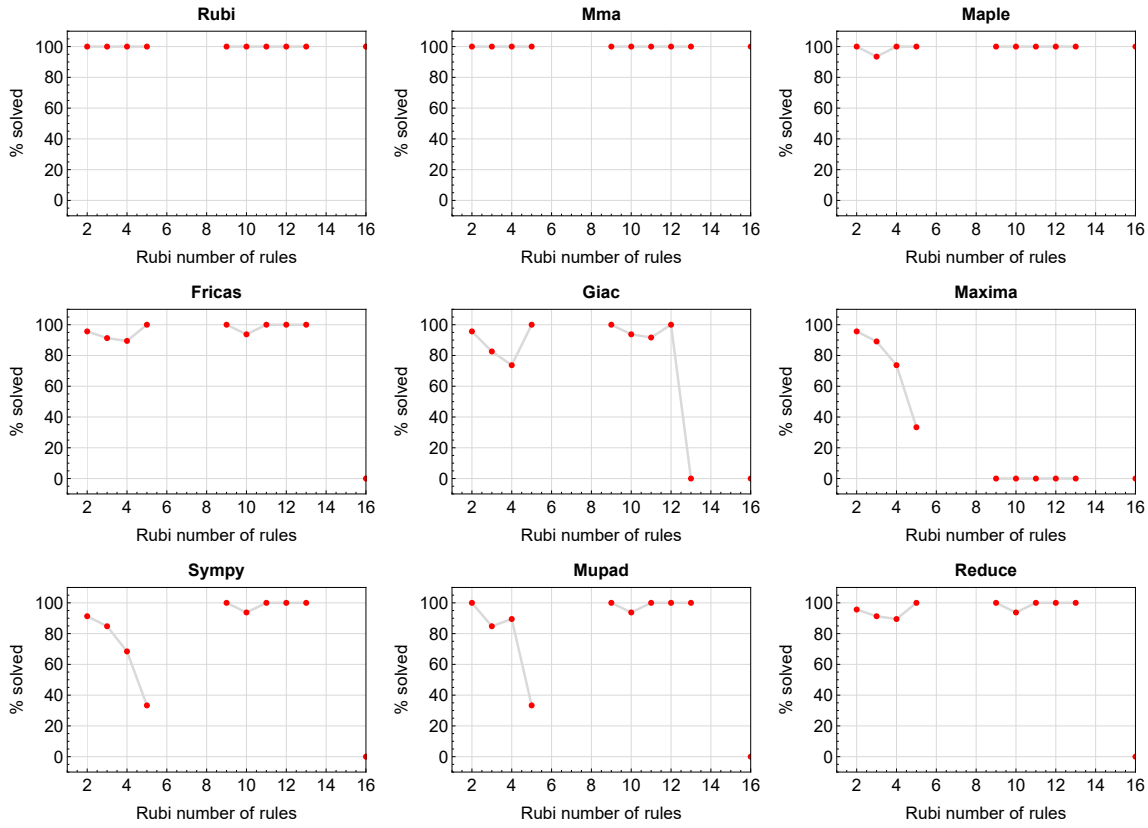


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

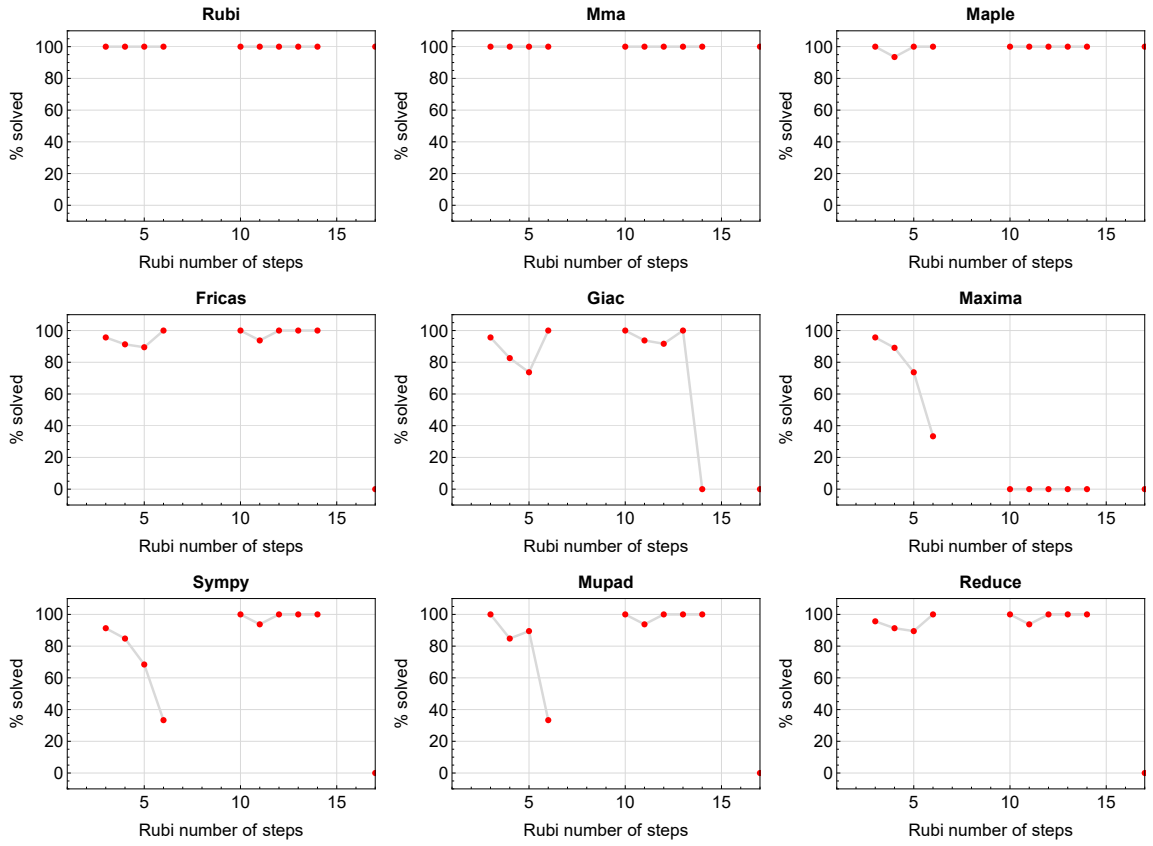


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

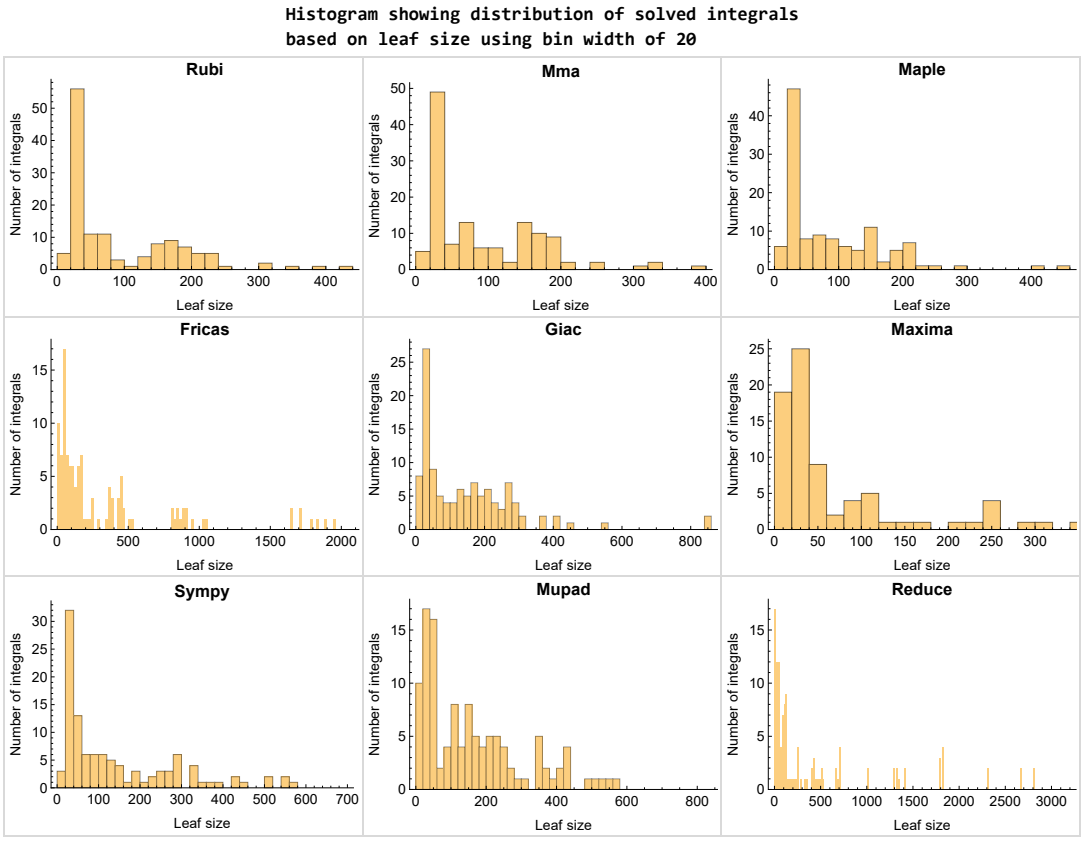


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

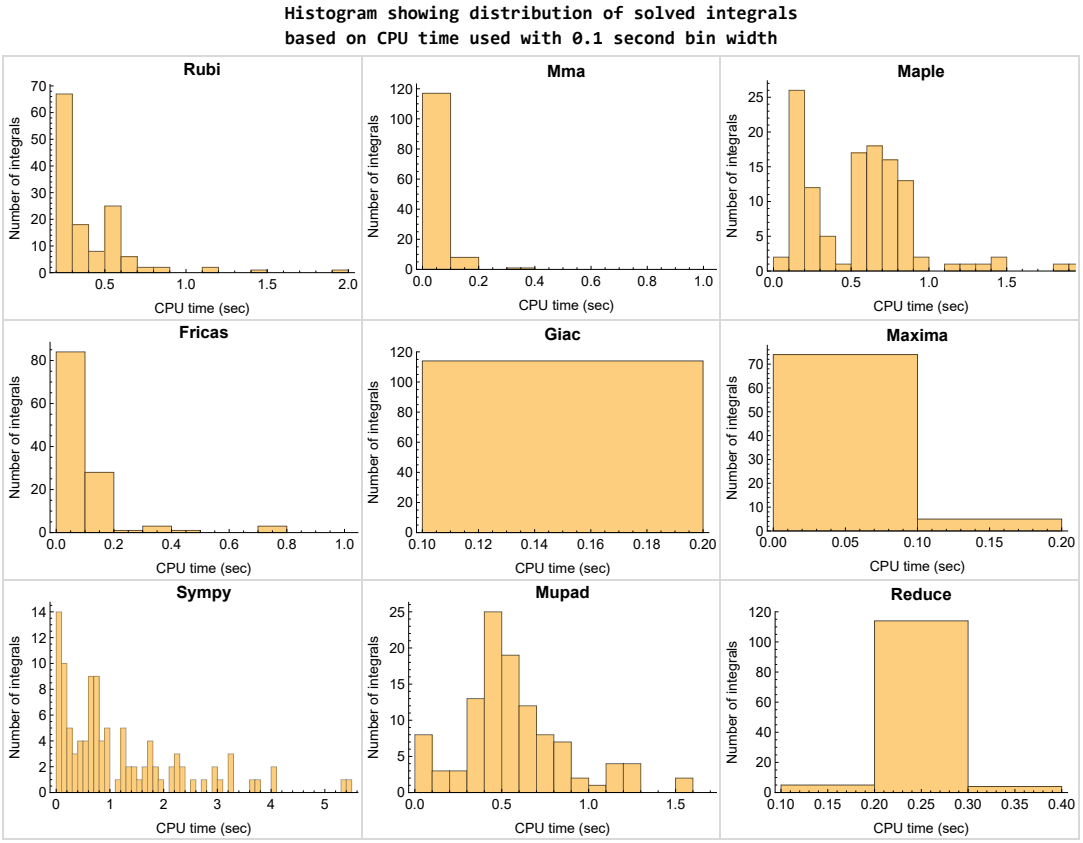


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

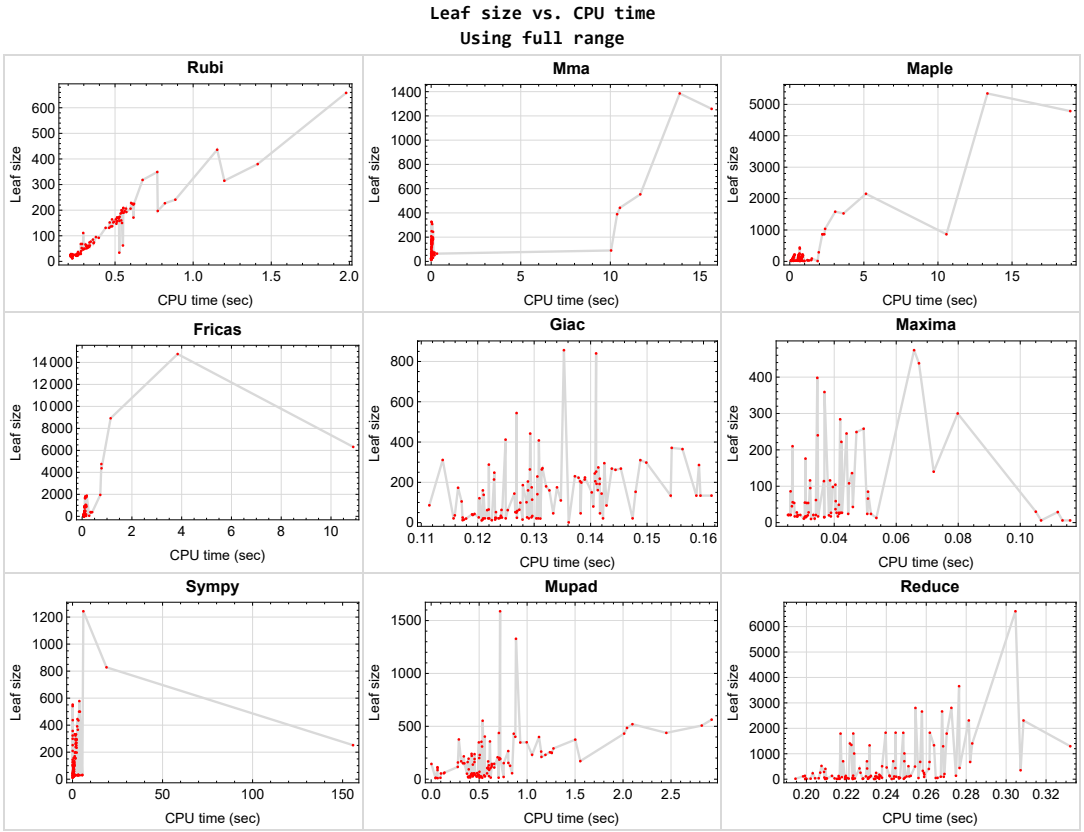


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{129, 130, 131, 132\}$

Mathematica $\{123, 124, 125, 127, 128\}$

Maple $\{7, 128, 129, 130, 131, 132\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

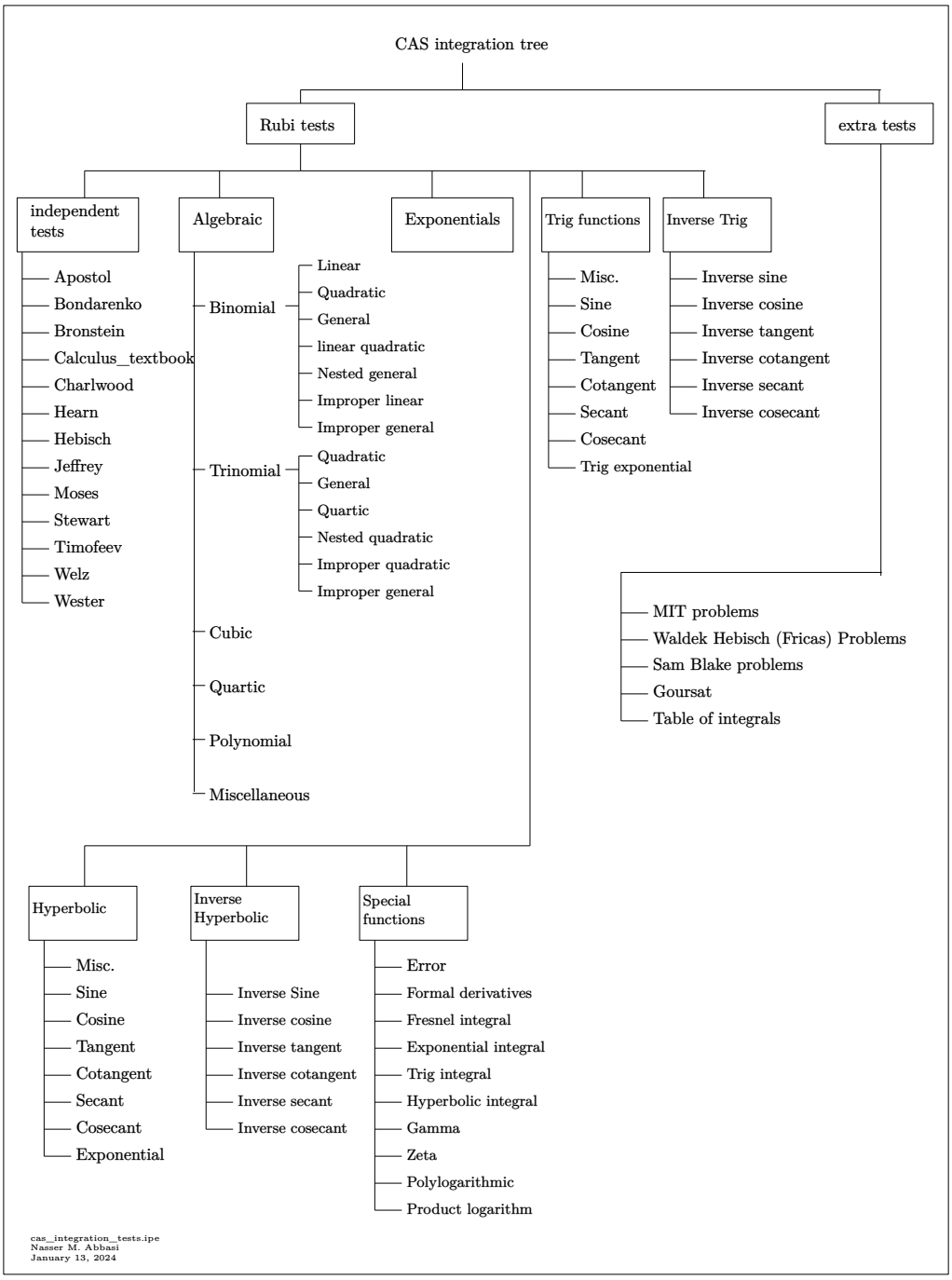
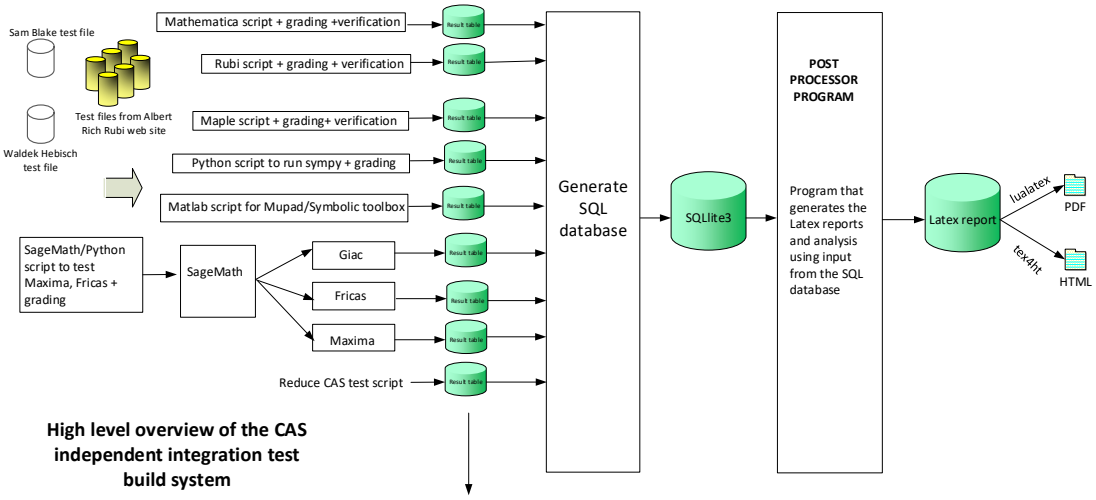


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 27
Mma 27
Maple 28
Fricas 28
Maxima 29
Giac 29
Mupad 30
Sympy 30
Reduce 31

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 119, 129, 130, 131, 132 }

B grade { 40, 41, 42, 52, 53, 54, 55, 56, 57, 110, 111, 112 }

C grade { 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 50, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 89, 92, 95, 97, 100, 103, 105, 108, 109, 110, 111, 112, 113, 114, 115 }

B grade { 46, 52, 53, 54, 55, 56, 57 }

C grade { 7, 11, 58, 59, 61, 62, 64, 65, 67, 68, 70, 71, 73, 74, 76, 77, 79, 80, 82, 83, 85, 86, 88, 90, 91, 93, 94, 96, 98, 99, 101, 102, 104, 106, 107, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132 }

F normal fail { 47, 49, 51 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 7, 8, 9, 10, 18, 19, 20, 25, 26, 36, 37, 39, 43, 44, 45, 50, 58, 59, 61, 62, 63, 64, 65, 85, 86, 87, 88, 89, 90, 91, 119 }

B grade { 1, 2, 5, 6, 12, 13, 14, 15, 16, 17, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 40, 41, 42, 46, 48, 52, 53, 54, 55, 56, 57, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 129, 130, 131, 132 }

C grade { 11, 116, 117, 118, 120, 121, 122 }

F normal fail { 47, 49, 51, 123, 124, 125, 126, 127 }

F(-1) timedout fail { 128 }

F(-2) exception fail { }

Maxima

A grade { 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 50, 60, 63, 66, 69, 72, 78, 87, 89, 92, 97, 109, 110, 111, 112, 113, 114, 115 }

B grade { 1, 2, 3, 12, 13, 17, 29, 40, 41, 42, 46, 48, 52, 53, 54, 55, 56, 57, 75, 81, 84, 95, 100, 103, 105, 108 }

C grade { 11 }

F normal fail { 47, 49, 51, 58, 59, 61, 62, 64, 65, 67, 68, 70, 71, 73, 74, 76, 77, 79, 80, 82, 83, 85, 86, 88, 90, 91, 93, 94, 96, 98, 99, 101, 102, 104, 106, 107, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20, 24, 25, 26, 28, 30, 36, 37, 38, 39, 43, 44, 45, 48, 50, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 119 }

B grade { 12, 13, 14, 21, 22, 23, 29, 31, 32, 33, 34, 35, 40, 41, 42, 46, 52, 53, 54, 55, 56, 57, 83, 108, 109, 129, 130, 131, 132 }

C grade { 11 }

F normal fail { 47, 49, 51, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 27, 120 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 126 }

C grade { }

F normal fail { }

F(-1) timedout fail { 26, 36, 37, 47, 49, 51, 123, 124, 125, 127, 128, 129, 130, 131, 132 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 24, 25, 26, 27, 28, 29, 33, 34, 35, 36, 38, 43, 44, 45, 58, 59, 61, 62, 63, 64, 65, 67, 68, 70, 71, 73, 74, 76, 77, 79, 80, 82, 85, 86, 88, 89, 90, 91, 93, 94, 96, 98, 99, 101, 102, 104, 116, 117, 118, 119 }

B grade { 1, 2, 3, 18, 19, 20, 21, 22, 23, 30, 31, 32, 37, 39, 40, 41, 42, 50, 52, 53, 54, 55, 56, 57, 60, 66, 69, 72, 75, 78, 81, 83, 84, 87, 92, 95, 97, 100, 103, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115 }

C grade { }

F normal fail { 12, 13, 51, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132 }

F(-1) timedout fail { 46, 47, 48, 49, 109, 120, 121, 122 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132 }

C grade { }

F normal fail { 47, 49, 51, 123, 124, 125, 126, 127, 128 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	54	103	828	21	59	36
N.S.	1	0.83	0.83	0.73	1.80	3.43	27.60	0.70	1.97	1.20
time (sec)	N/A	0.219	0.016	0.746	0.031	0.069	18.853	0.122	0.238	0.552

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	22	54	67	366	21	35	36
N.S.	1	0.89	0.89	0.79	1.93	2.39	13.07	0.75	1.25	1.29
time (sec)	N/A	0.223	0.006	0.694	0.027	0.066	2.928	0.121	0.204	0.440

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	29	54	41	71	36	12	21
N.S.	1	1.00	1.24	1.16	2.16	1.64	2.84	1.44	0.48	0.84
time (sec)	N/A	0.218	0.010	0.689	0.032	0.065	0.655	0.116	0.200	0.755

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	15	42	26	34	15	29
N.S.	1	1.00	1.00	0.96	0.54	1.50	0.93	1.21	0.54	1.04
time (sec)	N/A	0.220	0.004	0.826	0.031	0.072	0.774	0.124	0.228	0.602

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	17	69	29	0	28	26
N.S.	1	0.83	0.83	0.73	0.57	2.30	0.97	0.00	0.93	0.87
time (sec)	N/A	0.219	0.005	0.845	0.039	0.064	2.359	0.000	0.246	0.476

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	17	97	29	0	50	26
N.S.	1	0.83	0.83	0.73	0.57	3.23	0.97	0.00	1.67	0.87
time (sec)	N/A	0.220	0.008	0.839	0.035	0.075	4.006	0.000	0.231	0.510

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	16	30	9	19	26	8	13
N.S.	1	1.00	1.25	0.80	1.50	0.45	0.95	1.30	0.40	0.65
time (sec)	N/A	0.215	0.006	0.240	0.105	0.062	0.758	0.120	0.219	0.503

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	24	29	9	20	25	7	13
N.S.	1	1.00	1.25	1.20	1.45	0.45	1.00	1.25	0.35	0.65
time (sec)	N/A	0.219	0.004	0.086	0.112	0.063	0.674	0.117	0.199	0.097

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	6	8	22	15	8	14
N.S.	1	1.00	1.00	0.88	0.23	0.31	0.85	0.58	0.31	0.54
time (sec)	N/A	0.217	0.004	0.497	0.116	0.095	0.782	0.121	0.223	0.703

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	6	8	22	15	8	14
N.S.	1	1.00	1.00	0.96	0.23	0.31	0.85	0.58	0.31	0.54
time (sec)	N/A	0.225	0.004	0.507	0.114	0.072	0.609	0.123	0.234	0.532

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	10	6	6	24	23	8	15
N.S.	1	1.00	1.00	0.36	0.21	0.21	0.86	0.82	0.29	0.54
time (sec)	N/A	0.223	0.004	0.686	0.107	0.076	0.572	0.123	0.209	0.621

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	108	135	0	856	98	110
N.S.	1	0.83	0.83	0.73	3.60	4.50	0.00	28.53	3.27	3.67
time (sec)	N/A	0.232	0.019	0.177	0.045	0.078	0.000	0.135	0.245	0.658

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	22	66	83	0	408	63	58
N.S.	1	0.89	0.89	0.79	2.36	2.96	0.00	14.57	2.25	2.07
time (sec)	N/A	0.231	0.007	0.138	0.051	0.087	0.000	0.131	0.231	0.479

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	24	44	29	121	29	22
N.S.	1	1.00	1.00	0.88	0.96	1.76	1.16	4.84	1.16	0.88
time (sec)	N/A	0.225	0.003	0.130	0.051	0.081	0.913	0.120	0.224	0.384

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	27	61	31	23	18	26
N.S.	1	1.00	1.00	0.96	1.17	2.65	1.35	1.00	0.78	1.13
time (sec)	N/A	0.229	0.005	0.146	0.041	0.088	1.228	0.124	0.195	0.409

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	43	109	32	26	47	26
N.S.	1	0.83	0.83	0.73	1.43	3.63	1.07	0.87	1.57	0.87
time (sec)	N/A	0.226	0.006	0.145	0.046	0.067	3.206	0.130	0.265	0.522

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	85	151	32	26	80	26
N.S.	1	0.83	0.83	0.73	2.83	5.03	1.07	0.87	2.67	0.87
time (sec)	N/A	0.229	0.009	0.148	0.051	0.076	5.456	0.129	0.218	0.560

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	17	27	41	24	25	17
N.S.	1	1.00	1.00	1.05	0.81	1.29	1.95	1.14	1.19	0.81
time (sec)	N/A	0.230	0.007	0.118	0.027	0.097	0.473	0.127	0.228	0.407

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	37	24	16	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.95	1.26	0.84	0.89
time (sec)	N/A	0.220	0.003	0.104	0.031	0.065	0.288	0.123	0.223	0.388

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	17	21	36	21	13	22
N.S.	1	1.00	1.00	1.16	0.89	1.11	1.89	1.11	0.68	1.16
time (sec)	N/A	0.223	0.006	0.066	0.028	0.070	0.129	0.129	0.235	0.398

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	17	35	39	62	21	40
N.S.	1	1.00	1.00	1.05	0.81	1.67	1.86	2.95	1.00	1.90
time (sec)	N/A	0.218	0.008	0.120	0.041	0.068	0.279	0.122	0.202	0.415

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	17	46	39	114	32	54
N.S.	1	1.00	1.00	1.05	0.81	2.19	1.86	5.43	1.52	2.57
time (sec)	N/A	0.219	0.009	0.107	0.031	0.065	0.414	0.130	0.210	0.442

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	17	57	39	186	43	26
N.S.	1	1.00	1.00	1.05	0.81	2.71	1.86	8.86	2.05	1.24
time (sec)	N/A	0.219	0.014	0.124	0.028	0.089	0.698	0.128	0.220	0.506

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	49	60	29	21	50	26
N.S.	1	0.83	0.83	0.73	1.63	2.00	0.97	0.70	1.67	0.87
time (sec)	N/A	0.223	0.007	0.324	0.041	0.069	5.350	0.122	0.266	0.433

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	22	27	36	29	21	26	24
N.S.	1	0.89	0.89	0.79	0.96	1.29	1.04	0.75	0.93	0.86
time (sec)	N/A	0.220	0.005	0.119	0.039	0.074	1.437	0.121	0.210	0.397

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	13	37	32	20	12	0
N.S.	1	1.00	1.00	0.96	0.46	1.32	1.14	0.71	0.43	0.00
time (sec)	N/A	0.226	0.006	0.115	0.054	0.067	0.783	0.118	0.215	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	29	15	48	27	0	15	42
N.S.	1	1.00	1.28	1.16	0.60	1.92	1.08	0.00	0.60	1.68
time (sec)	N/A	0.226	0.009	0.133	0.037	0.068	0.783	0.000	0.217	0.424

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	37	70	27	43	37	70
N.S.	1	0.83	0.83	0.73	1.23	2.33	0.90	1.43	1.23	2.33
time (sec)	N/A	0.225	0.010	0.136	0.040	0.086	1.221	0.119	0.206	0.503

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	59	92	27	63	59	98
N.S.	1	0.83	0.83	0.73	1.97	3.07	0.90	2.10	1.97	3.27
time (sec)	N/A	0.222	0.011	0.151	0.040	0.076	3.040	0.124	0.236	0.596

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	24	93	1243	52	80	55
N.S.	1	0.83	0.83	0.73	0.80	3.10	41.43	1.73	2.67	1.83
time (sec)	N/A	0.220	0.007	0.522	0.045	0.067	6.002	0.123	0.219	0.451

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	22	24	58	333	52	45	24
N.S.	1	0.89	0.89	0.79	0.86	2.07	11.89	1.86	1.61	0.86
time (sec)	N/A	0.215	0.005	0.171	0.042	0.070	1.254	0.127	0.256	0.400

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	43	87	50	15	52
N.S.	1	1.00	1.00	0.96	1.04	1.87	3.78	2.17	0.65	2.26
time (sec)	N/A	0.218	0.004	0.142	0.052	0.078	0.393	0.127	0.231	0.373

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	27	79	31	144	32	26
N.S.	1	1.00	1.00	0.88	1.08	3.16	1.24	5.76	1.28	1.04
time (sec)	N/A	0.218	0.009	0.112	0.043	0.074	0.897	0.126	0.214	0.473

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	27	112	31	412	65	110
N.S.	1	0.83	0.83	0.73	0.90	3.73	1.03	13.73	2.17	3.67
time (sec)	N/A	0.220	0.013	0.123	0.042	0.073	1.360	0.125	0.209	0.592

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	25	25	22	27	145	31	840	98	152
N.S.	1	0.83	0.83	0.73	0.90	4.83	1.03	28.00	3.27	5.07
time (sec)	N/A	0.218	0.009	0.132	0.042	0.094	2.992	0.141	0.267	0.757

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	21	37	32	33	24	0
N.S.	1	1.00	1.26	1.07	0.78	1.37	1.19	1.22	0.89	0.00
time (sec)	N/A	0.217	0.013	0.200	0.028	0.164	3.257	0.130	0.221	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	27	34	22	15	17	58	15	13	0
N.S.	1	0.84	1.06	0.69	0.47	0.53	1.81	0.47	0.41	0.00
time (sec)	N/A	0.224	0.012	0.376	0.030	0.068	0.842	0.118	0.216	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	29	21	51	32	1	24	31
N.S.	1	1.00	1.26	1.07	0.78	1.89	1.19	0.04	0.89	1.15
time (sec)	N/A	0.224	0.018	0.204	0.034	0.284	1.249	0.136	0.200	0.485

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	27	34	22	15	15	80	15	15	42
N.S.	1	0.79	1.00	0.65	0.44	0.44	2.35	0.44	0.44	1.24
time (sec)	N/A	0.220	0.015	0.352	0.037	0.062	0.551	0.128	0.236	0.497

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	77	41	86	86	99	86	94	86
N.S.	1	0.94	2.48	1.32	2.77	2.77	3.19	2.77	3.03	2.77
time (sec)	N/A	0.255	0.016	0.728	0.026	0.062	0.053	0.111	0.269	0.066

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	49	172	41	176	176	209	173	205	162
N.S.	1	0.96	3.37	0.80	3.45	3.45	4.10	3.39	4.02	3.18
time (sec)	N/A	0.297	0.020	1.450	0.031	0.066	0.062	0.117	0.269	0.451

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	249	41	284	284	357	288	350	237
N.S.	1	1.02	5.19	0.85	5.92	5.92	7.44	6.00	7.29	4.94
time (sec)	N/A	0.304	0.025	1.188	0.042	0.081	0.088	0.122	0.307	0.452

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	10	11	10	11	11	10
N.S.	1	1.00	1.00	1.00	0.83	0.92	0.83	0.92	0.92	0.83
time (sec)	N/A	0.227	0.003	1.879	0.033	0.067	0.053	0.121	0.243	0.067

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	11	12	12	12	14	13
N.S.	1	1.00	1.00	1.00	0.85	0.92	0.92	0.92	1.08	1.00
time (sec)	N/A	0.226	0.003	0.986	0.030	0.059	0.071	0.117	0.249	0.049

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	11	22	22	12	24	11
N.S.	1	1.00	1.00	1.00	0.85	1.69	1.69	0.92	1.85	0.85
time (sec)	N/A	0.225	0.003	0.981	0.034	0.066	0.068	0.122	0.244	0.045

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	73	289	300	438	0	264	496	401
N.S.	1	0.91	0.78	3.11	3.23	4.71	0.00	2.84	5.33	4.31
time (sec)	N/A	0.359	0.188	1.956	0.080	0.093	0.000	0.131	0.254	0.877

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	68	68	0	0	0	0	0	0	0
N.S.	1	1.24	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.171	0.000	0.000	0.000	0.000	0.000	0.267	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	58	51	91	140	183	0	99	183	168
N.S.	1	0.94	0.82	1.47	2.26	2.95	0.00	1.60	2.95	2.71
time (sec)	N/A	0.316	0.149	1.485	0.072	0.087	0.000	0.128	0.259	0.594

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	68	68	0	0	0	0	0	775	0
N.S.	1	1.24	1.24	0.00	0.00	0.00	0.00	0.00	14.09	0.00
time (sec)	N/A	0.289	0.125	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	28	56	252	38	57	55
N.S.	1	1.00	1.00	0.97	0.93	1.87	8.40	1.27	1.90	1.83
time (sec)	N/A	0.259	0.064	1.302	0.028	0.087	155.772	0.119	0.249	0.424

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	223	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.29	0.00
time (sec)	N/A	0.287	0.189	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	98	104	95	95	107	105	106	93
N.S.	1	0.94	3.16	3.35	3.06	3.06	3.45	3.39	3.42	3.00
time (sec)	N/A	0.270	0.010	0.661	0.032	0.067	0.039	0.117	0.235	0.391

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	46	203	234	210	210	252	248	246	191
N.S.	1	0.90	3.98	4.59	4.12	4.12	4.94	4.86	4.82	3.75
time (sec)	N/A	0.295	0.018	0.665	0.027	0.074	0.063	0.123	0.259	0.452

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	323	406	359	359	437	442	437	309
N.S.	1	0.89	4.55	5.72	5.06	5.06	6.15	6.23	6.15	4.35
time (sec)	N/A	0.315	0.034	0.679	0.037	0.096	0.104	0.129	0.245	0.530

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	32	102	110	116	116	144	138	109	113
N.S.	1	0.86	2.76	2.97	3.14	3.14	3.89	3.73	2.95	3.05
time (sec)	N/A	0.280	0.004	0.643	0.032	0.067	0.044	0.121	0.234	0.066

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	49	207	250	240	240	323	311	249	221
N.S.	1	0.82	3.45	4.17	4.00	4.00	5.38	5.18	4.15	3.68
time (sec)	N/A	0.304	0.007	0.654	0.035	0.070	0.059	0.114	0.235	0.423

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	66	327	441	398	398	552	544	440	348
N.S.	1	0.80	3.94	5.31	4.80	4.80	6.65	6.55	5.30	4.19
time (sec)	N/A	0.311	0.016	0.669	0.035	0.088	0.072	0.127	0.277	0.497

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	157	159	92	0	164	46	160	139	169
N.S.	1	1.01	1.02	0.59	0.00	1.05	0.29	1.03	0.89	1.08
time (sec)	N/A	0.514	0.035	0.568	0.000	0.091	0.300	0.133	0.223	0.313

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	147	142	78	0	135	27	180	123	158
N.S.	1	1.02	0.99	0.54	0.00	0.94	0.19	1.25	0.85	1.10
time (sec)	N/A	0.509	0.016	0.546	0.000	0.081	0.188	0.132	0.237	0.279

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	42	42	21	74	42
N.S.	1	1.00	1.00	0.95	0.91	1.91	1.91	0.95	3.36	1.91
time (sec)	N/A	0.254	0.009	0.537	0.038	0.068	0.182	0.125	0.246	0.458

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	136	114	76	0	452	29	141	109	150
N.S.	1	0.97	0.81	0.54	0.00	3.23	0.21	1.01	0.78	1.07
time (sec)	N/A	0.472	0.018	0.197	0.000	0.087	0.164	0.131	0.244	0.346

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	131	116	71	0	442	26	160	111	144
N.S.	1	0.94	0.83	0.51	0.00	3.16	0.19	1.14	0.79	1.03
time (sec)	N/A	0.440	0.017	0.186	0.000	0.091	0.161	0.129	0.230	0.632

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	37	36	54	56	51	49	58	86	56
N.S.	1	1.03	1.00	1.50	1.56	1.42	1.36	1.61	2.39	1.56
time (sec)	N/A	0.263	0.013	0.716	0.027	0.068	0.289	0.127	0.232	0.133

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	155	140	86	0	150	44	175	254	138
N.S.	1	1.01	0.91	0.56	0.00	0.97	0.29	1.14	1.65	0.90
time (sec)	N/A	0.493	0.056	0.766	0.000	0.082	0.411	0.130	0.227	0.435

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	152	139	87	0	236	61	193	403	201
N.S.	1	0.97	0.89	0.56	0.00	1.51	0.39	1.24	2.58	1.29
time (sec)	N/A	0.489	0.044	0.740	0.000	0.099	0.584	0.141	0.261	0.712

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	51	44	75	98	158	100	41	420	104
N.S.	1	0.91	0.79	1.34	1.75	2.82	1.79	0.73	7.50	1.86
time (sec)	N/A	0.319	0.021	0.734	0.040	0.078	1.188	0.119	0.230	0.618

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	152	141	0	838	109	205	703	240
N.S.	1	0.99	0.88	0.82	0.00	4.87	0.63	1.19	4.09	1.40
time (sec)	N/A	0.514	0.064	0.567	0.000	0.098	0.846	0.138	0.230	0.421

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	164	151	124	0	810	92	214	677	185
N.S.	1	0.96	0.89	0.73	0.00	4.76	0.54	1.26	3.98	1.09
time (sec)	N/A	0.505	0.043	0.558	0.000	0.093	0.772	0.139	0.262	0.710

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	52	58	21	43	43
N.S.	1	1.00	1.00	0.96	0.91	2.26	2.52	0.91	1.87	1.87
time (sec)	N/A	0.250	0.008	0.556	0.029	0.068	0.724	0.131	0.221	0.487

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	171	152	144	0	852	105	205	705	216
N.S.	1	0.99	0.88	0.84	0.00	4.95	0.61	1.19	4.10	1.26
time (sec)	N/A	0.513	0.055	0.246	0.000	0.098	0.714	0.141	0.218	0.340

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	164	151	127	0	808	92	214	677	207
N.S.	1	0.96	0.89	0.75	0.00	4.75	0.54	1.26	3.98	1.22
time (sec)	N/A	0.481	0.038	0.227	0.000	0.119	0.640	0.123	0.282	0.692

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	54	48	100	104	169	110	101	508	105
N.S.	1	0.92	0.81	1.69	1.76	2.86	1.86	1.71	8.61	1.78
time (sec)	N/A	0.325	0.018	0.786	0.040	0.079	0.973	0.129	0.238	0.585

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	190	168	154	0	384	170	204	1009	231
N.S.	1	1.01	0.89	0.82	0.00	2.04	0.90	1.09	5.37	1.23
time (sec)	N/A	0.544	0.059	0.793	0.000	0.086	1.204	0.129	0.224	1.052

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	187	166	140	0	509	199	244	1330	255
N.S.	1	0.99	0.88	0.74	0.00	2.71	1.06	1.30	7.07	1.36
time (sec)	N/A	0.536	0.050	0.806	0.000	0.120	1.607	0.141	0.232	1.238

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	71	60	126	222	431	250	65	1293	211
N.S.	1	0.89	0.75	1.58	2.78	5.39	3.12	0.81	16.16	2.64
time (sec)	N/A	0.362	0.048	0.783	0.042	0.084	2.135	0.122	0.268	1.150

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	206	182	214	0	1706	287	274	1825	399
N.S.	1	1.00	0.89	1.04	0.00	8.32	1.40	1.34	8.90	1.95
time (sec)	N/A	0.599	0.104	0.602	0.000	0.123	1.792	0.141	0.248	1.124

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	197	179	186	0	1650	260	268	1793	347
N.S.	1	0.98	0.89	0.93	0.00	8.21	1.29	1.33	8.92	1.73
time (sec)	N/A	0.561	0.082	0.587	0.000	0.124	1.702	0.144	0.270	0.929

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	133	153	21	132	131
N.S.	1	1.00	1.00	0.96	0.91	5.78	6.65	0.91	5.74	5.70
time (sec)	N/A	0.260	0.009	0.579	0.025	0.076	1.458	0.142	0.252	0.601

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	206	180	214	0	1708	296	268	1828	403
N.S.	1	1.02	0.89	1.06	0.00	8.46	1.47	1.33	9.05	2.00
time (sec)	N/A	0.570	0.071	0.299	0.000	0.121	1.590	0.145	0.262	0.559

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	197	176	185	0	1646	267	264	1793	354
N.S.	1	0.99	0.89	0.93	0.00	8.31	1.35	1.33	9.06	1.79
time (sec)	N/A	0.544	0.067	0.286	0.000	0.117	1.854	0.129	0.217	0.524

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	74	63	143	245	467	269	144	1402	230
N.S.	1	0.90	0.77	1.74	2.99	5.70	3.28	1.76	17.10	2.80
time (sec)	N/A	0.365	0.034	0.788	0.044	0.102	2.200	0.142	0.222	1.191

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	225	196	215	0	855	391	223	2307	431
N.S.	1	1.03	0.89	0.98	0.00	3.90	1.79	1.02	10.53	1.97
time (sec)	N/A	0.621	0.091	0.840	0.000	0.161	2.577	0.138	0.309	2.011

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	222	192	188	0	1024	435	365	2802	520
N.S.	1	1.02	0.88	0.87	0.00	4.72	2.00	1.68	12.91	2.40
time (sec)	N/A	0.617	0.101	0.852	0.000	0.164	3.260	0.156	0.255	2.098

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	92	80	161	438	889	500	80	2661	438
N.S.	1	0.91	0.79	1.59	4.34	8.80	4.95	0.79	26.35	4.34
time (sec)	N/A	0.396	0.076	0.869	0.067	0.124	3.649	0.140	0.258	2.451

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	160	163	96	0	179	66	175	142	151
N.S.	1	0.95	0.97	0.57	0.00	1.07	0.39	1.04	0.85	0.90
time (sec)	N/A	0.540	0.018	0.663	0.000	0.122	0.270	0.134	0.242	0.331

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	150	145	82	0	147	44	192	126	162
N.S.	1	0.96	0.93	0.53	0.00	0.94	0.28	1.23	0.81	1.04
time (sec)	N/A	0.545	0.008	0.661	0.000	0.080	0.228	0.142	0.216	0.380

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	46	45	45	46	46	77	45
N.S.	1	1.00	1.00	1.84	1.80	1.80	1.84	1.84	3.08	1.80
time (sec)	N/A	0.264	0.005	0.540	0.026	0.082	0.180	0.133	0.236	0.425

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	137	115	77	0	458	41	150	110	127
N.S.	1	0.96	0.80	0.54	0.00	3.20	0.29	1.05	0.77	0.89
time (sec)	N/A	0.482	0.006	0.219	0.000	0.097	0.184	0.140	0.252	0.822

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	40	42	57	62	54	53	64	89	62
N.S.	1	0.95	1.00	1.36	1.48	1.29	1.26	1.52	2.12	1.48
time (sec)	N/A	0.283	0.012	0.715	0.034	0.082	0.331	0.127	0.259	0.139

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	158	143	96	0	156	54	229	257	154
N.S.	1	0.95	0.86	0.58	0.00	0.94	0.33	1.38	1.55	0.93
time (sec)	N/A	0.528	0.032	0.769	0.000	0.141	0.553	0.131	0.207	0.804

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	155	142	91	0	245	75	199	406	200
N.S.	1	0.92	0.85	0.54	0.00	1.46	0.45	1.18	2.42	1.19
time (sec)	N/A	0.510	0.031	0.785	0.000	0.088	0.694	0.138	0.209	0.501

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	54	47	79	116	170	121	85	423	118
N.S.	1	0.83	0.72	1.22	1.78	2.62	1.86	1.31	6.51	1.82
time (sec)	N/A	0.325	0.017	0.752	0.039	0.082	0.910	0.143	0.227	0.630

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	174	155	145	0	940	133	224	706	266
N.S.	1	0.95	0.84	0.79	0.00	5.11	0.72	1.22	3.84	1.45
time (sec)	N/A	0.529	0.025	0.585	0.000	0.103	0.976	0.139	0.249	0.803

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	167	154	128	0	906	112	230	680	194
N.S.	1	0.92	0.85	0.70	0.00	4.98	0.62	1.26	3.74	1.07
time (sec)	N/A	0.524	0.011	0.579	0.000	0.102	0.746	0.137	0.257	0.390

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	47	55	55	60	46	46	46
N.S.	1	1.00	1.00	1.81	2.12	2.12	2.31	1.77	1.77	1.77
time (sec)	N/A	0.257	0.007	0.553	0.031	0.079	0.665	0.138	0.216	0.418

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	172	153	146	0	894	122	218	706	192
N.S.	1	0.98	0.87	0.83	0.00	5.08	0.69	1.24	4.01	1.09
time (sec)	N/A	0.525	0.032	0.265	0.000	0.096	0.669	0.142	0.243	0.721

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	57	51	109	114	173	122	110	511	116
N.S.	1	0.84	0.75	1.60	1.68	2.54	1.79	1.62	7.51	1.71
time (sec)	N/A	0.335	0.014	0.744	0.037	0.076	0.938	0.135	0.251	0.277

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	193	171	158	0	399	197	270	1012	261
N.S.	1	0.95	0.84	0.78	0.00	1.97	0.97	1.33	4.99	1.29
time (sec)	N/A	0.573	0.031	0.815	0.000	0.094	1.310	0.131	0.249	1.147

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	190	169	144	0	527	233	253	1333	291
N.S.	1	0.94	0.83	0.71	0.00	2.60	1.15	1.25	6.57	1.43
time (sec)	N/A	0.555	0.040	0.863	0.000	0.109	2.249	0.141	0.264	1.275

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	74	63	130	249	452	294	161	1296	250
N.S.	1	0.80	0.68	1.41	2.71	4.91	3.20	1.75	14.09	2.72
time (sec)	N/A	0.337	0.039	0.806	0.047	0.129	2.318	0.141	0.332	1.256

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	209	185	218	0	1886	332	310	1828	430
N.S.	1	0.95	0.84	0.99	0.00	8.57	1.51	1.41	8.31	1.95
time (sec)	N/A	0.553	0.086	0.623	0.000	0.186	1.970	0.149	0.240	0.861

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	200	182	190	0	1822	298	298	1796	349
N.S.	1	0.93	0.84	0.88	0.00	8.44	1.38	1.38	8.31	1.62
time (sec)	N/A	0.541	0.013	0.617	0.000	0.128	1.891	0.150	0.223	0.996

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	47	136	136	155	46	135	134
N.S.	1	1.00	1.00	1.81	5.23	5.23	5.96	1.77	5.19	5.15
time (sec)	N/A	0.245	0.008	0.580	0.046	0.118	1.711	0.142	0.275	0.631

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	181	216	0	1780	323	286	1829	357
N.S.	1	1.00	0.87	1.04	0.00	8.60	1.56	1.38	8.84	1.72
time (sec)	N/A	0.553	0.009	0.340	0.000	0.182	1.647	0.159	0.245	0.613

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	77	66	148	258	474	292	153	1405	249
N.S.	1	0.82	0.70	1.57	2.74	5.04	3.11	1.63	14.95	2.65
time (sec)	N/A	0.345	0.032	0.784	0.049	0.092	2.250	0.148	0.283	1.257

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	228	199	219	0	879	445	295	2310	485
N.S.	1	0.96	0.84	0.92	0.00	3.71	1.88	1.24	9.75	2.05
time (sec)	N/A	0.603	0.063	0.870	0.000	0.161	2.717	0.142	0.281	2.041

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	225	195	192	0	1051	500	371	2805	563
N.S.	1	0.96	0.83	0.82	0.00	4.47	2.13	1.58	11.94	2.40
time (sec)	N/A	0.610	0.082	0.897	0.000	0.139	4.090	0.154	0.273	2.923

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	95	83	165	474	919	578	262	2664	507
N.S.	1	0.82	0.72	1.42	4.09	7.92	4.98	2.26	22.97	4.37
time (sec)	N/A	0.378	0.033	0.855	0.066	0.169	3.782	0.144	0.268	2.820

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	28	104	0	62	105	88
N.S.	1	1.00	1.00	0.97	0.93	3.47	0.00	2.07	3.50	2.93
time (sec)	N/A	0.268	0.100	1.216	0.032	0.112	0.000	0.125	0.250	0.605

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	29	80	22	21	108	126	25	117	107
N.S.	1	1.26	3.48	0.96	0.91	4.70	5.48	1.09	5.09	4.65
time (sec)	N/A	0.281	0.014	0.629	0.025	0.072	0.035	0.124	0.242	0.067

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	172	22	21	250	299	21	286	229
N.S.	1	1.00	7.48	0.96	0.91	10.87	13.00	0.91	12.43	9.96
time (sec)	N/A	0.257	0.027	0.625	0.026	0.086	0.061	0.124	0.209	0.506

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	308	22	21	432	541	21	523	376
N.S.	1	1.00	13.39	0.96	0.91	18.78	23.52	0.91	22.74	16.35
time (sec)	N/A	0.255	0.056	0.642	0.033	0.105	0.078	0.116	0.208	0.289

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	54	56	20	54	54
N.S.	1	1.00	1.00	0.95	0.91	2.45	2.55	0.91	2.45	2.45
time (sec)	N/A	0.255	0.008	0.556	0.031	0.095	0.174	0.131	0.247	0.102

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	66	73	21	55	55
N.S.	1	1.00	1.00	0.96	0.91	2.87	3.17	0.91	2.39	2.39
time (sec)	N/A	0.256	0.009	0.582	0.025	0.083	0.847	0.130	0.223	0.583

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	172	197	21	173	171
N.S.	1	1.00	1.00	0.96	0.91	7.48	8.57	0.91	7.52	7.43
time (sec)	N/A	0.255	0.011	0.620	0.026	0.161	2.291	0.147	0.237	1.555

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	227	132	108	0	6315	238	0	460	374
N.S.	1	0.97	0.56	0.46	0.00	26.99	1.02	0.00	1.97	1.60
time (sec)	N/A	0.819	0.038	0.201	0.000	10.892	1.749	0.000	0.240	1.502

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	197	81	74	0	4759	158	0	327	437
N.S.	1	0.94	0.39	0.35	0.00	22.66	0.75	0.00	1.56	2.08
time (sec)	N/A	0.773	0.021	0.198	0.000	0.774	0.624	0.000	0.258	0.706

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	171	79	72	0	1950	83	0	232	145
N.S.	1	0.95	0.44	0.40	0.00	10.83	0.46	0.00	1.29	0.81
time (sec)	N/A	0.618	0.018	0.188	0.000	0.730	0.417	0.000	0.203	0.657

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	131	116	71	0	442	26	160	111	144
N.S.	1	0.94	0.83	0.51	0.00	3.16	0.19	1.14	0.79	1.03
time (sec)	N/A	0.462	0.009	0.178	0.000	0.110	0.160	0.121	0.198	0.003

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	241	119	105	0	4370	0	0	1347	553
N.S.	1	1.08	0.53	0.47	0.00	19.51	0.00	0.00	6.01	2.47
time (sec)	N/A	0.886	0.036	0.250	0.000	0.780	0.000	0.000	0.223	0.537

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	315	173	143	0	8919	0	0	3657	1588
N.S.	1	1.01	0.55	0.46	0.00	28.59	0.00	0.00	11.72	5.09
time (sec)	N/A	1.200	0.064	0.275	0.000	1.146	0.000	0.000	0.276	0.719

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	380	244	200	0	14765	0	0	6603	1328
N.S.	1	0.98	0.63	0.52	0.00	38.05	0.00	0.00	17.02	3.42
time (sec)	N/A	1.413	0.104	0.312	0.000	3.838	0.000	0.000	0.305	0.884

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	349	1258	4787	0	0	0	0	403	0
N.S.	1	0.97	3.48	13.26	0.00	0.00	0.00	0.00	1.12	0.00
time (sec)	N/A	0.770	15.652	18.929	0.000	0.000	0.000	0.000	0.300	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	318	553	2153	0	0	0	0	473	0
N.S.	1	0.98	1.70	6.60	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.676	11.671	5.142	0.000	0.000	0.000	0.000	0.325	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	152	389	1528	0	0	0	0	186	0
N.S.	1	0.99	2.53	9.92	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.466	10.377	3.628	0.000	0.000	0.000	0.000	0.294	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	90	1036	0	0	0	0	96	56
N.S.	1	1.00	0.81	9.33	0.00	0.00	0.00	0.00	0.86	0.50
time (sec)	N/A	0.296	10.040	2.385	0.000	0.000	0.000	0.000	0.288	0.843

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	436	442	1582	0	0	0	0	19	0
N.S.	1	1.06	1.07	3.83	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.154	10.534	3.066	0.000	0.000	0.000	0.000	200.040	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	629	658	1385	5348	0	0	0	0	19	0
N.S.	1	1.05	2.20	8.50	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.979	13.871	13.339	0.000	0.000	0.000	0.000	200.037	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	61	64	864	0	379	0	134	131	0
N.S.	1	1.11	1.16	15.71	0.00	6.89	0.00	2.44	2.38	0.00
time (sec)	N/A	0.328	0.324	10.571	0.000	0.342	0.000	0.154	0.213	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	62	64	864	0	379	0	134	131	0
N.S.	1	1.13	1.16	15.71	0.00	6.89	0.00	2.44	2.38	0.00
time (sec)	N/A	0.339	0.003	2.330	0.000	0.405	0.000	0.161	0.220	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	34	64	864	0	379	0	134	131	0
N.S.	1	0.62	1.16	15.71	0.00	6.89	0.00	2.44	2.38	0.00
time (sec)	N/A	0.527	0.006	2.227	0.000	0.358	0.000	0.159	0.232	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	B	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	62	64	864	0	379	0	134	131	0
N.S.	1	1.11	1.14	15.43	0.00	6.77	0.00	2.39	2.34	0.00
time (sec)	N/A	0.551	0.004	2.195	0.000	0.348	0.000	0.159	0.199	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [80] had the largest ratio of [.846153999999999962]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	0.83	13	0.231
2	A	4	3	0.89	13	0.231
3	A	4	3	1.00	13	0.231
4	A	4	3	1.00	13	0.231
5	A	4	3	0.83	13	0.231
6	A	4	3	0.83	13	0.231
7	A	4	3	1.00	11	0.273
8	A	4	3	1.00	11	0.273
9	A	4	3	1.00	11	0.273
10	A	4	3	1.00	11	0.273
11	A	4	3	1.00	13	0.231
12	A	4	3	0.83	13	0.231
13	A	4	3	0.89	13	0.231
14	A	4	3	1.00	13	0.231
15	A	4	3	1.00	13	0.231
16	A	4	3	0.83	13	0.231
17	A	4	3	0.83	13	0.231
18	A	3	2	1.00	13	0.154
19	A	3	2	1.00	13	0.154
20	A	3	2	1.00	13	0.154
21	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	2	1.00	13	0.154
23	A	3	2	1.00	13	0.154
24	A	4	3	0.83	13	0.231
25	A	4	3	0.89	13	0.231
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	13	0.231
28	A	4	3	0.83	13	0.231
29	A	4	3	0.83	13	0.231
30	A	4	3	0.83	13	0.231
31	A	4	3	0.89	13	0.231
32	A	4	3	1.00	13	0.231
33	A	4	3	1.00	13	0.231
34	A	4	3	0.83	13	0.231
35	A	4	3	0.83	13	0.231
36	A	4	3	1.00	15	0.200
37	A	4	3	0.84	15	0.200
38	A	4	3	1.00	15	0.200
39	A	4	3	0.79	15	0.200
40	A	4	3	0.94	19	0.158
41	A	5	4	0.96	21	0.190
42	A	5	4	1.02	21	0.190
43	A	3	2	1.00	13	0.154
44	A	3	2	1.00	13	0.154
45	A	3	2	1.00	13	0.154
46	A	5	4	0.91	21	0.190
47	A	4	3	1.24	21	0.143
48	A	5	4	0.94	21	0.190
49	A	4	3	1.24	21	0.143
50	A	3	2	1.00	19	0.105
51	A	4	3	1.00	21	0.143
52	A	4	3	0.94	19	0.158
53	A	4	3	0.90	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	0.89	21	0.143
55	A	4	3	0.86	22	0.136
56	A	4	3	0.82	24	0.125
57	A	4	3	0.80	24	0.125
58	A	11	10	1.01	21	0.476
59	A	11	10	1.02	21	0.476
60	A	3	2	1.00	21	0.095
61	A	10	9	0.97	19	0.474
62	A	10	9	0.94	13	0.692
63	A	6	5	1.03	21	0.238
64	A	11	10	1.01	21	0.476
65	A	11	10	0.97	21	0.476
66	A	5	4	0.91	21	0.190
67	A	11	10	0.99	21	0.476
68	A	11	10	0.96	21	0.476
69	A	3	2	1.00	21	0.095
70	A	11	10	0.99	19	0.526
71	A	11	10	0.96	13	0.769
72	A	5	4	0.92	21	0.190
73	A	12	11	1.01	21	0.524
74	A	12	11	0.99	21	0.524
75	A	5	4	0.89	21	0.190
76	A	12	11	1.00	21	0.524
77	A	12	11	0.98	21	0.524
78	A	3	2	1.00	21	0.095
79	A	12	11	1.02	19	0.579
80	A	12	11	0.99	13	0.846
81	A	5	4	0.90	21	0.190
82	A	13	12	1.03	21	0.571
83	A	13	12	1.02	21	0.571
84	A	5	4	0.91	21	0.190
85	A	11	10	0.95	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	10	0.96	24	0.417
87	A	3	2	1.00	24	0.083
88	A	10	9	0.96	22	0.409
89	A	6	5	0.95	24	0.208
90	A	11	10	0.95	24	0.417
91	A	11	10	0.92	24	0.417
92	A	5	4	0.83	24	0.167
93	A	11	10	0.95	24	0.417
94	A	11	10	0.92	24	0.417
95	A	3	2	1.00	24	0.083
96	A	11	10	0.98	22	0.455
97	A	5	4	0.84	24	0.167
98	A	12	11	0.95	24	0.458
99	A	12	11	0.94	24	0.458
100	A	5	4	0.80	24	0.167
101	A	12	11	0.95	24	0.458
102	A	12	11	0.93	24	0.458
103	A	3	2	1.00	24	0.083
104	A	12	11	1.00	22	0.500
105	A	5	4	0.82	24	0.167
106	A	13	12	0.96	24	0.500
107	A	13	12	0.96	24	0.500
108	A	5	4	0.82	24	0.167
109	A	3	2	1.00	21	0.095
110	A	4	3	1.26	19	0.158
111	A	3	2	1.00	21	0.095
112	A	3	2	1.00	21	0.095
113	A	3	2	1.00	21	0.095
114	A	3	2	1.00	21	0.095
115	A	3	2	1.00	21	0.095
116	A	5	4	0.97	17	0.235
117	A	14	13	0.94	17	0.765

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	12	11	0.95	15	0.733
119	A	10	9	0.94	13	0.692
120	A	5	4	1.08	17	0.235
121	A	4	3	1.01	17	0.176
122	A	5	4	0.98	17	0.235
123	A	5	4	0.97	19	0.211
124	A	4	3	0.98	19	0.158
125	A	5	4	0.99	17	0.235
126	A	3	2	1.00	15	0.133
127	A	11	10	1.06	19	0.526
128	A	17	16	1.05	19	0.842
129	A	6	5	1.11	27	0.185
130	A	6	5	1.13	28	0.179
131	A	6	5	0.62	49	0.102
132	A	6	5	1.11	50	0.100

CHAPTER 3

Listing of integrals

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3.115 $\int \frac{(c+dx)^3}{(a+b(c+dx)^4)^3}\,dx$ . . . 963
3.116 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$ . . . 969
3.117 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$ . . . 976
3.118 $\int \frac{x}{a+b(c+dx)^3}\,dx$ . . . 986
3.119 $\int \frac{1}{a+b(c+dx)^3}\,dx$ . . . 995
3.120 $\int \frac{1}{x(a+b(c+dx)^3)}\,dx$ . . . 1004
3.121 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$ . . . 1011

3.122 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$ 1018
3.123 $\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx$ 1026
3.124 $\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx$ 1034
3.125 $\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$ 1041
3.126 $\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx$ 1048
3.127 $\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx$ 1054
3.128 $\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx$ 1063
3.129 $\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx$ 1075
3.130 $\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx$ 1082
3.131 $\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$ 1089
3.132 $\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$ 1097

3.1 $\int \left(c(a+bx)^2\right)^{5/2}\,dx$

Optimal result 76
Mathematica [A] (verified) 76
Rubi [A] (verified) 77
Maple [A] (verified) 78
Fricas [B] (verification not implemented) 78
Sympy [B] (verification not implemented) 79
Maxima [B] (verification not implemented) 80
Giac [A] (verification not implemented) 80
Mupad [B] (verification not implemented) 80
Reduce [B] (verification not implemented) 81

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{c^2(a+bx)^5\sqrt{c(a+bx)^2}}{6b}$$

output
```
1/6*c^2*(b*x+a)^5*(c*(b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{(a+bx)\left(c(a+bx)^2\right)^{5/2}}{6b}$$

input
```
Integrate[(c*(a + b*x)^2)^(5/2),x]
```

output
```
((a + b*x)*(c*(a + b*x)^2)^(5/2))/(6*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^2\right)^{5/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^2\right)^{5/2} \int (a+bx)^5 d(a+bx)}{b(a+bx)^5}$$

$$\downarrow 15$$

$$\frac{(a+bx)\left(c(a+bx)^2\right)^{5/2}}{6b}$$

input
```
Int[(c*(a + b*x)^2)^(5/2),x]
```

output
```
((a + b*x)*(c*(a + b*x)^2)^(5/2))/(6*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\left(c(bx+a)^2\right)^{\frac{5}{2}}(bx+a)}{6b}$	22
risch	$\frac{c^2(bx+a)^5\sqrt{c(bx+a)^2}}{6b}$	27
gosper	$\frac{x\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)\left(c(bx+a)^2\right)^{\frac{5}{2}}}{6(bx+a)^5}$	73
orering	$\frac{x\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)\left(c(bx+a)^2\right)^{\frac{5}{2}}}{6(bx+a)^5}$	73
trager	$\frac{c^2x\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)\sqrt{b^2cx^2+2abcx+a^2c}}{6bx+6a}$	87

input
```
int((c*(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*(b*x+a)^2)^(5/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.43

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{\left(b^5c^2x^6+6\,ab^4c^2x^5+15\,a^2b^3c^2x^4+20\,a^3b^2c^2x^3+15\,a^4bc^2x^2+6\,a^5c^2x\right)\sqrt{b^2cx^2+2\,abcx+}}{6\,(bx+a)}$$

input
```
integrate((c*(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(b^5*c^2*x^6 + 6*a*b^4*c^2*x^5 + 15*a^2*b^3*c^2*x^4 + 20*a^3*b^2*c^2*x
^3 + 15*a^4*b*c^2*x^2 + 6*a^5*c^2*x)*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(
b*x + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(24) = 48$.

Time = 18.85 (sec) , antiderivative size = 828, normalized size of antiderivative = 27.60

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((c*(b*x+a)**2)**(5/2),x)
```

output
```
a**4*c**2*Piecewise(((a/(2*b) + x/2)*sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2
), Ne(b**2*c, 0)), ((a**2*c + 2*a*b*c*x)**(3/2)/(3*a*b*c), Ne(a*b*c, 0)),
(x*sqrt(a**2*c), True)) + 4*a**3*b*c**2*Piecewise(((-a**2/(6*b**2) + a*x/(
6*b) + x**2/3)*sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2), Ne(b**2*c, 0)), ((-
a**2*c*(a**2*c + 2*a*b*c*x)**(3/2)/3 + (a**2*c + 2*a*b*c*x)**(5/2)/5)/(2*a
**2*b**2*c**2), Ne(a*b*c, 0)), (x**2*sqrt(a**2*c)/2, True)) + 6*a**2*b**2*
c**2*Piecewise((sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2)*(a**3/(12*b**3) - a
**2*x/(12*b**2) + a*x**2/(12*b) + x**3/4), Ne(b**2*c, 0)), ((a**4*c**2*(a*
*2*c + 2*a*b*c*x)**(3/2)/3 - 2*a**2*c*(a**2*c + 2*a*b*c*x)**(5/2)/5 + (a**
2*c + 2*a*b*c*x)**(7/2)/7)/(4*a**3*b**3*c**3), Ne(a*b*c, 0)), (x**3*sqrt(a
**2*c)/3, True)) + 4*a*b**3*c**2*Piecewise((sqrt(a**2*c + 2*a*b*c*x + b**2
*c*x**2)*(-a**4/(20*b**4) + a**3*x/(20*b**3) - a**2*x**2/(20*b**2) + a*x**
3/(20*b) + x**4/5), Ne(b**2*c, 0)), ((-a**6*c**3*(a**2*c + 2*a*b*c*x)**(3/
2)/3 + 3*a**4*c**2*(a**2*c + 2*a*b*c*x)**(5/2)/5 - 3*a**2*c*(a**2*c + 2*a*
b*c*x)**(7/2)/7 + (a**2*c + 2*a*b*c*x)**(9/2)/9)/(8*a**4*b**4*c**4), Ne(a*
b*c, 0)), (x**4*sqrt(a**2*c)/4, True)) + b**4*c**2*Piecewise((sqrt(a**2*c
+ 2*a*b*c*x + b**2*c*x**2)*(a**5/(30*b**5) - a**4*x/(30*b**4) + a**3*x**2/
(30*b**3) - a**2*x**3/(30*b**2) + a*x**4/(30*b) + x**5/6), Ne(b**2*c, 0)),
 ((a**8*c**4*(a**2*c + 2*a*b*c*x)**(3/2)/3 - 4*a**6*c**3*(a**2*c + 2*a*b*c
*x)**(5/2)/5 + 6*a**4*c**2*(a**2*c + 2*a*b*c*x)**(7/2)/7 - 4*a**2*c*(a*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{1}{6}\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{5}{2}}x + \frac{\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{5}{2}}a}{6\,b}$$

input
```
integrate((c*(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(5/2)*x + 1/6*(b^2*c*x^2 + 2*a*b*c*x +
 a^2*c)^(5/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{(bx+a)^6 c^{\frac{5}{2}}\operatorname{sgn}(bx+a)}{6\,b}$$

input
```
integrate((c*(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
1/6*(b*x + a)^6*c^(5/2)*sgn(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{(x\,b^2+a\,b)\,\left(c\,a^2+2\,c\,a\,b\,x+c\,b^2\,x^2\right)^{5/2}}{6\,b^2}$$

input
```
int((c*(a + b*x)^2)^(5/2),x)
```

output
```
((a*b + b^2*x)*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(5/2))/(6*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \left(c(a+bx)^2\right)^{5/2}\,dx = \frac{\sqrt{c}\,c^2x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)}{6}$$

input

```
int((c*(b*x+a)^2)^(5/2),x)
```

output

```
(sqrt(c)*c**2*x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x
**3 + 6*a*b**4*x**4 + b**5*x**5))/6
```

3.2 $\int \left(c(a+bx)^2\right)^{3/2}\,dx$

Optimal result 82
Mathematica [A] (verified) 82
Rubi [A] (verified) 83
Maple [A] (verified) 84
Fricas [B] (verification not implemented) 84
Sympy [B] (verification not implemented) 85
Maxima [B] (verification not implemented) 86
Giac [A] (verification not implemented) 86
Mupad [B] (verification not implemented) 86
Reduce [B] (verification not implemented) 87

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{c(a+bx)^3\sqrt{c(a+bx)^2}}{4b}$$

output
```
1/4*c*(b*x+a)^3*(c*(b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{(a+bx)\left(c(a+bx)^2\right)^{3/2}}{4b}$$

input
```
Integrate[(c*(a + b*x)^2)^(3/2),x]
```

output
```
((a + b*x)*(c*(a + b*x)^2)^(3/2))/(4*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^2\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^2\right)^{3/2} \int (a+bx)^3 d(a+bx)}{b(a+bx)^3}$$

$$\downarrow 15$$

$$\frac{(a+bx)\left(c(a+bx)^2\right)^{3/2}}{4b}$$

input
```
Int[(c*(a + b*x)^2)^(3/2),x]
```

output
```
((a + b*x)*(c*(a + b*x)^2)^(3/2))/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\left(c(bx+a)^2\right)^{\frac{3}{2}}(bx+a)}{4b}$	22
risch	$\frac{c(bx+a)^3\sqrt{c(bx+a)^2}}{4b}$	25
gosper	$\frac{x\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)\left(c(bx+a)^2\right)^{\frac{3}{2}}}{4(bx+a)^3}$	51
orering	$\frac{x\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)\left(c(bx+a)^2\right)^{\frac{3}{2}}}{4(bx+a)^3}$	51
trager	$\frac{cx\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)\sqrt{b^2cx^2+2abcx+a^2c}}{4bx+4a}$	63

input

```
int((c*(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/4*(c*(b*x+a)^2)^(3/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{(b^3cx^4+4\,ab^2cx^3+6\,a^2bcx^2+4\,a^3cx)\sqrt{b^2cx^2+2\,abcx+a^2c}}{4\,(bx+a)}$$

input

```
integrate((c*(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(b^3*c*x^4 + 4*a*b^2*c*x^3 + 6*a^2*b*c*x^2 + 4*a^3*c*x)*sqrt(b^2*c*x^2
 + 2*a*b*c*x + a^2*c)/(b*x + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(22) = 44$.

Time = 2.93 (sec) , antiderivative size = 366, normalized size of antiderivative = 13.07

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = a^2c\left(\begin{cases}\left(\frac{a}{2b}+\frac{x}{2}\right)\sqrt{a^2c+2abcx+b^2cx^2} & \text{for } b^2c\neq 0\\ \frac{(a^2c+2abcx)^{\frac{3}{2}}}{3abc} & \text{for } abc\neq 0\\ x\sqrt{a^2c} & \text{otherwise}\end{cases}\right) + 2abc\left(\begin{cases}\left(-\frac{a^2}{6b^2}+\frac{ax}{6b}+\frac{x^2}{3}\right)\sqrt{a^2c+2abcx+b^2cx^2} & \text{for } b^2c\neq 0\\ \frac{-\frac{a^2c\left(a^2c+2abcx\right)^{\frac{3}{2}}}{3}+\frac{\left(a^2c+2abcx\right)^{\frac{5}{2}}}{5}}{2a^2b^2c^2} & \text{for } abc\neq 0\\ \frac{x^2\sqrt{a^2c}}{2} & \text{otherwise}\end{cases}\right) + b^2c\left(\begin{cases}\sqrt{a^2c+2abcx+b^2cx^2}\left(\frac{a^3}{12b^3}-\frac{a^2x}{12b^2}+\frac{ax^2}{12b}+\frac{x^3}{4}\right) & \text{for } b^2c\neq 0\\ \frac{\frac{a^4c^2\left(a^2c+2abcx\right)^{\frac{3}{2}}}{3}-\frac{2a^2c\left(a^2c+2abcx\right)^{\frac{5}{2}}}{5}+\frac{\left(a^2c+2abcx\right)^{\frac{7}{2}}}{7}}{4a^3b^3c^3} & \text{for } abc\neq 0\\ \frac{x^3\sqrt{a^2c}}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c*(b*x+a)**2)**(3/2),x)
```

output
```
a**2*c*Piecewise(((a/(2*b) + x/2)*sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2),
Ne(b**2*c, 0)), ((a**2*c + 2*a*b*c*x)**(3/2)/(3*a*b*c), Ne(a*b*c, 0)), (x*
sqrt(a**2*c), True)) + 2*a*b*c*Piecewise(((-a**2/(6*b**2) + a*x/(6*b) + x*
*2/3)*sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2), Ne(b**2*c, 0)), ((-a**2*c*(a
**2*c + 2*a*b*c*x)**(3/2)/3 + (a**2*c + 2*a*b*c*x)**(5/2)/5)/(2*a**2*b**2*
c**2), Ne(a*b*c, 0)), (x**2*sqrt(a**2*c)/2, True)) + b**2*c*Piecewise((sqr
t(a**2*c + 2*a*b*c*x + b**2*c*x**2)*(a**3/(12*b**3) - a**2*x/(12*b**2) + a
*x**2/(12*b) + x**3/4), Ne(b**2*c, 0)), ((a**4*c**2*(a**2*c + 2*a*b*c*x)**
(3/2)/3 - 2*a**2*c*(a**2*c + 2*a*b*c*x)**(5/2)/5 + (a**2*c + 2*a*b*c*x)**(
7/2)/7)/(4*a**3*b**3*c**3), Ne(a*b*c, 0)), (x**3*sqrt(a**2*c)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{1}{4}\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{3}{2}}x + \frac{\left(b^2cx^2+2\,abcx+a^2c\right)^{\frac{3}{2}}a}{4\,b}$$

input
```
integrate((c*(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^2*c*x^2 + 2*a*b*c*x + a^2*c)^(3/2)*x + 1/4*(b^2*c*x^2 + 2*a*b*c*x +
 a^2*c)^(3/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{(bx+a)^4 c^{\frac{3}{2}}\operatorname{sgn}(bx+a)}{4\,b}$$

input
```
integrate((c*(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(b*x + a)^4*c^(3/2)*sgn(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{\left(x\,b^2+a\,b\right)\,\left(c\,a^2+2\,c\,a\,b\,x+c\,b^2\,x^2\right)^{3/2}}{4\,b^2}$$

input
```
int((c*(a + b*x)^2)^(3/2),x)
```

output
```
((a*b + b^2*x)*(a^2*c + b^2*c*x^2 + 2*a*b*c*x)^(3/2))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \left(c(a+bx)^2\right)^{3/2}\,dx = \frac{\sqrt{c}\,cx(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4}$$

input
```
int((c*(b*x+a)^2)^(3/2),x)
```

output
```
(sqrt(c)*c*x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/4
```

3.3 $\int \sqrt{c(a+bx)^2}\,dx$

Optimal result 88
Mathematica [A] (verified) 88
Rubi [A] (verified) 89
Maple [A] (verified) 90
Fricas [A] (verification not implemented) 90
Sympy [B] (verification not implemented) 91
Maxima [B] (verification not implemented) 91
Giac [A] (verification not implemented) 92
Mupad [B] (verification not implemented) 92
Reduce [B] (verification not implemented) 92

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{(a+bx)\sqrt{c(a+bx)^2}}{2b}$$

output
```
1/2*(b*x+a)*(c*(b*x+a)^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{cx(a+bx)(2a+bx)}{2\sqrt{c(a+bx)^2}}$$

input
```
Integrate[Sqrt[c*(a + b*x)^2],x]
```

output
```
(c*x*(a + b*x)*(2*a + b*x))/(2*Sqrt[c*(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c(a+bx)^2}\,dx$$

$$\downarrow 239$$

$$\frac{\int \sqrt{c(a+bx)^2}\,d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\sqrt{c(a+bx)^2}\int (a+bx)\,d(a+bx)}{b(a+bx)}$$

$$\downarrow 15$$

$$\frac{(a+bx)\sqrt{c(a+bx)^2}}{2b}$$

input
```
Int[Sqrt[c*(a + b*x)^2],x]
```

output
```
((a + b*x)*Sqrt[c*(a + b*x)^2])/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{x(bx+2a)\sqrt{c(bx+a)^2}}{2bx+2a}$	29
default	$\frac{x(bx+2a)\sqrt{c(bx+a)^2}}{2bx+2a}$	29
orering	$\frac{x(bx+2a)\sqrt{c(bx+a)^2}}{2bx+2a}$	29
trager	$\frac{x(bx+2a)\sqrt{b^2cx^2+2abcx+a^2c}}{2bx+2a}$	40
risch	$\frac{\sqrt{c(bx+a)^2}\,bx^2}{2bx+2a}+\frac{\sqrt{c(bx+a)^2}\,ax}{bx+a}$	47

input
```
int((c*(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)*(c*(b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{\sqrt{b^2cx^2+2\,abcx+a^2c}(bx^2+2\,ax)}{2\,(bx+a)}$$

input
```
integrate((c*(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*(b*x^2 + 2*a*x)/(b*x + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.

Time = 0.65 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \sqrt{c(a+bx)^2}\,dx = \begin{cases} \left(\frac{a}{2b} + \frac{x}{2}\right)\sqrt{a^2c + 2abcx + b^2cx^2} & \text{for } b^2c \neq 0 \\ \frac{(a^2c+2abcx)^{\frac{3}{2}}}{3abc} & \text{for } abc \neq 0 \\ x\sqrt{a^2c} & \text{otherwise} \end{cases}$$

input
```
integrate((c*(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((a/(2*b) + x/2)*sqrt(a**2*c + 2*a*b*c*x + b**2*c*x**2), Ne(b**2
*c, 0)), ((a**2*c + 2*a*b*c*x)**(3/2)/(3*a*b*c), Ne(a*b*c, 0)), (x*sqrt(a*
*2*c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(21) = 42$.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{1}{2}\sqrt{b^2cx^2 + 2\,abcx + a^2c}x + \frac{\sqrt{b^2cx^2 + 2\,abcx + a^2c}a}{2\,b}$$

input
```
integrate((c*(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*x + 1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x
 + a^2*c)*a/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{1}{2}\left((bx^2+2\,ax)\mathrm{sgn}(bx+a) + \frac{a^2\mathrm{sgn}(bx+a)}{b}\right)\sqrt{c}$$

input
```
integrate((c*(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*((b*x^2 + 2*a*x)*sgn(b*x + a) + a^2*sgn(b*x + a)/b)*sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{\sqrt{c\,(a+b\,x)^2}\,(a+b\,x)}{2\,b}$$

input
```
int((c*(a + b*x)^2)^(1/2),x)
```

output
```
((c*(a + b*x)^2)^(1/2)*(a + b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \sqrt{c(a+bx)^2}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2}$$

input
```
int((c*(b*x+a)^2)^(1/2),x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/2
```

3.4 $\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx$

Optimal result . . . 93
Mathematica [A] (verified) . . . 93
Rubi [A] (verified) . . . 94
Maple [A] (verified) . . . 95
Fricas [A] (verification not implemented) . . . 95
Sympy [A] (verification not implemented) . . . 96
Maxima [A] (verification not implemented) . . . 96
Giac [A] (verification not implemented) . . . 96
Mupad [B] (verification not implemented) . . . 97
Reduce [B] (verification not implemented) . . . 97

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

output
```
(b*x+a)*ln(b*x+a)/b/(c*(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

input
```
Integrate[1/Sqrt[c*(a + b*x)^2],x]
```

output
```
((a + b*x)*Log[a + b*x])/(b*Sqrt[c*(a + b*x)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{\sqrt{c(a+bx)^2}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)\int \frac{1}{a+bx}\,d(a+bx)}{b\sqrt{c(a+bx)^2}}$$
$$\downarrow 14$$
$$\frac{(a+bx)\log(a+bx)}{b\sqrt{c(a+bx)^2}}$$

input
```
Int[1/Sqrt[c*(a + b*x)^2],x]
```

output
```
((a + b*x)*Log[a + b*x])/(b*Sqrt[c*(a + b*x)^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{(bx+a)\ln(bx+a)}{b\sqrt{c(bx+a)^2}}$	27
risch	$\frac{(bx+a)\ln(bx+a)}{b\sqrt{c(bx+a)^2}}$	27

input

```
int(1/(c*(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(b*x+a)*ln(b*x+a)/b/(c*(b*x+a)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\sqrt{b^2cx^2+2\,abcx+a^2c}\log\left(bx+a\right)}{b^2cx+abc}$$

input

```
integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output

```
sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*log(b*x + a)/(b^2*c*x + a*b*c)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{\sqrt{b^2c\left(\frac{a}{b}+x\right)^2}}$$

input
```
integrate(1/(c*(b*x+a)**2)**(1/2),x)
```

output
```
(a/b + x)*log(a/b + x)/sqrt(b**2*c*(a/b + x)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\log\left(x+\frac{a}{b}\right)}{b\sqrt{c}}$$

input
```
integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(x + a/b)/(b*sqrt(c))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\log\left(|bx+a|\sqrt{|c|}|\mathrm{sgn}(bx+a)|\right)}{b\sqrt{c}\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(1/(c*(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(b*x + a)*sqrt(abs(c))*abs(sgn(b*x + a)))/(b*sqrt(c)*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\ln\left(\sqrt{c}\,(a+b\,x)+\sqrt{c\,(a+b\,x)^2}\right)}{b\sqrt{c}}$$

input
```
int(1/(c*(a + b*x)^2)^(1/2),x)
```

output
```
log(c^(1/2)*(a + b*x) + (c*(a + b*x)^2)^(1/2))/(b*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{c(a+bx)^2}}\,dx = \frac{\sqrt{c}\log(bx+a)}{bc}$$

input
```
int(1/(c*(b*x+a)^2)^(1/2),x)
```

output
```
(sqrt(c)*log(a + b*x))/(b*c)
```

3.5 $\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx$

Optimal result . . . 98
Mathematica [A] (verified) . . . 98
Rubi [A] (verified) . . . 99
Maple [A] (verified) . . . 100
Fricas [B] (verification not implemented) . . . 100
Sympy [A] (verification not implemented) . . . 101
Maxima [A] (verification not implemented) . . . 101
Giac [F(-2)] . . . 102
Mupad [B] (verification not implemented) . . . 102
Reduce [B] (verification not implemented) . . . 102

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx = -\frac{1}{2bc(a+bx)\sqrt{c(a+bx)^2}}$$

output
```
-1/2/b/c/(b*x+a)/(c*(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx = -\frac{a+bx}{2b\left(c(a+bx)^2\right)^{3/2}}$$

input
```
Integrate[(c*(a + b*x)^2)^(-3/2),x]
```

output
```
-1/2*(a + b*x)/(b*(c*(a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{(c(a+bx)^2)^{3/2}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)^3 \int \frac{1}{(a+bx)^3}\,d(a+bx)}{b\left(c(a+bx)^2\right)^{3/2}}$$
$$\downarrow 15$$
$$-\frac{a+bx}{2b\left(c(a+bx)^2\right)^{3/2}}$$

input
```
Int[(c*(a + b*x)^2)^(-3/2),x]
```

output
```
-1/2*(a + b*x)/(b*(c*(a + b*x)^2)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{bx+a}{2b\left(c(bx+a)^2\right)^{\frac{3}{2}}}$	22
default	$-\frac{bx+a}{2b\left(c(bx+a)^2\right)^{\frac{3}{2}}}$	22
orering	$-\frac{bx+a}{2b\left(c(bx+a)^2\right)^{\frac{3}{2}}}$	22
risch	$-\frac{1}{2bc(bx+a)\sqrt{c(bx+a)^2}}$	27
trager	$\frac{(bx+2a)x\sqrt{b^2cx^2+2abcx+a^2c}}{2a^2c^2(bx+a)^3}$	46

input

```
int(1/(c*(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(b*x+a)/b/(c*(b*x+a)^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(c(a+bx)^2\right)^{3/2}}\,dx = -\frac{\sqrt{b^2cx^2+2\,abcx+a^2c}}{2\left(b^4c^2x^3+3\,ab^3c^2x^2+3\,a^2b^2c^2x+a^3bc^2\right)}$$

input

```
integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(b^4*c^2*x^3 + 3*a*b^3*c^2*x^2 +
3*a^2*b^2*c^2*x + a^3*b*c^2)
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx = \begin{cases} -\frac{\frac{a}{b}+x}{2\left(c(a+bx)^2\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(a^2c)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x+a)**2)**(3/2),x)
```

output
```
Piecewise((-(a/b + x)/(2*(c*(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(a**2*c)*
*(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx = -\frac{1}{2\,b^3c^{\frac{3}{2}}\left(x+\frac{a}{b}\right)^2}$$

input
```
integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/(b^3*c^(3/2)*(x + a/b)^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx = -\frac{\sqrt{c\,(a+b\,x)^2}}{2\,b\,c^2\,(a+b\,x)^3}$$

input
```
int(1/(c*(a + b*x)^2)^(3/2),x)
```

output
```
-(c*(a + b*x)^2)^(1/2)/(2*b*c^2*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{(c(a+bx)^2)^{3/2}}\,dx = -\frac{\sqrt{c}}{2b\,c^2\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/(c*(b*x+a)^2)^(3/2),x)
```

output
```
( - sqrt(c))/(2*b*c**2*(a**2 + 2*a*b*x + b**2*x**2))
```

3.6 $\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx$

Optimal result . . . 103
Mathematica [A] (verified) . . . 103
Rubi [A] (verified) . . . 104
Maple [A] (verified) . . . 105
Fricas [B] (verification not implemented) . . . 105
Sympy [A] (verification not implemented) . . . 106
Maxima [A] (verification not implemented) . . . 106
Giac [F(-2)] . . . 107
Mupad [B] (verification not implemented) . . . 107
Reduce [B] (verification not implemented) . . . 107

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx = -\frac{1}{4bc^2(a+bx)^3\sqrt{c(a+bx)^2}}$$

output
```
-1/4/b/c^2/(b*x+a)^3/(c*(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx = -\frac{a+bx}{4b\left(c(a+bx)^2\right)^{5/2}}$$

input
```
Integrate[(c*(a + b*x)^2)^(-5/2),x]
```

output
```
-1/4*(a + b*x)/(b*(c*(a + b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{(c(a+bx)^2)^{5/2}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)^5 \int \frac{1}{(a+bx)^5}\,d(a+bx)}{b\,(c(a+bx)^2)^{5/2}}$$
$$\downarrow 15$$
$$-\frac{a+bx}{4b\,(c(a+bx)^2)^{5/2}}$$

input
```
Int[(c*(a + b*x)^2)^(-5/2),x]
```

output
```
-1/4*(a + b*x)/(b*(c*(a + b*x)^2)^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{bx+a}{4b\left(c(bx+a)^2\right)^{\frac{5}{2}}}$	22
default	$-\frac{bx+a}{4b\left(c(bx+a)^2\right)^{\frac{5}{2}}}$	22
orering	$-\frac{bx+a}{4b\left(c(bx+a)^2\right)^{\frac{5}{2}}}$	22
risch	$-\frac{1}{4bc^2(bx+a)^3\sqrt{c(bx+a)^2}}$	27
trager	$\frac{(b^3x^3+4ab^2x^2+6a^2bx+4a^3)x\sqrt{b^2cx^2+2abcx+a^2c}}{4c^3a^4(bx+a)^5}$	68

input

```
int(1/(c*(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4*(b*x+a)/b/(c*(b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.23

$$\int \frac{1}{\left(c(a+bx)^2\right)^{5/2}}\,dx = -\frac{\sqrt{b^2cx^2+2\,abcx+a^2c}}{4\left(b^6c^3x^5+5\,ab^5c^3x^4+10\,a^2b^4c^3x^3+10\,a^3b^3c^3x^2+5\,a^4b^2c^3x+a^5bc^3\right)}$$

input

```
integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/(b^6*c^3*x^5 + 5*a*b^5*c^3*x^4 +
10*a^2*b^4*c^3*x^3 + 10*a^3*b^3*c^3*x^2 + 5*a^4*b^2*c^3*x + a^5*b*c^3)
```

Sympy [A] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx = \begin{cases} -\frac{\frac{a}{b}+x}{4\left(c(a+bx)^2\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(a^2c)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x+a)**2)**(5/2),x)
```

output
```
Piecewise((-(a/b + x)/(4*(c*(a + b*x)**2)**(5/2)), Ne(b, 0)), (x/(a**2*c)*
*(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.57

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx = -\frac{1}{4\,b^5c^{\frac{5}{2}}\left(x+\frac{a}{b}\right)^4}$$

input
```
integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4/(b^5*c^(5/2)*(x + a/b)^4)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx = -\frac{\sqrt{c\,(a+b\,x)^2}}{4\,b\,c^3\,(a+b\,x)^5}$$

input
```
int(1/(c*(a + b*x)^2)^(5/2),x)
```

output
```
-(c*(a + b*x)^2)^(1/2)/(4*b*c^3*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \frac{1}{(c(a+bx)^2)^{5/2}}\,dx = -\frac{\sqrt{c}}{4b\,c^3\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int(1/(c*(b*x+a)^2)^(5/2),x)
```

output

```
( - sqrt(c))/(4*b*c**3*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
*3 + b**4*x**4))
```

3.7 $\int \sqrt{(3+5x)^2}\,dx$

Optimal result 109
Mathematica [A] (verified) 109
Rubi [A] (verified) 110
Maple [C] (warning: unable to verify) 111
Fricas [A] (verification not implemented) 111
Sympy [A] (verification not implemented) 112
Maxima [A] (verification not implemented) 112
Giac [A] (verification not implemented) 112
Mupad [B] (verification not implemented) 113
Reduce [B] (verification not implemented) 113

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \sqrt{(3+5x)^2}\,dx = \frac{1}{10}(3+5x)\sqrt{(3+5x)^2}$$

output
```
1/10*(3+5*x)*((3+5*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \sqrt{(3+5x)^2}\,dx = \frac{x\sqrt{(3+5x)^2}(6+5x)}{6+10x}$$

input
```
Integrate[Sqrt[(3 + 5*x)^2],x]
```

output
```
(x*Sqrt[(3 + 5*x)^2]*(6 + 5*x))/(6 + 10*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(5x+3)^2}\,dx$$

$$\downarrow 239$$

$$\frac{1}{5}\int \sqrt{(5x+3)^2}d(5x+3)$$

$$\downarrow 20$$

$$\frac{\sqrt{(5x+3)^2}\int (5x+3)d(5x+3)}{5(5x+3)}$$

$$\downarrow 15$$

$$\frac{1}{10}(5x+3)\sqrt{(5x+3)^2}$$

input
```
Int[Sqrt[(3 + 5*x)^2],x]
```

output
```
((3 + 5*x)*Sqrt[(3 + 5*x)^2])/10
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\operatorname{csgn}(3+5x)(3+5x)^2}{10}$	16
gosper	$\frac{x(5x+6)\sqrt{(3+5x)^2}}{6+10x}$	25
orering	$\frac{x(5x+6)\sqrt{(3+5x)^2}}{6+10x}$	25
risch	$\frac{5\sqrt{(3+5x)^2}\,x^2}{2(3+5x)}+\frac{3\sqrt{(3+5x)^2}\,x}{3+5x}$	42

input

```
int(((3+5*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/10*csgn(3+5*x)*(3+5*x)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int \sqrt{(3+5x)^2}\,dx = \frac{5}{2}\,x^2 + 3\,x$$

input

```
integrate(((3+5*x)^2)^(1/2),x, algorithm="fricas")
```

output

```
5/2*x^2 + 3*x
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{(3+5x)^2}\,dx = \left(\frac{x}{2}+\frac{3}{10}\right)\sqrt{25x^2+30x+9}$$

input
```
integrate(((3+5*x)**2)**(1/2),x)
```

output
```
(x/2 + 3/10)*sqrt(25*x**2 + 30*x + 9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \sqrt{(3+5x)^2}\,dx = \frac{1}{2}\sqrt{25\,x^2+30\,x+9}x+\frac{3}{10}\sqrt{25\,x^2+30\,x+9}$$

input
```
integrate(((3+5*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(25*x^2 + 30*x + 9)*x + 3/10*sqrt(25*x^2 + 30*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \sqrt{(3+5x)^2}\,dx = \frac{1}{2}\left(5\,x^2+6\,x\right)\mathrm{sgn}(5\,x+3)+\frac{9}{10}\,\mathrm{sgn}(5\,x+3)$$

input
```
integrate(((3+5*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(5*x^2 + 6*x)*sgn(5*x + 3) + 9/10*sgn(5*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \sqrt{(3+5x)^2}\,dx = \frac{|5\,x+3|\,(5\,x+3)}{10}$$

input
```
int(((5*x + 3)^2)^(1/2),x)
```

output
```
(abs(5*x + 3)*(5*x + 3))/10
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.40

$$\int \sqrt{(3+5x)^2}\,dx = \frac{x(5x+6)}{2}$$

input
```
int(((3+5*x)^2)^(1/2),x)
```

output
```
(x*(5*x + 6))/2
```

3.8 $\int \sqrt{(6+10x)^2}\,dx$

Optimal result 114
Mathematica [A] (verified) 114
Rubi [A] (verified) 115
Maple [A] (verified) 116
Fricas [A] (verification not implemented) 116
Sympy [A] (verification not implemented) 117
Maxima [A] (verification not implemented) 117
Giac [A] (verification not implemented) 117
Mupad [B] (verification not implemented) 118
Reduce [B] (verification not implemented) 118

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \sqrt{(6+10x)^2}\,dx = \frac{1}{5}(3+5x)\sqrt{(3+5x)^2}$$

output
```
1/5*(3+5*x)*((3+5*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \sqrt{(6+10x)^2}\,dx = \frac{x\sqrt{(3+5x)^2}(6+5x)}{3+5x}$$

input
```
Integrate[Sqrt[(6 + 10*x)^2],x]
```

output
```
(x*Sqrt[(3 + 5*x)^2]*(6 + 5*x))/(3 + 5*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(10x+6)^2}\,dx$$

$$\downarrow 239$$

$$\frac{1}{10}\int \sqrt{(10x+6)^2}\,d(10x+6)$$

$$\downarrow 20$$

$$\frac{\sqrt{(10x+6)^2}\int (10x+6)\,d(10x+6)}{10(10x+6)}$$

$$\downarrow 15$$

$$\frac{1}{20}(10x+6)\sqrt{(10x+6)^2}$$

input
```
Int[Sqrt[(6 + 10*x)^2],x]
```

output
```
((6 + 10*x)*Sqrt[(6 + 10*x)^2])/20
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
gosper	$\frac{x(5x+6)\sqrt{(3+5x)^2}}{3+5x}$	24
orering	$\frac{x(5x+6)\sqrt{(6+10x)^2}}{6+10x}$	25
default	$\frac{\sqrt{(3+5x)^2}\sqrt{4}\,x(5x+6)}{6+10x}$	28
risch	$\frac{5\sqrt{(3+5x)^2}\,x^2}{3+5x}+\frac{6\sqrt{(3+5x)^2}\,x}{3+5x}$	42

input

```
int(((6+10*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
x*(5*x+6)*((3+5*x)^2)^(1/2)/(3+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int \sqrt{(6+10x)^2}\,dx = 5\,x^2+6\,x$$

input

```
integrate(((6+10*x)^2)^(1/2),x, algorithm="fricas")
```

output

```
5*x^2 + 6*x
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{(6+10x)^2}\,dx = 2\left(\frac{x}{2}+\frac{3}{10}\right)\sqrt{25x^2+30x+9}$$

input
```
integrate(((6+10*x)**2)**(1/2),x)
```

output
```
2*(x/2 + 3/10)*sqrt(25*x**2 + 30*x + 9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \sqrt{(6+10x)^2}\,dx = \sqrt{25\,x^2+30\,x+9}\,x+\frac{3}{5}\sqrt{25\,x^2+30\,x+9}$$

input
```
integrate(((6+10*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(25*x^2 + 30*x + 9)*x + 3/5*sqrt(25*x^2 + 30*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \sqrt{(6+10x)^2}\,dx = \left(5\,x^2+6\,x\right)\mathrm{sgn}(5\,x+3)+\frac{9}{5}\,\mathrm{sgn}(5\,x+3)$$

input
```
integrate(((6+10*x)^2)^(1/2),x, algorithm="giac")
```

output
```
(5*x^2 + 6*x)*sgn(5*x + 3) + 9/5*sgn(5*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \sqrt{(6+10x)^2}\,dx = \frac{|10\,x+6|\,(5\,x+3)}{10}$$

input
```
int(((10*x + 6)^2)^(1/2),x)
```

output
```
(abs(10*x + 6)*(5*x + 3))/10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

$$\int \sqrt{(6+10x)^2}\,dx = x(5x+6)$$

input
```
int(((6+10*x)^2)^(1/2),x)
```

output
```
x*(5*x + 6)
```

3.9 $\int \frac{1}{\sqrt{(3+5x)^2}}\,dx$

Optimal result 119
Mathematica [A] (verified) 119
Rubi [A] (verified) 120
Maple [A] (verified) 121
Fricas [A] (verification not implemented) 121
Sympy [A] (verification not implemented) 122
Maxima [A] (verification not implemented) 122
Giac [A] (verification not implemented) 122
Mupad [B] (verification not implemented) 123
Reduce [B] (verification not implemented) 123

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{(3+5x)\log(3+5x)}{5\sqrt{(3+5x)^2}}$$

output
```
1/5*(3+5*x)*ln(3+5*x)/((3+5*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{(3+5x)\log(3+5x)}{5\sqrt{(3+5x)^2}}$$

input
```
Integrate[1/Sqrt[(3 + 5*x)^2],x]
```

output
```
((3 + 5*x)*Log[3 + 5*x])/(5*Sqrt[(3 + 5*x)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(5x+3)^2}}\,dx$$

$$\downarrow 239$$

$$\frac{1}{5}\int \frac{1}{\sqrt{(5x+3)^2}}d(5x+3)$$

$$\downarrow 20$$

$$\frac{(5x+3)\int \frac{1}{5x+3}d(5x+3)}{5\sqrt{(5x+3)^2}}$$

$$\downarrow 14$$

$$\frac{(5x+3)\log(5x+3)}{5\sqrt{(5x+3)^2}}$$

input
```
Int[1/Sqrt[(3 + 5*x)^2],x]
```

output
```
((3 + 5*x)*Log[3 + 5*x])/(5*Sqrt[(3 + 5*x)^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{(3+5x)\ln(3+5x)}{5\sqrt{(3+5x)^2}}$	23
risch	$\frac{\sqrt{(3+5x)^2}\,\ln(3+5x)}{15+25x}$	25
meijerg	$\frac{3\ln(1+\frac{5x}{3})}{5\sqrt{(3+5x)^2}}+\frac{x\ln(1+\frac{5x}{3})}{\sqrt{(3+5x)^2}}$	36

input

```
int(1/((3+5*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/5*(3+5*x)*ln(3+5*x)/((3+5*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{1}{5}\log(5\,x+3)$$

input

```
integrate(1/((3+5*x)^2)^(1/2),x, algorithm="fricas")
```

output

```
1/5*log(5*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{\left(x+\frac{3}{5}\right)\log\left(x+\frac{3}{5}\right)}{5\sqrt{\left(x+\frac{3}{5}\right)^2}}$$

input
```
integrate(1/((3+5*x)**2)**(1/2),x)
```

output
```
(x + 3/5)*log(x + 3/5)/(5*sqrt((x + 3/5)**2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{1}{5}\log\left(x+\frac{3}{5}\right)$$

input
```
integrate(1/((3+5*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*log(x + 3/5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{1}{5}\log\left(|5\,x+3|\right)\operatorname{sgn}(5\,x+3)$$

input
```
integrate(1/((3+5*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*log(abs(5*x + 3))*sgn(5*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{\ln(5\,x+3)\,\operatorname{sign}(5\,x+3)}{5}$$

input
```
int(1/((5*x + 3)^2)^(1/2),x)
```

output
```
(log(5*x + 3)*sign(5*x + 3))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{(3+5x)^2}}\,dx = \frac{\log(5x+3)}{5}$$

input
```
int(1/((3+5*x)^2)^(1/2),x)
```

output
```
log(5*x + 3)/5
```

3.10 $\int \frac{1}{\sqrt{(6+10x)^2}}\,dx$

Optimal result 124
Mathematica [A] (verified) 124
Rubi [A] (verified) 125
Maple [A] (verified) 126
Fricas [A] (verification not implemented) 126
Sympy [A] (verification not implemented) 127
Maxima [A] (verification not implemented) 127
Giac [A] (verification not implemented) 127
Mupad [B] (verification not implemented) 128
Reduce [B] (verification not implemented) 128

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{(3+5x)\log(3+5x)}{10\sqrt{(3+5x)^2}}$$

output
```
1/10*(3+5*x)*ln(3+5*x)/((3+5*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{(6+10x)\log(6+10x)}{10\sqrt{(6+10x)^2}}$$

input
```
Integrate[1/Sqrt[(6 + 10*x)^2],x]
```

output
```
((6 + 10*x)*Log[6 + 10*x])/(10*Sqrt[(6 + 10*x)^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {239, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(10x+6)^2}}\,dx$$

$$\downarrow 239$$

$$\frac{1}{10}\int \frac{1}{\sqrt{(10x+6)^2}}\,d(10x+6)$$

$$\downarrow 20$$

$$\frac{(10x+6)\int \frac{1}{10x+6}\,d(10x+6)}{10\sqrt{(10x+6)^2}}$$

$$\downarrow 14$$

$$\frac{(10x+6)\log(10x+6)}{10\sqrt{(10x+6)^2}}$$

input
```
Int[1/Sqrt[(6 + 10*x)^2],x]
```

output
```
((6 + 10*x)*Log[6 + 10*x])/(10*Sqrt[(6 + 10*x)^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\sqrt{(3+5x)^2}\,\ln(3+5x)}{30+50x}$	25
default	$\frac{(3+5x)\sqrt{4}\,\ln(3+5x)}{20\sqrt{(3+5x)^2}}$	26
meijerg	$\frac{3\ln\left(1+\frac{5x}{3}\right)}{5\sqrt{(6+10x)^2}}+\frac{x\ln\left(1+\frac{5x}{3}\right)}{\sqrt{(6+10x)^2}}$	36

input
```
int(1/((6+10*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*((3+5*x)^2)^(1/2)/(3+5*x)*ln(3+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{1}{10}\log(5x+3)$$

input
```
integrate(1/((6+10*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/10*log(5*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{\left(x+\frac{3}{5}\right)\log\left(x+\frac{3}{5}\right)}{10\sqrt{\left(x+\frac{3}{5}\right)^2}}$$

input
```
integrate(1/((6+10*x)**2)**(1/2),x)
```

output
```
(x + 3/5)*log(x + 3/5)/(10*sqrt((x + 3/5)**2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{1}{10}\log\left(x+\frac{3}{5}\right)$$

input
```
integrate(1/((6+10*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/10*log(x + 3/5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{1}{10}\log\left(|5\,x+3|\right)\operatorname{sgn}(5\,x+3)$$

input
```
integrate(1/((6+10*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/10*log(abs(5*x + 3))*sgn(5*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{\ln(10x+6)\ \mathrm{sign}(10x+6)}{10}$$

input
```
int(1/((10*x + 6)^2)^(1/2),x)
```

output
```
(log(10*x + 6)*sign(10*x + 6))/10
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{(6+10x)^2}}\,dx = \frac{\log(5x+3)}{10}$$

input
```
int(1/((6+10*x)^2)^(1/2),x)
```

output
```
log(5*x + 3)/10
```

3.11 $\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx$

Optimal result . . . 129
Mathematica [A] (verified) . . . 129
Rubi [A] (verified) . . . 130
Maple [C] (verified) . . . 131
Fricas [C] (verification not implemented) . . . 131
Sympy [A] (verification not implemented) . . . 132
Maxima [C] (verification not implemented) . . . 132
Giac [C] (verification not implemented) . . . 132
Mupad [B] (verification not implemented) . . . 133
Reduce [B] (verification not implemented) . . . 133

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

output
```
1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

input
```
Integrate[1/Sqrt[-(2 + 3*x)^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-(3x+2)^2}}\,dx$$

$$\downarrow 239$$

$$\frac{1}{3}\int \frac{1}{\sqrt{-(3x+2)^2}}d(3x+2)$$

$$\downarrow 20$$

$$\frac{(3x+2)\int \frac{1}{3x+2}d(3x+2)}{3\sqrt{-(3x+2)^2}}$$

$$\downarrow 14$$

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

input
```
Int[1/Sqrt[-(2 + 3*x)^2],x]
```

output
```
((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.36

method	result	size
meijerg	$-\frac{i \ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^2}}$	25
risch	$\frac{(2+3x)\ln(2+3x)}{3\sqrt{-(2+3x)^2}}$	25

input
```
int(1/(-(2+3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*ln(1+3/2*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = -\frac{1}{3}i \log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*I*log(x + 2/3)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{\left(x+\frac{2}{3}\right)\log\left(x+\frac{2}{3}\right)}{3\sqrt{-\left(x+\frac{2}{3}\right)^2}}$$

input
```
integrate(1/(-(2+3*x)**2)**(1/2),x)
```

output
```
(x + 2/3)*log(x + 2/3)/(3*sqrt(-(x + 2/3)**2))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*I*log(x + 2/3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{i\,\log\left((-3i\,x-2i)\mathrm{sgn}(-3\,x-2)\right)}{3\,\mathrm{sgn}\left(-3\,x-2\right)}$$

input
```
integrate(1/(-(2+3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*log((-3*I*x - 2*I)*sgn(-3*x - 2))/sgn(-3*x - 2)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = -\frac{\ln(-3\,x-2)\ \mathrm{sign}(3\,x+2)\ 1\mathrm{i}}{3}$$

input
```
int(1/(-(3*x + 2)^2)^(1/2),x)
```

output
```
-(log(- 3*x - 2)*sign(3*x + 2)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{-(2+3x)^2}}\,dx = \frac{\log(3x+2)}{3}$$

input
```
int(1/(-(2+3*x)^2)^(1/2),x)
```

output
```
log(3*x + 2)/3
```

3.12 $\int (c(a+bx)^3)^{5/2}\,dx$

Optimal result . . . 134
Mathematica [A] (verified) . . . 134
Rubi [A] (verified) . . . 135
Maple [A] (verified) . . . 136
Fricas [B] (verification not implemented) . . . 136
Sympy [F] . . . 137
Maxima [B] (verification not implemented) . . . 137
Giac [B] (verification not implemented) . . . 138
Mupad [B] (verification not implemented) . . . 139
Reduce [B] (verification not implemented) . . . 139

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int (c(a+bx)^3)^{5/2}\,dx = \frac{2c^2(a+bx)^7\sqrt{c(a+bx)^3}}{17b}$$

output
```
2/17*c^2*(b*x+a)^7*(c*(b*x+a)^3)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int (c(a+bx)^3)^{5/2}\,dx = \frac{2(a+bx)\left(c(a+bx)^3\right)^{5/2}}{17b}$$

input
```
Integrate[(c*(a + b*x)^3)^(5/2),x]
```

output
```
(2*(a + b*x)*(c*(a + b*x)^3)^(5/2))/(17*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^3\right)^{5/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^3\right)^{5/2} \int (a+bx)^{15/2} d(a+bx)}{b(a+bx)^{15/2}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)\left(c(a+bx)^3\right)^{5/2}}{17b}$$

input
```
Int[(c*(a + b*x)^3)^(5/2),x]
```

output
```
(2*(a + b*x)*(c*(a + b*x)^3)^(5/2))/(17*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{2(bx+a)\left(c(bx+a)^3\right)^{\frac{5}{2}}}{17b}$	22
orering	$\frac{2(bx+a)\left(c(bx+a)^3\right)^{\frac{5}{2}}}{17b}$	22
default	$\frac{2\left(c(bx+a)^3\right)^{\frac{5}{2}}(bcx+ac)^{\frac{17}{2}}}{17(bx+a)^5(c(bx+a))^{\frac{5}{2}}c^6b}$	46
risch	$\frac{2c^2\sqrt{c(bx+a)^3}\left(b^8x^8+8a\,x^7b^7+28a^2x^6b^6+56a^3x^5b^5+70a^4x^4b^4+56a^5b^3x^3+28a^6x^2b^2+8a^7xb+a^8\right)}{17(bx+a)b}$	109
trager	$\frac{2c^2\left(b^7x^7+7a\,b^6x^6+21a^2b^5x^5+35a^3b^4x^4+35a^4b^3x^3+21a^5b^2x^2+7a^6bx+a^7\right)\sqrt{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx+a^3c}}{17b}$	114

input
```
int((c*(b*x+a)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*(b*x+a)*(c*(b*x+a)^3)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.50

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \frac{2\left(b^7c^2x^7+7\,ab^6c^2x^6+21\,a^2b^5c^2x^5+35\,a^3b^4c^2x^4+35\,a^4b^3c^2x^3+21\,a^5b^2c^2x^2+7\,a^6bc^2x+\right.}{17\,b}$$

input
```
integrate((c*(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/17*(b^7*c^2*x^7 + 7*a*b^6*c^2*x^6 + 21*a^2*b^5*c^2*x^5 + 35*a^3*b^4*c^2*
x^4 + 35*a^4*b^3*c^2*x^3 + 21*a^5*b^2*c^2*x^2 + 7*a^6*b*c^2*x + a^7*c^2)*s
qrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/b
```

Sympy [F]

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \int \left(c(a+bx)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*(b*x+a)**3)**(5/2),x)
```

output
```
Integral((c*(a + b*x)**3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.60

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \frac{2\left(b^7c^{\frac{5}{2}}x^7 + 7\,ab^6c^{\frac{5}{2}}x^6 + 21\,a^2b^5c^{\frac{5}{2}}x^5 + 35\,a^3b^4c^{\frac{5}{2}}x^4 + 35\,a^4b^3c^{\frac{5}{2}}x^3 + 21\,a^5b^2c^{\frac{5}{2}}x^2 + 7\,a^6bc^{\frac{5}{2}}x\right.}{17\,b}$$

input
```
integrate((c*(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

output
```
2/17*(b^7*c^(5/2)*x^7 + 7*a*b^6*c^(5/2)*x^6 + 21*a^2*b^5*c^(5/2)*x^5 + 35*
a^3*b^4*c^(5/2)*x^4 + 35*a^4*b^3*c^(5/2)*x^3 + 21*a^5*b^2*c^(5/2)*x^2 + 7*
a^6*b*c^(5/2)*x + a^7*c^(5/2))*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(26) = 52$.

Time = 0.14 (sec) , antiderivative size = 856, normalized size of antiderivative = 28.53

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((c*(b*x+a)^3)^(5/2),x, algorithm="giac")
```

output

```
2/109395*(109395*sqrt(b*c*x + a*c)*a^8*sgn(b*x + a) - 291720*(3*sqrt(b*c*x
 + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^7*sgn(b*x + a)/c + 204204*(15*sqrt(b*
c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a
^6*sgn(b*x + a)/c^2 - 175032*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a
*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^
5*sgn(b*x + a)/c^3 + 24310*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7
/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^4*sgn(b*x + a)/c^4 - 8840*(693*sqrt(b*
c*x + a*c)*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)
^(5/2)*a^3*c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)
*a*c - 63*(b*c*x + a*c)^(11/2))*a^3*sgn(b*x + a)/c^5 + 1020*(3003*sqrt(b*c
*x + a*c)*a^6*c^6 - 6006*(b*c*x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^
(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2
)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a^2*
sgn(b*x + a)/c^6 - 136*(6435*sqrt(b*c*x + a*c)*a^7*c^7 - 15015*(b*c*x + a*
c)^(3/2)*a^6*c^6 + 27027*(b*c*x + a*c)^(5/2)*a^5*c^5 - 32175*(b*c*x + a*c)
^(7/2)*a^4*c^4 + 25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 12285*(b*c*x + a*c)^(
11/2)*a^2*c^2 + 3465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*c*x + a*c)^(15/2))*
a*sgn(b*x + a)/c^7 + (109395*sqrt(b*c*x + a*c)*a^8*c^8 - 291720*(b*c*x + a
*c)^(3/2)*a^7*c^7 + 612612*(b*c*x + a*c)^(5/2)*a^6*c^6 - 875160*(b*c*x ...
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.67

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \sqrt{c\,(a+b\,x)^3}\left(\frac{14\,a^6\,c^2\,x}{17}+\frac{2\,a^7\,c^2}{17\,b}+\frac{2\,b^6\,c^2\,x^7}{17}+\frac{42\,a^5\,b\,c^2\,x^2}{17}+\frac{14\,a\,b^5\,c^2\,x^6}{17}+\frac{70\,a^4\,b^2\,c^2\,x^3}{17}+\frac{70\,a^3\,b^3\,c^2\,x^4}{17}+\frac{42\,a^2\,b^4\,c^2\,x^5}{17}\right)$$

input
```
int((c*(a + b*x)^3)^(5/2),x)
```

output
```
(c*(a + b*x)^3)^(1/2)*((14*a^6*c^2*x)/17 + (2*a^7*c^2)/(17*b) + (2*b^6*c^2
*x^7)/17 + (42*a^5*b*c^2*x^2)/17 + (14*a*b^5*c^2*x^6)/17 + (70*a^4*b^2*c^2
*x^3)/17 + (70*a^3*b^3*c^2*x^4)/17 + (42*a^2*b^4*c^2*x^5)/17)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.27

$$\int \left(c(a+bx)^3\right)^{5/2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c^2(b^8x^8+8a\,b^7x^7+28a^2b^6x^6+56a^3b^5x^5+70a^4b^4x^4+56a^5b^3x^3+28a^6b^2x^2+\cdots}{17b}$$

input
```
int((c*(b*x+a)^3)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*c**2*(a**8 + 8*a**7*b*x + 28*a**6*b**2*x**2 + 56*
a**5*b**3*x**3 + 70*a**4*b**4*x**4 + 56*a**3*b**5*x**5 + 28*a**2*b**6*x**6
 + 8*a*b**7*x**7 + b**8*x**8))/(17*b)
```

3.13 $\int \left(c(a+bx)^3\right)^{3/2}\,dx$

Optimal result 140
Mathematica [A] (verified) 140
Rubi [A] (verified) 141
Maple [A] (verified) 142
Fricas [B] (verification not implemented) 142
Sympy [F] 143
Maxima [B] (verification not implemented) 143
Giac [B] (verification not implemented) 144
Mupad [B] (verification not implemented) 144
Reduce [B] (verification not implemented) 145

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \frac{2c(a+bx)^4\sqrt{c(a+bx)^3}}{11b}$$

output
```
2/11*c*(b*x+a)^4*(c*(b*x+a)^3)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \frac{2(a+bx)\left(c(a+bx)^3\right)^{3/2}}{11b}$$

input
```
Integrate[(c*(a + b*x)^3)^(3/2),x]
```

output
```
(2*(a + b*x)*(c*(a + b*x)^3)^(3/2))/(11*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^3\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^3\right)^{3/2} \int (a+bx)^{9/2} d(a+bx)}{b(a+bx)^{9/2}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)\left(c(a+bx)^3\right)^{3/2}}{11b}$$

input
```
Int[(c*(a + b*x)^3)^(3/2),x]
```

output
```
(2*(a + b*x)*(c*(a + b*x)^3)^(3/2))/(11*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{2(bx+a)\left(c(bx+a)^3\right)^{\frac{3}{2}}}{11b}$	22
orering	$\frac{2(bx+a)\left(c(bx+a)^3\right)^{\frac{3}{2}}}{11b}$	22
default	$\frac{2\left(c(bx+a)^3\right)^{\frac{3}{2}}(bcx+ac)^{\frac{11}{2}}}{11(bx+a)^3(c(bx+a))^{\frac{3}{2}}bc^4}$	46
risch	$\frac{2c\sqrt{c(bx+a)^3}\,(b^5x^5+5ab^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}{11(bx+a)b}$	74
trager	$\frac{2c(b^4x^4+4ax^3b^3+6a^2b^2x^2+4a^3bx+a^4)\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{11b}$	79

input

```
int((c*(b*x+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/11*(b*x+a)*(c*(b*x+a)^3)^(3/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.96

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \frac{2\,(b^4cx^4+4\,ab^3cx^3+6\,a^2b^2cx^2+4\,a^3bcx+a^4c)\sqrt{b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c}}{11\,b}$$

input

```
integrate((c*(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/11*(b^4*c*x^4 + 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 4*a^3*b*c*x + a^4*c)*s
qrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/b
```

Sympy [F]

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \int \left(c(a+bx)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*(b*x+a)**3)**(3/2),x)
```

output
```
Integral((c*(a + b*x)**3)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time $= 0.05$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 2.36$

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \frac{2\left(b^4c^{\frac{3}{2}}x^4 + 4\,ab^3c^{\frac{3}{2}}x^3 + 6\,a^2b^2c^{\frac{3}{2}}x^2 + 4\,a^3bc^{\frac{3}{2}}x + a^4c^{\frac{3}{2}}\right)(bx+a)^{\frac{3}{2}}}{11\,b}$$

input
```
integrate((c*(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
2/11*(b^4*c^(3/2)*x^4 + 4*a*b^3*c^(3/2)*x^3 + 6*a^2*b^2*c^(3/2)*x^2 + 4*a^
3*b*c^(3/2)*x + a^4*c^(3/2))*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(24) = 48.

Time = 0.13 (sec) , antiderivative size = 408, normalized size of antiderivative = 14.57

$$\int (c(a+bx)^3)^{3/2}\,dx = \frac{2\left(693\sqrt{bcx+ac}a^5\mathrm{sgn}(bx+a) - \frac{1155\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^4\mathrm{sgn}(bx+a)}{c} + \frac{462\left(15\sqrt{bcx+ac}a^2c^2-1\right.}{}\right.$$

input
```
integrate((c*(b*x+a)^3)^(3/2),x, algorithm="giac")
```

output
```
2/693*(693*sqrt(b*c*x + a*c)*a^5*sgn(b*x + a) - 1155*(3*sqrt(b*c*x + a*c)*
a*c - (b*c*x + a*c)^(3/2))*a^4*sgn(b*x + a)/c + 462*(15*sqrt(b*c*x + a*c)*
a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3*sgn(b*x
+ a)/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*
c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^2*sgn(b*x + a)
/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3
 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c
*x + a*c)^(9/2))*a*sgn(b*x + a)/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 115
5*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b*
c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))*sgn(b*x + a)/c^5)*c/b
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int (c(a+bx)^3)^{3/2}\,dx = \sqrt{c\,(a+b\,x)^3}\left(\frac{2\,a^4\,c}{11\,b} + \frac{2\,b^3\,c\,x^4}{11} + \frac{8\,a^3\,c\,x}{11} + \frac{12\,a^2\,b\,c\,x^2}{11} + \frac{8\,a\,b^2\,c\,x^3}{11}\right)$$

input
```
int((c*(a + b*x)^3)^(3/2),x)
```

output
```
(c*(a + b*x)^3)^(1/2)*((2*a^4*c)/(11*b) + (2*b^3*c*x^4)/11 + (8*a^3*c*x)/1
1 + (12*a^2*b*c*x^2)/11 + (8*a*b^2*c*x^3)/11)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \left(c(a+bx)^3\right)^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,c(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}{11b}$$

input
```
int((c*(b*x+a)^3)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*c*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**
2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))/(11*b)
```

3.14 $\int \sqrt{c(a+bx)^3}\,dx$

Optimal result 146
Mathematica [A] (verified) 146
Rubi [A] (verified) 147
Maple [A] (verified) 148
Fricas [B] (verification not implemented) 148
Sympy [A] (verification not implemented) 149
Maxima [A] (verification not implemented) 149
Giac [B] (verification not implemented) 149
Mupad [B] (verification not implemented) 150
Reduce [B] (verification not implemented) 150

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \sqrt{c(a+bx)^3}\,dx = \frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

output
```
2/5*(b*x+a)*(c*(b*x+a)^3)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{c(a+bx)^3}\,dx = \frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

input
```
Integrate[Sqrt[c*(a + b*x)^3],x]
```

output
```
(2*(a + b*x)*Sqrt[c*(a + b*x)^3])/(5*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c(a+bx)^3}\,dx$$

$$\downarrow 239$$

$$\frac{\int \sqrt{c(a+bx)^3}\,d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\sqrt{c(a+bx)^3}\int (a+bx)^{3/2}\,d(a+bx)}{b(a+bx)^{3/2}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)\sqrt{c(a+bx)^3}}{5b}$$

input
```
Int[Sqrt[c*(a + b*x)^3],x]
```

output
```
(2*(a + b*x)*Sqrt[c*(a + b*x)^3])/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{2(bx+a)\sqrt{c(bx+a)^3}}{5b}$	22
orering	$\frac{2(bx+a)\sqrt{c(bx+a)^3}}{5b}$	22
risch	$\frac{2\sqrt{c(bx+a)^3}\,(b^2x^2+2abx+a^2)}{5(bx+a)b}$	40
trager	$\frac{2(bx+a)\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{5b}$	45
default	$\frac{2\sqrt{c(bx+a)^3}\,(bcx+ac)^{\frac{5}{2}}}{5(bx+a)\sqrt{c(bx+a)}\,bc^2}$	46

input
```
int((c*(b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)*(c*(b*x+a)^3)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \sqrt{c(a+bx)^3}\,dx = \frac{2\sqrt{b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c}(bx+a)}{5\,b}$$

input
```
integrate((c*(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \sqrt{c(a+bx)^3}\,dx = \begin{cases} \frac{2\sqrt{c(a+bx)^3}\left(\frac{a}{b}+x\right)}{5} & \text{for } b \neq 0 \\ x\sqrt{a^3 c} & \text{otherwise} \end{cases}$$

input
```
integrate((c*(b*x+a)**3)**(1/2),x)
```

output
```
Piecewise((2*sqrt(c*(a + b*x)**3)*(a/b + x)/5, Ne(b, 0)), (x*sqrt(a**3*c),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sqrt{c(a+bx)^3}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^{\frac{3}{2}}}{5\,b}$$

input
```
integrate((c*(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(21) = 42.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

$$\int \sqrt{c(a+bx)^3}\,dx$$

$$= \frac{2\left(15\sqrt{bcx+ac}a^2\mathrm{sgn}(bx+a) - \frac{10\left(3\sqrt{bcx+ac}ac - (bcx+ac)^{\frac{3}{2}}\right)a\mathrm{sgn}(bx+a)}{c} + \frac{\left(15\sqrt{bcx+ac}a^2c^2 - 10\,(bcx+ac)^{\frac{3}{2}}ac + 3\,(bcx+ac\right.}{c^2}}{15\,b}$$

input
```
integrate((c*(b*x+a)^3)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*c*x + a*c)*a^2*sgn(b*x + a) - 10*(3*sqrt(b*c*x + a*c)*a*c
- (b*c*x + a*c)^(3/2))*a*sgn(b*x + a)/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 -
10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*sgn(b*x + a)/c^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{c(a+bx)^3}\,dx = \sqrt{c\,(a+b\,x)^3}\left(\frac{2\,x}{5}+\frac{2\,a}{5\,b}\right)$$

input
```
int((c*(a + b*x)^3)^(1/2),x)
```

output
```
(c*(a + b*x)^3)^(1/2)*((2*x)/5 + (2*a)/(5*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \sqrt{c(a+bx)^3}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5b}$$

input
```
int((c*(b*x+a)^3)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b)
```

3.15 $\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx$

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Mathematica [A] (verified) . . . 151
Rubi [A] (verified) . . . 152
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Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

output
```
(-2*b*x-2*a)/b/(c*(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

input
```
Integrate[1/Sqrt[c*(a + b*x)^3],x]
```

output
```
(-2*(a + b*x))/(b*Sqrt[c*(a + b*x)^3])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{\sqrt{c(a+bx)^3}}d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)^{3/2}\int \frac{1}{(a+bx)^{3/2}}d(a+bx)}{b\sqrt{c(a+bx)^3}}$$
$$\downarrow 15$$
$$-\frac{2(a+bx)}{b\sqrt{c(a+bx)^3}}$$

input
```
Int[1/Sqrt[c*(a + b*x)^3],x]
```

output
```
(-2*(a + b*x))/(b*Sqrt[c*(a + b*x)^3])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{2(bx+a)}{b\sqrt{c(bx+a)^3}}$	22
orering	$-\frac{2(bx+a)}{b\sqrt{c(bx+a)^3}}$	22
default	$-\frac{2(bx+a)\sqrt{c(bx+a)}}{\sqrt{c(bx+a)^3}\,b\sqrt{bcx+ac}}$	41
trager	$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{c(bx+a)^2b}$	50

input

```
int(1/(c*(b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2*(b*x+a)/b/(c*(b*x+a)^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2\sqrt{b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c}}{b^3cx^2+2\,ab^2cx+a^2bc}$$

input

```
integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

output

```
-2*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^3*c*x^2 + 2*a*
b^2*c*x + a^2*b*c)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = \begin{cases} -\frac{2\left(\frac{a}{b}+x\right)}{\sqrt{c(a+bx)^3}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a^3 c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x+a)**3)**(1/2),x)
```

output
```
Piecewise((-2*(a/b + x)/sqrt(c*(a + b*x)**3), Ne(b, 0)), (x/sqrt(a**3*c),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)}{(bx+a)^{\frac{3}{2}}bc}$$

input
```
integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

output
```
-2*(b*sqrt(c)*x + a*sqrt(c))/((b*x + a)^(3/2)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2}{\sqrt{bcx+ac}b\operatorname{sgn}(bx+a)}$$

input
```
integrate(1/(c*(b*x+a)^3)^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(b*c*x + a*c)*b*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2\sqrt{c\,(a+b\,x)^3}}{b\,c\,(a+b\,x)^2}$$

input
```
int(1/(c*(a + b*x)^3)^(1/2),x)
```

output
```
-(2*(c*(a + b*x)^3)^(1/2))/(b*c*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{c(a+bx)^3}}\,dx = -\frac{2\sqrt{c}}{\sqrt{bx+a}\,bc}$$

input
```
int(1/(c*(b*x+a)^3)^(1/2),x)
```

output
```
( - 2*sqrt(c))/(sqrt(a + b*x)*b*c)
```

3.16 $\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx$

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Giac [A] (verification not implemented) 160
Mupad [B] (verification not implemented) 160
Reduce [B] (verification not implemented) 160

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx = -\frac{2}{7bc(a+bx)^2\sqrt{c(a+bx)^3}}$$

output
```
-2/7/b/c/(b*x+a)^2/(c*(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx = -\frac{2(a+bx)}{7b\left(c(a+bx)^3\right)^{3/2}}$$

input
```
Integrate[(c*(a + b*x)^3)^(-3/2),x]
```

output
```
(-2*(a + b*x))/(7*b*(c*(a + b*x)^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{(a+bx)^{9/2}\int \frac{1}{(a+bx)^{9/2}}\,d(a+bx)}{b\left(c(a+bx)^3\right)^{3/2}}$$

$$\downarrow 15$$

$$-\frac{2(a+bx)}{7b\left(c(a+bx)^3\right)^{3/2}}$$

input
```
Int[(c*(a + b*x)^3)^(-3/2),x]
```

output
```
(-2*(a + b*x))/(7*b*(c*(a + b*x)^3)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2(bx+a)}{7b\left(c(bx+a)^3\right)^{\frac{3}{2}}}$	22
orering	$-\frac{2(bx+a)}{7b\left(c(bx+a)^3\right)^{\frac{3}{2}}}$	22
default	$-\frac{2(bx+a)^3(c(bx+a))^{\frac{3}{2}}c^2}{7\left(c(bx+a)^3\right)^{\frac{3}{2}}b(bcx+ac)^{\frac{7}{2}}}$	46
trager	$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{7c^2(bx+a)^5b}$	50

input

```
int(1/(c*(b*x+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/7*(b*x+a)/b/(c*(b*x+a)^3)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.63

$$\int \frac{1}{\left(c(a+bx)^3\right)^{3/2}}\,dx = -\frac{2\sqrt{b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c}}{7\left(b^6c^2x^5+5\,ab^5c^2x^4+10\,a^2b^4c^2x^3+10\,a^3b^3c^2x^2+5\,a^4b^2c^2x+a^5bc^2\right)}$$

input

```
integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^6*c^2*x^5 +
5*a*b^5*c^2*x^4 + 10*a^2*b^4*c^2*x^3 + 10*a^3*b^3*c^2*x^2 + 5*a^4*b^2*c^2*
x + a^5*b*c^2)
```

Sympy [A] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(c(a+bx)^3)^{3/2}}\,dx = \begin{cases} -\frac{2\left(\frac{a}{b}+x\right)}{7\left(c(a+bx)^3\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(a^3c)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x+a)**3)**(3/2),x)
```

output
```
Piecewise((-2*(a/b + x)/(7*(c*(a + b*x)**3)**(3/2)), Ne(b, 0)), (x/(a**3*c
)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c(a+bx)^3)^{3/2}}\,dx = -\frac{2\sqrt{c}}{7\left(b^3c^2x^2+2\,ab^2c^2x+a^2bc^2\right)(bx+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
-2/7*sqrt(c)/((b^3*c^2*x^2 + 2*a*b^2*c^2*x + a^2*b*c^2)*(b*x + a)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^3)^{3/2}}\,dx = -\frac{2\,c^2}{7\,(bcx+ac)^{\frac{7}{2}}\,b\,\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="giac")
```

output
```
-2/7*c^2/((b*c*x + a*c)^(7/2)*b*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^3)^{3/2}}\,dx = -\frac{2\sqrt{c\,(a+b\,x)^3}}{7\,b\,c^2\,(a+b\,x)^5}$$

input
```
int(1/(c*(a + b*x)^3)^(3/2),x)
```

output
```
-(2*(c*(a + b*x)^3)^(1/2))/(7*b*c^2*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \frac{1}{(c(a+bx)^3)^{3/2}}\,dx = -\frac{2\sqrt{c}}{7\sqrt{bx+a}\,b\,c^2\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int(1/(c*(b*x+a)^3)^(3/2),x)
```

output
```
( - 2*sqrt(c))/(7*sqrt(a + b*x)*b*c**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3))
```

3.17 $\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx$

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Mupad [B] (verification not implemented) 165
Reduce [B] (verification not implemented) 165

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx = -\frac{2}{13bc^2(a+bx)^5\sqrt{c(a+bx)^3}}$$

output
```
-2/13/b/c^2/(b*x+a)^5/(c*(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx = -\frac{2(a+bx)}{13b\left(c(a+bx)^3\right)^{5/2}}$$

input
```
Integrate[(c*(a + b*x)^3)^(-5/2),x]
```

output
```
(-2*(a + b*x))/(13*b*(c*(a + b*x)^3)^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{(c(a+bx)^3)^{5/2}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)^{15/2}\int \frac{1}{(a+bx)^{15/2}}\,d(a+bx)}{b\,(c(a+bx)^3)^{5/2}}$$
$$\downarrow 15$$
$$-\frac{2(a+bx)}{13b\,(c(a+bx)^3)^{5/2}}$$

input
```
Int[(c*(a + b*x)^3)^(-5/2),x]
```

output
```
(-2*(a + b*x))/(13*b*(c*(a + b*x)^3)^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2(bx+a)}{13b\left(c(bx+a)^3\right)^{\frac{5}{2}}}$	22
orering	$-\frac{2(bx+a)}{13b\left(c(bx+a)^3\right)^{\frac{5}{2}}}$	22
default	$-\frac{2(bx+a)^5(c(bx+a))^{\frac{5}{2}}c^4}{13\left(c(bx+a)^3\right)^{\frac{5}{2}}b(bcx+ac)^{\frac{13}{2}}}$	46
trager	$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{13c^3(bx+a)^8b}$	50

input

```
int(1/(c*(b*x+a)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/13*(b*x+a)/b/(c*(b*x+a)^3)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.03

$$\int \frac{1}{\left(c(a+bx)^3\right)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{b^3cx^3+3ab^2cx^2+3a^2bcx+a^3c}}{13\left(b^9c^3x^8+8ab^8c^3x^7+28a^2b^7c^3x^6+56a^3b^6c^3x^5+70a^4b^5c^3x^4+56a^5b^4c^3x^3+28a^6b^3c^3x^2+8a^7b^2c^3x\right.}$$

input

```
integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

output
```
-2/13*sqrt(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/(b^9*c^3*x^8 +
 8*a*b^8*c^3*x^7 + 28*a^2*b^7*c^3*x^6 + 56*a^3*b^6*c^3*x^5 + 70*a^4*b^5*c^
3*x^4 + 56*a^5*b^4*c^3*x^3 + 28*a^6*b^3*c^3*x^2 + 8*a^7*b^2*c^3*x + a^8*b*
c^3)
```

Sympy [A] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx = \begin{cases} -\frac{2\left(\frac{a}{b}+x\right)}{13\left(c(a+bx)^3\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{(a^3c)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x+a)**3)**(5/2),x)
```

output
```
Piecewise((-2*(a/b + x)/(13*(c*(a + b*x)**3)**(5/2)), Ne(b, 0)), (x/(a**3*
c)**(5/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx = -\frac{2\sqrt{c}}{13\left(b^6c^3x^5 + 5\,ab^5c^3x^4 + 10\,a^2b^4c^3x^3 + 10\,a^3b^3c^3x^2 + 5\,a^4b^2c^3x + a^5bc^3\right)(bx+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

output
```
-2/13*sqrt(c)/((b^6*c^3*x^5 + 5*a*b^5*c^3*x^4 + 10*a^2*b^4*c^3*x^3 + 10*a^
3*b^3*c^3*x^2 + 5*a^4*b^2*c^3*x + a^5*b*c^3)*(b*x + a)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx = -\frac{2\,c^4}{13\,(bcx+ac)^{\frac{13}{2}}\,b\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="giac")
```

output
```
-2/13*c^4/((b*c*x + a*c)^(13/2)*b*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx = -\frac{2\sqrt{c\,(a+b\,x)^3}}{13\,b\,c^3\,(a+b\,x)^8}$$

input
```
int(1/(c*(a + b*x)^3)^(5/2),x)
```

output
```
-(2*(c*(a + b*x)^3)^(1/2))/(13*b*c^3*(a + b*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \frac{1}{(c(a+bx)^3)^{5/2}}\,dx = -\frac{2\sqrt{c}}{13\sqrt{bx+a}\,b\,c^3\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6)}$$

input
```
int(1/(c*(b*x+a)^3)^(5/2),x)
```

output

```
( - 2*sqrt(c))/(13*sqrt(a + b*x)*b*c**3*(a**6 + 6*a**5*b*x + 15*a**4*b**2*
x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.18 $\int \left(\frac{c}{a+bx}\right)^{5/2} dx$

Optimal result 167
Mathematica [A] (verified) 167
Rubi [A] (verified) 168
Maple [A] (verified) 169
Fricas [A] (verification not implemented) 169
Sympy [B] (verification not implemented) 170
Maxima [A] (verification not implemented) 170
Giac [A] (verification not implemented) 170
Mupad [B] (verification not implemented) 171
Reduce [B] (verification not implemented) 171

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2c\left(\frac{c}{a+bx}\right)^{3/2}}{3b}$$

output
```
-2/3*c*(c/(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2c\left(\frac{c}{a+bx}\right)^{3/2}}{3b}$$

input
```
Integrate[(c/(a + b*x))^(5/2),x]
```

output
```
(-2*c*(c/(a + b*x))^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{a+bx}\right)^{5/2} d(a+bx)}{b}$$

$$\downarrow 19$$

$$-\frac{2c\left(\frac{c}{a+bx}\right)^{3/2}}{3b}$$

input
```
Int[(c/(a + b*x))^(5/2),x]
```

output
```
(-2*c*(c/(a + b*x))^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}{3b}$	22
default	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}{3b}$	22
orering	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}{3b}$	22
trager	$-\frac{2c^2\sqrt{\frac{c}{bx+a}}}{3b(bx+a)}$	27

input
```
int((c/(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(c/(b*x+a))^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2\,c^2\sqrt{\frac{c}{bx+a}}}{3\,(b^2x+ab)}$$

input
```
integrate((c/(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*c^2*sqrt(c/(b*x + a))/(b^2*x + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = \begin{cases} -\frac{2a\left(\frac{c}{a+bx}\right)^{\frac{5}{2}}}{3b} - \frac{2x\left(\frac{c}{a+bx}\right)^{\frac{5}{2}}}{3} & \text{for } b \neq 0 \\ x\left(\frac{c}{a}\right)^{\frac{5}{2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a))**(5/2),x)
```

output
```
Piecewise((-2*a*(c/(a + b*x))**(5/2)/(3*b) - 2*x*(c/(a + b*x))**(5/2)/3, N
e(b, 0)), (x*(c/a)**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2\,c\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((c/(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3*c*(c/(b*x + a))^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2\,c^4 \mathrm{sgn}(bx+a)}{3\,(bcx+ac)^{\frac{3}{2}}b}$$

input
```
integrate((c/(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/3*c^4*sgn(b*x + a)/((b*c*x + a*c)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2\,c\left(\frac{c}{a+b\,x}\right)^{3/2}}{3\,b}$$

input
```
int((c/(a + b*x))^(5/2),x)
```

output
```
-(2*c*(c/(a + b*x))^(3/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \left(\frac{c}{a+bx}\right)^{5/2} dx = -\frac{2\sqrt{c}\,c^2}{3\sqrt{bx+a}\,b\,(bx+a)}$$

input
```
int((c/(b*x+a))^(5/2),x)
```

output
```
( - 2*sqrt(c)*c**2)/(3*sqrt(a + b*x)*b*(a + b*x))
```

3.19 $\int \left(\frac{c}{a+bx}\right)^{3/2} dx$

Optimal result . . . 172
Mathematica [A] (verified) . . . 172
Rubi [A] (verified) . . . 173
Maple [A] (verified) . . . 174
Fricas [A] (verification not implemented) . . . 174
Sympy [B] (verification not implemented) . . . 175
Maxima [A] (verification not implemented) . . . 175
Giac [A] (verification not implemented) . . . 175
Mupad [B] (verification not implemented) . . . 176
Reduce [B] (verification not implemented) . . . 176

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

output
```
-2*c*(c/(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

input
```
Integrate[(c/(a + b*x))^(3/2),x]
```

output
```
(-2*c*Sqrt[c/(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{a+bx}\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 19$$

$$-\frac{2c\sqrt{\frac{c}{a+bx}}}{b}$$

input
```
Int[(c/(a + b*x))^(3/2),x]
```

output
```
(-2*c*Sqrt[c/(a + b*x)])/b
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
trager	$-\frac{2c\sqrt{\frac{c}{bx+a}}}{b}$	18
gosper	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{b}$	22
default	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{b}$	22
orering	$-\frac{2(bx+a)\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}{b}$	22

input
```
int((c/(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(c/(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2\,c\sqrt{\frac{c}{bx+a}}}{b}$$

input
```
integrate((c/(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*c*sqrt(c/(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = \begin{cases} -\frac{2a\left(\frac{c}{a+bx}\right)^{\frac{3}{2}}}{b} - 2x\left(\frac{c}{a+bx}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ x\left(\frac{c}{a}\right)^{\frac{3}{2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a))**(3/2),x)
```

output
```
Piecewise((-2*a*(c/(a + b*x))**(3/2)/b - 2*x*(c/(a + b*x))**(3/2), Ne(b, 0
)), (x*(c/a)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2\,c\sqrt{\frac{c}{bx+a}}}{b}$$

input
```
integrate((c/(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2*c*sqrt(c/(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2\,c^2\mathrm{sgn}(bx+a)}{\sqrt{bcx+acb}}$$

input
```
integrate((c/(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2*c^2*sgn(b*x + a)/(sqrt(b*c*x + a*c)*b)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2\,c\,\sqrt{\frac{c}{a+b\,x}}}{b}$$

input
```
int((c/(a + b*x))^(3/2),x)
```

output
```
-(2*c*(c/(a + b*x))^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(\frac{c}{a+bx}\right)^{3/2} dx = -\frac{2\sqrt{c}\,c}{\sqrt{bx+a}\,b}$$

input
```
int((c/(b*x+a))^(3/2),x)
```

output
```
( - 2*sqrt(c)*c)/(sqrt(a + b*x)*b)
```

3.20 $\int \sqrt{\frac{c}{a+bx}}\,dx$

Optimal result . . . 177
Mathematica [A] (verified) . . . 177
Rubi [A] (verified) . . . 178
Maple [A] (verified) . . . 179
Fricas [A] (verification not implemented) . . . 179
Sympy [B] (verification not implemented) . . . 180
Maxima [A] (verification not implemented) . . . 180
Giac [A] (verification not implemented) . . . 180
Mupad [B] (verification not implemented) . . . 181
Reduce [B] (verification not implemented) . . . 181

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2c}{b\sqrt{\frac{c}{a+bx}}}$$

output
```
2*c/b/(c/(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2c}{b\sqrt{\frac{c}{a+bx}}}$$

input
```
Integrate[Sqrt[c/(a + b*x)],x]
```

output
```
(2*c)/(b*Sqrt[c/(a + b*x)])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{c}{a+bx}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \sqrt{\frac{c}{a+bx}}\,d(a+bx)}{b}$$
$$\downarrow 19$$
$$\frac{2c}{b\sqrt{\frac{c}{a+bx}}}$$

input
```
Int[Sqrt[c/(a + b*x)],x]
```

output
```
(2*c)/(b*Sqrt[c/(a + b*x)])
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$	22
default	$\frac{2(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$	22
trager	$\frac{2(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$	22
risch	$\frac{2(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$	22
orering	$\frac{2(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$	22

input
```
int((c/(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*(c/(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2\,(bx+a)\sqrt{\frac{c}{bx+a}}}{b}$$

input
```
integrate((c/(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2*(b*x + a)*sqrt(c/(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \begin{cases} \frac{2a\sqrt{\frac{c}{a+bx}}}{b} + 2x\sqrt{\frac{c}{a+bx}} & \text{for } b \neq 0 \\ x\sqrt{\frac{c}{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a))**(1/2),x)
```

output
```
Piecewise((2*a*sqrt(c/(a + b*x))/b + 2*x*sqrt(c/(a + b*x)), Ne(b, 0)), (x*
sqrt(c/a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2\,c}{b\sqrt{\frac{c}{bx+a}}}$$

input
```
integrate((c/(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2*c/(b*sqrt(c/(b*x + a)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2\sqrt{bcx+ac}\operatorname{sgn}(bx+a)}{b}$$

input
```
integrate((c/(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*c*x + a*c)*sgn(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \sqrt{\frac{c}{a+b\,x}}\left(2\,x+\frac{2\,a}{b}\right)$$

input
```
int((c/(a + b*x))^(1/2),x)
```

output
```
(c/(a + b*x))^(1/2)*(2*x + (2*a)/b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \sqrt{\frac{c}{a+bx}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}}{b}$$

input
```
int((c/(b*x+a))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x))/b
```

3.21 $\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx$

Optimal result 182
Mathematica [A] (verified) 182
Rubi [A] (verified) 183
Maple [A] (verified) 184
Fricas [B] (verification not implemented) 184
Sympy [B] (verification not implemented) 185
Maxima [A] (verification not implemented) 185
Giac [B] (verification not implemented) 185
Mupad [B] (verification not implemented) 186
Reduce [B] (verification not implemented) 186

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2c}{3b\left(\frac{c}{a+bx}\right)^{3/2}}$$

output
```
2/3*c/b/(c/(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2c}{3b\left(\frac{c}{a+bx}\right)^{3/2}}$$

input
```
Integrate[1/Sqrt[c/(a + b*x)],x]
```

output
```
(2*c)/(3*b*(c/(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{\sqrt{\frac{c}{a+bx}}} d(a+bx)}{b}$$
$$\downarrow 19$$
$$\frac{2c}{3b\left(\frac{c}{a+bx}\right)^{3/2}}$$

input
```
Int[1/Sqrt[c/(a + b*x)],x]
```

output
```
(2*c)/(3*b*(c/(a + b*x))^(3/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\frac{2a}{3}+\frac{2bx}{3}}{b\sqrt{\frac{c}{bx+a}}}$	22
default	$\frac{\frac{2a}{3}+\frac{2bx}{3}}{b\sqrt{\frac{c}{bx+a}}}$	22
risch	$\frac{\frac{2a}{3}+\frac{2bx}{3}}{b\sqrt{\frac{c}{bx+a}}}$	22
orering	$\frac{\frac{2a}{3}+\frac{2bx}{3}}{b\sqrt{\frac{c}{bx+a}}}$	22
trager	$\frac{2\sqrt{\frac{c}{bx+a}}\,(bx+a)^2}{3cb}$	27

input
```
int(1/(c/(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)/b/(c/(b*x+a))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{\frac{c}{bx+a}}}{3\,bc}$$

input
```
integrate(1/(c/(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(c/(b*x + a))/(b*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \begin{cases} \frac{2a}{3b\sqrt{\frac{c}{a+bx}}} + \frac{2x}{3\sqrt{\frac{c}{a+bx}}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\frac{c}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a))**(1/2),x)
```

output
```
Piecewise((2*a/(3*b*sqrt(c/(a + b*x))) + 2*x/(3*sqrt(c/(a + b*x))), Ne(b,
0)), (x/sqrt(c/a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2\,c}{3\,b\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
2/3*c/(b*(c/(b*x + a))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.95

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2\left(3\sqrt{bcx+ac}a - \frac{3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(1/(c/(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2
))/c)/(b*c*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \sqrt{\frac{c}{a+b\,x}}\left(\frac{2\,b\,x^2}{3\,c}+\frac{2\,a^2}{3\,b\,c}+\frac{4\,a\,x}{3\,c}\right)$$

input
```
int(1/(c/(a + b*x))^(1/2),x)
```

output
```
(c/(a + b*x))^(1/2)*((2*b*x^2)/(3*c) + (2*a^2)/(3*b*c) + (4*a*x)/(3*c))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{c}{a+bx}}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(bx+a)}{3bc}$$

input
```
int(1/(c/(b*x+a))^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a + b*x))/(3*b*c)
```

3.22 $\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx$

Optimal result 187
Mathematica [A] (verified) 187
Rubi [A] (verified) 188
Maple [A] (verified) 189
Fricas [B] (verification not implemented) 189
Sympy [B] (verification not implemented) 190
Maxima [A] (verification not implemented) 190
Giac [B] (verification not implemented) 190
Mupad [B] (verification not implemented) 191
Reduce [B] (verification not implemented) 191

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \frac{2c}{5b\left(\frac{c}{a+bx}\right)^{5/2}}$$

output
```
2/5*c/b/(c/(b*x+a))^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \frac{2c}{5b\left(\frac{c}{a+bx}\right)^{5/2}}$$

input
```
Integrate[(c/(a + b*x))^(-3/2),x]
```

output
```
(2*c)/(5*b*(c/(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}} d(a+bx)}{b}$$

$$\downarrow 19$$

$$\frac{2c}{5b\left(\frac{c}{a+bx}\right)^{5/2}}$$

input
```
Int[(c/(a + b*x))^(-3/2),x]
```

output
```
(2*c)/(5*b*(c/(a + b*x))^(5/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}$	22
default	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}$	22
orering	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\left(\frac{c}{bx+a}\right)^{\frac{3}{2}}}$	22
risch	$\frac{\frac{2}{5}a^2+\frac{4}{5}abx+\frac{2}{5}b^2x^2}{c\sqrt{\frac{c}{bx+a}}\,b}$	36
trager	$\frac{2(bx+a)(b^2x^2+2abx+a^2)\sqrt{\frac{c}{bx+a}}}{5c^2b}$	41

input
```
int(1/(c/(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)/b/(c/(b*x+a))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{\frac{c}{bx+a}}}{5\,bc^2}$$

input
```
integrate(1/(c/(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(c/(b*x + a))/(b*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \begin{cases} \frac{2a}{5b\left(\frac{c}{a+bx}\right)^{\frac{3}{2}}} + \frac{2x}{5\left(\frac{c}{a+bx}\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a))**(3/2),x)
```

output
```
Piecewise((2*a/(5*b*(c/(a + b*x))**(3/2)) + 2*x/(5*(c/(a + b*x))**(3/2)),
Ne(b, 0)), (x/(c/a)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \frac{2\,c}{5\,b\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(c/(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/5*c/(b*(c/(b*x + a))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.43

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{3/2}}\,dx = \frac{2\left(15\sqrt{bcx+ac}a^2 - \frac{10\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c} + \frac{15\sqrt{bcx+ac}a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc^2\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(1/(c/(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*
c)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c
 + 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^2*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int \frac{1}{\left(\frac{c}{a+b\,x}\right)^{3/2}}\,dx = \sqrt{\frac{c}{a+b\,x}}\left(\frac{6\,a^2\,x}{5\,c^2}+\frac{2\,a^3}{5\,b\,c^2}+\frac{2\,b^2\,x^3}{5\,c^2}+\frac{6\,a\,b\,x^2}{5\,c^2}\right)$$

input
```
int(1/(c/(a + b*x))^(3/2),x)
```

output
```
(c/(a + b*x))^(1/2)*((6*a^2*x)/(5*c^2) + (2*a^3)/(5*b*c^2) + (2*b^2*x^3)/(
5*c^2) + (6*a*b*x^2)/(5*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(\frac{c}{a+b\,x}\right)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x+a}\,(b^2x^2+2abx+a^2)}{5b\,c^2}$$

input
```
int(1/(c/(b*x+a))^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b*c**2)
```

3.23 $\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx$

Optimal result 192
Mathematica [A] (verified) 192
Rubi [A] (verified) 193
Maple [A] (verified) 194
Fricas [B] (verification not implemented) 194
Sympy [B] (verification not implemented) 195
Maxima [A] (verification not implemented) 195
Giac [B] (verification not implemented) 195
Mupad [B] (verification not implemented) 196
Reduce [B] (verification not implemented) 196

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2c}{7b\left(\frac{c}{a+bx}\right)^{7/2}}$$

output
```
2/7*c/b/(c/(b*x+a))^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2c}{7b\left(\frac{c}{a+bx}\right)^{7/2}}$$

input
```
Integrate[(c/(a + b*x))^(-5/2),x]
```

output
```
(2*c)/(7*b*(c/(a + b*x))^(7/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {239, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}} d(a+bx)}{b}$$

$$\downarrow 19$$

$$\frac{2c}{7b\left(\frac{c}{a+bx}\right)^{7/2}}$$

input
```
Int[(c/(a + b*x))^(-5/2),x]
```

output
```
(2*c)/(7*b*(c/(a + b*x))^(7/2))
```

Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\frac{2bx}{7}+\frac{2a}{7}}{b\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}$	22
default	$\frac{\frac{2bx}{7}+\frac{2a}{7}}{b\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}$	22
orering	$\frac{\frac{2bx}{7}+\frac{2a}{7}}{b\left(\frac{c}{bx+a}\right)^{\frac{5}{2}}}$	22
risch	$\frac{\frac{2}{7}a^3+\frac{6}{7}a^2bx+\frac{6}{7}ab^2x^2+\frac{2}{7}b^3x^3}{c^2\sqrt{\frac{c}{bx+a}}\,b}$	47
trager	$\frac{2(bx+a)\left(b^3x^3+3ab^2x^2+3a^2bx+a^3\right)\sqrt{\frac{c}{bx+a}}}{7c^3b}$	52

input
```
int(1/(c/(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x+a)/b/(c/(b*x+a))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.71

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2\left(b^4x^4+4ab^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)\sqrt{\frac{c}{bx+a}}}{7bc^3}$$

input
```
integrate(1/(c/(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/7*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(c/(b*x
+ a))/(b*c^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \begin{cases} \frac{2a}{7b\left(\frac{c}{a+bx}\right)^{\frac{5}{2}}} + \frac{2x}{7\left(\frac{c}{a+bx}\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*a/(7*b*(c/(a + b*x))**(5/2)) + 2*x/(7*(c/(a + b*x))**(5/2)),
Ne(b, 0)), (x/(c/a)**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2\,c}{7\,b\left(\frac{c}{bx+a}\right)^{\frac{7}{2}}}$$

input
```
integrate(1/(c/(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/7*c/(b*(c/(b*x + a))^(7/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2\left(35\sqrt{bcx+ac}a^3 - \frac{35\left(3\sqrt{bcx+ac}ac-(bcx+ac)^{\frac{3}{2}}\right)a^2}{c} + \frac{7\left(15\sqrt{bcx+ac}a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right.}{c^2}\right.}{35\,bc^3\mathrm{sgn}\,(bx+a)}$$

input
```
integrate(1/(c/(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*
c)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)
*a*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(
b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^
(7/2))/c^3)/(b*c^3*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2\sqrt{\frac{c}{a+b\,x}}\,(a+b\,x)^4}{7\,b\,c^3}$$

input
```
int(1/(c/(a + b*x))^(5/2),x)
```

output
```
(2*(c/(a + b*x))^(1/2)*(a + b*x)^4)/(7*b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(\frac{c}{a+bx}\right)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}{7b\,c^3}$$

input
```
int(1/(c/(b*x+a))^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/
(7*b*c**3)
```

3.24 $\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx$

Optimal result . . . 197
Mathematica [A] (verified) . . . 197
Rubi [A] (verified) . . . 198
Maple [A] (verified) . . . 199
Fricas [B] (verification not implemented) . . . 199
Sympy [A] (verification not implemented) . . . 200
Maxima [A] (verification not implemented) . . . 200
Giac [A] (verification not implemented) . . . 201
Mupad [B] (verification not implemented) . . . 201
Reduce [B] (verification not implemented) . . . 201

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{c^2\sqrt{\frac{c}{(a+bx)^2}}}{4b(a+bx)^3}$$

output
```
-1/4*c^2*(c/(b*x+a)^2)^(1/2)/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{\left(\frac{c}{(a+bx)^2}\right)^{5/2}(a+bx)}{4b}$$

input
```
Integrate[(c/(a + b*x)^2)^(5/2),x]
```

output
```
-1/4*((c/(a + b*x)^2)^(5/2)*(a + b*x))/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{(a+bx)^5 \left(\frac{c}{(a+bx)^2}\right)^{5/2} \int \frac{1}{(a+bx)^5} d(a+bx)}{b}$$

$$\downarrow 15$$

$$-\frac{(a+bx)\left(\frac{c}{(a+bx)^2}\right)^{5/2}}{4b}$$

input
```
Int[(c/(a + b*x)^2)^(5/2),x]
```

output
```
-1/4*((c/(a + b*x)^2)^(5/2)*(a + b*x))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}{4b}$	22
default	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}{4b}$	22
orering	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}{4b}$	22
risch	$-\frac{c^2\sqrt{\frac{c}{(bx+a)^2}}}{4b(bx+a)^3}$	27
trager	$\frac{c^2\left(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3\right)x\sqrt{\frac{c}{b^2x^2+2abx+a^2}}}{4a^4(bx+a)^3}$	68

input

```
int((c/(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/4*(b*x+a)/b*(c/(b*x+a)^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{c^2\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{4\left(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b\right)}$$

input

```
integrate((c/(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/4*c^2*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*
b^2*x + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = \begin{cases} -\frac{\left(\frac{c}{(a+bx)^2}\right)^{\frac{5}{2}}\left(\frac{a}{b}+x\right)}{4} & \text{for } b \neq 0 \\ x\left(\frac{c}{a^2}\right)^{\frac{5}{2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a)**2)**(5/2),x)
```

output
```
Piecewise((-(c/(a + b*x)**2)**(5/2)*(a/b + x)/4, Ne(b, 0)), (x*(c/a**2)**(
5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{c^{\frac{5}{2}}}{4\left(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b\right)}$$

input
```
integrate((c/(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*c^(5/2)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{c^{\frac{5}{2}}\operatorname{sgn}(bx+a)}{4\,(bx+a)^4 b}$$

input
```
integrate((c/(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*c^(5/2)*sgn(b*x + a)/((b*x + a)^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{c^2\sqrt{\frac{c}{(a+b\,x)^2}}}{4\,b\,(a+b\,x)^3}$$

input
```
int((c/(a + b*x)^2)^(5/2),x)
```

output
```
-(c^2*(c/(a + b*x)^2)^(1/2))/(4*b*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \left(\frac{c}{(a+bx)^2}\right)^{5/2} dx = -\frac{\sqrt{c}\,c^2}{4b\,(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)}$$

input
```
int((c/(b*x+a)^2)^(5/2),x)
```

output
```
( - sqrt(c)*c**2)/(4*b*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
**3 + b**4*x**4))
```

3.25 $\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx$

Optimal result 202
Mathematica [A] (verified) 202
Rubi [A] (verified) 203
Maple [A] (verified) 204
Fricas [A] (verification not implemented) 204
Sympy [A] (verification not implemented) 205
Maxima [A] (verification not implemented) 205
Giac [A] (verification not implemented) 205
Mupad [B] (verification not implemented) 206
Reduce [B] (verification not implemented) 206

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{(a+bx)^2}}}{2b(a+bx)}$$

output
```
-1/2*c*(c/(b*x+a)^2)^(1/2)/b/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{\left(\frac{c}{(a+bx)^2}\right)^{3/2}(a+bx)}{2b}$$

input
```
Integrate[(c/(a + b*x)^2)^(3/2),x]
```

output
```
-1/2*((c/(a + b*x)^2)^(3/2)*(a + b*x))/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{(a+bx)^3 \left(\frac{c}{(a+bx)^2}\right)^{3/2} \int \frac{1}{(a+bx)^3} d(a+bx)}{b}$$

$$\downarrow 15$$

$$-\frac{(a+bx)\left(\frac{c}{(a+bx)^2}\right)^{3/2}}{2b}$$

input
```
Int[(c/(a + b*x)^2)^(3/2),x]
```

output
```
-1/2*((c/(a + b*x)^2)^(3/2)*(a + b*x))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}{2b}$	22
default	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}{2b}$	22
orering	$-\frac{(bx+a)\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}{2b}$	22
risch	$-\frac{c\sqrt{\frac{c}{(bx+a)^2}}}{2b(bx+a)}$	25
trager	$\frac{c(bx+2a)x\sqrt{\frac{c}{b^2x^2+2abx+a^2}}}{2a^2(bx+a)}$	44

input
```
int((c/(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x+a)/b*(c/(b*x+a)^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{2\,(b^2x+ab)}$$

input
```
integrate((c/(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*c*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = \begin{cases} -\frac{\left(\frac{c}{(a+bx)^2}\right)^{\frac{3}{2}}\left(\frac{a}{b}+x\right)}{2} & \text{for } b \neq 0 \\ x\left(\frac{c}{a^2}\right)^{\frac{3}{2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a)**2)**(3/2),x)
```

output
```
Piecewise((-(c/(a + b*x)**2)**(3/2)*(a/b + x)/2, Ne(b, 0)), (x*(c/a**2)**(
3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{c^{\frac{3}{2}}}{2\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

input
```
integrate((c/(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*c^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{c^{\frac{3}{2}}\operatorname{sgn}(bx+a)}{2\left(bx+a\right)^2 b}$$

input
```
integrate((c/(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*c^(3/2)*sgn(b*x + a)/((b*x + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{(a+b\,x)^2}}}{2\,b\,(a+b\,x)}$$

input
```
int((c/(a + b*x)^2)^(3/2),x)
```

output
```
-(c*(c/(a + b*x)^2)^(1/2))/(2*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(\frac{c}{(a+bx)^2}\right)^{3/2} dx = -\frac{\sqrt{c}\,c}{2b\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((c/(b*x+a)^2)^(3/2),x)
```

output
```
( - sqrt(c)*c)/(2*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.26 $\int \sqrt{\frac{c}{(a+bx)^2}}\,dx$

Optimal result . . . 207
Mathematica [A] (verified) . . . 207
Rubi [A] (verified) . . . 208
Maple [A] (verified) . . . 209
Fricas [A] (verification not implemented) . . . 209
Sympy [A] (verification not implemented) . . . 210
Maxima [A] (verification not implemented) . . . 210
Giac [A] (verification not implemented) . . . 210
Mupad [F(-1)] . . . 211
Reduce [B] (verification not implemented) . . . 211

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{\sqrt{\frac{c}{(a+bx)^2}}(a+bx)\log(a+bx)}{b}$$

output
```
(c/(b*x+a)^2)^(1/2)*(b*x+a)*ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{\sqrt{\frac{c}{(a+bx)^2}}(a+bx)\log(a+bx)}{b}$$

input
```
Integrate[Sqrt[c/(a + b*x)^2],x]
```

output
```
(Sqrt[c/(a + b*x)^2]*(a + b*x)*Log[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \sqrt{\frac{c}{(a+bx)^2}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)\sqrt{\frac{c}{(a+bx)^2}}\int \frac{1}{a+bx}\,d(a+bx)}{b}$$
$$\downarrow 14$$
$$\frac{(a+bx)\sqrt{\frac{c}{(a+bx)^2}}\log(a+bx)}{b}$$

input
```
Int[Sqrt[c/(a + b*x)^2],x]
```

output
```
(Sqrt[c/(a + b*x)^2]*(a + b*x)*Log[a + b*x])/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{\frac{c}{(bx+a)^2}}\,(bx+a)\ln(bx+a)}{b}$	27
risch	$\frac{\sqrt{\frac{c}{(bx+a)^2}}\,(bx+a)\ln(bx+a)}{b}$	27

input
```
int((c/(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c/(b*x+a)^2)^(1/2)*(b*x+a)*ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{(bx+a)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}\log(bx+a)}{b}$$

input
```
integrate((c/(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
(b*x + a)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))*log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \begin{cases} \sqrt{\frac{c}{(a+bx)^2}}\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right) & \text{for } b \neq 0 \\ x\sqrt{\frac{c}{a^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise((sqrt(c/(a + b*x)**2)*(a/b + x)*log(a/b + x), Ne(b, 0)), (x*sqrt
(c/a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.46

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{\sqrt{c}\log\left(bx+a\right)}{b}$$

input
```
integrate((c/(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(c)*log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{\sqrt{c}\log\left(|bx+a|\right)\operatorname{sgn}(bx+a)}{b}$$

input
```
integrate((c/(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c)*log(abs(b*x + a))*sgn(b*x + a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \int \sqrt{\frac{c}{(a+b\,x)^2}}\,dx$$

input
```
int((c/(a + b*x)^2)^(1/2),x)
```

output
```
int((c/(a + b*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \sqrt{\frac{c}{(a+bx)^2}}\,dx = \frac{\sqrt{c}\log(bx+a)}{b}$$

input
```
int((c/(b*x+a)^2)^(1/2),x)
```

output
```
(sqrt(c)*log(a + b*x))/b
```

3.27 $\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx$

Optimal result . . . 212
Mathematica [A] (verified) . . . 212
Rubi [A] (verified) . . . 213
Maple [A] (verified) . . . 214
Fricas [B] (verification not implemented) . . . 215
Sympy [A] (verification not implemented) . . . 215
Maxima [A] (verification not implemented) . . . 215
Giac [F(-2)] . . . 216
Mupad [B] (verification not implemented) . . . 216
Reduce [B] (verification not implemented) . . . 217

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \frac{a+bx}{2b\sqrt{\frac{c}{(a+bx)^2}}}$$

output
```
1/2*(b*x+a)/b/(c/(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \frac{x(2a+bx)}{2\sqrt{\frac{c}{(a+bx)^2}}(a+bx)}$$

input
```
Integrate[1/Sqrt[c/(a + b*x)^2],x]
```

output
```
(x*(2*a + b*x))/(2*Sqrt[c/(a + b*x)^2]*(a + b*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)\,d(a+bx)}{b(a+bx)\sqrt{\frac{c}{(a+bx)^2}}}$$

$$\downarrow 15$$

$$\frac{a+bx}{2b\sqrt{\frac{c}{(a+bx)^2}}}$$

input
```
Int[1/Sqrt[c/(a + b*x)^2],x]
```

output
```
(a + b*x)/(2*b*Sqrt[c/(a + b*x)^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{x(bx+2a)}{2(bx+a)\sqrt{\frac{c}{(bx+a)^2}}}$	29
default	$\frac{x(bx+2a)}{2(bx+a)\sqrt{\frac{c}{(bx+a)^2}}}$	29
orering	$\frac{x(bx+2a)}{2(bx+a)\sqrt{\frac{c}{(bx+a)^2}}}$	29
trager	$\frac{(bx+a)(bx+2a)x\sqrt{\frac{c}{b^2x^2+2abx+a^2}}}{2c}$	41
risch	$\frac{b\,x^2}{2\sqrt{\frac{c}{(bx+a)^2}}\,(bx+a)}+\frac{ax}{\sqrt{\frac{c}{(bx+a)^2}}\,(bx+a)}$	47

input
```
int(1/(c/(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)/(b*x+a)/(c/(b*x+a)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \frac{(b^2x^3+3\,abx^2+2\,a^2x)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{2\,c}$$

input
```
integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^3 + 3*a*b*x^2 + 2*a^2*x)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/c
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \begin{cases} \frac{\frac{a}{b}+x}{2\sqrt{\frac{c}{(a+bx)^2}}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\frac{c}{a^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**2)**(1/2),x)
```

output
```
Piecewise(((a/b + x)/(2*sqrt(c/(a + b*x)**2)), Ne(b, 0)), (x/sqrt(c/a**2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \frac{bx^2+2\,ax}{2\sqrt{c}}$$

input
```
integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)/sqrt(c)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c/(b*x+a)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \sqrt{\frac{c}{(a+b\,x)^2}}\left(\frac{a^2\,x}{c}+\frac{b^2\,x^3}{2\,c}+\frac{3\,a\,b\,x^2}{2\,c}\right)$$

input
```
int(1/(c/(a + b*x)^2)^(1/2),x)
```

output
```
(c/(a + b*x)^2)^(1/2)*((a^2*x)/c + (b^2*x^3)/(2*c) + (3*a*b*x^2)/(2*c))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^2}}}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2c}$$

input
```
int(1/(c/(b*x+a)^2)^(1/2),x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/(2*c)
```

3.28 $\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx$

Optimal result 218
Mathematica [A] (verified) 218
Rubi [A] (verified) 219
Maple [A] (verified) 220
Fricas [B] (verification not implemented) 221
Sympy [A] (verification not implemented) 221
Maxima [A] (verification not implemented) 222
Giac [A] (verification not implemented) 222
Mupad [B] (verification not implemented) 222
Reduce [B] (verification not implemented) 223

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{(a+bx)^3}{4bc\sqrt{\frac{c}{(a+bx)^2}}}$$

output
```
1/4*(b*x+a)^3/b/c/(c/(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{a+bx}{4b\left(\frac{c}{(a+bx)^2}\right)^{3/2}}$$

input
```
Integrate[(c/(a + b*x)^2)^(-3/2),x]
```

output
```
(a + b*x)/(4*b*(c/(a + b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)^3 d(a+bx)}{b(a+bx)^3 \left(\frac{c}{(a+bx)^2}\right)^{3/2}}$$

$$\downarrow 15$$

$$\frac{a+bx}{4b\left(\frac{c}{(a+bx)^2}\right)^{3/2}}$$

input
```
Int[(c/(a + b*x)^2)^(-3/2),x]
```

output
```
(a + b*x)/(4*b*(c/(a + b*x)^2)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{bx+a}{4\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}b}$	22
risch	$\frac{(bx+a)^3}{4bc\sqrt{\frac{c}{(bx+a)^2}}}$	27
gosper	$\frac{x\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)}{4(bx+a)^3\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}$	51
orering	$\frac{x\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)}{4(bx+a)^3\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}}$	51
trager	$\frac{(bx+a)\left(b^3x^3+4ab^2x^2+6a^2bx+4a^3\right)x\sqrt{\frac{c}{b^2x^2+2abx+a^2}}}{4c^2}$	63

input
```
int(1/(c/(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(c/(b*x+a)^2)^(3/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.33

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{(b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 4\,a^4x)\sqrt{\frac{c}{b^2x^2+2\,abx+a^2}}}{4\,c^2}$$

input
```
integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 4*a^4*x)*sqrt
(c/(b^2*x^2 + 2*a*b*x + a^2))/c^2
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \begin{cases} \frac{\frac{a}{b}+x}{4\left(\frac{c}{(a+bx)^2}\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a^2}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**2)**(3/2),x)
```

output
```
Piecewise(((a/b + x)/(4*(c/(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(c/a**2)**
(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{2\,(bx^2 + 2\,ax)a^2 + (bx^2 + 2\,ax)^2 b}{4\,c^{\frac{3}{2}}\mathrm{sgn}\,(bx + a)}$$

input
```
integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*(2*(b*x^2 + 2*a*x)*a^2 + (b*x^2 + 2*a*x)^2*b)/(c^(3/2)*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.33

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \sqrt{\frac{c}{(a+b\,x)^2}}\left(\frac{a^4\,x}{c^2} + \frac{b^4\,x^5}{4\,c^2} + \frac{5\,a^3\,b\,x^2}{2\,c^2} + \frac{5\,a\,b^3\,x^4}{4\,c^2} + \frac{5\,a^2\,b^2\,x^3}{2\,c^2}\right)$$

input
```
int(1/(c/(a + b*x)^2)^(3/2),x)
```

output
```
(c/(a + b*x)^2)^(1/2)*((a^4*x)/c^2 + (b^4*x^5)/(4*c^2) + (5*a^3*b*x^2)/(2*
c^2) + (5*a*b^3*x^4)/(4*c^2) + (5*a^2*b^2*x^3)/(2*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}\,x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4c^2}$$

input
```
int(1/(c/(b*x+a)^2)^(3/2),x)
```

output
```
(sqrt(c)*x*(4*a**3 + 6*a**2*b*x + 4*a*b**2*x**2 + b**3*x**3))/(4*c**2)
```

3.29 $\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx$

Optimal result . . . 224
Mathematica [A] (verified) . . . 224
Rubi [A] (verified) . . . 225
Maple [A] (verified) . . . 226
Fricas [B] (verification not implemented) . . . 227
Sympy [A] (verification not implemented) . . . 227
Maxima [B] (verification not implemented) . . . 228
Giac [B] (verification not implemented) . . . 228
Mupad [B] (verification not implemented) . . . 229
Reduce [B] (verification not implemented) . . . 229

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{(a+bx)^5}{6bc^2\sqrt{\frac{c}{(a+bx)^2}}}$$

output
```
1/6*(b*x+a)^5/b/c^2/(c/(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{a+bx}{6b\left(\frac{c}{(a+bx)^2}\right)^{5/2}}$$

input
```
Integrate[(c/(a + b*x)^2)^(-5/2),x]
```

output
```
(a + b*x)/(6*b*(c/(a + b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)^5 d(a+bx)}{b(a+bx)^5 \left(\frac{c}{(a+bx)^2}\right)^{5/2}}$$

$$\downarrow 15$$

$$\frac{a+bx}{6b\left(\frac{c}{(a+bx)^2}\right)^{5/2}}$$

input
```
Int[(c/(a + b*x)^2)^(-5/2),x]
```

output
```
(a + b*x)/(6*b*(c/(a + b*x)^2)^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{bx+a}{6\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}b}$	22
risch	$\frac{(bx+a)^5}{6bc^2\sqrt{\frac{c}{(bx+a)^2}}}$	27
gosper	$\frac{x\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)}{6(bx+a)^5\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}$	73
orering	$\frac{x\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)}{6(bx+a)^5\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}}$	73
trager	$\frac{(bx+a)\left(b^5x^5+6ab^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5\right)x\sqrt{\frac{c}{b^2x^2+2abx+a^2}}}{6c^3}$	85

input
```
int(1/(c/(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(c/(b*x+a)^2)^(5/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.07

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{(b^6x^7 + 7\,ab^5x^6 + 21\,a^2b^4x^5 + 35\,a^3b^3x^4 + 35\,a^4b^2x^3 + 21\,a^5bx^2 + 6\,a^6x)\sqrt{\frac{c}{b^2x^2+2\,abx+a}}}{6\,c^3}$$

input
```
integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(b^6*x^7 + 7*a*b^5*x^6 + 21*a^2*b^4*x^5 + 35*a^3*b^3*x^4 + 35*a^4*b^2*
x^3 + 21*a^5*b*x^2 + 6*a^6*x)*sqrt(c/(b^2*x^2 + 2*a*b*x + a^2))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \begin{cases} \frac{\frac{a}{b}+x}{6\left(\frac{c}{(a+bx)^2}\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a^2}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**2)**(5/2),x)
```

output
```
Piecewise(((a/b + x)/(6*(c/(a + b*x)**2)**(5/2)), Ne(b, 0)), (x/(c/a**2)**
(5/2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{b^5x^6 + 6\,ab^4x^5 + 15\,a^2b^3x^4 + 20\,a^3b^2x^3 + 15\,a^4bx^2 + 6\,a^5x}{6\,c^{\frac{5}{2}}}$$

input
```
integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^
2 + 6*a^5*x)/c^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{3\,(bx^2 + 2\,ax)a^4 + 3\,(bx^2 + 2\,ax)^2a^2b + (bx^2 + 2\,ax)^3b^2}{6\,c^{\frac{5}{2}}\operatorname{sgn}(bx + a)}$$

input
```
integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="giac")
```

output
```
1/6*(3*(b*x^2 + 2*a*x)*a^4 + 3*(b*x^2 + 2*a*x)^2*a^2*b + (b*x^2 + 2*a*x)^3
*b^2)/(c^(5/2)*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \sqrt{\frac{c}{(a+b\,x)^2}}\left(\frac{a^6\,x}{c^3}+\frac{b^6\,x^7}{6\,c^3}+\frac{7\,a^5\,b\,x^2}{2\,c^3}+\frac{7\,a\,b^5\,x^6}{6\,c^3}+\frac{35\,a^4\,b^2\,x^3}{6\,c^3}+\frac{35\,a^3\,b^3\,x^4}{6\,c^3}+\frac{7\,a^2\,b^4\,x^5}{2\,c^3}\right)$$

input
```
int(1/(c/(a + b*x)^2)^(5/2),x)
```

output
```
(c/(a + b*x)^2)^(1/2)*((a^6*x)/c^3 + (b^6*x^7)/(6*c^3) + (7*a^5*b*x^2)/(2*
c^3) + (7*a*b^5*x^6)/(6*c^3) + (35*a^4*b^2*x^3)/(6*c^3) + (35*a^3*b^3*x^4)
/(6*c^3) + (7*a^2*b^4*x^5)/(2*c^3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}}\,dx = \frac{\sqrt{c}\,x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)}{6c^3}$$

input
```
int(1/(c/(b*x+a)^2)^(5/2),x)
```

output
```
(sqrt(c)*x*(6*a**5 + 15*a**4*b*x + 20*a**3*b**2*x**2 + 15*a**2*b**3*x**3 +
 6*a*b**4*x**4 + b**5*x**5))/(6*c**3)
```

3.30 $\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx$

Optimal result 230
Mathematica [A] (verified) 230
Rubi [A] (verified) 231
Maple [A] (verified) 232
Fricas [B] (verification not implemented) 232
Sympy [B] (verification not implemented) 233
Maxima [A] (verification not implemented) 234
Giac [A] (verification not implemented) 234
Mupad [B] (verification not implemented) 234
Reduce [B] (verification not implemented) 235

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2c^2\sqrt{\frac{c}{(a+bx)^3}}}{13b(a+bx)^5}$$

output
```
-2/13*c^2*(c/(b*x+a)^3)^(1/2)/b/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2\left(\frac{c}{(a+bx)^3}\right)^{5/2}(a+bx)}{13b}$$

input
```
Integrate[(c/(a + b*x)^3)^(5/2),x]
```

output
```
(-2*(c/(a + b*x)^3)^(5/2)*(a + b*x))/(13*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{(a+bx)^{15/2}\left(\frac{c}{(a+bx)^3}\right)^{5/2} \int \frac{1}{(a+bx)^{15/2}} d(a+bx)}{b}$$

$$\downarrow 15$$

$$-\frac{2(a+bx)\left(\frac{c}{(a+bx)^3}\right)^{5/2}}{13b}$$

input
```
Int[(c/(a + b*x)^3)^(5/2),x]
```

output
```
(-2*(c/(a + b*x)^3)^(5/2)*(a + b*x))/(13*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}{13b}$	22
default	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}{13b}$	22
orering	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}{13b}$	22
trager	$-\frac{2c^2\sqrt{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{13b(bx+a)^5}$	49

input

```
int((c/(b*x+a)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/13*(b*x+a)*(c/(b*x+a)^3)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.10

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2\,c^2\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{13\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

input

```
integrate((c/(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

output

```
-2/13*c^2*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b^6*x^5 + 5*a
*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(27) = 54$.

Time = 6.00 (sec) , antiderivative size = 1243, normalized size of antiderivative = 41.43

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = \text{Too large to display}$$

input

```
integrate((c/(b*x+a)**3)**(5/2),x)
```

output

```
Piecewise((-638*a**6*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**
(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x*
*3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3828*a**5*b*x*(c/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a
**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 957
0*a**4*b**2*x**2*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(5/2
)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 +
 455*a*b**5*x**4 + 91*b**6*x**5) - 12760*a**3*b**3*x**3*(c/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 91
0*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) -
1170*a**3*c*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(91
*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*
a*b**5*x**4 + 91*b**6*x**5) - 9570*a**2*b**4*x**4*(c/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3
*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3510*a
**2*b*c*x*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(91*a
**5*b + 455*a**4*b**2*x + 910*a**3*b**3*x**2 + 910*a**2*b**4*x**3 + 455*a*
b**5*x**4 + 91*b**6*x**5) - 3828*a*b**5*x**5*(c/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3))**(5/2)/(91*a**5*b + 455*a**4*b**2*x + 910*a**3*b**3
*x**2 + 910*a**2*b**4*x**3 + 455*a*b**5*x**4 + 91*b**6*x**5) - 3510*a*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2\left(bc^{\frac{5}{2}}x + ac^{\frac{5}{2}}\right)}{13\,(bx+a)^{\frac{15}{2}}b}$$

input
```
integrate((c/(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

output
```
-2/13*(b*c^(5/2)*x + a*c^(5/2))/((b*x + a)^(15/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2\,c^9 \mathrm{sgn}(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\,\mathrm{sgn}(bx+a)}{13\,(bcx+ac)^{\frac{13}{2}}b}$$

input
```
integrate((c/(b*x+a)^3)^(5/2),x, algorithm="giac")
```

output
```
-2/13*c^9*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/((b*c*
x + a*c)^(13/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx = -\frac{2\,b^2\,c^2\,\sqrt{\frac{c}{(a+b\,x)^3}}\left(x^3 + \frac{a^3}{b^3} + \frac{3\,a\,x^2}{b} + \frac{3\,a^2\,x}{b^2}\right)}{13\,(a+b\,x)^8}$$

input
```
int((c/(a + b*x)^3)^(5/2),x)
```

output
```
-(2*b^2*c^2*(c/(a + b*x)^3)^(1/2)*(x^3 + a^3/b^3 + (3*a*x^2)/b + (3*a^2*x)
/b^2))/(13*(a + b*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \left(\frac{c}{(a+bx)^3}\right)^{5/2} dx =$$
$$-\frac{2\sqrt{c}\,c^2}{13\sqrt{bx+a}\,b\left(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6a^5bx+a^6\right)}$$

input
```
int((c/(b*x+a)^3)^(5/2),x)
```

output
```
( - 2*sqrt(c)*c**2)/(13*sqrt(a + b*x)*b*(a**6 + 6*a**5*b*x + 15*a**4*b**2*
x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.31 $\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx$

Optimal result 236
Mathematica [A] (verified) 236
Rubi [A] (verified) 237
Maple [A] (verified) 238
Fricas [B] (verification not implemented) 238
Sympy [B] (verification not implemented) 239
Maxima [A] (verification not implemented) 239
Giac [B] (verification not implemented) 240
Mupad [B] (verification not implemented) 240
Reduce [B] (verification not implemented) 240

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2c\sqrt{\frac{c}{(a+bx)^3}}}{7b(a+bx)^2}$$

output
```
-2/7*c*(c/(b*x+a)^3)^(1/2)/b/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\left(\frac{c}{(a+bx)^3}\right)^{3/2}(a+bx)}{7b}$$

input
```
Integrate[(c/(a + b*x)^3)^(3/2),x]
```

output
```
(-2*(c/(a + b*x)^3)^(3/2)*(a + b*x))/(7*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{(a+bx)^{9/2}\left(\frac{c}{(a+bx)^3}\right)^{3/2} \int \frac{1}{(a+bx)^{9/2}} d(a+bx)}{b}$$

$$\downarrow 15$$

$$-\frac{2(a+bx)\left(\frac{c}{(a+bx)^3}\right)^{3/2}}{7b}$$

input
```
Int[(c/(a + b*x)^3)^(3/2),x]
```

output
```
(-2*(c/(a + b*x)^3)^(3/2)*(a + b*x))/(7*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}{7b}$	22
default	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}{7b}$	22
orering	$-\frac{2(bx+a)\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}{7b}$	22
trager	$-\frac{2c\sqrt{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{7b(bx+a)^2}$	47

input
```
int((c/(b*x+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x+a)*(c/(b*x+a)^3)^(3/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\,c\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{7\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate((c/(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
-2/7*c*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b^3*x^2 + 2*a*b^
2*x + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(26) = 52$.

Time = 1.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 11.89

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = \begin{cases} -\frac{67a^3\left(\frac{c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right)^{\frac{3}{2}}}{14a^2b+28ab^2x+14b^3x^2} - \frac{201a^2bx\left(\frac{c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right)^{\frac{3}{2}}}{14a^2b+28ab^2x+14b^3x^2} - \frac{201ab^2x^2\left(\frac{c}{a^3+3a^2bx+3ab^2x} \right.}{14a^2b+28ab^2x+14b} \\ x\left(\frac{c}{a^3}\right)^{\frac{3}{2}} \end{cases}$$

input
```
integrate((c/(b*x+a)**3)**(3/2),x)
```

output
```
Piecewise((-67*a**3*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(
3/2)/(14*a**2*b + 28*a*b**2*x + 14*b**3*x**2) - 201*a**2*b*x*(c/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(14*a**2*b + 28*a*b**2*x + 1
4*b**3*x**2) - 201*a*b**2*x**2*(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3))**(3/2)/(14*a**2*b + 28*a*b**2*x + 14*b**3*x**2) - 67*b**3*x**3*(c
/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))**(3/2)/(14*a**2*b + 28*a
*b**2*x + 14*b**3*x**2) + 63*c*sqrt(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3))/(14*a**2*b + 28*a*b**2*x + 14*b**3*x**2), Ne(b, 0)), (x*(c/a*
*3)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\left(bc^{\frac{3}{2}}x + ac^{\frac{3}{2}}\right)}{7\,(bx+a)^{\frac{9}{2}}b}$$

input
```
integrate((c/(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
-2/7*(b*c^(3/2)*x + a*c^(3/2))/((b*x + a)^(9/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\,c^5 \mathrm{sgn}(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\,\mathrm{sgn}(bx+a)}{7\,(bcx+ac)^{\frac{7}{2}}b}$$

input
```
integrate((c/(b*x+a)^3)^(3/2),x, algorithm="giac")
```

output
```
-2/7*c^5*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/((b*c*x
 + a*c)^(7/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\,c\,\sqrt{\frac{c}{(a+b\,x)^3}}}{7\,b\,(a+b\,x)^2}$$

input
```
int((c/(a + b*x)^3)^(3/2),x)
```

output
```
-(2*c*(c/(a + b*x)^3)^(1/2))/(7*b*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \left(\frac{c}{(a+bx)^3}\right)^{3/2} dx = -\frac{2\sqrt{c}\,c}{7\sqrt{bx+a}\,b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((c/(b*x+a)^3)^(3/2),x)
```

output

```
( - 2*sqrt(c)*c)/(7*sqrt(a + b*x)*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b
**3*x**3))
```

3.32 $\int \sqrt{\frac{c}{(a+bx)^3}}\,dx$

Optimal result 242
Mathematica [A] (verified) 242
Rubi [A] (verified) 243
Maple [A] (verified) 244
Fricas [B] (verification not implemented) 244
Sympy [B] (verification not implemented) 245
Maxima [A] (verification not implemented) 245
Giac [B] (verification not implemented) 245
Mupad [B] (verification not implemented) 246
Reduce [B] (verification not implemented) 246

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\sqrt{\frac{c}{(a+bx)^3}}(a+bx)}{b}$$

output
```
-2*(c/(b*x+a)^3)^(1/2)*(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\sqrt{\frac{c}{(a+bx)^3}}(a+bx)}{b}$$

input
```
Integrate[Sqrt[c/(a + b*x)^3],x]
```

output
```
(-2*Sqrt[c/(a + b*x)^3]*(a + b*x))/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx$$
$$\downarrow 239$$
$$\frac{\int \sqrt{\frac{c}{(a+bx)^3}}\,d(a+bx)}{b}$$
$$\downarrow 20$$
$$\frac{(a+bx)^{3/2}\sqrt{\frac{c}{(a+bx)^3}}\int \frac{1}{(a+bx)^{3/2}}\,d(a+bx)}{b}$$
$$\downarrow 15$$
$$-\frac{2(a+bx)\sqrt{\frac{c}{(a+bx)^3}}}{b}$$

input
```
Int[Sqrt[c/(a + b*x)^3],x]
```

output
```
(-2*Sqrt[c/(a + b*x)^3]*(a + b*x))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{2\sqrt{\frac{c}{(bx+a)^3}}\,(bx+a)}{b}$	22
default	$-\frac{2\sqrt{\frac{c}{(bx+a)^3}}\,(bx+a)}{b}$	22
orering	$-\frac{2\sqrt{\frac{c}{(bx+a)^3}}\,(bx+a)}{b}$	22
trager	$-\frac{2(bx+a)\sqrt{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{b}$	44

input
```
int((c/(b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c/(b*x+a)^3)^(1/2)*(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\,(bx+a)\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{b}$$

input
```
integrate((c/(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

output
```
-2*(b*x + a)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(20) = 40$.

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = \begin{cases} -\frac{2a\sqrt{\frac{c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}}}{b} - 2x\sqrt{\frac{c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}} & \text{for } b \neq 0 \\ x\sqrt{\frac{c}{a^3}} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x+a)**3)**(1/2),x)
```

output
```
Piecewise((-2*a*sqrt(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/b
- 2*x*sqrt(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)), Ne(b, 0)),
(x*sqrt(c/a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)}{(bx+a)^{\frac{3}{2}}b}$$

input
```
integrate((c/(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

output
```
-2*(b*sqrt(c)*x + a*sqrt(c))/((b*x + a)^(3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\,c\mathrm{sgn}(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3)\,\mathrm{sgn}(bx+a)}{\sqrt{bcx + ac}b}$$

input
```
integrate((c/(b*x+a)^3)^(1/2),x, algorithm="giac")
```

output
```
-2*c*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sgn(b*x + a)/(sqrt(b*c*x
 + a*c)*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\,b^2\,\sqrt{\frac{c}{(a+b\,x)^3}}\left(x^3+\frac{a^3}{b^3}+\frac{3\,a\,x^2}{b}+\frac{3\,a^2\,x}{b^2}\right)}{(a+b\,x)^2}$$

input
```
int((c/(a + b*x)^3)^(1/2),x)
```

output
```
-(2*b^2*(c/(a + b*x)^3)^(1/2)*(x^3 + a^3/b^3 + (3*a*x^2)/b + (3*a^2*x)/b^2
))/(a + b*x)^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{\frac{c}{(a+bx)^3}}\,dx = -\frac{2\sqrt{c}}{\sqrt{bx+a}\,b}$$

input
```
int((c/(b*x+a)^3)^(1/2),x)
```

output
```
( - 2*sqrt(c))/(sqrt(a + b*x)*b)
```

3.33 $\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx$

Optimal result . . . 247
Mathematica [A] (verified) . . . 247
Rubi [A] (verified) . . . 248
Maple [A] (verified) . . . 249
Fricas [B] (verification not implemented) . . . 250
Sympy [A] (verification not implemented) . . . 250
Maxima [A] (verification not implemented) . . . 251
Giac [B] (verification not implemented) . . . 251
Mupad [B] (verification not implemented) . . . 252
Reduce [B] (verification not implemented) . . . 252

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

output
```
2/5*(b*x+a)/b/(c/(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

input
```
Integrate[1/Sqrt[c/(a + b*x)^3],x]
```

output
```
(2*(a + b*x))/(5*b*Sqrt[c/(a + b*x)^3])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)^{3/2} d(a+bx)}{b(a+bx)^{3/2}\sqrt{\frac{c}{(a+bx)^3}}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)}{5b\sqrt{\frac{c}{(a+bx)^3}}}$$

input
```
Int[1/Sqrt[c/(a + b*x)^3],x]
```

output
```
(2*(a + b*x))/(5*b*Sqrt[c/(a + b*x)^3])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\sqrt{\frac{c}{(bx+a)^3}}}$	22
default	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\sqrt{\frac{c}{(bx+a)^3}}}$	22
orering	$\frac{\frac{2bx}{5}+\frac{2a}{5}}{b\sqrt{\frac{c}{(bx+a)^3}}}$	22
risch	$\frac{\frac{2}{5}a^2+\frac{4}{5}abx+\frac{2}{5}b^2x^2}{\sqrt{\frac{c}{(bx+a)^3}}\,(bx+a)b}$	40
trager	$\frac{2\left(b^2x^2+2abx+a^2\right)^2\sqrt{\frac{c}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}}{5cb}$	60

input
```
int(1/(c/(b*x+a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)/b/(c/(b*x+a)^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2\left(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4\right)\sqrt{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{5\,bc}$$

input
```
integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(c/(b^3*
x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \begin{cases} \frac{2\left(\frac{a}{b}+x\right)}{5\sqrt{\frac{c}{(a+bx)^3}}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\frac{c}{a^3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**3)**(1/2),x)
```

output
```
Piecewise((2*(a/b + x)/(5*sqrt(c/(a + b*x)**3)), Ne(b, 0)), (x/sqrt(c/a**3
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^{\frac{3}{2}}}{5\,bc}$$

input
```
integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(3/2)/(b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(21) = 42.

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.76

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx$$
$$= \frac{2\left(15\sqrt{bcx+ac}a^2 - \frac{10\left(3\sqrt{bcx+ac}ac - (bcx+ac)^{\frac{3}{2}}\right)a}{c} + \frac{15\sqrt{bcx+ac}a^2c^2 - 10\,(bcx+ac)^{\frac{3}{2}}ac + 3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc\mathrm{sgn}\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\mathrm{sgn}\left(bx+a\right)}$$

input
```
integrate(1/(c/(b*x+a)^3)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*
c)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c
 + 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
+ a^3)*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2\sqrt{\frac{c}{(a+b\,x)^3}}\,(a+b\,x)^4}{5\,b\,c}$$

input
```
int(1/(c/(a + b*x)^3)^(1/2),x)
```

output
```
(2*(c/(a + b*x)^3)^(1/2)*(a + b*x)^4)/(5*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{\frac{c}{(a+bx)^3}}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^2x^2+2abx+a^2)}{5bc}$$

input
```
int(1/(c/(b*x+a)^3)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**2 + 2*a*b*x + b**2*x**2))/(5*b*c)
```

3.34 $\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx$

Optimal result . . . 253
Mathematica [A] (verified) . . . 253
Rubi [A] (verified) . . . 254
Maple [A] (verified) . . . 255
Fricas [B] (verification not implemented) . . . 256
Sympy [A] (verification not implemented) . . . 256
Maxima [A] (verification not implemented) . . . 257
Giac [B] (verification not implemented) . . . 257
Mupad [B] (verification not implemented) . . . 258
Reduce [B] (verification not implemented) . . . 258

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2(a+bx)^4}{11bc\sqrt{\frac{c}{(a+bx)^3}}}$$

output
```
2/11*(b*x+a)^4/b/c/(c/(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2(a+bx)}{11b\left(\frac{c}{(a+bx)^3}\right)^{3/2}}$$

input
```
Integrate[(c/(a + b*x)^3)^(-3/2),x]
```

output
```
(2*(a + b*x))/(11*b*(c/(a + b*x)^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)^{9/2} d(a+bx)}{b(a+bx)^{9/2}\left(\frac{c}{(a+bx)^3}\right)^{3/2}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)}{11b\left(\frac{c}{(a+bx)^3}\right)^{3/2}}$$

input
```
Int[(c/(a + b*x)^3)^(-3/2),x]
```

output
```
(2*(a + b*x))/(11*b*(c/(a + b*x)^3)^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{\frac{2bx}{11}+\frac{2a}{11}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}$	22
default	$\frac{\frac{2bx}{11}+\frac{2a}{11}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}$	22
orering	$\frac{\frac{2bx}{11}+\frac{2a}{11}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}}$	22
risch	$\frac{\frac{2}{11}a^5+\frac{10}{11}a^4bx+\frac{20}{11}a^3b^2x^2+\frac{20}{11}a^2b^3x^3+\frac{10}{11}ab^4x^4+\frac{2}{11}b^5x^5}{c(bx+a)\sqrt{\frac{c}{(bx+a)^3}}\,b}$	76
trager	$\frac{2\left(b^2x^2+2abx+a^2\right)\left(b^5x^5+5ab^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5\right)\sqrt{\frac{c}{b^3x^3+3ab^2x^2+3a^2bx+a^3}}}{11c^2b}$	107

input
```
int(1/(c/(b*x+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*(b*x+a)/b/(c/(b*x+a)^3)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.73

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2\left(b^7x^7+7\,ab^6x^6+21\,a^2b^5x^5+35\,a^3b^4x^4+35\,a^4b^3x^3+21\,a^5b^2x^2+7\,a^6bx+a^7\right)\sqrt{\frac{}{b^3}}}{11\,bc^2}$$

input
```
integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
2/11*(b^7*x^7 + 7*a*b^6*x^6 + 21*a^2*b^5*x^5 + 35*a^3*b^4*x^4 + 35*a^4*b^3
*x^3 + 21*a^5*b^2*x^2 + 7*a^6*b*x + a^7)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3
*a^2*b*x + a^3))/(b*c^2)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{a}{b}+x\right)}{11\left(\frac{c}{(a+bx)^3}\right)^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a^3}\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**3)**(3/2),x)
```

output
```
Piecewise((2*(a/b + x)/(11*(c/(a + b*x)**3)**(3/2)), Ne(b, 0)), (x/(c/a**3
)**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^{\frac{9}{2}}}{11\,bc^2}$$

input
```
integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
2/11*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(9/2)/(b*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 412, normalized size of antiderivative = 13.73

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2\left(693\sqrt{bcx+ac}a^5 - \frac{1155\left(3\sqrt{bcx+ac}ac - (bcx+ac)^{\frac{3}{2}}\right)a^4}{c} + \frac{462\left(15\sqrt{bcx+ac}a^2c^2 - 10\,(bcx+ac)^{\frac{3}{2}}ac + 3\right.}{c^2}\right.$$

input
```
integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="giac")
```

output
```
2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x
+ a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)
^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^
3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b
*c*x + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*
x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*
c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*
c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3
- 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*
x + a*c)^(11/2))/c^5)/(b*c^2*sgn(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*
sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.67

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \sqrt{\frac{c}{(a+b\,x)^3}}\left(\frac{14\,a^6\,x}{11\,c^2}+\frac{2\,a^7}{11\,b\,c^2}+\frac{2\,b^6\,x^7}{11\,c^2}+\frac{42\,a^5\,b\,x^2}{11\,c^2}+\frac{14\,a\,b^5\,x^6}{11\,c^2}+\frac{70\,a^4\,b^2\,x^3}{11\,c^2}+\frac{70\,a^3\,b^3\,x^4}{11\,c^2}+\frac{42\,a^2\,b^4\,x^5}{11\,c^2}\right)$$

input
```
int(1/(c/(a + b*x)^3)^(3/2),x)
```

output
```
(c/(a + b*x)^3)^(1/2)*((14*a^6*x)/(11*c^2) + (2*a^7)/(11*b*c^2) + (2*b^6*x
^7)/(11*c^2) + (42*a^5*b*x^2)/(11*c^2) + (14*a*b^5*x^6)/(11*c^2) + (70*a^4
*b^2*x^3)/(11*c^2) + (70*a^3*b^3*x^4)/(11*c^2) + (42*a^2*b^4*x^5)/(11*c^2)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}{11b\,c^2}$$

input
```
int(1/(c/(b*x+a)^3)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*
b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))/(11*b*c**2)
```

3.35 $\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx$

Optimal result 259
Mathematica [A] (verified) 259
Rubi [A] (verified) 260
Maple [A] (verified) 261
Fricas [B] (verification not implemented) 262
Sympy [A] (verification not implemented) 262
Maxima [A] (verification not implemented) 263
Giac [B] (verification not implemented) 263
Mupad [B] (verification not implemented) 264
Reduce [B] (verification not implemented) 265

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \frac{2(a+bx)^7}{17bc^2\sqrt{\frac{c}{(a+bx)^3}}}$$

output
```
2/17*(b*x+a)^7/b/c^2/(c/(b*x+a)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \frac{2(a+bx)}{17b\left(\frac{c}{(a+bx)^3}\right)^{5/2}}$$

input
```
Integrate[(c/(a + b*x)^3)^(-5/2),x]
```

output
```
(2*(a + b*x))/(17*b*(c/(a + b*x)^3)^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx)^{15/2} d(a+bx)}{b(a+bx)^{15/2}\left(\frac{c}{(a+bx)^3}\right)^{5/2}}$$

$$\downarrow 15$$

$$\frac{2(a+bx)}{17b\left(\frac{c}{(a+bx)^3}\right)^{5/2}}$$

input
```
Int[(c/(a + b*x)^3)^(-5/2),x]
```

output
```
(2*(a + b*x))/(17*b*(c/(a + b*x)^3)^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{\frac{2bx}{17}+\frac{2a}{17}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}$
default	$\frac{\frac{2bx}{17}+\frac{2a}{17}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}$
orering	$\frac{\frac{2bx}{17}+\frac{2a}{17}}{b\left(\frac{c}{(bx+a)^3}\right)^{\frac{5}{2}}}$
risch	$\frac{\frac{2}{17}a^8+\frac{16}{17}a^7xb+\frac{56}{17}a^6x^2b^2+\frac{112}{17}a^5b^3x^3+\frac{140}{17}a^4x^4b^4+\frac{112}{17}a^3x^5b^5+\frac{56}{17}a^2x^6b^6+\frac{16}{17}a\,x^7b^7+\frac{2}{17}b^8x^8}{c^2(bx+a)\sqrt{\frac{c}{(bx+a)^3}}\,b}$
trager	$\frac{2\left(b^2x^2+2abx+a^2\right)\left(b^8x^8+8a\,x^7b^7+28a^2x^6b^6+56a^3x^5b^5+70a^4x^4b^4+56a^5b^3x^3+28a^6x^2b^2+8a^7xb+a^8\right)\sqrt{\frac{c}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}}{17c^3b}$

input
```
int(1/(c/(b*x+a)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*(b*x+a)/b/(c/(b*x+a)^3)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.83

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \frac{2\left(b^{10}x^{10}+10\,ab^9x^9+45\,a^2b^8x^8+120\,a^3b^7x^7+210\,a^4b^6x^6+252\,a^5b^5x^5+210\,a^6b^4x^4\right.}{17\,bc^3}$$

input
```
integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="fricas")
```

output
```
2/17*(b^10*x^10 + 10*a*b^9*x^9 + 45*a^2*b^8*x^8 + 120*a^3*b^7*x^7 + 210*a^
4*b^6*x^6 + 252*a^5*b^5*x^5 + 210*a^6*b^4*x^4 + 120*a^7*b^3*x^3 + 45*a^8*b
^2*x^2 + 10*a^9*b*x + a^10)*sqrt(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^
3))/(b*c^3)
```

Sympy [A] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{a}{b}+x\right)}{17\left(\frac{c}{(a+bx)^3}\right)^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a^3}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**3)**(5/2),x)
```

output
```
Piecewise((2*(a/b + x)/(17*(c/(a + b*x)**3)**(5/2)), Ne(b, 0)), (x/(c/a**3
)**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \frac{2\left(b\sqrt{c}x + a\sqrt{c}\right)(bx+a)^{\frac{15}{2}}}{17\,bc^3}$$

input
```
integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="maxima")
```

output
```
2/17*(b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^(15/2)/(b*c^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(26) = 52.

Time = 0.14 (sec) , antiderivative size = 840, normalized size of antiderivative = 28.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="giac")
```

output

```
2/109395*(109395*sqrt(b*c*x + a*c)*a^8 - 291720*(3*sqrt(b*c*x + a*c)*a*c -
 (b*c*x + a*c)^(3/2))*a^7/c + 204204*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b
*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^6/c^2 - 175032*(35*sqrt(b
*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5
/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^5/c^3 + 24310*(315*sqrt(b*c*x + a*c)*a^
4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2
- 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^4/c^4 - 8840*(69
3*sqrt(b*c*x + a*c)*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c
*x + a*c)^(5/2)*a^3*c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a
*c)^(9/2)*a*c - 63*(b*c*x + a*c)^(11/2))*a^3/c^5 + 1020*(3003*sqrt(b*c*x +
 a*c)*a^6*c^6 - 6006*(b*c*x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2
)*a^4*c^4 - 8580*(b*c*x + a*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^
2*c^2 - 1638*(b*c*x + a*c)^(11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a^2/c^6
- 136*(6435*sqrt(b*c*x + a*c)*a^7*c^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6
+ 27027*(b*c*x + a*c)^(5/2)*a^5*c^5 - 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 +
25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 12285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 3
465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*c*x + a*c)^(15/2))*a/c^7 + (109395*s
qrt(b*c*x + a*c)*a^8*c^8 - 291720*(b*c*x + a*c)^(3/2)*a^7*c^7 + 612612*(b*
c*x + a*c)^(5/2)*a^6*c^6 - 875160*(b*c*x + a*c)^(7/2)*a^5*c^5 + 850850*(b*
c*x + a*c)^(9/2)*a^4*c^4 - 556920*(b*c*x + a*c)^(11/2)*a^3*c^3 + 235620...
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.07

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \sqrt{\frac{c}{(a+b\,x)^3}}\left(\frac{20\,a^9\,x}{17\,c^3}+\frac{2\,a^{10}}{17\,b\,c^3}+\frac{2\,b^9\,x^{10}}{17\,c^3}+\frac{90\,a^8\,b\,x^2}{17\,c^3}+\frac{20\,a\,b^8\,x^9}{17\,c^3}+\frac{240\,a^7\,b^2\,x^3}{17\,c^3}+\frac{420\,a^6\,b^3\,x^4}{17\,c^3}+\frac{504\,a^5\,b^4\,x^5}{17\,c^3}+\frac{420\,a^4\,b^5\,x^6}{17\,c^3}+\frac{240\,a^3\,b^6\,x^7}{17\,c^3}+\frac{90\,a^2\,b^7\,x^8}{17\,c^3}\right)$$

input

```
int(1/(c/(a + b*x)^3)^(5/2),x)
```

output
```
(c/(a + b*x)^3)^(1/2)*((20*a^9*x)/(17*c^3) + (2*a^10)/(17*b*c^3) + (2*b^9*
x^10)/(17*c^3) + (90*a^8*b*x^2)/(17*c^3) + (20*a*b^8*x^9)/(17*c^3) + (240*
a^7*b^2*x^3)/(17*c^3) + (420*a^6*b^3*x^4)/(17*c^3) + (504*a^5*b^4*x^5)/(17
*c^3) + (420*a^4*b^5*x^6)/(17*c^3) + (240*a^3*b^6*x^7)/(17*c^3) + (90*a^2*
b^7*x^8)/(17*c^3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{bx+a}\,(b^8x^8+8a\,b^7x^7+28a^2b^6x^6+56a^3b^5x^5+70a^4b^4x^4+56a^5b^3x^3+28a^6b^2}{17b\,c^3}$$

input
```
int(1/(c/(b*x+a)^3)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x)*(a**8 + 8*a**7*b*x + 28*a**6*b**2*x**2 + 56*a**5*
b**3*x**3 + 70*a**4*b**4*x**4 + 56*a**3*b**5*x**5 + 28*a**2*b**6*x**6 + 8*
a*b**7*x**7 + b**8*x**8))/(17*b*c**3)
```

3.36 $\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx$

Optimal result . . . 266
Mathematica [A] (verified) . . . 266
Rubi [A] (verified) . . . 267
Maple [A] (verified) . . . 268
Fricas [A] (verification not implemented) . . . 268
Sympy [A] (verification not implemented) . . . 269
Maxima [A] (verification not implemented) . . . 269
Giac [A] (verification not implemented) . . . 269
Mupad [F(-1)] . . . 270
Reduce [B] (verification not implemented) . . . 270

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{(a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2b}$$

output
```
1/2*(b*x+a)*(c*(b*x+a)^(3/2))^(2/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{x\left(c(a+bx)^{3/2}\right)^{2/3}(2a+bx)}{2(a+bx)}$$

input
```
Integrate[(c*(a + b*x)^(3/2))^(2/3),x]
```

output
```
(x*(c*(a + b*x)^(3/2))^(2/3)*(2*a + b*x))/(2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^{3/2}\right)^{2/3} dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^{3/2}\right)^{2/3} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^{3/2}\right)^{2/3} \int (a+bx) d(a+bx)}{b(a+bx)}$$

$$\downarrow 15$$

$$\frac{(a+bx)\left(c(a+bx)^{3/2}\right)^{2/3}}{2b}$$

input
```
Int[(c*(a + b*x)^(3/2))^(2/3),x]
```

output
```
((a + b*x)*(c*(a + b*x)^(3/2))^(2/3))/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{x(bx+2a)\left(c(bx+a)^{\frac{3}{2}}\right)^{\frac{2}{3}}}{2bx+2a}$	29
orering	$\frac{x(bx+2a)\left(c(bx+a)^{\frac{3}{2}}\right)^{\frac{2}{3}}}{2bx+2a}$	29

input

```
int((c*(b*x+a)^(3/2))^(2/3),x,method=_RETURNVERBOSE)
```

output

```
1/2*x*(b*x+2*a)*(c*(b*x+a)^(3/2))^(2/3)/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{(bx^2+2\,ax)\left((bcx+ac)\sqrt{bx+a}\right)^{\frac{2}{3}}}{2\,(bx+a)}$$

input

```
integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="fricas")
```

output

```
1/2*(b*x^2 + 2*a*x)*((b*c*x + a*c)*sqrt(b*x + a))^(2/3)/(b*x + a)
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \begin{cases} \frac{\left(c(a+bx)^{\frac{3}{2}}\right)^{\frac{2}{3}}(a+bx)}{2b} & \text{for } b \neq 0 \\ x\left(a^{\frac{3}{2}}c\right)^{\frac{2}{3}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*(b*x+a)**(3/2))**(2/3),x)
```

output
```
Piecewise(((c*(a + b*x)**(3/2))**(2/3)*(a + b*x)/(2*b), Ne(b, 0)), (x*(a**
(3/2)*c)**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{\left((bx+a)^{\frac{3}{2}}c\right)^{\frac{2}{3}}(bx+a)}{2\,b}$$

input
```
integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/2*((b*x + a)^(3/2)*c)^(2/3)*(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{\left(\sqrt{bx+a}abcx + \sqrt{bx+a}ac\right)^{\frac{2}{3}}(bx+a)}{b}$$

input
```
integrate((c*(b*x+a)^(3/2))^(2/3),x, algorithm="giac")
```

output
```
(sqrt(b*x + a)*b*c*x + sqrt(b*x + a)*a*c)^(2/3)*(b*x + a)/b
```

Mupad [F(-1)]

Timed out.

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \int \left(c\,(a+b\,x)^{3/2}\right)^{2/3}\,dx$$

input
```
int((c*(a + b*x)^(3/2))^(2/3),x)
```

output
```
int((c*(a + b*x)^(3/2))^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(c(a+bx)^{3/2}\right)^{2/3}\,dx = \frac{c^{\frac{2}{3}}(b^2x^2+2abx+a^2)}{2b}$$

input
```
int((c*(b*x+a)^(3/2))^(2/3),x)
```

output
```
(c**(2/3)*(a**2 + 2*a*b*x + b**2*x**2))/(2*b)
```

3.37 $\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx$

Optimal result . . . 271
Mathematica [A] (verified) . . . 271
Rubi [A] (verified) . . . 272
Maple [A] (verified) . . . 273
Fricas [A] (verification not implemented) . . . 273
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Maxima [A] (verification not implemented) . . . 274
Giac [A] (verification not implemented) . . . 274
Mupad [F(-1)] . . . 275
Reduce [B] (verification not implemented) . . . 275

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{c\sqrt{c(a+bx)^{2/3}}(a+bx)^{5/3}}{2b}$$

output
```
1/2*c*(c*(b*x+a)^(2/3))^(1/2)*(b*x+a)^(5/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{x\left(c(a+bx)^{2/3}\right)^{3/2}(2a+bx)}{2(a+bx)}$$

input
```
Integrate[(c*(a + b*x)^(2/3))^(3/2),x]
```

output
```
(x*(c*(a + b*x)^(2/3))^(3/2)*(2*a + b*x))/(2*(a + b*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c(a+bx)^{2/3}\right)^{3/2} dx$$

$$\downarrow 239$$

$$\frac{\int \left(c(a+bx)^{2/3}\right)^{3/2} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\left(c(a+bx)^{2/3}\right)^{3/2} \int (a+bx) d(a+bx)}{b(a+bx)}$$

$$\downarrow 15$$

$$\frac{(a+bx)\left(c(a+bx)^{2/3}\right)^{3/2}}{2b}$$

input
```
Int[(c*(a + b*x)^(2/3))^(3/2),x]
```

output
```
((c*(a + b*x)^(2/3))^(3/2)*(a + b*x))/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\left(c(bx+a)^{\frac{2}{3}}\right)^{\frac{3}{2}}(bx+a)}{2b}$	22
gosper	$\frac{x(bx+2a)\left(c(bx+a)^{\frac{2}{3}}\right)^{\frac{3}{2}}}{2bx+2a}$	29
default	$\frac{x(bx+2a)\left(c(bx+a)^{\frac{2}{3}}\right)^{\frac{3}{2}}}{2bx+2a}$	29
orering	$\frac{x(bx+2a)\left(c(bx+a)^{\frac{2}{3}}\right)^{\frac{3}{2}}}{2bx+2a}$	29

input

```
int((c*(b*x+a)^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/b*(c*(b*x+a)^(2/3))^(3/2)*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{1}{2}\left(bcx^2+2\,acx\right)\sqrt{c}$$

input

```
integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="fricas")
```

output

```
1/2*(b*c*x^2 + 2*a*c*x)*sqrt(c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

Time = 0.84 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.81

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \begin{cases} \frac{2ax\left(c(a+bx)^{\frac{2}{3}}\right)^{\frac{3}{2}}}{2a+2bx} + \frac{bx^2\left(c(a+bx)^{\frac{2}{3}}\right)^{\frac{3}{2}}}{2a+2bx} & \text{for } a \neq -bx \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((c*(b*x+a)**(2/3))**(3/2),x)
```

output
```
Piecewise((2*a*x*(c*(a + b*x)**(2/3))**(3/2)/(2*a + 2*b*x) + b*x**2*(c*(a
+ b*x)**(2/3))**(3/2)/(2*a + 2*b*x), Ne(a, -b*x)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}$$

input
```
integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}$$

input
```
integrate((c*(b*x+a)^(2/3))^(3/2),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)*c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \int \left(c\,(a+b\,x)^{2/3}\right)^{3/2}\,dx$$

input
```
int((c*(a + b*x)^(2/3))^(3/2),x)
```

output
```
int((c*(a + b*x)^(2/3))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

$$\int \left(c(a+bx)^{2/3}\right)^{3/2}\,dx = \frac{\sqrt{c}\,cx(bx+2a)}{2}$$

input
```
int((c*(b*x+a)^(2/3))^(3/2),x)
```

output
```
(sqrt(c)*c*x*(2*a + b*x))/2
```

3.38 $\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}} dx$

Optimal result 276
Mathematica [A] (verified) 276
Rubi [A] (verified) 277
Maple [A] (verified) 278
Fricas [B] (verification not implemented) 279
Sympy [A] (verification not implemented) 279
Maxima [A] (verification not implemented) 280
Giac [A] (verification not implemented) 280
Mupad [B] (verification not implemented) 280
Reduce [B] (verification not implemented) 281

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}} dx = \frac{a+bx}{2b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

output
```
1/2*(b*x+a)/b/(c/(b*x+a)^(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}} dx = \frac{x(2a+bx)}{2\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}(a+bx)}$$

input
```
Integrate[(c/(a + b*x)^(3/2))^(-2/3),x]
```

output
```
(x*(2*a + b*x))/(2*(c/(a + b*x)^(3/2))^(2/3)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx) d(a+bx)}{b(a+bx)\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

$$\downarrow 15$$

$$\frac{a+bx}{2b\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}$$

input
```
Int[(c/(a + b*x)^(3/2))^(-2/3),x]
```

output
```
(a + b*x)/(2*b*(c/(a + b*x)^(3/2))^(2/3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{x(bx+2a)}{2(bx+a)\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$	29
orering	$\frac{x(bx+2a)}{2(bx+a)\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$	29

input
```
int(1/(c/(b*x+a)^(3/2))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x+2*a)/(b*x+a)/(c/(b*x+a)^(3/2))^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(21) = 42$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx = \frac{(bx^2+2\,ax)\sqrt{bx+a}\left(\frac{\sqrt{bx+a}c}{b^2x^2+2\,abx+a^2}\right)^{\frac{1}{3}}}{2\,c}$$

input
```
integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 + 2*a*x)*sqrt(b*x + a)*(sqrt(b*x + a)*c/(b^2*x^2 + 2*a*b*x + a^
2))^(1/3)/c
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx = \begin{cases} \frac{a+bx}{2b\left(\frac{c}{(a+bx)^{\frac{3}{2}}}\right)^{\frac{2}{3}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\frac{c}{a^{\frac{3}{2}}}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c/(b*x+a)**(3/2))**(2/3),x)
```

output
```
Piecewise(((a + b*x)/(2*b*(c/(a + b*x)**(3/2))**(2/3)), Ne(b, 0)), (x/(c/a
**(3/2))**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx = \frac{bx+a}{2\,b\left(\frac{c}{(bx+a)^{\frac{3}{2}}}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)/(b*(c/(b*x + a)^(3/2))^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.04

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx = +\infty$$

input
```
integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\frac{c}{(a+b\,x)^{3/2}}\right)^{2/3}}\,dx = \frac{x\left(\frac{c}{(a+b\,x)^{3/2}}\right)^{1/3}(2\,a+b\,x)\,\sqrt{a+b\,x}}{2\,c}$$

input
```
int(1/(c/(a + b*x)^(3/2))^(2/3),x)
```

output
```
(x*(c/(a + b*x)^(3/2))^(1/3)*(2*a + b*x)*(a + b*x)^(1/2))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{3/2}}\right)^{2/3}}\,dx = \frac{b^2x^2+2abx+a^2}{2c^{\frac{2}{3}}b}$$

input
```
int(1/(c/(b*x+a)^(3/2))^(2/3),x)
```

output
```
(a**2 + 2*a*b*x + b**2*x**2)/(2*c**(2/3)*b)
```

3.39 $\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}} dx$

Optimal result 282
Mathematica [A] (verified) 282
Rubi [A] (verified) 283
Maple [A] (verified) 284
Fricas [A] (verification not implemented) 285
Sympy [B] (verification not implemented) 285
Maxima [A] (verification not implemented) 286
Giac [A] (verification not implemented) 286
Mupad [B] (verification not implemented) 286
Reduce [B] (verification not implemented) 287

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}} dx = \frac{(a+bx)^{5/3}}{2bc\sqrt{\frac{c}{(a+bx)^{2/3}}}}$$

output
```
1/2*(b*x+a)^(5/3)/b/c/(c/(b*x+a)^(2/3))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}} dx = \frac{x(2a+bx)}{2\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}(a+bx)}$$

input
```
Integrate[(c/(a + b*x)^(2/3))^(-3/2),x]
```

output
```
(x*(2*a + b*x))/(2*(c/(a + b*x)^(2/3))^(3/2)*(a + b*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}} d(a+bx)}{b}$$

$$\downarrow 20$$

$$\frac{\int (a+bx) d(a+bx)}{b(a+bx)\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}$$

$$\downarrow 15$$

$$\frac{a+bx}{2b\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}$$

input
```
Int[(c/(a + b*x)^(2/3))^(-3/2),x]
```

output
```
(a + b*x)/(2*b*(c/(a + b*x)^(2/3))^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{bx+a}{2b\left(\frac{c}{(bx+a)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$	22
gosper	$\frac{x(bx+2a)}{2(bx+a)\left(\frac{c}{(bx+a)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$	29
default	$\frac{x(bx+2a)}{2(bx+a)\left(\frac{c}{(bx+a)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$	29
orering	$\frac{x(bx+2a)}{2(bx+a)\left(\frac{c}{(bx+a)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$	29

input
```
int(1/(c/(b*x+a)^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(c/(b*x+a)^(2/3))^(3/2)*(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \frac{2ax}{2a\left(\frac{c}{(a+bx)^{\frac{2}{3}}}\right)^{\frac{3}{2}} + 2bx\left(\frac{c}{(a+bx)^{\frac{2}{3}}}\right)^{\frac{3}{2}}} + \frac{bx^2}{2a\left(\frac{c}{(a+bx)^{\frac{2}{3}}}\right)^{\frac{3}{2}} + 2bx\left(\frac{c}{(a+bx)^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a)**(2/3))**(3/2),x)
```

output
```
2*a*x/(2*a*(c/(a + b*x)**(2/3))**(3/2) + 2*b*x*(c/(a + b*x)**(2/3))**(3/2)
) + b*x**2/(2*a*(c/(a + b*x)**(2/3))**(3/2) + 2*b*x*(c/(a + b*x)**(2/3))**
(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \frac{bx^2+2\,ax}{2\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c/(b*x+a)^(2/3))^(3/2),x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2*a*x)/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \sqrt{\frac{c}{(a+b\,x)^{2/3}}}\left(\frac{a\,x\,(a+b\,x)^{1/3}}{c^2}+\frac{b\,x^2\,(a+b\,x)^{1/3}}{2\,c^2}\right)$$

input
```
int(1/(c/(a + b*x)^(2/3))^(3/2),x)
```

output
```
(c/(a + b*x)^(2/3))^(1/2)*((a*x*(a + b*x)^(1/3))/c^2 + (b*x^2*(a + b*x)^(1
/3))/(2*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(\frac{c}{(a+bx)^{2/3}}\right)^{3/2}}\,dx = \frac{\sqrt{c}\,x(bx+2a)}{2c^2}$$

input
```
int(1/(c/(b*x+a)^(2/3))^(3/2),x)
```

output
```
(sqrt(c)*x*(2*a + b*x))/(2*c**2)
```

3.40 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)\, dx$

Optimal result 288
Mathematica [B] (verified) 288
Rubi [A] (verified) 289
Maple [A] (verified) 290
Fricas [B] (verification not implemented) 290
Sympy [B] (verification not implemented) 291
Maxima [B] (verification not implemented) 291
Giac [B] (verification not implemented) 292
Mupad [B] (verification not implemented) 292
Reduce [B] (verification not implemented) 293

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)\, dx = \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^6}{6d}$$

output
```
1/4*a*(d*x+c)^4/d+1/6*b*(d*x+c)^6/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. $2(31) = 62$.

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)\, dx = \frac{1}{12}x(2c+dx)\left(3a\left(2c^2+2cdx+d^2x^2\right) + 2b\left(3c^4+6c^3dx+7c^2d^2x^2+4cd^3x^3+d^4x^4\right)\right)$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^2),x]
```

output
```
(x*(2*c + d*x)*(3*a*(2*c^2 + 2*c*d*x + d^2*x^2) + 2*b*(3*c^4 + 6*c^3*d*x +
 7*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)))/12
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {895, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)\, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^2+a\right) d(c+dx)}{d}$$

$$\downarrow 244$$

$$\frac{\int \left(b(c+dx)^5+a(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a(c+dx)^4+\frac{1}{6}b(c+dx)^6}{d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^2),x]
```

output
```
((a*(c + d*x)^4)/4 + (b*(c + d*x)^6)/6)/d
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\frac{a\left(a+b(xd+c)^2\right)^2}{2}-\frac{\left(a+b(xd+c)^2\right)^3}{3}}{2db^2}$
norman	$\frac{bd^5x^6}{6}+cd^4bx^5+\left(\frac{5}{2}c^2d^3b+\frac{1}{4}ad^3\right)x^4+\left(\frac{10}{3}c^3bd^2+acd^2\right)x^3+\left(\frac{5}{2}c^4bd+\frac{3}{2}ac^2d\right)x^2+(bc^$
parallelrisch	$\frac{1}{6}bd^5x^6+cd^4bx^5+\frac{5}{2}d^3bc^2x^4+\frac{1}{4}ad^3x^4+\frac{10}{3}d^2bc^3x^3+d^2acx^3+\frac{5}{2}dbc^4x^2+\frac{3}{2}dac^2x^2+b$
gosper	$\frac{x\left(2bd^5x^5+12cd^4bx^4+30x^3c^2d^3b+40x^2c^3bd^2+3ad^3x^3+30xc^4bd+12x^2acd^2+12bc^5+18xac^2d+12c^3a\right)}{12}$
risch	$-\frac{a^3}{12db^2}+d^2acx^3+\frac{3dac^2x^2}{2}+ac^3x+\frac{ad^3x^4}{4}+cd^4bx^5+\frac{5d^3bc^2x^4}{2}+\frac{10d^2bc^3x^3}{3}+\frac{5dbc^4x^2}{2}+bc^5$
orering	$\frac{x\left(2bd^5x^5+12cd^4bx^4+30x^3c^2d^3b+40x^2c^3bd^2+3ad^3x^3+30xc^4bd+12x^2acd^2+12bc^5+18xac^2d+12c^3a\right)\left(a+b(xd+c)^2\right)}{12bd^2x^2+24bcdx+12bc^2+12a}$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/b^2*(1/2*a*(a+b*(d*x+c)^2)^2-1/3*(a+b*(d*x+c)^2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(27) = 54.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx = \frac{1}{6}bd^5x^6+bcd^4x^5+\frac{1}{4}\left(10bc^2+a\right)d^3x^4 + \frac{1}{3}\left(10bc^3+3ac\right)d^2x^3 + \frac{1}{2}\left(5bc^4+3ac^2\right)dx^2+\left(bc^5+ac^3\right)x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/6*b*d^5*x^6 + b*c*d^4*x^5 + 1/4*(10*b*c^2 + a)*d^3*x^4 + 1/3*(10*b*c^3 +
 3*a*c)*d^2*x^3 + 1/2*(5*b*c^4 + 3*a*c^2)*d*x^2 + (b*c^5 + a*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(22) = 44$.

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.19

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx = bcd^4x^5 + \frac{bd^5x^6}{6} + x^4\left(\frac{ad^3}{4} + \frac{5bc^2d^3}{2}\right) + x^3\left(acd^2 + \frac{10bc^3d^2}{3}\right) + x^2 \cdot \left(\frac{3ac^2d}{2} + \frac{5bc^4d}{2}\right) + x\left(ac^3 + bc^5\right)$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**2),x)
```

output
```
b*c*d**4*x**5 + b*d**5*x**6/6 + x**4*(a*d**3/4 + 5*b*c**2*d**3/2) + x**3*(
a*c*d**2 + 10*b*c**3*d**2/3) + x**2*(3*a*c**2*d/2 + 5*b*c**4*d/2) + x*(a*c
**3 + b*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx = \frac{1}{6}\,bd^5x^6 + bcd^4x^5 + \frac{1}{4}\left(10\,bc^2 + a\right)d^3x^4 + \frac{1}{3}\left(10\,bc^3 + 3\,ac\right)d^2x^3 + \frac{1}{2}\left(5\,bc^4 + 3\,ac^2\right)dx^2 + \left(bc^5 + ac^3\right)x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/6*b*d^5*x^6 + b*c*d^4*x^5 + 1/4*(10*b*c^2 + a)*d^3*x^4 + 1/3*(10*b*c^3 +
 3*a*c)*d^2*x^3 + 1/2*(5*b*c^4 + 3*a*c^2)*d*x^2 + (b*c^5 + a*c^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(27) = 54$.

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx = \frac{1}{2}\left(dx^2+2\,cx\right)bc^4 + \frac{1}{2}\left(dx^2+2\,cx\right)^2 bc^2 d + \frac{1}{6}\left(dx^2+2\,cx\right)^3 bd^2 + \frac{1}{2}\left(dx^2+2\,cx\right)ac^2 + \frac{1}{4}\left(dx^2+2\,cx\right)^2 ad$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(d*x^2 + 2*c*x)*b*c^4 + 1/2*(d*x^2 + 2*c*x)^2*b*c^2*d + 1/6*(d*x^2 + 2
*c*x)^3*b*d^2 + 1/2*(d*x^2 + 2*c*x)*a*c^2 + 1/4*(d*x^2 + 2*c*x)^2*a*d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx = x\left(b\,c^5+a\,c^3\right) + \frac{d^3\,x^4\left(10\,b\,c^2+a\right)}{4} + \frac{b\,d^5\,x^6}{6} + \frac{c^2\,d\,x^2\left(5\,b\,c^2+3\,a\right)}{2} + \frac{c\,d^2\,x^3\left(10\,b\,c^2+3\,a\right)}{3} + b\,c\,d^4\,x^5$$

input
```
int((a + b*(c + d*x)^2)*(c + d*x)^3,x)
```

output
```
x*(a*c^3 + b*c^5) + (d^3*x^4*(a + 10*b*c^2))/4 + (b*d^5*x^6)/6 + (c^2*d*x^
2*(3*a + 5*b*c^2))/2 + (c*d^2*x^3*(3*a + 10*b*c^2))/3 + b*c*d^4*x^5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.03

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)\,dx$$
$$=\frac{x(2b\,d^5x^5+12bc\,d^4x^4+30b\,c^2d^3x^3+40b\,c^3d^2x^2+3a\,d^3x^3+30b\,c^4dx+12ac\,d^2x^2+12b\,c^5+18a\,c^2dx+}{12}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2),x)
```

output
```
(x*(12*a*c**3 + 18*a*c**2*d*x + 12*a*c*d**2*x**2 + 3*a*d**3*x**3 + 12*b*c*
**5 + 30*b*c**4*d*x + 40*b*c**3*d**2*x**2 + 30*b*c**2*d**3*x**3 + 12*b*c*d*
**4*x**4 + 2*b*d**5*x**5))/12
```

3.41 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^2 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 51

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^2 \, dx = \frac{a^2(c+dx)^4}{4d} + \frac{ab(c+dx)^6}{3d} + \frac{b^2(c+dx)^8}{8d}$$

output

```
1/4*a^2*(d*x+c)^4/d+1/3*a*b*(d*x+c)^6/d+1/8*b^2*(d*x+c)^8/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(51) = 102$.

Time = 0.02 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.37

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^2\right)^2 \, dx = {} & c^3\left(a+bc^2\right)^2 x + \frac{1}{2}c^2\left(3a^2+10abc^2+7b^2c^4\right) dx^2 \\ & + \frac{1}{3}c\left(3a^2+20abc^2+21b^2c^4\right) d^2x^3 \\ & + \frac{1}{4}\left(a^2+20abc^2+35b^2c^4\right) d^3x^4 + bc\left(2a+7bc^2\right) d^4x^5 \\ & + \frac{1}{6}b\left(2a+21bc^2\right) d^5x^6 + b^2cd^6x^7 + \frac{1}{8}b^2d^7x^8\end{aligned}$$

input

```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^2,x]
```

output
```
c^3*(a + b*c^2)^2*x + (c^2*(3*a^2 + 10*a*b*c^2 + 7*b^2*c^4)*d*x^2)/2 + (c*
(3*a^2 + 20*a*b*c^2 + 21*b^2*c^4)*d^2*x^3)/3 + ((a^2 + 20*a*b*c^2 + 35*b^2
*c^4)*d^3*x^4)/4 + b*c*(2*a + 7*b*c^2)*d^4*x^5 + (b*(2*a + 21*b*c^2)*d^5*x
^6)/6 + b^2*c*d^6*x^7 + (b^2*d^7*x^8)/8
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^2 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^2+a\right)^2 d(c+dx)}{d}$$

$$\downarrow 243$$

$$\frac{\int (c+dx)^2 \left(b(c+dx)^2+a\right)^2 d(c+dx)^2}{2d}$$

$$\downarrow 49$$

$$\frac{\int \left(b^2(c+dx)^6+2ab(c+dx)^4+a^2(c+dx)^2\right) d(c+dx)^2}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}a^2(c+dx)^4+\frac{2}{3}ab(c+dx)^6+\frac{1}{4}b^2(c+dx)^8}{2d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^2)^2,x]
```

output
```
((a^2*(c + d*x)^4)/2 + (2*a*b*(c + d*x)^6)/3 + (b^2*(c + d*x)^8)/4)/(2*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\frac{a\left(a+b(xd+c)^2\right)^3}{3}-\frac{\left(a+b(xd+c)^2\right)^4}{4}}{2db^2}$
norman	$\frac{d^7b^2x^8}{8}+cd^6b^2x^7+\left(\frac{7}{2}b^2c^2d^5+\frac{1}{3}abd^5\right)x^6+\left(7c^3b^2d^4+2abcd^4\right)x^5+\left(\frac{35}{4}c^4b^2d^3+5abc^2d^3+$
gosper	$\frac{x(3d^7b^2x^7+24cd^6b^2x^6+84x^5b^2c^2d^5+168b^2c^3d^4x^4+8x^5abd^5+210x^3c^4b^2d^3+48abcd^4x^4+168x^2b^2c^5d^2+120x^3abc^2d^3+84x}{24}$
parallelrisch	$b^2c^7x+2bac^5x+a^2c^3x+\frac{7}{2}db^2c^6x^2+5dbac^4x^2+\frac{3}{2}a^2c^2dx^2+7d^2b^2c^5x^3+\frac{20}{3}d^2bac^3x^3+$
orering	$\frac{x(3d^7b^2x^7+24cd^6b^2x^6+84x^5b^2c^2d^5+168b^2c^3d^4x^4+8x^5abd^5+210x^3c^4b^2d^3+48abcd^4x^4+168x^2b^2c^5d^2+120x^3abc^2d^3+84x}{24(bd^2x^2+2bcdx+b}$
risch	$\frac{3a^2c^2dx^2}{2}-\frac{a^4}{24db^2}+\frac{a^2d^3x^4}{4}+2d^4bacx^5+5d^3bac^2x^4+\frac{20d^2bac^3x^3}{3}+5dbac^4x^2+a^2cd^2x^3+2b$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/b^2*(1/3*a*(a+b*(d*x+c)^2)^3-1/4*(a+b*(d*x+c)^2)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(45) = 90$.

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.45

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx = {} & \frac{1}{8}\,b^2d^7x^8 + b^2cd^6x^7 + \frac{1}{6}\left(21\,b^2c^2+2\,ab\right)d^5x^6 \\ & + \left(7\,b^2c^3+2\,abc\right)d^4x^5 \\ & + \frac{1}{4}\left(35\,b^2c^4+20\,abc^2+a^2\right)d^3x^4 \\ & + \frac{1}{3}\left(21\,b^2c^5+20\,abc^3+3\,a^2c\right)d^2x^3 \\ & + \frac{1}{2}\left(7\,b^2c^6+10\,abc^4+3\,a^2c^2\right)dx^2 \\ & + \left(b^2c^7+2\,abc^5+a^2c^3\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*d^7*x^8 + b^2*c*d^6*x^7 + 1/6*(21*b^2*c^2 + 2*a*b)*d^5*x^6 + (7*b^
2*c^3 + 2*a*b*c)*d^4*x^5 + 1/4*(35*b^2*c^4 + 20*a*b*c^2 + a^2)*d^3*x^4 + 1
/3*(21*b^2*c^5 + 20*a*b*c^3 + 3*a^2*c)*d^2*x^3 + 1/2*(7*b^2*c^6 + 10*a*b*c
^4 + 3*a^2*c^2)*d*x^2 + (b^2*c^7 + 2*a*b*c^5 + a^2*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.10

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx = {} & b^2cd^6x^7+\frac{b^2d^7x^8}{8}+x^6\left(\frac{abd^5}{3}+\frac{7b^2c^2d^5}{2}\right)\\ & +x^5\cdot\left(2abcd^4+7b^2c^3d^4\right)\\ & +x^4\left(\frac{a^2d^3}{4}+5abc^2d^3+\frac{35b^2c^4d^3}{4}\right)\\ & +x^3\left(a^2cd^2+\frac{20abc^3d^2}{3}+7b^2c^5d^2\right)\\ & +x^2\cdot\left(\frac{3a^2c^2d}{2}+5abc^4d+\frac{7b^2c^6d}{2}\right)\\ & +x\left(a^2c^3+2abc^5+b^2c^7\right)\end{aligned}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**2)**2,x)
```

output
```
b**2*c*d**6*x**7 + b**2*d**7*x**8/8 + x**6*(a*b*d**5/3 + 7*b**2*c**2*d**5/
2) + x**5*(2*a*b*c*d**4 + 7*b**2*c**3*d**4) + x**4*(a**2*d**3/4 + 5*a*b*c*
*2*d**3 + 35*b**2*c**4*d**3/4) + x**3*(a**2*c*d**2 + 20*a*b*c**3*d**2/3 +
7*b**2*c**5*d**2) + x**2*(3*a**2*c**2*d/2 + 5*a*b*c**4*d + 7*b**2*c**6*d/2
) + x*(a**2*c**3 + 2*a*b*c**5 + b**2*c**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(45) = 90.

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.45

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx = {} & \frac{1}{8}\,b^2d^7x^8+b^2cd^6x^7+\frac{1}{6}\left(21\,b^2c^2+2\,ab\right)d^5x^6\\ & +\left(7\,b^2c^3+2\,abc\right)d^4x^5\\ & +\frac{1}{4}\left(35\,b^2c^4+20\,abc^2+a^2\right)d^3x^4\\ & +\frac{1}{3}\left(21\,b^2c^5+20\,abc^3+3\,a^2c\right)d^2x^3\\ & +\frac{1}{2}\left(7\,b^2c^6+10\,abc^4+3\,a^2c^2\right)dx^2\\ & +\left(b^2c^7+2\,abc^5+a^2c^3\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*d^7*x^8 + b^2*c*d^6*x^7 + 1/6*(21*b^2*c^2 + 2*a*b)*d^5*x^6 + (7*b^
2*c^3 + 2*a*b*c)*d^4*x^5 + 1/4*(35*b^2*c^4 + 20*a*b*c^2 + a^2)*d^3*x^4 + 1
/3*(21*b^2*c^5 + 20*a*b*c^3 + 3*a^2*c)*d^2*x^3 + 1/2*(7*b^2*c^6 + 10*a*b*c
^4 + 3*a^2*c^2)*d*x^2 + (b^2*c^7 + 2*a*b*c^5 + a^2*c^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(45) = 90$.

Time $= 0.12$ (sec) , antiderivative size $= 173$, normalized size of antiderivative $= 3.39$

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx = {} & \frac{1}{2}\left(dx^2+2\,cx\right)b^2c^6+\frac{3}{4}\left(dx^2+2\,cx\right)^2b^2c^4d \\ & +\frac{1}{2}\left(dx^2+2\,cx\right)^3b^2c^2d^2 \\ & +\frac{1}{8}\left(dx^2+2\,cx\right)^4b^2d^3+\left(dx^2+2\,cx\right)abc^4 \\ & +\left(dx^2+2\,cx\right)^2abc^2d+\frac{1}{3}\left(dx^2+2\,cx\right)^3abd^2 \\ & +\frac{1}{2}\left(dx^2+2\,cx\right)a^2c^2+\frac{1}{4}\left(dx^2+2\,cx\right)^2a^2d\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(d*x^2 + 2*c*x)*b^2*c^6 + 3/4*(d*x^2 + 2*c*x)^2*b^2*c^4*d + 1/2*(d*x^2
 + 2*c*x)^3*b^2*c^2*d^2 + 1/8*(d*x^2 + 2*c*x)^4*b^2*d^3 + (d*x^2 + 2*c*x)*
a*b*c^4 + (d*x^2 + 2*c*x)^2*a*b*c^2*d + 1/3*(d*x^2 + 2*c*x)^3*a*b*d^2 + 1/
2*(d*x^2 + 2*c*x)*a^2*c^2 + 1/4*(d*x^2 + 2*c*x)^2*a^2*d
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.18

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx = {} & c^3\,x\left(b\,c^2+a\right)^2+\frac{b^2\,d^7\,x^8}{8}\\ & +\frac{d^3\,x^4\left(a^2+20\,a\,b\,c^2+35\,b^2\,c^4\right)}{4}\\ & +\frac{b\,d^5\,x^6\left(21\,b\,c^2+2\,a\right)}{6}+b^2\,c\,d^6\,x^7\\ & +\frac{c^2\,d\,x^2\left(3\,a^2+10\,a\,b\,c^2+7\,b^2\,c^4\right)}{2}\\ & +\frac{c\,d^2\,x^3\left(3\,a^2+20\,a\,b\,c^2+21\,b^2\,c^4\right)}{3}\\ & +b\,c\,d^4\,x^5\left(7\,b\,c^2+2\,a\right)\end{aligned}$$

input
```
int((a + b*(c + d*x)^2)^2*(c + d*x)^3,x)
```

output
```
c^3*x*(a + b*c^2)^2 + (b^2*d^7*x^8)/8 + (d^3*x^4*(a^2 + 35*b^2*c^4 + 20*a*
b*c^2))/4 + (b*d^5*x^6*(2*a + 21*b*c^2))/6 + b^2*c*d^6*x^7 + (c^2*d*x^2*(3
*a^2 + 7*b^2*c^4 + 10*a*b*c^2))/2 + (c*d^2*x^3*(3*a^2 + 21*b^2*c^4 + 20*a*
b*c^2))/3 + b*c*d^4*x^5*(2*a + 7*b*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.02

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)^2\,dx$$
$$= \frac{x(3b^2d^7x^7+24b^2cd^6x^6+84b^2c^2d^5x^5+168b^2c^3d^4x^4+8abd^5x^5+210b^2c^4d^3x^3+48abcd^4x^4+168b^2c^5d^2}{}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2)^2,x)
```

output

```
(x*(24*a**2*c**3 + 36*a**2*c**2*d*x + 24*a**2*c*d**2*x**2 + 6*a**2*d**3*x*
*3 + 48*a*b*c**5 + 120*a*b*c**4*d*x + 160*a*b*c**3*d**2*x**2 + 120*a*b*c**
2*d**3*x**3 + 48*a*b*c*d**4*x**4 + 8*a*b*d**5*x**5 + 24*b**2*c**7 + 84*b**
2*c**6*d*x + 168*b**2*c**5*d**2*x**2 + 210*b**2*c**4*d**3*x**3 + 168*b**2*
c**3*d**4*x**4 + 84*b**2*c**2*d**5*x**5 + 24*b**2*c*d**6*x**6 + 3*b**2*d**
7*x**7))/24
```

3.42 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 \, dx = -\frac{a(a+b(c+dx)^2)^4}{8b^2d} + \frac{(a+b(c+dx)^2)^5}{10b^2d}$$

output
```
-1/8*a*(a+b*(d*x+c)^2)^4/b^2/d+1/10*(a+b*(d*x+c)^2)^5/b^2/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(48) = 96.

Time = 0.02 (sec) , antiderivative size = 249, normalized size of antiderivative = 5.19

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 \, dx = {} & c^3\left(a+bc^2\right)^3 x + \frac{3}{2}c^2\left(a+bc^2\right)^2\left(a+3bc^2\right) dx^2 \\ & + c\left(a^3+10a^2bc^2+21ab^2c^4+12b^3c^6\right) d^2x^3 \\ & + \frac{1}{4}\left(a^3+30a^2bc^2+105ab^2c^4+84b^3c^6\right) d^3x^4 \\ & + \frac{3}{5}bc\left(5a^2+35abc^2+42b^2c^4\right) d^4x^5 \\ & + \frac{1}{2}b\left(a^2+21abc^2+42b^2c^4\right) d^5x^6 + 3b^2c\left(a+4bc^2\right) d^6x^7 \\ & + \frac{3}{8}b^2\left(a+12bc^2\right) d^7x^8 + b^3cd^8x^9 + \frac{1}{10}b^3d^9x^{10}\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^3,x]
```

output
```
c^3*(a + b*c^2)^3*x + (3*c^2*(a + b*c^2)^2*(a + 3*b*c^2)*d*x^2)/2 + c*(a^3
 + 10*a^2*b*c^2 + 21*a*b^2*c^4 + 12*b^3*c^6)*d^2*x^3 + ((a^3 + 30*a^2*b*c^
2 + 105*a*b^2*c^4 + 84*b^3*c^6)*d^3*x^4)/4 + (3*b*c*(5*a^2 + 35*a*b*c^2 +
42*b^2*c^4)*d^4*x^5)/5 + (b*(a^2 + 21*a*b*c^2 + 42*b^2*c^4)*d^5*x^6)/2 + 3
*b^2*c*(a + 4*b*c^2)*d^6*x^7 + (3*b^2*(a + 12*b*c^2)*d^7*x^8)/8 + b^3*c*d^
8*x^9 + (b^3*d^9*x^10)/10
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^3 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^2+a\right)^3 d(c+dx)}{d}$$

$$\downarrow 243$$

$$\frac{\int (c+dx)^2 \left(b(c+dx)^2+a\right)^3 d(c+dx)^2}{2d}$$

$$\downarrow 49$$

$$\frac{\int \left(\frac{(b(c+dx)^2+a)^4}{b} - \frac{a(b(c+dx)^2+a)^3}{b}\right) d(c+dx)^2}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{(a+b(c+dx)^2)^5}{5b^2} - \frac{a(a+b(c+dx)^2)^4}{4b^2}}{2d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^2)^3,x]
```

output
```
(-1/4*(a*(a + b*(c + d*x)^2)^4)/b^2 + (a + b*(c + d*x)^2)^5/(5*b^2))/(2*d)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result
default	$-\frac{\frac{a\left(a+b(xd+c)^2\right)^4}{4}-\frac{\left(a+b(xd+c)^2\right)^5}{5}}{2db^2}$
norman	$\frac{d^9b^3x^{10}}{10}+cd^8b^3x^9+\left(\frac{9}{2}c^2d^7b^3+\frac{3}{8}ab^2d^7\right)x^8+\left(12c^3b^3d^6+3ab^2cd^6\right)x^7+\left(21c^4b^3d^5+\frac{21}{2}ab$
gosper	$\frac{x\left(4d^9b^3x^9+40cd^8b^3x^8+180x^7c^2d^7b^3+480b^3c^3d^6x^6+15x^7ab^2d^7+840x^5c^4b^3d^5+120ab^2cd^6x^6+1008x^4b^3c^5d^4+420x^5ab^2c\right.}{}$
parallelrisch	$3ba^2c^5x+\frac{3}{2}da^3c^2x^2+\frac{15}{2}dba^2c^4x^2+d^2a^3cx^3+a^3c^3x+\frac{21}{2}d^5b^2ac^2x^6+21d^4b^2ac^3x^5+\frac{105}{4}$
orering	$\frac{x\left(4d^9b^3x^9+40cd^8b^3x^8+180x^7c^2d^7b^3+480b^3c^3d^6x^6+15x^7ab^2d^7+840x^5c^4b^3d^5+120ab^2cd^6x^6+1008x^4b^3c^5d^4+420x^5ab^2c\right.}{}$
risch	$3ba^2c^5x+\frac{3da^3c^2x^2}{2}+\frac{15dba^2c^4x^2}{2}+d^2a^3cx^3+a^3c^3x+\frac{21d^5b^2ac^2x^6}{2}+21d^4b^2ac^3x^5+\frac{105d^3b^2ac^4}{4}$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/b^2*(1/4*a*(a+b*(d*x+c)^2)^4-1/5*(a+b*(d*x+c)^2)^5)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(44) = 88$.

Time = 0.08 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.92

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx = {} & \frac{1}{10}b^3d^9x^{10}+b^3cd^8x^9+\frac{3}{8}\left(12b^3c^2+ab^2\right)d^7x^8\\ & +3\left(4b^3c^3+ab^2c\right)d^6x^7\\ & +\frac{1}{2}\left(42b^3c^4+21ab^2c^2+a^2b\right)d^5x^6\\ & +\frac{3}{5}\left(42b^3c^5+35ab^2c^3+5a^2bc\right)d^4x^5\\ & +\frac{1}{4}\left(84b^3c^6+105ab^2c^4+30a^2bc^2+a^3\right)d^3x^4\\ & +\left(12b^3c^7+21ab^2c^5+10a^2bc^3+a^3c\right)d^2x^3\\ & +\frac{3}{2}\left(3b^3c^8+7ab^2c^6+5a^2bc^4+a^3c^2\right)dx^2\\ & +\left(b^3c^9+3ab^2c^7+3a^2bc^5+a^3c^3\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

output

```
1/10*b^3*d^9*x^10 + b^3*c*d^8*x^9 + 3/8*(12*b^3*c^2 + a*b^2)*d^7*x^8 + 3*(
4*b^3*c^3 + a*b^2*c)*d^6*x^7 + 1/2*(42*b^3*c^4 + 21*a*b^2*c^2 + a^2*b)*d^5
*x^6 + 3/5*(42*b^3*c^5 + 35*a*b^2*c^3 + 5*a^2*b*c)*d^4*x^5 + 1/4*(84*b^3*c
^6 + 105*a*b^2*c^4 + 30*a^2*b*c^2 + a^3)*d^3*x^4 + (12*b^3*c^7 + 21*a*b^2*
c^5 + 10*a^2*b*c^3 + a^3*c)*d^2*x^3 + 3/2*(3*b^3*c^8 + 7*a*b^2*c^6 + 5*a^2
*b*c^4 + a^3*c^2)*d*x^2 + (b^3*c^9 + 3*a*b^2*c^7 + 3*a^2*b*c^5 + a^3*c^3)*
x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 7.44

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx = {} & b^3cd^8x^9 + \frac{b^3d^9x^{10}}{10} + x^8\cdot\left(\frac{3ab^2d^7}{8} + \frac{9b^3c^2d^7}{2}\right) \\ & + x^7\cdot\left(3ab^2cd^6 + 12b^3c^3d^6\right) \\ & + x^6\left(\frac{a^2bd^5}{2} + \frac{21ab^2c^2d^5}{2} + 21b^3c^4d^5\right) \\ & + x^5\cdot\left(3a^2bcd^4 + 21ab^2c^3d^4 + \frac{126b^3c^5d^4}{5}\right) \\ & + x^4\left(\frac{a^3d^3}{4} + \frac{15a^2bc^2d^3}{2} + \frac{105ab^2c^4d^3}{4} + 21b^3c^6d^3\right) \\ & + x^3\left(a^3cd^2 + 10a^2bc^3d^2 + 21ab^2c^5d^2 + 12b^3c^7d^2\right) \\ & + x^2\cdot\left(\frac{3a^3c^2d}{2} + \frac{15a^2bc^4d}{2} + \frac{21ab^2c^6d}{2} + \frac{9b^3c^8d}{2}\right) \\ & + x\left(a^3c^3 + 3a^2bc^5 + 3ab^2c^7 + b^3c^9\right)\end{aligned}$$

input

```
integrate((d*x+c)**3*(a+b*(d*x+c)**2)**3,x)
```

output

```
b**3*c*d**8*x**9 + b**3*d**9*x**10/10 + x**8*(3*a*b**2*d**7/8 + 9*b**3*c**
2*d**7/2) + x**7*(3*a*b**2*c*d**6 + 12*b**3*c**3*d**6) + x**6*(a**2*b*d**5
/2 + 21*a*b**2*c**2*d**5/2 + 21*b**3*c**4*d**5) + x**5*(3*a**2*b*c*d**4 +
21*a*b**2*c**3*d**4 + 126*b**3*c**5*d**4/5) + x**4*(a**3*d**3/4 + 15*a**2*
b*c**2*d**3/2 + 105*a*b**2*c**4*d**3/4 + 21*b**3*c**6*d**3) + x**3*(a**3*c
*d**2 + 10*a**2*b*c**3*d**2 + 21*a*b**2*c**5*d**2 + 12*b**3*c**7*d**2) + x
**2*(3*a**3*c**2*d/2 + 15*a**2*b*c**4*d/2 + 21*a*b**2*c**6*d/2 + 9*b**3*c*
*8*d/2) + x*(a**3*c**3 + 3*a**2*b*c**5 + 3*a*b**2*c**7 + b**3*c**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.92

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx = {} & \frac{1}{10}\,b^3d^9x^{10} + b^3cd^8x^9 + \frac{3}{8}\left(12\,b^3c^2+ab^2\right)d^7x^8 \\ & + 3\left(4\,b^3c^3+ab^2c\right)d^6x^7 \\ & + \frac{1}{2}\left(42\,b^3c^4+21\,ab^2c^2+a^2b\right)d^5x^6 \\ & + \frac{3}{5}\left(42\,b^3c^5+35\,ab^2c^3+5\,a^2bc\right)d^4x^5 \\ & + \frac{1}{4}\left(84\,b^3c^6+105\,ab^2c^4+30\,a^2bc^2+a^3\right)d^3x^4 \\ & + \left(12\,b^3c^7+21\,ab^2c^5+10\,a^2bc^3+a^3c\right)d^2x^3 \\ & + \frac{3}{2}\left(3\,b^3c^8+7\,ab^2c^6+5\,a^2bc^4+a^3c^2\right)dx^2 \\ & + \left(b^3c^9+3\,ab^2c^7+3\,a^2bc^5+a^3c^3\right)x\end{aligned}$$

input

```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
1/10*b^3*d^9*x^10 + b^3*c*d^8*x^9 + 3/8*(12*b^3*c^2 + a*b^2)*d^7*x^8 + 3*(
4*b^3*c^3 + a*b^2*c)*d^6*x^7 + 1/2*(42*b^3*c^4 + 21*a*b^2*c^2 + a^2*b)*d^5
*x^6 + 3/5*(42*b^3*c^5 + 35*a*b^2*c^3 + 5*a^2*b*c)*d^4*x^5 + 1/4*(84*b^3*c
^6 + 105*a*b^2*c^4 + 30*a^2*b*c^2 + a^3)*d^3*x^4 + (12*b^3*c^7 + 21*a*b^2*
c^5 + 10*a^2*b*c^3 + a^3*c)*d^2*x^3 + 3/2*(3*b^3*c^8 + 7*a*b^2*c^6 + 5*a^2
*b*c^4 + a^3*c^2)*d*x^2 + (b^3*c^9 + 3*a*b^2*c^7 + 3*a^2*b*c^5 + a^3*c^3)*
x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(44) = 88$.

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 6.00

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx = {} & \frac{1}{2}\left(dx^2+2\,cx\right)b^3c^8+\left(dx^2+2\,cx\right)^2b^3c^6d \\ & +\left(dx^2+2\,cx\right)^3b^3c^4d^2+\frac{1}{2}\left(dx^2+2\,cx\right)^4b^3c^2d^3 \\ & +\frac{1}{10}\left(dx^2+2\,cx\right)^5b^3d^4+\frac{3}{2}\left(dx^2+2\,cx\right)ab^2c^6 \\ & +\frac{9}{4}\left(dx^2+2\,cx\right)^2ab^2c^4d+\frac{3}{2}\left(dx^2+2\,cx\right)^3ab^2c^2d^2 \\ & +\frac{3}{8}\left(dx^2+2\,cx\right)^4ab^2d^3+\frac{3}{2}\left(dx^2+2\,cx\right)a^2bc^4 \\ & +\frac{3}{2}\left(dx^2+2\,cx\right)^2a^2bc^2d+\frac{1}{2}\left(dx^2+2\,cx\right)^3a^2bd^2 \\ & +\frac{1}{2}\left(dx^2+2\,cx\right)a^3c^2+\frac{1}{4}\left(dx^2+2\,cx\right)^2a^3d\end{aligned}$$

input

```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

output

```
1/2*(d*x^2 + 2*c*x)*b^3*c^8 + (d*x^2 + 2*c*x)^2*b^3*c^6*d + (d*x^2 + 2*c*x
)^3*b^3*c^4*d^2 + 1/2*(d*x^2 + 2*c*x)^4*b^3*c^2*d^3 + 1/10*(d*x^2 + 2*c*x)
^5*b^3*d^4 + 3/2*(d*x^2 + 2*c*x)*a*b^2*c^6 + 9/4*(d*x^2 + 2*c*x)^2*a*b^2*c
^4*d + 3/2*(d*x^2 + 2*c*x)^3*a*b^2*c^2*d^2 + 3/8*(d*x^2 + 2*c*x)^4*a*b^2*d
^3 + 3/2*(d*x^2 + 2*c*x)*a^2*b*c^4 + 3/2*(d*x^2 + 2*c*x)^2*a^2*b*c^2*d + 1
/2*(d*x^2 + 2*c*x)^3*a^2*b*d^2 + 1/2*(d*x^2 + 2*c*x)*a^3*c^2 + 1/4*(d*x^2
+ 2*c*x)^2*a^3*d
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.94

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx &= c^3\,x\left(b\,c^2+a\right)^3\\ &+\frac{d^3\,x^4\left(a^3+30\,a^2\,b\,c^2+105\,a\,b^2\,c^4+84\,b^3\,c^6\right)}{4}\\ &+\frac{b^3\,d^9\,x^{10}}{10}+\frac{3\,b^2\,d^7\,x^8\left(12\,b\,c^2+a\right)}{8}\\ &+b^3\,c\,d^8\,x^9+\frac{b\,d^5\,x^6\left(a^2+21\,a\,b\,c^2+42\,b^2\,c^4\right)}{2}\\ &+c\,d^2\,x^3\left(a^3+10\,a^2\,b\,c^2+21\,a\,b^2\,c^4+12\,b^3\,c^6\right)\\ &+\frac{3\,b\,c\,d^4\,x^5\left(5\,a^2+35\,a\,b\,c^2+42\,b^2\,c^4\right)}{5}\\ &+\frac{3\,c^2\,d\,x^2\left(b\,c^2+a\right)^2\left(3\,b\,c^2+a\right)}{2}\\ &+3\,b^2\,c\,d^6\,x^7\left(4\,b\,c^2+a\right)\end{aligned}$$

input

```
int((a + b*(c + d*x)^2)^3*(c + d*x)^3,x)
```

output

```
c^3*x*(a + b*c^2)^3 + (d^3*x^4*(a^3 + 84*b^3*c^6 + 30*a^2*b*c^2 + 105*a*b^
2*c^4))/4 + (b^3*d^9*x^10)/10 + (3*b^2*d^7*x^8*(a + 12*b*c^2))/8 + b^3*c*d
^8*x^9 + (b*d^5*x^6*(a^2 + 42*b^2*c^4 + 21*a*b*c^2))/2 + c*d^2*x^3*(a^3 +
12*b^3*c^6 + 10*a^2*b*c^2 + 21*a*b^2*c^4) + (3*b*c*d^4*x^5*(5*a^2 + 42*b^2
*c^4 + 35*a*b*c^2))/5 + (3*c^2*d*x^2*(a + b*c^2)^2*(a + 3*b*c^2))/2 + 3*b^
2*c*d^6*x^7*(a + 4*b*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 350, normalized size of antiderivative = 7.29

$$\int (c+dx)^3\left(a+b(c+dx)^2\right)^3\,dx$$
$$=\frac{x(4b^3d^9x^9+40b^3c\,d^8x^8+180b^3c^2d^7x^7+480b^3c^3d^6x^6+15a\,b^2d^7x^7+840b^3c^4d^5x^5+120a\,b^2c\,d^6x^6+100$$

input

```
int((d*x+c)^3*(a+b*(d*x+c)^2)^3,x)
```

output

```
(x*(40*a**3*c**3 + 60*a**3*c**2*d*x + 40*a**3*c*d**2*x**2 + 10*a**3*d**3*x
**3 + 120*a**2*b*c**5 + 300*a**2*b*c**4*d*x + 400*a**2*b*c**3*d**2*x**2 +
300*a**2*b*c**2*d**3*x**3 + 120*a**2*b*c*d**4*x**4 + 20*a**2*b*d**5*x**5 +
 120*a*b**2*c**7 + 420*a*b**2*c**6*d*x + 840*a*b**2*c**5*d**2*x**2 + 1050*
a*b**2*c**4*d**3*x**3 + 840*a*b**2*c**3*d**4*x**4 + 420*a*b**2*c**2*d**5*x
**5 + 120*a*b**2*c*d**6*x**6 + 15*a*b**2*d**7*x**7 + 40*b**3*c**9 + 180*b*
*3*c**8*d*x + 480*b**3*c**7*d**2*x**2 + 840*b**3*c**6*d**3*x**3 + 1008*b**
3*c**5*d**4*x**4 + 840*b**3*c**4*d**5*x**5 + 480*b**3*c**3*d**6*x**6 + 180
*b**3*c**2*d**7*x**7 + 40*b**3*c*d**8*x**8 + 4*b**3*d**9*x**9))/40
```

3.43 $\int \frac{2+x}{1+(2+x)^2}\,dx$

Optimal result 311
Mathematica [A] (verified) 311
Rubi [A] (verified) 312
Maple [A] (verified) 313
Fricas [A] (verification not implemented) 313
Sympy [A] (verification not implemented) 313
Maxima [A] (verification not implemented) 314
Giac [A] (verification not implemented) 314
Mupad [B] (verification not implemented) 314
Reduce [B] (verification not implemented) 315

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{1}{2}\log\left(1+(2+x)^2\right)$$

output
```
1/2*ln(1+(2+x)^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{1}{2}\log\left(1+(2+x)^2\right)$$

input
```
Integrate[(2 + x)/(1 + (2 + x)^2),x]
```

output
```
Log[1 + (2 + x)^2]/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {895, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(x+2)^2+1}\,dx$$
$$\downarrow 895$$
$$\int \frac{x+2}{(x+2)^2+1} d(x+2)$$
$$\downarrow 240$$
$$\frac{1}{2}\log\left((x+2)^2+1\right)$$

input
```
Int[(2 + x)/(1 + (2 + x)^2),x]
```

output
```
Log[1 + (2 + x)^2]/2
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\ln(x^2+4x+5)}{2}$	12
norman	$\frac{\ln(x^2+4x+5)}{2}$	12
risch	$\frac{\ln(x^2+4x+5)}{2}$	12
parallelrisch	$\frac{\ln(x^2+4x+5)}{2}$	12

input
```
int((2+x)/(1+(2+x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+4*x+5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{1}{2}\log\left(x^2+4\,x+5\right)$$

input
```
integrate((2+x)/(1+(2+x)^2),x, algorithm="fricas")
```

output
```
1/2*log(x^2 + 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{\log\left(x^2+4x+5\right)}{2}$$

input
```
integrate((2+x)/(1+(2+x)**2),x)
```

output
```
log(x**2 + 4*x + 5)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{1}{2}\log\left((x+2)^2+1\right)$$

input
```
integrate((2+x)/(1+(2+x)^2),x, algorithm="maxima")
```

output
```
1/2*log((x + 2)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{1}{2}\log\left(x^2+4\,x+5\right)$$

input
```
integrate((2+x)/(1+(2+x)^2),x, algorithm="giac")
```

output
```
1/2*log(x^2 + 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{\ln\left((x+2)^2+1\right)}{2}$$

input
```
int((x + 2)/((x + 2)^2 + 1),x)
```

output
```
log((x + 2)^2 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{1+(2+x)^2}\,dx = \frac{\log(x^2+4x+5)}{2}$$

input
```
int((2+x)/(1+(2+x)^2),x)
```

output
```
log(x**2 + 4*x + 5)/2
```

3.44 $\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx$

Optimal result 316
Mathematica [A] (verified) 316
Rubi [A] (verified) 317
Maple [A] (verified) 318
Fricas [A] (verification not implemented) 318
Sympy [A] (verification not implemented) 319
Maxima [A] (verification not implemented) 319
Giac [A] (verification not implemented) 319
Mupad [B] (verification not implemented) 320
Reduce [B] (verification not implemented) 320

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left(1+(2+x)^2\right)}$$

output
```
-1/2/(1+(2+x)^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left(1+(2+x)^2\right)}$$

input
```
Integrate[(2 + x)/(1 + (2 + x)^2)^2,x]
```

output
```
-1/2*1/(1 + (2 + x)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {895, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{\left((x+2)^2+1\right)^2}\,dx$$
$$\downarrow 895$$
$$\int \frac{x+2}{\left((x+2)^2+1\right)^2} d(x+2)$$
$$\downarrow 241$$
$$-\frac{1}{2\left((x+2)^2+1\right)}$$

input
```
Int[(2 + x)/(1 + (2 + x)^2)^2,x]
```

output
```
-1/2*1/(1 + (2 + x)^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{2(x^2+4x+5)}$	13
default	$-\frac{1}{2(x^2+4x+5)}$	13
norman	$-\frac{1}{2(x^2+4x+5)}$	13
risch	$-\frac{1}{2(x^2+4x+5)}$	13
parallelrisch	$-\frac{1}{2(x^2+4x+5)}$	13
orering	$-\frac{x^2+4x+5}{2\left(1+(2+x)^2\right)^2}$	20

input
```
int((2+x)/(1+(2+x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2+4*x+5)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left(x^2+4\,x+5\right)}$$

input
```
integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="fricas")
```

output
```
-1/2/(x^2 + 4*x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2x^2+8x+10}$$

input
```
integrate((2+x)/(1+(2+x)**2)**2,x)
```

output
```
-1/(2*x**2 + 8*x + 10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left((x+2)^2+1\right)}$$

input
```
integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2/((x + 2)^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left(x^2+4\,x+5\right)}$$

input
```
integrate((2+x)/(1+(2+x)^2)^2,x, algorithm="giac")
```

output
```
-1/2/(x^2 + 4*x + 5)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2\left((x+2)^2+1\right)}$$

input
```
int((x + 2)/((x + 2)^2 + 1)^2,x)
```

output
```
-1/(2*((x + 2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^2}\,dx = -\frac{1}{2x^2+8x+10}$$

input
```
int((2+x)/(1+(2+x)^2)^2,x)
```

output
```
( - 1)/(2*(x**2 + 4*x + 5))
```

3.45 $\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx$

Optimal result 321
Mathematica [A] (verified) 321
Rubi [A] (verified) 322
Maple [A] (verified) 323
Fricas [A] (verification not implemented) 323
Sympy [A] (verification not implemented) 324
Maxima [A] (verification not implemented) 324
Giac [A] (verification not implemented) 324
Mupad [B] (verification not implemented) 325
Reduce [B] (verification not implemented) 325

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4\left(1+(2+x)^2\right)^2}$$

output
```
-1/4/(1+(2+x)^2)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4\left(1+(2+x)^2\right)^2}$$

input
```
Integrate[(2 + x)/(1 + (2 + x)^2)^3,x]
```

output
```
-1/4*1/(1 + (2 + x)^2)^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {895, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{((x+2)^2+1)^3}\,dx$$
$$\downarrow 895$$
$$\int \frac{x+2}{((x+2)^2+1)^3}\,d(x+2)$$
$$\downarrow 241$$
$$-\frac{1}{4\left((x+2)^2+1\right)^2}$$

input
```
Int[(2 + x)/(1 + (2 + x)^2)^3,x]
```

output
```
-1/4*1/(1 + (2 + x)^2)^2
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{4(x^2+4x+5)^2}$	13
default	$-\frac{1}{4(x^2+4x+5)^2}$	13
norman	$-\frac{1}{4(x^2+4x+5)^2}$	13
risch	$-\frac{1}{4(x^2+4x+5)^2}$	13
parallelrisch	$-\frac{1}{4(x^2+4x+5)^2}$	13
orering	$-\frac{x^2+4x+5}{4\left(1+(2+x)^2\right)^3}$	20

input
```
int((2+x)/(1+(2+x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^2+4*x+5)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{2+x}{(1+(2+x)^2)^3}\,dx = -\frac{1}{4\left(x^4+8\,x^3+26\,x^2+40\,x+25\right)}$$

input
```
integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="fricas")
```

output
```
-1/4/(x^4 + 8*x^3 + 26*x^2 + 40*x + 25)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4x^4+32x^3+104x^2+160x+100}$$

input
```
integrate((2+x)/(1+(2+x)**2)**3,x)
```

output
```
-1/(4*x**4 + 32*x**3 + 104*x**2 + 160*x + 100)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4\left((x+2)^2+1\right)^2}$$

input
```
integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="maxima")
```

output
```
-1/4/((x + 2)^2 + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4\left(x^2+4\,x+5\right)^2}$$

input
```
integrate((2+x)/(1+(2+x)^2)^3,x, algorithm="giac")
```

output
```
-1/4/(x^2 + 4*x + 5)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4\left((x+2)^2+1\right)^2}$$

input
```
int((x + 2)/((x + 2)^2 + 1)^3,x)
```

output
```
-1/(4*((x + 2)^2 + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{2+x}{\left(1+(2+x)^2\right)^3}\,dx = -\frac{1}{4x^4+32x^3+104x^2+160x+100}$$

input
```
int((2+x)/(1+(2+x)^2)^3,x)
```

output
```
( - 1)/(4*(x**4 + 8*x**3 + 26*x**2 + 40*x + 25))
```

3.46 $\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal result . . . 326
Mathematica [A] (verified) . . . 326
Rubi [A] (verified) . . . 327
Maple [B] (verified) . . . 328
Fricas [B] (verification not implemented) . . . 329
Sympy [F(-1)] . . . 330
Maxima [B] (verification not implemented) . . . 330
Giac [B] (verification not implemented) . . . 331
Mupad [B] (verification not implemented) . . . 332
Reduce [B] (verification not implemented) . . . 333

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx = \frac{a^2(a+b(c+dx)^2)^{1+p}}{2b^3d(1+p)} - \frac{a(a+b(c+dx)^2)^{2+p}}{b^3d(2+p)} + \frac{(a+b(c+dx)^2)^{3+p}}{2b^3d(3+p)}$$

output

```
1/2*a^2*(a+b*(d*x+c)^2)^(p+1)/b^3/d/(p+1)-a*(a+b*(d*x+c)^2)^(2+p)/b^3/d/(2
+p)+1/2*(a+b*(d*x+c)^2)^(3+p)/b^3/d/(3+p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx = \frac{(a+b(c+dx)^2)^{1+p}\left(\frac{a^2}{1+p} - \frac{2a(a+b(c+dx)^2)}{2+p} + \frac{(a+b(c+dx)^2)^2}{3+p}\right)}{2b^3d}$$

input

```
Integrate[(c + d*x)^5*(a + b*(c + d*x)^2)^p,x]
```

output
```
((a + b*(c + d*x)^2)^(1 + p)*(a^2/(1 + p) - (2*a*(a + b*(c + d*x)^2))/(2 +
 p) + (a + b*(c + d*x)^2)^2/(3 + p)))/(2*b^3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^5 \left(b(c+dx)^2+a\right)^p d(c+dx)}{d}$$

$$\downarrow 243$$

$$\frac{\int (c+dx)^4 \left(b(c+dx)^2+a\right)^p d(c+dx)^2}{2d}$$

$$\downarrow 53$$

$$\frac{\int \left(\frac{a^2 (b(c+dx)^2+a)^p}{b^2} - \frac{2a(b(c+dx)^2+a)^{p+1}}{b^2} + \frac{(b(c+dx)^2+a)^{p+2}}{b^2}\right) d(c+dx)^2}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2 (a+b(c+dx)^2)^{p+1}}{b^3(p+1)} - \frac{2a(a+b(c+dx)^2)^{p+2}}{b^3(p+2)} + \frac{(a+b(c+dx)^2)^{p+3}}{b^3(p+3)}}{2d}$$

input
```
Int[(c + d*x)^5*(a + b*(c + d*x)^2)^p,x]
```

output
```
((a^2*(a + b*(c + d*x)^2)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*(c + d*x)^2
)^(2 + p))/(b^3*(2 + p)) + (a + b*(c + d*x)^2)^(3 + p)/(b^3*(3 + p)))/(2*d
)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2  Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)  Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(89) = 178.

Time = 1.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.11

method	result
gosper	$\frac{(b d^2 x^2+2bcdx+b c^2+a)^{p+1}(b^2 d^4 p^2 x^4+4b^2 c d^3 p^2 x^3+3b^2 d^4 p x^4+6b^2 c^2 d^2 p^2 x^2+12b^2 c d^3 p x^3+2d^4 x^4 b^2+4b^2 c^3 d p^2 x+18b^2 c^2$
orering	$\frac{(b^2 d^4 p^2 x^4+4b^2 c d^3 p^2 x^3+3b^2 d^4 p x^4+6b^2 c^2 d^2 p^2 x^2+12b^2 c d^3 p x^3+2d^4 x^4 b^2+4b^2 c^3 d p^2 x+18b^2 c^2 d^2 p x^2+8c x^3 d^3 b^2+b^2 c^4 p^2+$
norman	$\frac{d^5 x^6 e^{p \ln\left(a+b(xd+c)^2\right)}}{6+2p}+\frac{3c d^4 x^5 e^{p \ln\left(a+b(xd+c)^2\right)}}{3+p}+\frac{(b^3 c^6 p^2+3b^3 c^6 p+a b^2 c^4 p^2+2b^3 c^6+a b^2 c^4 p-2a^2 b c^2 p+2a^3) e^{p \ln\left(a-\right.}}{2b^3(p^3+6p^2+11p+6)d}$
risch	$\frac{(b^3 d^6 p^2 x^6+6b^3 c d^5 p^2 x^5+3b^3 d^6 p x^6+15b^3 c^2 d^4 p^2 x^4+18b^3 c d^5 p x^5+2d^6 x^6 b^3+20b^3 c^3 d^3 p^2 x^3+45b^3 c^2 d^4 p x^4+12b^3 c d^5 x^5+a b$
parallelrisch	$\frac{4x\left(a+b(xd+c)^2\right)^p a b^2 c^3 d^2 p+4x\left(a+b(xd+c)^2\right)^p a b^2 c^3 d^2 p^2-4x\left(a+b(xd+c)^2\right)^p a^2 bc d^2 p+4x^3\left(a+b(xd+c)^2\right)^p a b^2 c d^4 p^2+$

input
```
int((d*x+c)^5*(a+b*(d*x+c)^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3/d/(p^3+6*p^2+11*p+6)*(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^(p+1)*(b^2*d^4*
p^2*x^4+4*b^2*c*d^3*p^2*x^3+3*b^2*d^4*p*x^4+6*b^2*c^2*d^2*p^2*x^2+12*b^2*c
*d^3*p*x^3+2*b^2*d^4*x^4+4*b^2*c^3*d*p^2*x+18*b^2*c^2*d^2*p*x^2+8*b^2*c*d^
3*x^3+b^2*c^4*p^2+12*b^2*c^3*d*p*x+12*b^2*c^2*d^2*x^2-2*a*b*d^2*p*x^2+3*b^
2*c^4*p+8*b^2*c^3*d*x-4*a*b*c*d*p*x-2*a*b*d^2*x^2+2*b^2*c^4-2*a*b*c^2*p-4*
a*b*c*d*x-2*a*b*c^2+2*a^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(89) = 178$.

Time $= 0.09$ (sec) , antiderivative size $= 438$, normalized size of antiderivative $= 4.71$

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \frac{(2\,b^3c^6 + (b^3d^6p^2 + 3\,b^3d^6p + 2\,b^3d^6)x^6 + 6\,(b^3cd^5p^2 + 3\,b^3cd^5p + 2\,b^3cd^5)x^5 + (30\,b^3c^2d^4 + (15\,b^3c^2 + ab^2$$

input
```
integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*c^6 + (b^3*d^6*p^2 + 3*b^3*d^6*p + 2*b^3*d^6)*x^6 + 6*(b^3*c*d^
5*p^2 + 3*b^3*c*d^5*p + 2*b^3*c*d^5)*x^5 + (30*b^3*c^2*d^4 + (15*b^3*c^2 +
 a*b^2)*d^4*p^2 + (45*b^3*c^2 + a*b^2)*d^4*p)*x^4 + 4*(10*b^3*c^3*d^3 + (5
*b^3*c^3 + a*b^2*c)*d^3*p^2 + (15*b^3*c^3 + a*b^2*c)*d^3*p)*x^3 + 2*a^3 +
(b^3*c^6 + a*b^2*c^4)*p^2 + (30*b^3*c^4*d^2 + 3*(5*b^3*c^4 + 2*a*b^2*c^2)*
d^2*p^2 + (45*b^3*c^4 + 6*a*b^2*c^2 - 2*a^2*b)*d^2*p)*x^2 + (3*b^3*c^6 + a
*b^2*c^4 - 2*a^2*b*c^2)*p + 2*(6*b^3*c^5*d + (3*b^3*c^5 + 2*a*b^2*c^3)*d*p
^2 + (9*b^3*c^5 + 2*a*b^2*c^3 - 2*a^2*b*c)*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + a)^p/(b^3*d*p^3 + 6*b^3*d*p^2 + 11*b^3*d*p + 6*b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**5*(a+b*(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(89) = 178$.

Time = 0.08 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.23

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \frac{((p^2+3p+2)b^3d^6x^6+6(p^2+3p+2)b^3cd^5x^5+(p^2+3p+2)b^3c^6+(p^2+p)ab^2c^4-2a^2bc^2p+(15(p^2$$

input
```
integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*b^3*d^6*x^6 + 6*(p^2 + 3*p + 2)*b^3*c*d^5*x^5 + (p^2
+ 3*p + 2)*b^3*c^6 + (p^2 + p)*a*b^2*c^4 - 2*a^2*b*c^2*p + (15*(p^2 + 3*p
+ 2)*b^3*c^2*d^4 + (p^2 + p)*a*b^2*d^4)*x^4 + 4*(5*(p^2 + 3*p + 2)*b^3*c^3
*d^3 + (p^2 + p)*a*b^2*c*d^3)*x^3 + 2*a^3 + (15*(p^2 + 3*p + 2)*b^3*c^4*d^
2 + 6*(p^2 + p)*a*b^2*c^2*d^2 - 2*a^2*b*d^2*p)*x^2 + 2*(3*(p^2 + 3*p + 2)*
b^3*c^5*d + 2*(p^2 + p)*a*b^2*c^3*d - 2*a^2*b*c*d*p)*x)*(b*d^2*x^2 + 2*b*c
*d*x + b*c^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(89) = 178$.

Time $= 0.13$ (sec) , antiderivative size $= 264$, normalized size of antiderivative $= 2.84$

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$

$$= \frac{(bd^2x^2+2\,bcdx+bc^2+a)^3(bd^2x^2+2\,bcdx+bc^2+a)^p p - 2\,(bd^2x^2+2\,bcdx+bc^2+a)^2(bd^2x^2+2\,bcdx}{}$$

$$+ \frac{(bd^2x^2+2\,bcdx+bc^2+a)^{p+1}a^2}{2\,b^3d(p+1)}$$

input
```
integrate((d*x+c)^5*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
1/2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2
+ a)^p*p - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2*(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + a)^p*a*p + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^3*(b*d^2*x^2 +
2*b*c*d*x + b*c^2 + a)^p - 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + a)^p*a)/(b^3*d*p^2 + 5*b^3*d*p + 6*b^3*d) + 1/2*
(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^(p + 1)*a^2/(b^3*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.31

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \left(a+b\left(c+dx\right)^2\right)^p \left(\frac{d^5 x^6 \left(p^2+3p+2\right)}{2\left(p^3+6p^2+11p+6\right)} + \frac{\left(bc^2+a\right)\left(2a^2-2abc^2p-2abc^2+b^2c^4p^2+3b^2c^4p+2b^2c^4\right)}{2b^3 d\left(p^3+6p^2+11p+6\right)} + \frac{cx\left(-2a^2p+2abc^2p^2+2abc^2p+3b^2c^4p^2+9b^2c^4p+6b^2c^4\right)}{b^2\left(p^3+6p^2+11p+6\right)} + \frac{dx^2\left(-2a^2p+6abc^2p^2+6abc^2p+15b^2c^4p^2+45b^2c^4p+30b^2c^4\right)}{2b^2\left(p^3+6p^2+11p+6\right)} + \frac{3cd^4x^5\left(p^2+3p+2\right)}{p^3+6p^2+11p+6} + \frac{d^3x^4(p+1)\left(ap+30bc^2+15bc^2p\right)}{2b\left(p^3+6p^2+11p+6\right)} + \frac{2cd^2x^3(p+1)\left(ap+10bc^2+5bc^2p\right)}{b\left(p^3+6p^2+11p+6\right)}\right)$$

input

```
int((a + b*(c + d*x)^2)^p*(c + d*x)^5,x)
```

output

```
(a + b*(c + d*x)^2)^p*((d^5*x^6*(3*p + p^2 + 2))/(2*(11*p + 6*p^2 + p^3 +
6)) + ((a + b*c^2)*(2*a^2 + 2*b^2*c^4 + 3*b^2*c^4*p + b^2*c^4*p^2 - 2*a*b*
c^2 - 2*a*b*c^2*p))/(2*b^3*d*(11*p + 6*p^2 + p^3 + 6)) + (c*x*(6*b^2*c^4 -
 2*a^2*p + 9*b^2*c^4*p + 3*b^2*c^4*p^2 + 2*a*b*c^2*p + 2*a*b*c^2*p^2))/(b^
2*(11*p + 6*p^2 + p^3 + 6)) + (d*x^2*(30*b^2*c^4 - 2*a^2*p + 45*b^2*c^4*p
+ 15*b^2*c^4*p^2 + 6*a*b*c^2*p + 6*a*b*c^2*p^2))/(2*b^2*(11*p + 6*p^2 + p^
3 + 6)) + (3*c*d^4*x^5*(3*p + p^2 + 2))/(11*p + 6*p^2 + p^3 + 6) + (d^3*x^
4*(p + 1)*(a*p + 30*b*c^2 + 15*b*c^2*p))/(2*b*(11*p + 6*p^2 + p^3 + 6)) +
(2*c*d^2*x^3*(p + 1)*(a*p + 10*b*c^2 + 5*b*c^2*p))/(b*(11*p + 6*p^2 + p^3
+ 6)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 496, normalized size of antiderivative = 5.33

$$\int (c+dx)^5 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \frac{(b\,d^2x^2+2bcdx+b\,c^2+a)^p \left(b^3d^6p^2x^6+6b^3c\,d^5p^2x^5+3b^3d^6p\,x^6+15b^3c^2d^4p^2x^4+18b^3c\,d^5p\,x^5+2b^3d^6x\right.}{}$$

input
```
int((d*x+c)^5*(a+b*(d*x+c)^2)^p,x)
```

output
```
((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*(2*a**3 - 2*a**2*b*c**2*p - 4*a
**2*b*c*d*p*x - 2*a**2*b*d**2*p*x**2 + a*b**2*c**4*p**2 + a*b**2*c**4*p +
4*a*b**2*c**3*d*p**2*x + 4*a*b**2*c**3*d*p*x + 6*a*b**2*c**2*d**2*p**2*x**
2 + 6*a*b**2*c**2*d**2*p*x**2 + 4*a*b**2*c*d**3*p**2*x**3 + 4*a*b**2*c*d**
3*p*x**3 + a*b**2*d**4*p**2*x**4 + a*b**2*d**4*p*x**4 + b**3*c**6*p**2 + 3
*b**3*c**6*p + 2*b**3*c**6 + 6*b**3*c**5*d*p**2*x + 18*b**3*c**5*d*p*x + 1
2*b**3*c**5*d*x + 15*b**3*c**4*d**2*p**2*x**2 + 45*b**3*c**4*d**2*p*x**2 +
 30*b**3*c**4*d**2*x**2 + 20*b**3*c**3*d**3*p**2*x**3 + 60*b**3*c**3*d**3*
p*x**3 + 40*b**3*c**3*d**3*x**3 + 15*b**3*c**2*d**4*p**2*x**4 + 45*b**3*c*
*2*d**4*p*x**4 + 30*b**3*c**2*d**4*x**4 + 6*b**3*c*d**5*p**2*x**5 + 18*b**
3*c*d**5*p*x**5 + 12*b**3*c*d**5*x**5 + b**3*d**6*p**2*x**6 + 3*b**3*d**6*
p*x**6 + 2*b**3*d**6*x**6))/(2*b**3*d*(p**3 + 6*p**2 + 11*p + 6))
```

3.47 $\int (c+dx)^4 (a+b(c+dx)^2)^p \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 55

$$\int (c+dx)^4 (a+b(c+dx)^2)^p \, dx = \frac{(c+dx)^5 (a+b(c+dx)^2)^{1+p} \text{Hypergeometric2F1}\left(1, \frac{7}{2}+p, \frac{7}{2}, -\frac{b(c+dx)^2}{a}\right)}{5ad}$$

output
```
1/5*(d*x+c)^5*(a+b*(d*x+c)^2)^(p+1)*hypergeom([1, 7/2+p],[7/2],-b*(d*x+c)^
2/a)/a/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int (c+dx)^4 (a+b(c+dx)^2)^p \, dx = \frac{(c+dx)^5 (a+b(c+dx)^2)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2}, -p, \frac{7}{2}, -\frac{b(c+dx)^2}{a}\right)}{5d}$$

input
```
Integrate[(c + d*x)^4*(a + b*(c + d*x)^2)^p,x]
```

output
```
((c + d*x)^5*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*(c
 + d*x)^2)/a)])/(5*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^4 \left(b(c+dx)^2+a\right)^p \, d(c+dx)}{d}$$

$$\downarrow 279$$

$$\frac{\left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \int (c+dx)^4 \left(\frac{b(c+dx)^2}{a}+1\right)^p d(c+dx)}{d}$$

$$\downarrow 278$$

$$\frac{(c+dx)^5 \left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2}, -p, \frac{7}{2}, -\frac{b(c+dx)^2}{a}\right)}{5d}$$

input
```
Int[(c + d*x)^4*(a + b*(c + d*x)^2)^p,x]
```

output
```
((c + d*x)^5*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*(c
 + d*x)^2)/a)])/(5*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int (xd+c)^4 \left(a+b(xd+c)^2\right)^p dx$$

input
```
int((d*x+c)^4*(a+b*(d*x+c)^2)^p,x)
```

output
```
int((d*x+c)^4*(a+b*(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^4 \left((dx+c)^2 b+a\right)^p dx$$

input
```
integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4)*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*(a+b*(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^4 \left((dx+c)^2 b+a\right)^p \, dx$$

input
```
integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^4*((d*x + c)^2*b + a)^p, x)
```

Giac [F]

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^4 \left((dx+c)^2 b+a\right)^p \, dx$$

input
```
integrate((d*x+c)^4*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*((d*x + c)^2*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \int \left(a+b\left(c+d\,x\right)^2\right)^p \left(c+d\,x\right)^4 dx$$

input
```
int((a + b*(c + d*x)^2)^p*(c + d*x)^4,x)
```

output
```
int((a + b*(c + d*x)^2)^p*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \left(a+b(c+dx)^2\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^4*(a+b*(d*x+c)^2)^p,x)
```

output

```
(3*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a**3 - 6*(a + b*c**2 + 2*b*c*
d*x + b*d**2*x**2)**p*a**2*b*c**2*p - 6*(a + b*c**2 + 2*b*c*d*x + b*d**2*x
**2)**p*a**2*b*c*d*p*x + 4*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a*b**
2*c**4*p**2 + 2*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a*b**2*c**4*p +
12*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a*b**2*c**3*d*p**2*x + 6*(a +
 b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a*b**2*c**3*d*p*x + 12*(a + b*c**2 +
 2*b*c*d*x + b*d**2*x**2)**p*a*b**2*c**2*d**2*p**2*x**2 + 6*(a + b*c**2 +
2*b*c*d*x + b*d**2*x**2)**p*a*b**2*c**2*d**2*p*x**2 + 4*(a + b*c**2 + 2*b*
c*d*x + b*d**2*x**2)**p*a*b**2*c*d**3*p**2*x**3 + 2*(a + b*c**2 + 2*b*c*d*
x + b*d**2*x**2)**p*a*b**2*c*d**3*p*x**3 + 4*(a + b*c**2 + 2*b*c*d*x + b*d
**2*x**2)**p*b**3*c**6*p**2 + 8*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*
b**3*c**6*p + 3*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*c**6 + 20*(
a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*c**5*d*p**2*x + 40*(a + b*c*
*2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*c**5*d*p*x + 15*(a + b*c**2 + 2*b*c*
d*x + b*d**2*x**2)**p*b**3*c**5*d*x + 40*(a + b*c**2 + 2*b*c*d*x + b*d**2*
x**2)**p*b**3*c**4*d**2*p**2*x**2 + 80*(a + b*c**2 + 2*b*c*d*x + b*d**2*x*
*2)**p*b**3*c**4*d**2*p*x**2 + 30*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**
p*b**3*c**4*d**2*x**2 + 40*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*
c**3*d**3*p**2*x**3 + 80*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*c*
*3*d**3*p*x**3 + 30*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**3*c**3...
```

3.48 $\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx$

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Rubi [A] (verified) . . . 341
Maple [A] (verified) . . . 342
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Reduce [B] (verification not implemented) . . . 345

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = -\frac{a(a+b(c+dx)^2)^{1+p}}{2b^2d(1+p)} + \frac{(a+b(c+dx)^2)^{2+p}}{2b^2d(2+p)}$$

output
```
-1/2*a*(a+b*(d*x+c)^2)^(p+1)/b^2/d/(p+1)+1/2*(a+b*(d*x+c)^2)^(2+p)/b^2/d/(
2+p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = \frac{(a+b(c+dx)^2)^{1+p}\left(-a+b(1+p)(c+dx)^2\right)}{2b^2d(1+p)(2+p)}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^2)^p,x]
```

output
```
((a + b*(c + d*x)^2)^(1 + p)*(-a + b*(1 + p)*(c + d*x)^2))/(2*b^2*d*(1 + p
)*(2 + p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^2+a\right)^p \, d(c+dx)}{d}$$

$$\downarrow 243$$

$$\frac{\int (c+dx)^2 \left(b(c+dx)^2+a\right)^p \, d(c+dx)^2}{2d}$$

$$\downarrow 53$$

$$\frac{\int \left(\frac{(b(c+dx)^2+a)^{p+1}}{b} - \frac{a(b(c+dx)^2+a)^p}{b}\right) d(c+dx)^2}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{(a+b(c+dx)^2)^{p+2}}{b^2(p+2)} - \frac{a(a+b(c+dx)^2)^{p+1}}{b^2(p+1)}}{2d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^2)^p,x]
```

output
```
(-((a*(a + b*(c + d*x)^2)^(1 + p))/(b^2*(1 + p))) + (a + b*(c + d*x)^2)^(2
 + p)/(b^2*(2 + p)))/(2*d)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2  Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)  Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result
gosper	$-\frac{\left(b d^2 x^2+2bcdx+b c^2+a\right)^{p+1}\left(-b d^2 p x^2-2bcdpx-b d^2 x^2-b c^2 p-2bcdx-b c^2+a\right)}{2b^2 d(p^2+3p+2)}$
orering	$-\frac{\left(-b d^2 p x^2-2bcdpx-b d^2 x^2-b c^2 p-2bcdx-b c^2+a\right)\left(b d^2 x^2+2bcdx+b c^2+a\right)\left(a+b(xd+c)^2\right)^p}{2d b^2(p^2+3p+2)}$
risch	$-\frac{\left(-b^2 d^4 p x^4-4b^2 c d^3 p x^3-d^4 x^4 b^2-6b^2 c^2 d^2 p x^2-4c x^3 d^3 b^2-4b^2 c^3 dpx-6b^2 c^2 d^2 x^2-ab d^2 p x^2-b^2 c^4 p-4d b^2 x c^3-2abcdpx\right.}{2(p+1)(2+p)d b^2}$
norman	$\frac{c(2b c^2 p+2b c^2+ap) x\, \mathrm{e}^{p \ln\left(a+b(xd+c)^2\right)}}{b(p^2+3p+2)}+\frac{d^3 x^4 \mathrm{e}^{p \ln\left(a+b(xd+c)^2\right)}}{4+2p}+\frac{2c d^2 x^3 \mathrm{e}^{p \ln\left(a+b(xd+c)^2\right)}}{2+p}-\frac{(-b^2 c^4 p-b^2 c^4-ab c^2 p$
parallelrisch	$\frac{x^4\left(a+b(xd+c)^2\right)^p b^2 d^5 p+d^5\left(a+b(xd+c)^2\right)^p x^4 b^2+4x^3\left(a+b(xd+c)^2\right)^p b^2 c d^4 p+4c d^4\left(a+b(xd+c)^2\right)^p x^3 b^2+6x^2\left(a+b(xd+$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/d/(p^2+3*p+2)*(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^(p+1)*(-b*d^2*p*x^2-2
*b*c*d*p*x-b*d^2*x^2-b*c^2*p-2*b*c*d*x-b*c^2+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.95

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \frac{(b^2c^4 + (b^2d^4p + b^2d^4)x^4 + 4\,(b^2cd^3p + b^2cd^3)x^3 + (6\,b^2c^2d^2 + (6\,b^2c^2 + ab)d^2p)x^2 - a^2 + (b^2c^4 + abc^2)p}{2\,(b^2dp^2 + 3\,b^2dp + 2\,b^2d)}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^4 + (b^2*d^4*p + b^2*d^4)*x^4 + 4*(b^2*c*d^3*p + b^2*c*d^3)*x^3
 + (6*b^2*c^2*d^2 + (6*b^2*c^2 + a*b)*d^2*p)*x^2 - a^2 + (b^2*c^4 + a*b*c^
2)*p + 2*(2*b^2*c^3*d + (2*b^2*c^3 + a*b*c)*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2 + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.26

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx$$
$$= \frac{(b^2d^4(p+1)x^4 + 4\,b^2cd^3(p+1)x^3 + b^2c^4(p+1) + abc^2p + (6\,b^2c^2d^2(p+1) + abd^2p)x^2 - a^2 + 2\,(2\,b^2c^3d}{2\,(p^2+3\,p+2)b^2d}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*d^4*(p + 1)*x^4 + 4*b^2*c*d^3*(p + 1)*x^3 + b^2*c^4*(p + 1) + a*b
*c^2*p + (6*b^2*c^2*d^2*(p + 1) + a*b*d^2*p)*x^2 - a^2 + 2*(2*b^2*c^3*d*(p
 + 1) + a*b*c*d*p)*x)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p/((p^2 + 3*p +
2)*b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = \frac{(bd^2x^2+2\,bcdx+bc^2+a)^2(bd^2x^2+2\,bcdx+bc^2+a)^p}{2\,b^2d(p+2)}$$
$$- \frac{(bd^2x^2+2\,bcdx+bc^2+a)^{p+1}a}{2\,b^2d(p+1)}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 +
 a)^p/(b^2*d*(p + 2)) - 1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^(p + 1)*a/
(b^2*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.71

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = \left(a+b\left(c+dx\right)^2\right)^p \left(\frac{d^3 x^4 (p+1)}{2\left(p^2+3p+2\right)} + \frac{\left(bc^2+a\right)\left(bc^2-a+bc^2p\right)}{2b^2 d\left(p^2+3p+2\right)} + \frac{cx\left(ap+2bc^2+2bc^2p\right)}{b\left(p^2+3p+2\right)} + \frac{2cd^2x^3(p+1)}{p^2+3p+2} + \frac{dx^2\left(ap+6bc^2+6bc^2p\right)}{2b\left(p^2+3p+2\right)}\right)$$

input
```
int((a + b*(c + d*x)^2)^p*(c + d*x)^3,x)
```

output
```
(a + b*(c + d*x)^2)^p*((d^3*x^4*(p + 1))/(2*(3*p + p^2 + 2)) + ((a + b*c^2
)*(b*c^2 - a + b*c^2*p))/(2*b^2*d*(3*p + p^2 + 2)) + (c*x*(a*p + 2*b*c^2 +
 2*b*c^2*p))/(b*(3*p + p^2 + 2)) + (2*c*d^2*x^3*(p + 1))/(3*p + p^2 + 2) +
 (d*x^2*(a*p + 6*b*c^2 + 6*b*c^2*p))/(2*b*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.95

$$\int (c+dx)^3 \left(a+b(c+dx)^2\right)^p \, dx = \frac{\left(bd^2x^2+2bcdx+bc^2+a\right)^p \left(b^2d^4p\,x^4+4b^2cd^3p\,x^3+b^2d^4x^4+6b^2c^2d^2p\,x^2+4b^2cd^3x^3+4b^2c^3dpx+6b^2\right.}{2b^2d\left(p^2+3p+2\right)}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^2)^p,x)
```

output
```
((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*( - a**2 + a*b*c**2*p + 2*a*b*c
*d*p*x + a*b*d**2*p*x**2 + b**2*c**4*p + b**2*c**4 + 4*b**2*c**3*d*p*x + 4
*b**2*c**3*d*x + 6*b**2*c**2*d**2*p*x**2 + 6*b**2*c**2*d**2*x**2 + 4*b**2*
c*d**3*p*x**3 + 4*b**2*c*d**3*x**3 + b**2*d**4*p*x**4 + b**2*d**4*x**4))/(
2*b**2*d*(p**2 + 3*p + 2))
```

3.49 $\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx$

Optimal result . . . 346
Mathematica [A] (verified) . . . 346
Rubi [A] (verified) . . . 347
Maple [F] . . . 348
Fricas [F] . . . 348
Sympy [F(-1)] . . . 349
Maxima [F] . . . 349
Giac [F] . . . 349
Mupad [F(-1)] . . . 350
Reduce [F] . . . 350

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^{1+p} \text{Hypergeometric2F1}\left(1, \frac{5}{2}+p, \frac{5}{2}, -\frac{b(c+dx)^2}{a}\right)}{3ad}$$

output

```
1/3*(d*x+c)^3*(a+b*(d*x+c)^2)^(p+1)*hypergeom([1, 5/2+p],[5/2],-b*(d*x+c)^
2/a)/a/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^p \left(1+\frac{b(c+dx)^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2}, -p, \frac{5}{2}, -\frac{b(c+dx)^2}{a}\right)}{3d}$$

input

```
Integrate[(c + d*x)^2*(a + b*(c + d*x)^2)^p,x]
```

output
```
((c + d*x)^3*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*(c
 + d*x)^2)/a)])/(3*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^2 \left(b(c+dx)^2+a\right)^p \, d(c+dx)}{d}$$

$$\downarrow 279$$

$$\frac{\left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \int (c+dx)^2 \left(\frac{b(c+dx)^2}{a}+1\right)^p d(c+dx)}{d}$$

$$\downarrow 278$$

$$\frac{(c+dx)^3 \left(a+b(c+dx)^2\right)^p \left(\frac{b(c+dx)^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2}, -p, \frac{5}{2}, -\frac{b(c+dx)^2}{a}\right)}{3d}$$

input
```
Int[(c + d*x)^2*(a + b*(c + d*x)^2)^p,x]
```

output
```
((c + d*x)^3*(a + b*(c + d*x)^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*(c
 + d*x)^2)/a)])/(3*d*(1 + (b*(c + d*x)^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int (xd+c)^2 \left(a+b(xd+c)^2\right)^p dx$$

input
```
int((d*x+c)^2*(a+b*(d*x+c)^2)^p,x)
```

output
```
int((d*x+c)^2*(a+b*(d*x+c)^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^2 \left((dx+c)^2 b+a\right)^p dx$$

input
```
integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p,
x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(a+b*(d*x+c)**2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^2 \left((dx+c)^2 b+a\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*((d*x + c)^2*b + a)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \int (dx+c)^2 \left((dx+c)^2 b+a\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*((d*x + c)^2*b + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx = \int \left(a+b\left(c+d\,x\right)^2\right)^p \left(c+d\,x\right)^2 dx$$

input
```
int((a + b*(c + d*x)^2)^p*(c + d*x)^2,x)
```

output
```
int((a + b*(c + d*x)^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+b(c+dx)^2\right)^p \, dx$$

$$= \frac{-(b\,d^2x^2+2bcdx+b\,c^2+a)^p\,a^2+2(b\,d^2x^2+2bcdx+b\,c^2+a)^p\,ab\,c^2p+2(b\,d^2x^2+2bcdx+b\,c^2+a)^p\,a}{}$$

input
```
int((d*x+c)^2*(a+b*(d*x+c)^2)^p,x)
```

output

```
( - (a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a**2 + 2*(a + b*c**2 + 2*b*c
*d*x + b*d**2*x**2)**p*a*b*c**2*p + 2*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**
2)**p*a*b*c*d*p*x + 2*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**2*c**4*
p + (a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**2*c**4 + 6*(a + b*c**2 +
2*b*c*d*x + b*d**2*x**2)**p*b**2*c**3*d*p*x + 3*(a + b*c**2 + 2*b*c*d*x +
b*d**2*x**2)**p*b**2*c**3*d*x + 6*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**
p*b**2*c**2*d**2*p*x**2 + 3*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**2
*c**2*d**2*x**2 + 2*(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**2*c*d**3*
p*x**3 + (a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*b**2*c*d**3*x**3 + 8*in
t(((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*x)/(4*a*p**2 + 8*a*p + 3*a +
4*b*c**2*p**2 + 8*b*c**2*p + 3*b*c**2 + 8*b*c*d*p**2*x + 16*b*c*d*p*x + 6*
b*c*d*x + 4*b*d**2*p**2*x**2 + 8*b*d**2*p*x**2 + 3*b*d**2*x**2),x)*a**2*b*
d**2*p**3 + 16*int(((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*x)/(4*a*p**2
 + 8*a*p + 3*a + 4*b*c**2*p**2 + 8*b*c**2*p + 3*b*c**2 + 8*b*c*d*p**2*x +
16*b*c*d*p*x + 6*b*c*d*x + 4*b*d**2*p**2*x**2 + 8*b*d**2*p*x**2 + 3*b*d**2
*x**2),x)*a**2*b*d**2*p**2 + 6*int(((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)
**p*x)/(4*a*p**2 + 8*a*p + 3*a + 4*b*c**2*p**2 + 8*b*c**2*p + 3*b*c**2 + 8
*b*c*d*p**2*x + 16*b*c*d*p*x + 6*b*c*d*x + 4*b*d**2*p**2*x**2 + 8*b*d**2*p
*x**2 + 3*b*d**2*x**2),x)*a**2*b*d**2*p)/(b**2*c*d*(4*p**2 + 8*p + 3))
```

3.50 $\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx$

Optimal result . . . 352
Mathematica [A] (verified) . . . 352
Rubi [A] (verified) . . . 353
Maple [A] (verified) . . . 354
Fricas [A] (verification not implemented) . . . 354
Sympy [B] (verification not implemented) . . . 355
Maxima [A] (verification not implemented) . . . 355
Giac [A] (verification not implemented) . . . 356
Mupad [B] (verification not implemented) . . . 356
Reduce [B] (verification not implemented) . . . 356

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \frac{\left(a+b(c+dx)^2\right)^{1+p}}{2bd(1+p)}$$

output
```
1/2*(a+b*(d*x+c)^2)^(p+1)/b/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \frac{\left(a+b(c+dx)^2\right)^{1+p}}{2bd(1+p)}$$

input
```
Integrate[(c + d*x)*(a + b*(c + d*x)^2)^p,x]
```

output
```
(a + b*(c + d*x)^2)^(1 + p)/(2*b*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {895, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)\left(b(c+dx)^2+a\right)^p\,d(c+dx)}{d}$$

$$\downarrow 241$$

$$\frac{\left(a+b(c+dx)^2\right)^{p+1}}{2bd(p+1)}$$

input
```
Int[(c + d*x)*(a + b*(c + d*x)^2)^p,x]
```

output
```
(a + b*(c + d*x)^2)^(1 + p)/(2*b*d*(1 + p))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\left(a+b(xd+c)^2\right)^{p+1}}{2bd(p+1)}$	29
default	$\frac{\left(a+b(xd+c)^2\right)^{p+1}}{2bd(p+1)}$	29
gosper	$\frac{(b\,d^2x^2+2bcdx+b\,c^2+a)^{p+1}}{2bd(p+1)}$	39
risch	$\frac{(b\,d^2x^2+2bcdx+b\,c^2+a)\left(a+b(xd+c)^2\right)^p}{2bd(p+1)}$	48
orering	$\frac{(b\,d^2x^2+2bcdx+b\,c^2+a)\left(a+b(xd+c)^2\right)^p}{2bd(p+1)}$	48
norman	$\frac{cx\,\mathrm{e}^{p\ln\left(a+b(xd+c)^2\right)}}{p+1}+\frac{d\,x^2\mathrm{e}^{p\ln\left(a+b(xd+c)^2\right)}}{2p+2}+\frac{(b\,c^2+a)\mathrm{e}^{p\ln\left(a+b(xd+c)^2\right)}}{2bd(p+1)}$	86
parallelrisch	$\frac{d^3\left(a+b(xd+c)^2\right)^p x^2b+2c\left(a+b(xd+c)^2\right)^p xb\,d^2+\left(a+b(xd+c)^2\right)^p b\,c^2d+\left(a+b(xd+c)^2\right)^p ad}{2b\,d^2(p+1)}$	92

input
```
int((d*x+c)*(a+b*(d*x+c)^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a+b*(d*x+c)^2)^(p+1)/b/d/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx=\frac{(bd^2x^2+2\,bcdx+bc^2+a)(bd^2x^2+2\,bcdx+bc^2+a)^p}{2\,(bdp+bd)}$$

input
```
integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="fricas")
```

output
```
1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)^p/(b*d*p + b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.

Time = 155.77 (sec) , antiderivative size = 252, normalized size of antiderivative = 8.40

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx$$

$$= \begin{cases} \frac{cx}{a} & \text{for } b \\ a^p\left(cx+\frac{dx^2}{2}\right) & \text{for } b \\ cx(a+bc^2)^p & \text{for } d \\ \frac{\log\left(\frac{c}{d}+x-\frac{\sqrt{-ab}}{bd}\right)}{2bd}+\frac{\log\left(\frac{c}{d}+x+\frac{\sqrt{-ab}}{bd}\right)}{2bd} & \text{for } p \\ \frac{a\left(a+bc^2+2bcdx+bd^2x^2\right)^p}{2bdp+2bd}+\frac{bc^2\left(a+bc^2+2bcdx+bd^2x^2\right)^p}{2bdp+2bd}+\frac{2bcdx\left(a+bc^2+2bcdx+bd^2x^2\right)^p}{2bdp+2bd}+\frac{bd^2x^2\left(a+bc^2+2bcdx+bd^2x^2\right)^p}{2bdp+2bd} & \text{other} \end{cases}$$

input
```
integrate((d*x+c)*(a+b*(d*x+c)**2)**p,x)
```

output
```
Piecewise((c*x/a, Eq(b, 0) & Eq(d, 0) & Eq(p, -1)), (a**p*(c*x + d*x**2/2)
, Eq(b, 0)), (c*x*(a + b*c**2)**p, Eq(d, 0)), (log(c/d + x - sqrt(-a*b)/(b
*d))/(2*b*d) + log(c/d + x + sqrt(-a*b)/(b*d))/(2*b*d), Eq(p, -1)), (a*(a
+ b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p/(2*b*d*p + 2*b*d) + b*c**2*(a + b*c
**2 + 2*b*c*d*x + b*d**2*x**2)**p/(2*b*d*p + 2*b*d) + 2*b*c*d*x*(a + b*c**
2 + 2*b*c*d*x + b*d**2*x**2)**p/(2*b*d*p + 2*b*d) + b*d**2*x**2*(a + b*c**
2 + 2*b*c*d*x + b*d**2*x**2)**p/(2*b*d*p + 2*b*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \frac{\left((dx+c)^2b+a\right)^{p+1}}{2\,bd(p+1)}$$

input
```
integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="maxima")
```

output
```
1/2*((d*x + c)^2*b + a)^(p + 1)/(b*d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \frac{(bd^2x^2+2\,bcdx+bc^2+a)^{p+1}}{2\,bd(p+1)}$$

input
```
integrate((d*x+c)*(a+b*(d*x+c)^2)^p,x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^(p + 1)/(b*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \left(a+b\left(c+d\,x\right)^2\right)^p\left(\frac{c\,x}{p+1}+\frac{d\,x^2}{2\,(p+1)}+\frac{b\,c^2+a}{2\,b\,d\,(p+1)}\right)$$

input
```
int((a + b*(c + d*x)^2)^p*(c + d*x),x)
```

output
```
(a + b*(c + d*x)^2)^p*((c*x)/(p + 1) + (d*x^2)/(2*(p + 1)) + (a + b*c^2)/(
2*b*d*(p + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int (c+dx)\left(a+b(c+dx)^2\right)^p\,dx = \frac{\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^p\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)}{2bd\,(p+1)}$$

input
```
int((d*x+c)*(a+b*(d*x+c)^2)^p,x)
```

output
```
((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*(a + b*c**2 + 2*b*c*d*x + b*d**
2*x**2))/(2*b*d*(p + 1))
```

3.51 $\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx$

Optimal result . . . 357
Mathematica [A] (verified) . . . 357
Rubi [A] (verified) . . . 358
Maple [F] . . . 359
Fricas [F] . . . 359
Sympy [F] . . . 360
Maxima [F] . . . 360
Giac [F] . . . 360
Mupad [F(-1)] . . . 361
Reduce [F] . . . 361

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = -\frac{(a+b(c+dx)^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c+dx)^2}{a}\right)}{2ad(1+p)}$$

output
```
-1/2*(a+b*(d*x+c)^2)^(p+1)*hypergeom([1, p+1],[2+p],1+b*(d*x+c)^2/a)/a/d/(
p+1)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = -\frac{(a+b(c+dx)^2)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b(c+dx)^2}{a}\right)}{2ad(1+p)}$$

input
```
Integrate[(a + b*(c + d*x)^2)^p/(c + d*x),x]
```

output
```
-1/2*((a + b*(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c + d*x)^2)/a])/(a*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(b(c+dx)^2+a)^p}{c+dx} d(c+dx)}{d}$$
$$\downarrow 243$$
$$\frac{\int \frac{(b(c+dx)^2+a)^p}{(c+dx)^2} d(c+dx)^2}{2d}$$
$$\downarrow 75$$
$$-\frac{(a+b(c+dx)^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{b(c+dx)^2}{a}+1\right)}{2ad(p+1)}$$

input
```
Int[(a + b*(c + d*x)^2)^p/(c + d*x),x]
```

output
```
-1/2*((a + b*(c + d*x)^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (
b*(c + d*x)^2)/a])/(a*d*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int \frac{\left(a+b(xd+c)^2\right)^p}{xd+c}dx$$

input
```
int((a+b*(d*x+c)^2)^p/(d*x+c),x)
```

output
```
int((a+b*(d*x+c)^2)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = \int \frac{\left((dx+c)^2b+a\right)^p}{dx+c}\,dx$$

input
```
integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^p/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = \int \frac{(a+bc^2+2bcdx+bd^2x^2)^p}{c+dx}\,dx$$

input
```
integrate((a+b*(d*x+c)**2)**p/(d*x+c),x)
```

output
```
Integral((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = \int \frac{\left((dx+c)^2b+a\right)^p}{dx+c}\,dx$$

input
```
integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(((d*x + c)^2*b + a)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = \int \frac{\left((dx+c)^2b+a\right)^p}{dx+c}\,dx$$

input
```
integrate((a+b*(d*x+c)^2)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate(((d*x + c)^2*b + a)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx = \int \frac{\left(a+b\left(c+d\,x\right)^2\right)^p}{c+d\,x}\,dx$$

input
```
int((a + b*(c + d*x)^2)^p/(c + d*x),x)
```

output
```
int((a + b*(c + d*x)^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+b(c+dx)^2)^p}{c+dx}\,dx$$
$$= \frac{\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^p a+\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^p b\,c^2-2\left(\int \frac{\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^p x^2}{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+adx+ac}dx\right.}{2b\,c^2dp}$$

input
```
int((a+b*(d*x+c)^2)^p/(d*x+c),x)
```

output
```
((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*a + (a + b*c**2 + 2*b*c*d*x + b
*d**2*x**2)**p*b*c**2 - 2*int(((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*x
**2)/(a*c + a*d*x + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)
,x)*a*b*d**3*p - 4*int(((a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)**p*x)/(a*c
+ a*d*x + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)*a*b*c*
d**2*p)/(2*b*c**2*d*p)
```

3.52 $\int (c+dx)^3 \left(a+b(c+dx)^3\right)\, dx$

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Optimal result

Integrand size = 19, antiderivative size = 31

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)\, dx = \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^7}{7d}$$

output
```
1/4*a*(d*x+c)^4/d+1/7*b*(d*x+c)^7/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(31) = 62.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.16

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)\, dx = c^3\left(a+bc^3\right)x + \frac{3}{2}c^2\left(a+2bc^3\right)dx^2 + c\left(a+5bc^3\right)d^2x^3 + \frac{1}{4}\left(a+20bc^3\right)d^3x^4 + 3bc^2d^4x^5 + bcd^5x^6 + \frac{1}{7}bd^6x^7$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^3),x]
```

output
```
c^3*(a + b*c^3)*x + (3*c^2*(a + 2*b*c^3)*d*x^2)/2 + c*(a + 5*b*c^3)*d^2*x^
3 + ((a + 20*b*c^3)*d^3*x^4)/4 + 3*b*c^2*d^4*x^5 + b*c*d^5*x^6 + (b*d^6*x^
7)/7
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)\, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^3+a\right) d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b(c+dx)^6+a(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a(c+dx)^4+\frac{1}{7}b(c+dx)^7}{d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^3),x]
```

output
```
((a*(c + d*x)^4)/4 + (b*(c + d*x)^7)/7)/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(27) = 54$.

Time $= 0.66$ (sec) , antiderivative size $= 104$, normalized size of antiderivative $= 3.35$

method	result
norman	$\frac{d^6 b\, x^7}{7} + c\, d^5 b\, x^6 + 3c^2 d^4 b\, x^5 + \left(5c^3 b\, d^3 + \frac{1}{4} a\, d^3\right) x^4 + \left(5c^4 b\, d^2 + ac\, d^2\right) x^3 + \left(3c^5 bd + \frac{3}{2} a\, c^2 d\right)$
risch	$\frac{1}{7} d^6 b\, x^7 + c\, d^5 b\, x^6 + 3c^2 d^4 b\, x^5 + 5x^4 c^3 b\, d^3 + \frac{1}{4} a\, d^3 x^4 + 5b\, c^4 d^2 x^3 + d^2 ac\, x^3 + 3x^2 c^5 bd + \frac{3}{2} d$
parallelrisch	$\frac{1}{7} d^6 b\, x^7 + c\, d^5 b\, x^6 + 3c^2 d^4 b\, x^5 + 5x^4 c^3 b\, d^3 + \frac{1}{4} a\, d^3 x^4 + 5b\, c^4 d^2 x^3 + d^2 ac\, x^3 + 3x^2 c^5 bd + \frac{3}{2} d$
gosper	$\frac{x\left(4d^6 b\, x^6+28c\, d^5 b\, x^5+84c^2 d^4 b\, x^4+140x^3 c^3 b\, d^3+140b\, c^4 d^2 x^2+84x\, c^5 bd+7a\, d^3 x^3+28b\, c^6+28x^2 ac\, d^2+42xa\, c^2 d+28c^3 a\right)}{28}$
default	$\frac{d^6 b\, x^7}{7} + c\, d^5 b\, x^6 + 3c^2 d^4 b\, x^5 + \frac{\left(19c^3 b\, d^3+d^3\left(c^3 b+a\right)\right) x^4}{4} + \frac{\left(12c^4 b\, d^2+3c\, d^2\left(c^3 b+a\right)\right) x^3}{3} + \frac{\left(3c^5 bd+3c^2 d\left(c^3 b+\right.\right.$
orering	$\frac{x\left(4d^6 b\, x^6+28c\, d^5 b\, x^5+84c^2 d^4 b\, x^4+140x^3 c^3 b\, d^3+140b\, c^4 d^2 x^2+84x\, c^5 bd+7a\, d^3 x^3+28b\, c^6+28x^2 ac\, d^2+42xa\, c^2 d+28c^3 a\right)\left(a-\right.}{28b\, d^3 x^3+84bc\, d^2 x^2+84dbx\, c^2+28c^3 b+28a}$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^6*b*x^7+c*d^5*b*x^6+3*c^2*d^4*b*x^5+(5*c^3*b*d^3+1/4*a*d^3)*x^4+(5*b
*c^4*d^2+a*c*d^2)*x^3+(3*c^5*b*d+3/2*a*c^2*d)*x^2+(b*c^6+a*c^3)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(27) = 54$.

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}bd^6x^7 + bcd^5x^6 + 3\,bc^2d^4x^5 + \frac{1}{4}\left(20\,bc^3+a\right)d^3x^4 + \left(5\,bc^4+ac\right)d^2x^3 + \frac{3}{2}\left(2\,bc^5+ac^2\right)dx^2 + \left(bc^6+ac^3\right)x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/7*b*d^6*x^7 + b*c*d^5*x^6 + 3*b*c^2*d^4*x^5 + 1/4*(20*b*c^3 + a)*d^3*x^4
 + (5*b*c^4 + a*c)*d^2*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*x^2 + (b*c^6 + a*c^3)
*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = 3bc^2d^4x^5 + bcd^5x^6 + \frac{bd^6x^7}{7} + x^4\left(\frac{ad^3}{4} + 5bc^3d^3\right) + x^3\left(acd^2 + 5bc^4d^2\right) + x^2\cdot\left(\frac{3ac^2d}{2} + 3bc^5d\right) + x\left(ac^3 + bc^6\right)$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**3),x)
```

output
```
3*b*c**2*d**4*x**5 + b*c*d**5*x**6 + b*d**6*x**7/7 + x**4*(a*d**3/4 + 5*b*
c**3*d**3) + x**3*(a*c*d**2 + 5*b*c**4*d**2) + x**2*(3*a*c**2*d/2 + 3*b*c*
*5*d) + x*(a*c**3 + b*c**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}bd^6x^7 + bcd^5x^6 + 3\,bc^2d^4x^5 + \frac{1}{4}\left(20\,bc^3+a\right)d^3x^4 + \left(5\,bc^4+ac\right)d^2x^3 + \frac{3}{2}\left(2\,bc^5+ac^2\right)dx^2 + \left(bc^6+ac^3\right)x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/7*b*d^6*x^7 + b*c*d^5*x^6 + 3*b*c^2*d^4*x^5 + 1/4*(20*b*c^3 + a)*d^3*x^4
 + (5*b*c^4 + a*c)*d^2*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*x^2 + (b*c^6 + a*c^3)
*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.39

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}bd^6x^7 + bcd^5x^6 + 3\,bc^2d^4x^5 + 5\,bc^3d^3x^4 + 5\,bc^4d^2x^3 + 3\,bc^5dx^2 + bc^6x + \frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/7*b*d^6*x^7 + b*c*d^5*x^6 + 3*b*c^2*d^4*x^5 + 5*b*c^3*d^3*x^4 + 5*b*c^4*
d^2*x^3 + 3*b*c^5*d*x^2 + b*c^6*x + 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^
2*d*x^2 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.00

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = x\left(b\,c^6+a\,c^3\right)+\frac{d^3\,x^4\,(20\,b\,c^3+a)}{4}+\frac{b\,d^6\,x^7}{7}+3\,b\,c^2\,d^4\,x^5 + \frac{3\,c^2\,d\,x^2\,(2\,b\,c^3+a)}{2}+c\,d^2\,x^3\left(5\,b\,c^3+a\right)+b\,c\,d^5\,x^6$$

input
```
int((a + b*(c + d*x)^3)*(c + d*x)^3,x)
```

output
```
x*(a*c^3 + b*c^6) + (d^3*x^4*(a + 20*b*c^3))/4 + (b*d^6*x^7)/7 + 3*b*c^2*d
^4*x^5 + (3*c^2*d*x^2*(a + 2*b*c^3))/2 + c*d^2*x^3*(a + 5*b*c^3) + b*c*d^5
*x^6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.42

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)\,dx = \frac{x(4b\,d^6x^6+28bc\,d^5x^5+84b\,c^2d^4x^4+140b\,c^3d^3x^3+140b\,c^4d^2x^2+84b\,c^5dx+7a\,d^3x^3+28b\,c^6+28ac\,d^2}{28}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^3),x)
```

output
```
(x*(28*a*c**3 + 42*a*c**2*d*x + 28*a*c*d**2*x**2 + 7*a*d**3*x**3 + 28*b*c*
*6 + 84*b*c**5*d*x + 140*b*c**4*d**2*x**2 + 140*b*c**3*d**3*x**3 + 84*b*c*
*2*d**4*x**4 + 28*b*c*d**5*x**5 + 4*b*d**6*x**6))/28
```

3.53 $\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 dx$

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Maple [B] (verified) 370
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Sympy [B] (verification not implemented) 372
Maxima [B] (verification not implemented) 372
Giac [B] (verification not implemented) 373
Mupad [B] (verification not implemented) 374
Reduce [B] (verification not implemented) 375

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 dx = \frac{a^2(c+dx)^4}{4d} + \frac{2ab(c+dx)^7}{7d} + \frac{b^2(c+dx)^{10}}{10d}$$

output
```
1/4*a^2*(d*x+c)^4/d+2/7*a*b*(d*x+c)^7/d+1/10*b^2*(d*x+c)^10/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 203 vs. 2(51) = 102.

Time = 0.02 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.98

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 dx = {} & c^3\left(a+bc^3\right)^2 x + \frac{3}{2}c^2\left(a^2+4abc^3+3b^2c^6\right) dx^2 \\ & + c\left(a^2+10abc^3+12b^2c^6\right) d^2x^3 \\ & + \frac{1}{4}\left(a^2+40abc^3+84b^2c^6\right) d^3x^4 \\ & + \frac{6}{5}bc^2\left(5a+21bc^3\right) d^4x^5 \\ & + bc\left(2a+21bc^3\right) d^5x^6 + \frac{2}{7}b\left(a+42bc^3\right) d^6x^7 \\ & + \frac{9}{2}b^2c^2d^7x^8 + b^2cd^8x^9 + \frac{1}{10}b^2d^9x^{10}\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^3)^2,x]
```

output
```
c^3*(a + b*c^3)^2*x + (3*c^2*(a^2 + 4*a*b*c^3 + 3*b^2*c^6)*d*x^2)/2 + c*(a
^2 + 10*a*b*c^3 + 12*b^2*c^6)*d^2*x^3 + ((a^2 + 40*a*b*c^3 + 84*b^2*c^6)*d
^3*x^4)/4 + (6*b*c^2*(5*a + 21*b*c^3)*d^4*x^5)/5 + b*c*(2*a + 21*b*c^3)*d^
5*x^6 + (2*b*(a + 42*b*c^3)*d^6*x^7)/7 + (9*b^2*c^2*d^7*x^8)/2 + b^2*c*d^8
*x^9 + (b^2*d^9*x^10)/10
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^3+a\right)^2 d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b^2(c+dx)^9+2ab(c+dx)^6+a^2(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a^2(c+dx)^4+\frac{2}{7}ab(c+dx)^7+\frac{1}{10}b^2(c+dx)^{10}}{d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^3)^2,x]
```

output
```
((a^2*(c + d*x)^4)/4 + (2*a*b*(c + d*x)^7)/7 + (b^2*(c + d*x)^10)/10)/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(45) = 90$.

Time = 0.66 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.59

method	result
norman	$\frac{d^9b^2x^{10}}{10} + c\,d^8b^2x^9 + \frac{9b^2c^2d^7x^8}{2} + \left(12c^3b^2d^6 + \frac{2}{7}ab\,d^6\right)x^7 + \left(21c^4b^2d^5 + 2abc\,d^5\right)x^6 + \left(\frac{126}{5}c^5b^2$
gosper	$\frac{x\left(14d^9b^2x^9+140c\,d^8b^2x^8+630b^2c^2d^7x^7+1680x^6c^3b^2d^6+2940b^2c^4d^5x^5+3528x^4c^5b^2d^4+40x^6ab\,d^6+2940x^3b^2c^6d^3+280abc\right.}{}$
risch	$\frac{1}{10}d^9b^2x^{10} + c\,d^8b^2x^9 + \frac{9}{2}b^2c^2d^7x^8 + 12x^7c^3b^2d^6 + \frac{2}{7}x^7ab\,d^6 + 21b^2c^4d^5x^6 + 2abc\,d^5x^6 + \frac{126}{5}$
parallelrisch	$\frac{1}{10}d^9b^2x^{10} + c\,d^8b^2x^9 + \frac{9}{2}b^2c^2d^7x^8 + 12x^7c^3b^2d^6 + \frac{2}{7}x^7ab\,d^6 + 21b^2c^4d^5x^6 + 2abc\,d^5x^6 + \frac{126}{5}$
orering	$\frac{x\left(14d^9b^2x^9+140c\,d^8b^2x^8+630b^2c^2d^7x^7+1680x^6c^3b^2d^6+2940b^2c^4d^5x^5+3528x^4c^5b^2d^4+40x^6ab\,d^6+2940x^3b^2c^6d^3+280abc\right.}{}$
default	$\frac{d^9b^2x^{10}}{10} + c\,d^8b^2x^9 + \frac{9b^2c^2d^7x^8}{2} + \frac{\left(64c^3b^2d^6+d^3\left(2\left(c^3b+a\right)b\,d^3+18b^2c^3d^3\right)\right)x^7}{7} + \frac{\left(51c^4b^2d^5+3c\,d^2\left(2\left(c^3b+a\right)b\,d^3-$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*d^9*b^2*x^10+c*d^8*b^2*x^9+9/2*b^2*c^2*d^7*x^8+(12*c^3*b^2*d^6+2/7*a*
b*d^6)*x^7+(21*b^2*c^4*d^5+2*a*b*c*d^5)*x^6+(126/5*c^5*b^2*d^4+6*a*b*c^2*d
^4)*x^5+(21*b^2*c^6*d^3+10*a*b*c^3*d^3+1/4*a^2*d^3)*x^4+(12*b^2*c^7*d^2+10
*a*b*c^4*d^2+a^2*c*d^2)*x^3+(9/2*b^2*c^8*d+6*a*b*c^5*d+3/2*a^2*c^2*d)*x^2+
(b^2*c^9+2*a*b*c^6+a^2*c^3)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(45) = 90$.

Time $= 0.07$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 4.12$

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & \frac{1}{10}\,b^2d^9x^{10}+b^2cd^8x^9+\frac{9}{2}\,b^2c^2d^7x^8+\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6x^7\\ & +\left(21\,b^2c^4+2\,abc\right)d^5x^6+\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4x^5\\ & +\frac{1}{4}\left(84\,b^2c^6+40\,abc^3+a^2\right)d^3x^4\\ & +\left(12\,b^2c^7+10\,abc^4+a^2c\right)d^2x^3\\ & +\frac{3}{2}\left(3\,b^2c^8+4\,abc^5+a^2c^2\right)dx^2\\ & +\left(b^2c^9+2\,abc^6+a^2c^3\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*d^9*x^10 + b^2*c*d^8*x^9 + 9/2*b^2*c^2*d^7*x^8 + 2/7*(42*b^2*c^3
+ a*b)*d^6*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*x^6 + 6/5*(21*b^2*c^5 + 5*a*b*
c^2)*d^4*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)*d^3*x^4 + (12*b^2*c^7 +
 10*a*b*c^4 + a^2*c)*d^2*x^3 + 3/2*(3*b^2*c^8 + 4*a*b*c^5 + a^2*c^2)*d*x^2
 + (b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.94

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & \frac{9b^2c^2d^7x^8}{2} + b^2cd^8x^9 + \frac{b^2d^9x^{10}}{10} + x^7 \\ & \cdot\left(\frac{2abd^6}{7} + 12b^2c^3d^6\right) + x^6\cdot\left(2abcd^5 + 21b^2c^4d^5\right) \\ & + x^5\cdot\left(6abc^2d^4 + \frac{126b^2c^5d^4}{5}\right) \\ & + x^4\left(\frac{a^2d^3}{4} + 10abc^3d^3 + 21b^2c^6d^3\right) \\ & + x^3\left(a^2cd^2 + 10abc^4d^2 + 12b^2c^7d^2\right) \\ & + x^2\cdot\left(\frac{3a^2c^2d}{2} + 6abc^5d + \frac{9b^2c^8d}{2}\right) \\ & + x\left(a^2c^3 + 2abc^6 + b^2c^9\right)\end{aligned}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**3)**2,x)
```

output
```
9*b**2*c**2*d**7*x**8/2 + b**2*c*d**8*x**9 + b**2*d**9*x**10/10 + x**7*(2*
a*b*d**6/7 + 12*b**2*c**3*d**6) + x**6*(2*a*b*c*d**5 + 21*b**2*c**4*d**5)
+ x**5*(6*a*b*c**2*d**4 + 126*b**2*c**5*d**4/5) + x**4*(a**2*d**3/4 + 10*a
*b*c**3*d**3 + 21*b**2*c**6*d**3) + x**3*(a**2*c*d**2 + 10*a*b*c**4*d**2 +
 12*b**2*c**7*d**2) + x**2*(3*a**2*c**2*d/2 + 6*a*b*c**5*d + 9*b**2*c**8*d
/2) + x*(a**2*c**3 + 2*a*b*c**6 + b**2*c**9)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(45) = 90$.

Time = 0.03 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.12

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx = \frac{1}{10}b^2d^9x^{10}+b^2cd^8x^9+\frac{9}{2}b^2c^2d^7x^8+\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6x^7 + \left(21\,b^2c^4+2\,abc\right)d^5x^6+\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4x^5 + \frac{1}{4}\left(84\,b^2c^6+40\,abc^3+a^2\right)d^3x^4 + \left(12\,b^2c^7+10\,abc^4+a^2c\right)d^2x^3 + \frac{3}{2}\left(3\,b^2c^8+4\,abc^5+a^2c^2\right)dx^2 + \left(b^2c^9+2\,abc^6+a^2c^3\right)x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*d^9*x^10 + b^2*c*d^8*x^9 + 9/2*b^2*c^2*d^7*x^8 + 2/7*(42*b^2*c^3
+ a*b)*d^6*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*x^6 + 6/5*(21*b^2*c^5 + 5*a*b*
c^2)*d^4*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)*d^3*x^4 + (12*b^2*c^7 +
 10*a*b*c^4 + a^2*c)*d^2*x^3 + 3/2*(3*b^2*c^8 + 4*a*b*c^5 + a^2*c^2)*d*x^2
 + (b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(45) = 90.

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.86

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx = \frac{1}{10}b^2d^9x^{10}+b^2cd^8x^9+\frac{9}{2}b^2c^2d^7x^8+12\,b^2c^3d^6x^7 + 21\,b^2c^4d^5x^6+\frac{126}{5}b^2c^5d^4x^5+21\,b^2c^6d^3x^4+\frac{2}{7}abd^6x^7 + 12\,b^2c^7d^2x^3+2\,abcd^5x^6+\frac{9}{2}b^2c^8dx^2+6\,abc^2d^4x^5 + b^2c^9x+10\,abc^3d^3x^4+10\,abc^4d^2x^3+6\,abc^5dx^2 + 2\,abc^6x+\frac{1}{4}a^2d^3x^4+a^2cd^2x^3+\frac{3}{2}a^2c^2dx^2+a^2c^3x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/10*b^2*d^9*x^10 + b^2*c*d^8*x^9 + 9/2*b^2*c^2*d^7*x^8 + 12*b^2*c^3*d^6*x
^7 + 21*b^2*c^4*d^5*x^6 + 126/5*b^2*c^5*d^4*x^5 + 21*b^2*c^6*d^3*x^4 + 2/7
*a*b*d^6*x^7 + 12*b^2*c^7*d^2*x^3 + 2*a*b*c*d^5*x^6 + 9/2*b^2*c^8*d*x^2 +
6*a*b*c^2*d^4*x^5 + b^2*c^9*x + 10*a*b*c^3*d^3*x^4 + 10*a*b*c^4*d^2*x^3 +
6*a*b*c^5*d*x^2 + 2*a*b*c^6*x + 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*
c^2*d*x^2 + a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.75

$$\int (c+dx)^3\left(a+b(c+dx)^3\right)^2\,dx = c^3\,x\left(b\,c^3+a\right)^2+\frac{b^2\,d^9\,x^{10}}{10}+\frac{d^3\,x^4\left(a^2+40\,a\,b\,c^3+84\,b^2\,c^6\right)}{4}+b^2\,c\,d^8\,x^9+\frac{3\,c^2\,d\,x^2\left(a^2+4\,a\,b\,c^3+3\,b^2\,c^6\right)}{2}+c\,d^2\,x^3\left(a^2+10\,a\,b\,c^3+12\,b^2\,c^6\right)+\frac{9\,b^2\,c^2\,d^7\,x^8}{2}+\frac{2\,b\,d^6\,x^7\left(42\,b\,c^3+a\right)}{7}+b\,c\,d^5\,x^6\left(21\,b\,c^3+2\,a\right)+\frac{6\,b\,c^2\,d^4\,x^5\left(21\,b\,c^3+5\,a\right)}{5}$$

input
```
int((a + b*(c + d*x)^3)^2*(c + d*x)^3,x)
```

output
```
c^3*x*(a + b*c^3)^2 + (b^2*d^9*x^10)/10 + (d^3*x^4*(a^2 + 84*b^2*c^6 + 40*
a*b*c^3))/4 + b^2*c*d^8*x^9 + (3*c^2*d*x^2*(a^2 + 3*b^2*c^6 + 4*a*b*c^3))/
2 + c*d^2*x^3*(a^2 + 12*b^2*c^6 + 10*a*b*c^3) + (9*b^2*c^2*d^7*x^8)/2 + (2
*b*d^6*x^7*(a + 42*b*c^3))/7 + b*c*d^5*x^6*(2*a + 21*b*c^3) + (6*b*c^2*d^4
*x^5*(5*a + 21*b*c^3))/5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 4.82

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^2 \, dx$$
$$= \frac{x(14b^2d^9x^9 + 140b^2c\,d^8x^8 + 630b^2c^2d^7x^7 + 1680b^2c^3d^6x^6 + 2940b^2c^4d^5x^5 + 3528b^2c^5d^4x^4 + 40ab\,d^6x^6 +}{}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^3)^2,x)
```

output
```
(x*(140*a**2*c**3 + 210*a**2*c**2*d*x + 140*a**2*c*d**2*x**2 + 35*a**2*d**
3*x**3 + 280*a*b*c**6 + 840*a*b*c**5*d*x + 1400*a*b*c**4*d**2*x**2 + 1400*
a*b*c**3*d**3*x**3 + 840*a*b*c**2*d**4*x**4 + 280*a*b*c*d**5*x**5 + 40*a*b
*d**6*x**6 + 140*b**2*c**9 + 630*b**2*c**8*d*x + 1680*b**2*c**7*d**2*x**2
+ 2940*b**2*c**6*d**3*x**3 + 3528*b**2*c**5*d**4*x**4 + 2940*b**2*c**4*d**
5*x**5 + 1680*b**2*c**3*d**6*x**6 + 630*b**2*c**2*d**7*x**7 + 140*b**2*c*d
**8*x**8 + 14*b**2*d**9*x**9))/140
```

3.54 $\int (c+dx)^3 \left(a+b(c+dx)^3\right)^3 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 71

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^3 \, dx = \frac{a^3(c+dx)^4}{4d} + \frac{3a^2b(c+dx)^7}{7d} + \frac{3ab^2(c+dx)^{10}}{10d} + \frac{b^3(c+dx)^{13}}{13d}$$

output

```
1/4*a^3*(d*x+c)^4/d+3/7*a^2*b*(d*x+c)^7/d+3/10*a*b^2*(d*x+c)^10/d+1/13*b^3
*(d*x+c)^13/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 323 vs. $2(71) = 142$.

Time $= 0.03$ (sec) , antiderivative size $= 323$, normalized size of antiderivative $= 4.55$

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx &= c^3\left(a+bc^3\right)^3 x+\frac{3}{2}c^2\left(a+bc^3\right)^2\left(a+4bc^3\right)dx^2\\ &+c\left(a^3+15a^2bc^3+36ab^2c^6+22b^3c^9\right)d^2x^3\\ &+\frac{1}{4}\left(a^3+60a^2bc^3+252ab^2c^6+220b^3c^9\right)d^3x^4\\ &+\frac{9}{5}bc^2\left(5a^2+42abc^3+55b^2c^6\right)d^4x^5\\ &+3bc\left(a^2+21abc^3+44b^2c^6\right)d^5x^6\\ &+\frac{3}{7}b\left(a^2+84abc^3+308b^2c^6\right)d^6x^7\\ &+\frac{9}{2}b^2c^2\left(3a+22bc^3\right)d^7x^8+b^2c\left(3a+55bc^3\right)d^8x^9\\ &+\frac{1}{10}b^2\left(3a+220bc^3\right)d^9x^{10}\\ &+6b^3c^2d^{10}x^{11}+b^3cd^{11}x^{12}+\frac{1}{13}b^3d^{12}x^{13}\end{aligned}$$

input

```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^3)^3,x]
```

output

```
c^3*(a + b*c^3)^3*x + (3*c^2*(a + b*c^3)^2*(a + 4*b*c^3)*d*x^2)/2 + c*(a^3
 + 15*a^2*b*c^3 + 36*a*b^2*c^6 + 22*b^3*c^9)*d^2*x^3 + ((a^3 + 60*a^2*b*c^
3 + 252*a*b^2*c^6 + 220*b^3*c^9)*d^3*x^4)/4 + (9*b*c^2*(5*a^2 + 42*a*b*c^3
 + 55*b^2*c^6)*d^4*x^5)/5 + 3*b*c*(a^2 + 21*a*b*c^3 + 44*b^2*c^6)*d^5*x^6
+ (3*b*(a^2 + 84*a*b*c^3 + 308*b^2*c^6)*d^6*x^7)/7 + (9*b^2*c^2*(3*a + 22*
b*c^3)*d^7*x^8)/2 + b^2*c*(3*a + 55*b*c^3)*d^8*x^9 + (b^2*(3*a + 220*b*c^3
)*d^9*x^10)/10 + 6*b^3*c^2*d^10*x^11 + b^3*c*d^11*x^12 + (b^3*d^12*x^13)/1
3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^3 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^3+a\right)^3 d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b^3(c+dx)^{12}+3ab^2(c+dx)^9+3a^2b(c+dx)^6+a^3(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a^3(c+dx)^4+\frac{3}{7}a^2b(c+dx)^7+\frac{3}{10}ab^2(c+dx)^{10}+\frac{1}{13}b^3(c+dx)^{13}}{d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^3)^3,x]
```

output
```
((a^3*(c + d*x)^4)/4 + (3*a^2*b*(c + d*x)^7)/7 + (3*a*b^2*(c + d*x)^10)/10
 + (b^3*(c + d*x)^13)/13)/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(63) = 126$.

Time = 0.68 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.72

method	result
norman	$\frac{d^{12}b^3x^{13}}{13} + c\,d^{11}b^3x^{12} + 6c^2d^{10}b^3x^{11} + \left(22c^3d^9b^3 + \frac{3}{10}a\,b^2d^9\right)x^{10} + \left(55c^4d^8b^3 + 3a\,b^2c\,d^8\right)x^9 +$
gosper	$\frac{x\left(140d^{12}b^3x^{12}+1820c\,d^{11}b^3x^{11}+10920c^2d^{10}b^3x^{10}+40040x^9c^3d^9b^3+100100b^3c^4d^8x^8+180180x^7c^5d^7b^3+546x^9a\,b^2d^9+240\right.}{}$
risch	$\frac{3}{2}d\,a^3c^2x^2 + 99x^5b^3c^8d^4 + 55x^4b^3c^9d^3 + 6x^2b^3c^{11}d + 55b^3c^4d^8x^9 + 132b^3c^7d^5x^6 + 22b^3c^{10}d^2$
parallelrisch	$\frac{3}{2}d\,a^3c^2x^2 + 99x^5b^3c^8d^4 + 55x^4b^3c^9d^3 + 6x^2b^3c^{11}d + 55b^3c^4d^8x^9 + 132b^3c^7d^5x^6 + 22b^3c^{10}d^2$
orering	$\frac{x\left(140d^{12}b^3x^{12}+1820c\,d^{11}b^3x^{11}+10920c^2d^{10}b^3x^{10}+40040x^9c^3d^9b^3+100100b^3c^4d^8x^8+180180x^7c^5d^7b^3+546x^9a\,b^2d^9+240\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^3*(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*d^12*b^3*x^13+c*d^11*b^3*x^12+6*c^2*d^10*b^3*x^11+(22*c^3*d^9*b^3+3/1
0*a*b^2*d^9)*x^10+(55*b^3*c^4*d^8+3*a*b^2*c*d^8)*x^9+(99*c^5*d^7*b^3+27/2*
a*b^2*c^2*d^7)*x^8+(132*b^3*c^6*d^6+36*a*b^2*c^3*d^6+3/7*a^2*b*d^6)*x^7+(1
32*b^3*c^7*d^5+63*a*b^2*c^4*d^5+3*a^2*b*c*d^5)*x^6+(99*b^3*c^8*d^4+378/5*a
*b^2*c^5*d^4+9*a^2*b*c^2*d^4)*x^5+(55*b^3*c^9*d^3+63*a*b^2*c^6*d^3+15*a^2*
b*c^3*d^3+1/4*a^3*d^3)*x^4+(22*b^3*c^10*d^2+36*a*b^2*c^7*d^2+15*a^2*b*c^4*
d^2+a^3*c*d^2)*x^3+(6*b^3*c^11*d+27/2*a*b^2*c^8*d+9*a^2*b*c^5*d+3/2*a^3*c^
2*d)*x^2+(b^3*c^12+3*a*b^2*c^9+3*a^2*b*c^6+a^3*c^3)*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.06

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & \frac{1}{13}b^3d^{12}x^{13} + b^3cd^{11}x^{12} + 6\,b^3c^2d^{10}x^{11} \\ & + \frac{1}{10}\left(220\,b^3c^3 + 3\,ab^2\right)d^9x^{10} \\ & + \left(55\,b^3c^4 + 3\,ab^2c\right)d^8x^9 + \frac{9}{2}\left(22\,b^3c^5 + 3\,ab^2c^2\right)d^7x^8 \\ & + \frac{3}{7}\left(308\,b^3c^6 + 84\,ab^2c^3 + a^2b\right)d^6x^7 \\ & + 3\left(44\,b^3c^7 + 21\,ab^2c^4 + a^2bc\right)d^5x^6 \\ & + \frac{9}{5}\left(55\,b^3c^8 + 42\,ab^2c^5 + 5\,a^2bc^2\right)d^4x^5 \\ & + \frac{1}{4}\left(220\,b^3c^9 + 252\,ab^2c^6 + 60\,a^2bc^3 + a^3\right)d^3x^4 \\ & + \left(22\,b^3c^{10} + 36\,ab^2c^7 + 15\,a^2bc^4 + a^3c\right)d^2x^3 \\ & + \frac{3}{2}\left(4\,b^3c^{11} + 9\,ab^2c^8 + 6\,a^2bc^5 + a^3c^2\right)dx^2 \\ & + \left(b^3c^{12} + 3\,ab^2c^9 + 3\,a^2bc^6 + a^3c^3\right)x\end{aligned}$$

input

```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
1/13*b^3*d^12*x^13 + b^3*c*d^11*x^12 + 6*b^3*c^2*d^10*x^11 + 1/10*(220*b^3
*c^3 + 3*a*b^2)*d^9*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*x^9 + 9/2*(22*b^3*
c^5 + 3*a*b^2*c^2)*d^7*x^8 + 3/7*(308*b^3*c^6 + 84*a*b^2*c^3 + a^2*b)*d^6*
x^7 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*d^5*x^6 + 9/5*(55*b^3*c^8 +
42*a*b^2*c^5 + 5*a^2*b*c^2)*d^4*x^5 + 1/4*(220*b^3*c^9 + 252*a*b^2*c^6 + 6
0*a^2*b*c^3 + a^3)*d^3*x^4 + (22*b^3*c^10 + 36*a*b^2*c^7 + 15*a^2*b*c^4 +
a^3*c)*d^2*x^3 + 3/2*(4*b^3*c^11 + 9*a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*
x^2 + (b^3*c^12 + 3*a*b^2*c^9 + 3*a^2*b*c^6 + a^3*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(60) = 120$.

Time = 0.10 (sec) , antiderivative size = 437, normalized size of antiderivative = 6.15

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & 6b^3c^2d^{10}x^{11} + b^3cd^{11}x^{12} + \frac{b^3d^{12}x^{13}}{13} + x^{10} \\ & \cdot\left(\frac{3ab^2d^9}{10} + 22b^3c^3d^9\right) + x^9\cdot\left(3ab^2cd^8 + 55b^3c^4d^8\right) \\ & + x^8\cdot\left(\frac{27ab^2c^2d^7}{2} + 99b^3c^5d^7\right) + x^7 \\ & \cdot\left(\frac{3a^2bd^6}{7} + 36ab^2c^3d^6 + 132b^3c^6d^6\right) + x^6 \\ & \cdot\left(3a^2bcd^5 + 63ab^2c^4d^5 + 132b^3c^7d^5\right) + x^5 \\ & \cdot\left(9a^2bc^2d^4 + \frac{378ab^2c^5d^4}{5} + 99b^3c^8d^4\right) \\ & + x^4\left(\frac{a^3d^3}{4} + 15a^2bc^3d^3 + 63ab^2c^6d^3 + 55b^3c^9d^3\right) \\ & + x^3\left(a^3cd^2 + 15a^2bc^4d^2 + 36ab^2c^7d^2 + 22b^3c^{10}d^2\right) \\ & + x^2\cdot\left(\frac{3a^3c^2d}{2} + 9a^2bc^5d + \frac{27ab^2c^8d}{2} + 6b^3c^{11}d\right) \\ & + x\left(a^3c^3 + 3a^2bc^6 + 3ab^2c^9 + b^3c^{12}\right)\end{aligned}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**3)**3,x)
```

output
```
6*b**3*c**2*d**10*x**11 + b**3*c*d**11*x**12 + b**3*d**12*x**13/13 + x**10
*(3*a*b**2*d**9/10 + 22*b**3*c**3*d**9) + x**9*(3*a*b**2*c*d**8 + 55*b**3*
c**4*d**8) + x**8*(27*a*b**2*c**2*d**7/2 + 99*b**3*c**5*d**7) + x**7*(3*a*
*2*b*d**6/7 + 36*a*b**2*c**3*d**6 + 132*b**3*c**6*d**6) + x**6*(3*a**2*b*c
*d**5 + 63*a*b**2*c**4*d**5 + 132*b**3*c**7*d**5) + x**5*(9*a**2*b*c**2*d*
*4 + 378*a*b**2*c**5*d**4/5 + 99*b**3*c**8*d**4) + x**4*(a**3*d**3/4 + 15*
a**2*b*c**3*d**3 + 63*a*b**2*c**6*d**3 + 55*b**3*c**9*d**3) + x**3*(a**3*c
*d**2 + 15*a**2*b*c**4*d**2 + 36*a*b**2*c**7*d**2 + 22*b**3*c**10*d**2) +
x**2*(3*a**3*c**2*d/2 + 9*a**2*b*c**5*d + 27*a*b**2*c**8*d/2 + 6*b**3*c**1
1*d) + x*(a**3*c**3 + 3*a**2*b*c**6 + 3*a*b**2*c**9 + b**3*c**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(63) = 126$.

Time = 0.04 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.06

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & \frac{1}{13}b^3d^{12}x^{13} + b^3cd^{11}x^{12} + 6\,b^3c^2d^{10}x^{11} \\ & + \frac{1}{10}\left(220\,b^3c^3 + 3\,ab^2\right)d^9x^{10} \\ & + \left(55\,b^3c^4 + 3\,ab^2c\right)d^8x^9 + \frac{9}{2}\left(22\,b^3c^5 + 3\,ab^2c^2\right)d^7x^8 \\ & + \frac{3}{7}\left(308\,b^3c^6 + 84\,ab^2c^3 + a^2b\right)d^6x^7 \\ & + 3\left(44\,b^3c^7 + 21\,ab^2c^4 + a^2bc\right)d^5x^6 \\ & + \frac{9}{5}\left(55\,b^3c^8 + 42\,ab^2c^5 + 5\,a^2bc^2\right)d^4x^5 \\ & + \frac{1}{4}\left(220\,b^3c^9 + 252\,ab^2c^6 + 60\,a^2bc^3 + a^3\right)d^3x^4 \\ & + \left(22\,b^3c^{10} + 36\,ab^2c^7 + 15\,a^2bc^4 + a^3c\right)d^2x^3 \\ & + \frac{3}{2}\left(4\,b^3c^{11} + 9\,ab^2c^8 + 6\,a^2bc^5 + a^3c^2\right)dx^2 \\ & + \left(b^3c^{12} + 3\,ab^2c^9 + 3\,a^2bc^6 + a^3c^3\right)x\end{aligned}$$

input

```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
1/13*b^3*d^12*x^13 + b^3*c*d^11*x^12 + 6*b^3*c^2*d^10*x^11 + 1/10*(220*b^3
*c^3 + 3*a*b^2)*d^9*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*x^9 + 9/2*(22*b^3*
c^5 + 3*a*b^2*c^2)*d^7*x^8 + 3/7*(308*b^3*c^6 + 84*a*b^2*c^3 + a^2*b)*d^6*
x^7 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*d^5*x^6 + 9/5*(55*b^3*c^8 +
42*a*b^2*c^5 + 5*a^2*b*c^2)*d^4*x^5 + 1/4*(220*b^3*c^9 + 252*a*b^2*c^6 + 6
0*a^2*b*c^3 + a^3)*d^3*x^4 + (22*b^3*c^10 + 36*a*b^2*c^7 + 15*a^2*b*c^4 +
a^3*c)*d^2*x^3 + 3/2*(4*b^3*c^11 + 9*a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*
x^2 + (b^3*c^12 + 3*a*b^2*c^9 + 3*a^2*b*c^6 + a^3*c^3)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(63) = 126.

Time = 0.13 (sec) , antiderivative size = 442, normalized size of antiderivative = 6.23

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & \frac{1}{13}b^3d^{12}x^{13}+b^3cd^{11}x^{12}+6\,b^3c^2d^{10}x^{11}+22\,b^3c^3d^9x^{10}\\ & +55\,b^3c^4d^8x^9+99\,b^3c^5d^7x^8+132\,b^3c^6d^6x^7\\ & +\frac{3}{10}ab^2d^9x^{10}+132\,b^3c^7d^5x^6+3\,ab^2cd^8x^9\\ & +99\,b^3c^8d^4x^5+\frac{27}{2}ab^2c^2d^7x^8+55\,b^3c^9d^3x^4\\ & +36\,ab^2c^3d^6x^7+22\,b^3c^{10}d^2x^3+63\,ab^2c^4d^5x^6\\ & +6\,b^3c^{11}dx^2+\frac{378}{5}ab^2c^5d^4x^5+b^3c^{12}x\\ & +63\,ab^2c^6d^3x^4+\frac{3}{7}a^2bd^6x^7+36\,ab^2c^7d^2x^3\\ & +3\,a^2bcd^5x^6+\frac{27}{2}ab^2c^8dx^2+9\,a^2bc^2d^4x^5+3\,ab^2c^9x\\ & +15\,a^2bc^3d^3x^4+15\,a^2bc^4d^2x^3+9\,a^2bc^5dx^2\\ & +3\,a^2bc^6x+\frac{1}{4}a^3d^3x^4+a^3cd^2x^3+\frac{3}{2}a^3c^2dx^2+a^3c^3x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/13*b^3*d^12*x^13 + b^3*c*d^11*x^12 + 6*b^3*c^2*d^10*x^11 + 22*b^3*c^3*d^
9*x^10 + 55*b^3*c^4*d^8*x^9 + 99*b^3*c^5*d^7*x^8 + 132*b^3*c^6*d^6*x^7 + 3
/10*a*b^2*d^9*x^10 + 132*b^3*c^7*d^5*x^6 + 3*a*b^2*c*d^8*x^9 + 99*b^3*c^8*
d^4*x^5 + 27/2*a*b^2*c^2*d^7*x^8 + 55*b^3*c^9*d^3*x^4 + 36*a*b^2*c^3*d^6*x
^7 + 22*b^3*c^10*d^2*x^3 + 63*a*b^2*c^4*d^5*x^6 + 6*b^3*c^11*d*x^2 + 378/5
*a*b^2*c^5*d^4*x^5 + b^3*c^12*x + 63*a*b^2*c^6*d^3*x^4 + 3/7*a^2*b*d^6*x^7
 + 36*a*b^2*c^7*d^2*x^3 + 3*a^2*b*c*d^5*x^6 + 27/2*a*b^2*c^8*d*x^2 + 9*a^2
*b*c^2*d^4*x^5 + 3*a*b^2*c^9*x + 15*a^2*b*c^3*d^3*x^4 + 15*a^2*b*c^4*d^2*x
^3 + 9*a^2*b*c^5*d*x^2 + 3*a^2*b*c^6*x + 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 +
 3/2*a^3*c^2*d*x^2 + a^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.35

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^3\right)^3\,dx &= c^3\,x\left(b\,c^3+a\right)^3\\ &+\frac{d^3\,x^4\left(a^3+60\,a^2\,b\,c^3+252\,a\,b^2\,c^6+220\,b^3\,c^9\right)}{4}\\ &+\frac{b^3\,d^{12}\,x^{13}}{13}+b^3\,c\,d^{11}\,x^{12}\\ &+\frac{3\,b\,d^6\,x^7\left(a^2+84\,a\,b\,c^3+308\,b^2\,c^6\right)}{7}\\ &+\frac{b^2\,d^9\,x^{10}\left(220\,b\,c^3+3\,a\right)}{10}+6\,b^3\,c^2\,d^{10}\,x^{11}\\ &+c\,d^2\,x^3\left(a^3+15\,a^2\,b\,c^3+36\,a\,b^2\,c^6+22\,b^3\,c^9\right)\\ &+\frac{9\,b^2\,c^2\,d^7\,x^8\left(22\,b\,c^3+3\,a\right)}{2}\\ &+\frac{3\,c^2\,d\,x^2\left(b\,c^3+a\right)^2\left(4\,b\,c^3+a\right)}{2}\\ &+\frac{9\,b\,c^2\,d^4\,x^5\left(5\,a^2+42\,a\,b\,c^3+55\,b^2\,c^6\right)}{5}\\ &+3\,b\,c\,d^5\,x^6\left(a^2+21\,a\,b\,c^3+44\,b^2\,c^6\right)\\ &+b^2\,c\,d^8\,x^9\left(55\,b\,c^3+3\,a\right)\end{aligned}$$

input

```
int((a + b*(c + d*x)^3)^3*(c + d*x)^3,x)
```

output

```
c^3*x*(a + b*c^3)^3 + (d^3*x^4*(a^3 + 220*b^3*c^9 + 60*a^2*b*c^3 + 252*a*b
^2*c^6))/4 + (b^3*d^12*x^13)/13 + b^3*c*d^11*x^12 + (3*b*d^6*x^7*(a^2 + 30
8*b^2*c^6 + 84*a*b*c^3))/7 + (b^2*d^9*x^10*(3*a + 220*b*c^3))/10 + 6*b^3*c
^2*d^10*x^11 + c*d^2*x^3*(a^3 + 22*b^3*c^9 + 15*a^2*b*c^3 + 36*a*b^2*c^6)
+ (9*b^2*c^2*d^7*x^8*(3*a + 22*b*c^3))/2 + (3*c^2*d*x^2*(a + b*c^3)^2*(a +
 4*b*c^3))/2 + (9*b*c^2*d^4*x^5*(5*a^2 + 55*b^2*c^6 + 42*a*b*c^3))/5 + 3*b
*c*d^5*x^6*(a^2 + 44*b^2*c^6 + 21*a*b*c^3) + b^2*c*d^8*x^9*(3*a + 55*b*c^3
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 437, normalized size of antiderivative = 6.15

$$\int (c+dx)^3 \left(a+b(c+dx)^3\right)^3 \, dx$$
$$= \frac{x(140b^3d^{12}x^{12} + 1820b^3cd^{11}x^{11} + 10920b^3c^2d^{10}x^{10} + 40040b^3c^3d^9x^9 + 100100b^3c^4d^8x^8 + 180180b^3c^5d^7x^7}$$

input

```
int((d*x+c)^3*(a+b*(d*x+c)^3)^3,x)
```

output

```
(x*(1820*a**3*c**3 + 2730*a**3*c**2*d*x + 1820*a**3*c*d**2*x**2 + 455*a**3
*d**3*x**3 + 5460*a**2*b*c**6 + 16380*a**2*b*c**5*d*x + 27300*a**2*b*c**4*
d**2*x**2 + 27300*a**2*b*c**3*d**3*x**3 + 16380*a**2*b*c**2*d**4*x**4 + 54
60*a**2*b*c*d**5*x**5 + 780*a**2*b*d**6*x**6 + 5460*a*b**2*c**9 + 24570*a*
b**2*c**8*d*x + 65520*a*b**2*c**7*d**2*x**2 + 114660*a*b**2*c**6*d**3*x**3
 + 137592*a*b**2*c**5*d**4*x**4 + 114660*a*b**2*c**4*d**5*x**5 + 65520*a*b
**2*c**3*d**6*x**6 + 24570*a*b**2*c**2*d**7*x**7 + 5460*a*b**2*c*d**8*x**8
 + 546*a*b**2*d**9*x**9 + 1820*b**3*c**12 + 10920*b**3*c**11*d*x + 40040*b
**3*c**10*d**2*x**2 + 100100*b**3*c**9*d**3*x**3 + 180180*b**3*c**8*d**4*x
**4 + 240240*b**3*c**7*d**5*x**5 + 240240*b**3*c**6*d**6*x**6 + 180180*b**
3*c**5*d**7*x**7 + 100100*b**3*c**4*d**8*x**8 + 40040*b**3*c**3*d**9*x**9
+ 10920*b**3*c**2*d**10*x**10 + 1820*b**3*c*d**11*x**11 + 140*b**3*d**12*x
**12))/1820
```

3.55 $\int (ce+dex)^3 \left(a+b(c+dx)^3\right)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 37

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)\, dx = \frac{ae^3(c+dx)^4}{4d} + \frac{be^3(c+dx)^7}{7d}$$

output
```
1/4*a*e^3*(d*x+c)^4/d+1/7*b*e^3*(d*x+c)^7/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(37) = 74.

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.76

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)\, dx = e^3\left(c^3\left(a+bc^3\right)x + \frac{3}{2}c^2\left(a+2bc^3\right)dx^2 + c\left(a+5bc^3\right)d^2x^3 + \frac{1}{4}\left(a+20bc^3\right)d^3x^4 + 3bc^2d^4x^5 + bcd^5x^6 + \frac{1}{7}bd^6x^7\right)$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3),x]
```

output
```
e^3*(c^3*(a + b*c^3)*x + (3*c^2*(a + 2*b*c^3)*d*x^2)/2 + c*(a + 5*b*c^3)*d
^2*x^3 + ((a + 20*b*c^3)*d^3*x^4)/4 + 3*b*c^2*d^4*x^5 + b*c*d^5*x^6 + (b*d
^6*x^7)/7)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)\, dx$$

$$\downarrow 895$$

$$\frac{e^3 \int (c+dx)^3 \left(b(c+dx)^3+a\right) d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{e^3 \int \left(b(c+dx)^6+a(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{e^3\left(\frac{1}{4}a(c+dx)^4+\frac{1}{7}b(c+dx)^7\right)}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3),x]
```

output
```
(e^3*((a*(c + d*x)^4)/4 + (b*(c + d*x)^7)/7))/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(33) = 66$.

Time $= 0.64$ (sec) , antiderivative size $= 110$, normalized size of antiderivative $= 2.97$

method	result
gosper	$\frac{e^3x\left(4d^6bx^6+28cd^5bx^5+84c^2d^4bx^4+140x^3c^3bd^3+140bc^4d^2x^2+84xc^5bd+7ad^3x^3+28bc^6+28x^2acd^2+42xac^2d+28c^3a\right)}{28}$
norman	$\left(5c^3e^3bd^3+\frac{1}{4}ad^3e^3\right)x^4+\left(3c^5e^3bd+\frac{3}{2}ac^2de^3\right)x^2+\left(bc^6e^3+ac^3e^3\right)x+\left(5c^4e^3bd^2+acd\right.$
risch	$\frac{1}{7}e^3d^6bx^7+ce^3d^5bx^6+3c^2e^3d^4bx^5+5e^3x^4c^3bd^3+\frac{1}{4}e^3ad^3x^4+5e^3bc^4d^2x^3+e^3d^2acx^3+$
parallelrisch	$\frac{1}{7}e^3d^6bx^7+ce^3d^5bx^6+3c^2e^3d^4bx^5+5e^3x^4c^3bd^3+\frac{1}{4}e^3ad^3x^4+5e^3bc^4d^2x^3+e^3d^2acx^3+$
default	$\frac{e^3d^6bx^7}{7}+ce^3d^5bx^6+3c^2e^3d^4bx^5+\frac{(19c^3e^3bd^3+e^3d^3(c^3b+a))x^4}{4}+\frac{(12c^4e^3bd^2+3ce^3d^2(c^3b+a))x^3}{3}+\frac{(3}{}$
orering	$\frac{x\left(4d^6bx^6+28cd^5bx^5+84c^2d^4bx^4+140x^3c^3bd^3+140bc^4d^2x^2+84xc^5bd+7ad^3x^3+28bc^6+28x^2acd^2+42xac^2d+28c^3a\right)(de}{28(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}$

input
```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/28*e^3*x*(4*b*d^6*x^6+28*b*c*d^5*x^5+84*b*c^2*d^4*x^4+140*b*c^3*d^3*x^3+
140*b*c^4*d^2*x^2+84*b*c^5*d*x+7*a*d^3*x^3+28*b*c^6+28*a*c*d^2*x^2+42*a*c^
2*d*x+28*a*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(33) = 66$.

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.14

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}\,bd^6e^3x^7 + bcd^5e^3x^6 + 3\,bc^2d^4e^3x^5 + \frac{1}{4}\left(20\,bc^3+a\right)d^3e^3x^4 + \left(5\,bc^4+ac\right)d^2e^3x^3 + \frac{3}{2}\left(2\,bc^5+ac^2\right)de^3x^2 + \left(bc^6+ac^3\right)e^3x$$

input
```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/7*b*d^6*e^3*x^7 + b*c*d^5*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5 + 1/4*(20*b*c^3
+ a)*d^3*e^3*x^4 + (5*b*c^4 + a*c)*d^2*e^3*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*e
^3*x^2 + (b*c^6 + a*c^3)*e^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.89

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = 3bc^2d^4e^3x^5 + bcd^5e^3x^6 + \frac{bd^6e^3x^7}{7} + x^4\left(\frac{ad^3e^3}{4} + 5bc^3d^3e^3\right) + x^3\left(acd^2e^3 + 5bc^4d^2e^3\right) + x^2\cdot\left(\frac{3ac^2de^3}{2} + 3bc^5de^3\right) + x\left(ac^3e^3 + bc^6e^3\right)$$

input
```
integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3),x)
```

output
```
3*b*c**2*d**4*e**3*x**5 + b*c*d**5*e**3*x**6 + b*d**6*e**3*x**7/7 + x**4*(
a*d**3*e**3/4 + 5*b*c**3*d**3*e**3) + x**3*(a*c*d**2*e**3 + 5*b*c**4*d**2*
e**3) + x**2*(3*a*c**2*d*e**3/2 + 3*b*c**5*d*e**3) + x*(a*c**3*e**3 + b*c*
*6*e**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(33) = 66$.

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.14

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}bd^6e^3x^7 + bcd^5e^3x^6 + 3\,bc^2d^4e^3x^5 + \frac{1}{4}\left(20\,bc^3+a\right)d^3e^3x^4 + \left(5\,bc^4+ac\right)d^2e^3x^3 + \frac{3}{2}\left(2\,bc^5+ac^2\right)de^3x^2 + \left(bc^6+ac^3\right)e^3x$$

input
```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/7*b*d^6*e^3*x^7 + b*c*d^5*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5 + 1/4*(20*b*c^3
+ a)*d^3*e^3*x^4 + (5*b*c^4 + a*c)*d^2*e^3*x^3 + 3/2*(2*b*c^5 + a*c^2)*d*e
^3*x^2 + (b*c^6 + a*c^3)*e^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(33) = 66$.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.73

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = \frac{1}{7}bd^6e^3x^7 + bcd^5e^3x^6 + 3\,bc^2d^4e^3x^5 + 5\,bc^3d^3e^3x^4 + 5\,bc^4d^2e^3x^3 + 3\,bc^5de^3x^2 + bc^6e^3x + \frac{1}{4}ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}ac^2de^3x^2 + ac^3e^3x$$

input
```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/7*b*d^6*e^3*x^7 + b*c*d^5*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5 + 5*b*c^3*d^3*e^
3*x^4 + 5*b*c^4*d^2*e^3*x^3 + 3*b*c^5*d*e^3*x^2 + b*c^6*e^3*x + 1/4*a*d^3*
e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + a*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.05

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = \frac{d^3e^3x^4\left(20bc^3+a\right)}{4} + \frac{bd^6e^3x^7}{7} + c^3e^3x\left(bc^3+a\right) + \frac{3c^2de^3x^2\left(2bc^3+a\right)}{2} + cd^2e^3x^3\left(5bc^3+a\right) + bcd^5e^3x^6 + 3bc^2d^4e^3x^5$$

input
```
int((c*e + d*e*x)^3*(a + b*(c + d*x)^3),x)
```

output
```
(d^3*e^3*x^4*(a + 20*b*c^3))/4 + (b*d^6*e^3*x^7)/7 + c^3*e^3*x*(a + b*c^3)
 + (3*c^2*d*e^3*x^2*(a + 2*b*c^3))/2 + c*d^2*e^3*x^3*(a + 5*b*c^3) + b*c*d
^5*e^3*x^6 + 3*b*c^2*d^4*e^3*x^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)\,dx = \frac{e^3x(4bd^6x^6+28bcd^5x^5+84bc^2d^4x^4+140bc^3d^3x^3+140bc^4d^2x^2+84bc^5dx+7ad^3x^3+28bc^6+28ac$$

input
```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3),x)
```

output
```
(e**3*x*(28*a*c**3 + 42*a*c**2*d*x + 28*a*c*d**2*x**2 + 7*a*d**3*x**3 + 28
*b*c**6 + 84*b*c**5*d*x + 140*b*c**4*d**2*x**2 + 140*b*c**3*d**3*x**3 + 84
*b*c**2*d**4*x**4 + 28*b*c*d**5*x**5 + 4*b*d**6*x**6))/28
```

3.56 $\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^2 \, dx$

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Optimal result

Integrand size = 24, antiderivative size = 60

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^2 \, dx = \frac{a^2e^3(c+dx)^4}{4d} + \frac{2abe^3(c+dx)^7}{7d} + \frac{b^2e^3(c+dx)^{10}}{10d}$$

output
```
1/4*a^2*e^3*(d*x+c)^4/d+2/7*a*b*e^3*(d*x+c)^7/d+1/10*b^2*e^3*(d*x+c)^10/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. $2(60) = 120$.

Time = 0.01 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.45

$$\begin{aligned}\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^2 \, dx = e^3\Bigg(& c^3\left(a+bc^3\right)^2 x + \frac{3}{2}c^2\left(a^2+4abc^3+3b^2c^6\right) dx^2 \\ & + c\left(a^2+10abc^3+12b^2c^6\right) d^2x^3 \\ & + \frac{1}{4}\left(a^2+40abc^3+84b^2c^6\right) d^3x^4 \\ & + \frac{6}{5}bc^2\left(5a+21bc^3\right) d^4x^5 + bc\left(2a+21bc^3\right) d^5x^6 \\ & + \frac{2}{7}b\left(a+42bc^3\right) d^6x^7 + \frac{9}{2}b^2c^2d^7x^8 + b^2cd^8x^9 \\ & + \frac{1}{10}b^2d^9x^{10}\Bigg)\end{aligned}$$

input
```
Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x]
```

output
```
e^3*(c^3*(a + b*c^3)^2*x + (3*c^2*(a^2 + 4*a*b*c^3 + 3*b^2*c^6)*d*x^2)/2 +
 c*(a^2 + 10*a*b*c^3 + 12*b^2*c^6)*d^2*x^3 + ((a^2 + 40*a*b*c^3 + 84*b^2*c
^6)*d^3*x^4)/4 + (6*b*c^2*(5*a + 21*b*c^3)*d^4*x^5)/5 + b*c*(2*a + 21*b*c^
3)*d^5*x^6 + (2*b*(a + 42*b*c^3)*d^6*x^7)/7 + (9*b^2*c^2*d^7*x^8)/2 + b^2*
c*d^8*x^9 + (b^2*d^9*x^10)/10)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^2 dx$$

$$\downarrow 895$$

$$\frac{e^3 \int (c+dx)^3 \left(b(c+dx)^3+a\right)^2 d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{e^3 \int \left(b^2(c+dx)^9+2ab(c+dx)^6+a^2(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{e^3\left(\frac{1}{4}a^2(c+dx)^4+\frac{2}{7}ab(c+dx)^7+\frac{1}{10}b^2(c+dx)^{10}\right)}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x]
```

output
```
(e^3*((a^2*(c + d*x)^4)/4 + (2*a*b*(c + d*x)^7)/7 + (b^2*(c + d*x)^10)/10)
)/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(54) = 108$.

Time $= 0.65$ (sec) , antiderivative size $= 250$, normalized size of antiderivative $= 4.17$

method	result
gosper	$\frac{e^3x\left(14d^9b^2x^9+140cd^8b^2x^8+630b^2c^2d^7x^7+1680x^6c^3b^2d^6+2940b^2c^4d^5x^5+3528x^4c^5b^2d^4+40x^6abd^6+2940x^3b^2c^6d^3+280a\right.}{}$
norman	$\left(12c^3e^3b^2d^6+\frac{2}{7}abd^6e^3\right)x^7+\left(\frac{126}{5}c^5e^3b^2d^4+6abc^2d^4e^3\right)x^5+\left(21b^2c^6d^3e^3+10abc^3d^3e^3+\right.$
risch	$\frac{1}{10}e^3d^9b^2x^{10}+ce^3d^8b^2x^9+\frac{9}{2}c^2e^3d^7b^2x^8+12e^3x^7c^3b^2d^6+\frac{2}{7}e^3x^7abd^6+21e^3b^2c^4d^5x^6+2e^3$
parallelrisch	$\frac{1}{10}e^3d^9b^2x^{10}+ce^3d^8b^2x^9+\frac{9}{2}c^2e^3d^7b^2x^8+12e^3x^7c^3b^2d^6+\frac{2}{7}e^3x^7abd^6+21e^3b^2c^4d^5x^6+2e^3$
orering	$\frac{x\left(14d^9b^2x^9+140cd^8b^2x^8+630b^2c^2d^7x^7+1680x^6c^3b^2d^6+2940b^2c^4d^5x^5+3528x^4c^5b^2d^4+40x^6abd^6+2940x^3b^2c^6d^3+280abc\right.}{}$
default	$\frac{e^3d^9b^2x^{10}}{10}+ce^3d^8b^2x^9+\frac{9c^2e^3d^7b^2x^8}{2}+\frac{\left(64c^3e^3b^2d^6+e^3d^3\left(2\left(c^3b+a\right)bd^3+18b^2c^3d^3\right)\right)x^7}{7}+\frac{(51c^4e^3b^2d^5+3ce^3d}{}$

input
```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/140*e^3*x*(14*b^2*d^9*x^9+140*b^2*c*d^8*x^8+630*b^2*c^2*d^7*x^7+1680*b^2
*c^3*d^6*x^6+2940*b^2*c^4*d^5*x^5+3528*b^2*c^5*d^4*x^4+40*a*b*d^6*x^6+2940
*b^2*c^6*d^3*x^3+280*a*b*c*d^5*x^5+1680*b^2*c^7*d^2*x^2+840*a*b*c^2*d^4*x^
4+630*b^2*c^8*d*x+1400*a*b*c^3*d^3*x^3+140*b^2*c^9+1400*a*b*c^4*d^2*x^2+84
0*a*b*c^5*d*x+35*a^2*d^3*x^3+280*a*b*c^6+140*a^2*c*d^2*x^2+210*a^2*c^2*d*x
+140*a^2*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(54) = 108$.

Time = 0.07 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.00

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & \frac{1}{10}\,b^2d^9e^3x^{10}+b^2cd^8e^3x^9+\frac{9}{2}\,b^2c^2d^7e^3x^8\\ & +\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6e^3x^7+\left(21\,b^2c^4+2\,abc\right)d^5e^3x^6\\ & +\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4e^3x^5\\ & +\frac{1}{4}\left(84\,b^2c^6+40\,abc^3+a^2\right)d^3e^3x^4\\ & +\left(12\,b^2c^7+10\,abc^4+a^2c\right)d^2e^3x^3\\ & +\frac{3}{2}\left(3\,b^2c^8+4\,abc^5+a^2c^2\right)de^3x^2\\ & +\left(b^2c^9+2\,abc^6+a^2c^3\right)e^3x\end{aligned}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*d^9*e^3*x^10 + b^2*c*d^8*e^3*x^9 + 9/2*b^2*c^2*d^7*e^3*x^8 + 2/7*
(42*b^2*c^3 + a*b)*d^6*e^3*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*e^3*x^6 + 6/5*
(21*b^2*c^5 + 5*a*b*c^2)*d^4*e^3*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)
*d^3*e^3*x^4 + (12*b^2*c^7 + 10*a*b*c^4 + a^2*c)*d^2*e^3*x^3 + 3/2*(3*b^2*
c^8 + 4*a*b*c^5 + a^2*c^2)*d*e^3*x^2 + (b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*e^3
*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(51) = 102$.

Time = 0.06 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.38

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & \frac{9b^2c^2d^7e^3x^8}{2} + b^2cd^8e^3x^9 + \frac{b^2d^9e^3x^{10}}{10} \\ & + x^7\cdot\left(\frac{2abd^6e^3}{7} + 12b^2c^3d^6e^3\right) \\ & + x^6\cdot\left(2abcd^5e^3 + 21b^2c^4d^5e^3\right) \\ & + x^5\cdot\left(6abc^2d^4e^3 + \frac{126b^2c^5d^4e^3}{5}\right) \\ & + x^4\left(\frac{a^2d^3e^3}{4} + 10abc^3d^3e^3 + 21b^2c^6d^3e^3\right) \\ & + x^3\left(a^2cd^2e^3 + 10abc^4d^2e^3 + 12b^2c^7d^2e^3\right) \\ & + x^2\cdot\left(\frac{3a^2c^2de^3}{2} + 6abc^5de^3 + \frac{9b^2c^8de^3}{2}\right) \\ & + x\left(a^2c^3e^3 + 2abc^6e^3 + b^2c^9e^3\right)\end{aligned}$$

input

```
integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3)**2,x)
```

output

```
9*b**2*c**2*d**7*e**3*x**8/2 + b**2*c*d**8*e**3*x**9 + b**2*d**9*e**3*x**1
0/10 + x**7*(2*a*b*d**6*e**3/7 + 12*b**2*c**3*d**6*e**3) + x**6*(2*a*b*c*d
**5*e**3 + 21*b**2*c**4*d**5*e**3) + x**5*(6*a*b*c**2*d**4*e**3 + 126*b**2
*c**5*d**4*e**3/5) + x**4*(a**2*d**3*e**3/4 + 10*a*b*c**3*d**3*e**3 + 21*b
**2*c**6*d**3*e**3) + x**3*(a**2*c*d**2*e**3 + 10*a*b*c**4*d**2*e**3 + 12*
b**2*c**7*d**2*e**3) + x**2*(3*a**2*c**2*d*e**3/2 + 6*a*b*c**5*d*e**3 + 9*
b**2*c**8*d*e**3/2) + x*(a**2*c**3*e**3 + 2*a*b*c**6*e**3 + b**2*c**9*e**3
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(54) = 108.

Time = 0.03 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.00

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & \frac{1}{10}b^2d^9e^3x^{10}+b^2cd^8e^3x^9+\frac{9}{2}b^2c^2d^7e^3x^8\\ & +\frac{2}{7}\left(42\,b^2c^3+ab\right)d^6e^3x^7+\left(21\,b^2c^4+2\,abc\right)d^5e^3x^6\\ & +\frac{6}{5}\left(21\,b^2c^5+5\,abc^2\right)d^4e^3x^5\\ & +\frac{1}{4}\left(84\,b^2c^6+40\,abc^3+a^2\right)d^3e^3x^4\\ & +\left(12\,b^2c^7+10\,abc^4+a^2c\right)d^2e^3x^3\\ & +\frac{3}{2}\left(3\,b^2c^8+4\,abc^5+a^2c^2\right)de^3x^2\\ & +\left(b^2c^9+2\,abc^6+a^2c^3\right)e^3x\end{aligned}$$

input
```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*d^9*e^3*x^10 + b^2*c*d^8*e^3*x^9 + 9/2*b^2*c^2*d^7*e^3*x^8 + 2/7*
(42*b^2*c^3 + a*b)*d^6*e^3*x^7 + (21*b^2*c^4 + 2*a*b*c)*d^5*e^3*x^6 + 6/5*
(21*b^2*c^5 + 5*a*b*c^2)*d^4*e^3*x^5 + 1/4*(84*b^2*c^6 + 40*a*b*c^3 + a^2)
*d^3*e^3*x^4 + (12*b^2*c^7 + 10*a*b*c^4 + a^2*c)*d^2*e^3*x^3 + 3/2*(3*b^2*
c^8 + 4*a*b*c^5 + a^2*c^2)*d*e^3*x^2 + (b^2*c^9 + 2*a*b*c^6 + a^2*c^3)*e^3
*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(54) = 108.

Time = 0.11 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.18

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx = \frac{1}{10}b^2d^9e^3x^{10} + b^2cd^8e^3x^9 + \frac{9}{2}b^2c^2d^7e^3x^8 + 12\,b^2c^3d^6e^3x^7 + 21\,b^2c^4d^5e^3x^6 + \frac{126}{5}b^2c^5d^4e^3x^5 + 21\,b^2c^6d^3e^3x^4 + \frac{2}{7}abd^6e^3x^7 + 12\,b^2c^7d^2e^3x^3 + 2\,abcd^5e^3x^6 + \frac{9}{2}b^2c^8de^3x^2 + 6\,abc^2d^4e^3x^5 + b^2c^9e^3x + 10\,abc^3d^3e^3x^4 + 10\,abc^4d^2e^3x^3 + 6\,abc^5de^3x^2 + 2\,abc^6e^3x + \frac{1}{4}a^2d^3e^3x^4 + a^2cd^2e^3x^3 + \frac{3}{2}a^2c^2de^3x^2 + a^2c^3e^3x$$

input

```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output

```
1/10*b^2*d^9*e^3*x^10 + b^2*c*d^8*e^3*x^9 + 9/2*b^2*c^2*d^7*e^3*x^8 + 12*b
^2*c^3*d^6*e^3*x^7 + 21*b^2*c^4*d^5*e^3*x^6 + 126/5*b^2*c^5*d^4*e^3*x^5 +
21*b^2*c^6*d^3*e^3*x^4 + 2/7*a*b*d^6*e^3*x^7 + 12*b^2*c^7*d^2*e^3*x^3 + 2*
a*b*c*d^5*e^3*x^6 + 9/2*b^2*c^8*d*e^3*x^2 + 6*a*b*c^2*d^4*e^3*x^5 + b^2*c^
9*e^3*x + 10*a*b*c^3*d^3*e^3*x^4 + 10*a*b*c^4*d^2*e^3*x^3 + 6*a*b*c^5*d*e^
3*x^2 + 2*a*b*c^6*e^3*x + 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^
2*c^2*d*e^3*x^2 + a^2*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.68

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx = {} & c^3\,e^3\,x\left(b\,c^3+a\right)^2+\frac{b^2\,d^9\,e^3\,x^{10}}{10}\\ & +\frac{d^3\,e^3\,x^4\left(a^2+40\,a\,b\,c^3+84\,b^2\,c^6\right)}{4}\\ & +\frac{3\,c^2\,d\,e^3\,x^2\left(a^2+4\,a\,b\,c^3+3\,b^2\,c^6\right)}{2}\\ & +c\,d^2\,e^3\,x^3\left(a^2+10\,a\,b\,c^3+12\,b^2\,c^6\right)\\ & +\frac{9\,b^2\,c^2\,d^7\,e^3\,x^8}{2}+\frac{2\,b\,d^6\,e^3\,x^7\left(42\,b\,c^3+a\right)}{7}\\ & +b^2\,c\,d^8\,e^3\,x^9+b\,c\,d^5\,e^3\,x^6\left(21\,b\,c^3+2\,a\right)\\ & +\frac{6\,b\,c^2\,d^4\,e^3\,x^5\left(21\,b\,c^3+5\,a\right)}{5}\end{aligned}$$

input
```
int((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2,x)
```

output
```
c^3*e^3*x*(a + b*c^3)^2 + (b^2*d^9*e^3*x^10)/10 + (d^3*e^3*x^4*(a^2 + 84*b
^2*c^6 + 40*a*b*c^3))/4 + (3*c^2*d*e^3*x^2*(a^2 + 3*b^2*c^6 + 4*a*b*c^3))/
2 + c*d^2*e^3*x^3*(a^2 + 12*b^2*c^6 + 10*a*b*c^3) + (9*b^2*c^2*d^7*e^3*x^8
)/2 + (2*b*d^6*e^3*x^7*(a + 42*b*c^3))/7 + b^2*c*d^8*e^3*x^9 + b*c*d^5*e^3
*x^6*(2*a + 21*b*c^3) + (6*b*c^2*d^4*e^3*x^5*(5*a + 21*b*c^3))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.15

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)^2\,dx$$
$$= \frac{e^3x(14b^2d^9x^9+140b^2c\,d^8x^8+630b^2c^2d^7x^7+1680b^2c^3d^6x^6+2940b^2c^4d^5x^5+3528b^2c^5d^4x^4+40ab\,d^6x^6}{}$$

input
```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^2,x)
```

output

```
(e**3*x*(140*a**2*c**3 + 210*a**2*c**2*d*x + 140*a**2*c*d**2*x**2 + 35*a**
2*d**3*x**3 + 280*a*b*c**6 + 840*a*b*c**5*d*x + 1400*a*b*c**4*d**2*x**2 +
1400*a*b*c**3*d**3*x**3 + 840*a*b*c**2*d**4*x**4 + 280*a*b*c*d**5*x**5 + 4
0*a*b*d**6*x**6 + 140*b**2*c**9 + 630*b**2*c**8*d*x + 1680*b**2*c**7*d**2*
x**2 + 2940*b**2*c**6*d**3*x**3 + 3528*b**2*c**5*d**4*x**4 + 2940*b**2*c**
4*d**5*x**5 + 1680*b**2*c**3*d**6*x**6 + 630*b**2*c**2*d**7*x**7 + 140*b**
2*c*d**8*x**8 + 14*b**2*d**9*x**9))/140
```

3.57 $\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^3 \, dx$

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Optimal result

Integrand size = 24, antiderivative size = 83

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^3 \, dx = \frac{a^3e^3(c+dx)^4}{4d} + \frac{3a^2be^3(c+dx)^7}{7d} + \frac{3ab^2e^3(c+dx)^{10}}{10d} + \frac{b^3e^3(c+dx)^{13}}{13d}$$

output

```
1/4*a^3*e^3*(d*x+c)^4/d+3/7*a^2*b*e^3*(d*x+c)^7/d+3/10*a*b^2*e^3*(d*x+c)^1
0/d+1/13*b^3*e^3*(d*x+c)^13/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(83) = 166$.

Time $= 0.02$ (sec) , antiderivative size $= 327$, normalized size of antiderivative $= 3.94$

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = e^3\Bigg(&c^3\left(a+bc^3\right)^3 x+\frac{3}{2}c^2\left(a+bc^3\right)^2\left(a+4bc^3\right)dx^2\\&+c\left(a^3+15a^2bc^3+36ab^2c^6+22b^3c^9\right)d^2x^3\\&+\frac{1}{4}\left(a^3+60a^2bc^3+252ab^2c^6+220b^3c^9\right)d^3x^4\\&+\frac{9}{5}bc^2\left(5a^2+42abc^3+55b^2c^6\right)d^4x^5\\&+3bc\left(a^2+21abc^3+44b^2c^6\right)d^5x^6\\&+\frac{3}{7}b\left(a^2+84abc^3+308b^2c^6\right)d^6x^7\\&+\frac{9}{2}b^2c^2\left(3a+22bc^3\right)d^7x^8+b^2c\left(3a+55bc^3\right)d^8x^9\\&+\frac{1}{10}b^2\left(3a+220bc^3\right)d^9x^{10}+6b^3c^2d^{10}x^{11}\\&+b^3cd^{11}x^{12}+\frac{1}{13}b^3d^{12}x^{13}\Bigg)\end{aligned}$$

input

```
Integrate[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x]
```

output

```
e^3*(c^3*(a + b*c^3)^3*x + (3*c^2*(a + b*c^3)^2*(a + 4*b*c^3)*d*x^2)/2 + c
*(a^3 + 15*a^2*b*c^3 + 36*a*b^2*c^6 + 22*b^3*c^9)*d^2*x^3 + ((a^3 + 60*a^2
*b*c^3 + 252*a*b^2*c^6 + 220*b^3*c^9)*d^3*x^4)/4 + (9*b*c^2*(5*a^2 + 42*a*
b*c^3 + 55*b^2*c^6)*d^4*x^5)/5 + 3*b*c*(a^2 + 21*a*b*c^3 + 44*b^2*c^6)*d^5
*x^6 + (3*b*(a^2 + 84*a*b*c^3 + 308*b^2*c^6)*d^6*x^7)/7 + (9*b^2*c^2*(3*a
+ 22*b*c^3)*d^7*x^8)/2 + b^2*c*(3*a + 55*b*c^3)*d^8*x^9 + (b^2*(3*a + 220*
b*c^3)*d^9*x^10)/10 + 6*b^3*c^2*d^10*x^11 + b^3*c*d^11*x^12 + (b^3*d^12*x^
13)/13)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ce+dex)^3 \left(a+b(c+dx)^3\right)^3 dx$$

$$\downarrow 895$$

$$\frac{e^3 \int (c+dx)^3 \left(b(c+dx)^3+a\right)^3 d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{e^3 \int \left(b^3(c+dx)^{12}+3ab^2(c+dx)^9+3a^2b(c+dx)^6+a^3(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{e^3\left(\frac{1}{4}a^3(c+dx)^4+\frac{3}{7}a^2b(c+dx)^7+\frac{3}{10}ab^2(c+dx)^{10}+\frac{1}{13}b^3(c+dx)^{13}\right)}{d}$$

input
```
Int[(c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x]
```

output
```
(e^3*((a^3*(c + d*x)^4)/4 + (3*a^2*b*(c + d*x)^7)/7 + (3*a*b^2*(c + d*x)^1
0)/10 + (b^3*(c + d*x)^13)/13))/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(75) = 150$.

Time = 0.67 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.31

method	result
gosper	$\frac{e^3x(140d^{12}b^3x^{12}+1820cd^{11}b^3x^{11}+10920c^2d^{10}b^3x^{10}+40040x^9c^3d^9b^3+100100b^3c^4d^8x^8+180180x^7c^5d^7b^3+546x^9ab^2d^9+24}{}$
orering	$\frac{x(140d^{12}b^3x^{12}+1820cd^{11}b^3x^{11}+10920c^2d^{10}b^3x^{10}+40040x^9c^3d^9b^3+100100b^3c^4d^8x^8+180180x^7c^5d^7b^3+546x^9ab^2d^9+240}{}$
norman	$\left(22c^3e^3d^9b^3+\frac{3}{10}ab^2d^9e^3\right)x^{10}+\left(99c^5e^3d^7b^3+\frac{27}{2}ab^2c^2d^7e^3\right)x^8+\left(132b^3c^6d^6e^3+36ab^2c^3d\right.$
risch	$3e^3ab^2cd^8x^9+\frac{27}{2}e^3x^8ab^2c^2d^7+36e^3x^7ab^2c^3d^6+\frac{378}{5}e^3x^5ab^2c^5d^4+ce^3d^{11}b^3x^{12}+c^3a^3e^3x$
parallelrisch	$3e^3ab^2cd^8x^9+\frac{27}{2}e^3x^8ab^2c^2d^7+36e^3x^7ab^2c^3d^6+\frac{378}{5}e^3x^5ab^2c^5d^4+ce^3d^{11}b^3x^{12}+c^3a^3e^3x$
default	Expression too large to display

input
```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1820*e^3*x*(140*b^3*d^12*x^12+1820*b^3*c*d^11*x^11+10920*b^3*c^2*d^10*x^
10+40040*b^3*c^3*d^9*x^9+100100*b^3*c^4*d^8*x^8+180180*b^3*c^5*d^7*x^7+546
*a*b^2*d^9*x^9+240240*b^3*c^6*d^6*x^6+5460*a*b^2*c*d^8*x^8+240240*b^3*c^7*
d^5*x^5+24570*a*b^2*c^2*d^7*x^7+180180*b^3*c^8*d^4*x^4+65520*a*b^2*c^3*d^6
*x^6+100100*b^3*c^9*d^3*x^3+114660*a*b^2*c^4*d^5*x^5+40040*b^3*c^10*d^2*x^
2+137592*a*b^2*c^5*d^4*x^4+10920*b^3*c^11*d*x+780*a^2*b*d^6*x^6+114660*a*b
^2*c^6*d^3*x^3+1820*b^3*c^12+5460*a^2*b*c*d^5*x^5+65520*a*b^2*c^7*d^2*x^2+
16380*a^2*b*c^2*d^4*x^4+24570*a*b^2*c^8*d*x+27300*a^2*b*c^3*d^3*x^3+5460*a
*b^2*c^9+27300*a^2*b*c^4*d^2*x^2+16380*a^2*b*c^5*d*x+455*a^3*d^3*x^3+5460*
a^2*b*c^6+1820*a^3*c*d^2*x^2+2730*a^3*c^2*d*x+1820*a^3*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(75) = 150.

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.80

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = {}& \frac{1}{13}b^3d^{12}e^3x^{13}+b^3cd^{11}e^3x^{12}+6\,b^3c^2d^{10}e^3x^{11}\\ &+\frac{1}{10}\left(220\,b^3c^3+3\,ab^2\right)d^9e^3x^{10}\\ &+\left(55\,b^3c^4+3\,ab^2c\right)d^8e^3x^9\\ &+\frac{9}{2}\left(22\,b^3c^5+3\,ab^2c^2\right)d^7e^3x^8\\ &+\frac{3}{7}\left(308\,b^3c^6+84\,ab^2c^3+a^2b\right)d^6e^3x^7\\ &+3\left(44\,b^3c^7+21\,ab^2c^4+a^2bc\right)d^5e^3x^6\\ &+\frac{9}{5}\left(55\,b^3c^8+42\,ab^2c^5+5\,a^2bc^2\right)d^4e^3x^5\\ &+\frac{1}{4}\left(220\,b^3c^9+252\,ab^2c^6+60\,a^2bc^3+a^3\right)d^3e^3x^4\\ &+\left(22\,b^3c^{10}+36\,ab^2c^7+15\,a^2bc^4+a^3c\right)d^2e^3x^3\\ &+\frac{3}{2}\left(4\,b^3c^{11}+9\,ab^2c^8+6\,a^2bc^5+a^3c^2\right)de^3x^2\\ &+\left(b^3c^{12}+3\,ab^2c^9+3\,a^2bc^6+a^3c^3\right)e^3x\end{aligned}$$

input

```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
1/13*b^3*d^12*e^3*x^13 + b^3*c*d^11*e^3*x^12 + 6*b^3*c^2*d^10*e^3*x^11 + 1
/10*(220*b^3*c^3 + 3*a*b^2)*d^9*e^3*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*e^
3*x^9 + 9/2*(22*b^3*c^5 + 3*a*b^2*c^2)*d^7*e^3*x^8 + 3/7*(308*b^3*c^6 + 84
*a*b^2*c^3 + a^2*b)*d^6*e^3*x^7 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*
d^5*e^3*x^6 + 9/5*(55*b^3*c^8 + 42*a*b^2*c^5 + 5*a^2*b*c^2)*d^4*e^3*x^5 +
1/4*(220*b^3*c^9 + 252*a*b^2*c^6 + 60*a^2*b*c^3 + a^3)*d^3*e^3*x^4 + (22*b
^3*c^10 + 36*a*b^2*c^7 + 15*a^2*b*c^4 + a^3*c)*d^2*e^3*x^3 + 3/2*(4*b^3*c^
11 + 9*a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*e^3*x^2 + (b^3*c^12 + 3*a*b^2*
c^9 + 3*a^2*b*c^6 + a^3*c^3)*e^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(73) = 146.

Time = 0.07 (sec) , antiderivative size = 552, normalized size of antiderivative = 6.65

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & 6b^3c^2d^{10}e^3x^{11}+b^3cd^{11}e^3x^{12}+\frac{b^3d^{12}e^3x^{13}}{13}\\ & +x^{10}\cdot\left(\frac{3ab^2d^9e^3}{10}+22b^3c^3d^9e^3\right)\\ & +x^9\cdot\left(3ab^2cd^8e^3+55b^3c^4d^8e^3\right)+x^8\\ & \cdot\left(\frac{27ab^2c^2d^7e^3}{2}+99b^3c^5d^7e^3\right)+x^7\\ & \cdot\left(\frac{3a^2bd^6e^3}{7}+36ab^2c^3d^6e^3+132b^3c^6d^6e^3\right)+x^6\\ & \cdot\left(3a^2bcd^5e^3+63ab^2c^4d^5e^3+132b^3c^7d^5e^3\right)+x^5\\ & \cdot\left(9a^2bc^2d^4e^3+\frac{378ab^2c^5d^4e^3}{5}+99b^3c^8d^4e^3\right)\\ & +x^4\left(\frac{a^3d^3e^3}{4}+15a^2bc^3d^3e^3+63ab^2c^6d^3e^3\right.\\ & \qquad\left.+55b^3c^9d^3e^3\right)+x^3\left(a^3cd^2e^3+15a^2bc^4d^2e^3\right.\\ & \qquad\left.+36ab^2c^7d^2e^3+22b^3c^{10}d^2e^3\right)+x^2\\ & \cdot\left(\frac{3a^3c^2de^3}{2}+9a^2bc^5de^3+\frac{27ab^2c^8de^3}{2}+6b^3c^{11}de^3\right)\\ & +x\left(a^3c^3e^3+3a^2bc^6e^3+3ab^2c^9e^3+b^3c^{12}e^3\right)\end{aligned}$$

input

```
integrate((d*e*x+c*e)**3*(a+b*(d*x+c)**3)**3,x)
```

output

```
6*b**3*c**2*d**10*e**3*x**11 + b**3*c*d**11*e**3*x**12 + b**3*d**12*e**3*x
**13/13 + x**10*(3*a*b**2*d**9*e**3/10 + 22*b**3*c**3*d**9*e**3) + x**9*(3
*a*b**2*c*d**8*e**3 + 55*b**3*c**4*d**8*e**3) + x**8*(27*a*b**2*c**2*d**7*
e**3/2 + 99*b**3*c**5*d**7*e**3) + x**7*(3*a**2*b*d**6*e**3/7 + 36*a*b**2*
c**3*d**6*e**3 + 132*b**3*c**6*d**6*e**3) + x**6*(3*a**2*b*c*d**5*e**3 + 6
3*a*b**2*c**4*d**5*e**3 + 132*b**3*c**7*d**5*e**3) + x**5*(9*a**2*b*c**2*d
**4*e**3 + 378*a*b**2*c**5*d**4*e**3/5 + 99*b**3*c**8*d**4*e**3) + x**4*(a
**3*d**3*e**3/4 + 15*a**2*b*c**3*d**3*e**3 + 63*a*b**2*c**6*d**3*e**3 + 55
*b**3*c**9*d**3*e**3) + x**3*(a**3*c*d**2*e**3 + 15*a**2*b*c**4*d**2*e**3
+ 36*a*b**2*c**7*d**2*e**3 + 22*b**3*c**10*d**2*e**3) + x**2*(3*a**3*c**2*
d*e**3/2 + 9*a**2*b*c**5*d*e**3 + 27*a*b**2*c**8*d*e**3/2 + 6*b**3*c**11*d
*e**3) + x*(a**3*c**3*e**3 + 3*a**2*b*c**6*e**3 + 3*a*b**2*c**9*e**3 + b**
3*c**12*e**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(75) = 150$.

Time $= 0.03$ (sec) , antiderivative size $= 398$, normalized size of antiderivative $= 4.80$

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = {}& \frac{1}{13}\,b^3d^{12}e^3x^{13} + b^3cd^{11}e^3x^{12} + 6\,b^3c^2d^{10}e^3x^{11}\\ &+ \frac{1}{10}\left(220\,b^3c^3+3\,ab^2\right)d^9e^3x^{10}\\ &+ \left(55\,b^3c^4+3\,ab^2c\right)d^8e^3x^9\\ &+ \frac{9}{2}\left(22\,b^3c^5+3\,ab^2c^2\right)d^7e^3x^8\\ &+ \frac{3}{7}\left(308\,b^3c^6+84\,ab^2c^3+a^2b\right)d^6e^3x^7\\ &+ 3\left(44\,b^3c^7+21\,ab^2c^4+a^2bc\right)d^5e^3x^6\\ &+ \frac{9}{5}\left(55\,b^3c^8+42\,ab^2c^5+5\,a^2bc^2\right)d^4e^3x^5\\ &+ \frac{1}{4}\left(220\,b^3c^9+252\,ab^2c^6+60\,a^2bc^3+a^3\right)d^3e^3x^4\\ &+ \left(22\,b^3c^{10}+36\,ab^2c^7+15\,a^2bc^4+a^3c\right)d^2e^3x^3\\ &+ \frac{3}{2}\left(4\,b^3c^{11}+9\,ab^2c^8+6\,a^2bc^5+a^3c^2\right)de^3x^2\\ &+ \left(b^3c^{12}+3\,ab^2c^9+3\,a^2bc^6+a^3c^3\right)e^3x\end{aligned}$$

input

```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
1/13*b^3*d^12*e^3*x^13 + b^3*c*d^11*e^3*x^12 + 6*b^3*c^2*d^10*e^3*x^11 + 1
/10*(220*b^3*c^3 + 3*a*b^2)*d^9*e^3*x^10 + (55*b^3*c^4 + 3*a*b^2*c)*d^8*e^
3*x^9 + 9/2*(22*b^3*c^5 + 3*a*b^2*c^2)*d^7*e^3*x^8 + 3/7*(308*b^3*c^6 + 84
*a*b^2*c^3 + a^2*b)*d^6*e^3*x^7 + 3*(44*b^3*c^7 + 21*a*b^2*c^4 + a^2*b*c)*
d^5*e^3*x^6 + 9/5*(55*b^3*c^8 + 42*a*b^2*c^5 + 5*a^2*b*c^2)*d^4*e^3*x^5 +
1/4*(220*b^3*c^9 + 252*a*b^2*c^6 + 60*a^2*b*c^3 + a^3)*d^3*e^3*x^4 + (22*b
^3*c^10 + 36*a*b^2*c^7 + 15*a^2*b*c^4 + a^3*c)*d^2*e^3*x^3 + 3/2*(4*b^3*c^
11 + 9*a*b^2*c^8 + 6*a^2*b*c^5 + a^3*c^2)*d*e^3*x^2 + (b^3*c^12 + 3*a*b^2*
c^9 + 3*a^2*b*c^6 + a^3*c^3)*e^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(75) = 150$.

Time $= 0.13$ (sec) , antiderivative size $= 544$, normalized size of antiderivative $= 6.55$

$$\begin{aligned}\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = {} & \frac{1}{13}b^3d^{12}e^3x^{13} + b^3cd^{11}e^3x^{12} + 6\,b^3c^2d^{10}e^3x^{11} \\ & + 22\,b^3c^3d^9e^3x^{10} + 55\,b^3c^4d^8e^3x^9 + 99\,b^3c^5d^7e^3x^8 \\ & + 132\,b^3c^6d^6e^3x^7 + \frac{3}{10}ab^2d^9e^3x^{10} + 132\,b^3c^7d^5e^3x^6 \\ & + 3\,ab^2cd^8e^3x^9 + 99\,b^3c^8d^4e^3x^5 + \frac{27}{2}ab^2c^2d^7e^3x^8 \\ & + 55\,b^3c^9d^3e^3x^4 + 36\,ab^2c^3d^6e^3x^7 + 22\,b^3c^{10}d^2e^3x^3 \\ & + 63\,ab^2c^4d^5e^3x^6 + 6\,b^3c^{11}de^3x^2 + \frac{378}{5}ab^2c^5d^4e^3x^5 \\ & + b^3c^{12}e^3x + 63\,ab^2c^6d^3e^3x^4 + \frac{3}{7}a^2bd^6e^3x^7 \\ & + 36\,ab^2c^7d^2e^3x^3 + 3\,a^2bcd^5e^3x^6 + \frac{27}{2}ab^2c^8de^3x^2 \\ & + 9\,a^2bc^2d^4e^3x^5 + 3\,ab^2c^9e^3x + 15\,a^2bc^3d^3e^3x^4 \\ & + 15\,a^2bc^4d^2e^3x^3 + 9\,a^2bc^5de^3x^2 + 3\,a^2bc^6e^3x \\ & + \frac{1}{4}a^3d^3e^3x^4 + a^3cd^2e^3x^3 + \frac{3}{2}a^3c^2de^3x^2 + a^3c^3e^3x\end{aligned}$$

input

```
integrate((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output

```
1/13*b^3*d^12*e^3*x^13 + b^3*c*d^11*e^3*x^12 + 6*b^3*c^2*d^10*e^3*x^11 + 2
2*b^3*c^3*d^9*e^3*x^10 + 55*b^3*c^4*d^8*e^3*x^9 + 99*b^3*c^5*d^7*e^3*x^8 +
 132*b^3*c^6*d^6*e^3*x^7 + 3/10*a*b^2*d^9*e^3*x^10 + 132*b^3*c^7*d^5*e^3*x
^6 + 3*a*b^2*c*d^8*e^3*x^9 + 99*b^3*c^8*d^4*e^3*x^5 + 27/2*a*b^2*c^2*d^7*e
^3*x^8 + 55*b^3*c^9*d^3*e^3*x^4 + 36*a*b^2*c^3*d^6*e^3*x^7 + 22*b^3*c^10*d
^2*e^3*x^3 + 63*a*b^2*c^4*d^5*e^3*x^6 + 6*b^3*c^11*d*e^3*x^2 + 378/5*a*b^2
*c^5*d^4*e^3*x^5 + b^3*c^12*e^3*x + 63*a*b^2*c^6*d^3*e^3*x^4 + 3/7*a^2*b*d
^6*e^3*x^7 + 36*a*b^2*c^7*d^2*e^3*x^3 + 3*a^2*b*c*d^5*e^3*x^6 + 27/2*a*b^2
*c^8*d*e^3*x^2 + 9*a^2*b*c^2*d^4*e^3*x^5 + 3*a*b^2*c^9*e^3*x + 15*a^2*b*c^
3*d^3*e^3*x^4 + 15*a^2*b*c^4*d^2*e^3*x^3 + 9*a^2*b*c^5*d*e^3*x^2 + 3*a^2*b
*c^6*e^3*x + 1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x
^2 + a^3*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.19

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx = c^3\,e^3\,x\left(b\,c^3+a\right)^3 + \frac{d^3\,e^3\,x^4\left(a^3+60\,a^2\,b\,c^3+252\,a\,b^2\,c^6+220\,b^3\,c^9\right)}{4} + \frac{b^3\,d^{12}\,e^3\,x^{13}}{13} + \frac{3\,b\,d^6\,e^3\,x^7\left(a^2+84\,a\,b\,c^3+308\,b^2\,c^6\right)}{7} + \frac{b^2\,d^9\,e^3\,x^{10}\left(220\,b\,c^3+3\,a\right)}{10} + 6\,b^3\,c^2\,d^{10}\,e^3\,x^{11} + c\,d^2\,e^3\,x^3\left(a^3+15\,a^2\,b\,c^3+36\,a\,b^2\,c^6+22\,b^3\,c^9\right) + b^3\,c\,d^{11}\,e^3\,x^{12} + \frac{3\,c^2\,d\,e^3\,x^2\left(b\,c^3+a\right)^2\left(4\,b\,c^3+a\right)}{2} + \frac{9\,b\,c^2\,d^4\,e^3\,x^5\left(5\,a^2+42\,a\,b\,c^3+55\,b^2\,c^6\right)}{5} + 3\,b\,c\,d^5\,e^3\,x^6\left(a^2+21\,a\,b\,c^3+44\,b^2\,c^6\right) + b^2\,c\,d^8\,e^3\,x^9\left(55\,b\,c^3+3\,a\right) + \frac{9\,b^2\,c^2\,d^7\,e^3\,x^8\left(22\,b\,c^3+3\,a\right)}{2}$$

input

```
int((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3,x)
```

output

```
c^3*e^3*x*(a + b*c^3)^3 + (d^3*e^3*x^4*(a^3 + 220*b^3*c^9 + 60*a^2*b*c^3 +
 252*a*b^2*c^6))/4 + (b^3*d^12*e^3*x^13)/13 + (3*b*d^6*e^3*x^7*(a^2 + 308*
b^2*c^6 + 84*a*b*c^3))/7 + (b^2*d^9*e^3*x^10*(3*a + 220*b*c^3))/10 + 6*b^3
*c^2*d^10*e^3*x^11 + c*d^2*e^3*x^3*(a^3 + 22*b^3*c^9 + 15*a^2*b*c^3 + 36*a
*b^2*c^6) + b^3*c*d^11*e^3*x^12 + (3*c^2*d*e^3*x^2*(a + b*c^3)^2*(a + 4*b*
c^3))/2 + (9*b*c^2*d^4*e^3*x^5*(5*a^2 + 55*b^2*c^6 + 42*a*b*c^3))/5 + 3*b*
c*d^5*e^3*x^6*(a^2 + 44*b^2*c^6 + 21*a*b*c^3) + b^2*c*d^8*e^3*x^9*(3*a + 5
5*b*c^3) + (9*b^2*c^2*d^7*e^3*x^8*(3*a + 22*b*c^3))/2
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 440, normalized size of antiderivative = 5.30

$$\int (ce+dex)^3\left(a+b(c+dx)^3\right)^3\,dx$$
$$=\frac{e^3x(140b^3d^{12}x^{12}+1820b^3cd^{11}x^{11}+10920b^3c^2d^{10}x^{10}+40040b^3c^3d^9x^9+100100b^3c^4d^8x^8+180180b^3c^5d^7$$

input

```
int((d*e*x+c*e)^3*(a+b*(d*x+c)^3)^3,x)
```

output

```
(e**3*x*(1820*a**3*c**3 + 2730*a**3*c**2*d*x + 1820*a**3*c*d**2*x**2 + 455
*a**3*d**3*x**3 + 5460*a**2*b*c**6 + 16380*a**2*b*c**5*d*x + 27300*a**2*b*
c**4*d**2*x**2 + 27300*a**2*b*c**3*d**3*x**3 + 16380*a**2*b*c**2*d**4*x**4
 + 5460*a**2*b*c*d**5*x**5 + 780*a**2*b*d**6*x**6 + 5460*a*b**2*c**9 + 245
70*a*b**2*c**8*d*x + 65520*a*b**2*c**7*d**2*x**2 + 114660*a*b**2*c**6*d**3
*x**3 + 137592*a*b**2*c**5*d**4*x**4 + 114660*a*b**2*c**4*d**5*x**5 + 6552
0*a*b**2*c**3*d**6*x**6 + 24570*a*b**2*c**2*d**7*x**7 + 5460*a*b**2*c*d**8
*x**8 + 546*a*b**2*d**9*x**9 + 1820*b**3*c**12 + 10920*b**3*c**11*d*x + 40
040*b**3*c**10*d**2*x**2 + 100100*b**3*c**9*d**3*x**3 + 180180*b**3*c**8*d
**4*x**4 + 240240*b**3*c**7*d**5*x**5 + 240240*b**3*c**6*d**6*x**6 + 18018
0*b**3*c**5*d**7*x**7 + 100100*b**3*c**4*d**8*x**8 + 40040*b**3*c**3*d**9*
x**9 + 10920*b**3*c**2*d**10*x**10 + 1820*b**3*c*d**11*x**11 + 140*b**3*d*
*12*x**12))/1820
```

3.58 $\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{(c+dx)^2}{2bd} + \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}$$

output

```
1/2*(d*x+c)^2/b/d+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/
2)/a^(1/3))*3^(1/2)/b^(5/3)/d+1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(5
/3)/d-1/6*a^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^
(5/3)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{(c+dx)^2}{2bd} - \frac{a^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}$$

input
```
Integrate[(c + d*x)^4/(a + b*(c + d*x)^3),x]
```

output
```
(c + d*x)^2/(2*b*d) - (a^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*(c +
 d*x)])/(3*b^(5/3)*d) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^4}{b(c+dx)^3+a}\,d(c+dx)}{d}$$
$$\downarrow 843$$

$$\frac{\frac{(c+dx)^2}{2b}-\frac{a\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{b}}{d}$$

$$\downarrow 821$$

$$\frac{\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 27$$

$$\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 1082$$

$$\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 217$$

$$\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

$$\downarrow 1103$$

$$\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}}{d}$$

input
```
Int[(c + d*x)^4/(a + b*(c + d*x)^3),x]
```

output
```
((c + d*x)^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^
(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/
b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/
(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b)/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\frac{1}{2}x^2d+cx}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)a}{3b^2d}$	92
risch	$\frac{d\,x^2}{2b}+\frac{cx}{b}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(-\_R_{d-c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3b^2d}$	96

input
```
int((d*x+c)^4/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*x^2*d+c*x)-1/3/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R
),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx$$

$$= \frac{3\,d^2x^2+6\,cdx-2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}(bdx+bc)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ad^2x^2+2\,acdx+ac^2-(bdx\right.}{6\,bd}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(3*d^2*x^2 + 6*c*d*x - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)
*(b*d*x + b*c)*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*d^2
*x^2 + 2*a*c*d*x + a*c^2 - (b*d*x + b*c)*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/
3)) + 2*(a^2/b^2)^(1/3)*log(a*d*x + a*c + b*(a^2/b^2)^(2/3)))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.29

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(x+\frac{9t^2b^3+ac}{ad}\right)\right)\right)}{d}+\frac{cx}{b}+\frac{dx^2}{2b}$$

input
```
integrate((d*x+c)**4/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(x + (9*_t**2*b**3 + a*c)/(
a*d))))/d + c*x/b + d*x**2/(2*b)
```

Maxima [F]

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \int \frac{(dx+c)^4}{(dx+c)^3 b+a}\,dx$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-a*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 +
a), x)/b + 1/2*(d*x^2 + 2*c*x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{bd^7x^2+2\,bcd^6x}{2\,b^2d^6} - \frac{2\sqrt{3}(a^2bd^{15})^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-(a^2b)^{\frac{2}{3}}\right)}{3\,(a^2b)^{\frac{2}{3}}}\right)+(a^2bd^{15})^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-(a^2b)^{\frac{2}{3}}\right)^2+3\,(a^2b)\right.}{6\,b^2d^6}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(b*d^7*x^2 + 2*b*c*d^6*x)/(b^2*d^6) - 1/6*(2*sqrt(3)*(a^2*b*d^15)^(1/3
)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))/(a^2*b)^(2/3))
+ (a^2*b*d^15)^(1/3)*log((2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))^2 + 3*(a^2*
b)^(4/3)) - 2*(a^2*b*d^15)^(1/3)*log(abs(a*b*d*x + a*b*c + (a^2*b)^(2/3)))
)/(b^2*d^6)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{d\,x^2}{2\,b} + \frac{c\,x}{b} + \frac{a^{2/3}\,\ln\left(b^{1/3}\,c + a^{1/3} + b^{1/3}\,d\,x\right)}{3\,b^{5/3}\,d} - \frac{a^{2/3}\,\ln\left(\frac{a^2\,c\,d^4}{b} + \frac{a^2\,d^5\,x}{b} + \frac{a^{7/3}\,d^4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{5/3}\,d} + \frac{a^{2/3}\,\ln\left(\frac{a^2\,c\,d^4}{b} + \frac{a^2\,d^5\,x}{b} + \frac{9\,a^{7/3}\,d^4\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{5/3}\,d}$$

input
```
int((c + d*x)^4/(a + b*(c + d*x)^3),x)
```

output
```
(d*x^2)/(2*b) + (c*x)/b + (a^(2/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))
/(3*b^(5/3)*d) - (a^(2/3)*log((a^2*c*d^4)/b + (a^2*d^5*x)/b + (a^(7/3)*d^4
*((3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3))*((3^(1/2)*1i)/2 + 1/2))/(3*b^(5/3)*d)
+ (a^(2/3)*log((a^2*c*d^4)/b + (a^2*d^5*x)/b + (9*a^(7/3)*d^4*((3^(1/2)*1i
)/6 - 1/6)^2)/b^(4/3))*((3^(1/2)*1i)/6 - 1/6))/(b^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^4}{a+b(c+dx)^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)a + 6b^{\frac{2}{3}}a^{\frac{1}{3}}cdx + 3b^{\frac{2}{3}}a^{\frac{1}{3}}d^2x^2 - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b\right.}{6b^{\frac{5}{3}}a^{\frac{1}{3}}d}$$

input
```
int((d*x+c)^4/(a+b*(d*x+c)^3),x)
```

output

```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*a + 6*b**(2/3)*a**(1/3)*c*d*x + 3*b**(2/3)*a**(1/3)*d**2*x**2 - log(a
**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*
b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a + 2*log(a**(1/3) + b**(1/3)*c + b**
(1/3)*d*x)*a)/(6*b**(2/3)*a**(1/3)*b*d)
```

3.59 $\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx = \frac{x}{b} + \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}d} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}$$

output

```
x/b+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^
(1/2)/b^(4/3)/d-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(4/3)/d+1/6*a^(1
/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx = \frac{6\sqrt[3]{b}c+6\sqrt[3]{b}dx-2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)+\sqrt[3]{a}\log\left(a^{2/3}-\sqrt{}\right.}{6b^{4/3}d}$$

input
```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^3),x]
```

output
```
(6*b^(1/3)*c + 6*b^(1/3)*d*x - 2*Sqrt[3]*a^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1
/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d
*x)] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)
^2])/(6*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{(c+dx)^3}{b(c+dx)^3+a} d(c+dx)}{d}$$

$$\downarrow 843$$

$$\frac{\frac{c+dx}{b} - \frac{a\int \frac{1}{b(c+dx)^3+a} d(c+dx)}{b}}{d}$$

$$\downarrow 750$$

$$\frac{\frac{c+dx}{b} - \frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}} d(c+dx)}{3a^{2/3}}\right)}{b}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 1082$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 217$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

$$\downarrow 1103$$

$$\frac{\frac{c+dx}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}}{d}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^3),x]
```

output
```
((c + d*x)/b - (a*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) +
(-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3))
- Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3
)))/(3*a^(2/3))))/b)/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{x}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)a}{3b^2d}$	78
risch	$\frac{x}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)a}{3b^2d}$	78

input
```
int((d*x+c)^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
x/b-1/3/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+
3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{6\,dx + 2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}(bdx+bc)\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3\,a}\right) - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(d^2x^2 + 2\,cdx + c^2 + (dx+c)\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \right.}{6\,bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(6*d*x + 2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*d*x + b*c)*(-
a/b)^(2/3) - sqrt(3)*a)/a) - (-a/b)^(1/3)*log(d^2*x^2 + 2*c*d*x + c^2 + (d
*x + c)*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^(1/3)*log(d*x + c - (-a/b)
^(1/3)))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.19

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx = \frac{\operatorname{RootSum}\left(27t^3b^4 + a, \left(t \mapsto t\log\left(x + \frac{-3tb+c}{d}\right)\right)\right)}{d} + \frac{x}{b}$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(x + (-3*_t*b + c)/d)))/d + x/
b
```

Maxima [F]

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx = \int \frac{(dx+c)^3}{(dx+c)^3 b+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-a*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/b
 + x/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx = \frac{x}{b} + \frac{2\sqrt{3}\left(-\frac{ad^6}{b}\right)^{\frac{1}{3}} \arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right) - \left(-\frac{ad^6}{b}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)^2 + \right.}{6\,bd^3}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
x/b + 1/6*(2*sqrt(3)*(-a*d^6/b)^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3
))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-a*d^6/b)^(1
/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 + 4*(b*
d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-a*d^6/b)^(1/3)*log(abs(-b*d*x - b*c +
 (-a*b^2)^(1/3))))/(b*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{x}{b} + \frac{(-a)^{1/3}\ln\left((-a)^{4/3} + a\,b^{1/3}\,c + a\,b^{1/3}\,d\,x\right)}{3\,b^{4/3}\,d}$$
$$- \frac{(-a)^{1/3}\ln\left(3\,a\,b\,c\,d^5 - 3\,(-a)^{4/3}\,b^{2/3}\,d^5\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + 3\,a\,b\,d^6\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}\,d}$$
$$+ \frac{(-a)^{1/3}\ln\left(9\,(-a)^{4/3}\,b^{2/3}\,d^5\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + 3\,a\,b\,c\,d^5 + 3\,a\,b\,d^6\,x\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{4/3}\,d}$$

input
```
int((c + d*x)^3/(a + b*(c + d*x)^3),x)
```

output
```
x/b + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*c + a*b^(1/3)*d*x))/(3*b^(4/3
)*d) - ((-a)^(1/3)*log(3*a*b*c*d^5 - 3*(-a)^(4/3)*b^(2/3)*d^5*((3^(1/2)*1i
)/2 + 1/2) + 3*a*b*d^6*x)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(4/3)*d) + ((-a)^(1
/3)*log(9*(-a)^(4/3)*b^(2/3)*d^5*((3^(1/2)*1i)/6 - 1/6) + 3*a*b*c*d^5 + 3*
a*b*d^6*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right) + a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right) - 2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}} -}{6b^{\frac{4}{3}}d}$$

input
```
int((d*x+c)^3/(a+b*(d*x+c)^3),x)
```

output

```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3))) + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**
(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) - 2*a**
(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x) + 6*b**(1/3)*d*x)/(6*b**(1
/3)*b*d)
```

3.60 $\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx$

Optimal result 431
Mathematica [A] (verified) 431
Rubi [A] (verified) 432
Maple [A] (verified) 433
Fricas [B] (verification not implemented) 433
Sympy [B] (verification not implemented) 434
Maxima [A] (verification not implemented) 434
Giac [A] (verification not implemented) 434
Mupad [B] (verification not implemented) 435
Reduce [B] (verification not implemented) 435

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

output
```
1/3*ln(a+b*(d*x+c)^3)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

input
```
Integrate[(c + d*x)^2/(a + b*(c + d*x)^3),x]
```

output
```
Log[a + b*(c + d*x)^3]/(3*b*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^2}{b(c+dx)^3+a} d(c+dx)}{d}$$
$$\downarrow 792$$
$$\frac{\log\left(a+b(c+dx)^3\right)}{3bd}$$

input
```
Int[(c + d*x)^2/(a + b*(c + d*x)^3),x]
```

output
```
Log[a + b*(c + d*x)^3]/(3*b*d)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(a+b(xd+c)^3\right)}{3bd}$	21
default	$\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}$	43
norman	$\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}$	43
risch	$\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}$	43
parallelrisch	$\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}$	43

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a+b*(d*x+c)^3)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{3bd}$$

input
```
integrate((d*x+c)**2/(a+b*(d*x+c)**3),x)
```

output
```
log(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*b*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left((dx+c)^3b+a\right)}{3\,bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/3*log((d*x + c)^3*b + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(\left|(dx+c)^3b+a\right|\right)}{3\,bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*log(abs((d*x + c)^3*b + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,b\,d}$$

input
```
int((c + d*x)^2/(a + b*(c + d*x)^3),x)
```

output
```
log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.36

$$\int \frac{(c+dx)^2}{a+b(c+dx)^3}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}c+b^{\frac{1}{3}}dx\right)}{3bd}$$

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**
2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) + log(a**(1/3) + b**(1/3)*c + b
**(1/3)*d*x))/(3*b*d)
```

3.61 $\int \frac{c+dx}{a+b(c+dx)^3}\,dx$

Optimal result . . . 436
Mathematica [A] (verified) . . . 436
Rubi [A] (verified) . . . 437
Maple [C] (verified) . . . 440
Fricas [A] (verification not implemented) . . . 441
Sympy [A] (verification not implemented) . . . 441
Maxima [F] . . . 442
Giac [A] (verification not implemented) . . . 442
Mupad [B] (verification not implemented) . . . 443
Reduce [B] (verification not implemented) . . . 443

Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}b^{2/3}d}$$

output

```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/
3)/b^(2/3)/d-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(1/3)/b^(2/3)/d+1/6*ln(a^(2
/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)\right.}{6\sqrt[3]{a}b^{2/3}d}$$

input
```
Integrate[(c + d*x)/(a + b*(c + d*x)^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*
Log[a^(1/3) + b^(1/3)*(c + d*x)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(6*a^(1/3)*b^(2/3)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {895, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{c+dx}{b(c+dx)^3+a} d(c+dx)}{d}$$

$$\downarrow 821$$

$$\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}} d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{d}$$

$$\downarrow 16$$

$$\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 1082$$

$$\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 217$$

$$\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

$$\downarrow 1103$$

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{d}$$

input
```
Int[(c + d*x)/(a + b*(c + d*x)^3),x]
```

output
```
(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*Arc
Tan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) -
 a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*
b^(1/3)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)} \frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	76
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)} \frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	76

input
```
int((d*x+c)/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3
+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.23

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2d^3x^3+6b^2cd^2x^2+6b^2c^2dx+2b^2c^3-ab+3\sqrt{\frac{1}{3}}\left(abdx+abc+2\left(d^2x^2+2cdx+c^2\right)(-ab^2)^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{}}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right.}{} \right.$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*
d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c +
 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b
^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a)) + (-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)
*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d), 1/6*(6*sqrt(1/3)*a*b*sqrt(-
(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt
(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*
log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.21

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = \frac{\operatorname{RootSum}\left(27t^3ab^2+1, \left(t \mapsto t\log\left(x+\frac{9t^2ab+c}{d}\right)\right)\right)}{d}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(x + (9*_t**2*a*b + c)/d)))/
d
```

Maxima [F]

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = \int \frac{dx+c}{(dx+c)^3 b+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((d*x + c)^3*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{c+dx}{a+b(c+dx)^3}\,dx = &-\frac{1}{3}\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\,\left(-a^2b\right)^{\frac{2}{3}}}\right)\\ &-\frac{1}{6}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2\right.\\ &\left.+3\left(-a^2b\right)^{\frac{4}{3}}\right)+\frac{1}{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left|abdx+abc+\left(-a^2b\right)^{\frac{2}{3}}\right|\right)\end{aligned}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*
c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) - 1/6*(-1/(a*b^2*d^3))^(1/3)*log((2*a*
b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) + 1/3*(-1/(a*b^2*d
^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3)))
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b^{1/3}c-(-a)^{1/3}+b^{1/3}dx\right)}{3(-a)^{1/3}b^{2/3}d} + \frac{\ln\left(bcd^4+bd^5x-\frac{(-a)^{1/3}b^{2/3}d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}d} - \frac{\ln\left(bcd^4+bd^5x-\frac{(-a)^{1/3}b^{2/3}d^4\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6(-a)^{1/3}b^{2/3}d}$$

input
```
int((c + d*x)/(a + b*(c + d*x)^3),x)
```

output
```
log(b^(1/3)*c - (-a)^(1/3) + b^(1/3)*d*x)/(3*(-a)^(1/3)*b^(2/3)*d) + (log(
b*c*d^4 + b*d^5*x - ((-a)^(1/3)*b^(2/3)*d^4*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2
)*1i - 1))/(6*(-a)^(1/3)*b^(2/3)*d) - (log(b*c*d^4 + b*d^5*x - ((-a)^(1/3)
*b^(2/3)*d^4*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*(-a)^(1/3)*b^(2/3
)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{a+b(c+dx)^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}c\right.}{6b^{\frac{2}{3}}a^{\frac{1}{3}}d}$$

input
```
int((d*x+c)/(a+b*(d*x+c)^3),x)
```

output

```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3))) + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**
(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) - 2*log(a**(1/3) + b**
(1/3)*c + b**(1/3)*d*x))/(6*b**(2/3)*a**(1/3)*d)
```

3.62 $\int \frac{1}{a+b(c+dx)^3}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{a+b(c+dx)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

output

```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/b^(1/3)/d+1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2
/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)\right.}{6a^{2/3}\sqrt[3]{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*
Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {239, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{b(c+dx)^3+a} d(c+dx)}{d}$$

$$\downarrow 750$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}} d(c+dx)}{3a^{2/3}}}{d}$$

$$\downarrow 16$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3a^{2/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1082$$

$$\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 217$$

$$\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1103$$

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

input
```
Int[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^
(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)} \frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)} \frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

input
```
int(1/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abd^2x^2+4\,abcdx+2\,abc^2+(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{}\right.}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.}{} \right.$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*
d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4
*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-
(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3
)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)
^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

Maxima [F]

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \int \frac{1}{(dx+c)^3b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^3*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^3}\,dx = {} & \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right)\\ & -\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2\right.\\ & \left.+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)\\ & +\frac{1}{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right|\right)\end{aligned}$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(s
qrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x
+ b*c + (a*b^2)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b^{1/3}c + a^{1/3} + b^{1/3}d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d} + \frac{\ln\left(3\,b^2\,c\,d^5 + 3\,b^2\,d^6\,x + \frac{3\,a^{1/3}\,b^{5/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d} - \frac{\ln\left(3\,b^2\,c\,d^5 + 3\,b^2\,d^6\,x - \frac{3\,a^{1/3}\,b^{5/3}\,d^5\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

input
```
int(1/(a + b*(c + d*x)^3),x)
```

output
```
log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*
c*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1
/3)*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*
d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right) - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right) + 2\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}c$$
$$6a^{\frac{2}{3}}b^{\frac{1}{3}}d$$

input
```
int(1/(a+b*(d*x+c)^3),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3))) - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)
*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) + 2*log(a**(
1/3) + b**(1/3)*c + b**(1/3)*d*x)))/(6*b**(1/3)*a*d)
```

3.63 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

output
```
ln(d*x+c)/a/d-1/3*ln(a+b*(d*x+c)^3)/a/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log(c+dx)}{ad} - \frac{\log\left(a+b(c+dx)^3\right)}{3ad}$$

input
```
Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)),x]
```

output
```
Log[c + d*x]/(a*d) - Log[a + b*(c + d*x)^3]/(3*a*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {895, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)}\,d(c+dx)}{d}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)}\,d(c+dx)^3}{3d}$$

$$\downarrow 47$$

$$\frac{\frac{\int \frac{1}{(c+dx)^3}\,d(c+dx)^3}{a} - \frac{b\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)^3}{a}}{3d}$$

$$\downarrow 14$$

$$\frac{\frac{\log\left((c+dx)^3\right)}{a} - \frac{b\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)^3}{a}}{3d}$$

$$\downarrow 16$$

$$\frac{\frac{\log\left((c+dx)^3\right)}{a} - \frac{\log\left(a+b(c+dx)^3\right)}{a}}{3d}$$

input
```
Int[1/((c + d*x)*(a + b*(c + d*x)^3)),x]
```

output
```
(Log[(c + d*x)^3]/a - Log[a + b*(c + d*x)^3]/a)/(3*d)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

method	result	size
parallelrisch	$\frac{3\ln(xd+c)-\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3ad}$	54
default	$\frac{\ln(xd+c)}{ad}-\frac{\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3ad}$	57
norman	$\frac{\ln(xd+c)}{ad}-\frac{\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3ad}$	57
risch	$\frac{\ln(xd+c)}{ad}-\frac{\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3ad}$	57

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*ln(d*x+c)-ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a))/a/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\,\log\left(dx+c\right)}{3\,ad}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/3*(log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*log(d*x
 + c))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log\left(\frac{c}{d}+x\right)}{ad} - \frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3ad}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)**3),x)
```

output
```
log(c/d + x)/(a*d) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/
(b*d**3))/(3*a*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,ad} + \frac{\log\left(dx+c\right)}{ad}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a*d) + log(
d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,ad} + \frac{\log\left(\left|dx+c\right|\right)}{ad}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a*d) +
 log(abs(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{\ln(c+d\,x)}{a\,d} - \frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)*(c + d*x)),x)
```

output
```
log(c + d*x)/(a*d) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x
^2)/(3*a*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}c+b^{\frac{1}{3}}dx\right)+3\log(dx+c)}{3ad}$$

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*
c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) - log(a**(1/3) + b**(1/3)*c
+ b**(1/3)*d*x) + 3*log(c + d*x))/(3*a*d)
```

3.64 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 154

$$\begin{aligned}\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx &= -\frac{1}{ad(c+dx)} + \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} \\ &\quad + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{4/3}d} \\ &\quad - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}d}\end{aligned}$$

output

```
-1/a/d/(d*x+c)+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/
a^(1/3))*3^(1/2)/a^(4/3)/d+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)
/d-1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/
3)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{-\frac{6\sqrt[3]{a}}{c+dx} - 2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) - \sqrt[3]{b}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+\right.}{6a^{4/3}d}$$

input
```
Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)),x]
```

output
```
((-6*a^(1/3))/(c + d*x) - 2*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(
c + d*x))/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)]
- b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/
(6*a^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}\,d(c+dx)}{d}$$
$$\downarrow 847$$
$$\frac{-\frac{b\int \frac{c+dx}{b(c+dx)^3+a}\,d(c+dx)}{a} - \frac{1}{a(c+dx)}}{d}$$
$$\downarrow 821$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 16$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 1142$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 25$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 27$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 1082$$

$$-\frac{b\left(\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 217$$

$$-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

$$\downarrow 1103$$

$$-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{d}$$

input

```
Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)),x]
```

output
```
(-(1/(a*(c + d*x))) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b
^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])
/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]
/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56

method	result	size
risch	$-\frac{1}{ad(xd+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4d^3\_Z^3-b\right)}\_R\ln\left(\left(-4a^4d^4\_R^3+3bd\right)x-4a^4cd^3\_R^3-a^3d^2\_R^2+3bc\right)\right)}{3}$	86
default	$-\frac{1}{ad(xd+c)}-\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R_{+c^2}}}{3da}$	92

input
```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/a/d/(d*x+c)+1/3*sum(_R*ln((-4*_R^3*a^4*d^4+3*b*d)*x-4*a^4*c*d^3*_R^3-a^
3*d^2*_R^2+3*b*c),_R=RootOf(_Z^3*a^4*d^3-b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx = -\frac{2\sqrt{3}(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bd^2x^2+2\,bcdx+bc^2-(a\right.}{6\left(ad^2x+acd\right)}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(d*x + c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(
1/3) - 1/3*sqrt(3)) + (d*x + c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*(d*x + c)*(b/a)^(1/3)
*log(b*d*x + b*c + a*(b/a)^(2/3)) + 6)/(a*d^2*x + a*c*d)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{acd+ad^2x} + \frac{\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(x+\frac{9t^2a^3+bc}{bd}\right)\right)\right)}{d}$$

input
```
integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3),x)
```

output
```
-1/(a*c*d + a*d**2*x) + RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(x + (
9*_t**2*a**3 + b*c)/(b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 +
a), x)/a - 1/(a*d^2*x + a*c*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)}\,dx = \frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}} \log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}} - \frac{1}{(dx+c)d}\right|\right)}{3\,a} - \frac{\sqrt{3}(a^2b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}} - \frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{3\,a^2d} - \frac{(a^2b)^{\frac{1}{3}} \log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}} - \frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d} + \frac{1}{(dx+c)^2d^2}\right)}{6\,a^2d} - \frac{1}{(dx+c)ad}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a - 1
/3*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/((d*x +
 c)*d))/(b/(a*d^3))^(1/3))/(a^2*d) - 1/6*(a^2*b)^(1/3)*log((b/(a*d^3))^(2/
3) - (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^2*d) - 1/((
d*x + c)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{1}{(c+dx)^2\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{4/3}\,d} - \frac{1}{a\,d\,(c+d\,x)}$$
$$- \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,d}$$
$$+ \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)*(c + d*x)^2),x)
```

output
```
(b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(3*a^(4/3)*d) - 1/(a*d*(c
 + d*x)) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(
1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d) + (b^(1/3)*log(2*b^(1/3)*c
 - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(
a^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.65

$$\int \frac{1}{(c+dx)^2\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,c^2+2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bcdx+6b^{\frac{2}{3}}a^{\frac{1}{3}}dx-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}d\right.$$

input
```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3),x)
```

output

```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*b*c**2 + 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3)))*b*c*d*x + 6*b**(2/3)*a**(1/3)*d*x - log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b*c**2 - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b
*c*d*x + 2*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**2 + 2*log(a**(1/
3) + b**(1/3)*c + b**(1/3)*d*x)*b*c*d*x)/(6*b**(2/3)*a**(1/3)*a*c*d*(c + d
*x))
```

3.65 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{2ad(c+dx)^2} + \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}d} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{5/3}d} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}d}$$

output

```
-1/2/a/d/(d*x+c)^2+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(5/3)/d-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(
5/3)/d+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a
^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{-\frac{3a^{2/3}}{(c+dx)^2} - 2\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{5/3}d}$$

input
```
Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)),x]
```

output
```
((-3*a^(2/3))/(c + d*x)^2 - 2*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)
*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)
] + b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]
)/(6*a^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}\,d(c+dx)}{d}$$
$$\downarrow 847$$
$$\frac{-\frac{b\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)}{a} - \frac{1}{2a(c+dx)^2}}{d}$$
$$\downarrow 750$$

$$\frac{-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{d}$$

$$\downarrow 16$$

$$\frac{-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{d}$$

$$\downarrow 1142$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{d}$$

$$\downarrow 25$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{d}$$

$$\downarrow 27$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{d}$$

$$\downarrow 1082$$

$$-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{d}$$

$$\downarrow 217$$

$$-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{d}$$

$$\downarrow 1103$$

$$-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{d}$$

input `Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)),x]`

output
```
(-1/2*1/(a*(c + d*x)^2) - (b*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*
b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]]
)/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2
]/(2*b^(1/3)))/(3*a^(2/3))))/a)/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{1}{2ad(xd+c)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3da}$	87
risch	$-\frac{1}{2ad(xd+c)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5d^3\_Z^3+b^2\right)}\_R\ln\left(\left(-4a^5d^4\_R^3-3db^2\right)x-4a^5cd^3\_R^3-a^2bd\_R-3b^2c\right)\right)}{3}$	87

input
```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/d/(d*x+c)^2-1/3/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(
_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.51

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$$

$$= \frac{2\sqrt{3}(d^2x^2+2\,cdx+c^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(adx+ac)\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-(d^2x^2+2\,cdx+c^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2\right.}{6\,(}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(d^2*x^2 + 2*c*d*x + c^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sq
rt(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - (d^2*x^2 + 2*c*d*x
+ c^2)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^
2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) + 2*(d^2*x^2 + 2*c*d*x
+ c^2)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/3)) - 3)/(a*d^3*
x^2 + 2*a*c*d^2*x + a*c^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$$

$$= -\frac{1}{2ac^2d+4acd^2x+2ad^3x^2}+\frac{\text{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(x+\frac{-3ta^2+bc}{bd}\right)\right)\right)}{d}$$

input
```
integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3),x)
```

output
```
-1/(2*a*c**2*d + 4*a*c*d**2*x + 2*a*d**3*x**2) + RootSum(27*_t**3*a**5 + b
**2, Lambda(_t, _t*log(x + (-3*_t*a**2 + b*c)/(b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a
 - 1/2/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)^2+\right.}{6\,a}$$
$$-\frac{1}{2\,(dx+c)^2 ad}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/6*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3
))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^
3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 +
 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b
*d*x - b*c + (-a*b^2)^(1/3))))/a - 1/2/((d*x + c)^2*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

$$\int \frac{1}{(c+dx)^3\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{b^{2/3}\,\ln\left(a^2\,b^{1/3}\,c-(-a)^{7/3}+a^2\,b^{1/3}\,d\,x\right)}{3\,(-a)^{5/3}\,d} - \frac{1}{2\,a\,d\,(c^2+2\,c\,d\,x+d^2\,x^2)}$$
$$- \frac{b^{2/3}\,\ln\left(3\,a^2\,b^3\,c\,d^5+3\,a^2\,b^3\,d^6\,x+3\,(-a)^{7/3}\,b^{8/3}\,d^5\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-a)^{5/3}\,d}$$
$$+ \frac{b^{2/3}\,\ln\left(3\,a^2\,b^3\,c\,d^5+3\,a^2\,b^3\,d^6\,x-9\,(-a)^{7/3}\,b^{8/3}\,d^5\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{(-a)^{5/3}\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)*(c + d*x)^3),x)
```

output
```
(b^(2/3)*log(a^2*b^(1/3)*c - (-a)^(7/3) + a^2*b^(1/3)*d*x))/(3*(-a)^(5/3)*
d) - 1/(2*a*d*(c^2 + d^2*x^2 + 2*c*d*x)) - (b^(2/3)*log(3*a^2*b^3*c*d^5 +
3*a^2*b^3*d^6*x + 3*(-a)^(7/3)*b^(8/3)*d^5*((3^(1/2)*1i)/2 + 1/2))*((3^(1/
2)*1i)/2 + 1/2))/(3*(-a)^(5/3)*d) + (b^(2/3)*log(3*a^2*b^3*c*d^5 + 3*a^2*b
^3*d^6*x - 9*(-a)^(7/3)*b^(8/3)*d^5*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/
6 - 1/6))/((-a)^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.58

$$\int \frac{1}{(c+dx)^3\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,c^2+4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bcdx+2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,d$$

input
```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3),x)
```

output

```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*b*c**2 + 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c*d*x + 2*a**(1/3)*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*d**2*x**2 + a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2
/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**2 + 2*a**(1/3)*log(
a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2
*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d*x + a**(1/3)*log(a**(2/3) - b*
*(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d
*x + b**(2/3)*d**2*x**2)*b*d**2*x**2 - 2*a**(1/3)*log(a**(1/3) + b**(1/3)*
c + b**(1/3)*d*x)*b*c**2 - 4*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)
*d*x)*b*c*d*x - 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*d**
2*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)*a**2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.66 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx$

Optimal result 480
Mathematica [A] (verified) 480
Rubi [A] (verified) 481
Maple [A] (verified) 482
Fricas [B] (verification not implemented) 483
Sympy [B] (verification not implemented) 483
Maxima [A] (verification not implemented) 484
Giac [A] (verification not implemented) 484
Mupad [B] (verification not implemented) 485
Reduce [B] (verification not implemented) 485

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3ad(c+dx)^3} - \frac{b\log(c+dx)}{a^2d} + \frac{b\log\left(a+b(c+dx)^3\right)}{3a^2d}$$

output

```
-1/3/a/d/(d*x+c)^3-b*ln(d*x+c)/a^2/d+1/3*b*ln(a+b*(d*x+c)^3)/a^2/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx = \frac{-\frac{a}{(c+dx)^3} - 3b\log(c+dx) + b\log\left(a+b(c+dx)^3\right)}{3a^2d}$$

input

```
Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)),x]
```

output

```
(-(a/(c + d*x)^3) - 3*b*Log[c + d*x] + b*Log[a + b*(c + d*x)^3])/(3*a^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)}\,d(c+dx)}{d}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)}\,d(c+dx)^3}{3d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{b^2}{a^2(b(c+dx)^3+a)} - \frac{b}{a^2(c+dx)^3} + \frac{1}{a(c+dx)^6}\right) d(c+dx)^3}{3d}$$

$$\downarrow 2009$$

$$\frac{-\frac{b\log((c+dx)^3)}{a^2} + \frac{b\log(a+b(c+dx)^3)}{a^2} - \frac{1}{a(c+dx)^3}}{3d}$$

input
```
Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)),x]
```

output
```
(-(1/(a*(c + d*x)^3)) - (b*Log[(c + d*x)^3])/a^2 + (b*Log[a + b*(c + d*x)^
3])/a^2)/(3*d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
default	$-\frac{1}{3ad(xd+c)^3}-\frac{b\ln(xd+c)}{a^2d}+\frac{b\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3a^2d}$
risch	$-\frac{1}{3ad(xd+c)^3}-\frac{b\ln(xd+c)}{a^2d}+\frac{b\ln(-bd^3x^3-3bcd^2x^2-3dbxc^2-c^3b-a)}{3a^2d}$
norman	$\frac{\frac{x}{ac}+\frac{dx^2}{c^2a}+\frac{d^2x^3}{3ac^3}}{(xd+c)^3}-\frac{b\ln(xd+c)}{a^2d}+\frac{b\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3a^2d}$
parallelrisch	$-\frac{3\ln(xd+c)x^3bd^5-\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)x^3bd^5+9\ln(xd+c)x^2bcd^4-3\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+$

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/d/(d*x+c)^3-b*ln(d*x+c)/a^2/d+1/3*b/a^2/d*ln(b*d^3*x^3+3*b*c*d^2*x^
2+3*b*c^2*d*x+b*c^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x^2+3\,b}{3\left(a^2d^4x^3+3\,a^2cd^3x^2+3\,a^2c^2d^2x+a^2c^3d\right)}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(b*d^3*x^3 + 3*b
*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*log(d*x + c) - a)/(a^2*d^4*x^3 + 3*a^2*c*d^3*x^2 + 3*a^2
*c^2*d^2*x + a^2*c^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3ac^3d+9ac^2d^2x+9acd^3x^2+3ad^4x^3} - \frac{b\log\left(\frac{c}{d}+x\right)}{a^2d} + \frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2d}$$

input
```
integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3),x)
```

output
```
-1/(3*a*c**3*d + 9*a*c**2*d**2*x + 9*a*c*d**3*x**2 + 3*a*d**4*x**3) - b*lo
g(c/d + x)/(a**2*d) + b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**
3)/(b*d**3))/(3*a**2*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.75

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3\left(ad^4x^3+3\,acd^3x^2+3\,ac^2d^2x+ac^3d\right)} + \frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2d} - \frac{b\log\left(dx+c\right)}{a^2d}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/3/(a*d^4*x^3 + 3*a*c*d^3*x^2 + 3*a*c^2*d^2*x + a*c^3*d) + 1/3*b*log(b*d
^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d) - b*log(d*x + c)
/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)}\,dx = \frac{b\log\left(\left|-b-\frac{a}{(dx+c)^3}\right|\right)}{3\,a^2d} - \frac{1}{3\left(dx+c\right)^3ad}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*b*log(abs(-b - a/(d*x + c)^3))/(a^2*d) - 1/3/((d*x + c)^3*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.86

$$\int \frac{1}{(c+dx)^4\,(a+b(c+dx)^3)}\,dx = \frac{b\,\ln\left(-b\,c^3-3\,b\,c^2\,d\,x-3\,b\,c\,d^2\,x^2-b\,d^3\,x^3-a\right)}{3\,a^2\,d} - \frac{b\,\ln\left(c+d\,x\right)}{a^2\,d} - \frac{1}{3\,\left(a\,c^3\,d+3\,a\,c^2\,d^2\,x+3\,a\,c\,d^3\,x^2+a\,d^4\,x^3\right)}$$

input
```
int(1/((a + b*(c + d*x)^3)*(c + d*x)^4),x)
```

output
```
(b*log(- a - b*c^3 - b*d^3*x^3 - 3*b*c^2*d*x - 3*b*c*d^2*x^2))/(3*a^2*d) -
 (b*log(c + d*x))/(a^2*d) - 1/(3*(a*d^4*x^3 + a*c^3*d + 3*a*c^2*d^2*x + 3*
a*c*d^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 420, normalized size of antiderivative = 7.50

$$\int \frac{1}{(c+dx)^4\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)b\,c^3+3\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}c\right.}{}$$

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3),x)
```

output

```
(log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**
2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**3 + 3*log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b*c**2*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c -
b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x
**2)*b*c*d**2*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3
)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3
 + log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 + 3*log(a**(1/3) + b**
(1/3)*c + b**(1/3)*d*x)*b*c**2*d*x + 3*log(a**(1/3) + b**(1/3)*c + b**(1/3
)*d*x)*b*c*d**2*x**2 + log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*d**3*x*
*3 - 3*log(c + d*x)*b*c**3 - 9*log(c + d*x)*b*c**2*d*x - 9*log(c + d*x)*b*
c*d**2*x**2 - 3*log(c + d*x)*b*d**3*x**3 - a)/(3*a**2*d*(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3))
```

3.67 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 487
Mathematica [A] (verified) . . . 488
Rubi [A] (verified) . . . 488
Maple [C] (verified) . . . 493
Fricas [B] (verification not implemented) . . . 493
Sympy [A] (verification not implemented) . . . 494
Maxima [F] . . . 495
Giac [A] (verification not implemented) . . . 495
Mupad [B] (verification not implemented) . . . 496
Reduce [B] (verification not implemented) . . . 496

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{5/3}d} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9\sqrt[3]{a}b^{5/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}b^{5/3}d}$$

output

```
-1/3*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+
c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(5/3)/d-2/9*ln(a^(1/3)+b^(1/3)*(d*x
+c))/a^(1/3)/b^(5/3)/d+1/9*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x
+c)^2)/a^(1/3)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\frac{3b^{2/3}(c+dx)^2}{a+b(c+dx)^3} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{\sqrt[3]{a}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{a}}}{9b^{5/3}d}$$

input
```
Integrate[(c + d*x)^4/(a + b*(c + d*x)^3)^2,x]
```

output
```
((-3*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3
) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(1/3) - (2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/a^(1/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2
/3)*(c + d*x)^2]/a^(1/3))/(9*b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^4}{(b(c+dx)^3+a)^2}\,d(c+dx)}{d}$$
$$\downarrow 817$$
$$\frac{2\int \frac{c+dx}{b(c+dx)^3+a}\,d(c+dx)}{3b} - \frac{(c+dx)^2}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 821$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 16$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1142$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx}{3b(a+b(c+}}{d}$$

$$\downarrow 25$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+d}}{d}$$

$$\downarrow 27$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)}{3b(a+b(c+}$$
$$d$$

$$\downarrow 1082$$

$$\frac{2\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)}$$
$$d$$

$$\downarrow 217$$

$$\frac{2\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 1103$$

$$\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}}{d}$$

input
```
Int[(c + d*x)^4/(a + b*(c + d*x)^3)^2,x]
```

output
```
(-1/3*(c + d*x)^2/(b*(a + b*(c + d*x)^3)) + (2*(-1/3*Log[a^(1/3) + b^(1/3)
*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*
x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
+ b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{-\frac{d x^2}{3b}-\frac{2cx}{3b}-\frac{c^2}{3db}}{b d^3 x^3+3bc d^2 x^2+3dbx c^2+c^3 b+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b d^3 \_Z^3+3bc d^2 \_Z^2+3b c^2 d \_Z+c^3 b+a\right)} \frac{\left(\_R_{d+c}\right) \ln\left(x-\_R\right)}{d^2 \_R^2+2cd \_R_{+c^2}}\right)}{9b^2 d}$	141
risch	$\frac{-\frac{d x^2}{3b}-\frac{2cx}{3b}-\frac{c^2}{3db}}{b d^3 x^3+3bc d^2 x^2+3dbx c^2+c^3 b+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b d^3 \_Z^3+3bc d^2 \_Z^2+3b c^2 d \_Z+c^3 b+a\right)} \frac{\left(\_R_{d+c}\right) \ln\left(x-\_R\right)}{d^2 \_R^2+2cd \_R_{+c^2}}\right)}{9b^2 d}$	141

input
```
int((d*x+c)^4/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*d*x^2/b-2/3*c*x/b-1/3*c^2/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+
b*c^3+a)+2/9/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf
(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(133) = 266.

Time = 0.10 (sec) , antiderivative size = 838, normalized size of antiderivative = 4.87

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
[-1/9*(3*a*b^2*d^2*x^2 + 6*a*b^2*c*d*x + 3*a*b^2*c^2 - 3*sqrt(1/3)*(a*b^2*
d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt((-
a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b
^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)*
(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/
3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b*
d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^
(2/3)) + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^
(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^
2 + 3*a*b^4*c^2*d^2*x + (a*b^4*c^3 + a^2*b^3)*d), -1/9*(3*a*b^2*d^2*x^2 +
6*a*b^2*c*d*x + 3*a*b^2*c^2 - 6*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x
^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(s
qrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (
b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2
)^(2/3)) + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2
)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*
x^2 + 3*a*b^4*c^2*d^2*x + (a*b^4*c^3 + a^2*b^3)*d)]
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{-c^2-2cdx-d^2x^2}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3} + \frac{\text{RootSum}\left(729t^3ab^5+8,\left(t\mapsto t\log\left(x+\frac{81t^2ab^3+4c}{4d}\right)\right)\right)}{d}$$

input

```
integrate((d*x+c)**4/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-c**2 - 2*c*d*x - d**2*x**2)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*
x + 9*b**2*c*d**3*x**2 + 3*b**2*d**4*x**3) + RootSum(729*_t**3*a*b**5 + 8,
 Lambda(_t, _t*log(x + (81*_t**2*a*b**3 + 4*c)/(4*d))))/d
```

Maxima [F]

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{(dx+c)^4}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(d^2*x^2 + 2*c*d*x + c^2)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*
d^2*x + (b^2*c^3 + a*b)*d) + 2/3*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^
2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{9\,b}$$
$$-\frac{d^2x^2+2\,cdx+c^2}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/9*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a
*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*b
*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3)
)^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/b - 1/3*(d^2*x^2 + 2*c
*d*x + c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{2\ln\left(\frac{4cd^4}{9b}-\frac{4(-a)^{1/3}d^4}{9b^{4/3}}+\frac{4d^5x}{9b}\right)}{9(-a)^{1/3}b^{5/3}d} - \frac{\frac{dx^2}{3b}+\frac{c^2}{3bd}+\frac{2cx}{3b}}{bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3+a} + \frac{\ln\left(\frac{4cd^4}{9b}+\frac{4d^5x}{9b}-\frac{(-a)^{1/3}d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{1/3}b^{5/3}d} - \frac{\ln\left(\frac{4cd^4}{9b}+\frac{4d^5x}{9b}-\frac{(-a)^{1/3}d^4\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9b^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{1/3}b^{5/3}d}$$

input
```
int((c + d*x)^4/(a + b*(c + d*x)^3)^2,x)
```

output
```
(2*log((4*c*d^4)/(9*b) - (4*(-a)^(1/3)*d^4)/(9*b^(4/3)) + (4*d^5*x)/(9*b))
)/(9*(-a)^(1/3)*b^(5/3)*d) - ((d*x^2)/(3*b) + c^2/(3*b*d) + (2*c*x)/(3*b))
/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (log((4*c*d^4)/(9
*b) + (4*d^5*x)/(9*b) - ((-a)^(1/3)*d^4*(3^(1/2)*1i - 1)^2)/(9*b^(4/3)))*(
3^(1/2)*1i - 1))/(9*(-a)^(1/3)*b^(5/3)*d) - (log((4*c*d^4)/(9*b) + (4*d^5*
x)/(9*b) - ((-a)^(1/3)*d^4*(3^(1/2)*1i + 1)^2)/(9*b^(4/3)))*(3^(1/2)*1i +
1))/(9*(-a)^(1/3)*b^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 703, normalized size of antiderivative = 4.09

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^4/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*a*b*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*b**2*c**4 - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d*x - 6*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**2*x**
2 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b**2*c*d**3*x**3 + b**(2/3)*a**(1/3)*a - 2*b**(2/3)*a**(1/3)*b*c**
3 - 3*b**(2/3)*a**(1/3)*b*c**2*d*x + b**(2/3)*a**(1/3)*b*d**3*x**3 + log(a
**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*
b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c + log(a**(2/3) - b**(1/3)*a**(1
/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3
)*d**2*x**2)*b**2*c**4 + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a
**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*
c**3*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**2*d**2*x**2
 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*d**3*x**3 - 2*log(a**(
1/3) + b**(1/3)*c + b**(1/3)*d*x)*a*b*c - 2*log(a**(1/3) + b**(1/3)*c + b*
*(1/3)*d*x)*b**2*c**4 - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b**2*c
**3*d*x - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b**2*c**2*d**2*x*...
```

3.68 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 498
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Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{c+dx}{3bd\left(a+b(c+dx)^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}$$

output

```
-1/3*(d*x+c)/b/d/(a+b*(d*x+c)^3)-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c)
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(4/3)/d+1/9*ln(a^(1/3)+b^(1/3)*(d*x+c
))/a^(2/3)/b^(4/3)/d-1/18*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(2/3)/b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{-\frac{6\sqrt[3]{b}(c+dx)}{a+b(c+dx)^3} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{2/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{2/3}}}{18b^{4/3}d}$$

input
```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^3)^2,x]
```

output
```
((-6*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3)
+ 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(2/3) + (2*Log[a^(1/3) + b^(1
/3)*(c + d*x)])/a^(2/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3
)*(c + d*x)^2]/a^(2/3))/(18*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {895, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{(c+dx)^3}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)}{3b} - \frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 750$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3}}{d}$$

$$\downarrow 1082$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1103$$

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}}{d}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^3)^2,x]
```

output
```
(-1/3*(c + d*x)/(b*(a + b*(c + d*x)^3)) + (Log[a^(1/3) + b^(1/3)*(c + d*x)
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1
/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3
)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{x}{3b}-\frac{c}{3db}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9b^2d}$	124
risch	$\frac{-\frac{x}{3b}-\frac{c}{3db}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9b^2d}$	124

input

```
int((d*x+c)^3/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output

```
(-1/3*x/b-1/3*c/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+1/9/b^2
/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*
d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(131) = 262$.

Time = 0.09 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.76

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
[-1/18*(6*a^2*b*d*x + 6*a^2*b*c - 3*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d
^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*log((
2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt
(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) -
 (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/
(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) + (b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b
*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*x
^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x +
a*b*c + (a^2*b)^(2/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3
*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d), -1/18*(6*a^2*b*d*x + 6*a^2*b*c -
6*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c
^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x +
 c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*
x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*x^3 +
3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c
 + (a^2*b)^(2/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*
d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{-c-dx}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3} + \frac{\operatorname{RootSum}\left(729t^3a^2b^4-1,\left(t\mapsto t\log\left(x+\frac{9tab+c}{d}\right)\right)\right)}{d}$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**3)**2,x)
```

output
```
(-c - d*x)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x
**2 + 3*b**2*d**4*x**3) + RootSum(729*_t**3*a**2*b**4 - 1, Lambda(_t, _t*l
og(x + (9*_t*a*b + c)/d)))/d
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{(dx+c)^3}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(d*x + c)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3
 + a*b)*d) + 1/3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3 + a), x)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}} \arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right) - \left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.}{18\,b}$$
$$- \frac{dx+c}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output

```
1/18*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x +
b*c + (a*b^2)^(1/3))))/b - 1/3*(d*x + c)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{2/3}\,b^{4/3}\,d} - \frac{\frac{x}{3\,b}+\frac{c}{3\,b\,d}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a} + \frac{\ln\left(b\,c\,d^5+b\,d^6\,x+\frac{a^{1/3}\,b^{2/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}\,d} - \frac{\ln\left(b\,c\,d^5+b\,d^6\,x-\frac{a^{1/3}\,b^{2/3}\,d^5\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}\,d}$$

input

```
int((c + d*x)^3/(a + b*(c + d*x)^3)^2,x)
```

output

```
log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(9*a^(2/3)*b^(4/3)*d) - (x/(3*b) +
c/(3*b*d))/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (log(b*
c*d^5 + b*d^6*x + (a^(1/3)*b^(2/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i -
1))/(18*a^(2/3)*b^(4/3)*d) - (log(b*c*d^5 + b*d^6*x - (a^(1/3)*b^(2/3)*d^5
*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(18*a^(2/3)*b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.98

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^3/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*
b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**3 - 6*a**(1/3)*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**2*d*x - 6*a**
(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b*c*d**2*x**2 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*d**3*x**3 - a**(1/3)*log(a**(2/3) -
 b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*
c*d*x + b**(2/3)*d**2*x**2)*a - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d*
*2*x**2)*b*c**3 - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c
**2*d*x - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3
)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d**2*x*
*2 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
 b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a + 2*a**(1/3)*log(a**(1/
3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 + 6*a**(1/3)*log(a**(1/3) + b**(1/3
)*c + b**(1/3)*d*x)*b*c**2*d*x + 6*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b*
*(1/3)*d*x)*b*c*d**2*x**2 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(...
```

3.69 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result 508
Mathematica [A] (verified) 508
Rubi [A] (verified) 509
Maple [A] (verified) 510
Fricas [B] (verification not implemented) 510
Sympy [B] (verification not implemented) 511
Maxima [A] (verification not implemented) 511
Giac [A] (verification not implemented) 511
Mupad [B] (verification not implemented) 512
Reduce [B] (verification not implemented) 512

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3bd\left(a+b(c+dx)^3\right)}$$

output
```
-1/3/b/d/(a+b*(d*x+c)^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3bd\left(a+b(c+dx)^3\right)}$$

input
```
Integrate[(c + d*x)^2/(a + b*(c + d*x)^3)^2,x]
```

output
```
-1/3*1/(b*d*(a + b*(c + d*x)^3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^2}{(b(c+dx)^3+a)^2}\,d(c+dx)}{d}$$
$$\downarrow 793$$
$$-\frac{1}{3bd\,(a+b(c+dx)^3)}$$

input
```
Int[(c + d*x)^2/(a + b*(c + d*x)^3)^2,x]
```

output
```
-1/3*1/(b*d*(a + b*(c + d*x)^3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{3bd\left(a+b(xd+c)^3\right)}$	22
gosper	$-\frac{1}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	44
default	$-\frac{1}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	44
norman	$-\frac{1}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	44
risch	$-\frac{1}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	44
parallelrisch	$-\frac{1}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	44
orering	$-\frac{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}{3bd\left(a+b(xd+c)^3\right)^2}$	55

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/d/(a+b*(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{1}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+(b^2c^3+ab)d\right)}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.72 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}$$

input
```
integrate((d*x+c)**2/(a+b*(d*x+c)**3)**2,x)
```

output
```
-1/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 + 3*
b**2*d**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3\left((dx+c)^3b+a\right)bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3/(((d*x + c)^3*b + a)*b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3\left((dx+c)^3b+a\right)bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/3/(((d*x + c)^3*b + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{1}{3\,b\,d\,\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}$$

input
```
int((c + d*x)^2/(a + b*(c + d*x)^3)^2,x)
```

output
```
-1/(3*b*d*(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{1}{3bd\,\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}$$

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3)^2,x)
```

output
```
( - 1)/(3*b*d*(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3))
```

3.70 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 513
Mathematica [A] (verified) . . . 514
Rubi [A] (verified) . . . 514
Maple [C] (verified) . . . 518
Fricas [B] (verification not implemented) . . . 519
Sympy [A] (verification not implemented) . . . 519
Maxima [F] . . . 520
Giac [A] (verification not implemented) . . . 520
Mupad [B] (verification not implemented) . . . 521
Reduce [B] (verification not implemented) . . . 522

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}$$

output

```
1/3*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/b^(2/3)/d-1/9*ln(a^(1/3)+b^(1/3)*(d*x+
c))/a^(4/3)/b^(2/3)/d+1/18*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x
+c)^2)/a^(4/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{\frac{6\sqrt[3]{a}(c+dx)^2}{a+b(c+dx)^3} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}}}{18a^{4/3}d}$$

input
```
Integrate[(c + d*x)/(a + b*(c + d*x)^3)^2,x]
```

output
```
((6*a^(1/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(2/3) - (2*Log[a^(1/3) + b^(
1/3)*(c + d*x)])/b^(2/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/
3)*(c + d*x)^2]/b^(2/3))/(18*a^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {895, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$
$$\downarrow 819$$
$$\frac{\int \frac{c+dx}{b(c+dx)^3+a}\,d(c+dx)}{3a} + \frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 821$$

$$\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1082$$

$$\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}\,d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}} - \frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx) - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{3a}$$

$$\Big\downarrow 217$$

$$\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx) - \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{3a}$$

$$\Big\downarrow 1103$$

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}} - \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{(c+dx)^2}{3a(a+b(c+dx)^3)}}{3a}$$

input
```
Int[(c + d*x)/(a + b*(c + d*x)^3)^2,x]
```

output
```
((c + d*x)^2/(3*a*(a + b*(c + d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c +
d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^
(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2
/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{d}{3a}x^2+\frac{2c}{3a}x+\frac{c^2}{3ad}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9abd}$	144
risch	$\frac{\frac{d}{3a}x^2+\frac{2c}{3a}x+\frac{c^2}{3ad}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9abd}$	144

input
```
int((d*x+c)/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+2/3/a*c*x+1/3*c^2/a/d)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b
*c^3+a)+1/9/a/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(
_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(133) = 266.

Time = 0.10 (sec) , antiderivative size = 852, normalized size of antiderivative = 4.95

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
[1/18*(6*a*b^2*d^2*x^2 + 12*a*b^2*c*d*x + 6*a*b^2*c^2 + 3*sqrt(1/3)*(a*b^2
*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt((
-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*
b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)
*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2
/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) + (b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2/3)*log(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)
^(2/3)) - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)
^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d
^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d), 1/18*(6*a*b^2*d
^2*x^2 + 12*a*b^2*c*d*x + 6*a*b^2*c^2 + 6*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b
^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(-a*b^2)^(1/3)/a
)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)
/a)/b) + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(-a*b^2)^(2
/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c)
 + (-a*b^2)^(2/3)) - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 +
a)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*
a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{c^2+2cdx+d^2x^2}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3} + \frac{\text{RootSum}\left(729t^3a^4b^2+1,\left(t\mapsto t\log\left(x+\frac{81t^2a^3b+c}{d}\right)\right)\right)}{d}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)**3)**2,x)
```

output
```
(c**2 + 2*c*d*x + d**2*x**2)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x
+ 9*a*b*c*d**3*x**2 + 3*a*b*d**4*x**3) + RootSum(729*_t**3*a**4*b**2 + 1,
Lambda(_t, _t*log(x + (81*_t**2*a**3*b + c)/d)))/d
```

Maxima [F]

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{dx+c}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3*(d^2*x^2 + 2*c*d*x + c^2)/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d
^2*x + (a*b*c^3 + a^2)*d) + 1/3*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2
*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{18\,a}$$
$$+\frac{d^2x^2+2\,cdx+c^2}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)ad}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output

```
-1/18*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*
a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*
b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3
))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a + 1/3*(d^2*x^2 + 2*
c*d*x + c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\frac{d\,x^2}{3\,a}+\frac{c^2}{3\,a\,d}+\frac{2\,c\,x}{3\,a}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a} - \frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{4/3}\,b^{2/3}\,d} - \frac{\ln\left(\frac{b^{2/3}\,d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{36\,a^{5/3}}+\frac{b\,c\,d^4}{9\,a^2}+\frac{b\,d^5\,x}{9\,a^2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,b^{2/3}\,d} + \frac{\ln\left(\frac{b^{2/3}\,d^4\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{36\,a^{5/3}}+\frac{b\,c\,d^4}{9\,a^2}+\frac{b\,d^5\,x}{9\,a^2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,b^{2/3}\,d}$$

input

```
int((c + d*x)/(a + b*(c + d*x)^3)^2,x)
```

output

```
((d*x^2)/(3*a) + c^2/(3*a*d) + (2*c*x)/(3*a))/(a + b*c^3 + b*d^3*x^3 + 3*b
*c^2*d*x + 3*b*c*d^2*x^2) - log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(9*a^(4
/3)*b^(2/3)*d) - (log((b^(2/3)*d^4*(3^(1/2)*1i - 1)^2)/(36*a^(5/3)) + (b*c
*d^4)/(9*a^2) + (b*d^5*x)/(9*a^2))*(3^(1/2)*1i - 1))/(18*a^(4/3)*b^(2/3)*d
) + (log((b^(2/3)*d^4*(3^(1/2)*1i + 1)^2)/(36*a^(5/3)) + (b*c*d^4)/(9*a^2)
 + (b*d^5*x)/(9*a^2))*(3^(1/2)*1i + 1))/(18*a^(4/3)*b^(2/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 705, normalized size of antiderivative = 4.10

$$\int \frac{c+dx}{(a+b(c+dx)^3)^2}\,dx = \text{Too large to display}$$

input

```
int((d*x+c)/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*a*b*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*b**2*c**4 - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d*x - 6*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**2*x**
2 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b**2*c*d**3*x**3 - 2*b**(2/3)*a**(1/3)*a + 4*b**(2/3)*a**(1/3)*b*c
**3 + 6*b**(2/3)*a**(1/3)*b*c**2*d*x - 2*b**(2/3)*a**(1/3)*b*d**3*x**3 + l
og(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
+ 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c + log(a**(2/3) - b**(1/3)*a
**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**
(2/3)*d**2*x**2)*b**2*c**4 + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b
**2*c**3*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**2*d**2*
x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/
3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*d**3*x**3 - 2*log(
a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a*b*c - 2*log(a**(1/3) + b**(1/3)*c
+ b**(1/3)*d*x)*b**2*c**4 - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*
*2*c**3*d*x - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b**2*c**2*d**...
```

3.71 $\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 523
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Optimal result

Integrand size = 13, antiderivative size = 170

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{c+dx}{3ad\left(a+b(c+dx)^3\right)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{5/3}\sqrt[3]{b}d}$$

output

```
1/3*(d*x+c)/a/d/(a+b*(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))
*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(1/3)/d+2/9*ln(a^(1/3)+b^(1/3)*(d*x+c)
)/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)
^2)/a^(5/3)/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx$$
$$= \frac{\frac{3a^{2/3}(c+dx)}{a+b(c+dx)^3} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{b}}}{9a^{5/3}d}$$

input
```
Integrate[(a + b*(c + d*x)^3)^(-2),x]
```

output
```
((3*a^(2/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1/3) +
 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(1/3) + (2*Log[a^(1/3) + b^(1/
3)*(c + d*x)])/b^(1/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)
*(c + d*x)^2]/b^(1/3))/(9*a^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {239, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx$$
$$\downarrow 239$$
$$\frac{\int \frac{1}{(b(c+dx)^3+a)^2}\,d(c+dx)}{d}$$
$$\downarrow 749$$
$$\frac{2\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)}{3a} + \frac{c+dx}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 750$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 16$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+dx)^3)}}{d}$$

$$\downarrow 1142$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+}}{d}$$

$$\downarrow 25$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+}}{d}$$

$$\downarrow 27$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+}$$

$$d$$

$$\downarrow 1082$$

$$\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+dx}$$

$$d$$

$$\downarrow 217$$

$$\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+dx)^3)}$$

$$d$$

$$\downarrow 1103$$

$$\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}}{3a^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a(a+b(c+dx)^3)}}{d}$$

input
```
Int[(a + b*(c + d*x)^3)^(-2),x]
```

output
```
((c + d*x)/(3*a*(a + b*(c + d*x)^3)) + (2*(Log[a^(1/3) + b^(1/3)*(c + d*x)
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1
/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3
)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 749

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x}{3a}+\frac{c}{3da}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9abd}$	127
risch	$\frac{\frac{x}{3a}+\frac{c}{3da}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9abd}$	127

input
```
int(1/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*x/a+1/3*c/d/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+2/9/a/b/
d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d
^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(131) = 262.

Time = 0.12 (sec) , antiderivative size = 808, normalized size of antiderivative = 4.75

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
[1/9*(3*a^2*b*d*x + 3*a^2*b*c + 3*sqrt(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2
*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*
a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1
/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (
a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c
*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*
b*c + (a^2*b)^(2/3)))/(a^3*b^2*d^4*x^3 + 3*a^3*b^2*c*d^3*x^2 + 3*a^3*b^2*c
^2*d^2*x + (a^3*b^2*c^3 + a^4*b)*d), 1/9*(3*a^2*b*d*x + 3*a^2*b*c + 6*sqrt
(1/3)*(a*b^2*d^3*x^3 + 3*a*b^2*c*d^2*x^2 + 3*a*b^2*c^2*d*x + a*b^2*c^3 + a
^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) -
(a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*
b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^
2*b)^(2/3)))/(a^3*b^2*d^4*x^3 + 3*a^3*b^2*c*d^3*x^2 + 3*a^3*b^2*c^2*d^2*x
+ (a^3*b^2*c^3 + a^4*b)*d)]
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.54

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{c+dx}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3} + \frac{\operatorname{RootSum}\left(729t^3a^5b-8,\left(t\mapsto t\log\left(x+\frac{9ta^2+2c}{2d}\right)\right)\right)}{d}$$

input

```
integrate(1/(a+b*(d*x+c)**3)**2,x)
```

output

```
(c + d*x)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x + 9*a*b*c*d**3*x**2
 + 3*a*b*d**4*x**3) + RootSum(729*_t**3*a**5*b - 8, Lambda(_t, _t*log(x +
(9*_t*a**2 + 2*c)/(2*d))))/d
```

Maxima [F]

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3*(d*x + c)/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3
+ a^2)*d) + 2/3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3 + a), x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx$$
$$= \frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}} \arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right) - \left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2 + 4\left(bdx\right.\right.}{9\,a}$$
$$+ \frac{dx+c}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)ad}$$

input
```
integrate(1/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))
/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))^
(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b
*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b
*c + (a*b^2)^(1/3))))/a + 1/3*(d*x + c)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx = \frac{\frac{x}{3a}+\frac{c}{3ad}}{b c^3+3 b c^2 d x+3 b c d^2 x^2+b d^3 x^3+a} + \frac{2\ln\left(b^{1/3}c+a^{1/3}+b^{1/3}d x\right)}{9a^{5/3}b^{1/3}d} + \frac{\ln\left(\frac{2b^2cd^5}{a}+\frac{2b^2d^6x}{a}+\frac{b^{5/3}d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9a^{5/3}b^{1/3}d} - \frac{\ln\left(\frac{2b^2cd^5}{a}+\frac{2b^2d^6x}{a}-\frac{b^{5/3}d^5\left(1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9a^{5/3}b^{1/3}d}$$

input
```
int(1/(a + b*(c + d*x)^3)^2,x)
```

output
```
(x/(3*a) + c/(3*a*d))/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2
) + (2*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(5/3)*b^(1/3)*d) + (lo
g((2*b^2*c*d^5)/a + (2*b^2*d^6*x)/a + (b^(5/3)*d^5*(3^(1/2)*1i - 1))/a^(2/
3))*(3^(1/2)*1i - 1))/(9*a^(5/3)*b^(1/3)*d) - (log((2*b^2*c*d^5)/a + (2*b^
2*d^6*x)/a - (b^(5/3)*d^5*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(9*
a^(5/3)*b^(1/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 677, normalized size of antiderivative = 3.98

$$\int \frac{1}{(a+b(c+dx)^3)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*
b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**3 - 6*a**(1/3)*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**2*d*x - 6*a**
(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b*c*d**2*x**2 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*d**3*x**3 - a**(1/3)*log(a**(2/3) -
 b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*
c*d*x + b**(2/3)*d**2*x**2)*a - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d*
*2*x**2)*b*c**3 - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c
**2*d*x - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3
)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d**2*x*
*2 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
 b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a + 2*a**(1/3)*log(a**(1/
3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 + 6*a**(1/3)*log(a**(1/3) + b**(1/3
)*c + b**(1/3)*d*x)*b*c**2*d*x + 6*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b*
*(1/3)*d*x)*b*c*d**2*x**2 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(...
```

3.72 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{1}{3ad\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^2 d} - \frac{\log\left(a+b(c+dx)^3\right)}{3a^2 d}$$

output

```
1/3/a/d/(a+b*(d*x+c)^3)+ln(d*x+c)/a^2/d-1/3*ln(a+b*(d*x+c)^3)/a^2/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\frac{a}{a+b(c+dx)^3} + 3\log(c+dx) - \log\left(a+b(c+dx)^3\right)}{3a^2 d}$$

input

```
Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)^2),x]
```

output

```
(a/(a + b*(c + d*x)^3) + 3*Log[c + d*x] - Log[a + b*(c + d*x)^3])/(3*a^2*d
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)^2}\,d(c+dx)}{d}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)^2}\,d(c+dx)^3}{3d}$$

$$\downarrow 54$$

$$\frac{\int \left(-\frac{b}{a^2(b(c+dx)^3+a)} - \frac{b}{a(b(c+dx)^3+a)^2} + \frac{1}{a^2(c+dx)^3}\right) d(c+dx)^3}{3d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(a+b(c+dx)^3)}{a^2} + \frac{\log((c+dx)^3)}{a^2} + \frac{1}{a(a+b(c+dx)^3)}}{3d}$$

input
```
Int[1/((c + d*x)*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(a*(a + b*(c + d*x)^3)) + Log[(c + d*x)^3]/a^2 - Log[a + b*(c + d*x)^3]
/a^2)/(3*d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

method	result
norman	$\frac{1}{3ad(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{\ln(xd+c)}{a^2d}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^2d}$
risch	$\frac{1}{3ad(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{\ln(xd+c)}{a^2d}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^2d}$
default	$\frac{\ln(xd+c)}{a^2d}-\frac{b\left(-\frac{a}{3db\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}+\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}\right)}{a^2}$
parallelrisch	$\frac{3\ln(xd+c)x^3b^2d^5-\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)x^3b^2d^5+9\ln(xd+c)x^2b^2c\,d^4-3\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+$

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a/d/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+ln(d*x+c)/a^2/d-1/3/
a^2/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(55) = 110$.

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.86

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x\right.}{3\left(a^2bd^4x^3+3\,a^2bcd^3x^2+3\,a^2bc^2d^2x+\left(a^2bc^3+a^3\right)d\right)}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*log(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3 + a)*log(d*x + c) - a)/(a^2*b*d^4*x^3 + 3*a^2*b*c*d^
3*x^2 + 3*a^2*b*c^2*d^2*x + (a^2*b*c^3 + a^3)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(46) = 92$.

Time = 0.97 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.86

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{1}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3} + \frac{\log\left(\frac{c}{d}+x\right)}{a^2d} - \frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2d}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)**3)**2,x)
```

output
```
1/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d**2*x + 9*a*b*c*d**3*x**2 + 3*a*b
*d**4*x**3) + log(c/d + x)/(a**2*d) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**
3 + (a + b*c**3)/(b*d**3))/(3*a**2*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{1}{3\left(abd^4x^3+3\,abcd^3x^2+3\,abc^2d^2x+(abc^3+a^2)d\right)} - \frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2d} + \frac{\log(dx+c)}{a^2d}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3/(a*b*d^4*x^3 + 3*a*b*c*d^3*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3 + a^2)*d)
- 1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d) + l
og(d*x + c)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.71

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^2d} + \frac{\log\left(|dx+c|\right)}{a^2d} + \frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)ad}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^2*d)
 + log(abs(d*x + c))/(a^2*d) + 1/3/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.78

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{1}{3\left(a^2 d + b a c^3 d + 3 b a c^2 d^2 x + 3 b a c d^3 x^2 + b a d^4 x^3\right)} - \frac{\ln\left(b c^3 + 3 b c^2 d x + 3 b c d^2 x^2 + b d^3 x^3 + a\right)}{3 a^2 d} + \frac{\ln(c + d x)}{a^2 d}$$

input
```
int(1/((a + b*(c + d*x)^3)^2*(c + d*x)),x)
```

output
```
1/(3*(a^2*d + a*b*c^3*d + a*b*d^4*x^3 + 3*a*b*c^2*d^2*x + 3*a*b*c*d^3*x^2)
) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*a^2*d) + l
og(c + d*x)/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 508, normalized size of antiderivative = 8.61

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right)a - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cd\right.}{}$$

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*
c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a - log(a**(2/3) - b**(1/3)*
a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b*
*(2/3)*d**2*x**2)*b*c**3 - 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c
**2*d*x - 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b
**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d**2*x**2 - log(
a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2
*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3 - log(a**(1/3) + b**(1/3
)*c + b**(1/3)*d*x)*a - log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 -
 3*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**2*d*x - 3*log(a**(1/3) +
 b**(1/3)*c + b**(1/3)*d*x)*b*c*d**2*x**2 - log(a**(1/3) + b**(1/3)*c + b*
*(1/3)*d*x)*b*d**3*x**3 + 3*log(c + d*x)*a + 3*log(c + d*x)*b*c**3 + 9*log
(c + d*x)*b*c**2*d*x + 9*log(c + d*x)*b*c*d**2*x**2 + 3*log(c + d*x)*b*d**
3*x**3 + a)/(3*a**2*d*(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**
3*x**3))
```

3.73 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 188

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{a^2 d(c+dx)} - \frac{b(c+dx)^2}{3a^2 d\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}d} + \frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{7/3}d} - \frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}d}$$

output

```
-1/a^2/d/(d*x+c)-1/3*b*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3)+4/9*b^(1/3)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/d+4/9*b^(
1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/d-2/9*b^(1/3)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\frac{9\sqrt[3]{a}}{c+dx} - \frac{3\sqrt[3]{a}b(c+dx)^2}{a+b(c+dx)^3} - 4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) - 2\sqrt[3]{b}\log\left(a\right.}{9a^{7/3}d}$$

input
```
Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)^2),x]
```

output
```
((-9*a^(1/3))/(c + d*x) - (3*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) -
4*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3)
)] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] - 2*b^(1/3)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*a^(7/3)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {895, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$
$$\downarrow 819$$
$$\frac{\frac{4\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}\,d(c+dx)}{3a} + \frac{1}{3a(c+dx)(a+b(c+dx)^3)}}{d}$$

$$\downarrow 847$$

$$\frac{4\left(-\frac{b\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 821$$

$$\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 16$$

$$\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 1142$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 25$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 27$$

$$\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 1082$$

$$\frac{4\left(-\frac{b\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 217$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b}\right)}{d}$$

$$\downarrow 1103$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)}\right)}{d}$$

input
```
Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(3*a*(c + d*x)*(a + b*(c + d*x)^3)) + (4*(-(1/(a*(c + d*x))) - (b*(-1/3
*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1
/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/
3))))/a))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{a^2 d(xd+c)} - \frac{b\left(\frac{\frac{x^2 d}{3}+\frac{2cx}{3}+\frac{c^2}{3d}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right.}{d^2\_R^2+2cd\_R+c}\right.}{9bd}\right.}{a^2}$
risch	$\frac{-\frac{4b\,d^2x^3}{3a^2}-\frac{4bcd\,x^2}{a^2}-\frac{4bx\,c^2}{a^2}-\frac{4c^3b+3a}{3a^2d}}{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)(xd+c)}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a^7d^3\_Z^3-b\right)}\_R\ln\left(\left(-4a^7d^4\_R^3+3bd\right)x-4a^7c\,d^3\_R^3-a^5d^2\right.\right.}{9}$

input
```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/d/(d*x+c)-b/a^2*((1/3*x^2*d+2/3*c*x+1/3*c^2/d)/(b*d^3*x^3+3*b*c*d^2
*x^2+3*b*c^2*d*x+b*c^3+a)+4/9/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(
x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(149) = 298$.

Time = 0.09 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.04

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx =$$

$$-\frac{12\,bd^3x^3+36\,bcd^2x^2+36\,bc^2dx+12\,bc^3+4\sqrt{3}(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+bc^4+(4\,bc^3+a)dx+}{}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
-1/9*(12*b*d^3*x^3 + 36*b*c*d^2*x^2 + 36*b*c^2*d*x + 12*b*c^3 + 4*sqrt(3)*
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x +
 a*c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3))
+ 2*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d
*x + a*c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b
/a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^
2 + b*c^4 + (4*b*c^3 + a)*d*x + a*c)*(b/a)^(1/3)*log(b*d*x + b*c + a*(b/a)
^(2/3)) + 9*a)/(a^2*b*d^5*x^4 + 4*a^2*b*c*d^4*x^3 + 6*a^2*b*c^2*d^3*x^2 +
(4*a^2*b*c^3 + a^3)*d^2*x + (a^2*b*c^4 + a^3*c)*d)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-3a-4bc^3-12bc^2dx-12bcd^2x^2-4bd^3x^3}{3a^3cd+3a^2bc^4d+18a^2bc^2d^3x^2+12a^2bcd^4x^3+3a^2bd^5x^4+x\left(3a^3d^2+12a^2bc^3d^2\right)} + \frac{\text{RootSum}\left(729t^3a^7-64b,\left(t\mapsto t\log\left(x+\frac{81t^2a^5+16bc}{16bd}\right)\right)\right)}{d}$$

input

```
integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-3*a - 4*b*c**3 - 12*b*c**2*d*x - 12*b*c*d**2*x**2 - 4*b*d**3*x**3)/(3*a*
*3*c*d + 3*a**2*b*c**4*d + 18*a**2*b*c**2*d**3*x**2 + 12*a**2*b*c*d**4*x**
3 + 3*a**2*b*d**5*x**4 + x*(3*a**3*d**2 + 12*a**2*b*c**3*d**2)) + RootSum(
729*_t**3*a**7 - 64*b, Lambda(_t, _t*log(x + (81*_t**2*a**5 + 16*b*c)/(16*
b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^2 (dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(4*b*d^3*x^3 + 12*b*c*d^2*x^2 + 12*b*c^2*d*x + 4*b*c^3 + 3*a)/(a^2*b*
d^5*x^4 + 4*a^2*b*c*d^4*x^3 + 6*a^2*b*c^2*d^3*x^2 + (4*a^2*b*c^3 + a^3)*d^
2*x + (a^2*b*c^4 + a^3*c)*d) - 4/3*b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx = {} & \frac{4\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{1}{(dx+c)d}\right|\right)}{9a^2} \\ & - \frac{4\sqrt{3}(a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{9a^3d} \\ & - \frac{2(a^2b)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d}+\frac{1}{(dx+c)^2d^2}\right)}{9a^3d} \\ & - \frac{1}{(dx+c)a^2d} - \frac{b}{3(dx+c)a^2\left(b+\frac{a}{(dx+c)^3}\right)d}\end{aligned}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
4/9*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a^2 -
 4/9*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/((d*x
 + c)*d))/(b/(a*d^3))^(1/3))/(a^3*d) - 2/9*(a^2*b)^(1/3)*log((b/(a*d^3))^(
2/3) - (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^3*d) - 1/
((d*x + c)*a^2*d) - 1/3*b/((d*x + c)*a^2*(b + a/(d*x + c)^3)*d)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx \\
&= \frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{7/3}\,d} \\
&\quad - \frac{\frac{4\,b\,c^3+3\,a}{3\,a^2\,d}+\frac{4\,b\,d^2\,x^3}{3\,a^2}+\frac{4\,b\,c^2\,x}{a^2}+\frac{4\,b\,c\,d\,x^2}{a^2}}{a\,c+x\,\left(4\,b\,d\,c^3+a\,d\right)+b\,c^4+b\,d^4\,x^4+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3} \\
&\quad - \frac{4\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}\,d} \\
&\quad + \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{7/3}\,d}
\end{aligned}$$

input
```
int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^2),x)
```

output
```
(4*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(7/3)*d) - ((3*a +
 4*b*c^3)/(3*a^2*d) + (4*b*d^2*x^3)/(3*a^2) + (4*b*c^2*x)/a^2 + (4*b*c*d*x
^2)/a^2)/(a*c + x*(a*d + 4*b*c^3*d) + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2
+ 4*b*c*d^3*x^3) - (4*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/
3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)*d) + (b^(1/3)*log(2
*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*2i)/9
 - 2/9))/(a^(7/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1009, normalized size of antiderivative = 5.37

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input

```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^2,x)
```

output

```
(4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*a*b*c**2 + 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*a*b*c*d*x + 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**5 + 16*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*d*x + 24*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*
b**2*c**3*d**2*x**2 + 16*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3
)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**3*x**3 + 4*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c*d**4*x**4 - 6*
b**(2/3)*a**(1/3)*a*c + 3*b**(2/3)*a**(1/3)*a*d*x - 9*b**(2/3)*a**(1/3)*b*
c**4 - 24*b**(2/3)*a**(1/3)*b*c**3*d*x - 18*b**(2/3)*a**(1/3)*b*c**2*d**2*
x**2 + 3*b**(2/3)*a**(1/3)*b*d**4*x**4 - 2*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*a*b*c**2 - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(
1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c*d*
x - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3
)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**5 - 8*log(a**(2/3)
 - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3
)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**4*d*x - 12*log(a**(2/3) - b**(1/3)*a
**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + ...
```

3.74 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 554
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Optimal result

Integrand size = 21, antiderivative size = 188

$$\begin{aligned}\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx &= -\frac{1}{2a^2d(c+dx)^2} - \frac{b(c+dx)}{3a^2d\left(a+b(c+dx)^3\right)} \\ &\quad + \frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}d} \\ &\quad - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{8/3}d} \\ &\quad + \frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}d}\end{aligned}$$

output

```
-1/2/a^2/d/(d*x+c)^2-1/3*b*(d*x+c)/a^2/d/(a+b*(d*x+c)^3)+5/9*b^(2/3)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/d-5/9*b
^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(8/3)/d+5/18*b^(2/3)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{-\frac{9a^{2/3}}{(c+dx)^2} - \frac{6a^{2/3}b(c+dx)}{a+b(c+dx)^3} - 10\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + 5b^{2/3}\log$$
$$18a^{8/3}d$$

input
```
Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)^2),x]
```

output
```
((-9*a^(2/3))/(c + d*x)^2 - (6*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) -
10*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3
))] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] + 5*b^(2/3)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(8/3)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {895, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$
$$\downarrow 819$$
$$\frac{5\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}\,d(c+dx)}{3a} + \frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 847$$

$$\frac{5\left(-\frac{b\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 750$$

$$\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 16$$

$$\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}$$
$$d$$

$$\downarrow 1142$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 25$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}}{d}$$

$$\downarrow 27$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}\right)}{3a}}{d}$$

$$\downarrow 1082$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}}{d}$$

$$\downarrow 217$$

$$\frac{\left(5\left(-\frac{\left(b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}_{(c+dx)}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3a^{2/3}\sqrt[3]{b}}\right)\right)}{a}\right)-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+}}{d}$$

$$\downarrow 1103$$

$$\frac{\left(5\left(-\frac{\left(b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3a^{2/3}\sqrt[3]{b}}\right)\right)}{a}\right)-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c-}}{d}$$

input
```
Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(3*a*(c + d*x)^2*(a + b*(c + d*x)^3)) + (5*(-1/2*1/(a*(c + d*x)^2) - (b
*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3))))
/a))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

method	result
default	$-\frac{1}{2a^2d(xd+c)^2}-\frac{b\left(\frac{\frac{x}{3}+\frac{c}{3d}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+}\right.}{9bd}\right.}{a^2}$
risch	$\frac{-\frac{5b\,d^2x^3}{6a^2}-\frac{5bcd\,x^2}{2a^2}-\frac{5bx\,c^2}{2a^2}-\frac{5c^3b+3a}{6d\,a^2}}{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)(xd+c)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^8d^3\_Z^3+b^2\right)}\_R\ln\left(\left(-4a^8d^4\_R^3-3d\,b^2\right)x-4a^8c\,d^3\_R^3-a\right.\right.}{9}$

input
```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/d/(d*x+c)^2-b/a^2*((1/3*x+1/3*c/d)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c
^2*d*x+b*c^3+a)+5/9/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_
Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(147) = 294$.

Time = 0.12 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.71

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx =$$

$$-\frac{15\,bd^3x^3+45\,bcd^2x^2+45\,bc^2dx+15\,bc^3-10\sqrt{3}(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+bc^5+(10\,bc^3+a)d}{}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
-1/18*(15*b*d^3*x^3 + 45*b*c*d^2*x^2 + 45*b*c^2*d*x + 15*b*c^3 - 10*sqrt(3
)*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d
^2*x^2 + a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqr
t(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*d^5*x^5 + 5*b*c
*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2 + (5*
b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2 + a^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 10*(b*d^
5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2
+ a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^
2/a^2)^(1/3)) + 9*a)/(a^2*b*d^6*x^5 + 5*a^2*b*c*d^5*x^4 + 10*a^2*b*c^2*d^4
*x^3 + (10*a^2*b*c^3 + a^3)*d^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*x + (a^2
*b*c^5 + a^3*c^2)*d)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{-3a-5bc^3-15bc^2dx-15bcd^2x^2-5bd^3x^3}{6a^3c^2d+6a^2bc^5d+60a^2bc^2d^4x^3+30a^2bcd^5x^4+6a^2bd^6x^5+x^2\cdot(6a^3d^3+60a^2bc^3d^3)+x\,(12a^3cd^2+30}$$
$$+\frac{\text{RootSum}\left(729t^3a^8+125b^2,\left(t\mapsto t\log\left(x+\frac{-9ta^3+5bc}{5bd}\right)\right)\right)}{d}$$

input

```
integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-3*a - 5*b*c**3 - 15*b*c**2*d*x - 15*b*c*d**2*x**2 - 5*b*d**3*x**3)/(6*a*
*3*c**2*d + 6*a**2*b*c**5*d + 60*a**2*b*c**2*d**4*x**3 + 30*a**2*b*c*d**5*
x**4 + 6*a**2*b*d**6*x**5 + x**2*(6*a**3*d**3 + 60*a**2*b*c**3*d**3) + x*(
12*a**3*c*d**2 + 30*a**2*b*c**4*d**2)) + RootSum(729*_t**3*a**8 + 125*b**2
, Lambda(_t, _t*log(x + (-9*_t*a**3 + 5*b*c)/(5*b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^2 (dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/6*(5*b*d^3*x^3 + 15*b*c*d^2*x^2 + 15*b*c^2*d*x + 5*b*c^3 + 3*a)/(a^2*b*
d^6*x^5 + 5*a^2*b*c*d^5*x^4 + 10*a^2*b*c^2*d^4*x^3 + (10*a^2*b*c^3 + a^3)*
d^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*x + (a^2*b*c^5 + a^3*c^2)*d) - 5/3*b
*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{5\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)\right.\right.}{18\,a^2}$$
$$-\frac{bdx+bc}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)a^2d}-\frac{1}{2\left(dx+c\right)^2a^2d}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
5/18*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d
^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2
+ 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-
b*d*x - b*c + (-a*b^2)^(1/3))))/a^2 - 1/3*(b*d*x + b*c)/((b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a^2*d) - 1/2/((d*x + c)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.36

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{\frac{5\,b\,c^3+3\,a}{6\,a^2\,d}+\frac{5\,b\,d^2\,x^3}{6\,a^2}+\frac{5\,b\,c^2\,x}{2\,a^2}+\frac{5\,b\,c\,d\,x^2}{2\,a^2}}{x^2\,\left(10\,b\,c^3\,d^2+a\,d^2\right)+a\,c^2+b\,c^5+x\,\left(5\,b\,d\,c^4+2\,a\,d\,c\right)+b\,d^5\,x^5+10\,b\,c^2\,d^3\,x^3+5\,b\,c\,d^4\,x^4}$$
$$-\frac{5\,b^{2/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{8/3}\,d}$$
$$+\frac{5\,b^{2/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}\,d}$$
$$-\frac{5\,b^{2/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^3),x)
```

output
```
(5*b^(2/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)
*((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3)*d) - (5*b^(2/3)*log(b^(1/3)*c + a^(1/3
) + b^(1/3)*d*x))/(9*a^(8/3)*d) - ((3*a + 5*b*c^3)/(6*a^2*d) + (5*b*d^2*x^
3)/(6*a^2) + (5*b*c^2*x)/(2*a^2) + (5*b*c*d*x^2)/(2*a^2))/(x^2*(a*d^2 + 10
*b*c^3*d^2) + a*c^2 + b*c^5 + x*(2*a*c*d + 5*b*c^4*d) + b*d^5*x^5 + 10*b*c
^2*d^3*x^3 + 5*b*c*d^4*x^4) - (5*b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3
)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1330, normalized size of antiderivative = 7.07

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^2,x)
```

output

```
(10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a*b*c**2 + 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c*d*x + 10*a**(1/3)*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*d**2
*x**2 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x
)/(a**(1/3)*sqrt(3)))*b**2*c**5 + 50*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*d*x + 100*a**(1/
3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*b**2*c**3*d**2*x**2 + 100*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**3*x**3 + 50*a**(1/3
)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3
)))*b**2*c*d**4*x**4 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*d**5*x**5 + 5*a**(1/3)*log(a**(2
/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(
2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c**2 + 10*a**(1/3)*log(a**(2/3) - b**
(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*
x + b**(2/3)*d**2*x**2)*a*b*c*d*x + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2
/3)*d**2*x**2)*a*b*d**2*x**2 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)
*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3...
```

3.75 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3a^2d(c+dx)^3} - \frac{b}{3a^2d\left(a+b(c+dx)^3\right)} - \frac{2b\log(c+dx)}{a^3d} + \frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3d}$$

output

```
-1/3/a^2/d/(d*x+c)^3-1/3*b/a^2/d/(a+b*(d*x+c)^3)-2*b*ln(d*x+c)/a^3/d+2/3*b
*ln(a+b*(d*x+c)^3)/a^3/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{a\left(\frac{1}{(c+dx)^3}+\frac{b}{a+b(c+dx)^3}\right)+6b\log(c+dx)-2b\log\left(a+b(c+dx)^3\right)}{3a^3d}$$

input

```
Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)^2),x]
```

output
```
-1/3*(a*((c + d*x)^(-3) + b/(a + b*(c + d*x)^3)) + 6*b*Log[c + d*x] - 2*b*
Log[a + b*(c + d*x)^3])/(a^3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^4 \left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)^2}\,d(c+dx)}{d}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)^2}\,d(c+dx)^3}{3d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{2b^2}{a^3(b(c+dx)^3+a)} + \frac{b^2}{a^2(b(c+dx)^3+a)^2} - \frac{2b}{a^3(c+dx)^3} + \frac{1}{a^2(c+dx)^6}\right) d(c+dx)^3}{3d}$$

$$\downarrow 2009$$

$$\frac{-\frac{2b\log\left((c+dx)^3\right)}{a^3} + \frac{2b\log\left(a+b(c+dx)^3\right)}{a^3} - \frac{b}{a^2(a+b(c+dx)^3)} - \frac{1}{a^2(c+dx)^3}}{3d}$$

input
```
Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)^2),x]
```

output
```
(-(1/(a^2*(c + d*x)^3)) - b/(a^2*(a + b*(c + d*x)^3)) - (2*b*Log[(c + d*x)
^3])/a^3 + (2*b*Log[a + b*(c + d*x)^3])/a^3)/(3*d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

method	result
default	$-\frac{1}{3a^2d(xd+c)^3}-\frac{2b\ln(xd+c)}{a^3d}+\frac{b^2\left(-\frac{a}{3db\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}+\frac{2\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{3bd}\right)}{a^3}$
risch	$\frac{-\frac{2bd^2x^3}{3a^2}-\frac{2bcdx^2}{a^2}-\frac{2bxc^2}{a^2}-\frac{2c^3b+a}{3da^2}}{(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}-\frac{2b\ln(xd+c)}{a^3d}+\frac{2b\ln(-bd^3x^3-3bcd^2x^2-3dbxc^2-c^3b-a)}{3a^3d}$
norman	$\frac{-\frac{2bcdx^2}{a^2}-\frac{2bxc^2}{a^2}+\frac{-2b^2c^3d^5-abd^5}{3a^2d^6b}-\frac{2bd^2x^3}{3a^2}}{(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}-\frac{2b\ln(xd+c)}{a^3d}+\frac{2b\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3a^3d}$
parallelrisch	$-\frac{18\ln(xd+c)x^2ab^2cd^7-6\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)x^2ab^2cd^7+18\ln(xd+c)xab^2c^2d^6-6\ln(bd^3x^3+3bcd^2x^2+$

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/d/(d*x+c)^3-2*b*ln(d*x+c)/a^3/d+b^2/a^3*(-1/3*a/d/b/(b*d^3*x^3+3*
b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+2/3/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^
2*d*x+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(74) = 148$.

Time = 0.08 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.39

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+a^2-2\,(b^2d^6x^6+6\,b^2cd^5x^5+15\,b^2c^2d^4x^4+b^2c^6+(20\,b^2c^3$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2 -
2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*
c^3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^
5 + a*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)
 + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b
^2*c^3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2
*c^5 + a*b*c^2)*d*x)*log(d*x + c))/(a^3*b*d^7*x^6 + 6*a^3*b*c*d^6*x^5 + 15
*a^3*b*c^2*d^5*x^4 + (20*a^3*b*c^3 + a^4)*d^4*x^3 + 3*(5*a^3*b*c^4 + a^4*c
)*d^3*x^2 + 3*(2*a^3*b*c^5 + a^4*c^2)*d^2*x + (a^3*b*c^6 + a^4*c^3)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(70) = 140$.

Time = 2.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.12

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-a-2bc^3-6bc^2dx-6bcd^2x^2-2bd^3x^3}{3a^3c^3d+3a^2bc^6d+45a^2bc^2d^5x^4+18a^2bcd^6x^5+3a^2bd^7x^6+x^3\cdot(3a^3d^4+60a^2bc^3d^4)+x^2\cdot(9a^3cd^3+45}$$
$$-\frac{2b\log\left(\frac{c}{d}+x\right)}{a^3d}+\frac{2b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^3d}$$

input
```
integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3)**2,x)
```

output
```
(-a - 2*b*c**3 - 6*b*c**2*d*x - 6*b*c*d**2*x**2 - 2*b*d**3*x**3)/(3*a**3*c
**3*d + 3*a**2*b*c**6*d + 45*a**2*b*c**2*d**5*x**4 + 18*a**2*b*c*d**6*x**5
 + 3*a**2*b*d**7*x**6 + x**3*(3*a**3*d**4 + 60*a**2*b*c**3*d**4) + x**2*(9
*a**3*c*d**3 + 45*a**2*b*c**4*d**3) + x*(9*a**3*c**2*d**2 + 18*a**2*b*c**5
*d**2)) - 2*b*log(c/d + x)/(a**3*d) + 2*b*log(3*c**2*x/d**2 + 3*c*x**2/d +
 x**3 + (a + b*c**3)/(b*d**3))/(3*a**3*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(74) = 148.

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.78

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+a}{3\left(a^2bd^7x^6+6\,a^2bcd^6x^5+15\,a^2bc^2d^5x^4+(20\,a^2bc^3+a^3)d^4x^3+3\,(5\,a^2bc^4+a^3c)d^3x^2+3\,(2\,a^2bc^5+a^\right.}$$
$$+\frac{2\,b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^3d}-\frac{2\,b\log\left(dx+c\right)}{a^3d}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + a)/(a^2*b*d^7*
x^6 + 6*a^2*b*c*d^6*x^5 + 15*a^2*b*c^2*d^5*x^4 + (20*a^2*b*c^3 + a^3)*d^4*
x^3 + 3*(5*a^2*b*c^4 + a^3*c)*d^3*x^2 + 3*(2*a^2*b*c^5 + a^3*c^2)*d^2*x +
(a^2*b*c^6 + a^3*c^3)*d) + 2/3*b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3 + a)/(a^3*d) - 2*b*log(d*x + c)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = \frac{2\,b\log\left(\left|-b-\frac{a}{(dx+c)^3}\right|\right)}{3\,a^3 d} + \frac{b^2}{3\,a^3\left(b+\frac{a}{(dx+c)^3}\right)d} - \frac{1}{3\,(dx+c)^3 a^2 d}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
2/3*b*log(abs(-b - a/(d*x + c)^3))/(a^3*d) + 1/3*b^2/(a^3*(b + a/(d*x + c)
^3)*d) - 1/3/((d*x + c)^3*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.64

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = \frac{2\,b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d} - \frac{\frac{2\,b\,c^3+a}{3\,a^2\,d}+\frac{2\,b\,d^2\,x^3}{3\,a^2}+\frac{2\,b\,c^2\,x}{a^2}+\frac{2\,b\,c\,d\,x^2}{a^2}}{x\,\left(6\,b\,d\,c^5+3\,a\,d\,c^2\right)+x^3\,\left(20\,b\,c^3\,d^3+a\,d^3\right)+a\,c^3+b\,c^6+x^2\,\left(15\,b\,c^4\,d^2+3\,a\,c\,d^2\right)+b\,d^6\,x^6+15\,b\,c^2} - \frac{2\,b\,\ln\left(c+d\,x\right)}{a^3\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)^2*(c + d*x)^4),x)
```

output
```
(2*b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^3*d) -
 ((a + 2*b*c^3)/(3*a^2*d) + (2*b*d^2*x^3)/(3*a^2) + (2*b*c^2*x)/a^2 + (2*b
*c*d*x^2)/a^2)/(x*(3*a*c^2*d + 6*b*c^5*d) + x^3*(a*d^3 + 20*b*c^3*d^3) + a
*c^3 + b*c^6 + x^2*(15*b*c^4*d^2 + 3*a*c*d^2) + b*d^6*x^6 + 15*b*c^2*d^4*x
^4 + 6*b*c*d^5*x^5) - (2*b*log(c + d*x))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1293, normalized size of antiderivative = 16.16

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^2,x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c**3 + 6*log(a**(2/3) - b
**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*
d*x + b**(2/3)*d**2*x**2)*a*b*c**2*d*x + 6*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*a*b*c*d**2*x**2 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a
*b*d**3*x**3 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**6 + 12*
log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**5*d*x + 30*log(a**(2/3)
- b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)
*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**4*d**2*x**2 + 40*log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b**2*c**3*d**3*x**3 + 30*log(a**(2/3) - b**(1/3)*a**
(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2
/3)*d**2*x**2)*b**2*c**2*d**4*x**4 + 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c
 - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**
2*x**2)*b**2*c*d**5*x**5 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*...
```

3.76 $\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}d} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}$$

output

```
-1/6*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)^2+1/9*(d*x+c)^2/a/b/d/(a+b*(d*x+c)^3)-1
/27*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3
)/b^(5/3)/d-1/27*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/b^(5/3)/d+1/54*ln(a^(
2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{-\frac{9b^{2/3}(c+dx)^2}{(a+b(c+dx)^3)^2} + \frac{6b^{2/3}(c+dx)^2}{a(a+b(c+dx)^3)} + \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{4/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)\right.}{a^{4/3}}}{54b^{5/3}d}$$

input

```
Integrate[(c + d*x)^4/(a + b*(c + d*x)^3)^3,x]
```

output

```
((-9*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (6*b^(2/3)*(c + d*x)^2)/
(a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x
))/(Sqrt[3]*a^(1/3))])/a^(4/3) - (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(4
/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(4/
3))/(54*b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {895, 817, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{(c+dx)^4}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{\frac{\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 819$$

$$\frac{\frac{\frac{\int \frac{c+dx}{b(c+dx)^3+a}\,d(c+dx)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 821$$

$$\frac{\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}\,d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-$$

$$\downarrow 25$$

$$\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}\,d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}$$
$$d$$

$$\downarrow 1082$$

$$\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-6$$
$$d$$

$$\downarrow 217$$

$$\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}$$
$$d$$

$$\downarrow 1103$$

$$\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}$$
$$d$$

input
```
Int[(c + d*x)^4/(a + b*(c + d*x)^3)^3,x]
```

output
```
(-1/6*(c + d*x)^2/(b*(a + b*(c + d*x)^3)^2) + ((c + d*x)^2/(3*a*(a + b*(c
+ d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-
((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3))
)/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x],
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{d^4x^5}{9a}+\frac{5cd^3x^4}{9a}+\frac{10c^2d^2x^3}{9a}-\frac{d\left(-20c^3b+a\right)x^2}{18ba}-\frac{c\left(-5c^3b+a\right)x}{9ba}-\frac{c^2\left(-2c^3b+a\right)}{18bda}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}}{27ab^2d}$
risch	$\frac{\frac{d^4x^5}{9a}+\frac{5cd^3x^4}{9a}+\frac{10c^2d^2x^3}{9a}-\frac{d\left(-20c^3b+a\right)x^2}{18ba}-\frac{c\left(-5c^3b+a\right)x}{9ba}-\frac{c^2\left(-2c^3b+a\right)}{18bda}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}}{27ab^2d}$

input
```
int((d*x+c)^4/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*d^4/a*x^5+5/9*c*d^3/a*x^4+10/9*c^2*d^2/a*x^3-1/18/b*d*(-20*b*c^3+a)/a
*x^2-1/9/b*c*(-5*b*c^3+a)/a*x-1/18/b*c^2/d*(-2*b*c^3+a)/a)/(b*d^3*x^3+3*b*
c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/27/a/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*
c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(164) = 328$.

Time $= 0.12$ (sec) , antiderivative size $= 1706$, normalized size of antiderivative $= 8.32$

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
[1/54*(6*a*b^3*d^5*x^5 + 30*a*b^3*c*d^4*x^4 + 60*a*b^3*c^2*d^3*x^3 + 6*a*b
^3*c^5 - 3*a^2*b^2*c^2 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*x^2 + 6*(5*a*b^3*c
^4 - a^2*b^2*c)*d*x + 3*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*
a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)
*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 +
a^2*b^2*c^2)*d*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*
x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(
d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^
(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a)) + (b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4
+ b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*
b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/
3)) - 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*
(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 +
 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2
)^(1/3)))/(a^2*b^5*d^7*x^6 + 6*a^2*b^5*c*d^6*x^5 + 15*a^2*b^5*c^2*d^5*x^4
+ 2*(10*a^2*b^5*c^3 + a^3*b^4)*d^4*x^3 + 3*(5*a^2*b^5*c^4 + 2*a^3*b^4*c)*d
^3*x^2 + 6*(a^2*b^5*c^5 + a^3*b^4*c^2)*d^2*x + (a^2*b^5*c^6 + 2*a^3*b^4*c^
3 + a^4*b^3)*d), 1/54*(6*a*b^3*d^5*x^5 + 30*a*b^3*c*d^4*x^4 + 60*a*b^3*...
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-ac^2 + 2bc^5 + 20bc^2d^3x^3 + 10bcd^4x^4 + 2bd^5x^5 + x^2(-ad^2 + 20}{18a^3bd + 36a^2b^2c^3d + 18ab^3c^6d + 270ab^3c^2d^5x^4 + 108ab^3cd^6x^5 + 18ab^3d^7x^6 + x^3\cdot(36a^2b^2d^4 + 360ab^3c^3} + \frac{\operatorname{RootSum}\left(19683t^3a^4b^5 + 1, \left(t \mapsto t\log\left(x + \frac{729t^2a^3b^3 + c}{d}\right)\right)\right)}{d}$$

input

```
integrate((d*x+c)**4/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-a*c**2 + 2*b*c**5 + 20*b*c**2*d**3*x**3 + 10*b*c*d**4*x**4 + 2*b*d**5*x*
*5 + x**2*(-a*d**2 + 20*b*c**3*d**2) + x*(-2*a*c*d + 10*b*c**4*d))/(18*a**
3*b*d + 36*a**2*b**2*c**3*d + 18*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4
 + 108*a*b**3*c*d**6*x**5 + 18*a*b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4
+ 360*a*b**3*c**3*d**4) + x**2*(108*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**
3) + x*(108*a**2*b**2*c**2*d**2 + 108*a*b**3*c**5*d**2)) + RootSum(19683*_
t**3*a**4*b**5 + 1, Lambda(_t, _t*log(x + (729*_t**2*a**3*b**3 + c)/d)))/d
```

Maxima [F]

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx = \int \frac{(dx+c)^4}{\left((dx+c)^3 b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/18*(2*b*d^5*x^5 + 10*b*c*d^4*x^4 + 20*b*c^2*d^3*x^3 + 2*b*c^5 + (20*b*c^
3 - a)*d^2*x^2 - a*c^2 + 2*(5*b*c^4 - a*c)*d*x)/(a*b^3*d^7*x^6 + 6*a*b^3*c
*d^6*x^5 + 15*a*b^3*c^2*d^5*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(
5*a*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (
a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*d) + 1/9*integrate((d*x + c)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{54\,ab}$$

$$+\frac{2\,bd^5x^5+10\,bcd^4x^4+20\,bc^2d^3x^3+20\,bc^3d^2x^2+10\,bc^4dx+2\,bc^5-ad^2x^2-2\,acdx-ac^2}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2abd}$$

input
```
integrate((d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/54*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*
a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*
b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3
))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/(a*b) + 1/18*(2*b*d^5
*x^5 + 10*b*c*d^4*x^4 + 20*b*c^2*d^3*x^3 + 20*b*c^3*d^2*x^2 + 10*b*c^4*d*x
 + 2*b*c^5 - a*d^2*x^2 - 2*a*c*d*x - a*c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{\frac{d^4x^5}{9a} - \frac{ac^2-2bc^5}{18abd} + \frac{5cd^3x^4}{9a} + \frac{10c^2d^2x^3}{9a} - \frac{cx\,(a-5bc^3)}{9ab} - \frac{dx^2\,(a-20bc^3)}{18ab}}{x^3\,(20b^2c^3d^3+2abd^3)+x^2\,(15b^2c^4d^2+6abcd^2)+a^2+x\,(6db^2c^5+6adbc^2)+b^2c^6+b^2d^6x^6+}$$
$$+\frac{\ln\left(a^{1/3}b+(-b)^{4/3}c+(-b)^{4/3}dx\right)}{27a^{4/3}(-b)^{5/3}d}$$
$$+\frac{\ln\left(\frac{cd^4}{81a^2b}+\frac{d^5x}{81a^2b}+\frac{d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{324a^{5/3}(-b)^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{54a^{4/3}(-b)^{5/3}d}$$
$$-\frac{\ln\left(\frac{cd^4}{81a^2b}+\frac{d^5x}{81a^2b}+\frac{d^4\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{324a^{5/3}(-b)^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{54a^{4/3}(-b)^{5/3}d}$$

input
```
int((c + d*x)^4/(a + b*(c + d*x)^3)^3,x)
```

output

```
((d^4*x^5)/(9*a) - (a*c^2 - 2*b*c^5)/(18*a*b*d) + (5*c*d^3*x^4)/(9*a) + (1
0*c^2*d^2*x^3)/(9*a) - (c*x*(a - 5*b*c^3))/(9*a*b) - (d*x^2*(a - 20*b*c^3)
)/(18*a*b))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*
b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2
*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) + log(a^(1/3)*b + (-b)^(4
/3)*c + (-b)^(4/3)*d*x)/(27*a^(4/3)*(-b)^(5/3)*d) + (log((c*d^4)/(81*a^2*b
) + (d^5*x)/(81*a^2*b) + (d^4*(3^(1/2)*1i - 1)^2)/(324*a^(5/3)*(-b)^(4/3))
)*(3^(1/2)*1i - 1))/(54*a^(4/3)*(-b)^(5/3)*d) - (log((c*d^4)/(81*a^2*b) +
(d^5*x)/(81*a^2*b) + (d^4*(3^(1/2)*1i + 1)^2)/(324*a^(5/3)*(-b)^(4/3)))*(3
^(1/2)*1i + 1))/(54*a^(4/3)*(-b)^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1825, normalized size of antiderivative = 8.90

$$\int \frac{(c+dx)^4}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^4/(a+b*(d*x+c)^3)^3,x)
```

output

```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*a**2*b*c - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4 - 12*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**3*d*x - 12*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*
*2*d**2*x**2 - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*a*b**2*c*d**3*x**3 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7 - 12*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**6*d*x
 - 30*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b**3*c**5*d**2*x**2 - 40*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2
*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**3*x**3 - 30*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**3*d*
*4*x**4 - 12*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*b**3*c**2*d**5*x**5 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**6*x**6 - b**(2/3)*a**(1
/3)*a**2 - 5*b**(2/3)*a**(1/3)*a*b*c**3 - 12*b**(2/3)*a**(1/3)*a*b*c**2*d*
x - 9*b**(2/3)*a**(1/3)*a*b*c*d**2*x**2 - 2*b**(2/3)*a**(1/3)*a*b*d**3*x**
3 + 5*b**(2/3)*a**(1/3)*b**2*c**6 + 24*b**(2/3)*a**(1/3)*b**2*c**5*d*x + 4
5*b**(2/3)*a**(1/3)*b**2*c**4*d**2*x**2 + 40*b**(2/3)*a**(1/3)*b**2*c**...
```

3.77 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 586
Mathematica [A] (verified) . . . 587
Rubi [A] (verified) . . . 587
Maple [C] (verified) . . . 592
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Giac [A] (verification not implemented) . . . 595
Mupad [B] (verification not implemented) . . . 595
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Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{c+dx}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{c+dx}{18abd\left(a+b(c+dx)^3\right)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}$$

output

```
-1/6*(d*x+c)/b/d/(a+b*(d*x+c)^3)^2+1/18*(d*x+c)/a/b/d/(a+b*(d*x+c)^3)-1/27
*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b
^(4/3)/d+1/27*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/b^(4/3)/d-1/54*ln(a^(2/3
)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-\frac{9\sqrt[3]{b}(c+dx)}{(a+b(c+dx)^3)^2}+\frac{3\sqrt[3]{b}(c+dx)}{a(a+b(c+dx)^3)}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)\right.}{a^{5/3}}}{54b^{4/3}d}$$

input
```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^3)^3,x]
```

output
```
((-9*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (3*b^(1/3)*(c + d*x))/(a*(
a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(
Sqrt[3]*a^(1/3))])/a^(5/3) + (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(5/3)
- Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(5/3))/
(54*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {895, 817, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{(c+dx)^3}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{\frac{\int \frac{1}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 749$$

$$\frac{\frac{2\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 750$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 16$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 1142$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)\right.}}{6b}$$

$$\downarrow 25$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}}{d}$$

$$\downarrow 27$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}}{d}$$

$$\downarrow 1082$$

$$\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}}{d}$$

$$\downarrow 217$$

$$\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}-\frac{c+dx}{6b(a+b(c+dx)^3}}{6b}}{d}$$

$$\downarrow 1103$$

$$\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}}{6b}}{d}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^3)^3,x]
```

output
```
(-1/6*(c + d*x)/(b*(a + b*(c + d*x)^3)^2) + ((c + d*x)/(3*a*(a + b*(c + d*
x)^3)) + (2*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[
a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3
*a^(2/3))))/(3*a))/(6*b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{d^3x^4}{18a}+\frac{2cd^2x^3}{9a}+\frac{c^2dx^2}{3a}-\frac{\left(-2c^3b+a\right)x}{9ba}-\frac{c\left(-c^3b+2a\right)}{18bda}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{27ab^2d}$
risch	$\frac{\frac{d^3x^4}{18a}+\frac{2cd^2x^3}{9a}+\frac{c^2dx^2}{3a}-\frac{\left(-2c^3b+a\right)x}{9ba}-\frac{c\left(-c^3b+2a\right)}{18bda}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{27ab^2d}$

input

```
int((d*x+c)^3/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output

```
(1/18/a*d^3*x^4+2/9*c/a*d^2*x^3+1/3*c^2/a*d*x^2-1/9/b*(-2*b*c^3+a)/a*x-1/1
8/b*c/d*(-b*c^3+2*a)/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/
27/a/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_
Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(160) = 320$.

Time = 0.12 (sec) , antiderivative size = 1650, normalized size of antiderivative = 8.21

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
[1/54*(3*a^2*b^2*d^4*x^4 + 12*a^2*b^2*c*d^3*x^3 + 18*a^2*b^2*c^2*d^2*x^2 +
 3*a^2*b^2*c^4 - 6*a^3*b*c + 6*(2*a^2*b^2*c^3 - a^3*b)*d*x + 3*sqrt(1/3)*(
a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a
^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b
^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt(-(a^2*b)^(1/
3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a
^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*
(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d
*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b^2*d
^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 +
a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 +
a*b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2
- (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b^2*d^6*x^6 + 6*b^2*c*d^
5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*
b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)
*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^3*b^4*d^7*x^6 + 6*
a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*
d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b
^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b^3*c^3 + a^5*b^2)*d), 1/54*(3*a^2*b^
2*d^4*x^4 + 12*a^2*b^2*c*d^3*x^3 + 18*a^2*b^2*c^2*d^2*x^2 + 3*a^2*b^2*c...
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-2ac+bc^4+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4+x(-}{18a^3bd+36a^2b^2c^3d+18ab^3c^6d+270ab^3c^2d^5x^4+108ab^3cd^6x^5+18ab^3d^7x^6+x^3\cdot(36a^2b^2d^4+360ab^3c^3}$$
$$+\frac{\text{RootSum}\left(19683t^3a^5b^4-1,\left(t\mapsto t\log\left(x+\frac{27ta^2b+c}{d}\right)\right)\right)}{d}$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-2*a*c + b*c**4 + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4 + x*
(-2*a*d + 4*b*c**3*d))/(18*a**3*b*d + 36*a**2*b**2*c**3*d + 18*a*b**3*c**6
*d + 270*a*b**3*c**2*d**5*x**4 + 108*a*b**3*c*d**6*x**5 + 18*a*b**3*d**7*x
**6 + x**3*(36*a**2*b**2*d**4 + 360*a*b**3*c**3*d**4) + x**2*(108*a**2*b**
2*c*d**3 + 270*a*b**3*c**4*d**3) + x*(108*a**2*b**2*c**2*d**2 + 108*a*b**3
*c**5*d**2)) + RootSum(19683*_t**3*a**5*b**4 - 1, Lambda(_t, _t*log(x + (2
7*_t*a**2*b + c)/d)))/d
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{(dx+c)^3}{\left((dx+c)^3b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/18*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + 2*(2*b*c^3 - a
)*d*x - 2*a*c)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 15*a*b^3*c^2*d^5*x^4 +
 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^
2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b
)*d) + 1/9*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 +
a), x)/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2+4\left(bdx\right.\right.}{54\,ab}$$

$$+\frac{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4-2\,adx-2\,ac}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2abd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/54*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x +
b*c + (a*b^2)^(1/3))))/(a*b) + 1/18*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d
^2*x^2 + 4*b*c^3*d*x + b*c^4 - 2*a*d*x - 2*a*c)/((b*d^3*x^3 + 3*b*c*d^2*x^
2 + 3*b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{d^3x^4}{18\,a}+\frac{c^2dx^2}{3\,a}+\frac{2\,cd^2x^3}{9\,a}-\frac{x\left(a-2\,bc^3\right)}{9\,ab}-\frac{2\,ac-bc^4}{18\,abd}}{x^3\left(20\,b^2c^3d^3+2\,ab\,d^3\right)+x^2\left(15\,b^2c^4d^2+6\,ab\,cd^2\right)+a^2+x\left(6\,db^2c^5+6\,adbc^2\right)+b^2c^6+b^2d^6x^6+}$$

$$+\frac{\ln\left(b^{1/3}c+a^{1/3}+b^{1/3}dx\right)}{27\,a^{5/3}b^{4/3}d}+\frac{\ln\left(\frac{bcd^5}{3\,a}+\frac{bd^6x}{3\,a}+\frac{b^{2/3}d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{5/3}b^{4/3}d}$$

$$-\frac{\ln\left(\frac{bcd^5}{3\,a}+\frac{bd^6x}{3\,a}-\frac{b^{2/3}d^5\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{5/3}b^{4/3}d}$$

input
```
int((c + d*x)^3/(a + b*(c + d*x)^3)^3,x)
```

output
```
((d^3*x^4)/(18*a) + (c^2*d*x^2)/(3*a) + (2*c*d^2*x^3)/(9*a) - (x*(a - 2*b*
c^3))/(9*a*b) - (2*a*c - b*c^4)/(18*a*b*d))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d
^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^
2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^
4*x^4) + log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(27*a^(5/3)*b^(4/3)*d) + (
log((b*c*d^5)/(3*a) + (b*d^6*x)/(3*a) + (b^(2/3)*d^5*(3^(1/2)*1i - 1))/(6*
a^(2/3)))*(3^(1/2)*1i - 1))/(54*a^(5/3)*b^(4/3)*d) - (log((b*c*d^5)/(3*a)
+ (b*d^6*x)/(3*a) - (b^(2/3)*d^5*(3^(1/2)*1i + 1))/(6*a^(2/3)))*(3^(1/2)*1
i + 1))/(54*a^(5/3)*b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1793, normalized size of antiderivative = 8.92

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3/(a+b*(d*x+c)^3)^3,x)
```

output

```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*a**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**3 - 12*a**(1/3)*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**2*d*x
 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*a*b*c*d**2*x**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*d**3*x**3 - 2*a**(1/3)
*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)
))*b**2*c**6 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1
/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**5*d*x - 30*a**(1/3)*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*d**2*
x**2 - 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)
/(a**(1/3)*sqrt(3)))*b**2*c**3*d**3*x**3 - 30*a**(1/3)*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**4*x*
*4 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*b**2*c*d**5*x**5 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) -
2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*d**6*x**6 - a**(1/
3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2 - 2*a**(1/3)*log(a**(2/3
) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**...
```

3.78 $\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result 598
Mathematica [A] (verified) 598
Rubi [A] (verified) 599
Maple [A] (verified) 600
Fricas [B] (verification not implemented) 600
Sympy [B] (verification not implemented) 601
Maxima [A] (verification not implemented) 601
Giac [A] (verification not implemented) 602
Mupad [B] (verification not implemented) 602
Reduce [B] (verification not implemented) 602

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}$$

output
```
-1/6/b/d/(a+b*(d*x+c)^3)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{1}{6bd\left(a+b(c+dx)^3\right)^2}$$

input
```
Integrate[(c + d*x)^2/(a + b*(c + d*x)^3)^3,x]
```

output
```
-1/6*1/(b*d*(a + b*(c + d*x)^3)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^2}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$
$$\downarrow 793$$
$$-\frac{1}{6bd\,(a+b(c+dx)^3)^2}$$

input
```
Int[(c + d*x)^2/(a + b*(c + d*x)^3)^3,x]
```

output
```
-1/6*1/(b*d*(a + b*(c + d*x)^3)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{6bd\left(a+b(xd+c)^3\right)^2}$	22
gosper	$-\frac{1}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	44
default	$-\frac{1}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	44
norman	$-\frac{1}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	44
risch	$-\frac{1}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	44
parallelrisch	$-\frac{1}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	44
orering	$-\frac{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}{6bd\left(a+b(xd+c)^3\right)^3}$	55

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b/d/(a+b*(d*x+c)^3)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 5.78

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{1}{6\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+2\left(10\,b^3c^3+ab^2\right)d^4x^3+3\left(5\,b^3c^4+2\,ab^2c\right)d^3x^2+6\left(b^3c^5+ab^2c^2\right.\right.}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-1/6/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c^3 +
 a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b^2*c
^2)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(19) = 38$.

Time = 1.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.65

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{1}{6a^2bd + 12ab^2c^3d + 6b^3c^6d + 90b^3c^2d^5x^4 + 36b^3cd^6x^5 + 6b^3d^7x^6 + x^3\cdot(12ab^2d^4 + 120b^3c^3d^4) + x^2\cdot(3$$

input
```
integrate((d*x+c)**2/(a+b*(d*x+c)**3)**3,x)
```

output
```
-1/(6*a**2*b*d + 12*a*b**2*c**3*d + 6*b**3*c**6*d + 90*b**3*c**2*d**5*x**4
 + 36*b**3*c*d**6*x**5 + 6*b**3*d**7*x**6 + x**3*(12*a*b**2*d**4 + 120*b**
3*c**3*d**4) + x**2*(36*a*b**2*c*d**3 + 90*b**3*c**4*d**3) + x*(36*a*b**2*
c**2*d**2 + 36*b**3*c**5*d**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{(a+b(c+dx)^3)^3}\,dx = -\frac{1}{6\left((dx+c)^3b+a\right)^2bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/6/(((d*x + c)^3*b + a)^2*b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{1}{6\left((dx+c)^3b+a\right)^2bd}$$

input
```
integrate((d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/6/(((d*x + c)^3*b + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.70

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx =$$

$$-\frac{1}{6\,b\,d\,\left(x^3\,\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\,\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\,\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\right.}$$

input
```
int((c + d*x)^2/(a + b*(c + d*x)^3)^3,x)
```

output
```
-1/(6*b*d*(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*
c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a
*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 5.74

$$\int \frac{(c+dx)^2}{\left(a+b(c+dx)^3\right)^3}\,dx =$$

$$-\frac{1}{6bd\left(b^2d^6x^6+6b^2c\,d^5x^5+15b^2c^2d^4x^4+20b^2c^3d^3x^3+15b^2c^4d^2x^2+6b^2c^5dx+2ab\,d^3x^3+b^2c^6+6abc\,d\right.}$$

input
```
int((d*x+c)^2/(a+b*(d*x+c)^3)^3,x)
```

output
```
( - 1)/(6*b*d*(a**2 + 2*a*b*c**3 + 6*a*b*c**2*d*x + 6*a*b*c*d**2*x**2 + 2*
a*b*d**3*x**3 + b**2*c**6 + 6*b**2*c**5*d*x + 15*b**2*c**4*d**2*x**2 + 20*
b**2*c**3*d**3*x**3 + 15*b**2*c**2*d**4*x**4 + 6*b**2*c*d**5*x**5 + b**2*d
**6*x**6))
```

3.79 $\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$

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Reduce [B] (verification not implemented) . . . 615

Optimal result

Integrand size = 19, antiderivative size = 202

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}d} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}$$

output

```
1/6*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)^2+2/9*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3)-2/
27*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)
/b^(2/3)/d-2/27*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/b^(2/3)/d+1/27*ln(a^(2
/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{9a^{4/3}(c+dx)^2}{(a+b(c+dx)^3)^2} + \frac{12\sqrt[3]{a}(c+dx)^2}{a+b(c+dx)^3} + \frac{4\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} - \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{b^{2/3}} + \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+\right.}{b^{2/3}}}{54a^{7/3}d}$$

input

```
Integrate[(c + d*x)/(a + b*(c + d*x)^3)^3,x]
```

output

```
((9*a^(4/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (12*a^(1/3)*(c + d*x)^2)/
(a + b*(c + d*x)^3) + (4*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(
Sqrt[3]*a^(1/3))])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(2/3)
+ (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^(2/
3))/(54*a^(7/3)*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {895, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^3}\,d(c+dx)}{d}$$

$$\downarrow 819$$

$$\frac{\frac{2\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 819$$

$$\frac{\frac{2\left(\frac{\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 821$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 16$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 1142$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 25$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}}{d}$$

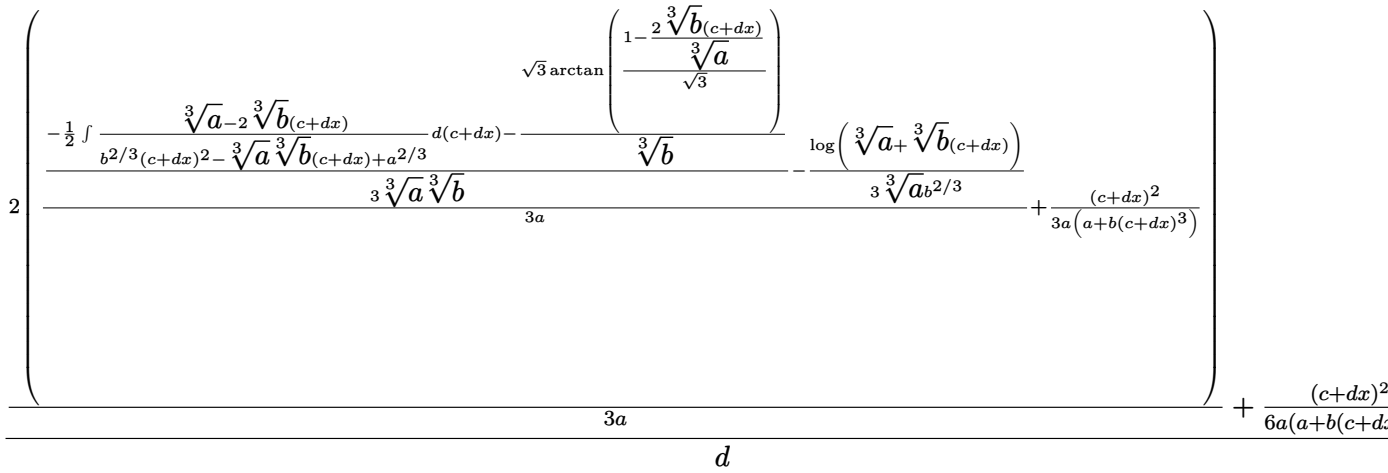
$$\downarrow 27$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}}{d}$$

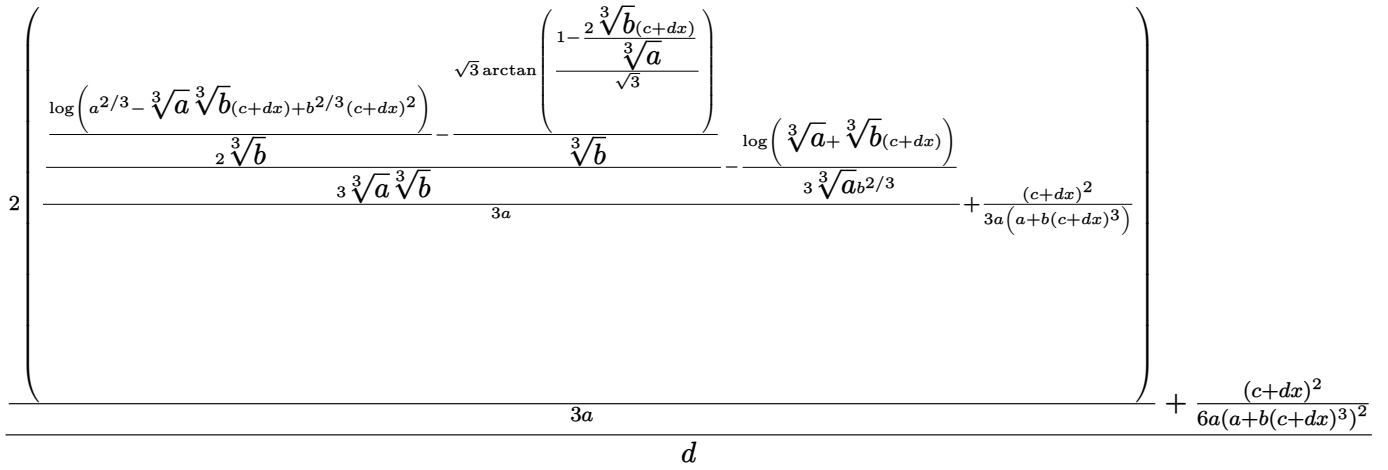
$$\downarrow 1082$$

$$\frac{2\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}}{d}$$

$$\downarrow 217$$



$$\downarrow 1103$$



input
```
Int[(c + d*x)/(a + b*(c + d*x)^3)^3,x]
```

output
```
((c + d*x)^2/(6*a*(a + b*(c + d*x)^3)^2) + (2*((c + d*x)^2/(3*a*(a + b*(c
+ d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-
((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3))
)/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p + 1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])   Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{2b\,d^4x^5}{9a^2}+\frac{10bc\,d^3x^4}{9a^2}+\frac{20b\,c^2d^2x^3}{9a^2}+\frac{d\left(40c^3b+7a\right)x^2}{18a^2}+\frac{c\left(10c^3b+7a\right)x}{9a^2}+\frac{c^2\left(4c^3b+7a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+\right.}\right.}{27a^2bd}$
risch	$\frac{\frac{2b\,d^4x^5}{9a^2}+\frac{10bc\,d^3x^4}{9a^2}+\frac{20b\,c^2d^2x^3}{9a^2}+\frac{d\left(40c^3b+7a\right)x^2}{18a^2}+\frac{c\left(10c^3b+7a\right)x}{9a^2}+\frac{c^2\left(4c^3b+7a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+\right.}\right.}{27a^2bd}$

input

```
int((d*x+c)/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output

```
(2/9*b*d^4/a^2*x^5+10/9*b*c*d^3/a^2*x^4+20/9*b*c^2*d^2/a^2*x^3+1/18*d*(40*
b*c^3+7*a)/a^2*x^2+1/9*c*(10*b*c^3+7*a)/a^2*x+1/18*c^2/d*(4*b*c^3+7*a)/a^2
)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+2/27/a^2/b/d*sum((_R*d+c
)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_
Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(161) = 322$.

Time = 0.12 (sec) , antiderivative size = 1708, normalized size of antiderivative = 8.46

$$\int \frac{c+dx}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
[1/54*(12*a*b^3*d^5*x^5 + 60*a*b^3*c*d^4*x^4 + 120*a*b^3*c^2*d^3*x^3 + 12*
a*b^3*c^5 + 21*a^2*b^2*c^2 + 3*(40*a*b^3*c^3 + 7*a^2*b^2)*d^2*x^2 + 6*(10*
a*b^3*c^4 + 7*a^2*b^2*c)*d*x + 6*sqrt(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*
x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 +
 a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b
^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b
^2*c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*
b*c + 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt(
(-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2
 + 3*b*c^2*d*x + b*c^3 + a)) + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c
^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2
*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*
log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (
-a*b^2)^(2/3)) - 4*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b
^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c
)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*(-a*b^2)^(2/3)*log(b*d*x + b*
c - (-a*b^2)^(1/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c
^2*d^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a
^4*b^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2
*a^4*b^3*c^3 + a^5*b^2)*d), 1/54*(12*a*b^3*d^5*x^5 + 60*a*b^3*c*d^4*x^4...
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.47

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{7ac^2+4bc^5+40bc^2d^3x^3+20bcd^4x^4+4bd^5x^5+x^2\cdot(7ad^2+4}{18a^4d+36a^3bc^3d+18a^2b^2c^6d+270a^2b^2c^2d^5x^4+108a^2b^2cd^6x^5+18a^2b^2d^7x^6+x^3\cdot(36a^3bd^4+360a^2b^2}$$
$$+\frac{\operatorname{RootSum}\left(19683t^3a^7b^2+8,\left(t\mapsto t\log\left(x+\frac{729t^2a^5b+4c}{4d}\right)\right)\right)}{d}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)**3)**3,x)
```

output
```
(7*a*c**2 + 4*b*c**5 + 40*b*c**2*d**3*x**3 + 20*b*c*d**4*x**4 + 4*b*d**5*x
**5 + x**2*(7*a*d**2 + 40*b*c**3*d**2) + x*(14*a*c*d + 20*b*c**4*d))/(18*a
**4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c**6*d + 270*a**2*b**2*c**2*d**5*x
**4 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2*b**2*d**7*x**6 + x**3*(36*a**3*b
*d**4 + 360*a**2*b**2*c**3*d**4) + x**2*(108*a**3*b*c*d**3 + 270*a**2*b**2
*c**4*d**3) + x*(108*a**3*b*c**2*d**2 + 108*a**2*b**2*c**5*d**2)) + RootSu
m(19683*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(x + (729*_t**2*a**5*b + 4*c
)/(4*d))))/d
```

Maxima [F]

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{dx+c}{\left((dx+c)^3b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*d^5*x^5 + 20*b*c*d^4*x^4 + 40*b*c^2*d^3*x^3 + 4*b*c^5 + (40*b*c^
3 + 7*a)*d^2*x^2 + 7*a*c^2 + 2*(10*b*c^4 + 7*a*c)*d*x)/(a^2*b^2*d^7*x^6 +
6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^3 + a^3*b)*
d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5 + a^3*b*c
^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) + 2/9*integrate((d*x + c)
/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.33

$$\int \frac{c+dx}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{1}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{27\,a^2}$$

$$+\frac{4\,bd^5x^5+20\,bcd^4x^4+40\,bc^2d^3x^3+40\,bc^3d^2x^2+20\,bc^4dx+4\,bc^5+7\,ad^2x^2+14\,acdx+7\,ac^2}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2a^2d}$$

input
```
integrate((d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/27*(2*sqrt(3)*(-1/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*
a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-1/(a*b^2*d^3))^(1/3)*log((2*a*
b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-1/(a*b^2*d^3
))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a^2 + 1/18*(4*b*d^5*x
^5 + 20*b*c*d^4*x^4 + 40*b*c^2*d^3*x^3 + 40*b*c^3*d^2*x^2 + 20*b*c^4*d*x +
 4*b*c^5 + 7*a*d^2*x^2 + 14*a*c*d*x + 7*a*c^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2
 + 3*b*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.00

$$\int \frac{c+dx}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{4bc^5+7ac^2}{18a^2d} + \frac{2bd^4x^5}{9a^2} + \frac{cx\left(10bc^3+7a\right)}{9a^2} + \frac{dx^2\left(40bc^3+7a\right)}{18a^2} + \frac{10bcd^3x^4}{9a^2} + \frac{20bc^2d^2}{9a^2}}{x^3\left(20b^2c^3d^3+2abd^3\right)+x^2\left(15b^2c^4d^2+6abcd^2\right)+a^2+x\left(6db^2c^5+6adbc^2\right)+b^2c^6+b^2d^6x^6+}$$

$$+\frac{2\ln\left(\frac{4bcd^4}{81a^4}-\frac{4b^{2/3}d^4}{81(-a)^{11/3}}+\frac{4bd^5x}{81a^4}\right)}{27(-a)^{7/3}b^{2/3}d}$$

$$+\frac{\ln\left(\frac{4bcd^4}{81a^4}-\frac{b^{2/3}d^4\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{81(-a)^{11/3}}+\frac{4bd^5x}{81a^4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{27(-a)^{7/3}b^{2/3}d}$$

$$-\frac{\ln\left(\frac{4bcd^4}{81a^4}-\frac{b^{2/3}d^4\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{81(-a)^{11/3}}+\frac{4bd^5x}{81a^4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{27(-a)^{7/3}b^{2/3}d}$$

input

```
int((c + d*x)/(a + b*(c + d*x)^3)^3,x)
```

output

```
((7*a*c^2 + 4*b*c^5)/(18*a^2*d) + (2*b*d^4*x^5)/(9*a^2) + (c*x*(7*a + 10*b
*c^3))/(9*a^2) + (d*x^2*(7*a + 40*b*c^3))/(18*a^2) + (10*b*c*d^3*x^4)/(9*a
^2) + (20*b*c^2*d^2*x^3)/(9*a^2))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*
(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2
*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) + (
2*log((4*b*c*d^4)/(81*a^4) - (4*b^(2/3)*d^4)/(81*(-a)^(11/3)) + (4*b*d^5*x
)/(81*a^4)))/(27*(-a)^(7/3)*b^(2/3)*d) + (log((4*b*c*d^4)/(81*a^4) - (b^(2
/3)*d^4*(3^(1/2)*1i - 1)^2)/(81*(-a)^(11/3)) + (4*b*d^5*x)/(81*a^4))*(3^(1
/2)*1i - 1))/(27*(-a)^(7/3)*b^(2/3)*d) - (log((4*b*c*d^4)/(81*a^4) - (b^(2
/3)*d^4*(3^(1/2)*1i + 1)^2)/(81*(-a)^(11/3)) + (4*b*d^5*x)/(81*a^4))*(3^(1
/2)*1i + 1))/(27*(-a)^(7/3)*b^(2/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1828, normalized size of antiderivative = 9.05

$$\int \frac{c+dx}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input

```
int((d*x+c)/(a+b*(d*x+c)^3)^3,x)
```

output

```
( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*a**2*b*c - 8*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4 - 24*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**3*d*x - 24*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*
*2*d**2*x**2 - 8*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*a*b**2*c*d**3*x**3 - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7 - 24*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**6*d*x
 - 60*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*b**3*c**5*d**2*x**2 - 80*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2
*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**3*x**3 - 60*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**3*d*
*4*x**4 - 24*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*b**3*c**2*d**5*x**5 - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**6*x**6 - 2*b**(2/3)*a**
(1/3)*a**2 + 17*b**(2/3)*a**(1/3)*a*b*c**3 + 30*b**(2/3)*a**(1/3)*a*b*c**2
*d*x + 9*b**(2/3)*a**(1/3)*a*b*c*d**2*x**2 - 4*b**(2/3)*a**(1/3)*a*b*d**3*
x**3 + 10*b**(2/3)*a**(1/3)*b**2*c**6 + 48*b**(2/3)*a**(1/3)*b**2*c**5*d*x
 + 90*b**(2/3)*a**(1/3)*b**2*c**4*d**2*x**2 + 80*b**(2/3)*a**(1/3)*b**2...
```

3.80 $\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 616
Mathematica [A] (verified) . . . 617
Rubi [A] (verified) . . . 617
Maple [C] (verified) . . . 623
Fricas [B] (verification not implemented) . . . 624
Sympy [A] (verification not implemented) . . . 625
Maxima [F] . . . 625
Giac [A] (verification not implemented) . . . 626
Mupad [B] (verification not implemented) . . . 626
Reduce [B] (verification not implemented) . . . 627

Optimal result

Integrand size = 13, antiderivative size = 198

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{c+dx}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{5(c+dx)}{18a^2d\left(a+b(c+dx)^3\right)} - \frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}d} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{8/3}\sqrt[3]{b}d} - \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{8/3}\sqrt[3]{b}d}$$

output

```
1/6*(d*x+c)/a/d/(a+b*(d*x+c)^3)^2+5/18*(d*x+c)/a^2/d/(a+b*(d*x+c)^3)-5/27*
arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^
(1/3)/d+5/27*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(8/3)/b^(1/3)/d-5/54*ln(a^(2/3)
-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{\frac{9a^{5/3}(c+dx)}{(a+b(c+dx)^3)^2} + \frac{15a^{2/3}(c+dx)}{a+b(c+dx)^3} + \frac{10\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{\sqrt[3]{b}} - \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)\right.}{\sqrt[3]{b}}}{54a^{8/3}d}$$

input
```
Integrate[(a + b*(c + d*x)^3)^(-3),x]
```

output
```
((9*a^(5/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (15*a^(2/3)*(c + d*x))/(a +
 b*(c + d*x)^3) + (10*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqr
t[3]*a^(1/3))])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(1/3) -
(5*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^(1/3)
)/(54*a^(8/3)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {239, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 749$$

$$\frac{\frac{5\int \frac{1}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{6a}+\frac{c+dx}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 749$$

$$\frac{\frac{5\left(\frac{2\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{c+dx}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 750$$

$$\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{c+dx}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{c+dx}{6a(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 1142$$

$$\frac{5\left(2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}\right)+\frac{c+dx}{3a\left(a+b(c+\right.}}{6a}}{d}$$

$$\downarrow 25$$

$$\frac{5\left(2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}\right)+\frac{c+dx}{3a\left(a+b(c+d\right.}}{6a}}{d}$$

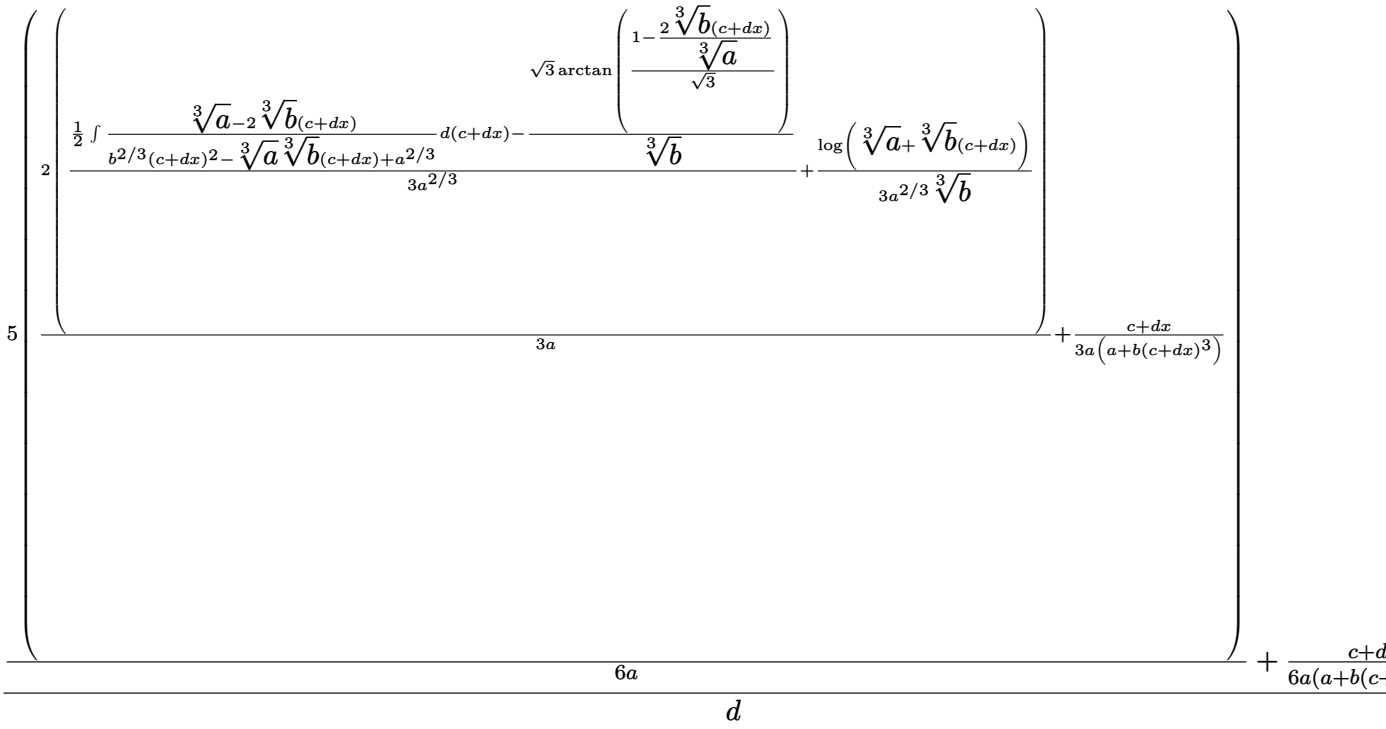
$$\downarrow 27$$

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+\right.}\right)}{6a}}{d}$$

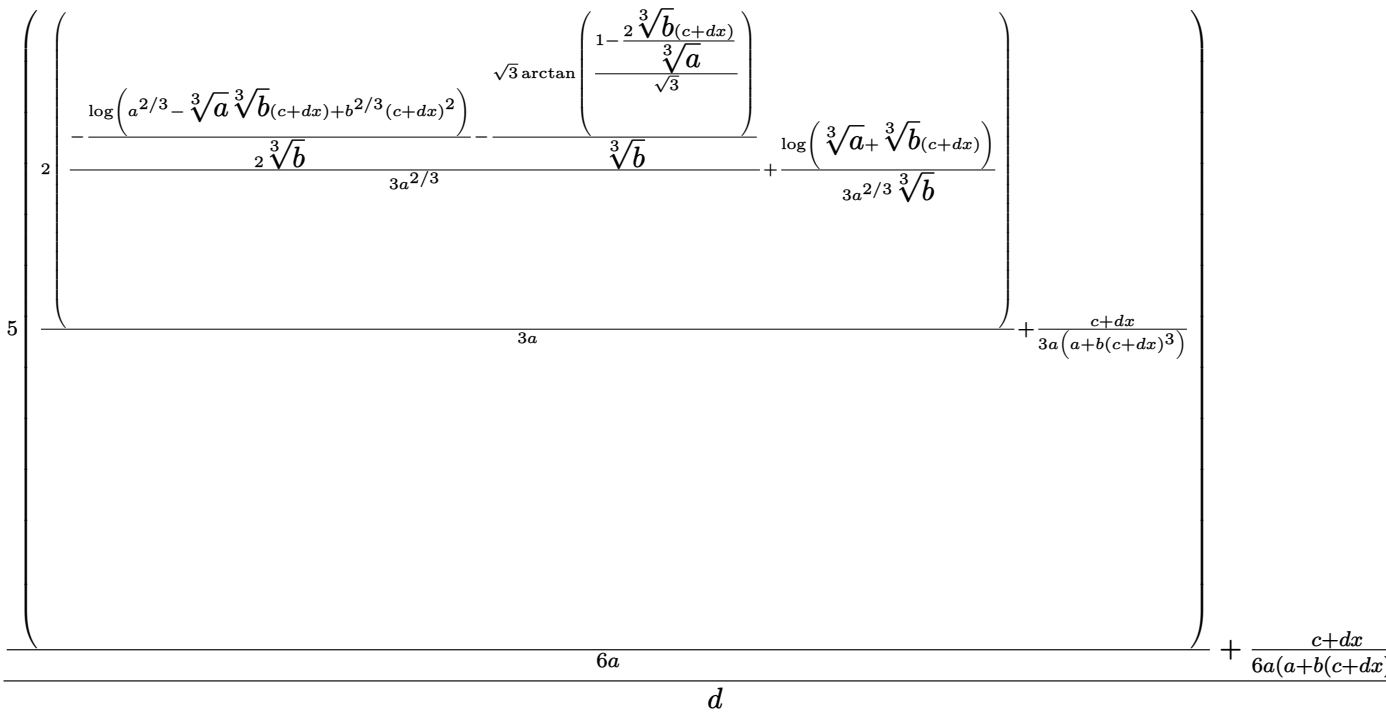
$$\downarrow 1082$$

$$\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^{3}\right.}\right)}{6a}}{d}$$

$$\downarrow 217$$



$\downarrow$ 1103



input
```
Int[(a + b*(c + d*x)^3)^(-3),x]
```

output
```
((c + d*x)/(6*a*(a + b*(c + d*x)^3)^2) + (5*((c + d*x)/(3*a*(a + b*(c + d*
x)^3)) + (2*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[
a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3
*a^(2/3))))/(3*a)))/(6*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{5b\,d^3x^4}{18a^2}+\frac{10bc\,d^2x^3}{9a^2}+\frac{5c^2bd\,x^2}{3a^2}+\frac{2\left(5c^3b+2a\right)x}{9a^2}+\frac{c\left(5c^3b+8a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_}{d^2\_R^2+2cd\_}\right.}{27a^2bd}$
risch	$\frac{\frac{5b\,d^3x^4}{18a^2}+\frac{10bc\,d^2x^3}{9a^2}+\frac{5c^2bd\,x^2}{3a^2}+\frac{2\left(5c^3b+2a\right)x}{9a^2}+\frac{c\left(5c^3b+8a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_}{d^2\_R^2+2cd\_}\right.}{27a^2bd}$

input
```
int(1/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/18/a^2*b*d^3*x^4+10/9/a^2*b*c*d^2*x^3+5/3*c^2/a^2*b*d*x^2+2/9*(5*b*c^3+
2*a)/a^2*x+1/18*c/d*(5*b*c^3+8*a)/a^2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*
x+b*c^3+a)^2+5/27/a^2/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf
(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(157) = 314$.

Time = 0.12 (sec) , antiderivative size = 1646, normalized size of antiderivative = 8.31

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
[1/54*(15*a^2*b^2*d^4*x^4 + 60*a^2*b^2*c*d^3*x^3 + 90*a^2*b^2*c^2*d^2*x^2
+ 15*a^2*b^2*c^4 + 24*a^3*b*c + 12*(5*a^2*b^2*c^3 + 2*a^3*b)*d*x + 15*sqrt
(1/3)*(a*b^3*d^6*x^6 + 6*a*b^3*c*d^5*x^5 + 15*a*b^3*c^2*d^4*x^4 + a*b^3*c^
6 + 2*a^2*b^2*c^3 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*x^3 + 3*(5*a*b^3*c^4 +
2*a^2*b^2*c)*d^2*x^2 + a^3*b + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*x)*sqrt(-(a^2
*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*
c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)
^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/
3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) -
 5*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b
^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b
^2*c^5 + a*b*c^2)*d*x + a^2)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 10*(b^2*d^6*x^6 +
6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b^2*c^3 + a*b)*d^3*
x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*
d*x + a^2)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^4*b^3*d^
7*x^6 + 6*a^4*b^3*c*d^6*x^5 + 15*a^4*b^3*c^2*d^5*x^4 + 2*(10*a^4*b^3*c^3 +
 a^5*b^2)*d^4*x^3 + 3*(5*a^4*b^3*c^4 + 2*a^5*b^2*c)*d^3*x^2 + 6*(a^4*b^3*c
^5 + a^5*b^2*c^2)*d^2*x + (a^4*b^3*c^6 + 2*a^5*b^2*c^3 + a^6*b)*d), 1/54*(
15*a^2*b^2*d^4*x^4 + 60*a^2*b^2*c*d^3*x^3 + 90*a^2*b^2*c^2*d^2*x^2 + 15...
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx$$
$$= \frac{8ac+5bc^4+30bc^2d^2x^2+20bcd^3x^3+5bd^4x^4+x}{18a^4d+36a^3bc^3d+18a^2b^2c^6d+270a^2b^2c^2d^5x^4+108a^2b^2cd^6x^5+18a^2b^2d^7x^6+x^3\cdot(36a^3bd^4+360a^2b^2}$$
$$+\frac{\text{RootSum}\left(19683t^3a^8b-125,\left(t\mapsto t\log\left(x+\frac{27ta^3+5c}{5d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**3)**3,x)
```

output
```
(8*a*c + 5*b*c**4 + 30*b*c**2*d**2*x**2 + 20*b*c*d**3*x**3 + 5*b*d**4*x**4
 + x*(8*a*d + 20*b*c**3*d))/(18*a**4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c
**6*d + 270*a**2*b**2*c**2*d**5*x**4 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2
*b**2*d**7*x**6 + x**3*(36*a**3*b*d**4 + 360*a**2*b**2*c**3*d**4) + x**2*(
108*a**3*b*c*d**3 + 270*a**2*b**2*c**4*d**3) + x*(108*a**3*b*c**2*d**2 + 1
08*a**2*b**2*c**5*d**2)) + RootSum(19683*_t**3*a**8*b - 125, Lambda(_t, _t
*log(x + (27*_t*a**3 + 5*c)/(5*d))))/d
```

Maxima [F]

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx = \int \frac{1}{\left((dx+c)^3b+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/18*(5*b*d^4*x^4 + 20*b*c*d^3*x^3 + 30*b*c^2*d^2*x^2 + 5*b*c^4 + 4*(5*b*c
^3 + 2*a)*d*x + 8*a*c)/(a^2*b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2
*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^3 + a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a
^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5 + a^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3
*b*c^3 + a^4)*d) + 5/9*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*
x + b*c^3 + a), x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{5\left(2\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right)-\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2+4\left(\right.\right.\right.}{54\,a^2}$$

$$+\frac{5\,bd^4x^4+20\,bcd^3x^3+30\,bc^2d^2x^2+20\,bc^3dx+5\,bc^4+8\,adx+8\,ac}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2a^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
5/54*(2*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3)
)/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x +
b*c + (a*b^2)^(1/3))))/a^2 + 1/18*(5*b*d^4*x^4 + 20*b*c*d^3*x^3 + 30*b*c^2
*d^2*x^2 + 20*b*c^3*d*x + 5*b*c^4 + 8*a*d*x + 8*a*c)/((b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{2x\left(5bc^3+2a\right)}{9a^2}+\frac{5bc^4+8ac}{18a^2d}+\frac{5bd^3x^4}{18a^2}+\frac{5bc^2dx^2}{3a^2}+\frac{10bcd^2x^3}{9a^2}}{x^3\left(20b^2c^3d^3+2abd^3\right)+x^2\left(15b^2c^4d^2+6abcd^2\right)+a^2+x\left(6db^2c^5+6adbc^2\right)+b^2c^6+b^2d^6x^6+}$$

$$+\frac{5\,\ln\left(b^{1/3}c+a^{1/3}+b^{1/3}dx\right)}{27\,a^{8/3}b^{1/3}d}$$

$$+\frac{\ln\left(\frac{5b^2cd^5}{3a^2}+\frac{5b^2d^6x}{3a^2}+\frac{b^{5/3}d^5\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{6a^{5/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{54\,a^{8/3}b^{1/3}d}$$

$$-\frac{\ln\left(\frac{5b^2cd^5}{3a^2}+\frac{5b^2d^6x}{3a^2}-\frac{b^{5/3}d^5\left(5+\sqrt{3}\,5\mathrm{i}\right)}{6a^{5/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{54\,a^{8/3}b^{1/3}d}$$

input
```
int(1/(a + b*(c + d*x)^3)^3,x)
```

output
```
((2*x*(2*a + 5*b*c^3))/(9*a^2) + (8*a*c + 5*b*c^4)/(18*a^2*d) + (5*b*d^3*x
^4)/(18*a^2) + (5*b*c^2*d*x^2)/(3*a^2) + (10*b*c*d^2*x^3)/(9*a^2))/(x^3*(2
0*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*
(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*
d^5*x^5 + 15*b^2*c^2*d^4*x^4) + (5*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))
/(27*a^(8/3)*b^(1/3)*d) + (log((5*b^2*c*d^5)/(3*a^2) + (5*b^2*d^6*x)/(3*a^
2) + (b^(5/3)*d^5*(3^(1/2)*5i - 5))/(6*a^(5/3)))*(3^(1/2)*5i - 5))/(54*a^(
8/3)*b^(1/3)*d) - (log((5*b^2*c*d^5)/(3*a^2) + (5*b^2*d^6*x)/(3*a^2) - (b^
(5/3)*d^5*(3^(1/2)*5i + 5))/(6*a^(5/3)))*(3^(1/2)*5i + 5))/(54*a^(8/3)*b^(
1/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1793, normalized size of antiderivative = 9.06

$$\int \frac{1}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*(d*x+c)^3)^3,x)
```

output

```
( - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3)))*a**2 - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c
 - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**3 - 60*a**(1/3)*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**2*d
*x - 60*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*a*b*c*d**2*x**2 - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) -
2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*d**3*x**3 - 10*a**(
1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqr
t(3)))*b**2*c**6 - 60*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b
**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**5*d*x - 150*a**(1/3)*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*
d**2*x**2 - 200*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3
)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d**3*x**3 - 150*a**(1/3)*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*
d**4*x**4 - 60*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*b**2*c*d**5*x**5 - 10*a**(1/3)*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*d**6*x**6
- 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2 - 10*a**(1/3)*
log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c...
```

3.81 $\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result 629
Mathematica [A] (verified) 629
Rubi [A] (verified) 630
Maple [A] (verified) 631
Fricas [B] (verification not implemented) 632
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Maxima [B] (verification not implemented) 633
Giac [A] (verification not implemented) 634
Mupad [B] (verification not implemented) 634
Reduce [B] (verification not implemented) 635

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{1}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{1}{3a^2d\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^3d} - \frac{\log\left(a+b(c+dx)^3\right)}{3a^3d}$$

output

```
1/6/a/d/(a+b*(d*x+c)^3)^2+1/3/a^2/d/(a+b*(d*x+c)^3)+ln(d*x+c)/a^3/d-1/3*ln
(a+b*(d*x+c)^3)/a^3/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\frac{a\left(a+2\left(a+b(c+dx)^3\right)\right)}{\left(a+b(c+dx)^3\right)^2} + 6\log(c+dx) - 2\log\left(a+b(c+dx)^3\right)}{6a^3d}$$

input

```
Integrate[1/((c + d*x)*(a + b*(c + d*x)^3)^3),x]
```

output
```
((a*(a + 2*(a + b*(c + d*x)^3)))/(a + b*(c + d*x)^3)^2 + 6*Log[c + d*x] -
2*Log[a + b*(c + d*x)^3])/(6*a^3*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$
$$\downarrow 798$$
$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)^3}\,d(c+dx)^3}{3d}$$
$$\downarrow 54$$
$$\frac{\int \left(-\frac{b}{a^3(b(c+dx)^3+a)} - \frac{b}{a^2(b(c+dx)^3+a)^2} - \frac{b}{a(b(c+dx)^3+a)^3} + \frac{1}{a^3(c+dx)^3}\right) d(c+dx)^3}{3d}$$
$$\downarrow 2009$$
$$\frac{-\frac{\log(a+b(c+dx)^3)}{a^3} + \frac{\log((c+dx)^3)}{a^3} + \frac{1}{a^2(a+b(c+dx)^3)} + \frac{1}{2a(a+b(c+dx)^3)^2}}{3d}$$

input
```
Int[1/((c + d*x)*(a + b*(c + d*x)^3)^3),x]
```

output
```
(1/(2*a*(a + b*(c + d*x)^3)^2) + 1/(a^2*(a + b*(c + d*x)^3)) + Log[(c + d*
x)^3]/a^3 - Log[a + b*(c + d*x)^3]/a^3)/(3*d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.74

method	result
risch	$\frac{\frac{b\,d^2x^3}{3a^2}+\frac{bcd\,x^2}{a^2}+\frac{bx\,c^2}{a^2}+\frac{2c^3b+3a}{6a^2d}}{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2}+\frac{\ln(xd+c)}{a^3d}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^3d}$
default	$\frac{\ln(xd+c)}{a^3d}-\frac{b\left(\frac{-\frac{a\,d^2x^3}{3}-acd\,x^2-a\,c^2x-\frac{a\left(2c^3b+3a\right)}{6db}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}\right)}{a^3}$
norman	$\frac{\frac{bcd\,x^2}{a^2}+\frac{bx\,c^2}{a^2}+\frac{2b^3c^3d^5+3a\,b^2d^5}{6a^2d^6b^2}+\frac{b\,d^2x^3}{3a^2}}{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2}+\frac{\ln(xd+c)}{a^3d}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^3d}$
parallelrisch	$\frac{-12\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)x^2a\,b^3c\,d^7-12\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)xa\,b^3c^2d^6+36\ln(xd+c)x^2a\,b$

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3/a^2*b*d^2*x^3+1/a^2*b*c*d*x^2+1/a^2*b*x*c^2+1/6*(2*b*c^3+3*a)/a^2/d)/
(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+ln(d*x+c)/a^3/d-1/3/a^3/d*
ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 467, normalized size of antiderivative = 5.70

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{2\,abd^3x^3 + 6\,abcd^2x^2 + 6\,abc^2dx + 2\,abc^3 + 3\,a^2 - 2\,(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + b^2c^6 + 2\,(10\,b^2c^3$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2 -
 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b
^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b
^2*c^5 + a*b*c^2)*d*x + a^2)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a) + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*
c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d
^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*log(d*x + c))/(a^3*b^2*d^7*x^6 +
 6*a^3*b^2*c*d^6*x^5 + 15*a^3*b^2*c^2*d^5*x^4 + 2*(10*a^3*b^2*c^3 + a^4*b)
*d^4*x^3 + 3*(5*a^3*b^2*c^4 + 2*a^4*b*c)*d^3*x^2 + 6*(a^3*b^2*c^5 + a^4*b*
c^2)*d^2*x + (a^3*b^2*c^6 + 2*a^4*b*c^3 + a^5)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(66) = 132$.

Time = 2.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.28

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{3a+2bc^3+6bc^2dx+6bcd^2x^2+2bd^3x^3}{6a^4d+12a^3bc^3d+6a^2b^2c^6d+90a^2b^2c^2d^5x^4+36a^2b^2cd^6x^5+6a^2b^2d^7x^6+x^3\cdot(12a^3bd^4+120a^2b^2c^3d^4)+}$$
$$+\frac{\log\left(\frac{c}{d}+x\right)}{a^3d}-\frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^3d}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)**3)**3,x)
```

output
```
(3*a + 2*b*c**3 + 6*b*c**2*d*x + 6*b*c*d**2*x**2 + 2*b*d**3*x**3)/(6*a**4*
d + 12*a**3*b*c**3*d + 6*a**2*b**2*c**6*d + 90*a**2*b**2*c**2*d**5*x**4 + 
36*a**2*b**2*c*d**6*x**5 + 6*a**2*b**2*d**7*x**6 + x**3*(12*a**3*b*d**4 + 
120*a**2*b**2*c**3*d**4) + x**2*(36*a**3*b*c*d**3 + 90*a**2*b**2*c**4*d**3
) + x*(36*a**3*b*c**2*d**2 + 36*a**2*b**2*c**5*d**2)) + log(c/d + x)/(a**3
*d) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a*
*3*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(76) = 152.

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.99

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+3\,a}{6\left(a^2b^2d^7x^6+6\,a^2b^2cd^6x^5+15\,a^2b^2c^2d^5x^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4x^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3x^2+6\left(a^2b^2\right.\right.}$$
$$-\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^3d}+\frac{\log\left(dx+c\right)}{a^3d}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/6*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + 3*a)/(a^2*b^2*d
^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2*c^3
+ a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2*c^5
+ a^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) - 1/3*log(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d) + log(d*x + c)/(a^3
*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.76

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^3d} + \frac{\log\left(|dx+c|\right)}{a^3d} + \frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+3\,a^2}{6\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2a^3d}$$

input
```
integrate(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^3*d)
 + log(abs(d*x + c))/(a^3*d) + 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*
b*c^2*d*x + 2*a*b*c^3 + 3*a^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a)^2*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.80

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{2\,bc^3+3\,a}{6\,a^2\,d}+\frac{b\,d^2\,x^3}{3\,a^2}+\frac{b\,c^2\,x}{a^2}+\frac{b\,c\,d\,x^2}{a^2}}{x^3\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\,d^6\,x^6+} - \frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d}+\frac{\ln\left(c+d\,x\right)}{a^3\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)^3*(c + d*x)),x)
```

output
```
((3*a + 2*b*c^3)/(6*a^2*d) + (b*d^2*x^3)/(3*a^2) + (b*c^2*x)/a^2 + (b*c*d*
x^2)/a^2)/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*
c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a
*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4) - log(a + b*c^3 + b*d^3*x^3
 + 3*b*c^2*d*x + 3*b*c*d^2*x^2)/(3*a^3*d) + log(c + d*x)/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1402, normalized size of antiderivative = 17.10

$$\int \frac{1}{(c+dx)\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)/(a+b*(d*x+c)^3)^3,x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3
)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2 - 4*log(a**(2/3) - b*
*(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d
*x + b**(2/3)*d**2*x**2)*a*b*c**3 - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c
 - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2
*x**2)*a*b*c**2*d*x - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**
(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c*d
**2*x**2 - 4*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*d**3*x**3 - 2*l
og(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
+ 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**6 - 12*log(a**(2/3) - b**
(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*
x + b**(2/3)*d**2*x**2)*b**2*c**5*d*x - 30*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*b**2*c**4*d**2*x**2 - 40*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b
**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x*
*2)*b**2*c**3*d**3*x**3 - 30*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**
2*c**2*d**4*x**4 - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/
3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*...
```

3.82 $\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 636
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Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = &-\frac{1}{a^3 d(c+dx)} - \frac{b(c+dx)^2}{6a^2 d\left(a+b(c+dx)^3\right)^2} \\ &- \frac{5b(c+dx)^2}{9a^3 d\left(a+b(c+dx)^3\right)} \\ &+ \frac{14\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}d} \\ &+ \frac{14\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{10/3}d} \\ &- \frac{7\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}d}\end{aligned}$$

output

```
-1/a^3/d/(d*x+c)-1/6*b*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3)^2-5/9*b*(d*x+c)^2/a
^3/d/(a+b*(d*x+c)^3)+14/27*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*
3^(1/2)/a^(1/3))*3^(1/2)/a^(10/3)/d+14/27*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+
c))/a^(10/3)/d-7/27*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*
x+c)^2)/a^(10/3)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{-\frac{54\sqrt[3]{a}}{c+dx} - \frac{9a^{4/3}b(c+dx)^2}{(a+b(c+dx)^3)^2} - \frac{30\sqrt[3]{a}b(c+dx)^2}{a+b(c+dx)^3} - 28\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 28\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+\right.}{54a^{10/3}d}$$

input
```
Integrate[1/((c + d*x)^2*(a + b*(c + d*x)^3)^3),x]
```

output
```
((-54*a^(1/3))/(c + d*x) - (9*a^(4/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3)^2
 - (30*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 28*Sqrt[3]*b^(1/3)*Arc
Tan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 28*b^(1/3)*Log[a
^(1/3) + b^(1/3)*(c + d*x)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c
+ d*x) + b^(2/3)*(c + d*x)^2])/(54*a^(10/3)*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {895, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$
$$\downarrow 819$$
$$\frac{\frac{7\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{6a} + \frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 819$$

$$\frac{7\left(\frac{4\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}d(c+dx)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}}{d}$$

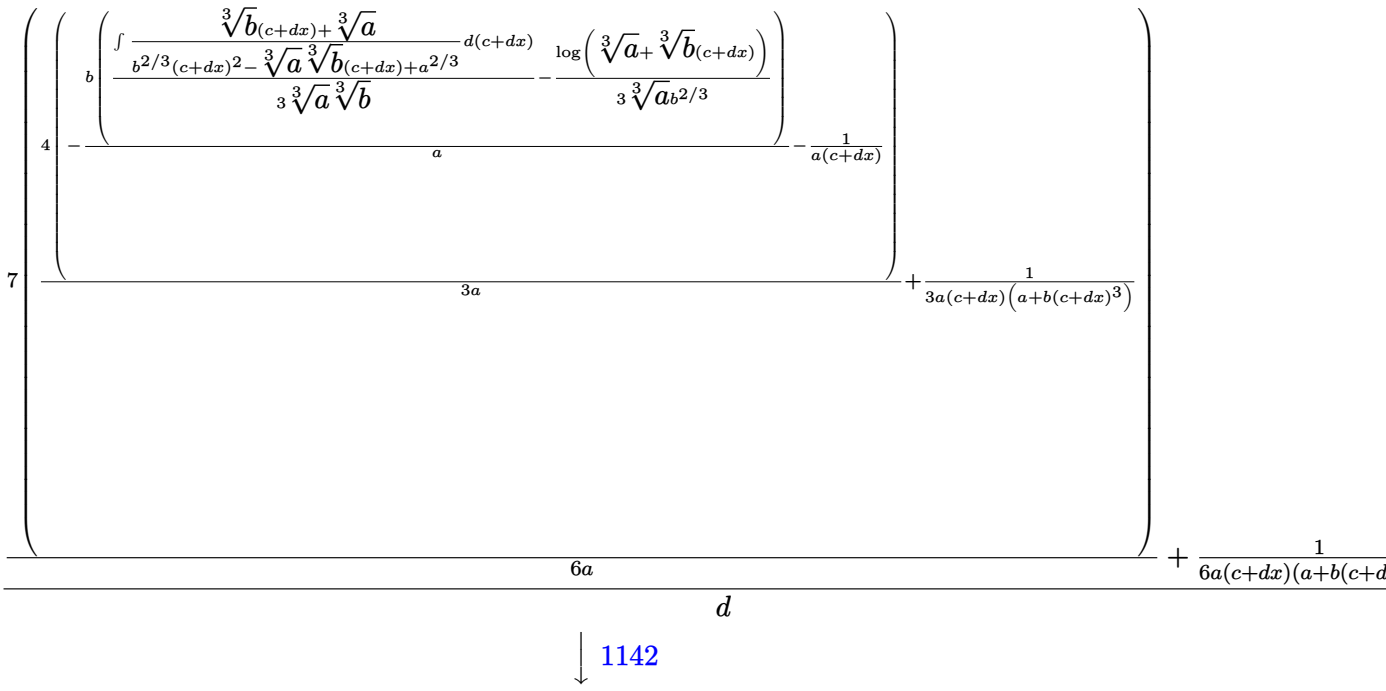
$$\downarrow 847$$

$$\frac{7\left(\frac{4\left(-\frac{b\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}}{d}$$

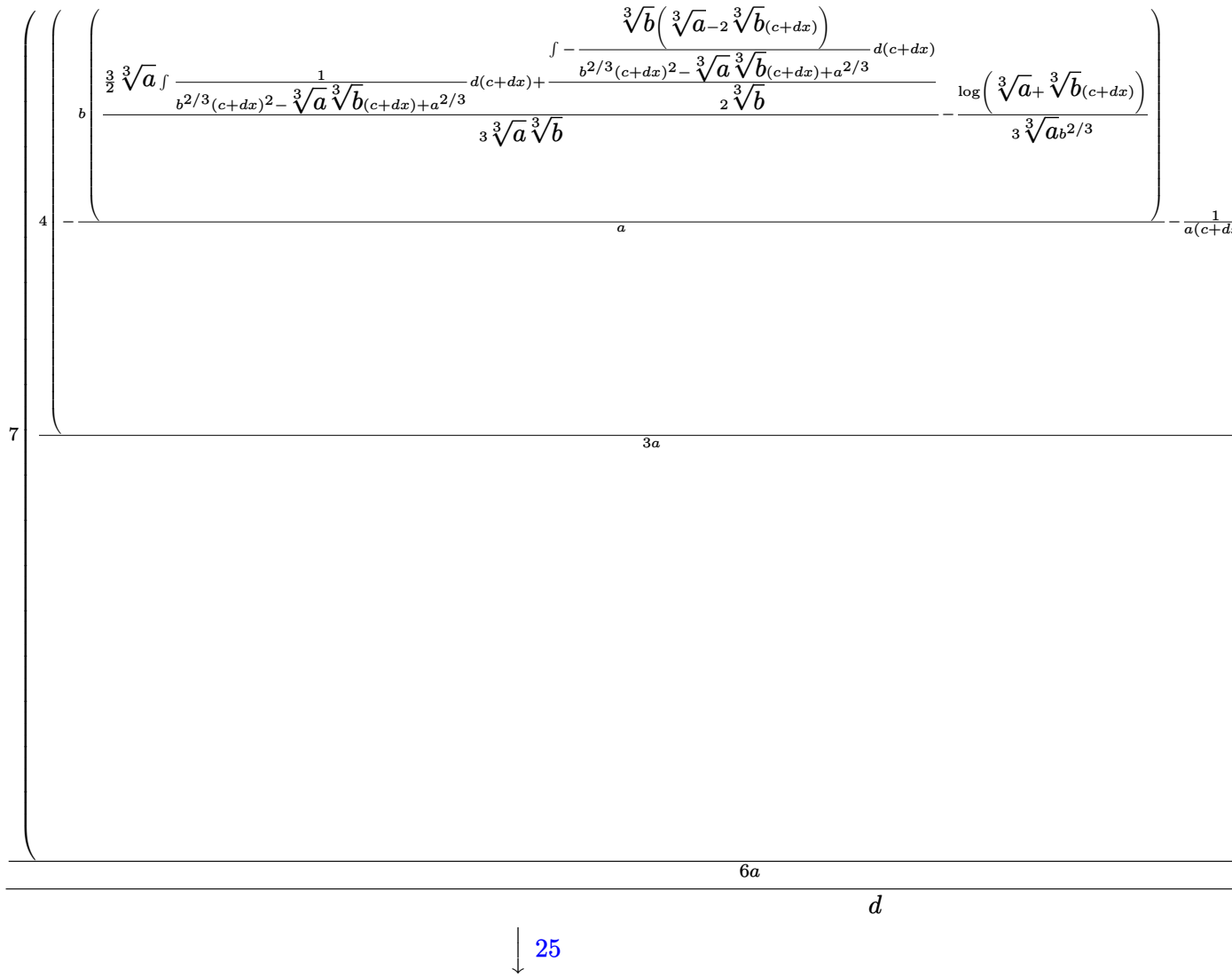
$$\downarrow 821$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(}}{d}$$

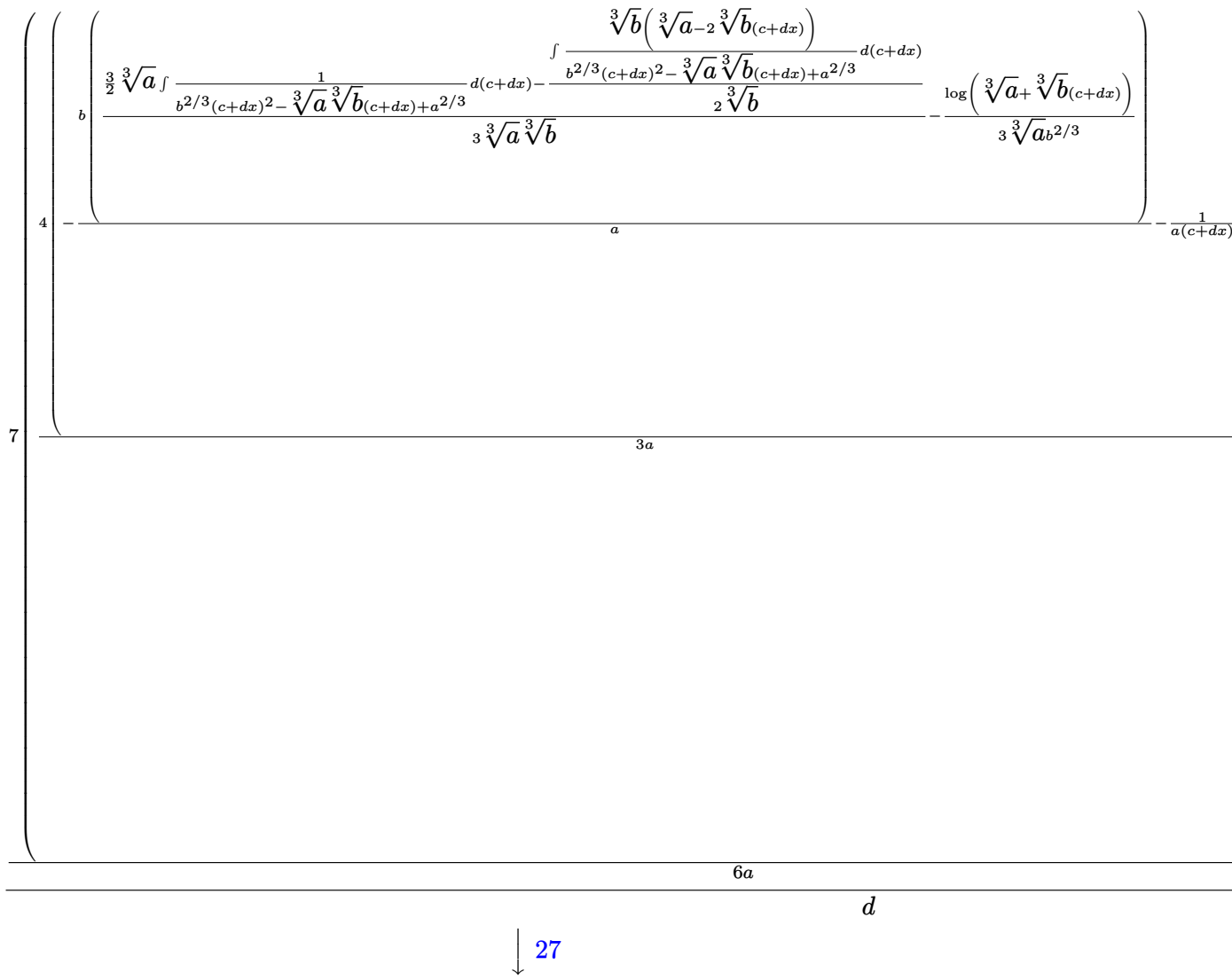
$$\downarrow 16$$



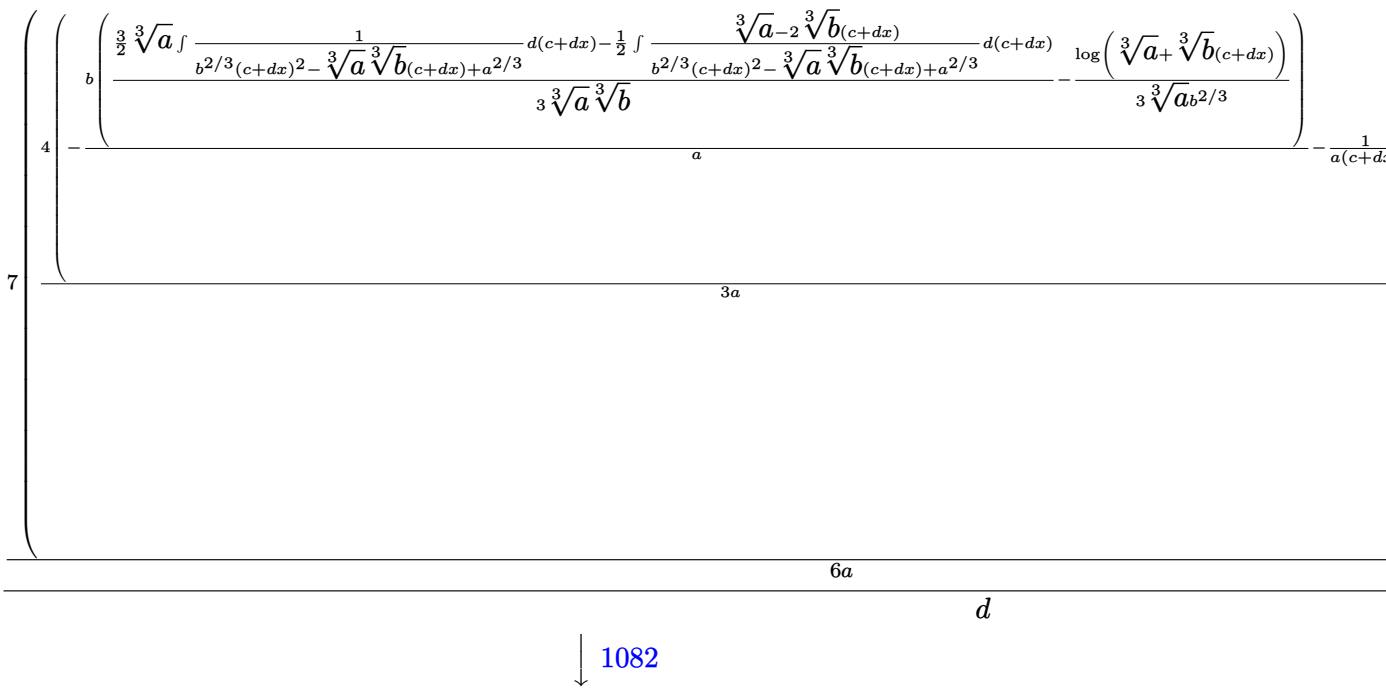
$$\downarrow 1142$$



$\downarrow$ 25

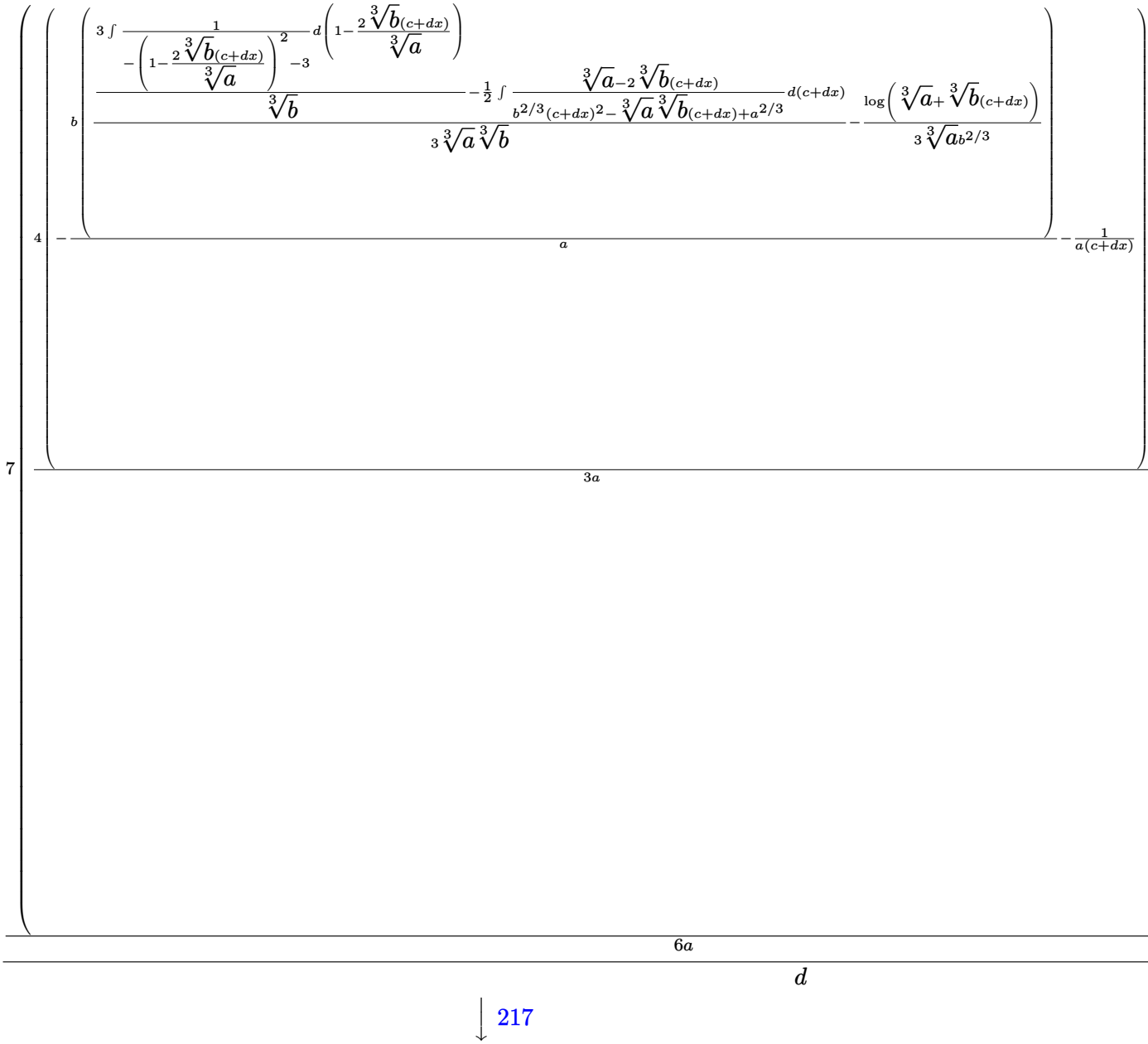


$\downarrow$ 27

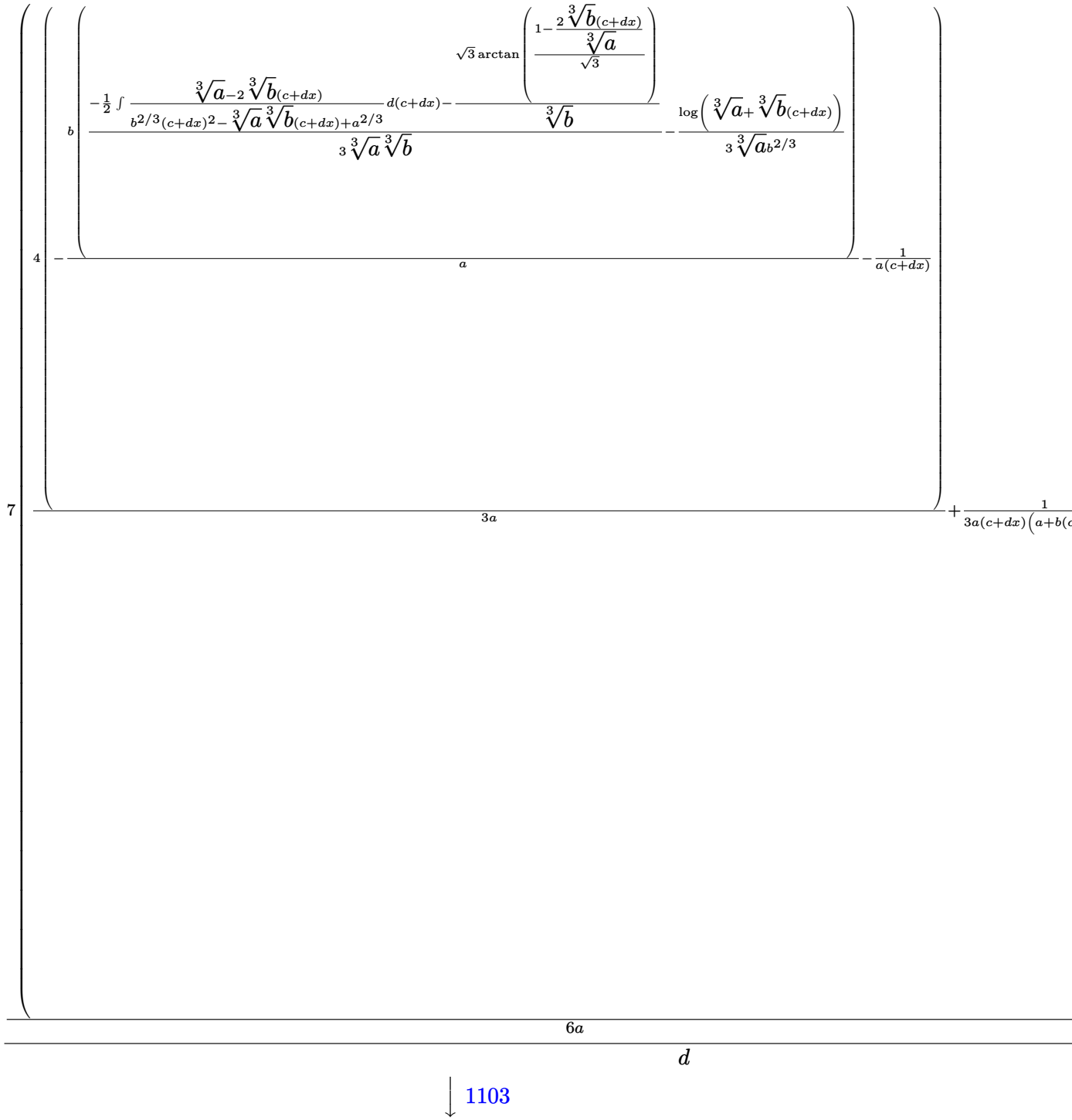


$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+d}\right.}{3a}\right.}{6a}}{d}$$

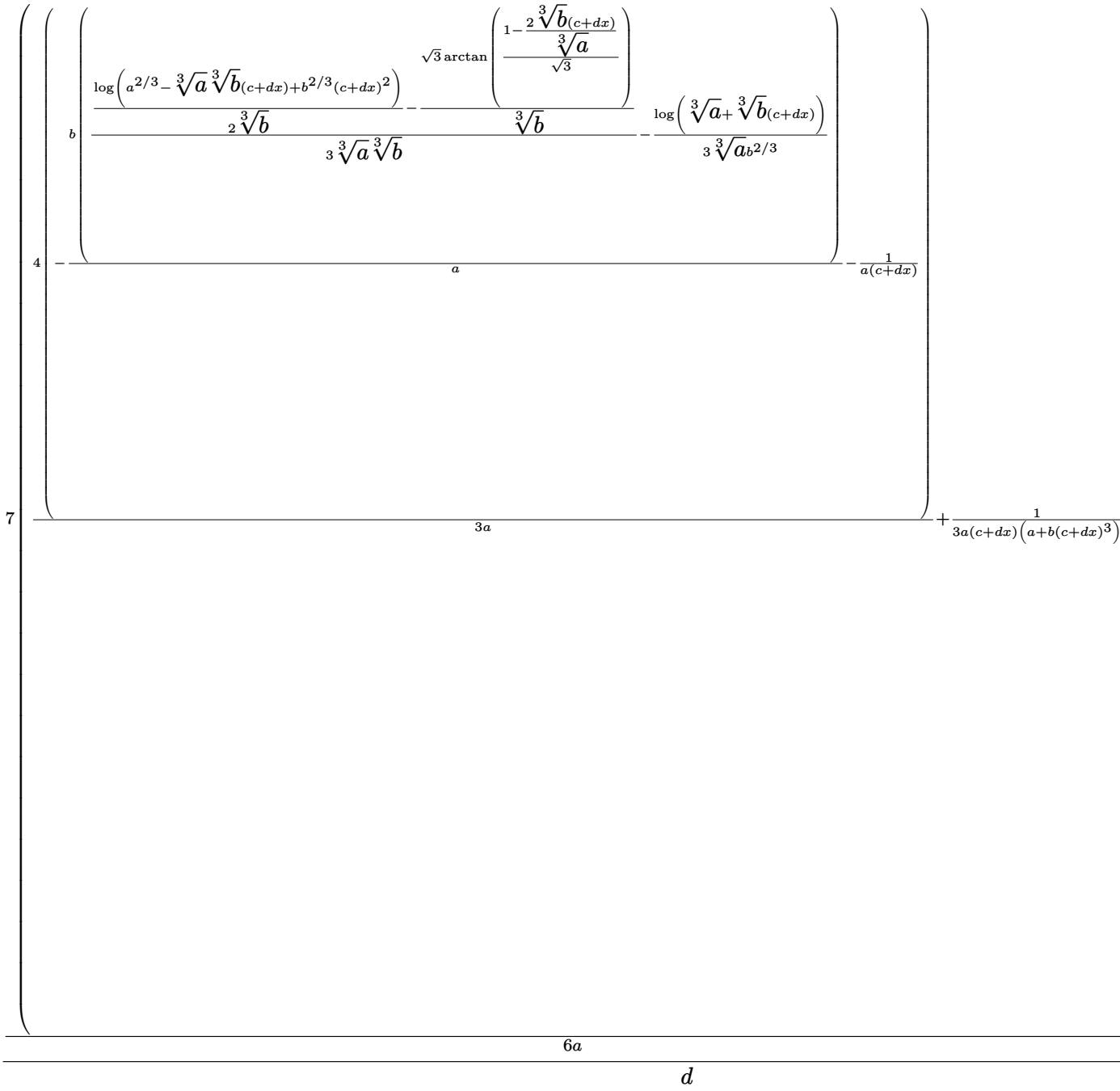
$\downarrow$ 1082



↓ 217



↓ 1103



input `Int[1/((c + d*x)^2*(a + b*(c + d*x)^3)^3),x]`

output
```
(1/(6*a*(c + d*x)*(a + b*(c + d*x)^3)^2) + (7*(1/(3*a*(c + d*x)*(a + b*(c
+ d*x)^3)) + (4*(-(1/(a*(c + d*x))) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*(c +
d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^
(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2
/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)))/(6*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

method	result
default	$-\frac{1}{a^3 d(xd+c)} - \frac{b\left(\frac{\frac{5d^4bx^5}{9}+\frac{25bcd^3x^4}{9}+\frac{50bc^2d^2x^3}{9}+\left(\frac{50}{9}bc^3d+\frac{13}{18}ad\right)x^2+\frac{c\left(25c^3b+13a\right)x}{9}+\frac{c^2\left(10c^3b+13a\right)}{18d}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{14\left(\sum_{R=\text{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3dbc\_Z+c^3b+a\right)}\right.}{a^3}$
risch	$\frac{-\frac{14b^2d^5x^6}{9a^3}-\frac{28b^2cd^4x^5}{3a^3}-\frac{70b^2c^2d^3x^4}{3a^3}-\frac{7bd^2\left(80c^3b+7a\right)x^3}{18a^3}-\frac{7bcd\left(20c^3b+7a\right)x^2}{6a^3}-\frac{7\left(8c^3b+7a\right)bc^2x}{6a^3}-\frac{28b^2c^6+49abc^3+18a^2}{18da^3}}{(xd+c)\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{14\left(\sum_{R}\right.}{}$

input
```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/d/(d*x+c)-1/a^3*b*((5/9*d^4*b*x^5+25/9*b*c*d^3*x^4+50/9*b*c^2*d^2*x
^3+(50/9*b*c^3*d+13/18*a*d)*x^2+1/9*c*(25*b*c^3+13*a)*x+1/18*c^2/d*(10*b*c
^3+13*a))/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+14/27/b/d*sum((_
R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^
2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(178) = 356$.

Time = 0.16 (sec) , antiderivative size = 855, normalized size of antiderivative = 3.90

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
-1/54*(84*b^2*d^6*x^6 + 504*b^2*c*d^5*x^5 + 1260*b^2*c^2*d^4*x^4 + 84*b^2*
c^6 + 21*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 147*a*b*c^3 + 63*(20*b^2*c^4 + 7*a
*b*c)*d^2*x^2 + 63*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 28*sqrt(3)*(b^2*d^7*x^7 +
 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4
*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c
^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*arcta
n(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*d^7*x^7 + 7*b
^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4
 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*
d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(b*d^2
*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28
*(b^2*d^7*x^7 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c
^3 + 2*a*b)*d^4*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^
2*c^5 + 4*a*b*c^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b
/a)^(1/3)*log(b*d*x + b*c + a*(b/a)^(2/3)) + 54*a^2)/(a^3*b^2*d^8*x^7 + 7*
a^3*b^2*c*d^7*x^6 + 21*a^3*b^2*c^2*d^6*x^5 + (35*a^3*b^2*c^3 + 2*a^4*b)*d^
5*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*x^3 + 3*(7*a^3*b^2*c^5 + 4*a^4*b*
c^2)*d^3*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*x + (a^3*b^2*c^7 +
2*a^4*b*c^4 + a^5*c)*d)
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.79

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-18a^2 - 49abc^3 - 28b^2c^6 - 420b^2c^2d^4x^4 - 168b^2cd^5x^5 - 28b^2d^6x^6 + x^3}{18a^5cd + 36a^4bc^4d + 18a^3b^2c^7d + 378a^3b^2c^2d^6x^5 + 126a^3b^2cd^7x^6 + 18a^3b^2d^8x^7 + x^4\cdot(36a^4bd^5 + 630a^3b^2c^3d^4 + 2a^5d^2) + x^3\cdot(36a^4bcd^4 + 630a^3b^2c^4d^3 + 4a^5cd) + x^2\cdot(18a^4bc^2d^3 + 378a^3b^2c^5d^2 + 3a^5c^2d) + x\cdot(18a^4bc^3d^2 + 126a^3b^2c^6d + 3a^5c^3) + 18a^3b^2c^8 + a^5c^4} + \frac{\operatorname{RootSum}\left(19683t^3a^{10} - 2744b, \left(t \mapsto t\log\left(x + \frac{729t^2a^7 + 196bc}{196bd}\right)\right)\right)}{d}$$

input

```
integrate(1/(d*x+c)**2/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-18*a**2 - 49*a*b*c**3 - 28*b**2*c**6 - 420*b**2*c**2*d**4*x**4 - 168*b**
2*c*d**5*x**5 - 28*b**2*d**6*x**6 + x**3*(-49*a*b*d**3 - 560*b**2*c**3*d**
3) + x**2*(-147*a*b*c*d**2 - 420*b**2*c**4*d**2) + x*(-147*a*b*c**2*d - 16
8*b**2*c**5*d))/(18*a**5*c*d + 36*a**4*b*c**4*d + 18*a**3*b**2*c**7*d + 37
8*a**3*b**2*c**2*d**6*x**5 + 126*a**3*b**2*c*d**7*x**6 + 18*a**3*b**2*d**8
*x**7 + x**4*(36*a**4*b*d**5 + 630*a**3*b**2*c**3*d**5) + x**3*(144*a**4*b
*c*d**4 + 630*a**3*b**2*c**4*d**4) + x**2*(216*a**4*b*c**2*d**3 + 378*a**3
*b**2*c**5*d**3) + x*(18*a**5*d**2 + 144*a**4*b*c**3*d**2 + 126*a**3*b**2*
c**6*d**2)) + RootSum(19683*_t**3*a**10 - 2744*b, Lambda(_t, _t*log(x + (7
29*_t**2*a**7 + 196*b*c)/(196*b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^3 (dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*d^6*x^6 + 168*b^2*c*d^5*x^5 + 420*b^2*c^2*d^4*x^4 + 28*b^2*c
^6 + 7*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 49*a*b*c^3 + 21*(20*b^2*c^4 + 7*a*b*
c)*d^2*x^2 + 21*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 18*a^2)/(a^3*b^2*d^8*x^7 + 7
*a^3*b^2*c*d^7*x^6 + 21*a^3*b^2*c^2*d^6*x^5 + (35*a^3*b^2*c^3 + 2*a^4*b)*d
^5*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*x^3 + 3*(7*a^3*b^2*c^5 + 4*a^4*b
*c^2)*d^3*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)*d^2*x + (a^3*b^2*c^7 +
 2*a^4*b*c^4 + a^5*c)*d) - 14/9*b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d
^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \frac{14\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{1}{(dx+c)d}\right|\right)}{27\,a^3} - \frac{14\sqrt{3}(a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}-\frac{2}{(dx+c)d}\right)}{3\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}\right)}{27\,a^4d} - \frac{7\,(a^2b)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3}\right)^{\frac{1}{3}}}{(dx+c)d}+\frac{1}{(dx+c)^2d^2}\right)}{27\,a^4d} - \frac{\frac{10\,b^2}{(dx+c)d}+\frac{13\,ab}{(dx+c)^4d}}{18\,a^3\left(b+\frac{a}{(dx+c)^3}\right)^2} - \frac{1}{(dx+c)a^3d}$$

input
```
integrate(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
14/27*(b/(a*d^3))^(1/3)*log(abs(-(b/(a*d^3))^(1/3) - 1/((d*x + c)*d)))/a^3
 - 14/27*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3))^(1/3) - 2/(
(d*x + c)*d))/(b/(a*d^3))^(1/3))/(a^4*d) - 7/27*(a^2*b)^(1/3)*log((b/(a*d^
3))^(2/3) - (b/(a*d^3))^(1/3)/((d*x + c)*d) + 1/((d*x + c)^2*d^2))/(a^4*d)
 - 1/18*(10*b^2/((d*x + c)*d) + 13*a*b/((d*x + c)^4*d))/(a^3*(b + a/(d*x +
 c)^3)^2) - 1/((d*x + c)*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.97

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \frac{14\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{27\,a^{10/3}\,d}$$
$$-\frac{\frac{18\,a^2+49\,a\,b\,c^3+28\,b^2\,c^6}{18\,a^3\,d}+\frac{7\,x^2\,\left(20\,d\,b^2\,c^4+7\,a\,d\,b\,c\right)}{6\,a^3}+\frac{7\,x\,\left(8\,b^2\,c^5+7\,a\,b\,c^2\right)}{6\,a^3}+\frac{7\,x^3\,\left(80\,b^2\,c^3}{18}}{x^4\,\left(35\,b^2\,c^3\,d^4+2\,a\,b\,d^4\right)+x\,\left(d\,a^2+8\,d\,a\,b\,c^3+7\,d\,b^2\,c^6\right)+a^2\,c+x^3\,\left(35\,b^2\,c^4\,d^3+8\,a\,b\,c\,d^3\right)+b^2\,c^7+}$$
$$+\frac{14\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{10/3}\,d}$$
$$-\frac{14\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{10/3}\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^2),x)
```

output
```
(14*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(10/3)*d) - ((18
*a^2 + 28*b^2*c^6 + 49*a*b*c^3)/(18*a^3*d) + (7*x^2*(20*b^2*c^4*d + 7*a*b*
c*d))/(6*a^3) + (7*x*(8*b^2*c^5 + 7*a*b*c^2))/(6*a^3) + (7*x^3*(80*b^2*c^3
*d^2 + 7*a*b*d^2))/(18*a^3) + (14*b^2*d^5*x^6)/(9*a^3) + (70*b^2*c^2*d^3*x
^4)/(3*a^3) + (28*b^2*c*d^4*x^5)/(3*a^3))/(x^4*(35*b^2*c^3*d^4 + 2*a*b*d^4
) + x*(a^2*d + 7*b^2*c^6*d + 8*a*b*c^3*d) + a^2*c + x^3*(35*b^2*c^4*d^3 +
8*a*b*c*d^3) + b^2*c^7 + x^2*(21*b^2*c^5*d^2 + 12*a*b*c^2*d^2) + b^2*d^7*x
^7 + 2*a*b*c^4 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5) + (14*b^(1/3)*log(2
*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2
 - 1/2))/(27*a^(10/3)*d) - (14*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(27*a^(10/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2307, normalized size of antiderivative = 10.53

$$\int \frac{1}{(c+dx)^2\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(a+b*(d*x+c)^3)^3,x)
```

output

```
(28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt
(3)))*a**2*b*c**2 + 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*
d*x)/(a**(1/3)*sqrt(3)))*a**2*b*c*d*x + 56*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**5 + 224*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*
*4*d*x + 336*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a*b**2*c**3*d**2*x**2 + 224*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2*d**3*x**3 + 56*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*
a*b**2*c*d**4*x**4 + 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**8 + 196*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7*d*x + 588*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**
6*d**2*x**2 + 980*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*b**3*c**5*d**3*x**3 + 980*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**4*x**4 + 588*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3))
)*b**3*c**3*d**5*x**5 + 196*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(
1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**2*d**6*x**6 + 28*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**7*x*...
```

3.83 $\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 654
Mathematica [A] (verified) . . . 655
Rubi [A] (verified) . . . 655
Maple [C] (verified) . . . 665
Fricas [B] (verification not implemented) . . . 666
Sympy [B] (verification not implemented) . . . 667
Maxima [F] . . . 668
Giac [B] (verification not implemented) . . . 669
Mupad [B] (verification not implemented) . . . 670
Reduce [B] (verification not implemented) . . . 671

Optimal result

Integrand size = 21, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = &-\frac{1}{2a^3d(c+dx)^2}-\frac{b(c+dx)}{6a^2d\left(a+b(c+dx)^3\right)^2}\\ &-\frac{11b(c+dx)}{18a^3d\left(a+b(c+dx)^3\right)}\\ &+\frac{20b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}d}\\ &-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{11/3}d}\\ &+\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}d}\end{aligned}$$

output

```
-1/2/a^3/d/(d*x+c)^2-1/6*b*(d*x+c)/a^2/d/(a+b*(d*x+c)^3)^2-11/18*b*(d*x+c)
/a^3/d/(a+b*(d*x+c)^3)+20/27*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c)
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(11/3)/d-20/27*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*
x+c))/a^(11/3)/d+10/27*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*
(d*x+c)^2)/a^(11/3)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}}{(c+dx)^2} - \frac{9a^{5/3}b(c+dx)}{\left(a+b(c+dx)^3\right)^2} - \frac{33a^{2/3}b(c+dx)}{a+b(c+dx)^3} - 40\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+\right.}{54a^{11/3}d}$$

input
```
Integrate[1/((c + d*x)^3*(a + b*(c + d*x)^3)^3),x]
```

output
```
((-27*a^(2/3))/(c + d*x)^2 - (9*a^(5/3)*b*(c + d*x))/(a + b*(c + d*x)^3)^2
 - (33*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 40*Sqrt[3]*b^(2/3)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 40*b^(2/3)*Log[a^(
1/3) + b^(1/3)*(c + d*x)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c +
d*x) + b^(2/3)*(c + d*x)^2])/(54*a^(11/3)*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {895, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^3}\,d(c+dx)}{d}$$

$$\downarrow 819$$

$$\frac{4\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{3a} + \frac{1}{6a(c+dx)^2\left(a+b(c+dx)^3\right)^2}$$

$$d$$

$$\downarrow 819$$

$$\frac{4\left(\frac{5\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}d(c+dx)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b(c+dx)^3)^2}}{d}$$

$$\downarrow 847$$

$$\frac{4\left(\frac{5\left(-\frac{b\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b(c+dx)^3)^2}}{d}$$

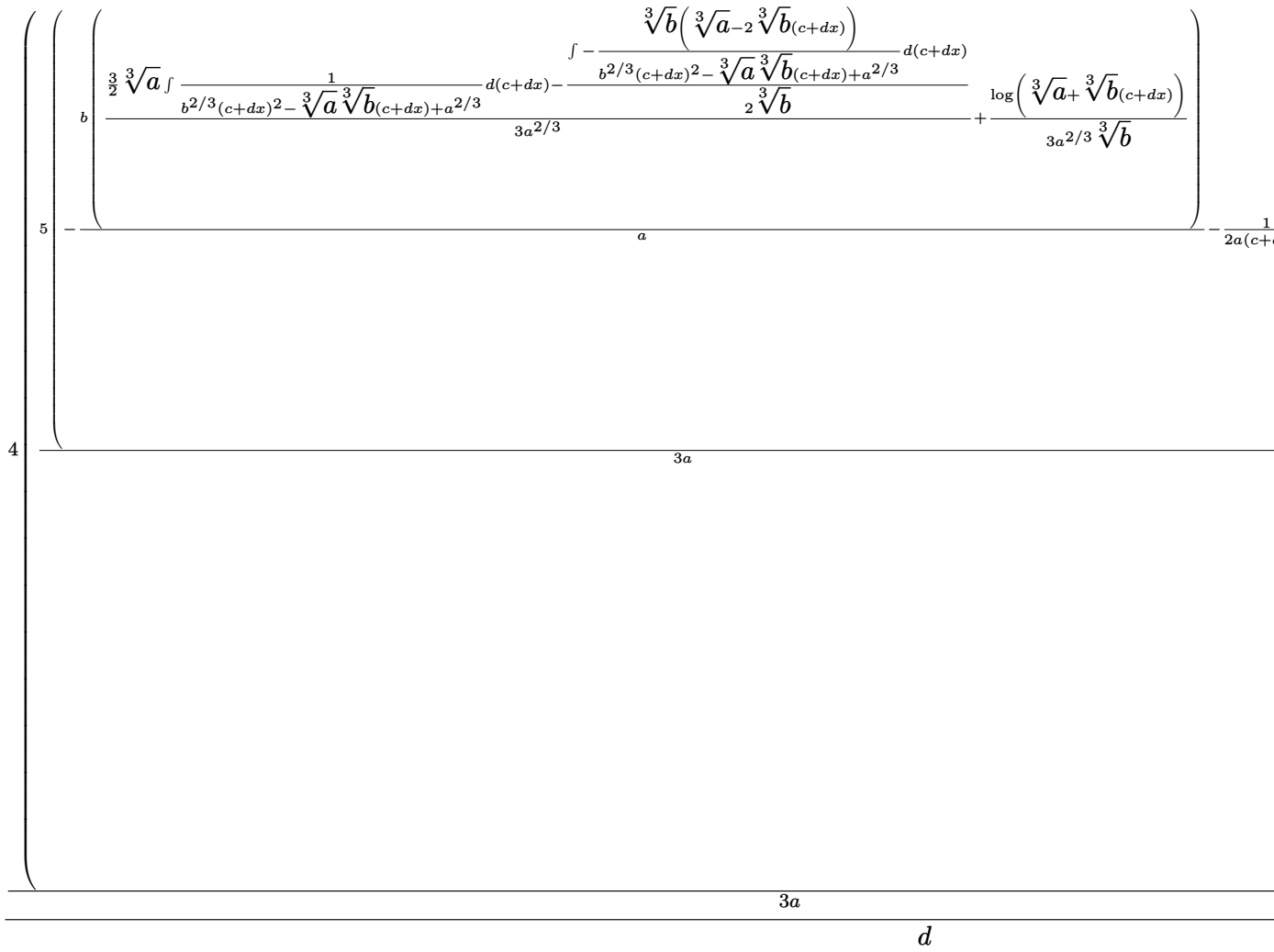
$$\downarrow 750$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(}}{d}$$

$$\downarrow 16$$

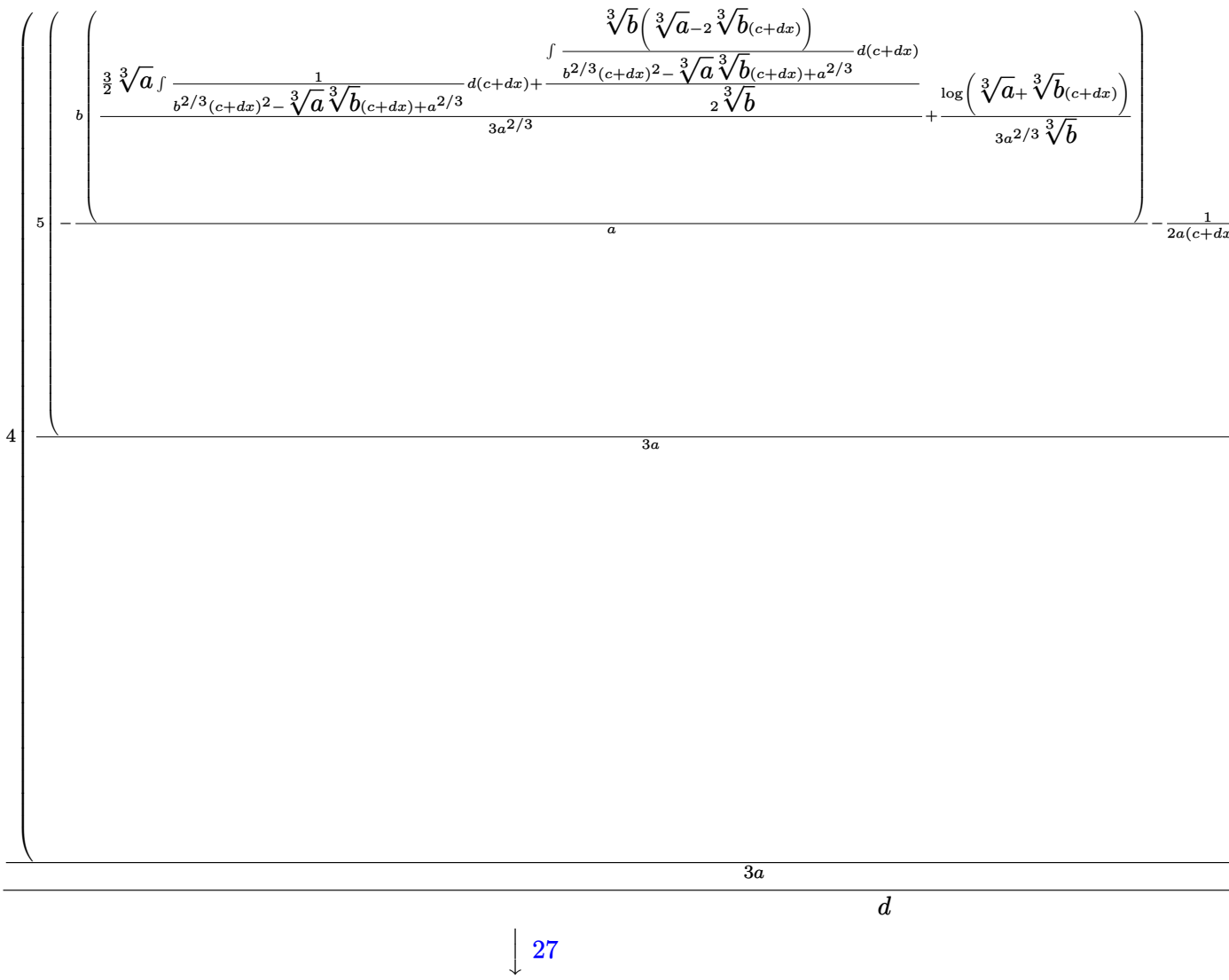
$$\frac{\left(4\left(\frac{\left(5\left(-\frac{\left(b\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b}\right)}{d}$$

$$\downarrow 1142$$

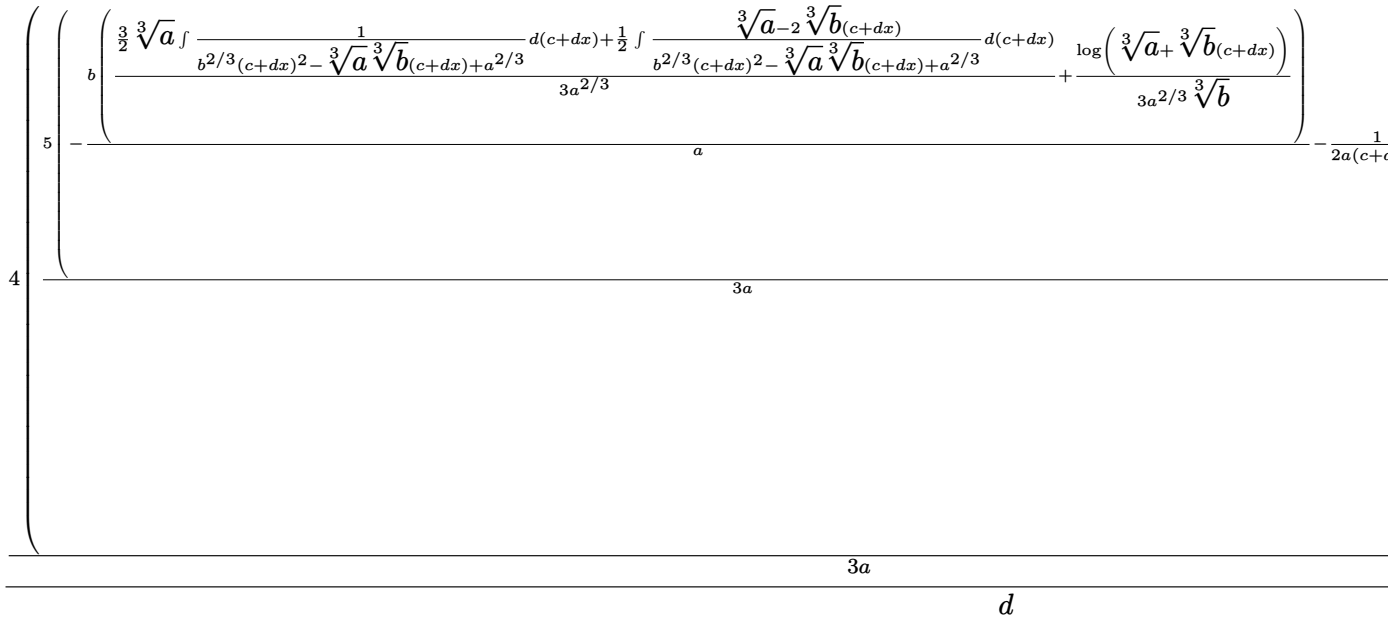


$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+}\right.}{3a}\right.}{3a}}{d}$$

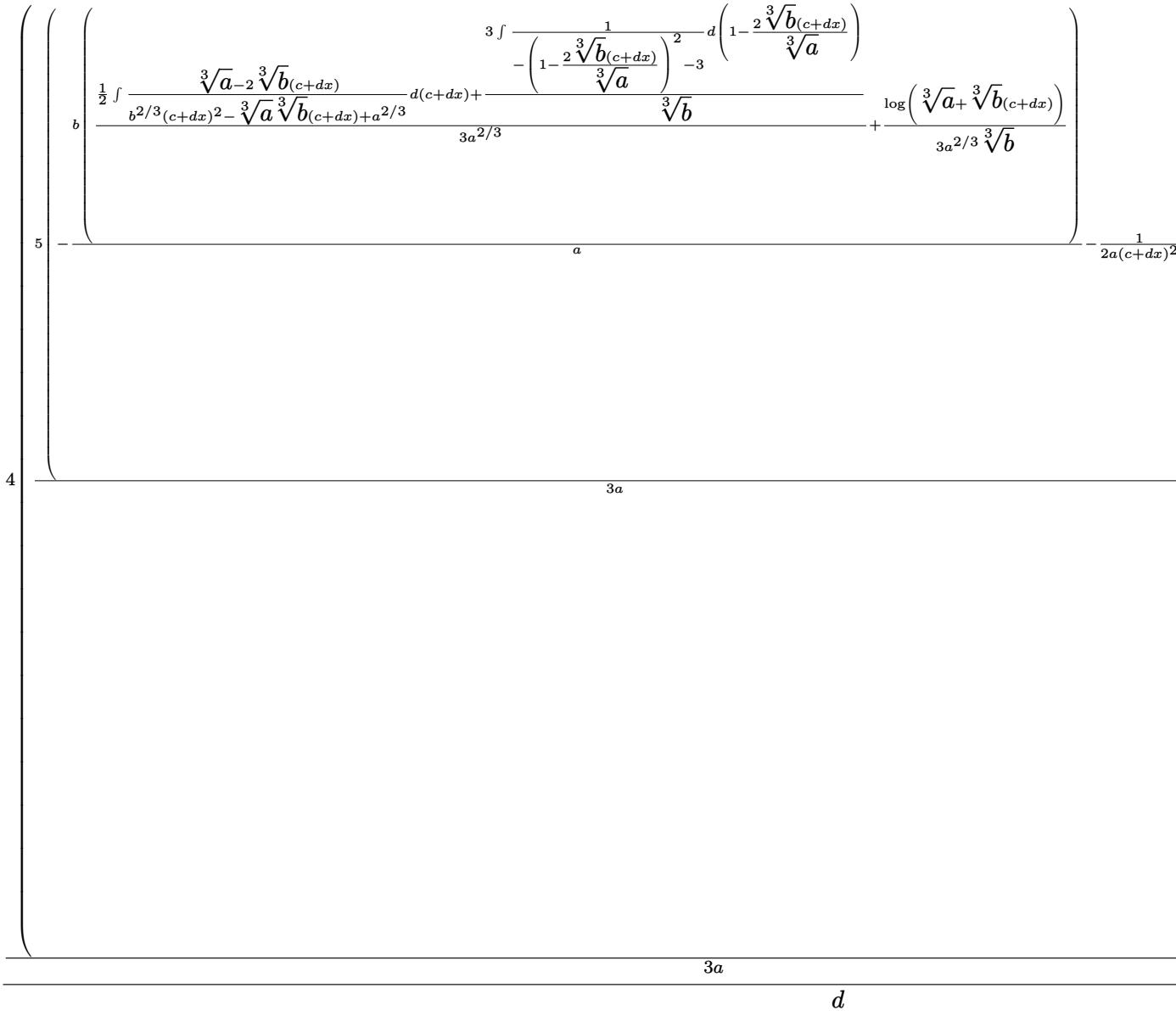
$\downarrow$ 25



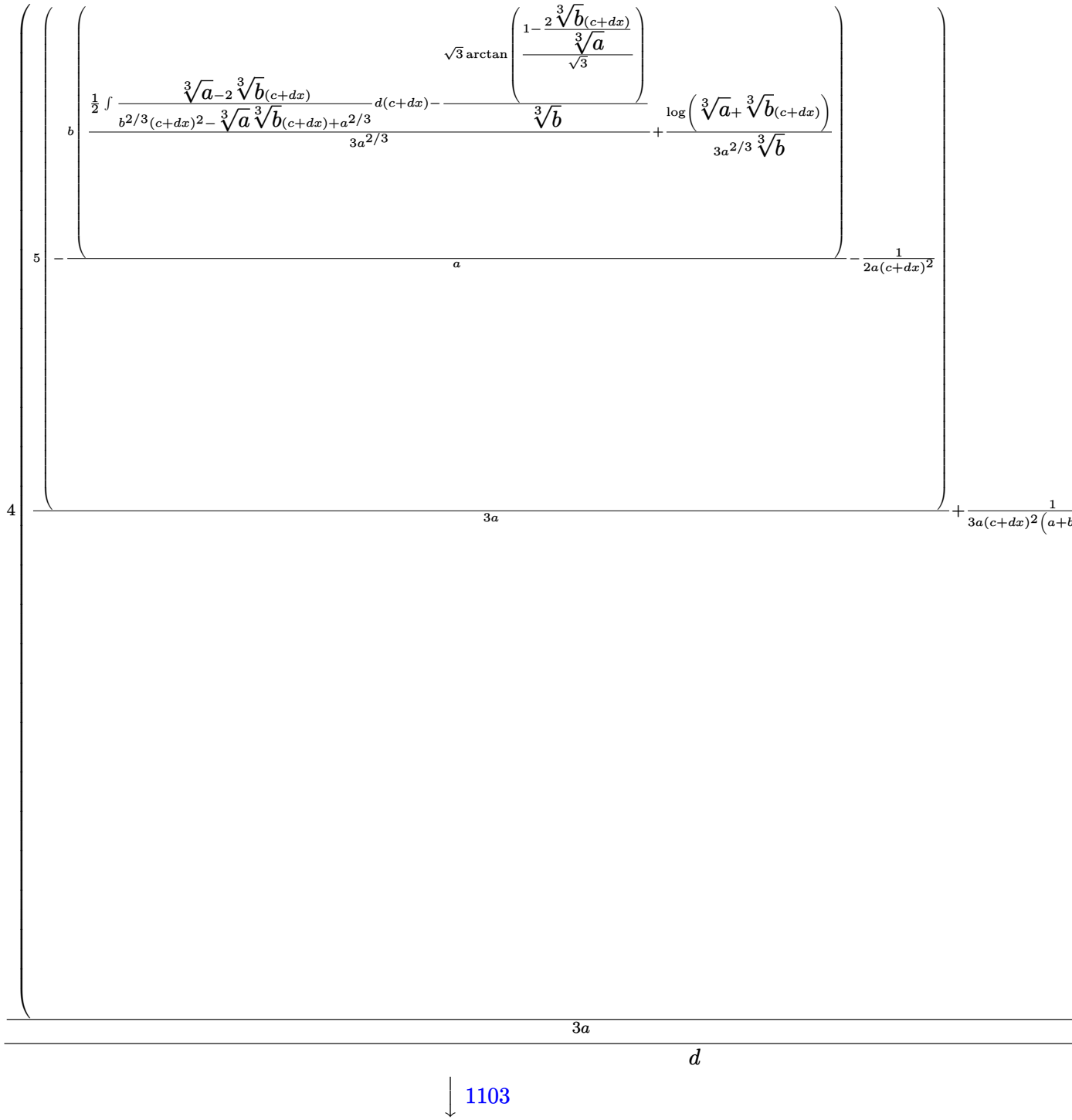
$\downarrow$ 27



$$\downarrow 1082$$

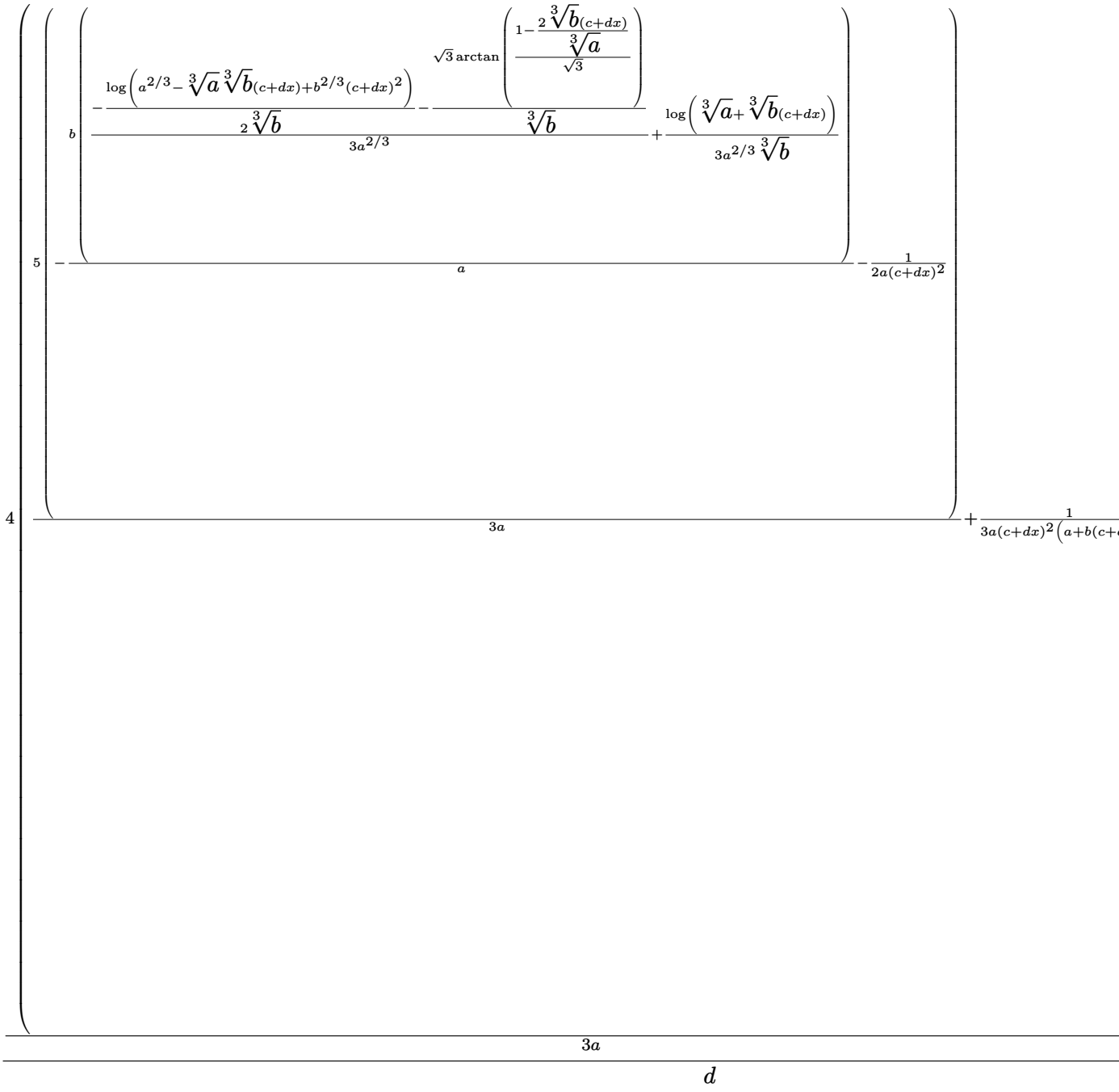


217



$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b\right.}\right.}{3a}}{d}$$

$\downarrow$ 1103



input `Int[1/((c + d*x)^3*(a + b*(c + d*x)^3)^3),x]`

output
```
(1/(6*a*(c + d*x)^2*(a + b*(c + d*x)^3)^2) + (4*(1/(3*a*(c + d*x)^2*(a + b
*(c + d*x)^3)) + (5*(-1/2*1/(a*(c + d*x)^2) - (b*(Log[a^(1/3) + b^(1/3)*(c
 + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x
))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a)))/(3*a))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.87

method	result
default	$-\frac{1}{2a^3d(xd+c)^2}-\frac{b\left(\frac{\frac{11b\,d^3x^4}{18}+\frac{22d^2x^3bc}{9}+\frac{11b\,c^2d\,x^2}{3}+\left(\frac{22c^3b}{9}+\frac{7a}{9}\right)x+\frac{c\left(11c^3b+14a\right)}{18d}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{20\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\right.}\right.}{27bd}\right.}{a^3}$
risch	$\frac{-\frac{10b^2d^5x^6}{9a^3}-\frac{20b^2c\,d^4x^5}{3a^3}-\frac{50b^2c^2d^3x^4}{3a^3}-\frac{8d^2b\left(25c^3b+2a\right)x^3}{9a^3}-\frac{2bcd\left(25c^3b+8a\right)x^2}{3a^3}-\frac{4b\,c^2\left(5c^3b+4a\right)x}{3a^3}-\frac{20b^2c^6+32ab\,c^3+9a^2}{18d\,a^3}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2(xd+c)^2}+\frac{20\left(\sum_{\_R=}\right.}{}$

input
```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3/d/(d*x+c)^2-1/a^3*b*((11/18*b*d^3*x^4+22/9*d^2*x^3*b*c+11/3*b*c^2
*d*x^2+(22/9*c^3*b+7/9*a)*x+1/18*c/d*(11*b*c^3+14*a))/(b*d^3*x^3+3*b*c*d^2
*x^2+3*b*c^2*d*x+b*c^3+a)^2+20/27/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_
R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(174) = 348$.

Time = 0.16 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.72

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
-1/54*(60*b^2*d^6*x^6 + 360*b^2*c*d^5*x^5 + 900*b^2*c^2*d^4*x^4 + 60*b^2*c
^6 + 48*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 96*a*b*c^3 + 36*(25*b^2*c^4 + 8*a*b
*c)*d^2*x^2 + 72*(5*b^2*c^5 + 4*a*b*c^2)*d*x - 40*sqrt(3)*(b^2*d^8*x^8 + 8
*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c
^8 + 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^
2)*d^3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*
c^7 + 5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(a*d*
x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 20*(b^2*d^8*x^8 + 8*b^2*c*d^7*
x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 + 10*(7*
b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^3*x^3
 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 + 5*a*b
*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + a^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 40*(b^2*d
^8*x^8 + 8*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x
^5 + b^2*c^8 + 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5
 + 5*a*b*c^2)*d^3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 +
 2*(4*b^2*c^7 + 5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c -
 a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*d^9*x^8 + 8*a^3*b^2*c*d^8*x^7 + 28
*a^3*b^2*c^2*d^7*x^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d^6*x^5 + 10*(7*a^3*b^2*
c^4 + a^4*b*c)*d^5*x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*b*c^2)*d^4*x^3 + (28...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(199) = 398$.

Time = 3.26 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.00

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-9a^2 - 32abc^3 - 20b^2c^6 - 300b^2c^2d^4x^4 - 12}{18a^5c^2d + 36a^4bc^5d + 18a^3b^2c^8d + 504a^3b^2c^2d^7x^6 + 144a^3b^2cd^8x^7 + 18a^3b^2d^9x^8 + x^5\cdot(36a^4bd^6 + 1008a} + \frac{\operatorname{RootSum}\left(19683t^3a^{11} + 8000b^2, \left(t \mapsto t\log\left(x + \frac{-27ta^4+20bc}{20bd}\right)\right)\right)}{d}$$

input

```
integrate(1/(d*x+c)**3/(a+b*(d*x+c)**3)**3,x)
```

output

```
(-9*a**2 - 32*a*b*c**3 - 20*b**2*c**6 - 300*b**2*c**2*d**4*x**4 - 120*b**2
*c*d**5*x**5 - 20*b**2*d**6*x**6 + x**3*(-32*a*b*d**3 - 400*b**2*c**3*d**3
) + x**2*(-96*a*b*c*d**2 - 300*b**2*c**4*d**2) + x*(-96*a*b*c**2*d - 120*b
**2*c**5*d))/(18*a**5*c**2*d + 36*a**4*b*c**5*d + 18*a**3*b**2*c**8*d + 50
4*a**3*b**2*c**2*d**7*x**6 + 144*a**3*b**2*c*d**8*x**7 + 18*a**3*b**2*d**9
*x**8 + x**5*(36*a**4*b*d**6 + 1008*a**3*b**2*c**3*d**6) + x**4*(180*a**4*
b*c*d**5 + 1260*a**3*b**2*c**4*d**5) + x**3*(360*a**4*b*c**2*d**4 + 1008*a
**3*b**2*c**5*d**4) + x**2*(18*a**5*d**3 + 360*a**4*b*c**3*d**3 + 504*a**3
*b**2*c**6*d**3) + x*(36*a**5*c*d**2 + 180*a**4*b*c**4*d**2 + 144*a**3*b**
2*c**7*d**2)) + RootSum(19683*_t**3*a**11 + 8000*b**2, Lambda(_t, _t*log(x
 + (-27*_t*a**4 + 20*b*c)/(20*b*d))))/d
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^3 (dx+c)^3}\,dx$$

input

```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
-1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 20*b^2*c
^6 + 16*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 32*a*b*c^3 + 12*(25*b^2*c^4 + 8*a*b
*c)*d^2*x^2 + 24*(5*b^2*c^5 + 4*a*b*c^2)*d*x + 9*a^2)/(a^3*b^2*d^9*x^8 + 8
*a^3*b^2*c*d^8*x^7 + 28*a^3*b^2*c^2*d^7*x^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d
^6*x^5 + 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*
b*c^2)*d^4*x^3 + (28*a^3*b^2*c^6 + 20*a^4*b*c^3 + a^5)*d^3*x^2 + 2*(4*a^3*
b^2*c^7 + 5*a^4*b*c^4 + a^5*c)*d^2*x + (a^3*b^2*c^8 + 2*a^4*b*c^5 + a^5*c^
2)*d) - 20/9*b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a), x)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(174) = 348.

Time = 0.16 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.68

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{10\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right.\right.\right.}{27\,a^3}$$

$$-\frac{20\,b^2d^6x^6+120\,b^2cd^5x^5+300\,b^2c^2d^4x^4+400\,b^2c^3d^3x^3+300\,b^2c^4d^2x^2+120\,b^2c^5dx+20\,b^2c^6+32\,abd}{18\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+adx+ac\right.}$$

input
```
integrate(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
10/27*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1
/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*
d^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2
 + 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(
-b*d*x - b*c + (-a*b^2)^(1/3))))/a^3 - 1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^
5*x^5 + 300*b^2*c^2*d^4*x^4 + 400*b^2*c^3*d^3*x^3 + 300*b^2*c^4*d^2*x^2 +
120*b^2*c^5*d*x + 20*b^2*c^6 + 32*a*b*d^3*x^3 + 96*a*b*c*d^2*x^2 + 96*a*b*
c^2*d*x + 32*a*b*c^3 + 9*a^2)/((b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^
2 + 4*b*c^3*d*x + b*c^4 + a*d*x + a*c)^2*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.40

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = \frac{20\,b^{2/3}\,\ln\left(a^6\,b^{1/3}\,c-(-a)^{19/3}+a^6\,b^{1/3}\,d\,x\right)}{27\,(-a)^{11/3}\,d}$$
$$-\frac{\frac{9\,a^2+32\,a\,b\,c^3+20\,b^2\,c^6}{18\,a^3\,d}+\frac{2\,x^2\,\left(25\,d\,b^2\,c^4+8\,a\,d\,b\,c\right)}{3\,a^3}+\frac{4\,x\,(5\,b^2}{}}{x^5\,\left(56\,b^2\,c^3\,d^5+2\,a\,b\,d^5\right)+x^4\,\left(70\,b^2\,c^4\,d^4+10\,a\,b\,c\,d^4\right)+x\,\left(2\,d\,a^2\,c+10\,d\,a\,b\,c^4+8\,d\,b^2\,c^7\right)+x^2\,\left(a^2\,d\right.}$$
$$+\frac{20\,b^{2/3}\,\ln\left(4860\,a^6\,b^3\,c\,d^5+4860\,a^6\,b^3\,d^6\,x-4860\,(-a)^{19/3}\,b^{8/3}\,d^5\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,(-a)^{11/3}\,d}$$
$$-\frac{20\,b^{2/3}\,\ln\left(4860\,a^6\,b^3\,c\,d^5+4860\,a^6\,b^3\,d^6\,x+4860\,(-a)^{19/3}\,b^{8/3}\,d^5\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,(-a)^{11/3}\,d}$$

input

```
int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^3),x)
```

output

```
(20*b^(2/3)*log(a^6*b^(1/3)*c - (-a)^(19/3) + a^6*b^(1/3)*d*x))/(27*(-a)^(
11/3)*d) - ((9*a^2 + 20*b^2*c^6 + 32*a*b*c^3)/(18*a^3*d) + (2*x^2*(25*b^2*
c^4*d + 8*a*b*c*d))/(3*a^3) + (4*x*(5*b^2*c^5 + 4*a*b*c^2))/(3*a^3) + (8*x
^3*(25*b^2*c^3*d^2 + 2*a*b*d^2))/(9*a^3) + (10*b^2*d^5*x^6)/(9*a^3) + (50*
b^2*c^2*d^3*x^4)/(3*a^3) + (20*b^2*c*d^4*x^5)/(3*a^3))/(x^5*(56*b^2*c^3*d^
5 + 2*a*b*d^5) + x^4*(70*b^2*c^4*d^4 + 10*a*b*c*d^4) + x*(8*b^2*c^7*d + 2*
a^2*c*d + 10*a*b*c^4*d) + x^2*(a^2*d^2 + 28*b^2*c^6*d^2 + 20*a*b*c^3*d^2)
+ a^2*c^2 + b^2*c^8 + x^3*(56*b^2*c^5*d^3 + 20*a*b*c^2*d^3) + b^2*d^8*x^8
+ 2*a*b*c^5 + 8*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6) + (20*b^(2/3)*log(4860
*a^6*b^3*c*d^5 + 4860*a^6*b^3*d^6*x - 4860*(-a)^(19/3)*b^(8/3)*d^5*((3^(1/
2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(27*(-a)^(11/3)*d) - (20*b^(2/3)*
log(4860*a^6*b^3*c*d^5 + 4860*a^6*b^3*d^6*x + 4860*(-a)^(19/3)*b^(8/3)*d^5
*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(27*(-a)^(11/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2802, normalized size of antiderivative = 12.91

$$\int \frac{1}{(c+dx)^3\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(d*x+c)^3/(a+b*(d*x+c)^3)^3,x)
```

output

```
(40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a**2*b*c**2 + 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**2*b*c*d*x + 40*a**(1/3)*sqrt
(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*
*2*b*d**2*x**2 + 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**
(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**5 + 400*a**(1/3)*sqrt(3)*atan((a*
*(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4*d*
x + 800*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*a*b**2*c**3*d**2*x**2 + 800*a**(1/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2*d**3
*x**3 + 400*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a*b**2*c*d**4*x**4 + 80*a**(1/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*d**5*x**5
 + 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*b**3*c**8 + 320*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7*d*x + 1120*a**(1/3)*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3))
)*b**3*c**6*d**2*x**2 + 2240*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**5*d**3*x**3 + 2800*a**(1/3
)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqr...
```

3.84 $\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result 672
Mathematica [A] (verified) 672
Rubi [A] (verified) 673
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Giac [A] (verification not implemented) 677
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Reduce [B] (verification not implemented) 678

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{1}{3a^3d(c+dx)^3} - \frac{b}{6a^2d\left(a+b(c+dx)^3\right)^2} - \frac{2b}{3a^3d\left(a+b(c+dx)^3\right)} - \frac{3b\log(c+dx)}{a^4d} + \frac{b\log\left(a+b(c+dx)^3\right)}{a^4d}$$

output

```
-1/3/a^3/d/(d*x+c)^3-1/6*b/a^2/d/(a+b*(d*x+c)^3)^2-2/3*b/a^3/d/(a+b*(d*x+c
)^3)-3*b*ln(d*x+c)/a^4/d+b*ln(a+b*(d*x+c)^3)/a^4/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{a\left(-\frac{2}{(c+dx)^3} - \frac{ab}{\left(a+b(c+dx)^3\right)^2} - \frac{4b}{a+b(c+dx)^3}\right) - 18b\log(c+dx) + 6b\log\left(a+b(c+dx)^3\right)}{6a^4d}$$

input
```
Integrate[1/((c + d*x)^4*(a + b*(c + d*x)^3)^3),x]
```

output
```
(a*(-2/(c + d*x)^3 - (a*b)/(a + b*(c + d*x)^3)^2 - (4*b)/(a + b*(c + d*x)^
3)) - 18*b*Log[c + d*x] + 6*b*Log[a + b*(c + d*x)^3])/(6*a^4*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)^3}\,d(c+dx)^3}{3d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{3b^2}{a^4(b(c+dx)^3+a)}+\frac{2b^2}{a^3(b(c+dx)^3+a)^2}+\frac{b^2}{a^2(b(c+dx)^3+a)^3}-\frac{3b}{a^4(c+dx)^3}+\frac{1}{a^3(c+dx)^6}\right)d(c+dx)^3}{3d}$$

$$\downarrow 2009$$

$$\frac{-\frac{3b\log\left((c+dx)^3\right)}{a^4}+\frac{3b\log\left(a+b(c+dx)^3\right)}{a^4}-\frac{2b}{a^3(a+b(c+dx)^3)}-\frac{1}{a^3(c+dx)^3}-\frac{b}{2a^2(a+b(c+dx)^3)^2}}{3d}$$

input
```
Int[1/((c + d*x)^4*(a + b*(c + d*x)^3)^3),x]
```

output
```
(-(1/(a^3*(c + d*x)^3)) - b/(2*a^2*(a + b*(c + d*x)^3)^2) - (2*b)/(a^3*(a
+ b*(c + d*x)^3)) - (3*b*Log[(c + d*x)^3])/a^4 + (3*b*Log[a + b*(c + d*x)^
3])/a^4)/(3*d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

method	result
default	$-\frac{1}{3a^3d(xd+c)^3}-\frac{3b\ln(xd+c)}{a^4d}+\frac{b^2\left(\frac{-\frac{2ad^2x^3}{3}-2acdx^2-2ac^2x-\frac{a\left(4c^3b+5a\right)}{6db}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{bd}\right)}{a^4}$
risch	$\frac{-\frac{b^2d^5x^6}{a^3}-\frac{6b^2cd^4x^5}{a^3}-\frac{15b^2c^2d^3x^4}{a^3}-\frac{d^2\left(40c^3b+3a\right)bx^3}{2a^3}-\frac{3bcd\left(10c^3b+3a\right)x^2}{2a^3}-\frac{3\left(4c^3b+3a\right)bc^2x}{2a^3}-\frac{6b^2c^6+9abc^3+2a^2}{6da^3}}{(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}-\frac{3b\ln(xd$
norman	$\frac{-\frac{b^2d^5x^6}{a^3}+\frac{-6b^4c^6d^8-9ab^3c^3d^8-2a^2b^2d^8}{6a^3d^9b^2}+\frac{\left(-40b^4c^3d^8-3ab^3d^8\right)x^3}{2a^3d^6b^2}+\frac{3c\left(-10b^4c^3d^8-3ab^3d^8\right)x^2}{2a^3d^7b^2}+\frac{3c^2\left(-4b^4c^3d^8-3ab^3d^8\right)x}{2a^3d^8b^2}-\frac{6b}{}}{(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}$
parallelrisch	Expression too large to display

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3/d/(d*x+c)^3-3*b*ln(d*x+c)/a^4/d+1/a^4*b^2*((-2/3*a*d^2*x^3-2*a*c*
d*x^2-2*a*c^2*x-1/6*a/d*(4*b*c^3+5*a)/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*
d*x+b*c^3+a)^2+1/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 889, normalized size of antiderivative = 8.80

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-1/6*(6*a*b^2*d^6*x^6 + 36*a*b^2*c*d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^
2*c^6 + 3*(40*a*b^2*c^3 + 3*a^2*b)*d^3*x^3 + 9*a^2*b*c^3 + 9*(10*a*b^2*c^4
 + 3*a^2*b*c)*d^2*x^2 + 2*a^3 + 9*(4*a*b^2*c^5 + 3*a^2*b*c^2)*d*x - 6*(b^3
*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d
^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(2
1*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3
*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*
b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a) + 18*(b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^
7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2
*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(21*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3
*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^
2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*lo
g(d*x + c))/(a^4*b^2*d^10*x^9 + 9*a^4*b^2*c*d^9*x^8 + 36*a^4*b^2*c^2*d^8*x
^7 + 2*(42*a^4*b^2*c^3 + a^5*b)*d^7*x^6 + 6*(21*a^4*b^2*c^4 + 2*a^5*b*c)*d
^6*x^5 + 6*(21*a^4*b^2*c^5 + 5*a^5*b*c^2)*d^5*x^4 + (84*a^4*b^2*c^6 + 40*a
^5*b*c^3 + a^6)*d^4*x^3 + 3*(12*a^4*b^2*c^7 + 10*a^5*b*c^4 + a^6*c)*d^3*x^
2 + 3*(3*a^4*b^2*c^8 + 4*a^5*b*c^5 + a^6*c^2)*d^2*x + (a^4*b^2*c^9 + 2*a^5
*b*c^6 + a^6*c^3)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(88) = 176.

Time = 3.65 (sec) , antiderivative size = 500, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-2a^2-9abc^3-6b^2c^6}{6a^5c^3d+12a^4bc^6d+6a^3b^2c^9d+216a^3b^2c^2d^8x^7+54a^3b^2cd^9x^8+6a^3b^2d^{10}x^9+x^6\cdot(12a^4bd^7+504a^3b^2c^3}$$
$$-\frac{3b\log\left(\frac{c}{d}+x\right)}{a^4d}+\frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{a^4d}$$

input
```
integrate(1/(d*x+c)**4/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-2*a**2 - 9*a*b*c**3 - 6*b**2*c**6 - 90*b**2*c**2*d**4*x**4 - 36*b**2*c*d
**5*x**5 - 6*b**2*d**6*x**6 + x**3*(-9*a*b*d**3 - 120*b**2*c**3*d**3) + x*
*2*(-27*a*b*c*d**2 - 90*b**2*c**4*d**2) + x*(-27*a*b*c**2*d - 36*b**2*c**5
*d))/(6*a**5*c**3*d + 12*a**4*b*c**6*d + 6*a**3*b**2*c**9*d + 216*a**3*b**
2*c**2*d**8*x**7 + 54*a**3*b**2*c*d**9*x**8 + 6*a**3*b**2*d**10*x**9 + x**
6*(12*a**4*b*d**7 + 504*a**3*b**2*c**3*d**7) + x**5*(72*a**4*b*c*d**6 + 75
6*a**3*b**2*c**4*d**6) + x**4*(180*a**4*b*c**2*d**5 + 756*a**3*b**2*c**5*d
**5) + x**3*(6*a**5*d**4 + 240*a**4*b*c**3*d**4 + 504*a**3*b**2*c**6*d**4)
 + x**2*(18*a**5*c*d**3 + 180*a**4*b*c**4*d**3 + 216*a**3*b**2*c**7*d**3)
+ x*(18*a**5*c**2*d**2 + 72*a**4*b*c**5*d**2 + 54*a**3*b**2*c**8*d**2)) -
3*b*log(c/d + x)/(a**4*d) + b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a +
 b*c**3)/(b*d**3))/(a**4*d)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(95) = 190.

Time = 0.07 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.34

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx =$$
$$-\frac{6\,b^2d^6x^6+36\,b^2cd^5x^5+90\,b^2c^2d^4x^4+}{6\left(a^3b^2d^{10}x^9+9\,a^3b^2cd^9x^8+36\,a^3b^2c^2d^8x^7+2\left(42\,a^3b^2c^3+a^4b\right)d^7x^6+6\left(21\,a^3b^2c^4+2\,a^4bc\right)d^6x^5+6\right.}$$
$$+\frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{a^4d}-\frac{3\,b\log\left(dx+c\right)}{a^4d}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/6*(6*b^2*d^6*x^6 + 36*b^2*c*d^5*x^5 + 90*b^2*c^2*d^4*x^4 + 6*b^2*c^6 +
3*(40*b^2*c^3 + 3*a*b)*d^3*x^3 + 9*a*b*c^3 + 9*(10*b^2*c^4 + 3*a*b*c)*d^2*
x^2 + 9*(4*b^2*c^5 + 3*a*b*c^2)*d*x + 2*a^2)/(a^3*b^2*d^10*x^9 + 9*a^3*b^2
*c*d^9*x^8 + 36*a^3*b^2*c^2*d^8*x^7 + 2*(42*a^3*b^2*c^3 + a^4*b)*d^7*x^6 +
 6*(21*a^3*b^2*c^4 + 2*a^4*b*c)*d^6*x^5 + 6*(21*a^3*b^2*c^5 + 5*a^4*b*c^2)
*d^5*x^4 + (84*a^3*b^2*c^6 + 40*a^4*b*c^3 + a^5)*d^4*x^3 + 3*(12*a^3*b^2*c
^7 + 10*a^4*b*c^4 + a^5*c)*d^3*x^2 + 3*(3*a^3*b^2*c^8 + 4*a^4*b*c^5 + a^5*
c^2)*d^2*x + (a^3*b^2*c^9 + 2*a^4*b*c^6 + a^5*c^3)*d) + b*log(b*d^3*x^3 +
3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^4*d) - 3*b*log(d*x + c)/(a^4*d
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{b\log\left(\left|-b-\frac{a}{(dx+c)^3}\right|\right)}{a^4 d} + \frac{5\,b^3+\frac{6\,ab^2}{(dx+c)^3}}{6\,a^4\left(b+\frac{a}{(dx+c)^3}\right)^2 d} - \frac{1}{3\,(dx+c)^3 a^3 d}$$

input
```
integrate(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
b*log(abs(-b - a/(d*x + c)^3))/(a^4*d) + 1/6*(5*b^3 + 6*a*b^2/(d*x + c)^3)
/(a^4*(b + a/(d*x + c)^3)^2*d) - 1/3/((d*x + c)^3*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.34

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{b\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{a^4\,d}$$
$$-\frac{\frac{2\,a^2+9\,a\,b\,c^3+6\,b^2\,c^6}{6\,a^3\,d}+\frac{3\,x^2}{}}{x\left(3\,d\,a^2\,c^2+12\,d\,a\,b\,c^5+9\,d\,b^2\,c^8\right)+x^6\left(84\,b^2\,c^3\,d^6+2\,a\,b\,d^6\right)+x^2\left(3\,a^2\,c\,d^2+30\,a\,b\,c^4\,d^2+36\,b^2\,c^7\,d\right.}$$
$$-\frac{3\,b\ln\left(c+d\,x\right)}{a^4\,d}$$

input
```
int(1/((a + b*(c + d*x)^3)^3*(c + d*x)^4),x)
```

output
```
(b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(a^4*d) - ((2
*a^2 + 6*b^2*c^6 + 9*a*b*c^3)/(6*a^3*d) + (3*x^2*(10*b^2*c^4*d + 3*a*b*c*d
))/(2*a^3) + (3*x*(4*b^2*c^5 + 3*a*b*c^2))/(2*a^3) + (x^3*(40*b^2*c^3*d^2
+ 3*a*b*d^2))/(2*a^3) + (b^2*d^5*x^6)/a^3 + (15*b^2*c^2*d^3*x^4)/a^3 + (6*
b^2*c*d^4*x^5)/a^3)/(x*(3*a^2*c^2*d + 9*b^2*c^8*d + 12*a*b*c^5*d) + x^6*(8
4*b^2*c^3*d^6 + 2*a*b*d^6) + x^2*(3*a^2*c*d^2 + 36*b^2*c^7*d^2 + 30*a*b*c^
4*d^2) + x^5*(126*b^2*c^4*d^5 + 12*a*b*c*d^5) + x^3*(a^2*d^3 + 84*b^2*c^6*
d^3 + 40*a*b*c^3*d^3) + a^2*c^3 + b^2*c^9 + x^4*(126*b^2*c^5*d^4 + 30*a*b*
c^2*d^4) + b^2*d^9*x^9 + 2*a*b*c^6 + 9*b^2*c*d^8*x^8 + 36*b^2*c^2*d^7*x^7)
 - (3*b*log(c + d*x))/(a^4*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2661, normalized size of antiderivative = 26.35

$$\int \frac{1}{(c+dx)^4\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^4/(a+b*(d*x+c)^3)^3,x)
```

output

```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2*b*c**3 + 18*log(a**(2/3)
 - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3
)*c*d*x + b**(2/3)*d**2*x**2)*a**2*b*c**2*d*x + 18*log(a**(2/3) - b**(1/3)
*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b
**(2/3)*d**2*x**2)*a**2*b*c*d**2*x**2 + 6*log(a**(2/3) - b**(1/3)*a**(1/3)
*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d
**2*x**2)*a**2*b*d**3*x**3 + 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1
/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*
a*b**2*c**6 + 72*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**5*d*x
 + 180*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/
3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**4*d**2*x**2 + 2
40*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**3*d**3*x**3 + 180*l
og(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
+ 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**2*d**4*x**4 + 72*log(a*
*(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b
**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c*d**5*x**5 + 12*log(a**(2/3) -
 b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/...
```

3.85 $\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$

Optimal result . . . 680
Mathematica [A] (verified) . . . 681
Rubi [A] (verified) . . . 681
Maple [C] (verified) . . . 687
Fricas [A] (verification not implemented) . . . 687
Sympy [A] (verification not implemented) . . . 688
Maxima [F] . . . 688
Giac [A] (verification not implemented) . . . 689
Mupad [B] (verification not implemented) . . . 689
Reduce [B] (verification not implemented) . . . 690

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx = \frac{e^4(c+dx)^2}{2bd} + \frac{a^{2/3}e^4 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{2/3}e^4 \log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}e^4 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}$$

output

```
1/2*e^4*(d*x+c)^2/b/d+1/3*a^(2/3)*e^4*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))*3^(1/2)/a^(1/3))*3^(1/2)/b^(5/3)/d+1/3*a^(2/3)*e^4*ln(a^(1/3)+b^(1/3)*(
d*x+c))/b^(5/3)/d-1/6*a^(2/3)*e^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/
3)*(d*x+c)^2)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.97

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx = e^4\left(\frac{(c+dx)^2}{2bd} - \frac{a^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{5/3}d} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{5/3}d}\right)$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3),x]
```

output
```
e^4*((c + d*x)^2/(2*b*d) - (a^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x)
)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*d) + (a^(2/3)*Log[a^(1/3) + b^(1/3)
*(c + d*x)])/(3*b^(5/3)*d) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d
*x) + b^(2/3)*(c + d*x)^2])/(6*b^(5/3)*d))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$$
$$\downarrow 895$$
$$\frac{e^4\int \frac{(c+dx)^4}{b(c+dx)^3+a}d(c+dx)}{d}$$

$$\downarrow 843$$

$$\frac{e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{b}\right)}{d}$$

$$\downarrow 821$$

$$\frac{e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{d}$$

$$\downarrow 1142$$

$$\frac{e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right.}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{d}$$

$$\downarrow 25$$

$$e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right.}{b}\right.$$

$$\Big/ d$$

$$\downarrow 27$$

$$e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right.}{b}\right.$$

$$\Big/ d$$

$$\downarrow 1082$$

$$e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$$\frac{}{d}$$

$$\downarrow 217$$

$$e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)$$

$$\frac{}{d}$$

$$\downarrow 1103$$

$$\frac{e^4\left(\frac{(c+dx)^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}\right)}{d}$$

input
```
Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3),x]
```

output
```
(e^4*((c + d*x)^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/
3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[
3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.57

method	result	size
default	$e^4\left(\frac{\frac{1}{2}x^2d+cx}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)a}{3b^2d}\right)$	96
risch	$\frac{e^4x^2d}{2b}+\frac{e^4cx}{b}+\frac{a\,e^4\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(-\_R_{d-c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3b^2d}$	105

input
```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
e^4*(1/b*(1/2*x^2*d+c*x)-1/3/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln
(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$$

$$=\frac{3\,d^2e^4x^2+6\,cde^4x-2\sqrt{3}e^4\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bdx+bc)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-e^4\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ad^2x^2+2\,acdx+ac\right.}{6\,bd}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(3*d^2*e^4*x^2 + 6*c*d*e^4*x - 2*sqrt(3)*e^4*(a^2/b^2)^(1/3)*arctan(1/
3*(2*sqrt(3)*(b*d*x + b*c)*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) - e^4*(a^2/b^2)
^(1/3)*log(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (b*d*x + b*c)*(a^2/b^2)^(2/3) +
 a*(a^2/b^2)^(1/3)) + 2*e^4*(a^2/b^2)^(1/3)*log(a*d*x + a*c + b*(a^2/b^2)^
(2/3)))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.39

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx = \frac{e^4\,\text{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(x+\frac{9t^2b^3e^8+ace^8}{ade^8}\right)\right)\right)}{d} + \frac{ce^4x}{b}+\frac{de^4x^2}{2b}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3),x)
```

output
```
e**4*RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(x + (9*_t**2*b**3*e**
8 + a*c*e**8)/(a*d*e**8))))/d + c*e**4*x/b + d*e**4*x**2/(2*b)
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx = \int \frac{(dex+ce)^4}{(dx+c)^3b+a}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-a*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a), x)/b + 1/2*(d*e^4*x^2 + 2*c*e^4*x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx = \frac{bd^7e^4x^2+2\,bcd^6e^4x}{2\,b^2d^6} - \frac{2\sqrt{3}(a^2bd^{15}e^{12})^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-(a^2b)^{\frac{2}{3}}\right)}{3\,(a^2b)^{\frac{2}{3}}}\right)+(a^2bd^{15}e^{12})^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-(a^2b)^{\frac{2}{3}}\right)^2+3\right.}{6\,b^2d^6}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(b*d^7*e^4*x^2 + 2*b*c*d^6*e^4*x)/(b^2*d^6) - 1/6*(2*sqrt(3)*(a^2*b*d^
15*e^12)^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*b*c - (a^2*b)^(2/3))/(a
^2*b)^(2/3)) + (a^2*b*d^15*e^12)^(1/3)*log((2*a*b*d*x + 2*a*b*c - (a^2*b)^
(2/3))^2 + 3*(a^2*b)^(4/3)) - 2*(a^2*b*d^15*e^12)^(1/3)*log(abs(a*b*d*x +
a*b*c + (a^2*b)^(2/3))))/(b^2*d^6)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx &= \frac{d\,e^4\,x^2}{2\,b}+\frac{c\,e^4\,x}{b}+\frac{a^{2/3}\,e^4\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,b^{5/3}\,d}\\ &+\frac{a^{2/3}\,e^4\,\ln\left(a^{1/3}-2\,b^{1/3}\,c-2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{5/3}\,d}\\ &-\frac{a^{2/3}\,e^4\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{5/3}\,d}\end{aligned}$$

input
```
int((c*e + d*e*x)^4/(a + b*(c + d*x)^3),x)
```

output
```
(d*e^4*x^2)/(2*b) + (c*e^4*x)/b + (a^(2/3)*e^4*log(b^(1/3)*c + a^(1/3) + b
^(1/3)*d*x))/(3*b^(5/3)*d) + (a^(2/3)*e^4*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*c + a^(1/3) - 2*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(5/3)*d) - (a^(
2/3)*e^4*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*(
(3^(1/2)*1i)/2 + 1/2))/(3*b^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \frac{(ce+dex)^4}{a+b(c+dx)^3}\,dx$$
$$= \frac{e^4\left(2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)a+6b^{\frac{2}{3}}a^{\frac{1}{3}}cdx+3b^{\frac{2}{3}}a^{\frac{1}{3}}d^2x^2-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx\right.\right.}{6b^{\frac{5}{3}}a^{\frac{1}{3}}d}$$

input
```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3),x)
```

output
```
(e**4*(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3)))*a + 6*b**(2/3)*a**(1/3)*c*d*x + 3*b**(2/3)*a**(1/3)*d**2*x**2 -
 log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**
2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a + 2*log(a**(1/3) + b**(1/3)*c
 + b**(1/3)*d*x)*a))/(6*b**(2/3)*a**(1/3)*b*d)
```

3.86 $\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$

Optimal result . . . 691
Mathematica [A] (verified) . . . 692
Rubi [A] (verified) . . . 692
Maple [C] (verified) . . . 698
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Sympy [A] (verification not implemented) . . . 699
Maxima [F] . . . 699
Giac [A] (verification not implemented) . . . 699
Mupad [B] (verification not implemented) . . . 700
Reduce [B] (verification not implemented) . . . 701

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx = \frac{e^3x}{b} + \frac{\sqrt[3]{a}e^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}d} - \frac{\sqrt[3]{a}e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3b^{4/3}d} + \frac{\sqrt[3]{a}e^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6b^{4/3}d}$$

output

```
e^3*x/b+1/3*a^(1/3)*e^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(
1/3))*3^(1/2)/b^(4/3)/d-1/3*a^(1/3)*e^3*ln(a^(1/3)+b^(1/3)*(d*x+c))/b^(4/3
)/d+1/6*a^(1/3)*e^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/
b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.93

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{e^3\left(6\sqrt[3]{b}c+6\sqrt[3]{b}dx-2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)+\sqrt[3]{a}\log\left(a^{2/3}\right.\right.}{6b^{4/3}d}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3),x]
```

output
```
(e^3*(6*b^(1/3)*c + 6*b^(1/3)*d*x - 2*Sqrt[3]*a^(1/3)*ArcTan[(-a^(1/3) + 2
*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*(
c + d*x)] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c +
 d*x)^2]))/(6*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$$
$$\downarrow 895$$
$$\frac{e^3\int \frac{(c+dx)^3}{b(c+dx)^3+a}d(c+dx)}{d}$$
$$\downarrow 843$$
$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{b}\right)}{d}$$

$$\downarrow 750$$

$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{b}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{d}$$

$$\downarrow 1142$$

$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{d}$$

$$\downarrow 25$$

$$e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$$\downarrow 27$$

$$e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)$$

$$\downarrow 1082$$

$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{d}$$

$$\downarrow 217$$

$$\frac{e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{d}$$

$$\downarrow 1103$$

$$e^3\left(\frac{c+dx}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)\Bigg/d$$

input
```
Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3),x]
```

output
```
(e^3*((c + d*x)/b - (a*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3
)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1
/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b
^(1/3)))/(3*a^(2/3))))/b))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

method	result	size
default	$e^3\left(\frac{x}{b}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)a}{3b^2d}\right)$	82
risch	$\frac{e^3x}{b}-\frac{a\,e^3\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3b^2d}$	84

input
```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
e^3*(x/b-1/3/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b
*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$$
$$=\frac{6\,de^3x+2\sqrt{3}e^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bdx+bc)\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-e^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(d^2x^2+2\,cdx+c^2+(dx+c)\left(-\right.\right.}{6\,bd}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(6*d*e^3*x + 2*sqrt(3)*e^3*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*d*x +
 b*c)*(-a/b)^(2/3) - sqrt(3)*a)/a) - e^3*(-a/b)^(1/3)*log(d^2*x^2 + 2*c*d*
x + c^2 + (d*x + c)*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*e^3*(-a/b)^(1/3)*log(
d*x + c - (-a/b)^(1/3)))/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx = \frac{e^3\,\text{RootSum}\left(27t^3b^4+a, \left(t \mapsto t\log\left(x+\frac{-3tbe^3+ce^3}{de^3}\right)\right)\right)}{d} + \frac{e^3x}{b}$$

input
```
integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3),x)
```

output
```
e**3*RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(x + (-3*_t*b*e**3 + c*e*
**3)/(d*e**3))))/d + e**3*x/b
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx = \int \frac{(dex+ce)^3}{(dx+c)^3b+a}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-a*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
x)/b + e^3*x/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.23

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx = \frac{e^3x}{b} + \frac{2\sqrt{3}\left(-\frac{ad^6e^9}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right) - \left(-\frac{ad^6e^9}{b}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)\right.}{6\,bd^3}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
e^3*x/b + 1/6*(2*sqrt(3)*(-a*d^6*e^9/b)^(1/3)*arctan(-(b*d*x + b*c - (-a*b
^2)^(1/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-a*d
^6*e^9/b)^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3
))^2 + 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-a*d^6*e^9/b)^(1/3)*log(ab
s(-b*d*x - b*c + (-a*b^2)^(1/3))))/(b*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{e^3\,x}{b} + \frac{(-a)^{1/3}\,e^3\,\ln\left((-a)^{4/3} + a\,b^{1/3}\,c + a\,b^{1/3}\,d\,x\right)}{3\,b^{4/3}\,d}$$
$$- \frac{(-a)^{1/3}\,e^3\,\ln\left(2\,a\,b^{1/3}\,c - (-a)^{4/3} + 2\,a\,b^{1/3}\,d\,x - \sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}\,d}$$
$$+ \frac{(-a)^{1/3}\,e^3\,\ln\left(2\,a\,b^{1/3}\,c - (-a)^{4/3} + 2\,a\,b^{1/3}\,d\,x + \sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{4/3}\,d}$$

input
```
int((c*e + d*e*x)^3/(a + b*(c + d*x)^3),x)
```

output
```
(e^3*x)/b + ((-a)^(1/3)*e^3*log((-a)^(4/3) + a*b^(1/3)*c + a*b^(1/3)*d*x))
/(3*b^(4/3)*d) - ((-a)^(1/3)*e^3*log(2*a*b^(1/3)*c - 3^(1/2)*(-a)^(4/3)*1i
 - (-a)^(4/3) + 2*a*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(4/3)*d) + (
(-a)^(1/3)*e^3*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*c + 2*
a*b^(1/3)*d*x)*((3^(1/2)*1i)/6 - 1/6))/(b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{(ce+dex)^3}{a+b(c+dx)^3}\,dx$$
$$= \frac{e^3\left(2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)-2a^{\frac{1}{3}}\log\left(\right.\right.}{6b^{\frac{4}{3}}d}$$

input
```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3),x)
```

output
```
(e**3*(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3))) + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) -
 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x) + 6*b**(1/3)*d*x))/(
6*b**(1/3)*b*d)
```

3.87 $\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx$

Optimal result 702
Mathematica [A] (verified) 702
Rubi [A] (verified) 703
Maple [A] (verified) 704
Fricas [A] (verification not implemented) 704
Sympy [B] (verification not implemented) 704
Maxima [A] (verification not implemented) 705
Giac [A] (verification not implemented) 705
Mupad [B] (verification not implemented) 706
Reduce [B] (verification not implemented) 706

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\log\left(a+b(c+dx)^3\right)}{3bd}$$

output
```
1/3*e^2*ln(a+b*(d*x+c)^3)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\log\left(a+b(c+dx)^3\right)}{3bd}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3),x]
```

output
```
(e^2*Log[a + b*(c + d*x)^3])/(3*b*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {895, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx$$

$$\downarrow 895$$

$$\frac{e^2 \int \frac{(c+dx)^2}{b(c+dx)^3+a} d(c+dx)}{d}$$

$$\downarrow 792$$

$$\frac{e^2 \log\left(a+b(c+dx)^3\right)}{3bd}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3),x]
```

output
```
(e^2*Log[a + b*(c + d*x)^3])/(3*b*d)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

method	result	size
default	$\frac{e^2 \ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3bd}$	46
norman	$\frac{e^2 \ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3bd}$	46
risch	$\frac{e^2 \ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3bd}$	46
parallelrisch	$\frac{e^2 \ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3bd}$	46

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2 \log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,bd}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*e^2*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2 \log\left(a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3\right)}{3bd}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3),x)
```

output
```
e**2*log(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*b*d
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,bd}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/3*e^2*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\log\left(|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a|\right)}{3\,bd}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*e^2*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(b*d
)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,b\,d}$$

input
```
int((c*e + d*e*x)^2/(a + b*(c + d*x)^3),x)
```

output
```
(e^2*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.08

$$\int \frac{(ce+dex)^2}{a+b(c+dx)^3}\,dx = \frac{e^2\left(\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}c+b^{\frac{1}{3}}dx\right)\right)}{3bd}$$

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3),x)
```

output
```
(e**2*(log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/
3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) + log(a**(1/3) + b**(1/3)
*c + b**(1/3)*d*x)))/(3*b*d)
```

3.88 $\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$

Optimal result . . . 707
Mathematica [A] (verified) . . . 707
Rubi [A] (verified) . . . 708
Maple [C] (verified) . . . 712
Fricas [A] (verification not implemented) . . . 712
Sympy [A] (verification not implemented) . . . 713
Maxima [F] . . . 713
Giac [A] (verification not implemented) . . . 714
Mupad [B] (verification not implemented) . . . 714
Reduce [B] (verification not implemented) . . . 715

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = -\frac{e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}d} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6\sqrt[3]{a}b^{2/3}d}$$

output

```
-1/3*e*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(
1/3)/b^(2/3)/d-1/3*e*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(1/3)/b^(2/3)/d+1/6*e*l
n(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = \frac{e\left(2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+\right.\right.}{6\sqrt[3]{a}b^{2/3}d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3),x]
```

output
```
(e*(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] -
 2*Log[a^(1/3) + b^(1/3)*(c + d*x)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d
*x) + b^(2/3)*(c + d*x)^2]))/(6*a^(1/3)*b^(2/3)*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {895, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$$

$$\downarrow 895$$

$$\frac{e\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{d}$$

$$\downarrow 821$$

$$\frac{e\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\downarrow 1142$$

$$e\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\downarrow 25$$

$$e\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\downarrow 27$$

$$e\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\downarrow 1082$$

$$e\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{d}$$

$$\downarrow 217$$

$$e\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)$$

$$\frac{}{d}$$

$$\downarrow 1103$$

$$e\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)$$

$$\frac{}{d}$$

input
```
Int[(c*e + d*e*x)/(a + b*(c + d*x)^3),x]
```

output
```
(e*(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*
ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3
) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/
3)*b^(1/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{e\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3bd}$	77
risch	$\frac{e\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3bd}$	77

input
```
int((d*e*x+c*e)/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*e/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d
^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.20

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$$

$$= \left[3\sqrt{\tfrac{1}{3}}abe\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2d^3x^3+6\,b^2cd^2x^2+6\,b^2c^2dx+2\,b^2c^3-ab+3\sqrt{\tfrac{1}{3}}\left(abdx+abc+2\left(d^2x^2+2\,cdx+c^2\right)(-ab^2)^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{}\right.\right.$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*e*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*
c*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c
 + 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a
*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a)) + (-a*b^2)^(2/3)*e*log(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(
2/3)*e*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d), 1/6*(6*sqrt(1/3)*a*b*
e*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/
3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*e*log(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(-a*b
^2)^(2/3)*e*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.29

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = \frac{e\,\text{RootSum}\left(27t^3ab^2+1, \left(t \mapsto t\log\left(x+\frac{9t^2abe^2+ce^2}{de^2}\right)\right)\right)}{d}$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)**3),x)
```

output
```
e*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(x + (9*_t**2*a*b*e**2 + c
*e**2)/(d*e**2))))/d
```

Maxima [F]

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = \int \frac{dex+ce}{(dx+c)^3b+a}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate((d*e*x + c*e)/((d*x + c)^3*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = -\frac{1}{3}\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\,\left(-a^2b\right)^{\frac{2}{3}}}\right) - \frac{1}{6}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}} \log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2 + 3\left(-a^2b\right)^{\frac{4}{3}}\right) + \frac{1}{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}} \log\left(\left|abdx+abc+\left(-a^2b\right)^{\frac{2}{3}}\right|\right)$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x + 2*a*
b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) - 1/6*(-e^3/(a*b^2*d^3))^(1/3)*log((
2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) + 1/3*(-e^3/(a
*b^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3)))
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx = -\frac{e\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{1/3}\,b^{2/3}\,d} + \frac{\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(e-\sqrt{3}\,e\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}\,d} + \frac{\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(e+\sqrt{3}\,e\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}\,d}$$

input
```
int((c*e + d*e*x)/(a + b*(c + d*x)^3),x)
```

output
```
(log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*(e - 3^(1
/2)*e*1i))/(6*a^(1/3)*b^(2/3)*d) - (e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*
x))/(3*a^(1/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3
) + 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i))/(6*a^(1/3)*b^(2/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\int \frac{ce+dex}{a+b(c+dx)^3}\,dx$$
$$= \frac{e\left(-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)-2\log\left(a^{\frac{1}{3}}+b\right.\right.}{6b^{\frac{2}{3}}a^{\frac{1}{3}}d}$$

input
```
int((d*e*x+c*e)/(a+b*(d*x+c)^3),x)
```

output
```
(e*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3))) + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) - 2*log(a**(1/3) +
b**(1/3)*c + b**(1/3)*d*x)))/(6*b**(2/3)*a**(1/3)*d)
```

3.89 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx$

Optimal result 716
Mathematica [A] (verified) 716
Rubi [A] (verified) 717
Maple [A] (verified) 718
Fricas [A] (verification not implemented) 719
Sympy [A] (verification not implemented) 719
Maxima [A] (verification not implemented) 720
Giac [A] (verification not implemented) 720
Mupad [B] (verification not implemented) 721
Reduce [B] (verification not implemented) 721

Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log(c+dx)}{ade} - \frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

output

```
ln(d*x+c)/a/d/e-1/3*ln(a+b*(d*x+c)^3)/a/d/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log(c+dx)}{ade} - \frac{\log\left(a+b(c+dx)^3\right)}{3ade}$$

input

```
Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x]
```

output

```
Log[c + d*x]/(a*d*e) - Log[a + b*(c + d*x)^3]/(3*a*d*e)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {895, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)}d(c+dx)}{de}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)}d(c+dx)^3}{3de}$$

$$\downarrow 47$$

$$\frac{\frac{\int \frac{1}{(c+dx)^3}d(c+dx)^3}{a} - \frac{b\int \frac{1}{b(c+dx)^3+a}d(c+dx)^3}{a}}{3de}$$

$$\downarrow 14$$

$$\frac{\frac{\log\left((c+dx)^3\right)}{a} - \frac{b\int \frac{1}{b(c+dx)^3+a}d(c+dx)^3}{a}}{3de}$$

$$\downarrow 16$$

$$\frac{\frac{\log\left((c+dx)^3\right)}{a} - \frac{\log\left(a+b(c+dx)^3\right)}{a}}{3de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x]
```

output
```
(Log[(c + d*x)^3]/a - Log[a + b*(c + d*x)^3]/a)/(3*d*e)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$\frac{3\ln(xd+c)-\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3ade}$	57
default	$\frac{\frac{\ln(xd+c)}{ad}-\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3ad}}{e}$	61
norman	$\frac{\ln(xd+c)}{ade}-\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3ade}$	63
risch	$\frac{\ln(xd+c)}{ade}-\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3ade}$	63

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*ln(d*x+c)-ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a))/a/d/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\,\log\left(dx+c\right)}{3\,ade}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/3*(log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*log(d*x
 + c))/(a*d*e)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{\log\left(\frac{c}{d}+x\right)}{ade} - \frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3ade}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3),x)
```

output
```
log(c/d + x)/(a*d*e) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3
)/(b*d**3))/(3*a*d*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,ade} + \frac{\log(dx+c)}{ade}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a*d*e) + lo
g(d*x + c)/(a*d*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,ade} + \frac{\log\left(\left|dx+c\right|\right)}{ade}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a*d*e)
 + log(abs(d*x + c))/(a*d*e)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{\ln(c+dx)}{ade} - \frac{\ln\left(bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3+a\right)}{3ade}$$

input
```
int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)),x)
```

output
```
log(c + d*x)/(a*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2
*x^2)/(3*a*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+b^{\frac{2}{3}}c^2+2b^{\frac{2}{3}}cdx+b^{\frac{2}{3}}d^2x^2\right)-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}c+b^{\frac{1}{3}}dx\right)+3\log(dx+c)}{3ade}$$

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*
c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) - log(a**(1/3) + b**(1/3)*c
+ b**(1/3)*d*x) + 3*log(c + d*x))/(3*a*d*e)
```

3.90 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal result . . . 722
Mathematica [A] (verified) . . . 723
Rubi [A] (verified) . . . 723
Maple [C] (verified) . . . 727
Fricas [A] (verification not implemented) . . . 728
Sympy [A] (verification not implemented) . . . 728
Maxima [F] . . . 729
Giac [A] (verification not implemented) . . . 729
Mupad [B] (verification not implemented) . . . 730
Reduce [B] (verification not implemented) . . . 730

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{ade^2(c+dx)} + \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}de^2} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{4/3}de^2} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{4/3}de^2}$$

output

```
-1/a/d/e^2/(d*x+c)+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(4/3)/d/e^2+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))
/a^(4/3)/d/e^2-1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x
+c)^2)/a^(4/3)/d/e^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{-\frac{6\sqrt[3]{a}}{c+dx} - 2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) - \sqrt[3]{b}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+\right.}{6a^{4/3}de^2}$$

input
```
Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x]
```

output
```
((-6*a^(1/3))/(c + d*x) - 2*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(
c + d*x))/(Sqrt[3]*a^(1/3))] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)]
- b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/
(6*a^(4/3)*d*e^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}\,d(c+dx)}{de^2}$$
$$\downarrow 847$$
$$\frac{-\frac{b\int \frac{c+dx}{b(c+dx)^3+a}\,d(c+dx)}{a} - \frac{1}{a(c+dx)}}{de^2}$$
$$\downarrow 821$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 16$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 1142$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 25$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 27$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 1082$$

$$-\frac{b\left(\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 217$$

$$-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

$$\downarrow 1103$$

$$-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}}{de^2}$$

input
```
Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x]
```

output
```
(-(1/(a*(c + d*x))) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b
^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])
/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]
/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)/(d*e^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.58

method	result	si
default	$\frac{-\frac{1}{ad(xd+c)}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3da}}{e^2}$	9
risch	$-\frac{1}{ad\,e^2(xd+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^4d^3e^6\_Z^3-b\right)}\_R\ln\left(\left(-4a^4d^4e^6\_R^3+3bd\right)x-4a^4c\,d^3e^6\_R^3-a^3d^2e^4\_R^2+3bc\right)\right)}{3}$	1

input
```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(-1/a/d/(d*x+c)-1/3/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),
_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx =$$
$$-\frac{2\sqrt{3}(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+(dx+c)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bd^2x^2+2\,bcdx+bc^2-(a\right.}{6\left(ad^2e^2x+acde^2\right)}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(d*x + c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(
1/3) - 1/3*sqrt(3)) + (d*x + c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*(d*x + c)*(b/a)^(1/3)
*log(b*d*x + b*c + a*(b/a)^(2/3)) + 6)/(a*d^2*e^2*x + a*c*d*e^2)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx$$
$$=-\frac{1}{acde^2+ad^2e^2x}+\frac{\operatorname{RootSum}\left(27t^3a^4-b,\left(t\mapsto t\log\left(x+\frac{9t^2a^3+bc}{bd}\right)\right)\right)}{de^2}$$

input
```
integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3),x)
```

output
```
-1/(a*c*d*e**2 + a*d**2*e**2*x) + RootSum(27*_t**3*a**4 - b, Lambda(_t, _t
*log(x + (9*_t**2*a**3 + b*c)/(b*d))))/(d*e**2)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)(dex+ce)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/(a*d^2*e^2*x + a*c*d*e^2) - b*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^
2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.38

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx = \frac{\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} \log\left(\left|-\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} - \frac{1}{(dex+ce)de}\right|\right)}{3\,a} - \frac{\sqrt{3}(a^2b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} - \frac{2}{(dex+ce)de}\right)}{3\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}\right)}{3\,a^2de^2} - \frac{(a^2b)^{\frac{1}{3}} \log\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{2}{3}} - \frac{\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}{(dex+ce)de} + \frac{1}{(dex+ce)^2d^2e^2}\right)}{6\,a^2de^2} - \frac{1}{(dex+ce)ade}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*(b/(a*d^3*e^6))^(1/3)*log(abs(-(b/(a*d^3*e^6))^(1/3) - 1/((d*e*x + c*e
)*d*e)))/a - 1/3*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3*e^6))
^(1/3) - 2/((d*e*x + c*e)*d*e))/(b/(a*d^3*e^6))^(1/3))/(a^2*d*e^2) - 1/6*(
a^2*b)^(1/3)*log((b/(a*d^3*e^6))^(2/3) - (b/(a*d^3*e^6))^(1/3)/((d*e*x + c
*e)*d*e) + 1/((d*e*x + c*e)^2*d^2*e^2))/(a^2*d*e^2) - 1/((d*e*x + c*e)*a*d
*e)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx \\
&= \frac{b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{4/3}\,d\,e^2} - \frac{1}{a\,d\,\left(c\,e^2+d\,e^2\,x\right)} \\
&\quad - \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,d\,e^2} \\
&\quad + \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}\,d\,e^2}
\end{aligned}$$

input
```
int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)),x)
```

output
```
(b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(3*a^(4/3)*d*e^2) - 1/(a*
d*(c*e^2 + d*e^2*x)) - (b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(
1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d*e^2) + (b^(1/3)
*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)
*1i)/6 - 1/6))/(a^(4/3)*d*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.55

$$\begin{aligned}
&\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)}\,dx \\
&= \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bc^2+2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bcdx+6b^{\frac{2}{3}}a^{\frac{1}{3}}dx-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}d\right.
\end{aligned}$$

input
```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3),x)
```

output

```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*b*c**2 + 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3)))*b*c*d*x + 6*b**(2/3)*a**(1/3)*d*x - log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b*c**2 - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b
*c*d*x + 2*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**2 + 2*log(a**(1/
3) + b**(1/3)*c + b**(1/3)*d*x)*b*c*d*x)/(6*b**(2/3)*a**(1/3)*a*c*d*e**2*(
c + d*x))
```

3.91 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$

Optimal result . . . 732
Mathematica [A] (verified) . . . 733
Rubi [A] (verified) . . . 733
Maple [C] (verified) . . . 737
Fricas [A] (verification not implemented) . . . 738
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Giac [A] (verification not implemented) . . . 739
Mupad [B] (verification not implemented) . . . 740
Reduce [B] (verification not implemented) . . . 740

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{2ade^3(c+dx)^2} + \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}de^3} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{5/3}de^3} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{5/3}de^3}$$

output

```
-1/2/a/d/e^3/(d*x+c)^2+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*
3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/d/e^3-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x
+c))/a^(5/3)/d/e^3+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*
(d*x+c)^2)/a^(5/3)/d/e^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{-\frac{3a^{2/3}}{(c+dx)^2} - 2\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{5/3}de^3}$$

input
```
Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x]
```

output
```
((-3*a^(2/3))/(c + d*x)^2 - 2*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)
*(c + d*x))/(Sqrt[3]*a^(1/3))] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)
] + b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]
)/(6*a^(5/3)*d*e^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}\,d(c+dx)}{de^3}$$
$$\downarrow 847$$
$$\frac{-\frac{b\int \frac{1}{b(c+dx)^3+a}\,d(c+dx)}{a} - \frac{1}{2a(c+dx)^2}}{de^3}$$
$$\downarrow 750$$

$$\frac{-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{de^3}$$

$$\downarrow 16$$

$$\frac{-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{de^3}$$

$$\downarrow 1142$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{de^3}$$

$$\downarrow 25$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{de^3}$$

$$\downarrow 27$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}}{de^3}$$

$$\downarrow 1082$$

$$-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{de^3}$$

$$\downarrow 217$$

$$-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{de^3}$$

$$\downarrow 1103$$

$$-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}}{de^3}$$

input

```
Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x]
```

output
```
(-1/2*1/(a*(c + d*x)^2) - (b*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*
b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]]
)/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2
]/(2*b^(1/3)))/(3*a^(2/3))))/a)/(d*e^3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

method	result
default	$\frac{-\frac{1}{2ad(xd+c)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3da}}{e^3}$
risch	$-\frac{1}{2ad\,e^3(xd+c)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5d^3e^9\_Z^3+b^2\right)}\_R\ln\left(\left(-4a^5d^4e^9\_R^3-3d\,b^2\right)x-4a^5c\,d^3e^9\_R^3-a^2bd\,e^3\_R-3b^2c\right)\right)}{3}$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(-1/2/a/d/(d*x+c)^2-1/3/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=
RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.46

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{2\sqrt{3}(d^2x^2+2\,cdx+c^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(adx+ac)\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-(d^2x^2+2\,cdx+c^2)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2\right.}{6\,(ad^3}$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(d^2*x^2 + 2*c*d*x + c^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sq
rt(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - (d^2*x^2 + 2*c*d*x
+ c^2)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + a^2*(-b^
2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) + 2*(d^2*x^2 + 2*c*d*x
+ c^2)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^2/a^2)^(1/3)) - 3)/(a*d^3*
e^3*x^2 + 2*a*c*d^2*e^3*x + a*c^2*d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.45

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= -\frac{1}{2ac^2de^3+4acd^2e^3x+2ad^3e^3x^2}$$
$$+\frac{\operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(x+\frac{-3ta^2+bc}{bd}\right)\right)\right)}{de^3}$$

input
```
integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3),x)
```

output
```
-1/(2*a*c**2*d*e**3 + 4*a*c*d**2*e**3*x + 2*a*d**3*e**3*x**2) + RootSum(27
*_t**3*a**5 + b**2, Lambda(_t, _t*log(x + (-3*_t*a**2 + b*c)/(b*d))))/(d*e
**3)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)(dex+ce)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/2/(a*d^3*e^3*x^2 + 2*a*c*d^2*e^3*x + a*c^2*d*e^3) - b*integrate(1/(b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a*e^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)^2+\right.}{6\,ae^3}$$
$$-\frac{1}{2\,(dx+c)^2 ade^3}$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/6*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/3
))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d^
3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2 +
 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(-b
*d*x - b*c + (-a*b^2)^(1/3))))/(a*e^3) - 1/2/((d*x + c)^2*a*d*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{b^{2/3}\,\ln\left(a^2\,b^{1/3}\,c-(-a)^{7/3}+a^2\,b^{1/3}\,d\,x\right)}{3\,(-a)^{5/3}\,d\,e^3} - \frac{1}{2\,a\,d\,\left(c^2\,e^3+2\,c\,d\,e^3\,x+d^2\,e^3\,x^2\right)}$$
$$- \frac{b^{2/3}\,\ln\left((-a)^{7/3}+2\,a^2\,b^{1/3}\,c+2\,a^2\,b^{1/3}\,d\,x+\sqrt{3}\,(-a)^{7/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-a)^{5/3}\,d\,e^3}$$
$$+ \frac{b^{2/3}\,\ln\left((-a)^{7/3}+2\,a^2\,b^{1/3}\,c+2\,a^2\,b^{1/3}\,d\,x-\sqrt{3}\,(-a)^{7/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{(-a)^{5/3}\,d\,e^3}$$

input
```
int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)),x)
```

output
```
(b^(2/3)*log(a^2*b^(1/3)*c - (-a)^(7/3) + a^2*b^(1/3)*d*x))/(3*(-a)^(5/3)*
d*e^3) - 1/(2*a*d*(c^2*e^3 + d^2*e^3*x^2 + 2*c*d*e^3*x)) - (b^(2/3)*log((-
a)^(7/3) + 3^(1/2)*(-a)^(7/3)*1i + 2*a^2*b^(1/3)*c + 2*a^2*b^(1/3)*d*x)*((
3^(1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)*d*e^3) + (b^(2/3)*log((-a)^(7/3) - 3^(
1/2)*(-a)^(7/3)*1i + 2*a^2*b^(1/3)*c + 2*a^2*b^(1/3)*d*x)*((3^(1/2)*1i)/6
- 1/6))/((-a)^(5/3)*d*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.42

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bc^2+4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bcdx+2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)bd$$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3),x)
```

output

```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*b*c**2 + 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c*d*x + 2*a**(1/3)*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*d**2*x**2 + a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2
/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**2 + 2*a**(1/3)*log(
a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2
*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d*x + a**(1/3)*log(a**(2/3) - b*
*(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d
*x + b**(2/3)*d**2*x**2)*b*d**2*x**2 - 2*a**(1/3)*log(a**(1/3) + b**(1/3)*
c + b**(1/3)*d*x)*b*c**2 - 4*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)
*d*x)*b*c*d*x - 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*d**
2*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)*a**2*d*e**3*(c**2 + 2*c*d*x + d**2*x**2
))
```

3.92 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx$

Optimal result . . . 742
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Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3ade^4(c+dx)^3} - \frac{b\log(c+dx)}{a^2de^4} + \frac{b\log\left(a+b(c+dx)^3\right)}{3a^2de^4}$$

output

```
-1/3/a/d/e^4/(d*x+c)^3-b*ln(d*x+c)/a^2/d/e^4+1/3*b*ln(a+b*(d*x+c)^3)/a^2/d
/e^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx = \frac{-\frac{a}{(c+dx)^3} - 3b\log(c+dx) + b\log\left(a+b(c+dx)^3\right)}{3a^2de^4}$$

input

```
Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x]
```

output

```
(-(a/(c + d*x)^3) - 3*b*Log[c + d*x] + b*Log[a + b*(c + d*x)^3])/(3*a^2*d*
e^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^4\,(a+b(c+dx)^3)}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)}d(c+dx)}{de^4}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)}d(c+dx)^3}{3de^4}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{b^2}{a^2(b(c+dx)^3+a)} - \frac{b}{a^2(c+dx)^3} + \frac{1}{a(c+dx)^6}\right) d(c+dx)^3}{3de^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{b\log((c+dx)^3)}{a^2} + \frac{b\log(a+b(c+dx)^3)}{a^2} - \frac{1}{a(c+dx)^3}}{3de^4}$$

input

```
Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x]
```

output

```
(-(1/(a*(c + d*x)^3)) - (b*Log[(c + d*x)^3])/a^2 + (b*Log[a + b*(c + d*x)^
3])/a^2)/(3*d*e^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

method	result
default	$\frac{-\frac{1}{3ad(xd+c)^3}-\frac{b\ln(xd+c)}{a^2d}+\frac{b\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{3a^2d}}{e^4}$
risch	$-\frac{1}{3ade^4(xd+c)^3}-\frac{b\ln(xd+c)}{a^2de^4}+\frac{b\ln(-bd^3x^3-3bcd^2x^2-3dbxc^2-c^3b-a)}{3a^2e^4d}$
norman	$\frac{\frac{x}{cea}+\frac{dx^2}{c^2ea}+\frac{d^2x^3}{3ec^3a}}{e^3(xd+c)^3}-\frac{b\ln(xd+c)}{a^2de^4}+\frac{b\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{3a^2e^4d}$
parallelrisch	$\frac{-3\ln(xd+c)x^3bd^5+\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)x^3bd^5-9\ln(xd+c)x^2bcd^4+3\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+\right.}{}$

input
```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(-1/3/a/d/(d*x+c)^3-b*ln(d*x+c)/a^2/d+1/3*b/a^2/d*ln(b*d^3*x^3+3*b*c
*d^2*x^2+3*b*c^2*d*x+b*c^3+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.62

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx$$
$$= \frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x^2+3\,b}{3\left(a^2d^4e^4x^3+3\,a^2cd^3e^4x^2+3\,a^2c^2d^2e^4x+a^2c^3de^4\right)}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(b*d^3*x^3 + 3*b
*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*log(d*x + c) - a)/(a^2*d^4*e^4*x^3 + 3*a^2*c*d^3*e^4*x^2
 + 3*a^2*c^2*d^2*e^4*x + a^2*c^3*d*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.

Time = 0.91 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.86

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3ac^3de^4+9ac^2d^2e^4x+9acd^3e^4x^2+3ad^4e^4x^3} - \frac{b\log\left(\frac{c}{d}+x\right)}{a^2de^4} + \frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2de^4}$$

input
```
integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3),x)
```

output
```
-1/(3*a*c**3*d*e**4 + 9*a*c**2*d**2*e**4*x + 9*a*c*d**3*e**4*x**2 + 3*a*d*
*4*e**4*x**3) - b*log(c/d + x)/(a**2*d*e**4) + b*log(3*c**2*x/d**2 + 3*c*x
**2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a**2*d*e**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx = -\frac{1}{3\left(ad^4e^4x^3+3\,acd^3e^4x^2+3\,ac^2d^2e^4x+ac^3de^4\right)} + \frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2de^4} - \frac{b\log(dx+c)}{a^2de^4}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/3/(a*d^4*e^4*x^3 + 3*a*c*d^3*e^4*x^2 + 3*a*c^2*d^2*e^4*x + a*c^3*d*e^4)
 + 1/3*b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^2*d*e
^4) - b*log(d*x + c)/(a^2*d*e^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)}\,dx = \frac{b\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^2de^4} - \frac{b\log\left(|dx+c|\right)}{a^2de^4} - \frac{1}{3\left(dx+c\right)^3ade^4}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*b*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^2*d
*e^4) - b*log(abs(d*x + c))/(a^2*d*e^4) - 1/3/((d*x + c)^3*a*d*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.82

$$\int \frac{1}{(ce+dex)^4\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^2\,d\,e^4}$$
$$- \frac{1}{3\,\left(a\,c^3\,d\,e^4+3\,a\,c^2\,d^2\,e^4\,x+3\,a\,c\,d^3\,e^4\,x^2+a\,d^4\,e^4\,x^3\right)} - \frac{b\,\ln\left(c+d\,x\right)}{a^2\,d\,e^4}$$

input
```
int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)),x)
```

output
```
(b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^2*d*e^4)
 - 1/(3*(a*d^4*e^4*x^3 + a*c^3*d*e^4 + 3*a*c^2*d^2*e^4*x + 3*a*c*d^3*e^4*x
^2)) - (b*log(c + d*x))/(a^2*d*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 423, normalized size of antiderivative = 6.51

$$\int \frac{1}{(ce+dex)^4\,(a+b(c+dx)^3)}\,dx$$
$$= \frac{\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right)b\,c^3 + 3\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}c\right.$$

input
```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3),x)
```

output

```
(log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**
2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**3 + 3*log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b*c**2*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c -
b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x
**2)*b*c*d**2*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3
)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3
 + log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 + 3*log(a**(1/3) + b**
(1/3)*c + b**(1/3)*d*x)*b*c**2*d*x + 3*log(a**(1/3) + b**(1/3)*c + b**(1/3
)*d*x)*b*c*d**2*x**2 + log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*d**3*x*
*3 - 3*log(c + d*x)*b*c**3 - 9*log(c + d*x)*b*c**2*d*x - 9*log(c + d*x)*b*
c*d**2*x**2 - 3*log(c + d*x)*b*d**3*x**3 - a)/(3*a**2*d*e**4*(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))
```

3.93 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 749
Mathematica [A] (verified) . . . 750
Rubi [A] (verified) . . . 750
Maple [C] (verified) . . . 756
Fricas [B] (verification not implemented) . . . 756
Sympy [A] (verification not implemented) . . . 757
Maxima [F] . . . 758
Giac [A] (verification not implemented) . . . 758
Mupad [B] (verification not implemented) . . . 759
Reduce [B] (verification not implemented) . . . 760

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^4(c+dx)^2}{3bd\left(a+b(c+dx)^3\right)} - \frac{2e^4\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{5/3}d} - \frac{2e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9\sqrt[3]{a}b^{5/3}d} + \frac{e^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{9\sqrt[3]{a}b^{5/3}d}$$

output

```
-1/3*e^4*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)-2/9*e^4*arctan(1/3*(a^(1/3)-2*b^(1/
3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(5/3)/d-2/9*e^4*ln(a^(1/3)+
b^(1/3)*(d*x+c))/a^(1/3)/b^(5/3)/d+1/9*e^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x
+c)+b^(2/3)*(d*x+c)^2)/a^(1/3)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{e^4\left(-\frac{3b^{2/3}(c+dx)^2}{a+b(c+dx)^3}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{\sqrt[3]{a}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{\sqrt[3]{a}}\right.}{9b^{5/3}d}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x]
```

output
```
(e^4*((-3*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a
^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(1/3) - (2*Log[a^(1/3)
 + b^(1/3)*(c + d*x)])/a^(1/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2]/a^(1/3)))/(9*b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{e^4\int \frac{(c+dx)^4}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{e^4\left(\frac{2\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 821$$

$$\frac{e^4\left(2\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^4\left(2\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 1142$$

$$\frac{e^4\left(2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(}{3b(a-}\right)}{d}$$

$$\downarrow 25$$

$$e^4\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c-}{3b(a+b}\right)}{d}$$

$$\downarrow 27$$

$$e^4\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(}{3b(a+}\right)}{d}$$

$$\downarrow 1082$$

$$e^4\left(\frac{2\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+d}{3b(a+b(c}\right)$$

$$\frac{}{d}$$

$$\downarrow 217$$

$$e^4\left(\frac{2\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}\right)$$

$$\frac{}{d}$$

$$\downarrow 1103$$

$$\frac{e^4\left(2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{(c+dx)^2}{3b(a+b(c+dx)^3)}\right)}{d}$$

input
```
Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x]
```

output
```
(e^4*(-1/3*(c + d*x)^2/(b*(a + b*(c + d*x)^3)) + (2*(-1/3*Log[a^(1/3) + b^
(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c
 + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c +
d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*b)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

method	result
default	$e^4\left(\frac{-\frac{d\,x^2}{3b}-\frac{2cx}{3b}-\frac{c^2}{3db}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R_{+c^2}}\right)}{9b^2d}\right)$
risch	$\frac{-\frac{d\,e^4x^2}{3b}-\frac{2e^4cx}{3b}-\frac{c^2e^4}{3db}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{2e^4\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R_{+c^2}}\right)}{9b^2d}$

input
```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
e^4*((-1/3*d*x^2/b-2/3*c*x/b-1/3*c^2/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2
*d*x+b*c^3+a)+2/9/b^2/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=R
ootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(145) = 290.

Time = 0.10 (sec) , antiderivative size = 940, normalized size of antiderivative = 5.11

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
[-1/9*(3*a*b^2*d^2*e^4*x^2 + 6*a*b^2*c*d*e^4*x + 3*a*b^2*c^2*e^4 - 3*sqrt(
1/3)*(a*b^2*d^3*e^4*x^3 + 3*a*b^2*c*d^2*e^4*x^2 + 3*a*b^2*c^2*d*e^4*x + (a
*b^2*c^3 + a^2*b)*e^4)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c
*d^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c
+ 2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*
b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a)) - (b*d^3*e^4*x^3 + 3*b*c*d^2*e^4*x^2 + 3*b*c^2*d*
e^4*x + (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) + 2*(b*d^3*e^4*x^3
+ 3*b*c*d^2*e^4*x^2 + 3*b*c^2*d*e^4*x + (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*lo
g(b*d*x + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*
b^4*c^2*d^2*x + (a*b^4*c^3 + a^2*b^3)*d), -1/9*(3*a*b^2*d^2*e^4*x^2 + 6*a*
b^2*c*d*e^4*x + 3*a*b^2*c^2*e^4 - 6*sqrt(1/3)*(a*b^2*d^3*e^4*x^3 + 3*a*b^2
*c*d^2*e^4*x^2 + 3*a*b^2*c^2*d*e^4*x + (a*b^2*c^3 + a^2*b)*e^4)*sqrt(-(-a*
b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-
a*b^2)^(1/3)/a)/b) - (b*d^3*e^4*x^3 + 3*b*c*d^2*e^4*x^2 + 3*b*c^2*d*e^4*x
+ (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^(2/3)) + 2*(b*d^3*e^4*x^3 + 3*b*
c*d^2*e^4*x^2 + 3*b*c^2*d*e^4*x + (b*c^3 + a)*e^4)*(-a*b^2)^(2/3)*log(b*d*
x + b*c - (-a*b^2)^(1/3)))/(a*b^4*d^4*x^3 + 3*a*b^4*c*d^3*x^2 + 3*a*b^4...
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx \\
&= \frac{-c^2e^4 - 2cde^4x - d^2e^4x^2}{3abd + 3b^2c^3d + 9b^2c^2d^2x + 9b^2cd^3x^2 + 3b^2d^4x^3} \\
&\quad + \frac{e^4\,\mathrm{RootSum}\left(729t^3ab^5 + 8, \left(t \mapsto t\log\left(x + \frac{81t^2ab^3e^8 + 4ce^8}{4de^8}\right)\right)\right)}{d}
\end{aligned}$$

input

```
integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3)**2,x)
```

output
```
(-c**2*e**4 - 2*c*d*e**4*x - d**2*e**4*x**2)/(3*a*b*d + 3*b**2*c**3*d + 9*
b**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 + 3*b**2*d**4*x**3) + e**4*RootSum(7
29*_t**3*a*b**5 + 8, Lambda(_t, _t*log(x + (81*_t**2*a*b**3*e**8 + 4*c*e**
8)/(4*d*e**8))))/d
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{(dex+ce)^4}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2/3*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3 + a), x)/b - 1/3*(d^2*e^4*x^2 + 2*c*d*e^4*x + c^2*e^4)/(b^2*d^4*x^3 + 3
*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.22

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+\right.}{9\,b}$$
$$-\frac{d^2e^4x^2+2\,cde^4x+c^2e^4}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output

```
-1/9*(2*sqrt(3)*(-e^12/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x +
2*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^12/(a*b^2*d^3))^(1/3)*log(
(2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^12/(a
*b^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/b - 1/3*(d^2*
e^4*x^2 + 2*c*d*e^4*x + c^2*e^4)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
 + b*c^3 + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{2\,e^4\,\ln\left(b^{1/3}\,c-(-a)^{1/3}+b^{1/3}\,d\,x\right)}{9\,(-a)^{1/3}\,b^{5/3}\,d} - \frac{\frac{d\,e^4\,x^2}{3\,b}+\frac{c^2\,e^4}{3\,b\,d}+\frac{2\,c\,e^4\,x}{3\,b}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}$$
$$- \frac{\ln\left(\frac{4\,c\,d^4\,e^8}{9\,b}-\frac{(-a)^{1/3}\,d^4\left(e^4+\sqrt{3}\,e^4\,1\mathrm{i}\right)^2}{9\,b^{4/3}}+\frac{4\,d^5\,e^8\,x}{9\,b}\right)\left(e^4+\sqrt{3}\,e^4\,1\mathrm{i}\right)}{9\,(-a)^{1/3}\,b^{5/3}\,d}$$
$$+ \frac{e^4\,\ln\left(\frac{4\,c\,d^4\,e^8}{9\,b}+\frac{4\,d^5\,e^8\,x}{9\,b}-\frac{9\,(-a)^{1/3}\,d^4\,e^8\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2}{b^{4/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{(-a)^{1/3}\,b^{5/3}\,d}$$

input

```
int((c*e + d*e*x)^4/(a + b*(c + d*x)^3)^2,x)
```

output

```
(2*e^4*log(b^(1/3)*c - (-a)^(1/3) + b^(1/3)*d*x))/(9*(-a)^(1/3)*b^(5/3)*d)
 - ((d*e^4*x^2)/(3*b) + (c^2*e^4)/(3*b*d) + (2*c*e^4*x)/(3*b))/(a + b*c^3
+ b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) - (log((4*c*d^4*e^8)/(9*b) - ((
-a)^(1/3)*d^4*(3^(1/2)*e^4*1i + e^4)^2)/(9*b^(4/3)) + (4*d^5*e^8*x)/(9*b))
*(3^(1/2)*e^4*1i + e^4))/(9*(-a)^(1/3)*b^(5/3)*d) + (e^4*log((4*c*d^4*e^8)
/(9*b) + (4*d^5*e^8*x)/(9*b) - (9*(-a)^(1/3)*d^4*e^8*((3^(1/2)*1i)/9 - 1/9
)^2)/b^(4/3))*((3^(1/2)*1i)/9 - 1/9))/((-a)^(1/3)*b^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 706, normalized size of antiderivative = 3.84

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^2}\,dx = \text{Too large to display}$$

input

```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x)
```

output

```
(e**4*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*a*b*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4 - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d*x - 6*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d*
*2*x**2 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*b**2*c*d**3*x**3 + b**(2/3)*a**(1/3)*a - 2*b**(2/3)*a**(1/3)
*b*c**3 - 3*b**(2/3)*a**(1/3)*b*c**2*d*x + b**(2/3)*a**(1/3)*b*d**3*x**3 +
 log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**
2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c + log(a**(2/3) - b**(1/3)
*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b
**(2/3)*d**2*x**2)*b**2*c**4 + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(
1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)
*b**2*c**3*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*
d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**2*d**
2*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(
2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*d**3*x**3 - 2*lo
g(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a*b*c - 2*log(a**(1/3) + b**(1/3)*
c + b**(1/3)*d*x)*b**2*c**4 - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*
b**2*c**3*d*x - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b**2*c**2*d...
```

3.94 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 761
Mathematica [A] (verified) . . . 762
Rubi [A] (verified) . . . 762
Maple [C] (verified) . . . 767
Fricas [B] (verification not implemented) . . . 767
Sympy [A] (verification not implemented) . . . 768
Maxima [F] . . . 769
Giac [A] (verification not implemented) . . . 769
Mupad [B] (verification not implemented) . . . 770
Reduce [B] (verification not implemented) . . . 770

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^3(c+dx)}{3bd\left(a+b(c+dx)^3\right)} - \frac{e^3 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}d} + \frac{e^3 \log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{2/3}b^{4/3}d} - \frac{e^3 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{2/3}b^{4/3}d}$$

output

```
-1/3*e^3*(d*x+c)/b/d/(a+b*(d*x+c)^3)-1/9*e^3*arctan(1/3*(a^(1/3)-2*b^(1/3)
*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(4/3)/d+1/9*e^3*ln(a^(1/3)+b^
(1/3)*(d*x+c))/a^(2/3)/b^(4/3)/d-1/18*e^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+
c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.85

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{e^3\left(-\frac{6\sqrt[3]{b}(c+dx)}{a+b(c+dx)^3}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{a^{2/3}}\right.}{18b^{4/3}d}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x]
```

output
```
(e^3*((-6*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(
1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/a^(2/3) + (2*Log[a^(1/3) +
 b^(1/3)*(c + d*x)])/a^(2/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b
^(2/3)*(c + d*x)^2]/a^(2/3)))/(18*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {895, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{e^3\int \frac{(c+dx)^3}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{e^3\left(\frac{\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 750$$

$$\frac{e^3\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^3\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 1142$$

$$\frac{e^3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 25$$

$$\frac{e^3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+d}\right)}{d}$$

$$\downarrow 27$$

$$e^3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+\right)$$

$$d$$

$$\downarrow 1082$$

$$e^3\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)}\right)$$

$$d$$

$$\downarrow 217$$

$$e^3\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)$$

$$d$$

$$\downarrow 1103$$

$$e^3\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}}{3a^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3b}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{c+dx}{3b(a+b(c+dx)^3)}\right)\Bigg/d$$

input
```
Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x]
```

output
```
(e^3*(-1/3*(c + d*x)/(b*(a + b*(c + d*x)^3)) + (Log[a^(1/3) + b^(1/3)*(c +
 d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))
/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b
^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895

```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

method	result	size
default	$e^3\left(\frac{-\frac{x}{3b}-\frac{c}{3db}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9b^2d}\right)$	128
risch	$\frac{-\frac{e^3x}{3b}-\frac{e^3c}{3bd}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{e^3\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9b^2d}$	133

input
```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
e^3*((-1/3*x/b-1/3*c/d/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+1/
9/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(143) = 286.

Time = 0.10 (sec) , antiderivative size = 906, normalized size of antiderivative = 4.98

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
[-1/18*(6*a^2*b*d*e^3*x + 6*a^2*b*c*e^3 - 3*sqrt(1/3)*(a*b^2*d^3*e^3*x^3 +
 3*a*b^2*c*d^2*e^3*x^2 + 3*a*b^2*c^2*d*e^3*x + (a*b^2*c^3 + a^2*b)*e^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x
+ 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 +
 (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^
2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a)) + (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + (b*c^3 + a
)*e^3)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/
3)*(d*x + c) + (a^2*b)^(1/3)*a) - 2*(b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3
*b*c^2*d*e^3*x + (b*c^3 + a)*e^3)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2
*b)^(2/3)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x +
 (a^2*b^3*c^3 + a^3*b^2)*d), -1/18*(6*a^2*b*d*e^3*x + 6*a^2*b*c*e^3 - 6*sq
rt(1/3)*(a*b^2*d^3*e^3*x^3 + 3*a*b^2*c*d^2*e^3*x^2 + 3*a*b^2*c^2*d*e^3*x +
 (a*b^2*c^3 + a^2*b)*e^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b
)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (b*d^3*e
^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + (b*c^3 + a)*e^3)*(a^2*b)^(2
/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a
^2*b)^(1/3)*a) - 2*(b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x +
(b*c^3 + a)*e^3)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*
b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 ...
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{-ce^3-de^3x}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3} + \frac{e^3\,\text{RootSum}\left(729t^3a^2b^4-1,\left(t\mapsto t\log\left(x+\frac{9tabe^3+ce^3}{de^3}\right)\right)\right)}{d}$$

input

```
integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-c*e**3 - d*e**3*x)/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**
2*c*d**3*x**2 + 3*b**2*d**4*x**3) + e**3*RootSum(729*_t**3*a**2*b**4 - 1,
Lambda(_t, _t*log(x + (9*_t*a*b*e**3 + c*e**3)/(d*e**3))))/d
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^2}\,dx = \int \frac{(dex+ce)^3}{\left((dx+c)^3 b+a\right)^2}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
 x)/b - 1/3*(d*e^3*x + c*e^3)/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d
^2*x + (b^2*c^3 + a*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^2}\,dx$$
$$= \frac{2\sqrt{3}\left(\frac{e^9}{a^2bd^3}\right)^{\frac{1}{3}} \arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right) - \left(\frac{e^9}{a^2bd^3}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2 + 4\left(bd\right.\right.}{18\,b}$$
$$- \frac{de^3x+ce^3}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/18*(2*sqrt(3)*(e^9/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (e^9/(a^2*b*d
^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 +
 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(e^9/(a^2*b*d^3))^(1/3)*log(abs(b*
d*x + b*c + (a*b^2)^(1/3))))/b - 1/3*(d*e^3*x + c*e^3)/((b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{e^3\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{2/3}\,b^{4/3}\,d} - \frac{\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(e^3+\sqrt{3}\,e^3\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}\,d} - \frac{\frac{e^3\,x}{3\,b}+\frac{c\,e^3}{3\,b\,d}}{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a} + \frac{e^3\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{2/3}\,b^{4/3}\,d}$$

input
```
int((c*e + d*e*x)^3/(a + b*(c + d*x)^3)^2,x)
```

output
```
(e^3*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(2/3)*b^(4/3)*d) - (log(
2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*(3^(1/2)*e^3*1
i + e^3))/(18*a^(2/3)*b^(4/3)*d) - ((e^3*x)/(3*b) + (c*e^3)/(3*b*d))/(a +
b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) + (e^3*log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(9*a
^(2/3)*b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.74

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x)
```

output

```
(e**3*( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**3 - 6*a**(1/3)*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**2*d*x -
 6*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*b*c*d**2*x**2 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*d**3*x**3 - a**(1/3)*log(a**(
2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**
(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a - a**(1/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2
/3)*d**2*x**2)*b*c**3 - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b*
*(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**
2)*b*c**2*d*x - 3*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a
**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d
**2*x**2 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)
*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3
+ 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a + 2*a**(1/3)*log(
a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 + 6*a**(1/3)*log(a**(1/3) + b
**(1/3)*c + b**(1/3)*d*x)*b*c**2*d*x + 6*a**(1/3)*log(a**(1/3) + b**(1/3)*
c + b**(1/3)*d*x)*b*c*d**2*x**2 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*c ...
```

3.95 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 26

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^2}{3bd\,(a+b(c+dx)^3)}$$

output
```
-1/3*e^2/b/d/(a+b*(d*x+c)^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^2}{3bd\,(a+b(c+dx)^3)}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x]
```

output
```
-1/3*e^2/(b*d*(a + b*(c + d*x)^3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^2}\,dx$$

$$\downarrow 895$$

$$\frac{e^2 \int \frac{(c+dx)^2}{(b(c+dx)^3+a)^2} d(c+dx)}{d}$$

$$\downarrow 793$$

$$-\frac{e^2}{3bd\,(a+b(c+dx)^3)}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x]
```

output
```
-1/3*e^2/(b*d*(a + b*(c + d*x)^3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

method	result	size
gosper	$-\frac{e^2}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	47
default	$-\frac{e^2}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	47
norman	$-\frac{e^2}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	47
risch	$-\frac{e^2}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	47
parallelrisch	$-\frac{e^2}{3(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)bd}$	47
orering	$-\frac{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)(dex+ce)^2}{3bd(xd+c)^2\left(a+b(xd+c)^3\right)^2}$	72

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*e^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^2}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+(b^2c^3+ab)d\right)}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3*e^2/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b
)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 0.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{e^2}{3abd+3b^2c^3d+9b^2c^2d^2x+9b^2cd^3x^2+3b^2d^4x^3}$$

input
```
integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3)**2,x)
```

output
```
-e**2/(3*a*b*d + 3*b**2*c**3*d + 9*b**2*c**2*d**2*x + 9*b**2*c*d**3*x**2 +
 3*b**2*d**4*x**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{e^2}{3\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+(b^2c^3+ab)d\right)}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*e^2/(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + (b^2*c^3 + a*b
)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^2}\,dx = -\frac{e^2}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)bd}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/3*e^2/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^2}{3\,b\,d\,\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}$$

input
```
int((c*e + d*e*x)^2/(a + b*(c + d*x)^3)^2,x)
```

output
```
-e^2/(3*b*d*(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{e^2}{3bd\,\left(b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a\right)}$$

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x)
```

output
```
( - e**2)/(3*b*d*(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**
3))
```

3.96 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 777
Mathematica [A] (verified) . . . 778
Rubi [A] (verified) . . . 778
Maple [C] (verified) . . . 783
Fricas [B] (verification not implemented) . . . 784
Sympy [A] (verification not implemented) . . . 785
Maxima [F] . . . 785
Giac [A] (verification not implemented) . . . 786
Mupad [B] (verification not implemented) . . . 786
Reduce [B] (verification not implemented) . . . 787

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{e(c+dx)^2}{3ad\left(a+b(c+dx)^3\right)} - \frac{e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}d} - \frac{e\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{4/3}b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{4/3}b^{2/3}d}$$

output

```
1/3*e*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)-1/9*e*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d
*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/b^(2/3)/d-1/9*e*ln(a^(1/3)+b^(1/3)
*(d*x+c))/a^(4/3)/b^(2/3)/d+1/18*e*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2
/3)*(d*x+c)^2)/a^(4/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{e\left(\frac{6\sqrt[3]{a}(c+dx)^2}{a+b(c+dx)^3}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{b^{2/3}}\right)}{18a^{4/3}d}$$

input
```
Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x]
```

output
```
(e*((6*a^(1/3)*(c + d*x)^2)/(a + b*(c + d*x)^3) + (2*Sqrt[3]*ArcTan[(-a^(1
/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/b^(2/3) - (2*Log[a^(1/3) +
b^(1/3)*(c + d*x)])/b^(2/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^
(2/3)*(c + d*x)^2]/b^(2/3)))/(18*a^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {895, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{e\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{d}$$

$$\downarrow 819$$

$$\frac{e\left(\frac{\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 821$$

$$\frac{e\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 1142$$

$$\frac{e\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 25$$

$$e\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)}\right)$$

$$\Big\downarrow 27$$

$$e\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+d}\right)$$

$$\Big\downarrow 1082$$

$$e\left(\frac{\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3}\right)$$

$$\Big\downarrow 217$$

$$e\left(\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}_{(c+dx)}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

$$\downarrow 1103$$

$$e\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a(a+b(c+dx)^3)}\right)}{d}$$

input

```
Int[(c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x]
```

output

```
(e*((c + d*x)^2/(3*a*(a + b*(c + d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c
 + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))
/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b
^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p + 1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])   Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.83

method	result	size
default	$e\left(\frac{\frac{dx^2}{3a}+\frac{2cx}{3a}+\frac{c^2}{3ad}}{bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{9abd}\right)$	146
risch	$\frac{\frac{dex^2}{3a}+\frac{2cex}{3a}+\frac{c^2e}{3da}}{bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a}+\frac{e\left(\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9abd}$	148

input

```
int((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output

```
e*((1/3*d/a*x^2+2/3/a*c*x+1/3*c^2/a/d)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*
x+b*c^3+a)+1/9/a/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=Root
Of(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(137) = 274$.

Time = 0.10 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.08

$$\int \frac{ce + dex}{(a + b(c + dx)^3)^2}\, dx = \text{Too large to display}$$

input

```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
[1/18*(6*a*b^2*d^2*e*x^2 + 12*a*b^2*c*d*e*x + 6*a*b^2*c^2*e + 3*sqrt(1/3)*
(a*b^2*d^3*e*x^3 + 3*a*b^2*c*d^2*e*x^2 + 3*a*b^2*c^2*d*e*x + (a*b^2*c^3 +
a^2*b)*e)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*
b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2
+ 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a)
 - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3 + a)) + (b*d^3*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*x + (b*c^3 + a)
*e)*(-a*b^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3
)*(b*d*x + b*c) + (-a*b^2)^(2/3)) - 2*(b*d^3*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b
*c^2*d*e*x + (b*c^3 + a)*e)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3
)))/(a^2*b^3*d^4*x^3 + 3*a^2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^
3*c^3 + a^3*b^2)*d), 1/18*(6*a*b^2*d^2*e*x^2 + 12*a*b^2*c*d*e*x + 6*a*b^2*
c^2*e + 6*sqrt(1/3)*(a*b^2*d^3*e*x^3 + 3*a*b^2*c*d^2*e*x^2 + 3*a*b^2*c^2*d
*e*x + (a*b^2*c^3 + a^2*b)*e)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
b*d*x + 2*b*c + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b*d^3*e*x^3
+ 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*x + (b*c^3 + a)*e)*(-a*b^2)^(2/3)*log(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) + (-a*b^2)^
(2/3)) - 2*(b*d^3*e*x^3 + 3*b*c*d^2*e*x^2 + 3*b*c^2*d*e*x + (b*c^3 + a)*e)
*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^3*d^4*x^3 + 3*a^
2*b^3*c*d^3*x^2 + 3*a^2*b^3*c^2*d^2*x + (a^2*b^3*c^3 + a^3*b^2)*d)]
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^2}\,dx = \frac{c^2e+2cdex+d^2ex^2}{3a^2d+3abc^3d+9abc^2d^2x+9abcd^3x^2+3abd^4x^3} + \frac{e\,\text{RootSum}\left(729t^3a^4b^2+1,\left(t\mapsto t\log\left(x+\frac{81t^2a^3be^2+ce^2}{de^2}\right)\right)\right)}{d}$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)**3)**2,x)
```

output
```
(c**2*e + 2*c*d*e*x + d**2*e*x**2)/(3*a**2*d + 3*a*b*c**3*d + 9*a*b*c**2*d
**2*x + 9*a*b*c*d**3*x**2 + 3*a*b*d**4*x**3) + e*RootSum(729*_t**3*a**4*b*
*2 + 1, Lambda(_t, _t*log(x + (81*_t**2*a**3*b*e**2 + c*e**2)/(d*e**2))))/
d
```

Maxima [F]

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^2}\,dx = \int \frac{dex+ce}{\left((dx+c)^3b+a\right)^2}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3*e*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a), x)/a + 1/3*(d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(a*b*d^4*x^3 + 3*a*b*c*d
^3*x^2 + 3*a*b*c^2*d^2*x + (a*b*c^3 + a^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.24

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{2\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{18\,a}$$

$$+\frac{d^2ex^2+2\,cdex+c^2e}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)ad}$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/18*(2*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x +
2*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^3/(a*b^2*d^3))^(1/3)*log((
2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^3/(a*b
^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a + 1/3*(d^2*e*
x^2 + 2*c*d*e*x + c^2*e)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
 + a)*a*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\frac{c^2e}{3ad}+\frac{2cex}{3a}+\frac{dex^2}{3a}}{bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3+a} - \frac{e\,\ln\left(b^{1/3}c+a^{1/3}+b^{1/3}dx\right)}{9a^{4/3}b^{2/3}d} + \frac{\ln\left(a^{1/3}-2b^{1/3}c-2b^{1/3}dx+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(e-\sqrt{3}\,e\,1\mathrm{i}\right)}{18a^{4/3}b^{2/3}d} + \frac{\ln\left(2b^{1/3}c-a^{1/3}+2b^{1/3}dx+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(e+\sqrt{3}\,e\,1\mathrm{i}\right)}{18a^{4/3}b^{2/3}d}$$

input
```
int((c*e + d*e*x)/(a + b*(c + d*x)^3)^2,x)
```

output
```
((c^2*e)/(3*a*d) + (2*c*e*x)/(3*a) + (d*e*x^2)/(3*a))/(a + b*c^3 + b*d^3*x
^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2) - (e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d
*x))/(9*a^(4/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/
3) - 2*b^(1/3)*d*x)*(e - 3^(1/2)*e*1i))/(18*a^(4/3)*b^(2/3)*d) + (log(3^(1
/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i)
)/(18*a^(4/3)*b^(2/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.01

$$\int \frac{ce + dex}{(a + b(c + dx)^3)^2}\, dx = \text{Too large to display}$$

input
```
int((d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x)
```

output
```
(e*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3)))*a*b*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*b**2*c**4 - 6*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c
 - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d*x - 6*sqrt(3)*atan((a**
(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**2*
x**2 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3)))*b**2*c*d**3*x**3 - 2*b**(2/3)*a**(1/3)*a + 4*b**(2/3)*a**(1/3)*
b*c**3 + 6*b**(2/3)*a**(1/3)*b*c**2*d*x - 2*b**(2/3)*a**(1/3)*b*d**3*x**3
+ log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c*
*2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c + log(a**(2/3) - b**(1/3
)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x +
b**(2/3)*d**2*x**2)*b**2*c**4 + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**
(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2
)*b**2*c**3*d*x + 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)
*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**2*d*
*2*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**
(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*d**3*x**3 - 2*l
og(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a*b*c - 2*log(a**(1/3) + b**(1/3)
*c + b**(1/3)*d*x)*b**2*c**4 - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)
*b**2*c**3*d*x - 6*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b**2*c**2*...
```

3.97 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{1}{3ade\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^2de} - \frac{\log\left(a+b(c+dx)^3\right)}{3a^2de}$$

output
```
1/3/a/d/e/(a+b*(d*x+c)^3)+ln(d*x+c)/a^2/d/e-1/3*ln(a+b*(d*x+c)^3)/a^2/d/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{\frac{a}{a+b(c+dx)^3} + 3\log(c+dx) - \log\left(a+b(c+dx)^3\right)}{3a^2de}$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x]
```

output
```
(a/(a + b*(c + d*x)^3) + 3*Log[c + d*x] - Log[a + b*(c + d*x)^3])/(3*a^2*d
*e)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)^2}\,d(c+dx)}{de}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)^2}\,d(c+dx)^3}{3de}$$

$$\downarrow 54$$

$$\frac{\int \left(-\frac{b}{a^2(b(c+dx)^3+a)} - \frac{b}{a(b(c+dx)^3+a)^2} + \frac{1}{a^2(c+dx)^3}\right) d(c+dx)^3}{3de}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(a+b(c+dx)^3)}{a^2} + \frac{\log((c+dx)^3)}{a^2} + \frac{1}{a(a+b(c+dx)^3)}}{3de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(a*(a + b*(c + d*x)^3)) + Log[(c + d*x)^3]/a^2 - Log[a + b*(c + d*x)^3]
/a^2)/(3*d*e)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.60

method	result
norman	$\frac{1}{3ade(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{\ln(xd+c)}{a^2de}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^2de}$
risch	$\frac{1}{3ade(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{\ln(xd+c)}{a^2de}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^2de}$
default	$\frac{\frac{\ln(xd+c)}{a^2d}-\frac{b\left(-\frac{a}{3db\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}+\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}\right)}{a^2}}{e}$
parallelrisch	$\frac{3\ln(xd+c)x^3b^2d^5-\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)x^3b^2d^5+9\ln(xd+c)x^2b^2c\,d^4-3\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+$

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a/d/e/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+ln(d*x+c)/a^2/d/e-
1/3/a^2/d/e*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.54

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)-3\left(bd^3x^3+3\,bcd^2x\right.}{3\left(a^2bd^4ex^3+3\,a^2bcd^3ex^2+3\,a^2bc^2d^2ex+(a^2bc^3+a^3)de\right)}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*log(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - 3*(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3 + a)*log(d*x + c) - a)/(a^2*b*d^4*e*x^3 + 3*a^2*b*c*
d^3*e*x^2 + 3*a^2*b*c^2*d^2*e*x + (a^2*b*c^3 + a^3)*d*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(51) = 102$.

Time = 0.94 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx = \frac{1}{3a^2de+3abc^3de+9abc^2d^2ex+9abcd^3ex^2+3abd^4ex^3} + \frac{\log\left(\frac{c}{d}+x\right)}{a^2de} - \frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^2de}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3)**2,x)
```

output
```
1/(3*a**2*d*e + 3*a*b*c**3*d*e + 9*a*b*c**2*d**2*e*x + 9*a*b*c*d**3*e*x**2
 + 3*a*b*d**4*e*x**3) + log(c/d + x)/(a**2*d*e) - log(3*c**2*x/d**2 + 3*c*
x**2/d + x**3 + (a + b*c**3)/(b*d**3))/(3*a**2*d*e)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\begin{aligned}&\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx \\ &= \frac{1}{3\left(abd^4ex^3+3\,abcd^3ex^2+3\,abc^2d^2ex+(abc^3+a^2)de\right)} \\ &\quad -\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^2de}+\frac{\log(dx+c)}{a^2de}\end{aligned}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/3/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*e*x^2 + 3*a*b*c^2*d^2*e*x + (a*b*c^3 + a^
2)*d*e) - 1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^
2*d*e) + log(d*x + c)/(a^2*d*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx &= -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^2de} \\ &\quad +\frac{\log\left(\left|dx+c\right|\right)}{a^2de} \\ &\quad +\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)ade}\end{aligned}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^2*d*
e) + log(abs(d*x + c))/(a^2*d*e) + 1/3/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3 + a)*a*d*e)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{1}{3\left(e\,a^2\,d+b\,e\,a\,c^3\,d+3\,b\,e\,a\,c^2\,d^2\,x+3\,b\,e\,a\,c\,d^3\,x^2+b\,e\,a\,d^4\,x^3\right)} + \frac{\ln(c+d\,x)}{a^2\,d\,e} - \frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^2\,d\,e}$$

input
```
int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^2),x)
```

output
```
1/(3*(a^2*d*e + a*b*d^4*e*x^3 + a*b*c^3*d*e + 3*a*b*c^2*d^2*e*x + 3*a*b*c*
d^3*e*x^2)) + log(c + d*x)/(a^2*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2
*d*x + 3*b*c*d^2*x^2)/(3*a^2*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 511, normalized size of antiderivative = 7.51

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right)a - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cd\right.}{}$$

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^2,x)
```

output

```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*
c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a - log(a**(2/3) - b**(1/3)*
a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b*
*(2/3)*d**2*x**2)*b*c**3 - 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c
**2*d*x - 3*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b
**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c*d**2*x**2 - log(
a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2
*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*d**3*x**3 - log(a**(1/3) + b**(1/3
)*c + b**(1/3)*d*x)*a - log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 -
 3*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**2*d*x - 3*log(a**(1/3) +
 b**(1/3)*c + b**(1/3)*d*x)*b*c*d**2*x**2 - log(a**(1/3) + b**(1/3)*c + b*
*(1/3)*d*x)*b*d**3*x**3 + 3*log(c + d*x)*a + 3*log(c + d*x)*b*c**3 + 9*log
(c + d*x)*b*c**2*d*x + 9*log(c + d*x)*b*c*d**2*x**2 + 3*log(c + d*x)*b*d**
3*x**3 + a)/(3*a**2*d*e*(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d
**3*x**3))
```

3.98 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 795
Mathematica [A] (verified) . . . 796
Rubi [A] (verified) . . . 796
Maple [C] (verified) . . . 802
Fricas [B] (verification not implemented) . . . 803
Sympy [A] (verification not implemented) . . . 804
Maxima [F] . . . 805
Giac [A] (verification not implemented) . . . 805
Mupad [B] (verification not implemented) . . . 806
Reduce [B] (verification not implemented) . . . 807

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{a^2de^2(c+dx)} - \frac{b(c+dx)^2}{3a^2de^2\left(a+b(c+dx)^3\right)} + \frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}de^2} + \frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{7/3}de^2} - \frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{9a^{7/3}de^2}$$

output

```
-1/a^2/d/e^2/(d*x+c)-1/3*b*(d*x+c)^2/a^2/d/e^2/(a+b*(d*x+c)^3)+4/9*b^(1/3)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/d
/e^2+4/9*b^(1/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/d/e^2-2/9*b^(1/3)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/d/e^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\frac{9\sqrt[3]{a}}{c+dx} - \frac{3\sqrt[3]{a}b(c+dx)^2}{a+b(c+dx)^3} - 4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) - 2\sqrt[3]{b}\log\left(a\right.}{9a^{7/3}de^2}$$

input
```
Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x]
```

output
```
((-9*a^(1/3))/(c + d*x) - (3*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) -
4*Sqrt[3]*b^(1/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3)
)] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] - 2*b^(1/3)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(9*a^(7/3)*d*e^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {895, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{de^2}$$
$$\downarrow 819$$
$$\frac{\frac{4\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}\,d(c+dx)}{3a} + \frac{1}{3a(c+dx)(a+b(c+dx)^3)}}{de^2}$$

$$\downarrow 847$$

$$\frac{4\left(-\frac{b\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$de^2$$

$$\downarrow 821$$

$$\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$de^2$$

$$\downarrow 16$$

$$\frac{4\left(-\frac{b\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)^3)}$$
$$de^2$$

$$\downarrow 1142$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}}{de^2}$$

$$\downarrow 25$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)-\frac{1}{a(c+dx)}\right)}{3a}}{de^2}$$

$$\downarrow 27$$

$$\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}}{de^2}$$

$$\downarrow 1082$$

$$\frac{4\left(-\frac{b\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}}{de^2}$$

$$\downarrow 217$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b}\right)}{de^2}$$

$$\downarrow 1103$$

$$\frac{\left(4\left(-\frac{\left(b\left(\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)(a+b(c+dx)}\right)}{de^2}$$

input
```
Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(3*a*(c + d*x)*(a + b*(c + d*x)^3)) + (4*(-(1/(a*(c + d*x))) - (b*(-1/3
*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(
1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1
/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/
3))))/a))/(3*a))/(d*e^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.78

method	result
default	$\frac{-\frac{1}{a^2 d(xd+c)}-\frac{b\left(\frac{\frac{x^2 d}{3}+\frac{2cx}{3}+\frac{c^2}{3d}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\left(\_R_{d+c}\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9bd}\right)}{e^2}$
risch	$\frac{-\frac{4b\,d^2x^3}{3a^2}-\frac{4bcd\,x^2}{a^2}-\frac{4bx\,c^2}{a^2}-\frac{4c^3b+3a}{3a^2d}}{e^2(xd+c)(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a^7d^3e^6\_Z^3-b\right)}\_R\ln\left(\left(-4a^7d^4e^6\_R^3+3bd\right)x-4a^7c\,d^3e^6\_R\right.\right.}{9}$

input
```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(-1/a^2/d/(d*x+c)-b/a^2*((1/3*x^2*d+2/3*c*x+1/3*c^2/d)/(b*d^3*x^3+3*
b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+4/9/b/d*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c
^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(164) = 328$.

Time $= 0.09$ (sec) , antiderivative size $= 399$, normalized size of antiderivative $= 1.97$

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx =$$

$$-\frac{12\,bd^3x^3+36\,bcd^2x^2+36\,bc^2dx+12\,bc^3+4\sqrt{3}(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+bc^4+(4\,bc^3+a)dx+}{}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
-1/9*(12*b*d^3*x^3 + 36*b*c*d^2*x^2 + 36*b*c^2*d*x + 12*b*c^3 + 4*sqrt(3)*
(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d*x +
 a*c)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3))
+ 2*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + b*c^4 + (4*b*c^3 + a)*d
*x + a*c)*(b/a)^(1/3)*log(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b
/a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^
2 + b*c^4 + (4*b*c^3 + a)*d*x + a*c)*(b/a)^(1/3)*log(b*d*x + b*c + a*(b/a)
^(2/3)) + 9*a)/(a^2*b*d^5*e^2*x^4 + 4*a^2*b*c*d^4*e^2*x^3 + 6*a^2*b*c^2*d^
3*e^2*x^2 + (4*a^2*b*c^3 + a^3)*d^2*e^2*x + (a^2*b*c^4 + a^3*c)*d*e^2)
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-3a-4bc^3-12bc^2dx-12bcd^2x^2-4bd^3x^3}{3a^3cde^2+3a^2bc^4de^2+18a^2bc^2d^3e^2x^2+12a^2bcd^4e^2x^3+3a^2bd^5e^2x^4+x\left(3a^3d^2e^2+12a^2bc^3d^2e^2\right)}$$
$$+\frac{\text{RootSum}\left(729t^3a^7-64b,\left(t\mapsto t\log\left(x+\frac{81t^2a^5+16bc}{16bd}\right)\right)\right)}{de^2}$$

input

```
integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-3*a - 4*b*c**3 - 12*b*c**2*d*x - 12*b*c*d**2*x**2 - 4*b*d**3*x**3)/(3*a*
*3*c*d*e**2 + 3*a**2*b*c**4*d*e**2 + 18*a**2*b*c**2*d**3*e**2*x**2 + 12*a*
*2*b*c*d**4*e**2*x**3 + 3*a**2*b*d**5*e**2*x**4 + x*(3*a**3*d**2*e**2 + 12
*a**2*b*c**3*d**2*e**2)) + RootSum(729*_t**3*a**7 - 64*b, Lambda(_t, _t*lo
g(x + (81*_t**2*a**5 + 16*b*c)/(16*b*d))))/(d*e**2)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^2 (dex+ce)^2}\,dx$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(4*b*d^3*x^3 + 12*b*c*d^2*x^2 + 12*b*c^2*d*x + 4*b*c^3 + 3*a)/(a^2*b*
d^5*e^2*x^4 + 4*a^2*b*c*d^4*e^2*x^3 + 6*a^2*b*c^2*d^3*e^2*x^2 + (4*a^2*b*c
^3 + a^3)*d^2*e^2*x + (a^2*b*c^4 + a^3*c)*d*e^2) - 4/3*b*integrate((d*x +
c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^2*e^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx \\
&= \frac{4\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} \log\left(\left|-\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} - \frac{1}{(dex+ce)de}\right|\right)}{9a^2} \\
&\quad - \frac{4\sqrt{3}(a^2b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}} - \frac{2}{(dex+ce)de}\right)}{3\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}\right)}{9a^3de^2} \\
&\quad - \frac{2(a^2b)^{\frac{1}{3}} \log\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{2}{3}} - \frac{\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}{(dex+ce)de} + \frac{1}{(dex+ce)^2d^2e^2}\right)}{9a^3de^2} \\
&\quad - \frac{1}{(dex+ce)a^2de} - \frac{b}{3(dex+ce)a^2\left(b+\frac{ae^3}{(dex+ce)^3}\right)de}
\end{aligned}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output

```
4/9*(b/(a*d^3*e^6))^(1/3)*log(abs(-(b/(a*d^3*e^6))^(1/3) - 1/((d*e*x + c*e
)*d*e)))/a^2 - 4/9*sqrt(3)*(a^2*b)^(1/3)*arctan(1/3*sqrt(3)*((b/(a*d^3*e^6
))^(1/3) - 2/((d*e*x + c*e)*d*e))/(b/(a*d^3*e^6))^(1/3))/(a^3*d*e^2) - 2/9
*(a^2*b)^(1/3)*log((b/(a*d^3*e^6))^(2/3) - (b/(a*d^3*e^6))^(1/3)/((d*e*x +
 c*e)*d*e) + 1/((d*e*x + c*e)^2*d^2*e^2))/(a^3*d*e^2) - 1/((d*e*x + c*e)*a
^2*d*e) - 1/3*b/((d*e*x + c*e)*a^2*(b + a*e^3/(d*e*x + c*e)^3)*d*e)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.29

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{7/3}\,d\,e^2}$$
$$- \frac{\frac{4\,b\,c^3+3\,a}{3\,a^2\,d}+\frac{4\,b\,d^2\,x^3}{3\,a^2}+\frac{4\,b\,c^2\,x}{a^2}+\frac{4\,b\,c\,d\,x^2}{a^2}}{x\,\left(4\,b\,d\,c^3\,e^2+a\,d\,e^2\right)+b\,c^4\,e^2+a\,c\,e^2+b\,d^4\,e^2\,x^4+4\,b\,c\,d^3\,e^2\,x^3+6\,b\,c^2\,d^2\,e^2\,x^2}$$
$$- \frac{4\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}\,d\,e^2}$$
$$+ \frac{b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{7/3}\,d\,e^2}$$

input

```
int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^2),x)
```

output

```
(4*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(9*a^(7/3)*d*e^2) - ((3
*a + 4*b*c^3)/(3*a^2*d) + (4*b*d^2*x^3)/(3*a^2) + (4*b*c^2*x)/a^2 + (4*b*c
*d*x^2)/a^2)/(x*(a*d*e^2 + 4*b*c^3*d*e^2) + b*c^4*e^2 + a*c*e^2 + b*d^4*e^
2*x^4 + 4*b*c*d^3*e^2*x^3 + 6*b*c^2*d^2*e^2*x^2) - (4*b^(1/3)*log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2)
)/(9*a^(7/3)*d*e^2) + (b^(1/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1
/3) + 2*b^(1/3)*d*x)*((3^(1/2)*2i)/9 - 2/9))/(a^(7/3)*d*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.99

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input

```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^2,x)
```

output

```
(4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*a*b*c**2 + 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*a*b*c*d*x + 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**5 + 16*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*d*x + 24*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*
b**2*c**3*d**2*x**2 + 16*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3
)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**3*x**3 + 4*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c*d**4*x**4 - 6*
b**(2/3)*a**(1/3)*a*c + 3*b**(2/3)*a**(1/3)*a*d*x - 9*b**(2/3)*a**(1/3)*b*
c**4 - 24*b**(2/3)*a**(1/3)*b*c**3*d*x - 18*b**(2/3)*a**(1/3)*b*c**2*d**2*
x**2 + 3*b**(2/3)*a**(1/3)*b*d**4*x**4 - 2*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*a*b*c**2 - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(
1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c*d*
x - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3
)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**5 - 8*log(a**(2/3)
 - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3
)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**4*d*x - 12*log(a**(2/3) - b**(1/3)*a
**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + ...
```

3.99 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 808
Mathematica [A] (verified) . . . 809
Rubi [A] (verified) . . . 809
Maple [C] (verified) . . . 815
Fricas [B] (verification not implemented) . . . 816
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Maxima [F] . . . 818
Giac [A] (verification not implemented) . . . 818
Mupad [B] (verification not implemented) . . . 819
Reduce [B] (verification not implemented) . . . 819

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= -\frac{1}{2a^2de^3(c+dx)^2} - \frac{b(c+dx)}{3a^2de^3\left(a+b(c+dx)^3\right)} + \frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}de^3}$$
$$- \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{9a^{8/3}de^3} + \frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{18a^{8/3}de^3}$$

output

```
-1/2/a^2/d/e^3/(d*x+c)^2-1/3*b*(d*x+c)/a^2/d/e^3/(a+b*(d*x+c)^3)+5/9*b^(2/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)
/d/e^3-5/9*b^(2/3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(8/3)/d/e^3+5/18*b^(2/3)*
ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(8/3)/d/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{-\frac{9a^{2/3}}{(c+dx)^2} - \frac{6a^{2/3}b(c+dx)}{a+b(c+dx)^3} - 10\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) + 5b^{2/3}\log$$
$$18a^{8/3}de^3$$

input
```
Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x]
```

output
```
((-9*a^(2/3))/(c + d*x)^2 - (6*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) -
10*Sqrt[3]*b^(2/3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3
))] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*(c + d*x)] + 5*b^(2/3)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(18*a^(8/3)*d*e^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {895, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{de^3}$$
$$\downarrow 819$$
$$\frac{5\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}\,d(c+dx)}{3a} + \frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}$$
$$de^3$$

$$\downarrow 847$$

$$\frac{5\left(-\frac{b\int\frac{1}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}}{de^3}$$

$$\downarrow 750$$

$$\frac{5\left(-\frac{b\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}}{de^3}$$

$$\downarrow 16$$

$$\frac{5\left(-\frac{b\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c+dx)^3)}}{de^3}$$

$$\downarrow 1142$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}\right)}{3a}}{de^3}$$

$$\downarrow 25$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}}{de^3}$$

$$\downarrow 27$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)}\right)}{3a}}{de^3}$$

$$\downarrow 1082$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}}{de^3}$$

$$\downarrow 217$$

$$\frac{\left(5\left(-\frac{\left(b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}_{(c+dx)}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3a^{2/3}\sqrt[3]{b}}\right)\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+}}{de^3}$$

$$\downarrow 1103$$

$$\frac{\left(5\left(-\frac{\left(b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}_{(c+dx)}+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}_{(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}_{(c+dx)}\right)}{3a^{2/3}\sqrt[3]{b}}\right)\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2(a+b(c-}}{de^3}$$

input
```
Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x]
```

output
```
(1/(3*a*(c + d*x)^2*(a + b*(c + d*x)^3)) + (5*(-1/2*1/(a*(c + d*x)^2) - (b
*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3))))
/a))/(3*a))/(d*e^3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.71

method	result
default	$-\frac{1}{2a^2 d(xd+c)^2}-\frac{b\left(\frac{\frac{x}{3}+\frac{c}{3d}}{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{9bd}\right)}{a^2}$
risch	$\frac{-\frac{5b\,d^2x^3}{6a^2}-\frac{5bcd\,x^2}{2a^2}-\frac{5bx\,c^2}{2a^2}-\frac{5c^3b+3a}{6d\,a^2}}{e^3(xd+c)^2(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^8d^3e^9\_Z^3+b^2\right)}\_R\ln\left(\left(-4a^8d^4e^9\_R^3-3db^2\right)x-4a^8cd^3e^9\right.\right.}{9}$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(-1/2/a^2/d/(d*x+c)^2-b/a^2*((1/3*x+1/3*c/d)/(b*d^3*x^3+3*b*c*d^2*x^
2+3*b*c^2*d*x+b*c^3+a)+5/9/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=R
ootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(162) = 324$.

Time $= 0.11$ (sec) , antiderivative size $= 527$, normalized size of antiderivative $= 2.60$

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx =$$

$$-\frac{15\,bd^3x^3+45\,bcd^2x^2+45\,bc^2dx+15\,bc^3-10\sqrt{3}(bd^5x^5+5\,bcd^4x^4+10\,bc^2d^3x^3+bc^5+(10\,bc^3+a)d$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
-1/18*(15*b*d^3*x^3 + 45*b*c*d^2*x^2 + 45*b*c^2*d*x + 15*b*c^3 - 10*sqrt(3
)*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d
^2*x^2 + a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqr
t(3)*(a*d*x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 5*(b*d^5*x^5 + 5*b*c
*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2 + a*c^2 + (5*
b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2 + a^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 10*(b*d^
5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + b*c^5 + (10*b*c^3 + a)*d^2*x^2
+ a*c^2 + (5*b*c^4 + 2*a*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c - a*(-b^
2/a^2)^(1/3)) + 9*a)/(a^2*b*d^6*e^3*x^5 + 5*a^2*b*c*d^5*e^3*x^4 + 10*a^2*b
*c^2*d^4*e^3*x^3 + (10*a^2*b*c^3 + a^3)*d^3*e^3*x^2 + (5*a^2*b*c^4 + 2*a^3
*c)*d^2*e^3*x + (a^2*b*c^5 + a^3*c^2)*d*e^3)
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{-3a-5bc^3-15bc^2dx-15bcd^2x^2-5bd^3x^3}{6a^3c^2de^3+6a^2bc^5de^3+60a^2bc^2d^4e^3x^3+30a^2bcd^5e^3x^4+6a^2bd^6e^3x^5+x^2\cdot\left(6a^3d^3e^3+60a^2bc^3d^3e^3\right)+x} + \frac{\operatorname{RootSum}\left(729t^3a^8+125b^2,\left(t\mapsto t\log\left(x+\frac{-9ta^3+5bc}{5bd}\right)\right)\right)}{de^3}$$

input

```
integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3)**2,x)
```

output

```
(-3*a - 5*b*c**3 - 15*b*c**2*d*x - 15*b*c*d**2*x**2 - 5*b*d**3*x**3)/(6*a*
*3*c**2*d*e**3 + 6*a**2*b*c**5*d*e**3 + 60*a**2*b*c**2*d**4*e**3*x**3 + 30
*a**2*b*c*d**5*e**3*x**4 + 6*a**2*b*d**6*e**3*x**5 + x**2*(6*a**3*d**3*e**
3 + 60*a**2*b*c**3*d**3*e**3) + x*(12*a**3*c*d**2*e**3 + 30*a**2*b*c**4*d*
*2*e**3)) + RootSum(729*_t**3*a**8 + 125*b**2, Lambda(_t, _t*log(x + (-9*_
t*a**3 + 5*b*c)/(5*b*d))))/(d*e**3)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^2 (dex+ce)^3}\,dx$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/6*(5*b*d^3*x^3 + 15*b*c*d^2*x^2 + 15*b*c^2*d*x + 5*b*c^3 + 3*a)/(a^2*b*
d^6*e^3*x^5 + 5*a^2*b*c*d^5*e^3*x^4 + 10*a^2*b*c^2*d^4*e^3*x^3 + (10*a^2*b
*c^3 + a^3)*d^3*e^3*x^2 + (5*a^2*b*c^4 + 2*a^3*c)*d^2*e^3*x + (a^2*b*c^5 +
 a^3*c^2)*d*e^3) - 5/3*b*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a), x)/(a^2*e^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx$$
$$= \frac{5\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right)\right.\right.}{18\,a^2e^3}$$
$$-\frac{bdx+bc}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)a^2de^3}-\frac{1}{2\left(dx+c\right)^2a^2de^3}$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
5/18*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*d
^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2
+ 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(b
*d*x + b*c - (-a*b^2)^(1/3))))/(a^2*e^3) - 1/3*(b*d*x + b*c)/((b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*a^2*d*e^3) - 1/2/((d*x + c)^2*a^
2*d*e^3)
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.43

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx =$$

$$-\frac{\frac{5\,b\,c^3+3\,a}{6\,a^2\,d}+\frac{5\,b\,d^2\,x^3}{6\,a^2}+\frac{5\,b\,c^2\,x}{2\,a^2}+\frac{5\,b\,c\,d\,x^2}{2\,a^2}}{x\left(5\,b\,d\,c^4\,e^3+2\,a\,d\,c\,e^3\right)+x^2\left(10\,b\,c^3\,d^2\,e^3+a\,d^2\,e^3\right)+a\,c^2\,e^3+b\,c^5\,e^3+b\,d^5\,e^3\,x^5+5\,b\,c\,d^4\,e^3\,x^4+10}$$

$$-\frac{5\,b^{2/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{9\,a^{8/3}\,d\,e^3}$$

$$+\frac{5\,b^{2/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}\,d\,e^3}$$

$$-\frac{5\,b^{2/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}\,d\,e^3}$$

input
```
int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^2),x)
```

output
```
(5*b^(2/3)*log(2*b^(1/3)*c - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)
*((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3)*d*e^3) - (5*b^(2/3)*log(b^(1/3)*c + a^
(1/3) + b^(1/3)*d*x))/(9*a^(8/3)*d*e^3) - ((3*a + 5*b*c^3)/(6*a^2*d) + (5*
b*d^2*x^3)/(6*a^2) + (5*b*c^2*x)/(2*a^2) + (5*b*c*d*x^2)/(2*a^2))/(x*(2*a*
c*d*e^3 + 5*b*c^4*d*e^3) + x^2*(a*d^2*e^3 + 10*b*c^3*d^2*e^3) + a*c^2*e^3
+ b*c^5*e^3 + b*d^5*e^3*x^5 + 5*b*c*d^4*e^3*x^4 + 10*b*c^2*d^3*e^3*x^3) -
(5*b^(2/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)
*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)*d*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1333, normalized size of antiderivative = 6.57

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^2,x)
```

output

```
(10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a*b*c**2 + 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c*d*x + 10*a**(1/3)*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*d**2
*x**2 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x
)/(a**(1/3)*sqrt(3)))*b**2*c**5 + 50*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4*d*x + 100*a**(1/
3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(
3)))*b**2*c**3*d**2*x**2 + 100*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d**3*x**3 + 50*a**(1/3
)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3
)))*b**2*c*d**4*x**4 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*d**5*x**5 + 5*a**(1/3)*log(a**(2
/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(
2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c**2 + 10*a**(1/3)*log(a**(2/3) - b**
(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*
x + b**(2/3)*d**2*x**2)*a*b*c*d*x + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2
/3)*d**2*x**2)*a*b*d**2*x**2 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)
*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3...
```

3.100 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx$

Optimal result . . . 821
Mathematica [A] (verified) . . . 821
Rubi [A] (verified) . . . 822
Maple [A] (verified) . . . 823
Fricas [B] (verification not implemented) . . . 824
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Maxima [B] (verification not implemented) . . . 825
Giac [A] (verification not implemented) . . . 826
Mupad [B] (verification not implemented) . . . 826
Reduce [B] (verification not implemented) . . . 827

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{1}{3a^2de^4(c+dx)^3} - \frac{b}{3a^2de^4\left(a+b(c+dx)^3\right)} - \frac{2b\log(c+dx)}{a^3de^4} + \frac{2b\log\left(a+b(c+dx)^3\right)}{3a^3de^4}$$

output

```
-1/3/a^2/d/e^4/(d*x+c)^3-1/3*b/a^2/d/e^4/(a+b*(d*x+c)^3)-2*b*ln(d*x+c)/a^3
/d/e^4+2/3*b*ln(a+b*(d*x+c)^3)/a^3/d/e^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx = -\frac{a\left(\frac{1}{(c+dx)^3}+\frac{b}{a+b(c+dx)^3}\right)+6b\log(c+dx)-2b\log\left(a+b(c+dx)^3\right)}{3a^3de^4}$$

input

```
Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x]
```

output
```
-1/3*(a*((c + d*x)^(-3) + b/(a + b*(c + d*x)^3)) + 6*b*Log[c + d*x] - 2*b*
Log[a + b*(c + d*x)^3])/(a^3*d*e^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)^2}d(c+dx)}{de^4}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)^2}d(c+dx)^3}{3de^4}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{2b^2}{a^3(b(c+dx)^3+a)}+\frac{b^2}{a^2(b(c+dx)^3+a)^2}-\frac{2b}{a^3(c+dx)^3}+\frac{1}{a^2(c+dx)^6}\right)d(c+dx)^3}{3de^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{2b\log\left((c+dx)^3\right)}{a^3}+\frac{2b\log\left(a+b(c+dx)^3\right)}{a^3}-\frac{b}{a^2(a+b(c+dx)^3)}-\frac{1}{a^2(c+dx)^3}}{3de^4}$$

input
```
Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x]
```

output
```
(-(1/(a^2*(c + d*x)^3)) - b/(a^2*(a + b*(c + d*x)^3)) - (2*b*Log[(c + d*x)
^3])/a^3 + (2*b*Log[a + b*(c + d*x)^3])/a^3)/(3*d*e^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.41

method	result
default	$\frac{-\frac{1}{3a^2d(xd+c)^3}-\frac{2b\ln(xd+c)}{a^3d}+\frac{b^2\left(-\frac{a}{3db\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}+\frac{2\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{3bd}\right)}{a^3}}{e^4}$
risch	$\frac{-\frac{2bd^2x^3}{3a^2}-\frac{2bcdx^2}{a^2}-\frac{2bxc^2}{a^2}-\frac{2c^3b+a}{3da^2}}{e^4(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}-\frac{2b\ln(xd+c)}{a^3de^4}+\frac{2b\ln(-bd^3x^3-3bcd^2x^2-3dbxc^2-c^3b-a)}{3a^3e^4d}$
norman	$\frac{-\frac{2cbdx^2}{a^2e}-\frac{2c^2bx}{a^2e}+\frac{-2b^2c^3d^5-ab^2d^5}{3a^2d^6eb}-\frac{2bd^2x^3}{3a^2e}}{e^3(xd+c)^3(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}-\frac{2b\ln(xd+c)}{a^3de^4}+\frac{2b\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)}{3a^3e^4d}$
parallelrisch	$\frac{-18\ln(xd+c)x^2ab^2cd^7+6\ln(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)x^2ab^2cd^7-18\ln(xd+c)xab^2c^2d^6+6\ln(bd^3x^3+3bcd^2x^2+$

input
```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(-1/3/a^2/d/(d*x+c)^3-2*b*ln(d*x+c)/a^3/d+b^2/a^3*(-1/3*a/d/b/(b*d^3
*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)+2/3/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2
+3*b*c^2*d*x+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(86) = 172$.

Time = 0.13 (sec) , antiderivative size = 452, normalized size of antiderivative = 4.91

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+a^2-2\left(b^2d^6x^6+6\,b^2cd^5x^5+15\,b^2c^2d^4x^4+b^2c^6+(20\,b^2c^3}{3\,(a^3}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2 -
2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b^2*
c^3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2*c^
5 + a*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)
 + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + (20*b
^2*c^3 + a*b)*d^3*x^3 + a*b*c^3 + 3*(5*b^2*c^4 + a*b*c)*d^2*x^2 + 3*(2*b^2
*c^5 + a*b*c^2)*d*x)*log(d*x + c))/(a^3*b*d^7*e^4*x^6 + 6*a^3*b*c*d^6*e^4*
x^5 + 15*a^3*b*c^2*d^5*e^4*x^4 + (20*a^3*b*c^3 + a^4)*d^4*e^4*x^3 + 3*(5*a
^3*b*c^4 + a^4*c)*d^3*e^4*x^2 + 3*(2*a^3*b*c^5 + a^4*c^2)*d^2*e^4*x + (a^3
*b*c^6 + a^4*c^3)*d*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(83) = 166$.

Time = 2.32 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.20

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx$$

$$= \frac{-a-2bc^3-6bc^2dx-6bcd^2x^2-2bd^3x^3}{3a^3c^3de^4+3a^2bc^6de^4+45a^2bc^2d^5e^4x^4+18a^2bcd^6e^4x^5+3a^2bd^7e^4x^6+x^3\cdot(3a^3d^4e^4+60a^2bc^3d^4e^4)+x^2}$$
$$-\frac{2b\log\left(\frac{c}{d}+x\right)}{a^3de^4}+\frac{2b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^3de^4}$$

input
```
integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3)**2,x)
```

output
```
(-a - 2*b*c**3 - 6*b*c**2*d*x - 6*b*c*d**2*x**2 - 2*b*d**3*x**3)/(3*a**3*c
**3*d*e**4 + 3*a**2*b*c**6*d*e**4 + 45*a**2*b*c**2*d**5*e**4*x**4 + 18*a**
2*b*c*d**6*e**4*x**5 + 3*a**2*b*d**7*e**4*x**6 + x**3*(3*a**3*d**4*e**4 +
60*a**2*b*c**3*d**4*e**4) + x**2*(9*a**3*c*d**3*e**4 + 45*a**2*b*c**4*d**3
*e**4) + x*(9*a**3*c**2*d**2*e**4 + 18*a**2*b*c**5*d**2*e**4)) - 2*b*log(c
/d + x)/(a**3*d*e**4) + 2*b*log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a + b
*c**3)/(b*d**3))/(3*a**3*d*e**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(86) = 172.

Time = 0.05 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.71

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx =$$
$$-\frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+a}{3\left(a^2bd^7e^4x^6+6\,a^2bcd^6e^4x^5+15\,a^2bc^2d^5e^4x^4+(20\,a^2bc^3+a^3)d^4e^4x^3+3\,(5\,a^2bc^4+a^3c)d^3e^4x^2+3\,(2\right.}$$
$$+\frac{2\,b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^3de^4}-\frac{2\,b\log\left(dx+c\right)}{a^3de^4}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/3*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + a)/(a^2*b*d^7*
e^4*x^6 + 6*a^2*b*c*d^6*e^4*x^5 + 15*a^2*b*c^2*d^5*e^4*x^4 + (20*a^2*b*c^3
 + a^3)*d^4*e^4*x^3 + 3*(5*a^2*b*c^4 + a^3*c)*d^3*e^4*x^2 + 3*(2*a^2*b*c^5
 + a^3*c^2)*d^2*e^4*x + (a^2*b*c^6 + a^3*c^3)*d*e^4) + 2/3*b*log(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d*e^4) - 2*b*log(d*x + c)
/(a^3*d*e^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.75

$$\begin{aligned}&\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx \\ &= \frac{2\,b\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^3de^4} - \frac{2\,b\log\left(\left|dx+c\right|\right)}{a^3de^4} \\ &\quad - \frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+a^2}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)(dx+c)^3a^3de^4}\end{aligned}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
2/3*b*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^3*d
*e^4) - 2*b*log(abs(d*x + c))/(a^3*d*e^4) - 1/3*(2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + a^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a)*(d*x + c)^3*a^3*d*e^4)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.72

$$\begin{aligned}&\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx = \frac{2\,b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d\,e^4} \\ &- \frac{\frac{2\,b\,c^3+a}{3\,a^2\,d}+\frac{2\,b\,d^2\,x^3}{3\,a^2}+\frac{2\,b\,c^2\,x}{a^2}+\frac{2\,b\,c\,d\,x^2}{a^2}}{x^2\,\left(15\,b\,c^4\,d^2\,e^4+3\,a\,c\,d^2\,e^4\right)+x\,\left(6\,b\,d\,c^5\,e^4+3\,a\,d\,c^2\,e^4\right)+x^3\,\left(20\,b\,c^3\,d^3\,e^4+a\,d^3\,e^4\right)+a\,c^3\,e^4+b\,c^6\,e} \\ &- \frac{2\,b\,\ln\left(c+d\,x\right)}{a^3\,d\,e^4}\end{aligned}$$

input
```
int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^2),x)
```

output
```
(2*b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(3*a^3*d*e^
4) - ((a + 2*b*c^3)/(3*a^2*d) + (2*b*d^2*x^3)/(3*a^2) + (2*b*c^2*x)/a^2 +
(2*b*c*d*x^2)/a^2)/(x^2*(15*b*c^4*d^2*e^4 + 3*a*c*d^2*e^4) + x*(3*a*c^2*d*
e^4 + 6*b*c^5*d*e^4) + x^3*(a*d^3*e^4 + 20*b*c^3*d^3*e^4) + a*c^3*e^4 + b*
c^6*e^4 + b*d^6*e^4*x^6 + 6*b*c*d^5*e^4*x^5 + 15*b*c^2*d^4*e^4*x^4) - (2*b
*log(c + d*x))/(a^3*d*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1296, normalized size of antiderivative = 14.09

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^2,x)
```

output

```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c**3 + 6*log(a**(2/3) - b
**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*
d*x + b**(2/3)*d**2*x**2)*a*b*c**2*d*x + 6*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*a*b*c*d**2*x**2 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/
3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a
*b*d**3*x**3 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**6 + 12*
log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**5*d*x + 30*log(a**(2/3)
- b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)
*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**4*d**2*x**2 + 40*log(a**(2/3) - b**(1
/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x
+ b**(2/3)*d**2*x**2)*b**2*c**3*d**3*x**3 + 30*log(a**(2/3) - b**(1/3)*a**
(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2
/3)*d**2*x**2)*b**2*c**2*d**4*x**4 + 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c
 - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**
2*x**2)*b**2*c*d**5*x**5 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*...
```

3.101 $\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 829
Mathematica [A] (verified) . . . 830
Rubi [A] (verified) . . . 830
Maple [C] (verified) . . . 836
Fricas [B] (verification not implemented) . . . 836
Sympy [A] (verification not implemented) . . . 837
Maxima [F] . . . 838
Giac [A] (verification not implemented) . . . 838
Mupad [B] (verification not implemented) . . . 839
Reduce [B] (verification not implemented) . . . 840

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{e^4(c+dx)^2}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^4(c+dx)^2}{9abd\left(a+b(c+dx)^3\right)} - \frac{e^4\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}d} - \frac{e^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{4/3}b^{5/3}d} + \frac{e^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{4/3}b^{5/3}d}$$

output

```
-1/6*e^4*(d*x+c)^2/b/d/(a+b*(d*x+c)^3)^2+1/9*e^4*(d*x+c)^2/a/b/d/(a+b*(d*x
+c)^3)-1/27*e^4*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^
(1/2)/a^(4/3)/b^(5/3)/d-1/27*e^4*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(4/3)/b^(5/
3)/d+1/54*e^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(4/3
)/b^(5/3)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{e^4\left(-\frac{9b^{2/3}(c+dx)^2}{(a+b(c+dx)^3)^2}+\frac{6b^{2/3}(c+dx)^2}{a(a+b(c+dx)^3)}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{4/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c\right.}{a^4}\right.}{54b^{5/3}d}$$

input
```
Integrate[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x]
```

output
```
(e^4*((-9*b^(2/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (6*b^(2/3)*(c + d*x
)^2)/(a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c
+ d*x))/(Sqrt[3]*a^(1/3))])/a^(4/3) - (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])
/a^(4/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/
a^(4/3)))/(54*b^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {895, 817, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^3}\,dx$$

$$\downarrow 895$$

$$\frac{e^4\int \frac{(c+dx)^4}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{e^4\left(\frac{\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 819$$

$$\frac{e^4\left(\frac{\frac{\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 821$$

$$\frac{e^4\left(\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^4\left(\frac{\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 1142$$

$$e^4\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)\right.}}{3b}\right)$$

$$d$$

$$\downarrow 25$$

$$e^4\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right.}}{3b}\right)$$

$$d$$

$$\downarrow 27$$

$$e^4\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)\right.}}{3b}\right)$$

$$d$$

$$\downarrow 1082$$

$$e^4\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}\right)$$

$$\Big\downarrow 217$$

$$e^4\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}-\frac{(c+dx)^2}{6b(a+b(c+dx)}\right)$$

$$\Big\downarrow 1103$$

$$e^4\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}}{3b}-\frac{(c+dx)^2}{6b(a+b(c+dx)^3)^2}\right)\Bigg/d$$

input
```
Int[(c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x]
```

output
```
(e^4*(-1/6*(c + d*x)^2/(b*(a + b*(c + d*x)^3)^2) + ((c + d*x)^2/(3*a*(a +
b*(c + d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3))
 + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3
)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(
1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*b)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99

method	result
default	$e^4\left(\frac{\frac{d^4x^5}{9a}+\frac{5cd^3x^4}{9a}+\frac{10c^2d^2x^3}{9a}-\frac{d\left(-20c^3b+a\right)x^2}{18ba}-\frac{c\left(-5c^3b+a\right)x}{9ba}-\frac{c^2\left(-2c^3b+a\right)}{18bda}}{(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{\sum}{27ab^2d}\right.$
risch	$\frac{\frac{d^4e^4x^5}{9a}+\frac{5cd^3e^4x^4}{9a}+\frac{10c^2d^2e^4x^3}{9a}-\frac{de^4\left(-20c^3b+a\right)x^2}{18ba}-\frac{ce^4\left(-5c^3b+a\right)x}{9ba}-\frac{c^2e^4\left(-2c^3b+a\right)}{18bda}}{(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{e^4\left(\sum\right.}{2}$

input

```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output

```
e^4*((1/9*d^4/a*x^5+5/9*c*d^3/a*x^4+10/9*c^2*d^2/a*x^3-1/18/b*d*(-20*b*c^3
+a)/a*x^2-1/9/b*c*(-5*b*c^3+a)/a*x-1/18/b*c^2/d*(-2*b*c^3+a)/a)/(b*d^3*x^3
+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/27/a/b^2/d*sum((_R*d+c)/(_R^2*d^2+
2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*
c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. 2(179) = 358.

Time = 0.19 (sec) , antiderivative size = 1886, normalized size of antiderivative = 8.57

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
[1/54*(6*a*b^3*d^5*e^4*x^5 + 30*a*b^3*c*d^4*e^4*x^4 + 60*a*b^3*c^2*d^3*e^4
*x^3 + 3*(20*a*b^3*c^3 - a^2*b^2)*d^2*e^4*x^2 + 6*(5*a*b^3*c^4 - a^2*b^2*c
)*d*e^4*x + 3*(2*a*b^3*c^5 - a^2*b^2*c^2)*e^4 + 3*sqrt(1/3)*(a*b^3*d^6*e^4
*x^6 + 6*a*b^3*c*d^5*e^4*x^5 + 15*a*b^3*c^2*d^4*e^4*x^4 + 2*(10*a*b^3*c^3
+ a^2*b^2)*d^3*e^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^2*e^4*x^2 + 6*(a*
b^3*c^5 + a^2*b^2*c^2)*d*e^4*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*e^4)*
sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d^2*x^2 + 6*b^2*c^2*d*
x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c + 2*(d^2*x^2 + 2*c*d*x
+ c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)
) + (b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4 + 2*(1
0*b^2*c^3 + a*b)*d^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^4*x^2 + 6*(b^
2*c^5 + a*b*c^2)*d*e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4)*(-a*b^2)^(2/3)
*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x + b*c) +
(-a*b^2)^(2/3)) - 2*(b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^
4*e^4*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e^4*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2
*e^4*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e^4*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^4
)*(-a*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)))/(a^2*b^5*d^7*x^6 + 6*a
^2*b^5*c*d^6*x^5 + 15*a^2*b^5*c^2*d^5*x^4 + 2*(10*a^2*b^5*c^3 + a^3*b^4)*d
^4*x^3 + 3*(5*a^2*b^5*c^4 + 2*a^3*b^4*c)*d^3*x^2 + 6*(a^2*b^5*c^5 + a^3...
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.51

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-ac^2e^4+2bc^5e^4+20bc^2d^3e^4x^3+10bcd^4e^4x^4+2bd^5e^4x^5+x^2(-ad^2e^4+}{18a^3bd+36a^2b^2c^3d+18ab^3c^6d+270ab^3c^2d^5x^4+108ab^3cd^6x^5+18ab^3d^7x^6+x^3\cdot(36a^2b^2d^4+360ab^3c^3}$$
$$+ \frac{e^4\,\mathrm{RootSum}\left(19683t^3a^4b^5+1,\left(t\mapsto t\log\left(x+\frac{729t^2a^3b^3e^8+ce^8}{de^8}\right)\right)\right)}{d}$$

input
```
integrate((d*e*x+c*e)**4/(a+b*(d*x+c)**3)**3,x)
```

output

```
(-a*c**2*e**4 + 2*b*c**5*e**4 + 20*b*c**2*d**3*e**4*x**3 + 10*b*c*d**4*e**
4*x**4 + 2*b*d**5*e**4*x**5 + x**2*(-a*d**2*e**4 + 20*b*c**3*d**2*e**4) +
x*(-2*a*c*d*e**4 + 10*b*c**4*d*e**4))/(18*a**3*b*d + 36*a**2*b**2*c**3*d +
 18*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4 + 108*a*b**3*c*d**6*x**5 + 1
8*a*b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4 + 360*a*b**3*c**3*d**4) + x**
2*(108*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**3) + x*(108*a**2*b**2*c**2*d*
*2 + 108*a*b**3*c**5*d**2)) + e**4*RootSum(19683*_t**3*a**4*b**5 + 1, Lamb
da(_t, _t*log(x + (729*_t**2*a**3*b**3*e**8 + c*e**8)/(d*e**8))))/d
```

Maxima [F]

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^3}\,dx = \int \frac{(dex+ce)^4}{\left((dx+c)^3 b+a\right)^3}\,dx$$

input

```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
1/9*e^4*integrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3 + a), x)/(a*b) + 1/18*(2*b*d^5*e^4*x^5 + 10*b*c*d^4*e^4*x^4 + 20*b*c^2*
d^3*e^4*x^3 + (20*b*c^3 - a)*d^2*e^4*x^2 + 2*(5*b*c^4 - a*c)*d*e^4*x + (2*
b*c^5 - a*c^2)*e^4)/(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5 + 15*a*b^3*c^2*d^5*
x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*(5*a*b^3*c^4 + 2*a^2*b^2*c)*d
^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x + (a*b^3*c^6 + 2*a^2*b^2*c^3 +
a^3*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.41

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^{12}}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{54\,ab}$$

$$+\frac{2\,bd^5e^4x^5+10\,bcd^4e^4x^4+20\,bc^2d^3e^4x^3+20\,bc^3d^2e^4x^2+10\,bc^4de^4x+2\,bc^5e^4-ad^2e^4x^2-2\,acde^4x-}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2abd}$$

input
```
integrate((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/54*(2*sqrt(3)*(-e^12/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x +
 2*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^12/(a*b^2*d^3))^(1/3)*log
((2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^12/(
a*b^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/(a*b) + 1/18
*(2*b*d^5*e^4*x^5 + 10*b*c*d^4*e^4*x^4 + 20*b*c^2*d^3*e^4*x^3 + 20*b*c^3*d
^2*e^4*x^2 + 10*b*c^4*d*e^4*x + 2*b*c^5*e^4 - a*d^2*e^4*x^2 - 2*a*c*d*e^4*
x - a*c^2*e^4)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a*
b*d)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.95

$$\int \frac{(ce+dex)^4}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{d^4e^4x^5}{9a} - \frac{ac^2e^4-2bc^5e^4}{18abd} + \frac{10c^2d^2e^4x^3}{9a} + \frac{5cd^3e^4x^4}{9a} - \frac{cx\left(ae^4-5bc^3e^4\right)}{9ab} - \frac{de^4x^2\left(a-\right.}{18a}}{x^3\left(20b^2c^3d^3+2abd^3\right)+x^2\left(15b^2c^4d^2+6abcd^2\right)+a^2+x\left(6db^2c^5+6adbc^2\right)+b^2c^6+b^2d^6x^6+}$$
$$- \frac{\ln\left(2(-b)^{4/3}c - a^{1/3}b + 2(-b)^{4/3}dx + \sqrt{3}a^{1/3}b\,1\mathrm{i}\right)\left(e^4+\sqrt{3}e^4\,1\mathrm{i}\right)}{54a^{4/3}(-b)^{5/3}d}$$
$$+ \frac{e^4\ln\left(a^{1/3}b+(-b)^{4/3}c+(-b)^{4/3}dx\right)}{27a^{4/3}(-b)^{5/3}d}$$
$$+ \frac{e^4\ln\left(\frac{d^4e^8\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{81a^{5/3}(-b)^{4/3}} + \frac{cd^4e^8}{81a^2b} + \frac{d^5e^8x}{81a^2b}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27a^{4/3}(-b)^{5/3}d}$$

input
```
int((c*e + d*e*x)^4/(a + b*(c + d*x)^3)^3,x)
```

output

```
((d^4*e^4*x^5)/(9*a) - (a*c^2*e^4 - 2*b*c^5*e^4)/(18*a*b*d) + (10*c^2*d^2*
e^4*x^3)/(9*a) + (5*c*d^3*e^4*x^4)/(9*a) - (c*x*(a*e^4 - 5*b*c^3*e^4))/(9*
a*b) - (d*e^4*x^2*(a - 20*b*c^3))/(18*a*b))/(x^3*(20*b^2*c^3*d^3 + 2*a*b*d
^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^
2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^
4*x^4) - (log(2*(-b)^(4/3)*c - a^(1/3)*b + 2*(-b)^(4/3)*d*x + 3^(1/2)*a^(1
/3)*b*1i)*(3^(1/2)*e^4*1i + e^4))/(54*a^(4/3)*(-b)^(5/3)*d) + (e^4*log(a^(
1/3)*b + (-b)^(4/3)*c + (-b)^(4/3)*d*x))/(27*a^(4/3)*(-b)^(5/3)*d) + (e^4*
log((d^4*e^8*((3^(1/2)*1i)/2 - 1/2)^2)/(81*a^(5/3)*(-b)^(4/3)) + (c*d^4*e^
8)/(81*a^2*b) + (d^5*e^8*x)/(81*a^2*b))*((3^(1/2)*1i)/2 - 1/2))/(27*a^(4/3
)*(-b)^(5/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1828, normalized size of antiderivative = 8.31

$$\int \frac{(ce+dex)^4}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input

```
int((d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x)
```

output

```
(e**4*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*a**2*b*c - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1
/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4 - 12*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**3*d*x - 12*sqrt(
3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b
**2*c**2*d**2*x**2 - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*
d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*d**3*x**3 - 2*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7 - 12*sqrt(3)*a
tan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*
*6*d*x - 30*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*b**3*c**5*d**2*x**2 - 40*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**3*x**3 - 30*sqrt(3)*
atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c
**3*d**4*x**4 - 12*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)
/(a**(1/3)*sqrt(3)))*b**3*c**2*d**5*x**5 - 2*sqrt(3)*atan((a**(1/3) - 2*b*
*(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**6*x**6 - b**(2/3)
*a**(1/3)*a**2 - 5*b**(2/3)*a**(1/3)*a*b*c**3 - 12*b**(2/3)*a**(1/3)*a*b*c
**2*d*x - 9*b**(2/3)*a**(1/3)*a*b*c*d**2*x**2 - 2*b**(2/3)*a**(1/3)*a*b*d*
*3*x**3 + 5*b**(2/3)*a**(1/3)*b**2*c**6 + 24*b**(2/3)*a**(1/3)*b**2*c**5*d
*x + 45*b**(2/3)*a**(1/3)*b**2*c**4*d**2*x**2 + 40*b**(2/3)*a**(1/3)*b*...
```

3.102 $\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 842
Mathematica [A] (verified) . . . 843
Rubi [A] (verified) . . . 843
Maple [C] (verified) . . . 849
Fricas [B] (verification not implemented) . . . 850
Sympy [A] (verification not implemented) . . . 851
Maxima [F] . . . 852
Giac [A] (verification not implemented) . . . 852
Mupad [B] (verification not implemented) . . . 853
Reduce [B] (verification not implemented) . . . 854

Optimal result

Integrand size = 24, antiderivative size = 216

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{e^3(c+dx)}{6bd\left(a+b(c+dx)^3\right)^2} + \frac{e^3(c+dx)}{18abd\left(a+b(c+dx)^3\right)} - \frac{e^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}d} + \frac{e^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{5/3}b^{4/3}d} - \frac{e^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{54a^{5/3}b^{4/3}d}$$

output

```
-1/6*e^3*(d*x+c)/b/d/(a+b*(d*x+c)^3)^2+1/18*e^3*(d*x+c)/a/b/d/(a+b*(d*x+c)
^3)-1/27*e^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/
2)/a^(5/3)/b^(4/3)/d+1/27*e^3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(5/3)/b^(4/3)/
d-1/54*e^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(5/3)/b
^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.84

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{e^3\left(-\frac{9\sqrt[3]{b}(c+dx)}{(a+b(c+dx)^3)^2}+\frac{3\sqrt[3]{b}(c+dx)}{a(a+b(c+dx)^3)}+\frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{a^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c\right.}{a^5}\right.}{54b^{4/3}d}$$

input
```
Integrate[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x]
```

output
```
(e^3*((-9*b^(1/3)*(c + d*x))/(a + b*(c + d*x)^3)^2 + (3*b^(1/3)*(c + d*x))
/(a*(a + b*(c + d*x)^3)) + (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*
x))/(Sqrt[3]*a^(1/3))])/a^(5/3) + (2*Log[a^(1/3) + b^(1/3)*(c + d*x)])/a^(
5/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/a^(5
/3)))/(54*b^(4/3)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {895, 817, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^3}\,dx$$

$$\downarrow 895$$

$$\frac{e^3\int \frac{(c+dx)^3}{(b(c+dx)^3+a)^3}\,d(c+dx)}{d}$$

$$\downarrow 817$$

$$\frac{e^3\left(\frac{\int \frac{1}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 749$$

$$\frac{e^3\left(\frac{\frac{2\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 750$$

$$\frac{e^3\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 16$$

$$\frac{e^3\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}}{6b}-\frac{c+dx}{6b(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 1142$$

$$e^3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{c+}{3a\left(a+b(}}{6b}\right)$$

$$\frac{}{d}$$

$$\downarrow 25$$

$$e^3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{c+dx}{3a\left(a+b(c+}}{6b}\right)$$

$$\frac{}{d}$$

$$\downarrow 27$$

$$e^3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+}{3a\left(a+b(}\right)}{6b}\right)}{d}$$

$$\downarrow 1082$$

$$e^3\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+d}\right)}{6b}\right)}{d}$$

$$\downarrow 217$$

$$e^3\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}\right)}{6b}-\frac{c+d}{6b(a+b(c+}$$

$$\Big\downarrow 1103$$

$$e^3\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{c+dx}{3a\left(a+b(c+dx)^3\right)}\right)}{6b}-\frac{c+dx}{6b(a+b(c+dx)}$$

input
```
Int[(c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x]
```

output
```
(e^3*(-1/6*(c + d*x)/(b*(a + b*(c + d*x)^3)^2) + ((c + d*x)/(3*a*(a + b*(c
 + d*x)^3)) + (2*(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (
-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) -
 Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)
))/(3*a^(2/3))))/(3*a))/(6*b)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

method	result
default	$e^3\left(\frac{\frac{d^3x^4}{18a}+\frac{2cd^2x^3}{9a}+\frac{c^2dx^2}{3a}-\frac{\left(-2c^3b+a\right)x}{9ba}-\frac{c\left(-c^3b+2a\right)}{18bda}}{(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R}}{27ab^2d}\right.$
risch	$\frac{\frac{d^3e^3x^4}{18a}+\frac{2cd^2e^3x^3}{9a}+\frac{e^3c^2dx^2}{3a}-\frac{e^3\left(-2c^3b+a\right)x}{9ba}-\frac{ce^3\left(-c^3b+2a\right)}{18bda}}{(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{e^3\left(\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+c^3b+a\right)}\frac{\ln\left(\right.}{d^2\_R}\right.$

input
```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
e^3*((1/18/a*d^3*x^4+2/9*c/a*d^2*x^3+1/3*c^2/a*d*x^2-1/9/b*(-2*b*c^3+a)/a*
x-1/18/b*c/d*(-b*c^3+2*a)/a)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)
^2+1/27/a/b^2/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^
3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(175) = 350$.

Time $= 0.13$ (sec) , antiderivative size $= 1822$, normalized size of antiderivative $= 8.44$

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
[1/54*(3*a^2*b^2*d^4*e^3*x^4 + 12*a^2*b^2*c*d^3*e^3*x^3 + 18*a^2*b^2*c^2*d
^2*e^3*x^2 + 6*(2*a^2*b^2*c^3 - a^3*b)*d*e^3*x + 3*(a^2*b^2*c^4 - 2*a^3*b*
c)*e^3 + 3*sqrt(1/3)*(a*b^3*d^6*e^3*x^6 + 6*a*b^3*c*d^5*e^3*x^5 + 15*a*b^3
*c^2*d^4*e^3*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e^3*x^3 + 3*(5*a*b^3*c^4
 + 2*a^2*b^2*c)*d^2*e^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e^3*x + (a*b^3
*c^6 + 2*a^2*b^2*c^3 + a^3*b)*e^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x
^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*
b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1
/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)) - (b^2*d^6*e^3*x^6 + 6*b^2*c*d
^5*e^3*x^5 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e^3*x^3 + 3
*(5*b^2*c^4 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e^3*x + (b^2*
c^6 + 2*a*b*c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(b^2*d^6*e^3*x^6
+ 6*b^2*c*d^5*e^3*x^5 + 15*b^2*c^2*d^4*e^3*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*
e^3*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*e^3*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e^
3*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e^3)*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c +
 (a^2*b)^(2/3)))/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d
^5*x^4 + 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b
^3*c)*d^3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*...
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.38

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{-2ace^3 + bc^4e^3 + 6bc^2d^2e^3x^2 + 4bcd^3e^3x^3 + bd^4e^3x^4 + x}{18a^3bd + 36a^2b^2c^3d + 18ab^3c^6d + 270ab^3c^2d^5x^4 + 108ab^3cd^6x^5 + 18ab^3d^7x^6 + x^3\cdot(36a^2b^2d^4 + 360ab^3c^3}$$
$$+ \frac{e^3\,\mathrm{RootSum}\left(19683t^3a^5b^4 - 1, \left(t \mapsto t\log\left(x + \frac{27ta^2be^3 + ce^3}{de^3}\right)\right)\right)}{d}$$

input

```
integrate((d*e*x+c*e)**3/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-2*a*c*e**3 + b*c**4*e**3 + 6*b*c**2*d**2*e**3*x**2 + 4*b*c*d**3*e**3*x**
3 + b*d**4*e**3*x**4 + x*(-2*a*d*e**3 + 4*b*c**3*d*e**3))/(18*a**3*b*d + 3
6*a**2*b**2*c**3*d + 18*a*b**3*c**6*d + 270*a*b**3*c**2*d**5*x**4 + 108*a*
b**3*c*d**6*x**5 + 18*a*b**3*d**7*x**6 + x**3*(36*a**2*b**2*d**4 + 360*a*b
**3*c**3*d**4) + x**2*(108*a**2*b**2*c*d**3 + 270*a*b**3*c**4*d**3) + x*(1
08*a**2*b**2*c**2*d**2 + 108*a*b**3*c**5*d**2)) + e**3*RootSum(19683*_t**3
*a**5*b**4 - 1, Lambda(_t, _t*log(x + (27*_t*a**2*b*e**3 + c*e**3)/(d*e**3
))))/d
```

Maxima [F]

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^3}\,dx = \int \frac{(dex+ce)^3}{\left((dx+c)^3 b+a\right)^3}\,dx$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/9*e^3*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a),
 x)/(a*b) + 1/18*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*x^3 + 6*b*c^2*d^2*e^3*x^2
+ 2*(2*b*c^3 - a)*d*e^3*x + (b*c^4 - 2*a*c)*e^3)/(a*b^3*d^7*x^6 + 6*a*b^3*
c*d^6*x^5 + 15*a*b^3*c^2*d^5*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^4*x^3 + 3*
(5*a*b^3*c^4 + 2*a^2*b^2*c)*d^3*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d^2*x +
(a*b^3*c^6 + 2*a^2*b^2*c^3 + a^3*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.38

$$\int \frac{(ce+dex)^3}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{2\sqrt{3}\left(\frac{e^9}{a^2bd^3}\right)^{\frac{1}{3}} \arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}}\right) - \left(\frac{e^9}{a^2bd^3}\right)^{\frac{1}{3}} \log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}(ab^2)^{\frac{1}{3}}\right)^2 + 4\left(bc\right.\right.}{54\,ab}$$

$$+ \frac{bd^4e^3x^4 + 4\,bcd^3e^3x^3 + 6\,bc^2d^2e^3x^2 + 4\,bc^3de^3x + bc^4e^3 - 2\,ade^3x - 2\,ace^3}{18\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3 + a\right)^2abd}$$

input
```
integrate((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/54*(2*sqrt(3)*(e^9/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/
3))/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - (e^9/(a^2*b*d
^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 +
 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 2*(e^9/(a^2*b*d^3))^(1/3)*log(abs(b*
d*x + b*c + (a*b^2)^(1/3))))/(a*b) + 1/18*(b*d^4*e^3*x^4 + 4*b*c*d^3*e^3*x
^3 + 6*b*c^2*d^2*e^3*x^2 + 4*b*c^3*d*e^3*x + b*c^4*e^3 - 2*a*d*e^3*x - 2*a
*c*e^3)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.62

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{\frac{b c^4 e^3 - 2 a c e^3}{18 a b d} + \frac{d^3 e^3 x^4}{18 a} - \frac{e^3 x\,(a - 2 b c^3)}{9 a b} + \frac{c^2 d e^3 x^2}{3 a} + \frac{2 c d^2 e^3 x^3}{9 a}}{x^3\,\left(20 b^2 c^3 d^3 + 2 a b d^3\right) + x^2\,\left(15 b^2 c^4 d^2 + 6 a b c d^2\right) + a^2 + x\,\left(6 d b^2 c^5 + 6 a d b c^2\right) + b^2 c^6 + b^2 d^6 x^6 +}$$
$$+ \frac{e^3\,\ln\left(b^{1/3} c + a^{1/3} + b^{1/3} d x\right)}{27 a^{5/3} b^{4/3} d}$$
$$- \frac{\ln\left(a^{1/3} - 2 b^{1/3} c - 2 b^{1/3} d x + \sqrt{3} a^{1/3}\,1\mathrm{i}\right)\,\left(e^3 + \sqrt{3} e^3\,1\mathrm{i}\right)}{54 a^{5/3} b^{4/3} d}$$
$$+ \frac{e^3\,\ln\left(2 b^{1/3} c - a^{1/3} + 2 b^{1/3} d x + \sqrt{3} a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27 a^{5/3} b^{4/3} d}$$

input
```
int((c*e + d*e*x)^3/(a + b*(c + d*x)^3)^3,x)
```

output
```
((b*c^4*e^3 - 2*a*c*e^3)/(18*a*b*d) + (d^3*e^3*x^4)/(18*a) - (e^3*x*(a - 2
*b*c^3))/(9*a*b) + (c^2*d*e^3*x^2)/(3*a) + (2*c*d^2*e^3*x^3)/(9*a))/(x^3*(
20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x
*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c
*d^5*x^5 + 15*b^2*c^2*d^4*x^4) + (e^3*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*
x))/(27*a^(5/3)*b^(4/3)*d) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/
3) - 2*b^(1/3)*d*x)*(3^(1/2)*e^3*1i + e^3))/(54*a^(5/3)*b^(4/3)*d) + (e^3*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*
1i)/2 - 1/2))/(27*a^(5/3)*b^(4/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1796, normalized size of antiderivative = 8.31

$$\int \frac{(ce+dex)^3}{\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input

```
int((d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x)
```

output

```
(e**3*( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**3 - 12*a**(1/3)*sqrt(3)*
atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c*
*2*d*x - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a*b*c*d**2*x**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3)
 - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*d**3*x**3 - 2*a*
*(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*s
qrt(3)))*b**2*c**6 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2
*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**5*d*x - 30*a**(1/3)*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**4
*d**2*x**2 - 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3
)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**3*d**3*x**3 - 30*a**(1/3)*sqrt(3)*atan(
(a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d
**4*x**4 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*
d*x)/(a**(1/3)*sqrt(3)))*b**2*c*d**5*x**5 - 2*a**(1/3)*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**2*d**6*x**6 -
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(
2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2 - 2*a**(1/3)*log(a
**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 +...
```

3.103 $\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result 855
Mathematica [A] (verified) 855
Rubi [A] (verified) 856
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Mupad [B] (verification not implemented) 859
Reduce [B] (verification not implemented) 859

Optimal result

Integrand size = 24, antiderivative size = 26

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

output
```
-1/6*e^2/b/d/(a+b*(d*x+c)^3)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(ce+dex)^2}{\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{e^2}{6bd\left(a+b(c+dx)^3\right)^2}$$

input
```
Integrate[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x]
```

output
```
-1/6*e^2/(b*d*(a + b*(c + d*x)^3)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx$$
$$\downarrow 895$$
$$\frac{e^2 \int \frac{(c+dx)^2}{(b(c+dx)^3+a)^3} d(c+dx)}{d}$$
$$\downarrow 793$$
$$-\frac{e^2}{6bd\,(a+b(c+dx)^3)^2}$$

input
```
Int[(c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x]
```

output
```
-1/6*e^2/(b*d*(a + b*(c + d*x)^3)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

method	result	size
gosper	$-\frac{e^2}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	47
default	$-\frac{e^2}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	47
norman	$-\frac{e^2}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	47
risch	$-\frac{e^2}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	47
parallelrisch	$-\frac{e^2}{6(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2bd}$	47
orering	$-\frac{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)(dex+ce)^2}{6bd(xd+c)^2\left(a+b(xd+c)^3\right)^3}$	72

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*e^2/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.23

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{e^2}{6\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+2\left(10\,b^3c^3+ab^2\right)d^4x^3+3\left(5\,b^3c^4+2\,ab^2c\right)d^3x^2+6\left(b^3c^5+ab^2c^2\right.\right.}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-1/6*e^2/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c
^3 + a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b
^2*c^2)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(20) = 40$.

Time = 1.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 5.96

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{e^2}{6a^2bd + 12ab^2c^3d + 6b^3c^6d + 90b^3c^2d^5x^4 + 36b^3cd^6x^5 + 6b^3d^7x^6 + x^3\cdot(12ab^2d^4 + 120b^3c^3d^4) + x^2\cdot(3$$

input
```
integrate((d*e*x+c*e)**2/(a+b*(d*x+c)**3)**3,x)
```

output
```
-e**2/(6*a**2*b*d + 12*a*b**2*c**3*d + 6*b**3*c**6*d + 90*b**3*c**2*d**5*x
**4 + 36*b**3*c*d**6*x**5 + 6*b**3*d**7*x**6 + x**3*(12*a*b**2*d**4 + 120*
b**3*c**3*d**4) + x**2*(36*a*b**2*c*d**3 + 90*b**3*c**4*d**3) + x*(36*a*b*
*2*c**2*d**2 + 36*b**3*c**5*d**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(24) = 48$.

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.23

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{e^2}{6\,(b^3d^7x^6 + 6\,b^3cd^6x^5 + 15\,b^3c^2d^5x^4 + 2\,(10\,b^3c^3 + ab^2)d^4x^3 + 3\,(5\,b^3c^4 + 2\,ab^2c)d^3x^2 + 6\,(b^3c^5 + ab^2c^2$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/6*e^2/(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 2*(10*b^3*c
^3 + a*b^2)*d^4*x^3 + 3*(5*b^3*c^4 + 2*a*b^2*c)*d^3*x^2 + 6*(b^3*c^5 + a*b
^2*c^2)*d^2*x + (b^3*c^6 + 2*a*b^2*c^3 + a^2*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx = -\frac{e^2}{6\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2bd}$$

input
```
integrate((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/6*e^2/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.15

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{e^2}{6\,b\,d\,\left(x^3\,\left(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3\right)+x^2\,\left(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2\right)+a^2+x\,\left(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2\right)+b^2\,c^6+b^2\right.}$$

input
```
int((c*e + d*e*x)^2/(a + b*(c + d*x)^3)^3,x)
```

output
```
-e^2/(6*b*d*(x^3*(20*b^2*c^3*d^3 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*
b*c*d^2) + a^2 + x*(6*b^2*c^5*d + 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2
*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.19

$$\int \frac{(ce+dex)^2}{(a+b(c+dx)^3)^3}\,dx =$$

$$-\frac{e^2}{6bd\left(b^2d^6x^6+6b^2c\,d^5x^5+15b^2c^2d^4x^4+20b^2c^3d^3x^3+15b^2c^4d^2x^2+6b^2c^5dx+2ab\,d^3x^3+b^2c^6+6abc\right.}$$

input
```
int((d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x)
```

output
```
( - e**2)/(6*b*d*(a**2 + 2*a*b*c**3 + 6*a*b*c**2*d*x + 6*a*b*c*d**2*x**2 +
 2*a*b*d**3*x**3 + b**2*c**6 + 6*b**2*c**5*d*x + 15*b**2*c**4*d**2*x**2 +
20*b**2*c**3*d**3*x**3 + 15*b**2*c**2*d**4*x**4 + 6*b**2*c*d**5*x**5 + b**
2*d**6*x**6))
```

3.104 $\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx = \frac{e(c+dx)^2}{6ad\left(a+b(c+dx)^3\right)^2} + \frac{2e(c+dx)^2}{9a^2d\left(a+b(c+dx)^3\right)} - \frac{2e\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}d} - \frac{2e\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{7/3}b^{2/3}d} + \frac{e\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{7/3}b^{2/3}d}$$

output

```
1/6*e*(d*x+c)^2/a/d/(a+b*(d*x+c)^3)^2+2/9*e*(d*x+c)^2/a^2/d/(a+b*(d*x+c)^3
)-2/27*e*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a
^(7/3)/b^(2/3)/d-2/27*e*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(7/3)/b^(2/3)/d+1/27
*e*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(7/3)/b^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{e\left(\frac{9a^{4/3}(c+dx)^2}{(a+b(c+dx)^3)^2}+\frac{12\sqrt[3]{a}(c+dx)^2}{a+b(c+dx)^3}+\frac{4\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{b^{2/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+\right.}{b^{2/3}}\right.}{54a^{7/3}d}$$

input

```
Integrate[(c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x]
```

output

```
(e*((9*a^(4/3)*(c + d*x)^2)/(a + b*(c + d*x)^3)^2 + (12*a^(1/3)*(c + d*x)^
2)/(a + b*(c + d*x)^3) + (4*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x)
)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*(c + d*x)])/b^(2/
3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/b^
(2/3)))/(54*a^(7/3)*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {895, 819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{e\int \frac{c+dx}{\left(b(c+dx)^3+a\right)^3}\,d(c+dx)}{d}$$

$$\downarrow 819$$

$$\frac{e\left(\frac{2\int\frac{c+dx}{\left(b(c+dx)^3+a\right)^2}d(c+dx)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 819$$

$$\frac{e\left(\frac{2\left(\frac{\int\frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}\right)}{d}$$

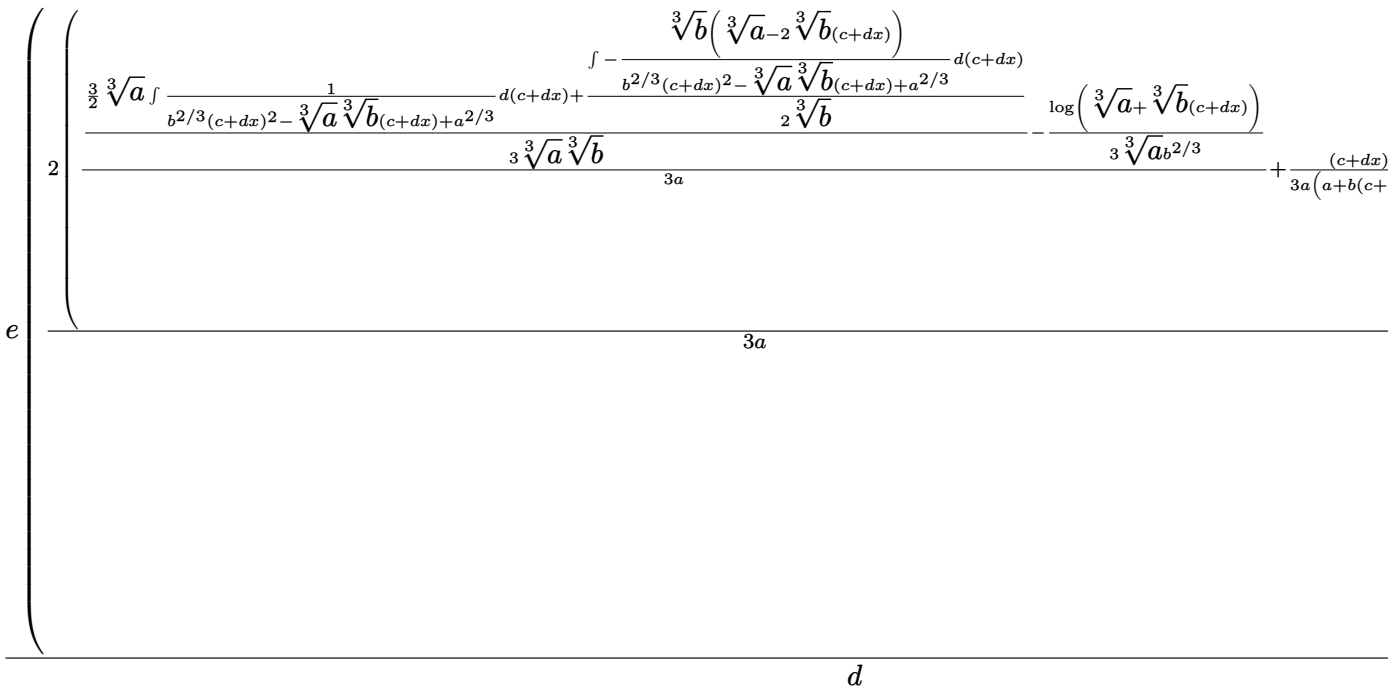
$$\downarrow 821$$

$$\frac{e\left(\frac{2\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}\right)}{d}$$

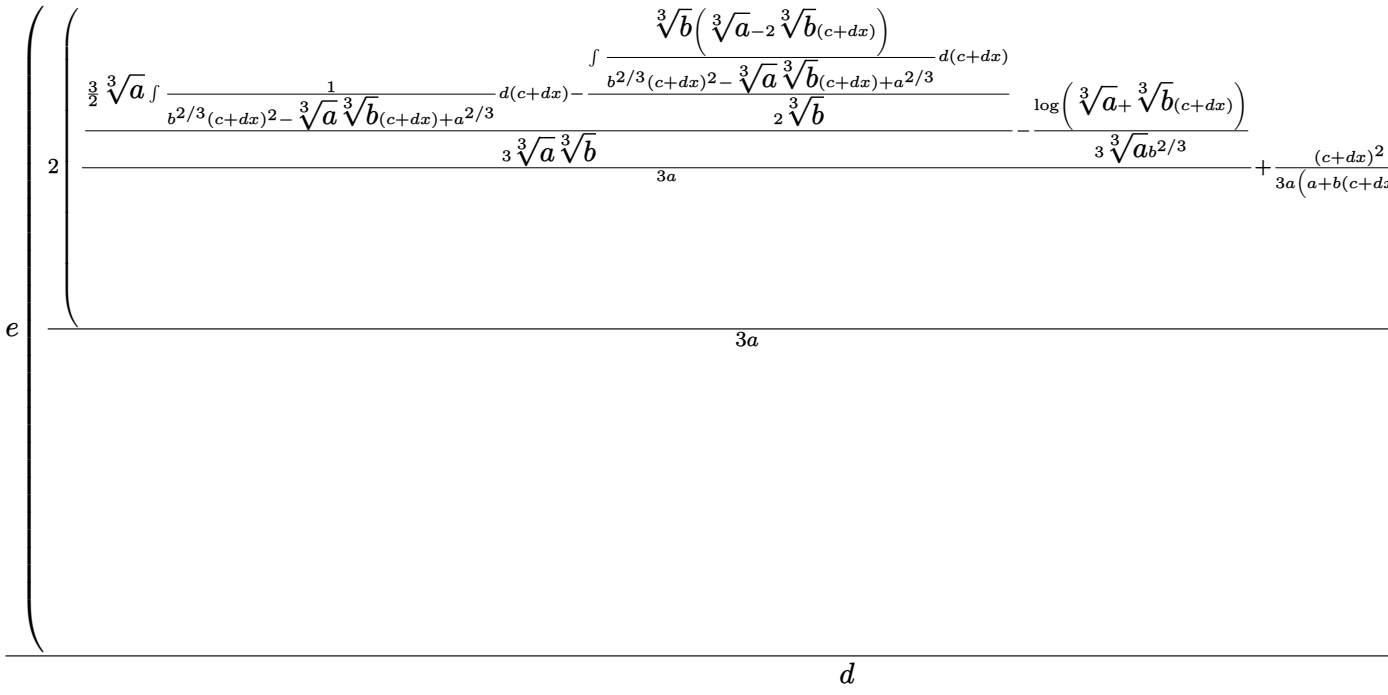
$$\downarrow 16$$

$$\frac{e\left(\frac{2\left(\frac{\int\frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+dx)^2}{6a(a+b(c+dx)^3)^2}\right)}{d}$$

$$\downarrow 1142$$



$\downarrow$ 25



$\downarrow$ 27

$$e\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)}{3a\left(a+b(c+\right.}\right.}{3a}\right.}{d}$$

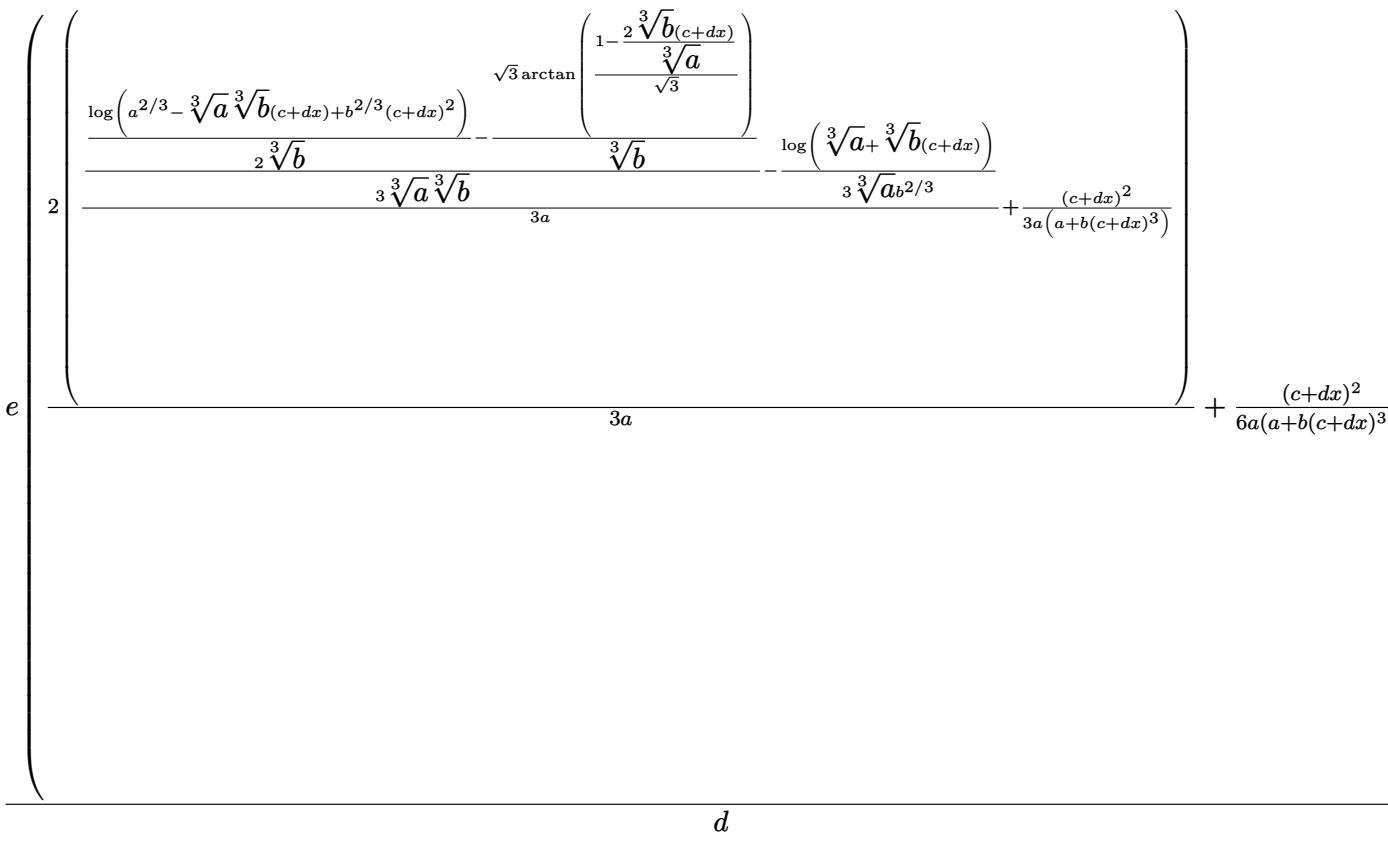
$\downarrow$ 1082

$$e\left(\frac{2\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right.}\right.}{3a}\right.}{d}$$

$\downarrow$ 217

$$\frac{e\left(\frac{2\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3a}+\frac{(c+dx)^2}{3a\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{(c+}{6a(a+b(}$$

$$\downarrow 1103$$



input
```
Int[(c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x]
```

output
```
(e*((c + d*x)^2/(6*a*(a + b*(c + d*x)^3)^2) + (2*((c + d*x)^2/(3*a*(a + b*
(c + d*x)^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3))
 + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/
3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p + 1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])   Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

method	result
default	$e\left(\frac{\frac{2b\,d^4x^5}{9a^2}+\frac{10bc\,d^3x^4}{9a^2}+\frac{20b\,c^2d^2x^3}{9a^2}+\frac{d\left(40c^3b+7a\right)x^2}{18a^2}+\frac{c\left(10c^3b+7a\right)x}{9a^2}+\frac{c^2\left(4c^3b+7a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\right.}\right.}{27a^2b}$
risch	$\frac{\frac{2b\,d^4e\,x^5}{9a^2}+\frac{10bc\,d^3e\,x^4}{9a^2}+\frac{20b\,c^2d^2e\,x^3}{9a^2}+\frac{de\left(40c^3b+7a\right)x^2}{18a^2}+\frac{ce\left(10c^3b+7a\right)x}{9a^2}+\frac{c^2e\left(4c^3b+7a\right)}{18d\,a^2}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{2e\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3b\right.}\right.}{27}$

input

```
int((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output

```
e*((2/9*b*d^4/a^2*x^5+10/9*b*c*d^3/a^2*x^4+20/9*b*c^2*d^2/a^2*x^3+1/18*d*(
40*b*c^3+7*a)/a^2*x^2+1/9*c*(10*b*c^3+7*a)/a^2*x+1/18*c^2/d*(4*b*c^3+7*a)/
a^2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+2/27/a^2/b/d*sum((_R*
d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+
3*_Z*b*c^2*d+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(166) = 332$.

Time = 0.18 (sec) , antiderivative size = 1780, normalized size of antiderivative = 8.60

$$\int \frac{ce + dex}{(a + b(c + dx)^3)^3}\, dx = \text{Too large to display}$$

input

```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
[1/54*(12*a*b^3*d^5*e*x^5 + 60*a*b^3*c*d^4*e*x^4 + 120*a*b^3*c^2*d^3*e*x^3
 + 3*(40*a*b^3*c^3 + 7*a^2*b^2)*d^2*e*x^2 + 6*(10*a*b^3*c^4 + 7*a^2*b^2*c)
*d*e*x + 6*sqrt(1/3)*(a*b^3*d^6*e*x^6 + 6*a*b^3*c*d^5*e*x^5 + 15*a*b^3*c^2
*d^4*e*x^4 + 2*(10*a*b^3*c^3 + a^2*b^2)*d^3*e*x^3 + 3*(5*a*b^3*c^4 + 2*a^2
*b^2*c)*d^2*e*x^2 + 6*(a*b^3*c^5 + a^2*b^2*c^2)*d*e*x + (a*b^3*c^6 + 2*a^2
*b^2*c^3 + a^3*b)*e)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*d^3*x^3 + 6*b^2*c*d
^2*x^2 + 6*b^2*c^2*d*x + 2*b^2*c^3 - a*b + 3*sqrt(1/3)*(a*b*d*x + a*b*c +
2*(d^2*x^2 + 2*c*d*x + c^2)*(-a*b^2)^(2/3) + (-a*b^2)^(1/3)*a)*sqrt((-a*b^
2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*(d*x + c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a)) + 2*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + 15*b^2*c^2
*d^4*e*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*
e*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e)*(-a*b
^2)^(2/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (-a*b^2)^(1/3)*(b*d*x
+ b*c) + (-a*b^2)^(2/3)) - 4*(b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + 15*b^2*c
^2*d^4*e*x^4 + 2*(10*b^2*c^3 + a*b)*d^3*e*x^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^
2*e*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*e*x + (b^2*c^6 + 2*a*b*c^3 + a^2)*e)*(-a
*b^2)^(2/3)*log(b*d*x + b*c - (-a*b^2)^(1/3)) + 3*(4*a*b^3*c^5 + 7*a^2*b^2
*c^2)*e)/(a^3*b^4*d^7*x^6 + 6*a^3*b^4*c*d^6*x^5 + 15*a^3*b^4*c^2*d^5*x^4 +
 2*(10*a^3*b^4*c^3 + a^4*b^3)*d^4*x^3 + 3*(5*a^3*b^4*c^4 + 2*a^4*b^3*c)*d^
3*x^2 + 6*(a^3*b^4*c^5 + a^4*b^3*c^2)*d^2*x + (a^3*b^4*c^6 + 2*a^4*b^3*...
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.56

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^3}\,dx$$
$$= \frac{7ac^2e+4bc^5e+40bc^2d^3ex^3+20bcd^4ex^4+4bd^5ex^5+x^2\cdot(7ad^2e+}{18a^4d+36a^3bc^3d+18a^2b^2c^6d+270a^2b^2c^2d^5x^4+108a^2b^2cd^6x^5+18a^2b^2d^7x^6+x^3\cdot(36a^3bd^4+360a^2b^2}$$
$$+\frac{e\,\mathrm{RootSum}\left(19683t^3a^7b^2+8,\left(t\mapsto t\log\left(x+\frac{729t^2a^5be^2+4ce^2}{4de^2}\right)\right)\right)}{d}$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)**3)**3,x)
```

output
```
(7*a*c**2*e + 4*b*c**5*e + 40*b*c**2*d**3*e*x**3 + 20*b*c*d**4*e*x**4 + 4*
b*d**5*e*x**5 + x**2*(7*a*d**2*e + 40*b*c**3*d**2*e) + x*(14*a*c*d*e + 20*
b*c**4*d*e))/(18*a**4*d + 36*a**3*b*c**3*d + 18*a**2*b**2*c**6*d + 270*a**
2*b**2*c**2*d**5*x**4 + 108*a**2*b**2*c*d**6*x**5 + 18*a**2*b**2*d**7*x**6
 + x**3*(36*a**3*b*d**4 + 360*a**2*b**2*c**3*d**4) + x**2*(108*a**3*b*c*d*
*3 + 270*a**2*b**2*c**4*d**3) + x*(108*a**3*b*c**2*d**2 + 108*a**2*b**2*c*
*5*d**2)) + e*RootSum(19683*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(x + (72
9*_t**2*a**5*b*e**2 + 4*c*e**2)/(4*d*e**2))))/d
```

Maxima [F]

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^3}\,dx = \int \frac{dex+ce}{\left((dx+c)^3b+a\right)^3}\,dx$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*d^5*e*x^5 + 20*b*c*d^4*e*x^4 + 40*b*c^2*d^3*e*x^3 + (40*b*c^3 +
7*a)*d^2*e*x^2 + 2*(10*b*c^4 + 7*a*c)*d*e*x + (4*b*c^5 + 7*a*c^2)*e)/(a^2*
b^2*d^7*x^6 + 6*a^2*b^2*c*d^6*x^5 + 15*a^2*b^2*c^2*d^5*x^4 + 2*(10*a^2*b^2
*c^3 + a^3*b)*d^4*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*x^2 + 6*(a^2*b^2
*c^5 + a^3*b*c^2)*d^2*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d) + 2/9*e*int
egrate((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)
/a^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.38

$$\int \frac{ce+dex}{\left(a+b(c+dx)^3\right)^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)}{3\left(-a^2b\right)^{\frac{2}{3}}}\right)+\left(-\frac{e^3}{ab^2d^3}\right)^{\frac{1}{3}}\log\left(\left(2\,abdx+2\,abc-\left(-a^2b\right)^{\frac{2}{3}}\right)^2+3\right.}{27\,a^2}$$

$$+\frac{4\,bd^5ex^5+20\,bcd^4ex^4+40\,bc^2d^3ex^3+40\,bc^3d^2ex^2+20\,bc^4dex+4\,bc^5e+7\,ad^2ex^2+14\,acdex+7\,ac^2}{18\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2a^2d}$$

input
```
integrate((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/27*(2*sqrt(3)*(-e^3/(a*b^2*d^3))^(1/3)*arctan(1/3*sqrt(3)*(2*a*b*d*x +
2*a*b*c - (-a^2*b)^(2/3))/(-a^2*b)^(2/3)) + (-e^3/(a*b^2*d^3))^(1/3)*log((
2*a*b*d*x + 2*a*b*c - (-a^2*b)^(2/3))^2 + 3*(-a^2*b)^(4/3)) - 2*(-e^3/(a*b
^2*d^3))^(1/3)*log(abs(a*b*d*x + a*b*c + (-a^2*b)^(2/3))))/a^2 + 1/18*(4*b
*d^5*e*x^5 + 20*b*c*d^4*e*x^4 + 40*b*c^2*d^3*e*x^3 + 40*b*c^3*d^2*e*x^2 +
20*b*c^4*d*e*x + 4*b*c^5*e + 7*a*d^2*e*x^2 + 14*a*c*d*e*x + 7*a*c^2*e)/((b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)^2*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.72

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^3}\,dx$$

$$= \frac{\frac{4\,b\,e\,c^5+7\,a\,e\,c^2}{18\,a^2\,d} + \frac{x^2\,(40\,b\,d\,e\,c^3+7\,a\,d\,e)}{18\,a^2} + \frac{c\,x\,(10\,b\,e\,c^3+7\,a\,e)}{9\,a^2} + \frac{2\,b\,d^4\,e\,x^5}{9\,a^2} + \frac{20\,b\,c^2\,d^2\,e\,x^3}{9\,a^2} + \frac{10}{}}{x^3\,(20\,b^2\,c^3\,d^3+2\,a\,b\,d^3)+x^2\,(15\,b^2\,c^4\,d^2+6\,a\,b\,c\,d^2)+a^2+x\,(6\,d\,b^2\,c^5+6\,a\,d\,b\,c^2)+b^2\,c^6+b^2\,d^6\,x^6+}$$

$$-\frac{2\,e\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{27\,a^{7/3}\,b^{2/3}\,d}$$

$$+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,c-2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(e-\sqrt{3}\,e\,1\mathrm{i}\right)}{27\,a^{7/3}\,b^{2/3}\,d}$$

$$+\frac{\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(e+\sqrt{3}\,e\,1\mathrm{i}\right)}{27\,a^{7/3}\,b^{2/3}\,d}$$

input
```
int((c*e + d*e*x)/(a + b*(c + d*x)^3)^3,x)
```

output
```
((7*a*c^2*e + 4*b*c^5*e)/(18*a^2*d) + (x^2*(7*a*d*e + 40*b*c^3*d*e))/(18*a
^2) + (c*x*(7*a*e + 10*b*c^3*e))/(9*a^2) + (2*b*d^4*e*x^5)/(9*a^2) + (20*b
*c^2*d^2*e*x^3)/(9*a^2) + (10*b*c*d^3*e*x^4)/(9*a^2))/(x^3*(20*b^2*c^3*d^3
 + 2*a*b*d^3) + x^2*(15*b^2*c^4*d^2 + 6*a*b*c*d^2) + a^2 + x*(6*b^2*c^5*d
+ 6*a*b*c^2*d) + b^2*c^6 + b^2*d^6*x^6 + 2*a*b*c^3 + 6*b^2*c*d^5*x^5 + 15*
b^2*c^2*d^4*x^4) - (2*e*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(7/3
)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*c + a^(1/3) - 2*b^(1/3)
*d*x)*(e - 3^(1/2)*e*1i))/(27*a^(7/3)*b^(2/3)*d) + (log(3^(1/2)*a^(1/3)*1i
 + 2*b^(1/3)*c - a^(1/3) + 2*b^(1/3)*d*x)*(e + 3^(1/2)*e*1i))/(27*a^(7/3)*
b^(2/3)*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1829, normalized size of antiderivative = 8.84

$$\int \frac{ce+dex}{(a+b(c+dx)^3)^3}\,dx = \text{Too large to display}$$

input
```
int((d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x)
```

output

```
(e*( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3)))*a**2*b*c - 8*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4 - 24*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**3*d*x - 24*sqrt(3)*
atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2
*c**2*d**2*x**2 - 8*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x
)/(a**(1/3)*sqrt(3)))*a*b**2*c*d**3*x**3 - 4*sqrt(3)*atan((a**(1/3) - 2*b*
*(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7 - 24*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**6*
d*x - 60*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)
*sqrt(3)))*b**3*c**5*d**2*x**2 - 80*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c
- 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**3*x**3 - 60*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**3
*d**4*x**4 - 24*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3)))*b**3*c**2*d**5*x**5 - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**6*x**6 - 2*b**(2/3)*
a**(1/3)*a**2 + 17*b**(2/3)*a**(1/3)*a*b*c**3 + 30*b**(2/3)*a**(1/3)*a*b*c
**2*d*x + 9*b**(2/3)*a**(1/3)*a*b*c*d**2*x**2 - 4*b**(2/3)*a**(1/3)*a*b*d*
*3*x**3 + 10*b**(2/3)*a**(1/3)*b**2*c**6 + 48*b**(2/3)*a**(1/3)*b**2*c**5*
d*x + 90*b**(2/3)*a**(1/3)*b**2*c**4*d**2*x**2 + 80*b**(2/3)*a**(1/3)*b...
```

3.105 $\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result 875
Mathematica [A] (verified) 875
Rubi [A] (verified) 876
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Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{1}{6ade\left(a+b(c+dx)^3\right)^2} + \frac{1}{3a^2de\left(a+b(c+dx)^3\right)} + \frac{\log(c+dx)}{a^3de} - \frac{\log\left(a+b(c+dx)^3\right)}{3a^3de}$$

output
```
1/6/a/d/e/(a+b*(d*x+c)^3)^2+1/3/a^2/d/e/(a+b*(d*x+c)^3)+ln(d*x+c)/a^3/d/e-
1/3*ln(a+b*(d*x+c)^3)/a^3/d/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\frac{a\left(a+2\left(a+b(c+dx)^3\right)\right)}{\left(a+b(c+dx)^3\right)^2} + 6\log(c+dx) - 2\log\left(a+b(c+dx)^3\right)}{6a^3de}$$

input
```
Integrate[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x]
```

output
```
((a*(a + 2*(a + b*(c + d*x)^3)))/(a + b*(c + d*x)^3)^2 + 6*Log[c + d*x] -
2*Log[a + b*(c + d*x)^3])/(6*a^3*d*e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)(b(c+dx)^3+a)^3}\,d(c+dx)}{de}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^3(b(c+dx)^3+a)^3}\,d(c+dx)^3}{3de}$$

$$\downarrow 54$$

$$\frac{\int \left(-\frac{b}{a^3(b(c+dx)^3+a)} - \frac{b}{a^2(b(c+dx)^3+a)^2} - \frac{b}{a(b(c+dx)^3+a)^3} + \frac{1}{a^3(c+dx)^3}\right) d(c+dx)^3}{3de}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(a+b(c+dx)^3)}{a^3} + \frac{\log((c+dx)^3)}{a^3} + \frac{1}{a^2(a+b(c+dx)^3)} + \frac{1}{2a(a+b(c+dx)^3)^2}}{3de}$$

input
```
Int[1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x]
```

output
```
(1/(2*a*(a + b*(c + d*x)^3)^2) + 1/(a^2*(a + b*(c + d*x)^3)) + Log[(c + d*
x)^3]/a^3 - Log[a + b*(c + d*x)^3]/a^3)/(3*d*e)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.57

method	result
default	$\frac{\frac{\ln(xd+c)}{a^3 d}-\frac{b\left(\frac{-\frac{a\,d^2x^3}{3}-acd\,x^2-a\,c^2x-\frac{a\left(2c^3b+3a\right)}{6db}}{\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)^2}+\frac{\ln\left(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a\right)}{3bd}\right)}{a^3}}{e}$
risch	$\frac{\frac{b\,d^2x^3}{3a^2}+\frac{bcd\,x^2}{a^2}+\frac{bx\,c^2}{a^2}+\frac{2c^3b+3a}{6a^2d}}{e(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2}+\frac{\ln(xd+c)}{a^3de}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^3ed}$
norman	$\frac{\frac{cbd\,x^2}{a^2e}+\frac{c^2bx}{a^2e}+\frac{2b^3c^3d^5+3a\,b^2d^5}{6a^2d^6eb^2}+\frac{b\,d^2x^3}{3a^2e}}{(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)^2}+\frac{\ln(xd+c)}{a^3de}-\frac{\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)}{3a^3ed}$
parallelrisch	$\frac{-12\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)x^2a\,b^3c\,d^7-12\ln(b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a)xa\,b^3c^2d^6+36\ln(xd+c)x^2a\,b$

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e*(ln(d*x+c)/a^3/d-1/a^3*b*((-1/3*a*d^2*x^3-a*c*d*x^2-a*c^2*x-1/6*a/d*(2
*b*c^3+3*a)/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+1/3/b/d*ln(
b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(88) = 176$.

Time = 0.09 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.04

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+3\,a^2-2\,(b^2d^6x^6+6\,b^2cd^5x^5+15\,b^2c^2d^4x^4+b^2c^6+2\,(10\,b^2c^3}{6\,(a}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2 -
 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*c^6 + 2*(10*b
^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d^2*x^2 + 6*(b
^2*c^5 + a*b*c^2)*d*x + a^2)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a) + 6*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + b^2*
c^6 + 2*(10*b^2*c^3 + a*b)*d^3*x^3 + 2*a*b*c^3 + 3*(5*b^2*c^4 + 2*a*b*c)*d
^2*x^2 + 6*(b^2*c^5 + a*b*c^2)*d*x + a^2)*log(d*x + c))/(a^3*b^2*d^7*e*x^6
 + 6*a^3*b^2*c*d^6*e*x^5 + 15*a^3*b^2*c^2*d^5*e*x^4 + 2*(10*a^3*b^2*c^3 +
a^4*b)*d^4*e*x^3 + 3*(5*a^3*b^2*c^4 + 2*a^4*b*c)*d^3*e*x^2 + 6*(a^3*b^2*c^
5 + a^4*b*c^2)*d^2*e*x + (a^3*b^2*c^6 + 2*a^4*b*c^3 + a^5)*d*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(73) = 146$.

Time = 2.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.11

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{3a+2bc^3+6bc^2dx+6bcd^2x^2+2b}{6a^4de+12a^3bc^3de+6a^2b^2c^6de+90a^2b^2c^2d^5ex^4+36a^2b^2cd^6ex^5+6a^2b^2d^7ex^6+x^3\cdot(12a^3bd^4e+120a^2b}$$
$$+\frac{\log\left(\frac{c}{d}+x\right)}{a^3de}-\frac{\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{3a^3de}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)**3)**3,x)
```

output
```
(3*a + 2*b*c**3 + 6*b*c**2*d*x + 6*b*c*d**2*x**2 + 2*b*d**3*x**3)/(6*a**4*
d*e + 12*a**3*b*c**3*d*e + 6*a**2*b**2*c**6*d*e + 90*a**2*b**2*c**2*d**5*e
*x**4 + 36*a**2*b**2*c*d**6*e*x**5 + 6*a**2*b**2*d**7*e*x**6 + x**3*(12*a*
*3*b*d**4*e + 120*a**2*b**2*c**3*d**4*e) + x**2*(36*a**3*b*c*d**3*e + 90*a
**2*b**2*c**4*d**3*e) + x*(36*a**3*b*c**2*d**2*e + 36*a**2*b**2*c**5*d**2*
e)) + log(c/d + x)/(a**3*d*e) - log(3*c**2*x/d**2 + 3*c*x**2/d + x**3 + (a
 + b*c**3)/(b*d**3))/(3*a**3*d*e)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(88) = 176.

Time = 0.05 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.74

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{2\,bd^3x^3+6\,bcd^2x^2+6\,bc^2dx+2\,bc^3+3\,a}{6\left(a^2b^2d^7ex^6+6\,a^2b^2cd^6ex^5+15\,a^2b^2c^2d^5ex^4+2\left(10\,a^2b^2c^3+a^3b\right)d^4ex^3+3\left(5\,a^2b^2c^4+2\,a^3bc\right)d^3ex^2+6\right.}$$
$$-\frac{\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{3\,a^3de}+\frac{\log\left(dx+c\right)}{a^3de}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/6*(2*b*d^3*x^3 + 6*b*c*d^2*x^2 + 6*b*c^2*d*x + 2*b*c^3 + 3*a)/(a^2*b^2*d
^7*e*x^6 + 6*a^2*b^2*c*d^6*e*x^5 + 15*a^2*b^2*c^2*d^5*e*x^4 + 2*(10*a^2*b^
2*c^3 + a^3*b)*d^4*e*x^3 + 3*(5*a^2*b^2*c^4 + 2*a^3*b*c)*d^3*e*x^2 + 6*(a^
2*b^2*c^5 + a^3*b*c^2)*d^2*e*x + (a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)*d*e) -
1/3*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(a^3*d*e) + l
og(d*x + c)/(a^3*d*e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{3\,a^3de} + \frac{\log\left(\left|dx+c\right|\right)}{a^3de} + \frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3+3\,a^2}{6\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2a^3de}$$

input
```
integrate(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^3*d*
e) + log(abs(d*x + c))/(a^3*d*e) + 1/6*(2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 +
6*a*b*c^2*d*x + 2*a*b*c^3 + 3*a^2)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d
*x + b*c^3 + a)^2*a^3*d*e)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = \frac{\frac{2\,bc^3+3\,a}{6\,a^2\,d}+\frac{b\,d^2\,x^3}{3\,a^2}+\frac{b\,c^2\,x}{a^2}+\frac{b\,c\,d\,x^2}{a^2}}{x^2\,\left(15\,e\,b^2\,c^4\,d^2+6\,a\,e\,b\,c\,d^2\right)+a^2\,e+x\,\left(6\,d\,e\,b^2\,c^5+6\,a\,d\,e\,b\,c^2\right)+x^3\,\left(20\,e\,b^2\,c^3\,d^3+2\,a\,e\,b\,d^3\right)+b^2\,c^6\,e} + \frac{\ln\left(c+d\,x\right)}{a^3\,d\,e}-\frac{\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{3\,a^3\,d\,e}$$

input
```
int(1/((c*e + d*e*x)*(a + b*(c + d*x)^3)^3),x)
```

output
```
((3*a + 2*b*c^3)/(6*a^2*d) + (b*d^2*x^3)/(3*a^2) + (b*c^2*x)/a^2 + (b*c*d*
x^2)/a^2)/(x^2*(15*b^2*c^4*d^2*e + 6*a*b*c*d^2*e) + a^2*e + x*(6*b^2*c^5*d
*e + 6*a*b*c^2*d*e) + x^3*(20*b^2*c^3*d^3*e + 2*a*b*d^3*e) + b^2*c^6*e + b
^2*d^6*e*x^6 + 2*a*b*c^3*e + 6*b^2*c*d^5*e*x^5 + 15*b^2*c^2*d^4*e*x^4) + l
og(c + d*x)/(a^3*d*e) - log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^
2*x^2)/(3*a^3*d*e)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1405, normalized size of antiderivative = 14.95

$$\int \frac{1}{(ce+dex)\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*e*x+c*e)/(a+b*(d*x+c)^3)^3,x)
```

output

```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3
)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2 - 4*log(a**(2/3) - b*
*(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d
*x + b**(2/3)*d**2*x**2)*a*b*c**3 - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c
- b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2
*x**2)*a*b*c**2*d*x - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**
(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c*d
**2*x**2 - 4*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x +
b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*d**3*x**3 - 2*l
og(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
+ 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c**6 - 12*log(a**(2/3) - b**
(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*
x + b**(2/3)*d**2*x**2)*b**2*c**5*d*x - 30*log(a**(2/3) - b**(1/3)*a**(1/3
)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*
d**2*x**2)*b**2*c**4*d**2*x**2 - 40*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b
**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x*
*2)*b**2*c**3*d**3*x**3 - 30*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)
*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**
2*c**2*d**4*x**4 - 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/
3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b**2*c*...
```

3.106 $\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 883
Mathematica [A] (verified) . . . 884
Rubi [A] (verified) . . . 884
Maple [C] (verified) . . . 894
Fricas [B] (verification not implemented) . . . 895
Sympy [B] (verification not implemented) . . . 896
Maxima [F] . . . 897
Giac [A] (verification not implemented) . . . 898
Mupad [B] (verification not implemented) . . . 899
Reduce [B] (verification not implemented) . . . 899

Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = &-\frac{1}{a^3de^2(c+dx)}-\frac{b(c+dx)^2}{6a^2de^2\left(a+b(c+dx)^3\right)^2}\\ &-\frac{5b(c+dx)^2}{9a^3de^2\left(a+b(c+dx)^3\right)}\\ &+\frac{14\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}de^2}\\ &+\frac{14\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{10/3}de^2}\\ &-\frac{7\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{10/3}de^2}\end{aligned}$$

output

```
-1/a^3/d/e^2/(d*x+c)-1/6*b*(d*x+c)^2/a^2/d/e^2/(a+b*(d*x+c)^3)^2-5/9*b*(d*
x+c)^2/a^3/d/e^2/(a+b*(d*x+c)^3)+14/27*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(10/3)/d/e^2+14/27*b^(1/3)*ln(a^(1/
3)+b^(1/3)*(d*x+c))/a^(10/3)/d/e^2-7/27*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(10/3)/d/e^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{-\frac{54\sqrt[3]{a}}{c+dx} - \frac{9a^{4/3}b(c+dx)^2}{(a+b(c+dx)^3)^2} - \frac{30\sqrt[3]{a}b(c+dx)^2}{a+b(c+dx)^3} - 28\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 28\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+\right.}{54a^{10/3}de^2}$$

input
```
Integrate[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x]
```

output
```
((-54*a^(1/3))/(c + d*x) - (9*a^(4/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3)^2
 - (30*a^(1/3)*b*(c + d*x)^2)/(a + b*(c + d*x)^3) - 28*Sqrt[3]*b^(1/3)*Arc
Tan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 28*b^(1/3)*Log[a
^(1/3) + b^(1/3)*(c + d*x)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c
+ d*x) + b^(2/3)*(c + d*x)^2])/(54*a^(10/3)*d*e^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {895, 819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{1}{(c+dx)^2(b(c+dx)^3+a)^3}\,d(c+dx)}{de^2}$$
$$\downarrow 819$$
$$\frac{7\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{6a} + \frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}$$
$$de^2$$

$$\downarrow 819$$

$$\frac{7\left(\frac{4\int \frac{1}{(c+dx)^2\left(b(c+dx)^3+a\right)}d(c+dx)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}}{de^2}$$

$$\downarrow 847$$

$$\frac{7\left(\frac{4\left(-\frac{b\int \frac{c+dx}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(c+dx)^3)^2}}{de^2}$$

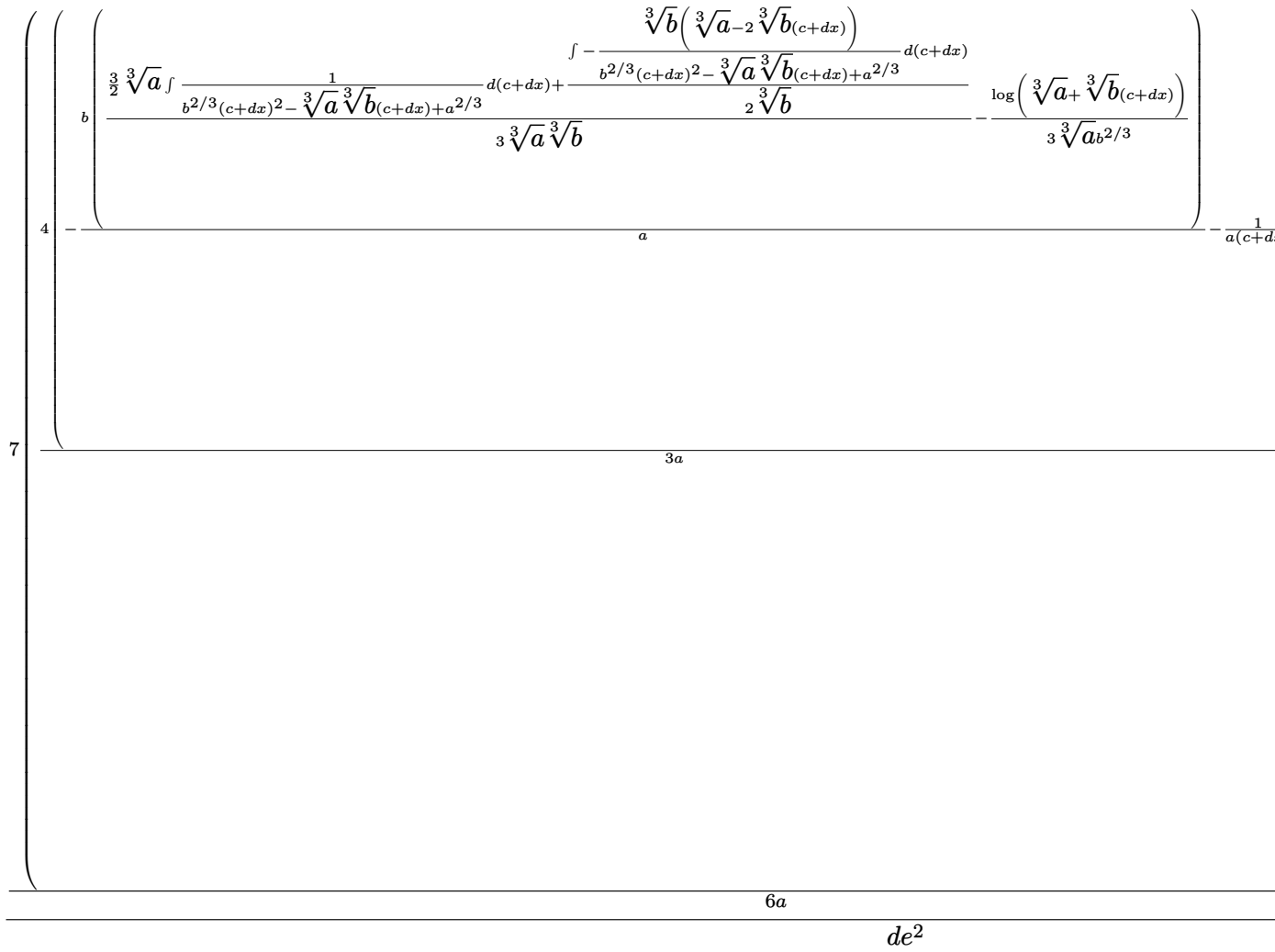
$$\downarrow 821$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(}}{de^2}$$

$$\downarrow 16$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}+\frac{1}{3a(c+dx)\left(a+b(c+dx)^3\right)}\right)}{6a}+\frac{1}{6a(c+dx)(a+b(c+d}}{de^2}$$

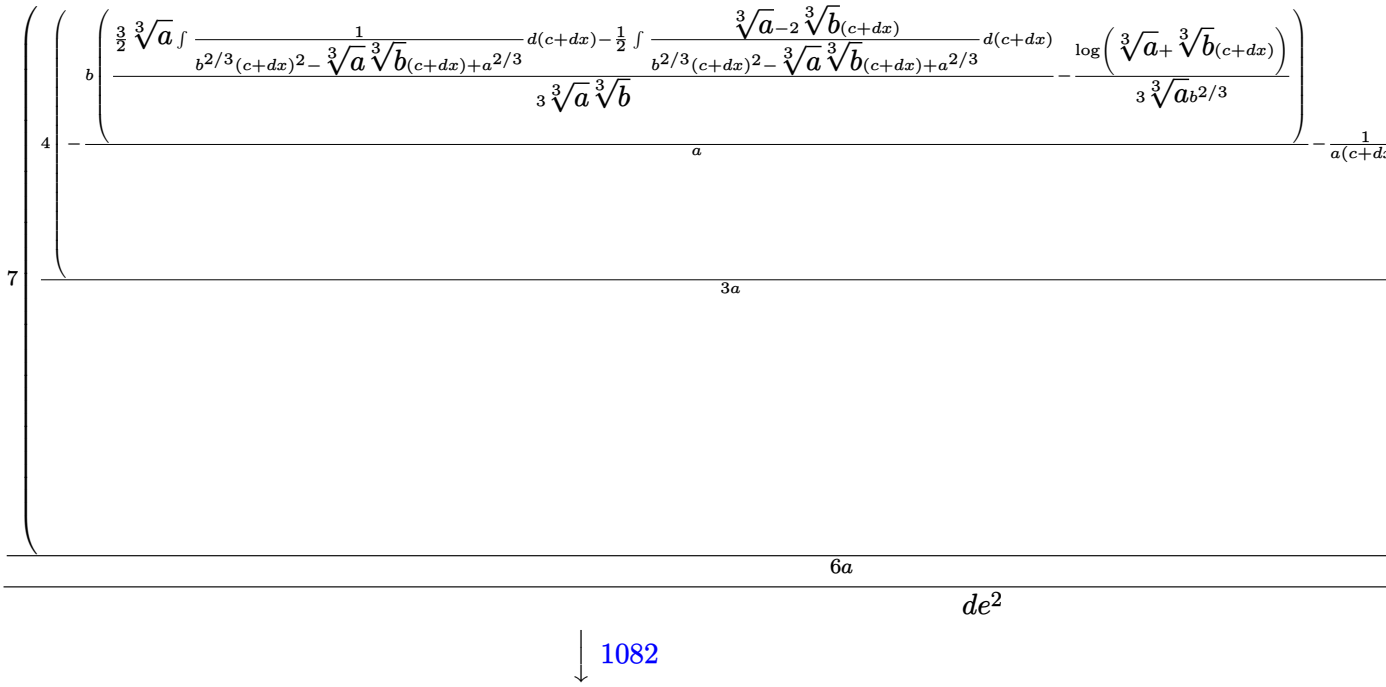
$$\downarrow 1142$$



$\downarrow$ 25

$$\frac{\left(7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}\right)}{6a}\right)}{de^2}$$

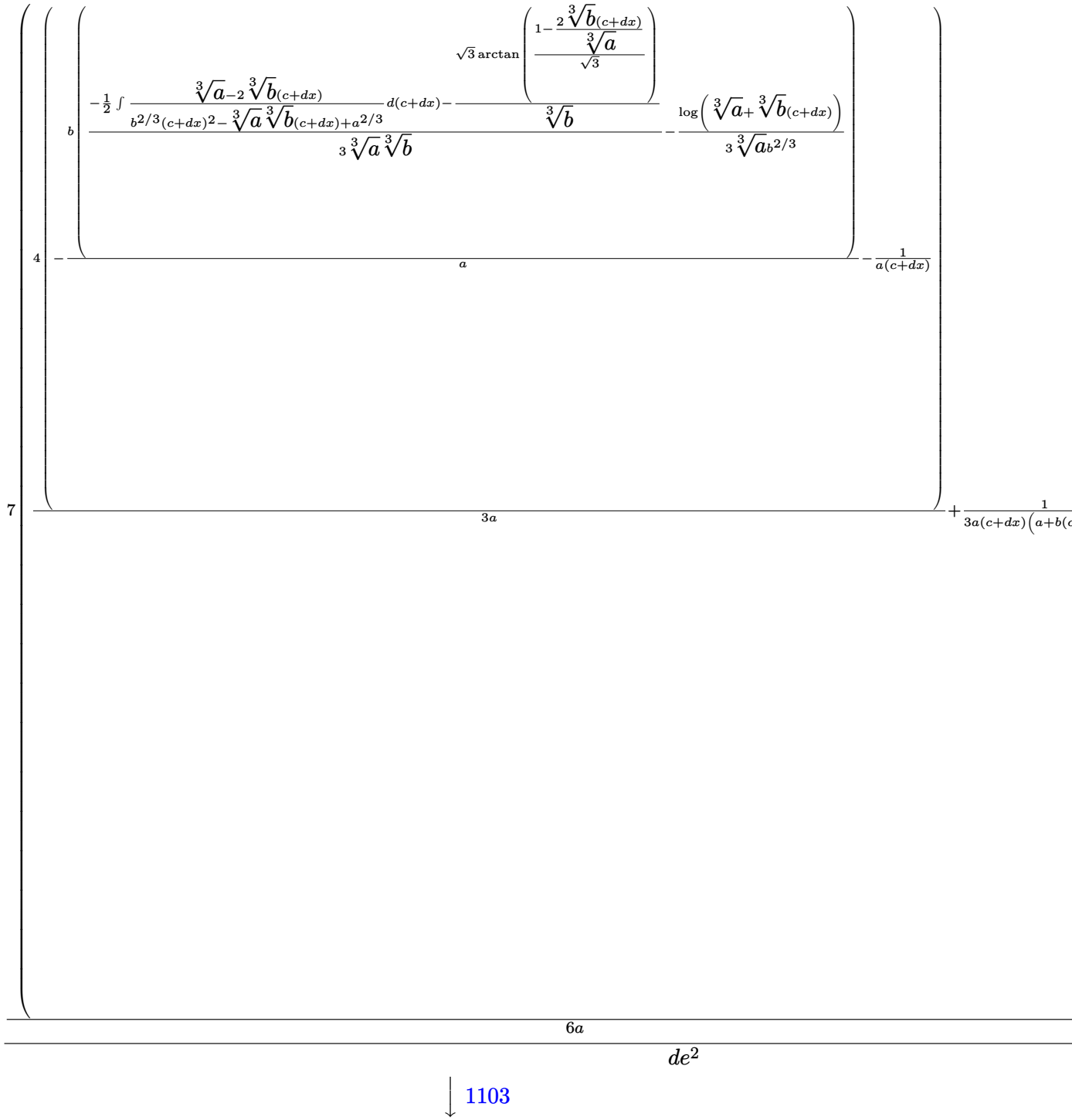
$$\downarrow 27$$



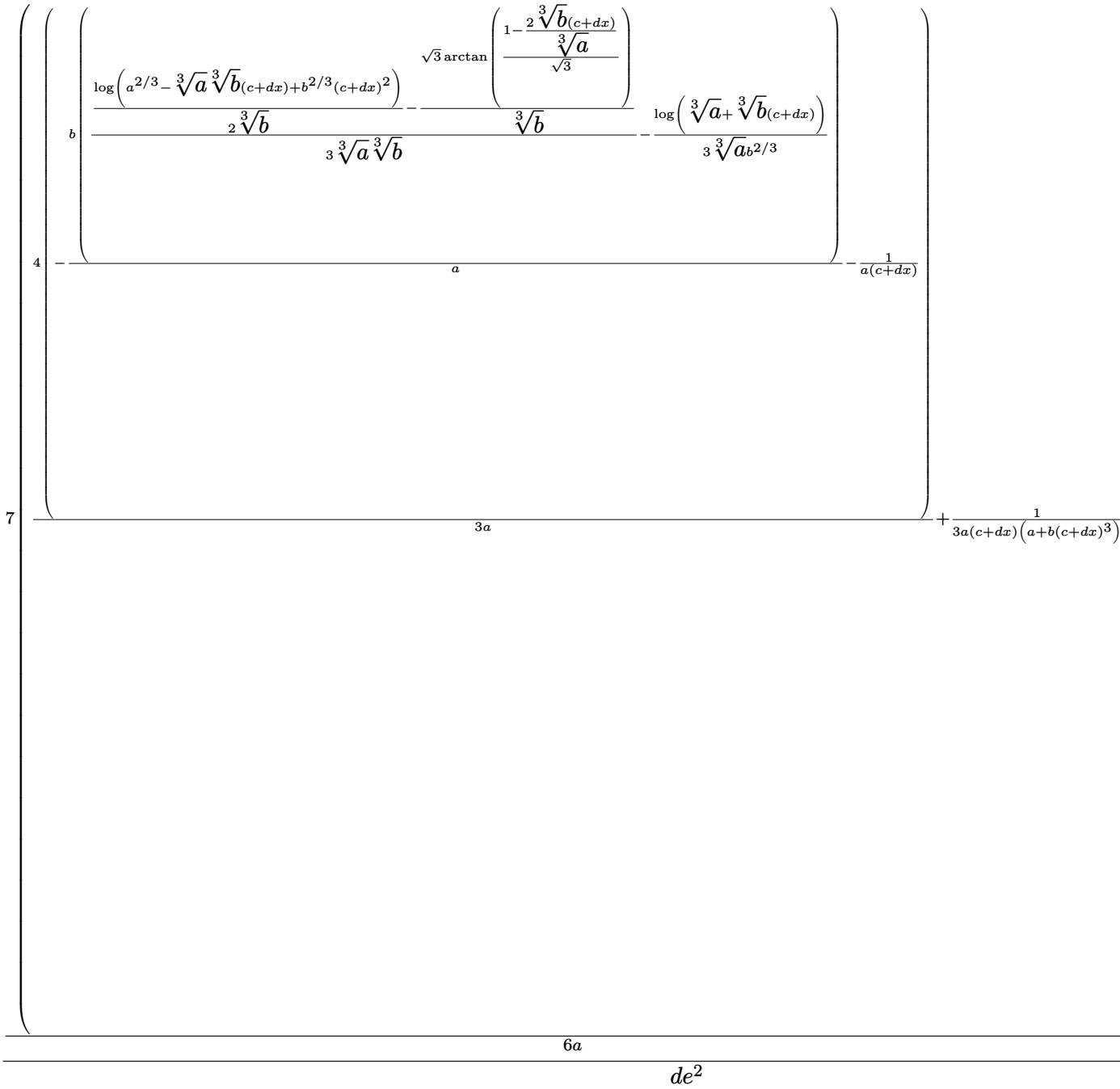
$$\downarrow 1082$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{a(c+dx)}\right)}{3a}\right)}{6a}}{de^2}$$

$\downarrow$ 217



$$\downarrow 1103$$



input `Int[1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x]`

output
```
(1/(6*a*(c + d*x)*(a + b*(c + d*x)^3)^2) + (7*(1/(3*a*(c + d*x)*(a + b*(c
+ d*x)^3)) + (4*(-(1/(a*(c + d*x))) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*(c +
d*x)]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x))/a^
(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2
/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)))/(6*a))/(d*
e^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.87 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

method	result
default	$\frac{-\frac{1}{a^3 d(xd+c)}-\frac{b\left(\frac{\frac{5d^4bx^5}{9}+\frac{25bcd^3x^4}{9}+\frac{50bc^2d^2x^3}{9}+\left(\frac{50}{9}bc^3d+\frac{13}{18}ad\right)x^2+\frac{c\left(25c^3b+13a\right)x}{9}+\frac{c^2\left(10c^3b+13a\right)}{18d}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{14\left(\underline{R}=\text{RootOf}\left(bd^3\underline{Z}^3+3\right.\right.\right.}{a^3}}{e^2}$
risch	$\frac{-\frac{14b^2d^5x^6}{9a^3}-\frac{28b^2cd^4x^5}{3a^3}-\frac{70b^2c^2d^3x^4}{3a^3}-\frac{7bd^2\left(80c^3b+7a\right)x^3}{18a^3}-\frac{7bcd\left(20c^3b+7a\right)x^2}{6a^3}-\frac{7\left(8c^3b+7a\right)bc^2x}{6a^3}-\frac{28b^2c^6+49abc^3+18a^2}{18da^3}}{e^2(xd+c)(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{14\left(\underline{R}\right.}{}$

input
```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(-1/a^3/d/(d*x+c)-1/a^3*b*((5/9*d^4*b*x^5+25/9*b*c*d^3*x^4+50/9*b*c^
2*d^2*x^3+(50/9*b*c^3*d+13/18*a*d)*x^2+1/9*c*(25*b*c^3+13*a)*x+1/18*c^2/d*
(10*b*c^3+13*a))/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+14/27/b/d
*sum((_R*d+c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(196) = 392$.

Time = 0.16 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.71

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
-1/54*(84*b^2*d^6*x^6 + 504*b^2*c*d^5*x^5 + 1260*b^2*c^2*d^4*x^4 + 84*b^2*
c^6 + 21*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 147*a*b*c^3 + 63*(20*b^2*c^4 + 7*a
*b*c)*d^2*x^2 + 63*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 28*sqrt(3)*(b^2*d^7*x^7 +
 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4
*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c
^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*arcta
n(2/3*sqrt(3)*(d*x + c)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*d^7*x^7 + 7*b
^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c^3 + 2*a*b)*d^4*x^4
 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^2*c^5 + 4*a*b*c^2)*
d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b/a)^(1/3)*log(b*d^2
*x^2 + 2*b*c*d*x + b*c^2 - (a*d*x + a*c)*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28
*(b^2*d^7*x^7 + 7*b^2*c*d^6*x^6 + 21*b^2*c^2*d^5*x^5 + b^2*c^7 + (35*b^2*c
^3 + 2*a*b)*d^4*x^4 + (35*b^2*c^4 + 8*a*b*c)*d^3*x^3 + 2*a*b*c^4 + 3*(7*b^
2*c^5 + 4*a*b*c^2)*d^2*x^2 + a^2*c + (7*b^2*c^6 + 8*a*b*c^3 + a^2)*d*x)*(b
/a)^(1/3)*log(b*d*x + b*c + a*(b/a)^(2/3)) + 54*a^2)/(a^3*b^2*d^8*e^2*x^7
+ 7*a^3*b^2*c*d^7*e^2*x^6 + 21*a^3*b^2*c^2*d^6*e^2*x^5 + (35*a^3*b^2*c^3 +
 2*a^4*b)*d^5*e^2*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*e^2*x^3 + 3*(7*a^
3*b^2*c^5 + 4*a^4*b*c^2)*d^3*e^2*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5)
*d^2*e^2*x + (a^3*b^2*c^7 + 2*a^4*b*c^4 + a^5*c)*d*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(219) = 438$.

Time = 2.72 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.88

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-18a^2 - 49abc^3 - 28b^2c^6 - 420b^2c^2d^4x^4 - 168b^2cd^5x^5 -}{18a^5cde^2 + 36a^4bc^4de^2 + 18a^3b^2c^7de^2 + 378a^3b^2c^2d^6e^2x^5 + 126a^3b^2cd^7e^2x^6 + 18a^3b^2d^8e^2x^7 + x^4\cdot(36a^4b} + \frac{\text{RootSum}\left(19683t^3a^{10} - 2744b, \left(t \mapsto t\log\left(x + \frac{729t^2a^7 + 196bc}{196bd}\right)\right)\right)}{de^2}$$

input

```
integrate(1/(d*e*x+c*e)**2/(a+b*(d*x+c)**3)**3,x)
```

output

```
(-18*a**2 - 49*a*b*c**3 - 28*b**2*c**6 - 420*b**2*c**2*d**4*x**4 - 168*b**
2*c*d**5*x**5 - 28*b**2*d**6*x**6 + x**3*(-49*a*b*d**3 - 560*b**2*c**3*d**
3) + x**2*(-147*a*b*c*d**2 - 420*b**2*c**4*d**2) + x*(-147*a*b*c**2*d - 16
8*b**2*c**5*d))/(18*a**5*c*d*e**2 + 36*a**4*b*c**4*d*e**2 + 18*a**3*b**2*c
**7*d*e**2 + 378*a**3*b**2*c**2*d**6*e**2*x**5 + 126*a**3*b**2*c*d**7*e**2
*x**6 + 18*a**3*b**2*d**8*e**2*x**7 + x**4*(36*a**4*b*d**5*e**2 + 630*a**3
*b**2*c**3*d**5*e**2) + x**3*(144*a**4*b*c*d**4*e**2 + 630*a**3*b**2*c**4*
d**4*e**2) + x**2*(216*a**4*b*c**2*d**3*e**2 + 378*a**3*b**2*c**5*d**3*e**
2) + x*(18*a**5*d**2*e**2 + 144*a**4*b*c**3*d**2*e**2 + 126*a**3*b**2*c**6
*d**2*e**2)) + RootSum(19683*_t**3*a**10 - 2744*b, Lambda(_t, _t*log(x + (
729*_t**2*a**7 + 196*b*c)/(196*b*d))))/(d*e**2)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^3 (dex+ce)^2}\,dx$$

input

```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
-1/18*(28*b^2*d^6*x^6 + 168*b^2*c*d^5*x^5 + 420*b^2*c^2*d^4*x^4 + 28*b^2*c
^6 + 7*(80*b^2*c^3 + 7*a*b)*d^3*x^3 + 49*a*b*c^3 + 21*(20*b^2*c^4 + 7*a*b*
c)*d^2*x^2 + 21*(8*b^2*c^5 + 7*a*b*c^2)*d*x + 18*a^2)/(a^3*b^2*d^8*e^2*x^7
 + 7*a^3*b^2*c*d^7*e^2*x^6 + 21*a^3*b^2*c^2*d^6*e^2*x^5 + (35*a^3*b^2*c^3
+ 2*a^4*b)*d^5*e^2*x^4 + (35*a^3*b^2*c^4 + 8*a^4*b*c)*d^4*e^2*x^3 + 3*(7*a
^3*b^2*c^5 + 4*a^4*b*c^2)*d^3*e^2*x^2 + (7*a^3*b^2*c^6 + 8*a^4*b*c^3 + a^5
)*d^2*e^2*x + (a^3*b^2*c^7 + 2*a^4*b*c^4 + a^5*c)*d*e^2) - 14/9*b*integrat
e((d*x + c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^3
*e^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.24

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx$$
$$= \frac{14\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}-\frac{1}{(dex+ce)de}\right|\right)}{27\,a^3} - \frac{\frac{10\,b^2}{(dex+ce)de}+\frac{13\,abe^2}{(dex+ce)^4d}}{18\,a^3\left(b+\frac{ae^3}{(dex+ce)^3}\right)^2}$$
$$- \frac{14\sqrt{3}(a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}-\frac{2}{(dex+ce)de}\right)}{3\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}\right)}{27\,a^4de^2}$$
$$- \frac{7\,(a^2b)^{\frac{1}{3}}\log\left(\left(\frac{b}{ad^3e^6}\right)^{\frac{2}{3}}-\frac{\left(\frac{b}{ad^3e^6}\right)^{\frac{1}{3}}}{(dex+ce)de}+\frac{1}{(dex+ce)^2d^2e^2}\right)}{27\,a^4de^2} - \frac{1}{(dex+ce)a^3de}$$

input
```
integrate(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
14/27*(b/(a*d^3*e^6))^(1/3)*log(abs(-(b/(a*d^3*e^6))^(1/3) - 1/((d*e*x + c
*e)*d*e)))/a^3 - 1/18*(10*b^2/((d*e*x + c*e)*d*e) + 13*a*b*e^2/((d*e*x + c
*e)^4*d))/(a^3*(b + a*e^3/(d*e*x + c*e)^3)^2) - 14/27*sqrt(3)*(a^2*b)^(1/3
)*arctan(1/3*sqrt(3)*((b/(a*d^3*e^6))^(1/3) - 2/((d*e*x + c*e)*d*e))/(b/(a
*d^3*e^6))^(1/3))/(a^4*d*e^2) - 7/27*(a^2*b)^(1/3)*log((b/(a*d^3*e^6))^(2/
3) - (b/(a*d^3*e^6))^(1/3)/((d*e*x + c*e)*d*e) + 1/((d*e*x + c*e)^2*d^2*e^
2))/(a^4*d*e^2) - 1/((d*e*x + c*e)*a^3*d*e)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.05

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = \frac{14\,b^{1/3}\,\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{27\,a^{10/3}\,d\,e^2}$$
$$-\frac{\frac{18\,a^2+49\,a\,b\,c^3+28\,b^2\,c^6}{18\,a^3\,d}+\frac{7\,x^2\,\left(20\,d\,b^2\,c^4+7\,a\,d\,b\,c\right)}{6\,a^3}+\frac{7\,x\,\left(8\,b^2\,c^5+\right.}{6\,a^3}}{x^3\,\left(35\,b^2\,c^4\,d^3\,e^2+8\,a\,b\,c\,d^3\,e^2\right)+x^2\,\left(21\,b^2\,c^5\,d^2\,e^2+12\,a\,b\,c^2\,d^2\,e^2\right)+x\,\left(d\,a^2\,e^2+8\,d\,a\,b\,c^3\,e^2+7\,d\,b^2\,c\right.}$$
$$+\frac{14\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{10/3}\,d\,e^2}$$
$$-\frac{14\,b^{1/3}\,\ln\left(2\,b^{1/3}\,c-a^{1/3}+2\,b^{1/3}\,d\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,a^{10/3}\,d\,e^2}$$

input
```
int(1/((c*e + d*e*x)^2*(a + b*(c + d*x)^3)^3),x)
```

output
```
(14*b^(1/3)*log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x))/(27*a^(10/3)*d*e^2) -
((18*a^2 + 28*b^2*c^6 + 49*a*b*c^3)/(18*a^3*d) + (7*x^2*(20*b^2*c^4*d + 7*
a*b*c*d))/(6*a^3) + (7*x*(8*b^2*c^5 + 7*a*b*c^2))/(6*a^3) + (7*x^3*(80*b^2
*c^3*d^2 + 7*a*b*d^2))/(18*a^3) + (14*b^2*d^5*x^6)/(9*a^3) + (70*b^2*c^2*d
^3*x^4)/(3*a^3) + (28*b^2*c*d^4*x^5)/(3*a^3))/(x^3*(35*b^2*c^4*d^3*e^2 + 8
*a*b*c*d^3*e^2) + x^2*(21*b^2*c^5*d^2*e^2 + 12*a*b*c^2*d^2*e^2) + x*(a^2*d
*e^2 + 7*b^2*c^6*d*e^2 + 8*a*b*c^3*d*e^2) + x^4*(35*b^2*c^3*d^4*e^2 + 2*a*
b*d^4*e^2) + a^2*c*e^2 + b^2*c^7*e^2 + b^2*d^7*e^2*x^7 + 2*a*b*c^4*e^2 + 2
1*b^2*c^2*d^5*e^2*x^5 + 7*b^2*c*d^6*e^2*x^6) + (14*b^(1/3)*log(2*b^(1/3)*c
 - 3^(1/2)*a^(1/3)*1i - a^(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 - 1/2))/(
27*a^(10/3)*d*e^2) - (14*b^(1/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*c - a^
(1/3) + 2*b^(1/3)*d*x)*((3^(1/2)*1i)/2 + 1/2))/(27*a^(10/3)*d*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2310, normalized size of antiderivative = 9.75

$$\int \frac{1}{(ce+dex)^2\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*e*x+c*e)^2/(a+b*(d*x+c)^3)^3,x)
```

output

```
(28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt
(3)))*a**2*b*c**2 + 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*
d*x)/(a**(1/3)*sqrt(3)))*a**2*b*c*d*x + 56*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**5 + 224*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*
*4*d*x + 336*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a*b**2*c**3*d**2*x**2 + 224*sqrt(3)*atan((a**(1/3) - 2*b**(
1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2*d**3*x**3 + 56*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*
a*b**2*c*d**4*x**4 + 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**8 + 196*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7*d*x + 588*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**
6*d**2*x**2 + 980*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*b**3*c**5*d**3*x**3 + 980*sqrt(3)*atan((a**(1/3) - 2*b
**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**4*d**4*x**4 + 588*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3))
)*b**3*c**3*d**5*x**5 + 196*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(
1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**2*d**6*x**6 + 28*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c*d**7*x*...
```

3.107 $\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 901
Mathematica [A] (verified) . . . 902
Rubi [A] (verified) . . . 902
Maple [C] (verified) . . . 912
Fricas [B] (verification not implemented) . . . 913
Sympy [B] (verification not implemented) . . . 914
Maxima [F] . . . 915
Giac [A] (verification not implemented) . . . 916
Mupad [B] (verification not implemented) . . . 916
Reduce [B] (verification not implemented) . . . 917

Optimal result

Integrand size = 24, antiderivative size = 235

$$\begin{aligned}
&\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx \\
&= -\frac{1}{2a^3de^3(c+dx)^2} - \frac{b(c+dx)}{6a^2de^3\left(a+b(c+dx)^3\right)^2} - \frac{11b(c+dx)}{18a^3de^3\left(a+b(c+dx)^3\right)} \\
&\quad + \frac{20b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}de^3} - \frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{27a^{11/3}de^3} \\
&\quad + \frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{27a^{11/3}de^3}
\end{aligned}$$

output

```
-1/2/a^3/d/e^3/(d*x+c)^2-1/6*b*(d*x+c)/a^2/d/e^3/(a+b*(d*x+c)^3)^2-11/18*b
*(d*x+c)/a^3/d/e^3/(a+b*(d*x+c)^3)+20/27*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(11/3)/d/e^3-20/27*b^(2/3)*ln(a^(
1/3)+b^(1/3)*(d*x+c))/a^(11/3)/d/e^3+10/27*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(11/3)/d/e^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}}{(c+dx)^2} - \frac{9a^{5/3}b(c+dx)}{\left(a+b(c+dx)^3\right)^2} - \frac{33a^{2/3}b(c+dx)}{a+b(c+dx)^3} - 40\sqrt{3}b^{2/3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) - 40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+\right.}{54a^{11/3}de^3}$$

input
```
Integrate[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x]
```

output
```
((-27*a^(2/3))/(c + d*x)^2 - (9*a^(5/3)*b*(c + d*x))/(a + b*(c + d*x)^3)^2
 - (33*a^(2/3)*b*(c + d*x))/(a + b*(c + d*x)^3) - 40*Sqrt[3]*b^(2/3)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] - 40*b^(2/3)*Log[a^(
1/3) + b^(1/3)*(c + d*x)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c +
d*x) + b^(2/3)*(c + d*x)^2])/(54*a^(11/3)*d*e^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {895, 819, 819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^3}\,d(c+dx)}{de^3}$$

$$\downarrow 819$$

$$\frac{4\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)^2}\,d(c+dx)}{3a} + \frac{1}{6a(c+dx)^2\left(a+b(c+dx)^3\right)^2}}{de^3}$$

$$\downarrow 819$$

$$\frac{4\left(\frac{5\int \frac{1}{(c+dx)^3\left(b(c+dx)^3+a\right)}d(c+dx)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b(c+dx)^3)^2}}{de^3}$$

$$\downarrow 847$$

$$\frac{4\left(\frac{5\left(-\frac{b\int \frac{1}{b(c+dx)^3+a}d(c+dx)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b(c+dx)^3)^2}}{de^3}$$

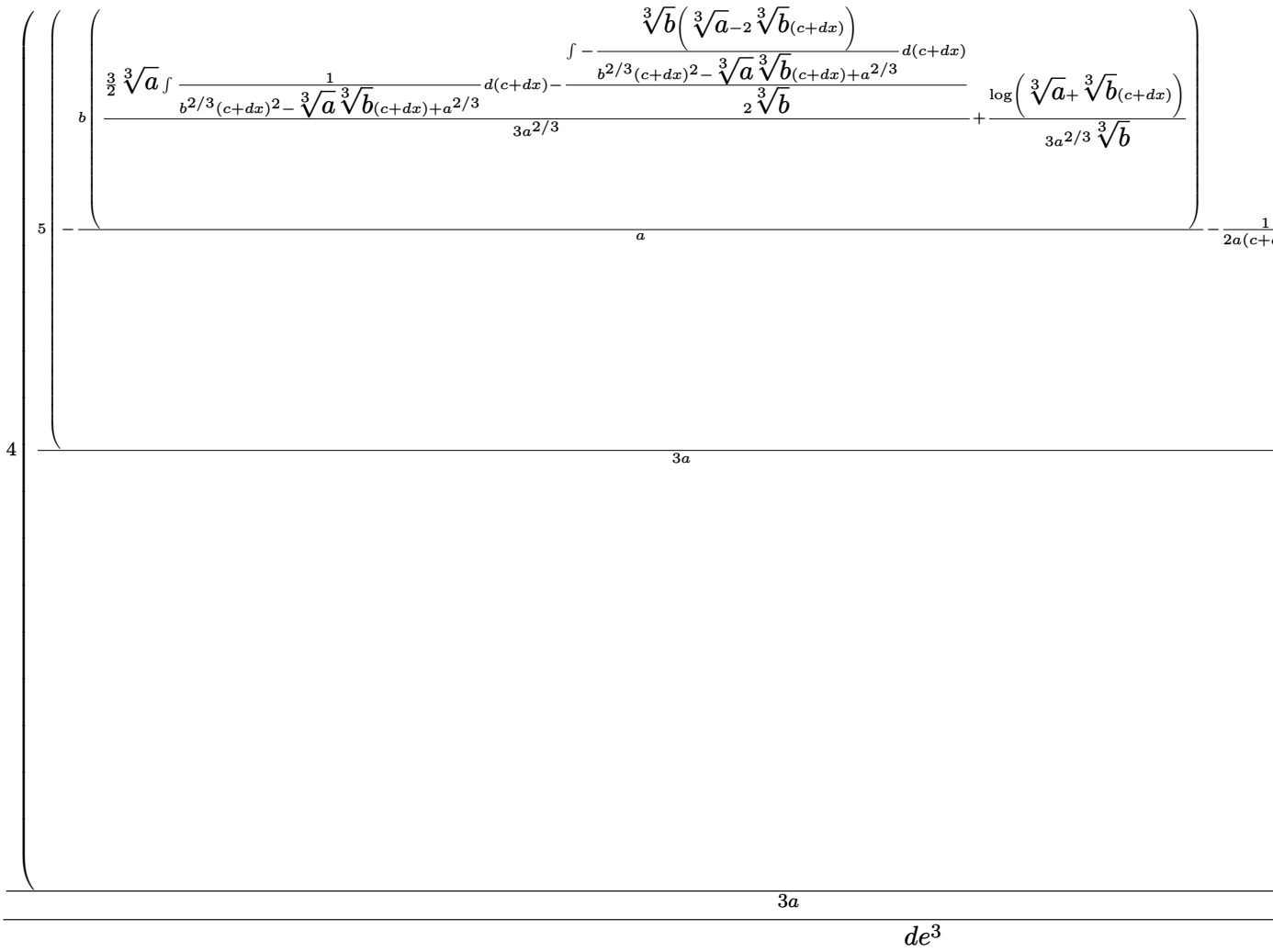
$$\downarrow 750$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(}}{de^3}$$

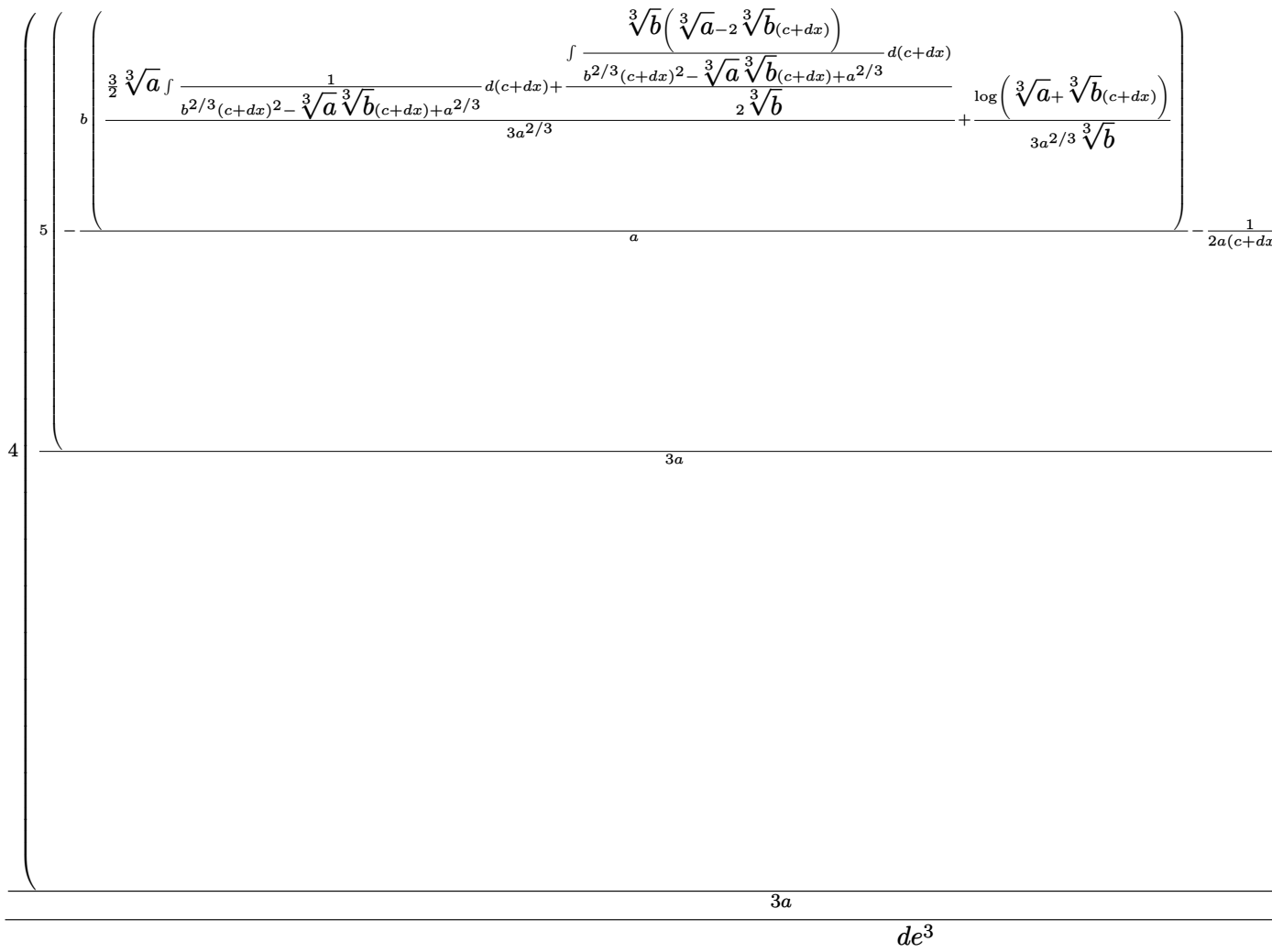
$$\downarrow 16$$

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}+\frac{1}{3a(c+dx)^2\left(a+b(c+dx)^3\right)}\right)}{3a}+\frac{1}{6a(c+dx)^2(a+b}}{de^3}$$

$$\downarrow 1142$$



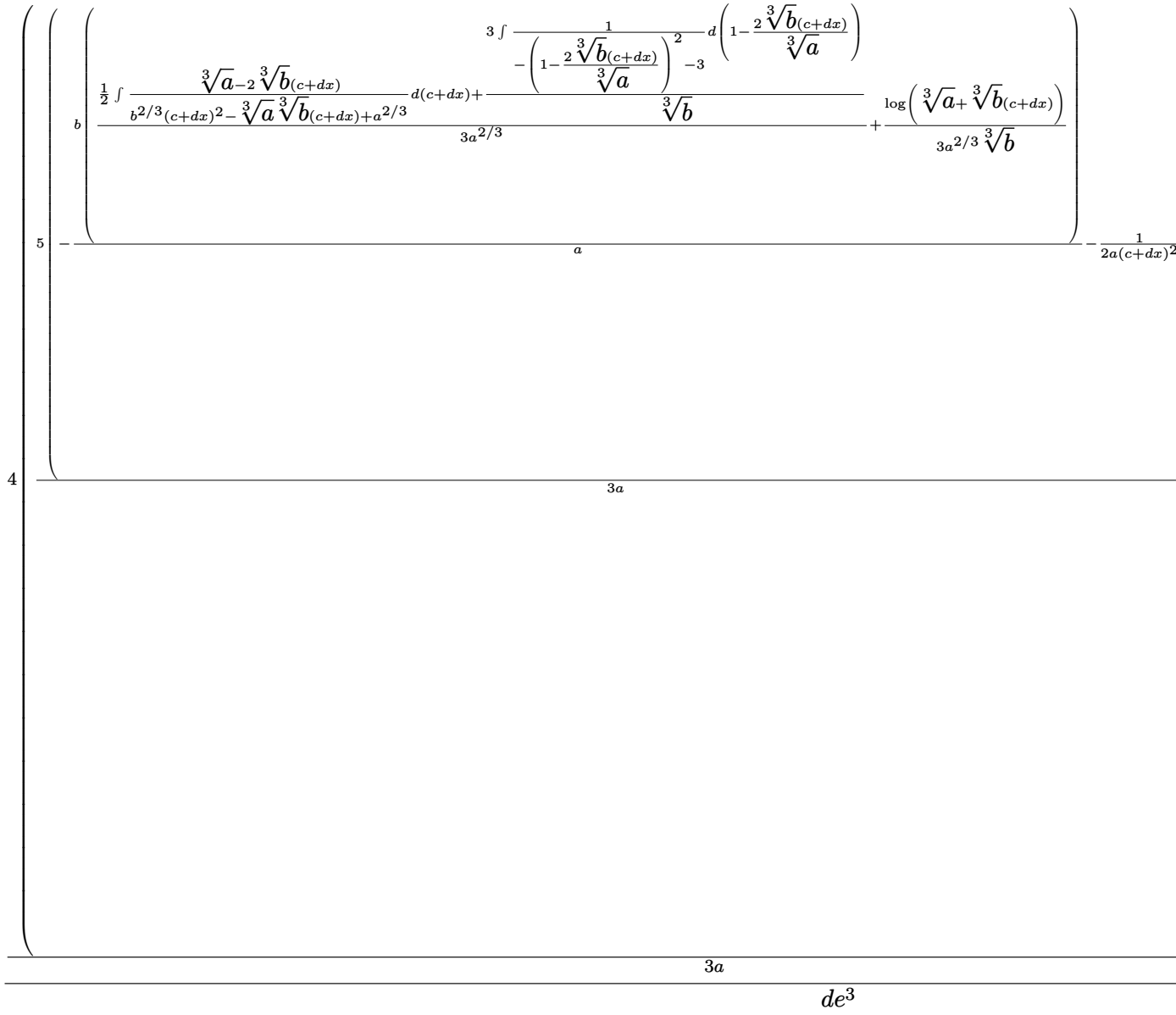
$\downarrow$ 25



$\downarrow$ 27

$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+d}\right.}{3a}\right.}{3a}}{de^3}$$

$\downarrow$ 1082

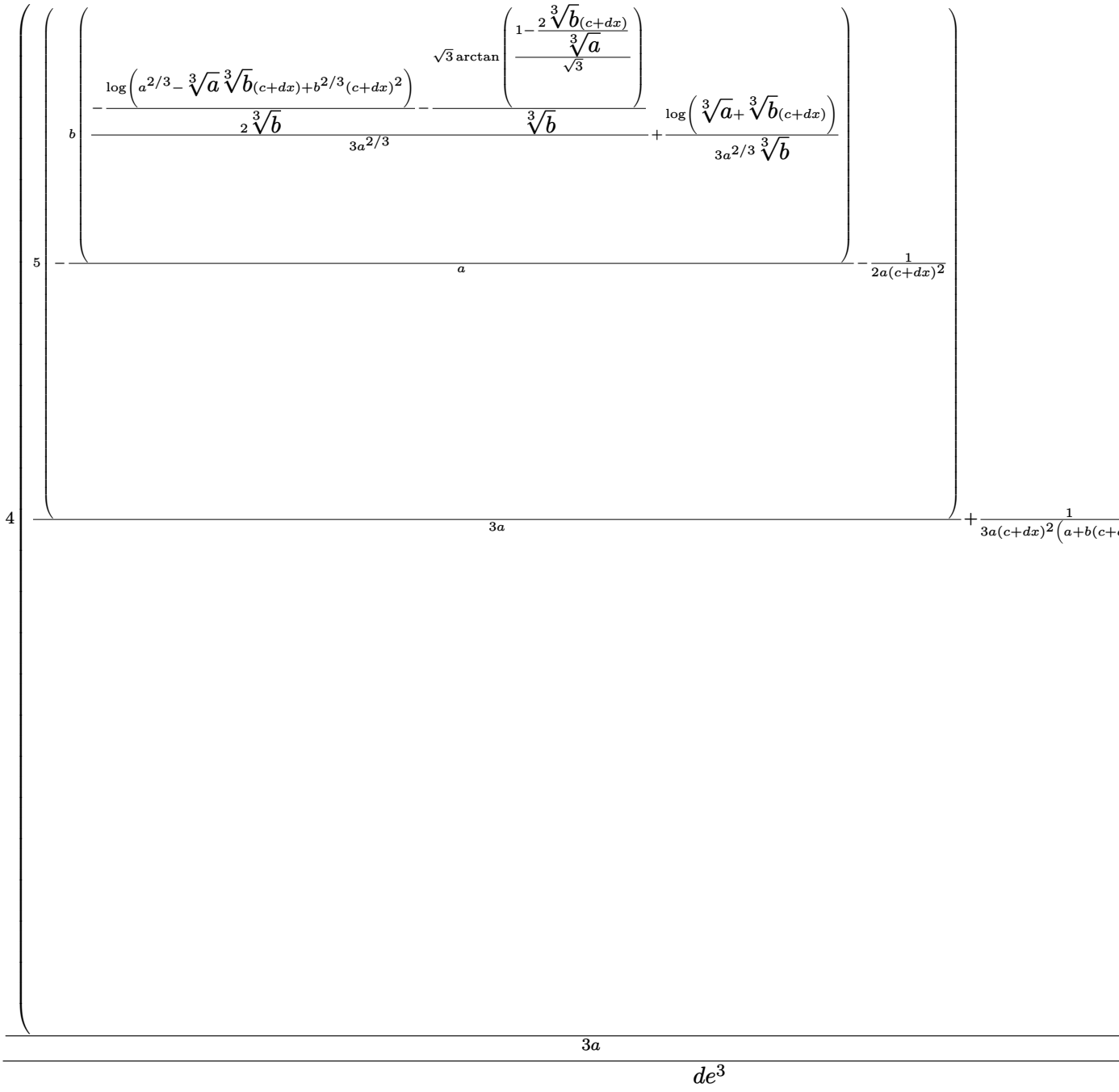


$$\frac{4\left(\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2a(c+dx)^2}\right)}{3a}\right)}{3a}}{de^3}$$

$\downarrow$ 217

$$\frac{\left(4\left(\frac{1}{3a(c+dx)^2\left(a+b\right.}+\frac{5\left(-\frac{1}{2a(c+dx)^2}-\frac{b\left(\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}\right)}{a}\right)}{3a}\right)}{3a}\right)}{de^3}$$

$$\downarrow 1103$$



input `Int[1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x]`

output
```
(1/(6*a*(c + d*x)^2*(a + b*(c + d*x)^3)^2) + (4*(1/(3*a*(c + d*x)^2*(a + b
*(c + d*x)^3)) + (5*(-1/2*1/(a*(c + d*x)^2) - (b*(Log[a^(1/3) + b^(1/3)*(c
 + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x
))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3))))/a))/(3*a)))/(3*a))/(d*e^3
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.82

method	result
default	$-\frac{1}{2a^3d(xd+c)^2}-\frac{b\left(\frac{\frac{11bd^3x^4}{18}+\frac{22d^2x^3bc}{9}+\frac{11bc^2dx^2}{3}+\left(\frac{22c^3b}{9}+\frac{7a}{9}\right)x+\frac{c\left(11c^3b+14a\right)}{18d}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{20\left(\sum_{\_R=\mathrm{RootOf}\left(bd^3\_Z^3+3bcd^2\_Z^2+3bc^2d\_Z+\right.}\right.}{27bd}\right.}{a^3}}{e^3}$
risch	$\frac{-\frac{10b^2d^5x^6}{9a^3}-\frac{20b^2cd^4x^5}{3a^3}-\frac{50b^2c^2d^3x^4}{3a^3}-\frac{8d^2b\left(25c^3b+2a\right)x^3}{9a^3}-\frac{2bcd\left(25c^3b+8a\right)x^2}{3a^3}-\frac{4bc^2\left(5c^3b+4a\right)x}{3a^3}-\frac{20b^2c^6+32abc^3+9a^2}{18da^3}}{e^3(xd+c)^2(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a)^2}+\frac{20\left(\_R=\right.}{}$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^3*(-1/2/a^3/d/(d*x+c)^2-1/a^3*b*((11/18*b*d^3*x^4+22/9*d^2*x^3*b*c+11/
3*b*c^2*d*x^2+(22/9*c^3*b+7/9*a)*x+1/18*c/d*(11*b*c^3+14*a))/(b*d^3*x^3+3*
b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^2+20/27/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)
*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(192) = 384$.

Time $= 0.14$ (sec) , antiderivative size $= 1051$, normalized size of antiderivative $= 4.47$

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
-1/54*(60*b^2*d^6*x^6 + 360*b^2*c*d^5*x^5 + 900*b^2*c^2*d^4*x^4 + 60*b^2*c
^6 + 48*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 96*a*b*c^3 + 36*(25*b^2*c^4 + 8*a*b
*c)*d^2*x^2 + 72*(5*b^2*c^5 + 4*a*b*c^2)*d*x - 40*sqrt(3)*(b^2*d^8*x^8 + 8
*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c
^8 + 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^
2)*d^3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*
c^7 + 5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*(a*d*
x + a*c)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 20*(b^2*d^8*x^8 + 8*b^2*c*d^7*
x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x^5 + b^2*c^8 + 10*(7*
b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5 + 5*a*b*c^2)*d^3*x^3
+ (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 + 2*(4*b^2*c^7 + 5*a*b
*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + a^2*(-b^2/a^2)^(2/3) + (a*b*d*x + a*b*c)*(-b^2/a^2)^(1/3)) - 40*(b^2*d
^8*x^8 + 8*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 2*(28*b^2*c^3 + a*b)*d^5*x
^5 + b^2*c^8 + 10*(7*b^2*c^4 + a*b*c)*d^4*x^4 + 2*a*b*c^5 + 4*(14*b^2*c^5
+ 5*a*b*c^2)*d^3*x^3 + (28*b^2*c^6 + 20*a*b*c^3 + a^2)*d^2*x^2 + a^2*c^2 +
 2*(4*b^2*c^7 + 5*a*b*c^4 + a^2*c)*d*x)*(-b^2/a^2)^(1/3)*log(b*d*x + b*c -
 a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*d^9*e^3*x^8 + 8*a^3*b^2*c*d^8*e^3*
x^7 + 28*a^3*b^2*c^2*d^7*e^3*x^6 + 2*(28*a^3*b^2*c^3 + a^4*b)*d^6*e^3*x^5
+ 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*e^3*x^4 + 4*(14*a^3*b^2*c^5 + 5*a^4*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(219) = 438$.

Time = 4.09 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.13

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{-9a^2 - 32abc^3 - 20b^2c^6 -}{18a^5c^2de^3 + 36a^4bc^5de^3 + 18a^3b^2c^8de^3 + 504a^3b^2c^2d^7e^3x^6 + 144a^3b^2cd^8e^3x^7 + 18a^3b^2d^9e^3x^8 + x^5\cdot(36a^4}$$
$$+ \frac{\operatorname{RootSum}\left(19683t^3a^{11} + 8000b^2, \left(t \mapsto t\log\left(x + \frac{-27ta^4+20bc}{20bd}\right)\right)\right)}{de^3}$$

input

```
integrate(1/(d*e*x+c*e)**3/(a+b*(d*x+c)**3)**3,x)
```

output

```
(-9*a**2 - 32*a*b*c**3 - 20*b**2*c**6 - 300*b**2*c**2*d**4*x**4 - 120*b**2
*c*d**5*x**5 - 20*b**2*d**6*x**6 + x**3*(-32*a*b*d**3 - 400*b**2*c**3*d**3
) + x**2*(-96*a*b*c*d**2 - 300*b**2*c**4*d**2) + x*(-96*a*b*c**2*d - 120*b
**2*c**5*d))/(18*a**5*c**2*d*e**3 + 36*a**4*b*c**5*d*e**3 + 18*a**3*b**2*c
**8*d*e**3 + 504*a**3*b**2*c**2*d**7*e**3*x**6 + 144*a**3*b**2*c*d**8*e**3
*x**7 + 18*a**3*b**2*d**9*e**3*x**8 + x**5*(36*a**4*b*d**6*e**3 + 1008*a**
3*b**2*c**3*d**6*e**3) + x**4*(180*a**4*b*c*d**5*e**3 + 1260*a**3*b**2*c**
4*d**5*e**3) + x**3*(360*a**4*b*c**2*d**4*e**3 + 1008*a**3*b**2*c**5*d**4*
e**3) + x**2*(18*a**5*d**3*e**3 + 360*a**4*b*c**3*d**3*e**3 + 504*a**3*b**
2*c**6*d**3*e**3) + x*(36*a**5*c*d**2*e**3 + 180*a**4*b*c**4*d**2*e**3 + 1
44*a**3*b**2*c**7*d**2*e**3)) + RootSum(19683*_t**3*a**11 + 8000*b**2, Lam
bda(_t, _t*log(x + (-27*_t*a**4 + 20*b*c)/(20*b*d))))/(d*e**3)
```

Maxima [F]

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)^3 (dex+ce)^3}\,dx$$

input

```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
-1/18*(20*b^2*d^6*x^6 + 120*b^2*c*d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 20*b^2*c
^6 + 16*(25*b^2*c^3 + 2*a*b)*d^3*x^3 + 32*a*b*c^3 + 12*(25*b^2*c^4 + 8*a*b
*c)*d^2*x^2 + 24*(5*b^2*c^5 + 4*a*b*c^2)*d*x + 9*a^2)/(a^3*b^2*d^9*e^3*x^8
 + 8*a^3*b^2*c*d^8*e^3*x^7 + 28*a^3*b^2*c^2*d^7*e^3*x^6 + 2*(28*a^3*b^2*c^
3 + a^4*b)*d^6*e^3*x^5 + 10*(7*a^3*b^2*c^4 + a^4*b*c)*d^5*e^3*x^4 + 4*(14*
a^3*b^2*c^5 + 5*a^4*b*c^2)*d^4*e^3*x^3 + (28*a^3*b^2*c^6 + 20*a^4*b*c^3 +
a^5)*d^3*e^3*x^2 + 2*(4*a^3*b^2*c^7 + 5*a^4*b*c^4 + a^5*c)*d^2*e^3*x + (a^
3*b^2*c^8 + 2*a^4*b*c^5 + a^5*c^2)*d*e^3) - 20/9*b*integrate(1/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(a^3*e^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.58

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{10\left(2\sqrt{3}\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc-(-ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}}\right)-\left(-\frac{b^2}{a^2d^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc+\sqrt{3}(-ab^2)^{\frac{1}{3}}\right.\right.\right.}{27\,a^3e^3}$$

$$-\frac{20\,b^2d^6x^6+120\,b^2cd^5x^5+300\,b^2c^2d^4x^4+400\,b^2c^3d^3x^3+300\,b^2c^4d^2x^2+120\,b^2c^5dx+20\,b^2c^6+32\,abd}{18\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+adx+ac\right)^2}$$

input
```
integrate(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
10/27*(2*sqrt(3)*(-b^2/(a^2*d^3))^(1/3)*arctan(-(b*d*x + b*c - (-a*b^2)^(1
/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))) - (-b^2/(a^2*
d^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c + sqrt(3)*(-a*b^2)^(1/3))^2
 + 4*(b*d*x + b*c - (-a*b^2)^(1/3))^2) + 2*(-b^2/(a^2*d^3))^(1/3)*log(abs(
-b*d*x - b*c + (-a*b^2)^(1/3))))/(a^3*e^3) - 1/18*(20*b^2*d^6*x^6 + 120*b^
2*c*d^5*x^5 + 300*b^2*c^2*d^4*x^4 + 400*b^2*c^3*d^3*x^3 + 300*b^2*c^4*d^2*
x^2 + 120*b^2*c^5*d*x + 20*b^2*c^6 + 32*a*b*d^3*x^3 + 96*a*b*c*d^2*x^2 + 9
6*a*b*c^2*d*x + 32*a*b*c^3 + 9*a^2)/((b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*
d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a*d*x + a*c)^2*a^3*d*e^3)
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.40

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx = \frac{20\,b^{2/3}\,\ln\left(a^6\,b^{1/3}\,c-(-a)^{19/3}+a^6\,b^{1/3}\,d\,x\right)}{27\,(-a)^{11/3}\,d\,e^3}$$

$$-\frac{\frac{9\,a^2+32\,a\,b\,c^3+20\,b^2\,c^6}{18\,a^3\,d}+\frac{2\,x^2\,(2}{x^4\,\left(70\,b^2\,c^4\,d^4\,e^3+10\,a\,b\,c\,d^4\,e^3\right)+x^2\,\left(a^2\,d^2\,e^3+20\,a\,b\,c^3\,d^2\,e^3+28\,b^2\,c^6\,d^2\,e^3\right)+x^3\,\left(56\,b^2\,c^5\,d^3\,e^3+20\right.}$$

$$+\frac{20\,b^{2/3}\,\ln\left((-a)^{19/3}+2\,a^6\,b^{1/3}\,c+2\,a^6\,b^{1/3}\,d\,x-\sqrt{3}\,(-a)^{19/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,(-a)^{11/3}\,d\,e^3}$$

$$-\frac{20\,b^{2/3}\,\ln\left((-a)^{19/3}+2\,a^6\,b^{1/3}\,c+2\,a^6\,b^{1/3}\,d\,x+\sqrt{3}\,(-a)^{19/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27\,(-a)^{11/3}\,d\,e^3}$$

input
```
int(1/((c*e + d*e*x)^3*(a + b*(c + d*x)^3)^3),x)
```

output
```
(20*b^(2/3)*log(a^6*b^(1/3)*c - (-a)^(19/3) + a^6*b^(1/3)*d*x))/(27*(-a)^(
11/3)*d*e^3) - ((9*a^2 + 20*b^2*c^6 + 32*a*b*c^3)/(18*a^3*d) + (2*x^2*(25*
b^2*c^4*d + 8*a*b*c*d))/(3*a^3) + (4*x*(5*b^2*c^5 + 4*a*b*c^2))/(3*a^3) +
(8*x^3*(25*b^2*c^3*d^2 + 2*a*b*d^2))/(9*a^3) + (10*b^2*d^5*x^6)/(9*a^3) +
(50*b^2*c^2*d^3*x^4)/(3*a^3) + (20*b^2*c*d^4*x^5)/(3*a^3))/(x^4*(70*b^2*c^
4*d^4*e^3 + 10*a*b*c*d^4*e^3) + x^2*(a^2*d^2*e^3 + 28*b^2*c^6*d^2*e^3 + 20
*a*b*c^3*d^2*e^3) + x^3*(56*b^2*c^5*d^3*e^3 + 20*a*b*c^2*d^3*e^3) + x^5*(5
6*b^2*c^3*d^5*e^3 + 2*a*b*d^5*e^3) + x*(8*b^2*c^7*d*e^3 + 2*a^2*c*d*e^3 +
10*a*b*c^4*d*e^3) + a^2*c^2*e^3 + b^2*c^8*e^3 + b^2*d^8*e^3*x^8 + 2*a*b*c^
5*e^3 + 28*b^2*c^2*d^6*e^3*x^6 + 8*b^2*c*d^7*e^3*x^7) + (20*b^(2/3)*log((-
a)^(19/3) - 3^(1/2)*(-a)^(19/3)*1i + 2*a^6*b^(1/3)*c + 2*a^6*b^(1/3)*d*x)*
((3^(1/2)*1i)/2 - 1/2))/(27*(-a)^(11/3)*d*e^3) - (20*b^(2/3)*log((-a)^(19/
3) + 3^(1/2)*(-a)^(19/3)*1i + 2*a^6*b^(1/3)*c + 2*a^6*b^(1/3)*d*x)*((3^(1/
2)*1i)/2 + 1/2))/(27*(-a)^(11/3)*d*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2805, normalized size of antiderivative = 11.94

$$\int \frac{1}{(ce+dex)^3\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(d*e*x+c*e)^3/(a+b*(d*x+c)^3)^3,x)
```

output

```
(40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a**2*b*c**2 + 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**2*b*c*d*x + 40*a**(1/3)*sqrt
(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*
*2*b*d**2*x**2 + 80*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**
(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**5 + 400*a**(1/3)*sqrt(3)*atan((a*
*(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**4*d*
x + 800*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*a*b**2*c**3*d**2*x**2 + 800*a**(1/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2*d**3
*x**3 + 400*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a*b**2*c*d**4*x**4 + 80*a**(1/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*d**5*x**5
 + 40*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a*
*(1/3)*sqrt(3)))*b**3*c**8 + 320*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**7*d*x + 1120*a**(1/3)*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3))
)*b**3*c**6*d**2*x**2 + 2240*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**5*d**3*x**3 + 2800*a**(1/3
)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqr...
```

3.108 $\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$

Optimal result . . . 919
Mathematica [A] (verified) . . . 919
Rubi [A] (verified) . . . 920
Maple [A] (verified) . . . 921
Fricas [B] (verification not implemented) . . . 922
Sympy [B] (verification not implemented) . . . 923
Maxima [B] (verification not implemented) . . . 923
Giac [B] (verification not implemented) . . . 924
Mupad [B] (verification not implemented) . . . 925
Reduce [B] (verification not implemented) . . . 926

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = -\frac{1}{3a^3de^4(c+dx)^3} - \frac{b}{6a^2de^4\left(a+b(c+dx)^3\right)^2} - \frac{2b}{3a^3de^4\left(a+b(c+dx)^3\right)} - \frac{3b\log(c+dx)}{a^4de^4} + \frac{b\log\left(a+b(c+dx)^3\right)}{a^4de^4}$$

output

```
-1/3/a^3/d/e^4/(d*x+c)^3-1/6*b/a^2/d/e^4/(a+b*(d*x+c)^3)^2-2/3*b/a^3/d/e^4
/(a+b*(d*x+c)^3)-3*b*ln(d*x+c)/a^4/d/e^4+b*ln(a+b*(d*x+c)^3)/a^4/d/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{a\left(-\frac{2}{(c+dx)^3} - \frac{ab}{\left(a+b(c+dx)^3\right)^2} - \frac{4b}{a+b(c+dx)^3}\right) - 18b\log(c+dx) + 6b\log\left(a+b(c+dx)^3\right)}{6a^4de^4}$$

input
```
Integrate[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x]
```

output
```
(a*(-2/(c + d*x)^3 - (a*b)/(a + b*(c + d*x)^3)^2 - (4*b)/(a + b*(c + d*x)^
3)) - 18*b*Log[c + d*x] + 6*b*Log[a + b*(c + d*x)^3])/(6*a^4*d*e^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {895, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$$

$$\downarrow 895$$

$$\frac{\int \frac{1}{(c+dx)^4(b(c+dx)^3+a)^3}\,d(c+dx)}{de^4}$$

$$\downarrow 798$$

$$\frac{\int \frac{1}{(c+dx)^6(b(c+dx)^3+a)^3}\,d(c+dx)^3}{3de^4}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{3b^2}{a^4(b(c+dx)^3+a)}+\frac{2b^2}{a^3(b(c+dx)^3+a)^2}+\frac{b^2}{a^2(b(c+dx)^3+a)^3}-\frac{3b}{a^4(c+dx)^3}+\frac{1}{a^3(c+dx)^6}\right)d(c+dx)^3}{3de^4}$$

$$\downarrow 2009$$

$$\frac{-\frac{3b\log\left((c+dx)^3\right)}{a^4}+\frac{3b\log\left(a+b(c+dx)^3\right)}{a^4}-\frac{2b}{a^3(a+b(c+dx)^3)}-\frac{1}{a^3(c+dx)^3}-\frac{b}{2a^2(a+b(c+dx)^3)^2}}{3de^4}$$

input
```
Int[1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x]
```

output
```
(-(1/(a^3*(c + d*x)^3)) - b/(2*a^2*(a + b*(c + d*x)^3)^2) - (2*b)/(a^3*(a
+ b*(c + d*x)^3)) - (3*b*Log[(c + d*x)^3])/a^4 + (3*b*Log[a + b*(c + d*x)^
3])/a^4)/(3*d*e^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.42

method	result
default	$\frac{-\frac{1}{3a^3d(xd+c)^3}-\frac{3b\ln(xd+c)}{a^4d}+\frac{b^2\left(\frac{-\frac{2ad^2x^3}{3}-2acdx^2-2ac^2x-\frac{a\left(4c^3b+5a\right)}{6db}}{\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}+\frac{\ln\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)}{bd}\right)}{a^4}}{e^4}$
risch	$\frac{-\frac{b^2d^5x^6}{a^3}-\frac{6b^2cd^4x^5}{a^3}-\frac{15b^2c^2d^3x^4}{a^3}-\frac{d^2\left(40c^3b+3a\right)bx^3}{2a^3}-\frac{3bcd\left(10c^3b+3a\right)x^2}{2a^3}-\frac{3\left(4c^3b+3a\right)bc^2x}{2a^3}-\frac{6b^2c^6+9abc^3+2a^2}{6da^3}}{e^4(xd+c)^3\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}-\frac{3b\ln(xd+$
norman	$\frac{-\frac{b^2d^5x^6}{a^3e}+\frac{-6b^4c^6d^8-9ab^3c^3d^8-2a^2b^2d^8}{6a^3d^9eb^2}+\frac{\left(-40b^4c^3d^8-3ab^3d^8\right)x^3}{2a^3d^6eb^2}+\frac{3c^2\left(-4b^4c^3d^8-3ab^3d^8\right)x}{2a^3d^8eb^2}+\frac{3c\left(-10b^4c^3d^8-3ab^3d^8\right)x^2}{2a^3d^7eb^2}-\frac{6c}{}}{e^3(xd+c)^3\left(bd^3x^3+3bcd^2x^2+3dbxc^2+c^3b+a\right)^2}$
parallelrisch	Expression too large to display

input
```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(-1/3/a^3/d/(d*x+c)^3-3*b*ln(d*x+c)/a^4/d+1/a^4*b^2*((-2/3*a*d^2*x^3
-2*a*c*d*x^2-2*a*c^2*x-1/6*a/d*(4*b*c^3+5*a)/b)/(b*d^3*x^3+3*b*c*d^2*x^2+3
*b*c^2*d*x+b*c^3+a)^2+1/b/d*ln(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(110) = 220$.

Time = 0.17 (sec) , antiderivative size = 919, normalized size of antiderivative = 7.92

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-1/6*(6*a*b^2*d^6*x^6 + 36*a*b^2*c*d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^
2*c^6 + 3*(40*a*b^2*c^3 + 3*a^2*b)*d^3*x^3 + 9*a^2*b*c^3 + 9*(10*a*b^2*c^4
 + 3*a^2*b*c)*d^2*x^2 + 2*a^3 + 9*(4*a*b^2*c^5 + 3*a^2*b*c^2)*d*x - 6*(b^3
*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d
^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(2
1*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3
*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*
b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a) + 18*(b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^
7*x^7 + 2*(42*b^3*c^3 + a*b^2)*d^6*x^6 + b^3*c^9 + 6*(21*b^3*c^4 + 2*a*b^2
*c)*d^5*x^5 + 2*a*b^2*c^6 + 6*(21*b^3*c^5 + 5*a*b^2*c^2)*d^4*x^4 + (84*b^3
*c^6 + 40*a*b^2*c^3 + a^2*b)*d^3*x^3 + a^2*b*c^3 + 3*(12*b^3*c^7 + 10*a*b^
2*c^4 + a^2*b*c)*d^2*x^2 + 3*(3*b^3*c^8 + 4*a*b^2*c^5 + a^2*b*c^2)*d*x)*lo
g(d*x + c))/(a^4*b^2*d^10*e^4*x^9 + 9*a^4*b^2*c*d^9*e^4*x^8 + 36*a^4*b^2*c
^2*d^8*e^4*x^7 + 2*(42*a^4*b^2*c^3 + a^5*b)*d^7*e^4*x^6 + 6*(21*a^4*b^2*c^
4 + 2*a^5*b*c)*d^6*e^4*x^5 + 6*(21*a^4*b^2*c^5 + 5*a^5*b*c^2)*d^5*e^4*x^4
+ (84*a^4*b^2*c^6 + 40*a^5*b*c^3 + a^6)*d^4*e^4*x^3 + 3*(12*a^4*b^2*c^7 +
10*a^5*b*c^4 + a^6*c)*d^3*e^4*x^2 + 3*(3*a^4*b^2*c^8 + 4*a^5*b*c^5 + a^6*c
^2)*d^2*e^4*x + (a^4*b^2*c^9 + 2*a^5*b*c^6 + a^6*c^3)*d*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(105) = 210.

Time = 3.78 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.98

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$$

$$= \frac{}{6a^5c^3de^4+12a^4bc^6de^4+6a^3b^2c^9de^4+216a^3b^2c^2d^8e^4x^7+54a^3b^2cd^9e^4x^8+6a^3b^2d^{10}e^4x^9+x^6\cdot(12a^4bd^7e}$$
$$-\frac{3b\log\left(\frac{c}{d}+x\right)}{a^4de^4}+\frac{b\log\left(\frac{3c^2x}{d^2}+\frac{3cx^2}{d}+x^3+\frac{a+bc^3}{bd^3}\right)}{a^4de^4}$$

input
```
integrate(1/(d*e*x+c*e)**4/(a+b*(d*x+c)**3)**3,x)
```

output
```
(-2*a**2 - 9*a*b*c**3 - 6*b**2*c**6 - 90*b**2*c**2*d**4*x**4 - 36*b**2*c*d
**5*x**5 - 6*b**2*d**6*x**6 + x**3*(-9*a*b*d**3 - 120*b**2*c**3*d**3) + x*
*2*(-27*a*b*c*d**2 - 90*b**2*c**4*d**2) + x*(-27*a*b*c**2*d - 36*b**2*c**5
*d))/(6*a**5*c**3*d*e**4 + 12*a**4*b*c**6*d*e**4 + 6*a**3*b**2*c**9*d*e**4
 + 216*a**3*b**2*c**2*d**8*e**4*x**7 + 54*a**3*b**2*c*d**9*e**4*x**8 + 6*a
**3*b**2*d**10*e**4*x**9 + x**6*(12*a**4*b*d**7*e**4 + 504*a**3*b**2*c**3*
d**7*e**4) + x**5*(72*a**4*b*c*d**6*e**4 + 756*a**3*b**2*c**4*d**6*e**4) +
 x**4*(180*a**4*b*c**2*d**5*e**4 + 756*a**3*b**2*c**5*d**5*e**4) + x**3*(6
*a**5*d**4*e**4 + 240*a**4*b*c**3*d**4*e**4 + 504*a**3*b**2*c**6*d**4*e**4
) + x**2*(18*a**5*c*d**3*e**4 + 180*a**4*b*c**4*d**3*e**4 + 216*a**3*b**2*
c**7*d**3*e**4) + x*(18*a**5*c**2*d**2*e**4 + 72*a**4*b*c**5*d**2*e**4 + 5
4*a**3*b**2*c**8*d**2*e**4)) - 3*b*log(c/d + x)/(a**4*d*e**4) + b*log(3*c*
*2*x/d**2 + 3*c*x**2/d + x**3 + (a + b*c**3)/(b*d**3))/(a**4*d*e**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(110) = 220.

Time = 0.07 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.09

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx =$$

$$-\frac{6\,b^2d^6x^6+36\,b^2cd^5x^5+90}{6\left(a^3b^2d^{10}e^4x^9+9\,a^3b^2cd^9e^4x^8+36\,a^3b^2c^2d^8e^4x^7+2\left(42\,a^3b^2c^3+a^4b\right)d^7e^4x^6+6\left(21\,a^3b^2c^4+2\,a^4bc\right)d}$$

$$+\frac{b\log\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)}{a^4de^4}-\frac{3\,b\log\left(dx+c\right)}{a^4de^4}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/6*(6*b^2*d^6*x^6 + 36*b^2*c*d^5*x^5 + 90*b^2*c^2*d^4*x^4 + 6*b^2*c^6 +
3*(40*b^2*c^3 + 3*a*b)*d^3*x^3 + 9*a*b*c^3 + 9*(10*b^2*c^4 + 3*a*b*c)*d^2*
x^2 + 9*(4*b^2*c^5 + 3*a*b*c^2)*d*x + 2*a^2)/(a^3*b^2*d^10*e^4*x^9 + 9*a^3
*b^2*c*d^9*e^4*x^8 + 36*a^3*b^2*c^2*d^8*e^4*x^7 + 2*(42*a^3*b^2*c^3 + a^4*
b)*d^7*e^4*x^6 + 6*(21*a^3*b^2*c^4 + 2*a^4*b*c)*d^6*e^4*x^5 + 6*(21*a^3*b^
2*c^5 + 5*a^4*b*c^2)*d^5*e^4*x^4 + (84*a^3*b^2*c^6 + 40*a^4*b*c^3 + a^5)*d
^4*e^4*x^3 + 3*(12*a^3*b^2*c^7 + 10*a^4*b*c^4 + a^5*c)*d^3*e^4*x^2 + 3*(3*
a^3*b^2*c^8 + 4*a^4*b*c^5 + a^5*c^2)*d^2*e^4*x + (a^3*b^2*c^9 + 2*a^4*b*c^
6 + a^5*c^3)*d*e^4) + b*log(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^
3 + a)/(a^4*d*e^4) - 3*b*log(d*x + c)/(a^4*d*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(110) = 220.

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.26

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx$$

$$=\frac{b\log\left(\left|bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right|\right)}{a^4de^4}-\frac{3\,b\log\left(\left|dx+c\right|\right)}{a^4de^4}$$

$$-\frac{6\,ab^2d^6x^6+36\,ab^2cd^5x^5+90\,ab^2c^2d^4x^4+6\,ab^2c^6+9\,a^2bc^3+3\left(40\,ab^2c^3d^3+3\,a^2bd^3\right)x^3+2\,a^3+9\left(10}{6\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a\right)^2(dx+c)^3a^4de}$$

input
```
integrate(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
b*log(abs(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(a^4*d*e^4
) - 3*b*log(abs(d*x + c))/(a^4*d*e^4) - 1/6*(6*a*b^2*d^6*x^6 + 36*a*b^2*c*
d^5*x^5 + 90*a*b^2*c^2*d^4*x^4 + 6*a*b^2*c^6 + 9*a^2*b*c^3 + 3*(40*a*b^2*c
^3*d^3 + 3*a^2*b*d^3)*x^3 + 2*a^3 + 9*(10*a*b^2*c^4*d^2 + 3*a^2*b*c*d^2)*x
^2 + 9*(4*a*b^2*c^5*d + 3*a^2*b*c^2*d)*x)/((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a)^2*(d*x + c)^3*a^4*d*e^4)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.37

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = \frac{b\,\ln\left(b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a\right)}{a^4\,d\,e^4}$$

$$-\frac{}{x^5\,\left(126\,b^2\,c^4\,d^5\,e^4+12\,a\,b\,c\,d^5\,e^4\right)+x^3\,\left(a^2\,d^3\,e^4+40\,a\,b\,c^3\,d^3\,e^4+84\,b^2\,c^6\,d^3\,e^4\right)+x\,\left(3\,d\,a^2\,c^2\,e^4+12\,c}$$

$$-\frac{3\,b\,\ln\left(c+d\,x\right)}{a^4\,d\,e^4}$$

input
```
int(1/((c*e + d*e*x)^4*(a + b*(c + d*x)^3)^3),x)
```

output
```
(b*log(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2))/(a^4*d*e^4) -
 ((2*a^2 + 6*b^2*c^6 + 9*a*b*c^3)/(6*a^3*d) + (3*x^2*(10*b^2*c^4*d + 3*a*b
*c*d))/(2*a^3) + (3*x*(4*b^2*c^5 + 3*a*b*c^2))/(2*a^3) + (x^3*(40*b^2*c^3*
d^2 + 3*a*b*d^2))/(2*a^3) + (b^2*d^5*x^6)/a^3 + (15*b^2*c^2*d^3*x^4)/a^3 +
 (6*b^2*c*d^4*x^5)/a^3)/(x^5*(126*b^2*c^4*d^5*e^4 + 12*a*b*c*d^5*e^4) + x^
3*(a^2*d^3*e^4 + 84*b^2*c^6*d^3*e^4 + 40*a*b*c^3*d^3*e^4) + x*(3*a^2*c^2*d
*e^4 + 9*b^2*c^8*d*e^4 + 12*a*b*c^5*d*e^4) + x^4*(126*b^2*c^5*d^4*e^4 + 30
*a*b*c^2*d^4*e^4) + x^2*(3*a^2*c*d^2*e^4 + 36*b^2*c^7*d^2*e^4 + 30*a*b*c^4
*d^2*e^4) + x^6*(84*b^2*c^3*d^6*e^4 + 2*a*b*d^6*e^4) + a^2*c^3*e^4 + b^2*c
^9*e^4 + b^2*d^9*e^4*x^9 + 2*a*b*c^6*e^4 + 36*b^2*c^2*d^7*e^4*x^7 + 9*b^2*
c*d^8*e^4*x^8) - (3*b*log(c + d*x))/(a^4*d*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2664, normalized size of antiderivative = 22.97

$$\int \frac{1}{(ce+dex)^4\left(a+b(c+dx)^3\right)^3}\,dx = \text{Too large to display}$$

input

```
int(1/(d*e*x+c*e)^4/(a+b*(d*x+c)^3)^3,x)
```

output

```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a**2*b*c**3 + 18*log(a**(2/3)
 - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3
)*c*d*x + b**(2/3)*d**2*x**2)*a**2*b*c**2*d*x + 18*log(a**(2/3) - b**(1/3)
*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b
**(2/3)*d**2*x**2)*a**2*b*c*d**2*x**2 + 6*log(a**(2/3) - b**(1/3)*a**(1/3)
*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d
**2*x**2)*a**2*b*d**3*x**3 + 12*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1
/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*
a*b**2*c**6 + 72*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**5*d*x
 + 180*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/
3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**4*d**2*x**2 + 2
40*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c
**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**3*d**3*x**3 + 180*l
og(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2
+ 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c**2*d**4*x**4 + 72*log(a*
*(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b
**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b**2*c*d**5*x**5 + 12*log(a**(2/3) -
 b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/...
```

3.109 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx$

Optimal result 927
Mathematica [A] (verified) 927
Rubi [A] (verified) 928
Maple [A] (verified) 929
Fricas [B] (verification not implemented) 929
Sympy [F(-1)] 930
Maxima [A] (verification not implemented) 930
Giac [B] (verification not implemented) 930
Mupad [B] (verification not implemented) 931
Reduce [B] (verification not implemented) 931

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \frac{\left(a+b(c+dx)^4\right)^{1+p}}{4bd(1+p)}$$

output
```
1/4*(a+b*(d*x+c)^4)^(p+1)/b/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \frac{\left(a+b(c+dx)^4\right)^{1+p}}{4bd(1+p)}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^p,x]
```

output
```
(a + b*(c + d*x)^4)^(1 + p)/(4*b*d*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^4+a\right)^p \, d(c+dx)}{d}$$

$$\downarrow 793$$

$$\frac{\left(a+b(c+dx)^4\right)^{p+1}}{4bd(p+1)}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^4)^p,x]
```

output
```
(a + b*(c + d*x)^4)^(1 + p)/(4*b*d*(1 + p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\left(a+b(xd+c)^4\right)^{p+1}}{4bd(p+1)}$
default	$\frac{\left(a+b(xd+c)^4\right)^{p+1}}{4bd(p+1)}$
gosper	$\frac{(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^{p+1}}{4bd(p+1)}$
risch	$\frac{(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)\left(a+b(xd+c)^4\right)^p}{4bd(p+1)}$
orering	$\frac{(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)\left(a+b(xd+c)^4\right)^p}{4bd(p+1)}$
parallelrisch	$\frac{d^7\left(a+b(xd+c)^4\right)^p x^4b+4c\,d^6\left(a+b(xd+c)^4\right)^p x^3b+6c^2d^5\left(a+b(xd+c)^4\right)^p x^2b+4c^3\left(a+b(xd+c)^4\right)^p x\,d^4b+\left(a+b(xd+c\right.}{4d^4b(p+1)}$
norman	$\frac{c^3x\,\mathrm{e}^{p\ln\left(a+b(xd+c)^4\right)}}{p+1}+\frac{c\,d^2x^3\mathrm{e}^{p\ln\left(a+b(xd+c)^4\right)}}{p+1}+\frac{d^3x^4\mathrm{e}^{p\ln\left(a+b(xd+c)^4\right)}}{4p+4}+\frac{3c^2d\,x^2\mathrm{e}^{p\ln\left(a+b(xd+c)^4\right)}}{2(p+1)}+\frac{(b\,c^4-$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a+b*(d*x+c)^4)^(p+1)/b/d/(p+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)^p\,dx$$
$$=\frac{(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a)(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a)^p}{4\,(bdp+bd)}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="fricas")
```

output
```
1/4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a
)*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a)^
p/(b*d*p + b*d)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**4)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \frac{\left((dx+c)^4 b+a\right)^{p+1}}{4\,bd(p+1)}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="maxima")
```

output
```
1/4*((d*x + c)^4*b + a)^(p + 1)/(b*d*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \frac{\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a\right)^{p+1}}{4\,bd(p+1)}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^p,x, algorithm="giac")
```

output
```
1/4*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a
)^(p + 1)/(b*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.93

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \left(a+b\,(c+d\,x)^4\right)^p \left(\frac{d^3\,x^4}{4\,(p+1)} + \frac{c^3\,x}{p+1} + \frac{b\,c^4+a}{4\,b\,d\,(p+1)} + \frac{3\,c^2\,d\,x^2}{2\,(p+1)} + \frac{c\,d^2\,x^3}{p+1}\right)$$

input
```
int((a + b*(c + d*x)^4)^p*(c + d*x)^3,x)
```

output
```
(a + b*(c + d*x)^4)^p*((d^3*x^4)/(4*(p + 1)) + (c^3*x)/(p + 1) + (a + b*c^
4)/(4*b*d*(p + 1)) + (3*c^2*d*x^2)/(2*(p + 1)) + (c*d^2*x^3)/(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.50

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^p \, dx = \frac{\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)^p \left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}{4bd\,(p+1)}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4)^p,x)
```

output
```
((a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**
4*x**4)**p*(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x*
*3 + b*d**4*x**4))/(4*b*d*(p + 1))
```

3.110 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx$

Optimal result 932
Mathematica [B] (verified) 932
Rubi [A] (verified) 933
Maple [A] (verified) 934
Fricas [B] (verification not implemented) 934
Sympy [B] (verification not implemented) 935
Maxima [A] (verification not implemented) 935
Giac [A] (verification not implemented) 936
Mupad [B] (verification not implemented) 936
Reduce [B] (verification not implemented) 937

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx = \frac{\left(a+b(c+dx)^4\right)^2}{8bd}$$

output
```
1/8*(a+b*(d*x+c)^4)^2/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(23) = 46.

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx = \frac{1}{8}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\left(2a + b\left(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\right)\right)$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^4),x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(2*a + b*(2*c^4 + 4*c^3*d*x
 + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4)))/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {895, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^4+a\right) d(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b(c+dx)^7+a(c+dx)^3\right) d(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}a(c+dx)^4+\frac{1}{8}b(c+dx)^8}{d}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^4),x]
```

output
```
((a*(c + d*x)^4)/4 + (b*(c + d*x)^8)/8)/d
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(a+b(xd+c)^4\right)^2}{8bd}$
norman	$\frac{d^7 b x^8}{8} + c d^6 b x^7 + \frac{7c^2 d^5 b x^6}{2} + 7c^3 d^4 b x^5 + \left(\frac{35}{4} c^4 b d^3 + \frac{1}{4} a d^3\right) x^4 + \left(7c^5 b d^2 + ac d^2\right) x^3 + \left(\frac{7}{2} c^6$
gosper	$\frac{x\left(d^7 b x^7+8bc d^6 x^6+28c^2 d^5 b x^5+56b c^3 d^4 x^4+70x^3 c^4 b d^3+56b c^5 d^2 x^2+28x c^6 bd+8b c^7+2a d^3 x^3+8ac d^2 x^2+12xa c^2 d+8c^3 a\right.}{8}$
parallelrisch	$b c^7 x + a c^3 x + \frac{7}{2} bd c^6 x^2 + \frac{3}{2} da c^2 x^2 + 7b d^2 c^5 x^3 + d^2 ac x^3 + \frac{35}{4} b d^3 c^4 x^4 + \frac{1}{4} a d^3 x^4 + 7c^3 d^4 b$
risch	$\frac{d^7 b x^8}{8} + c d^6 b x^7 + \frac{7c^2 d^5 b x^6}{2} + 7c^3 d^4 b x^5 + \frac{35b d^3 c^4 x^4}{4} + 7b d^2 c^5 x^3 + \frac{7bd c^6 x^2}{2} + b c^7 x + \frac{a d^3 x^4}{4} + \frac{b}{8}$
orering	$\frac{x\left(d^7 b x^7+8bc d^6 x^6+28c^2 d^5 b x^5+56b c^3 d^4 x^4+70x^3 c^4 b d^3+56b c^5 d^2 x^2+28x c^6 bd+8b c^7+2a d^3 x^3+8ac d^2 x^2+12xa c^2 d+8c^3 a\right.}{8b d^4 x^4+32bc d^3 x^3+48b c^2 d^2 x^2+32b c^3 dx+8b c^4+8a}$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*(a+b*(d*x+c)^4)^2/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.70

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)\,dx = \frac{1}{8} bd^7 x^8 + bcd^6 x^7 + \frac{7}{2} bc^2 d^5 x^6 + 7\,bc^3 d^4 x^5 + \frac{1}{4}\left(35\,bc^4 + a\right) d^3 x^4 + \left(7\,bc^5 + ac\right) d^2 x^3 + \frac{1}{2}\left(7\,bc^6 + 3\,ac^2\right) dx^2 + \left(bc^7 + ac^3\right) x$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/8*b*d^7*x^8 + b*c*d^6*x^7 + 7/2*b*c^2*d^5*x^6 + 7*b*c^3*d^4*x^5 + 1/4*(3
5*b*c^4 + a)*d^3*x^4 + (7*b*c^5 + a*c)*d^2*x^3 + 1/2*(7*b*c^6 + 3*a*c^2)*d
*x^2 + (b*c^7 + a*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.48

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)\,dx = 7bc^3d^4x^5 + \frac{7bc^2d^5x^6}{2} + bcd^6x^7 + \frac{bd^7x^8}{8} + x^4\left(\frac{ad^3}{4} + \frac{35bc^4d^3}{4}\right) + x^3\left(acd^2 + 7bc^5d^2\right) + x^2\cdot\left(\frac{3ac^2d}{2} + \frac{7bc^6d}{2}\right) + x\left(ac^3 + bc^7\right)$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**4),x)
```

output
```
7*b*c**3*d**4*x**5 + 7*b*c**2*d**5*x**6/2 + b*c*d**6*x**7 + b*d**7*x**8/8
+ x**4*(a*d**3/4 + 35*b*c**4*d**3/4) + x**3*(a*c*d**2 + 7*b*c**5*d**2) + x
**2*(3*a*c**2*d/2 + 7*b*c**6*d/2) + x*(a*c**3 + b*c**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)\,dx = \frac{\left((dx+c)^4b+a\right)^2}{8\,bd}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/8*((d*x + c)^4*b + a)^2/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx = \frac{(dx+c)^8 b + 2\,(dx+c)^4 a}{8\,d}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
1/8*((d*x + c)^8*b + 2*(d*x + c)^4*a)/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.65

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)\, dx = {} & x\left(b\,c^7 + a\,c^3\right) + \frac{d^3\,x^4\,(35\,b\,c^4+a)}{4} + \frac{b\,d^7\,x^8}{8} \\ & + \frac{c^2\,d\,x^2\,(7\,b\,c^4+3\,a)}{2} + 7\,b\,c^3\,d^4\,x^5 \\ & + \frac{7\,b\,c^2\,d^5\,x^6}{2} + c\,d^2\,x^3\left(7\,b\,c^4+a\right) + b\,c\,d^6\,x^7\end{aligned}$$

input
```
int((a + b*(c + d*x)^4)*(c + d*x)^3,x)
```

output
```
x*(a*c^3 + b*c^7) + (d^3*x^4*(a + 35*b*c^4))/4 + (b*d^7*x^8)/8 + (c^2*d*x^
2*(3*a + 7*b*c^4))/2 + 7*b*c^3*d^4*x^5 + (7*b*c^2*d^5*x^6)/2 + c*d^2*x^3*(
a + 7*b*c^4) + b*c*d^6*x^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 5.09

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)\,dx$$
$$=\frac{x(b\,d^7x^7+8bc\,d^6x^6+28b\,c^2d^5x^5+56b\,c^3d^4x^4+70b\,c^4d^3x^3+56b\,c^5d^2x^2+28b\,c^6dx+8b\,c^7+2a\,d^3x^3+}{8}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4),x)
```

output
```
(x*(8*a*c**3 + 12*a*c**2*d*x + 8*a*c*d**2*x**2 + 2*a*d**3*x**3 + 8*b*c**7
+ 28*b*c**6*d*x + 56*b*c**5*d**2*x**2 + 70*b*c**4*d**3*x**3 + 56*b*c**3*d*
*4*x**4 + 28*b*c**2*d**5*x**5 + 8*b*c*d**6*x**6 + b*d**7*x**7))/8
```

3.111 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx$

Optimal result 938
Mathematica [B] (verified) 938
Rubi [A] (verified) 939
Maple [A] (verified) 940
Fricas [B] (verification not implemented) 941
Sympy [B] (verification not implemented) 941
Maxima [A] (verification not implemented) 942
Giac [A] (verification not implemented) 943
Mupad [B] (verification not implemented) 943
Reduce [B] (verification not implemented) 944

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx = \frac{\left(a+b(c+dx)^4\right)^3}{12bd}$$

output
```
1/12*(a+b*(d*x+c)^4)^3/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 7.48

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx = \frac{1}{12}x\big(4c^3+6c^2dx+4cd^2x^2+d^3x^3\big)\big(3a^2 \\ +3ab\big(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\big) \\ +b^2\big(3c^8+12c^7dx+34c^6d^2x^2+60c^5d^3x^3+71c^4d^4x^4 \\ +56c^3d^5x^5+28c^2d^6x^6+8cd^7x^7+d^8x^8\big)\big)\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^2,x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(3*a^2 + 3*a*b*(2*c^4 + 4*c
^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4) + b^2*(3*c^8 + 12*c^7*d*x
+ 34*c^6*d^2*x^2 + 60*c^5*d^3*x^3 + 71*c^4*d^4*x^4 + 56*c^3*d^5*x^5 + 28*c
^2*d^6*x^6 + 8*c*d^7*x^7 + d^8*x^8)))/12
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^4+a\right)^2 d(c+dx)}{d}$$

$$\downarrow 793$$

$$\frac{\left(a+b(c+dx)^4\right)^3}{12bd}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^4)^2,x]
```

output
```
(a + b*(c + d*x)^4)^3/(12*b*d)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(a+b(xd+c)^4\right)^3}{12bd}$
norman	$\frac{d^{11}b^2x^{12}}{12}+c\,d^{10}b^2x^{11}+\frac{11c^2d^9b^2x^{10}}{2}+\frac{55c^3d^8b^2x^9}{3}+\left(\frac{165}{4}c^4b^2d^7+\frac{1}{4}ab\,d^7\right)x^8+(66c^5b^2d^6+2abc$
gosper	$\frac{x\left(d^{11}b^2x^{11}+12c\,d^{10}b^2x^{10}+66c^2d^9b^2x^9+220c^3d^8b^2x^8+495x^7c^4b^2d^7+792b^2c^5d^6x^6+924b^2c^6d^5x^5+792b^2c^7d^4x^4+3ab\,d^7x^7+\right.}{}$
parallelrisch	$\frac{3}{2}a^2c^2d\,x^2+\frac{165}{4}b^2d^7c^4x^8+66b^2d^6x^7c^5+77b^2d^5x^6c^6+66b^2d^4c^7x^5+\frac{1}{4}b\,d^7a\,x^8+\frac{165}{4}b^2d^3x^4c$
risch	$\frac{3a^2c^2d\,x^2}{2}+\frac{165b^2d^7c^4x^8}{4}+66b^2d^6x^7c^5+77b^2d^5x^6c^6+66b^2d^4c^7x^5+\frac{b\,d^7a\,x^8}{4}+\frac{165b^2d^3x^4c^8}{4}+\frac{55b^2c$
orering	$\frac{x\left(d^{11}b^2x^{11}+12c\,d^{10}b^2x^{10}+66c^2d^9b^2x^9+220c^3d^8b^2x^8+495x^7c^4b^2d^7+792b^2c^5d^6x^6+924b^2c^6d^5x^5+792b^2c^7d^4x^4+3ab\,d^7x^7+\right.}{}$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(a+b*(d*x+c)^4)^3/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 10.87

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^4\right)^2\,dx = {} & \frac{1}{12}\,b^2d^{11}x^{12} + b^2cd^{10}x^{11} + \frac{11}{2}\,b^2c^2d^9x^{10} + \frac{55}{3}\,b^2c^3d^8x^9 \\ & + \frac{1}{4}\left(165\,b^2c^4 + ab\right)d^7x^8 + 2\left(33\,b^2c^5 + abc\right)d^6x^7 \\ & + 7\left(11\,b^2c^6 + abc^2\right)d^5x^6 + 2\left(33\,b^2c^7 + 7\,abc^3\right)d^4x^5 \\ & + \frac{1}{4}\left(165\,b^2c^8 + 70\,abc^4 + a^2\right)d^3x^4 \\ & + \frac{1}{3}\left(55\,b^2c^9 + 42\,abc^5 + 3\,a^2c\right)d^2x^3 \\ & + \frac{1}{2}\left(11\,b^2c^{10} + 14\,abc^6 + 3\,a^2c^2\right)dx^2 \\ & + \left(b^2c^{11} + 2\,abc^7 + a^2c^3\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*d^11*x^12 + b^2*c*d^10*x^11 + 11/2*b^2*c^2*d^9*x^10 + 55/3*b^2*c^
3*d^8*x^9 + 1/4*(165*b^2*c^4 + a*b)*d^7*x^8 + 2*(33*b^2*c^5 + a*b*c)*d^6*x
^7 + 7*(11*b^2*c^6 + a*b*c^2)*d^5*x^6 + 2*(33*b^2*c^7 + 7*a*b*c^3)*d^4*x^5
 + 1/4*(165*b^2*c^8 + 70*a*b*c^4 + a^2)*d^3*x^4 + 1/3*(55*b^2*c^9 + 42*a*b
*c^5 + 3*a^2*c)*d^2*x^3 + 1/2*(11*b^2*c^10 + 14*a*b*c^6 + 3*a^2*c^2)*d*x^2
 + (b^2*c^11 + 2*a*b*c^7 + a^2*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 299, normalized size of antiderivative = 13.00

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^4\right)^2\,dx = {} & \frac{55b^2c^3d^8x^9}{3}+\frac{11b^2c^2d^9x^{10}}{2}+b^2cd^{10}x^{11}\\ & +\frac{b^2d^{11}x^{12}}{12}+x^8\left(\frac{abd^7}{4}+\frac{165b^2c^4d^7}{4}\right)\\ & +x^7\cdot\left(2abcd^6+66b^2c^5d^6\right)+x^6\\ & \cdot\left(7abc^2d^5+77b^2c^6d^5\right)+x^5\cdot\left(14abc^3d^4+66b^2c^7d^4\right)\\ & +x^4\left(\frac{a^2d^3}{4}+\frac{35abc^4d^3}{2}+\frac{165b^2c^8d^3}{4}\right)\\ & +x^3\left(a^2cd^2+14abc^5d^2+\frac{55b^2c^9d^2}{3}\right)\\ & +x^2\cdot\left(\frac{3a^2c^2d}{2}+7abc^6d+\frac{11b^2c^{10}d}{2}\right)\\ & +x\left(a^2c^3+2abc^7+b^2c^{11}\right)\end{aligned}$$

input
```
integrate((d*x+c)**3*(a+b*(d*x+c)**4)**2,x)
```

output
```
55*b**2*c**3*d**8*x**9/3 + 11*b**2*c**2*d**9*x**10/2 + b**2*c*d**10*x**11
+ b**2*d**11*x**12/12 + x**8*(a*b*d**7/4 + 165*b**2*c**4*d**7/4) + x**7*(2
*a*b*c*d**6 + 66*b**2*c**5*d**6) + x**6*(7*a*b*c**2*d**5 + 77*b**2*c**6*d*
*5) + x**5*(14*a*b*c**3*d**4 + 66*b**2*c**7*d**4) + x**4*(a**2*d**3/4 + 35
*a*b*c**4*d**3/2 + 165*b**2*c**8*d**3/4) + x**3*(a**2*c*d**2 + 14*a*b*c**5
*d**2 + 55*b**2*c**9*d**2/3) + x**2*(3*a**2*c**2*d/2 + 7*a*b*c**6*d + 11*b
**2*c**10*d/2) + x*(a**2*c**3 + 2*a*b*c**7 + b**2*c**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (c+dx)^3\left(a+b(c+dx)^4\right)^2\,dx = \frac{\left((dx+c)^4b+a\right)^3}{12\,bd}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
1/12*((d*x + c)^4*b + a)^3/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx = \frac{\left((dx+c)^4 b + a\right)^3}{12\,bd}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
1/12*((d*x + c)^4*b + a)^3/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 229, normalized size of antiderivative = 9.96

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx = {} & c^3\,x\left(b\,c^4+a\right)^2 + \frac{b^2\,d^{11}\,x^{12}}{12} \\ & + \frac{d^3\,x^4\left(a^2+70\,a\,b\,c^4+165\,b^2\,c^8\right)}{4} \\ & + b^2\,c\,d^{10}\,x^{11} + \frac{55\,b^2\,c^3\,d^8\,x^9}{3} \\ & + \frac{11\,b^2\,c^2\,d^9\,x^{10}}{2} + \frac{b\,d^7\,x^8\left(165\,b\,c^4+a\right)}{4} \\ & + \frac{c^2\,d\,x^2\left(3\,a^2+14\,a\,b\,c^4+11\,b^2\,c^8\right)}{2} \\ & + \frac{c\,d^2\,x^3\left(3\,a^2+42\,a\,b\,c^4+55\,b^2\,c^8\right)}{3} \\ & + 2\,b\,c\,d^6\,x^7\left(33\,b\,c^4+a\right) + 7\,b\,c^2\,d^5\,x^6\left(11\,b\,c^4+a\right) \\ & + 2\,b\,c^3\,d^4\,x^5\left(33\,b\,c^4+7\,a\right)\end{aligned}$$

input
```
int((a + b*(c + d*x)^4)^2*(c + d*x)^3,x)
```

output
```
c^3*x*(a + b*c^4)^2 + (b^2*d^11*x^12)/12 + (d^3*x^4*(a^2 + 165*b^2*c^8 + 7
0*a*b*c^4))/4 + b^2*c*d^10*x^11 + (55*b^2*c^3*d^8*x^9)/3 + (11*b^2*c^2*d^9
*x^10)/2 + (b*d^7*x^8*(a + 165*b*c^4))/4 + (c^2*d*x^2*(3*a^2 + 11*b^2*c^8
+ 14*a*b*c^4))/2 + (c*d^2*x^3*(3*a^2 + 55*b^2*c^8 + 42*a*b*c^4))/3 + 2*b*c
*d^6*x^7*(a + 33*b*c^4) + 7*b*c^2*d^5*x^6*(a + 11*b*c^4) + 2*b*c^3*d^4*x^5
*(7*a + 33*b*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 12.43

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^2 \, dx$$
$$= \frac{x(b^2d^{11}x^{11} + 12b^2c\,d^{10}x^{10} + 66b^2c^2d^9x^9 + 220b^2c^3d^8x^8 + 495b^2c^4d^7x^7 + 792b^2c^5d^6x^6 + 924b^2c^6d^5x^5 + 79}{}$$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4)^2,x)
```

output
```
(x*(12*a**2*c**3 + 18*a**2*c**2*d*x + 12*a**2*c*d**2*x**2 + 3*a**2*d**3*x*
*3 + 24*a*b*c**7 + 84*a*b*c**6*d*x + 168*a*b*c**5*d**2*x**2 + 210*a*b*c**4
*d**3*x**3 + 168*a*b*c**3*d**4*x**4 + 84*a*b*c**2*d**5*x**5 + 24*a*b*c*d**
6*x**6 + 3*a*b*d**7*x**7 + 12*b**2*c**11 + 66*b**2*c**10*d*x + 220*b**2*c*
*9*d**2*x**2 + 495*b**2*c**8*d**3*x**3 + 792*b**2*c**7*d**4*x**4 + 924*b**
2*c**6*d**5*x**5 + 792*b**2*c**5*d**6*x**6 + 495*b**2*c**4*d**7*x**7 + 220
*b**2*c**3*d**8*x**8 + 66*b**2*c**2*d**9*x**9 + 12*b**2*c*d**10*x**10 + b*
*2*d**11*x**11))/12
```

3.112 $\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx$

Optimal result . . . 945
Mathematica [B] (verified) . . . 945
Rubi [A] (verified) . . . 946
Maple [A] (verified) . . . 947
Fricas [B] (verification not implemented) . . . 948
Sympy [B] (verification not implemented) . . . 949
Maxima [A] (verification not implemented) . . . 950
Giac [A] (verification not implemented) . . . 950
Mupad [B] (verification not implemented) . . . 951
Reduce [B] (verification not implemented) . . . 952

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx = \frac{\left(a+b(c+dx)^4\right)^4}{16bd}$$

output
```
1/16*(a+b*(d*x+c)^4)^4/b/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. $2(23) = 46$.

Time = 0.06 (sec) , antiderivative size = 308, normalized size of antiderivative = 13.39

$$\begin{aligned}\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx = \frac{1}{16}x\big(&4c^3+6c^2dx+4cd^2x^2+d^3x^3\big)\big(4a^3\\ &+6a^2b\big(2c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4\big)\\ &+4ab^2\big(3c^8+12c^7dx+34c^6d^2x^2+60c^5d^3x^3\\ &+71c^4d^4x^4+56c^3d^5x^5+28c^2d^6x^6+8cd^7x^7+d^8x^8\big)\\ &+b^3\big(4c^{12}+24c^{11}dx+100c^{10}d^2x^2+280c^9d^3x^3\\ &+566c^8d^4x^4+848c^7d^5x^5+952c^6d^6x^6+800c^5d^7x^7\\ &+496c^4d^8x^8+220c^3d^9x^9+66c^2d^{10}x^{10}+12cd^{11}x^{11}\\ &+d^{12}x^{12}\big)\big)\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a + b*(c + d*x)^4)^3,x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(4*a^3 + 6*a^2*b*(2*c^4 + 4
*c^3*d*x + 6*c^2*d^2*x^2 + 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^2*(3*c^8 + 12*c^
7*d*x + 34*c^6*d^2*x^2 + 60*c^5*d^3*x^3 + 71*c^4*d^4*x^4 + 56*c^3*d^5*x^5
+ 28*c^2*d^6*x^6 + 8*c*d^7*x^7 + d^8*x^8) + b^3*(4*c^12 + 24*c^11*d*x + 10
0*c^10*d^2*x^2 + 280*c^9*d^3*x^3 + 566*c^8*d^4*x^4 + 848*c^7*d^5*x^5 + 952
*c^6*d^6*x^6 + 800*c^5*d^7*x^7 + 496*c^4*d^8*x^8 + 220*c^3*d^9*x^9 + 66*c^
2*d^10*x^10 + 12*c*d^11*x^11 + d^12*x^12)))/16
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 dx$$

$$\downarrow 895$$

$$\frac{\int (c+dx)^3 \left(b(c+dx)^4+a\right)^3 d(c+dx)}{d}$$

$$\downarrow 793$$

$$\frac{\left(a+b(c+dx)^4\right)^4}{16bd}$$

input
```
Int[(c + d*x)^3*(a + b*(c + d*x)^4)^3,x]
```

output
```
(a + b*(c + d*x)^4)^4/(16*b*d)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(a+b(xd+c)^4\right)^4}{16bd}$
norman	$\left(b^3c^{15}+3ab^2c^{11}+3a^2bc^7+c^3a^3\right)x+\left(\frac{15}{2}b^3c^{14}d+\frac{33}{2}ab^2c^{10}d+\frac{21}{2}a^2bc^6d+\frac{3}{2}a^3c^2d\right)x^2+($
gosper	$\frac{x\left(d^{15}b^3x^{15}+16cd^{14}b^3x^{14}+120c^2d^{13}b^3x^{13}+560c^3d^{12}b^3x^{12}+1820x^{11}c^4d^{11}b^3+4368b^3c^5d^{10}x^{10}+8008x^9c^6d^9b^3+11440b^3c^7d\right.}{}$
parallelrisch	$\frac{495}{4}b^2d^3ac^8x^4+3bd^6a^2cx^7+55b^2d^2ac^9x^3+\frac{21}{2}bd^5a^2c^2x^6+\frac{33}{2}b^2dac^{10}x^2+21bd^4a^2c^3x^5+$
orering	$\frac{x\left(d^{15}b^3x^{15}+16cd^{14}b^3x^{14}+120c^2d^{13}b^3x^{13}+560c^3d^{12}b^3x^{12}+1820x^{11}c^4d^{11}b^3+4368b^3c^5d^{10}x^{10}+8008x^9c^6d^9b^3+11440b^3c^7d\right.}{}$
risch	$\frac{495b^2d^3ac^8x^4}{4}+3bd^6a^2cx^7+55b^2d^2ac^9x^3+\frac{21bd^5a^2c^2x^6}{2}+\frac{33b^2dac^{10}x^2}{2}+21bd^4a^2c^3x^5+3b^2ac$

input
```
int((d*x+c)^3*(a+b*(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(a+b*(d*x+c)^4)^4/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(21) = 42$.

Time = 0.10 (sec) , antiderivative size = 432, normalized size of antiderivative = 18.78

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^4\right)^3\,dx = {} & \frac{1}{16}b^3d^{15}x^{16}+b^3cd^{14}x^{15}+\frac{15}{2}b^3c^2d^{13}x^{14}+35\,b^3c^3d^{12}x^{13}\\ & +\frac{1}{4}\left(455\,b^3c^4+ab^2\right)d^{11}x^{12}+3\left(91\,b^3c^5+ab^2c\right)d^{10}x^{11}\\ & +\frac{11}{2}\left(91\,b^3c^6+3\,ab^2c^2\right)d^9x^{10}\\ & +55\left(13\,b^3c^7+ab^2c^3\right)d^8x^9\\ & +\frac{3}{8}\left(2145\,b^3c^8+330\,ab^2c^4+a^2b\right)d^7x^8\\ & +\left(715\,b^3c^9+198\,ab^2c^5+3\,a^2bc\right)d^6x^7\\ & +\frac{7}{2}\left(143\,b^3c^{10}+66\,ab^2c^6+3\,a^2bc^2\right)d^5x^6\\ & +3\left(91\,b^3c^{11}+66\,ab^2c^7+7\,a^2bc^3\right)d^4x^5\\ & +\frac{1}{4}\left(455\,b^3c^{12}+495\,ab^2c^8+105\,a^2bc^4+a^3\right)d^3x^4\\ & +\left(35\,b^3c^{13}+55\,ab^2c^9+21\,a^2bc^5+a^3c\right)d^2x^3\\ & +\frac{3}{2}\left(5\,b^3c^{14}+11\,ab^2c^{10}+7\,a^2bc^6+a^3c^2\right)dx^2\\ & +\left(b^3c^{15}+3\,ab^2c^{11}+3\,a^2bc^7+a^3c^3\right)x\end{aligned}$$

input

```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="fricas")
```

output

```
1/16*b^3*d^15*x^16 + b^3*c*d^14*x^15 + 15/2*b^3*c^2*d^13*x^14 + 35*b^3*c^3
*d^12*x^13 + 1/4*(455*b^3*c^4 + a*b^2)*d^11*x^12 + 3*(91*b^3*c^5 + a*b^2*c
)*d^10*x^11 + 11/2*(91*b^3*c^6 + 3*a*b^2*c^2)*d^9*x^10 + 55*(13*b^3*c^7 +
a*b^2*c^3)*d^8*x^9 + 3/8*(2145*b^3*c^8 + 330*a*b^2*c^4 + a^2*b)*d^7*x^8 +
(715*b^3*c^9 + 198*a*b^2*c^5 + 3*a^2*b*c)*d^6*x^7 + 7/2*(143*b^3*c^10 + 66
*a*b^2*c^6 + 3*a^2*b*c^2)*d^5*x^6 + 3*(91*b^3*c^11 + 66*a*b^2*c^7 + 7*a^2*
b*c^3)*d^4*x^5 + 1/4*(455*b^3*c^12 + 495*a*b^2*c^8 + 105*a^2*b*c^4 + a^3)*
d^3*x^4 + (35*b^3*c^13 + 55*a*b^2*c^9 + 21*a^2*b*c^5 + a^3*c)*d^2*x^3 + 3/
2*(5*b^3*c^14 + 11*a*b^2*c^10 + 7*a^2*b*c^6 + a^3*c^2)*d*x^2 + (b^3*c^15 +
 3*a*b^2*c^11 + 3*a^2*b*c^7 + a^3*c^3)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(15) = 30$.

Time = 0.08 (sec) , antiderivative size = 541, normalized size of antiderivative = 23.52

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^4\right)^3\,dx = {} & 35b^3c^3d^{12}x^{13} + \frac{15b^3c^2d^{13}x^{14}}{2} + b^3cd^{14}x^{15} \\ & + \frac{b^3d^{15}x^{16}}{16} + x^{12}\left(\frac{ab^2d^{11}}{4} + \frac{455b^3c^4d^{11}}{4}\right) \\ & + x^{11}\cdot\left(3ab^2cd^{10} + 273b^3c^5d^{10}\right) \\ & + x^{10}\cdot\left(\frac{33ab^2c^2d^9}{2} + \frac{1001b^3c^6d^9}{2}\right) \\ & + x^9\cdot\left(55ab^2c^3d^8 + 715b^3c^7d^8\right) + x^8 \\ & \cdot\left(\frac{3a^2bd^7}{8} + \frac{495ab^2c^4d^7}{4} + \frac{6435b^3c^8d^7}{8}\right) + x^7 \\ & \cdot\left(3a^2bcd^6 + 198ab^2c^5d^6 + 715b^3c^9d^6\right) + x^6 \\ & \cdot\left(\frac{21a^2bc^2d^5}{2} + 231ab^2c^6d^5 + \frac{1001b^3c^{10}d^5}{2}\right) + x^5 \\ & \cdot\left(21a^2bc^3d^4 + 198ab^2c^7d^4 + 273b^3c^{11}d^4\right) + x^4\left(\frac{a^3d^3}{4}\right. \\ & \qquad \left. + \frac{105a^2bc^4d^3}{4} + \frac{495ab^2c^8d^3}{4} + \frac{455b^3c^{12}d^3}{4}\right) \\ & + x^3\left(a^3cd^2 + 21a^2bc^5d^2 + 55ab^2c^9d^2 + 35b^3c^{13}d^2\right) \\ & + x^2\cdot\left(\frac{3a^3c^2d}{2} + \frac{21a^2bc^6d}{2} + \frac{33ab^2c^{10}d}{2} + \frac{15b^3c^{14}d}{2}\right) \\ & + x\left(a^3c^3 + 3a^2bc^7 + 3ab^2c^{11} + b^3c^{15}\right)\end{aligned}$$

input

```
integrate((d*x+c)**3*(a+b*(d*x+c)**4)**3,x)
```

output
```
35*b**3*c**3*d**12*x**13 + 15*b**3*c**2*d**13*x**14/2 + b**3*c*d**14*x**15
 + b**3*d**15*x**16/16 + x**12*(a*b**2*d**11/4 + 455*b**3*c**4*d**11/4) +
x**11*(3*a*b**2*c*d**10 + 273*b**3*c**5*d**10) + x**10*(33*a*b**2*c**2*d**
9/2 + 1001*b**3*c**6*d**9/2) + x**9*(55*a*b**2*c**3*d**8 + 715*b**3*c**7*d
**8) + x**8*(3*a**2*b*d**7/8 + 495*a*b**2*c**4*d**7/4 + 6435*b**3*c**8*d**
7/8) + x**7*(3*a**2*b*c*d**6 + 198*a*b**2*c**5*d**6 + 715*b**3*c**9*d**6)
+ x**6*(21*a**2*b*c**2*d**5/2 + 231*a*b**2*c**6*d**5 + 1001*b**3*c**10*d**
5/2) + x**5*(21*a**2*b*c**3*d**4 + 198*a*b**2*c**7*d**4 + 273*b**3*c**11*d
**4) + x**4*(a**3*d**3/4 + 105*a**2*b*c**4*d**3/4 + 495*a*b**2*c**8*d**3/4
 + 455*b**3*c**12*d**3/4) + x**3*(a**3*c*d**2 + 21*a**2*b*c**5*d**2 + 55*a
*b**2*c**9*d**2 + 35*b**3*c**13*d**2) + x**2*(3*a**3*c**2*d/2 + 21*a**2*b*
c**6*d/2 + 33*a*b**2*c**10*d/2 + 15*b**3*c**14*d/2) + x*(a**3*c**3 + 3*a**
2*b*c**7 + 3*a*b**2*c**11 + b**3*c**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx = \frac{\left((dx+c)^4 b + a\right)^4}{16\, bd}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
1/16*((d*x + c)^4*b + a)^4/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx = \frac{\left((dx+c)^4 b + a\right)^4}{16\, bd}$$

input
```
integrate((d*x+c)^3*(a+b*(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
1/16*((d*x + c)^4*b + a)^4/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 16.35

$$\begin{aligned}\int (c+dx)^3\left(a+b(c+dx)^4\right)^3\,dx &= c^3\,x\left(b\,c^4+a\right)^3\\ &+\frac{d^3\,x^4\left(a^3+105\,a^2\,b\,c^4+495\,a\,b^2\,c^8+455\,b^3\,c^{12}\right)}{4}\\ &+\frac{b^3\,d^{15}\,x^{16}}{16}+\frac{b^2\,d^{11}\,x^{12}\left(455\,b\,c^4+a\right)}{4}+b^3\,c\,d^{14}\,x^{15}\\ &+\frac{3\,b\,d^7\,x^8\left(a^2+330\,a\,b\,c^4+2145\,b^2\,c^8\right)}{8}\\ &+35\,b^3\,c^3\,d^{12}\,x^{13}+\frac{15\,b^3\,c^2\,d^{13}\,x^{14}}{2}\\ &+c\,d^2\,x^3\left(a^3+21\,a^2\,b\,c^4+55\,a\,b^2\,c^8+35\,b^3\,c^{12}\right)\\ &+b\,c\,d^6\,x^7\left(3\,a^2+198\,a\,b\,c^4+715\,b^2\,c^8\right)\\ &+\frac{11\,b^2\,c^2\,d^9\,x^{10}\left(91\,b\,c^4+3\,a\right)}{2}\\ &+\frac{3\,c^2\,d\,x^2\left(b\,c^4+a\right)^2\left(5\,b\,c^4+a\right)}{2}\\ &+3\,b^2\,c\,d^{10}\,x^{11}\left(91\,b\,c^4+a\right)\\ &+3\,b\,c^3\,d^4\,x^5\left(7\,a^2+66\,a\,b\,c^4+91\,b^2\,c^8\right)\\ &+\frac{7\,b\,c^2\,d^5\,x^6\left(3\,a^2+66\,a\,b\,c^4+143\,b^2\,c^8\right)}{2}\\ &+55\,b^2\,c^3\,d^8\,x^9\left(13\,b\,c^4+a\right)\end{aligned}$$

input

```
int((a + b*(c + d*x)^4)^3*(c + d*x)^3,x)
```

output

```
c^3*x*(a + b*c^4)^3 + (d^3*x^4*(a^3 + 455*b^3*c^12 + 105*a^2*b*c^4 + 495*a
*b^2*c^8))/4 + (b^3*d^15*x^16)/16 + (b^2*d^11*x^12*(a + 455*b*c^4))/4 + b^
3*c*d^14*x^15 + (3*b*d^7*x^8*(a^2 + 2145*b^2*c^8 + 330*a*b*c^4))/8 + 35*b^
3*c^3*d^12*x^13 + (15*b^3*c^2*d^13*x^14)/2 + c*d^2*x^3*(a^3 + 35*b^3*c^12
+ 21*a^2*b*c^4 + 55*a*b^2*c^8) + b*c*d^6*x^7*(3*a^2 + 715*b^2*c^8 + 198*a*
b*c^4) + (11*b^2*c^2*d^9*x^10*(3*a + 91*b*c^4))/2 + (3*c^2*d*x^2*(a + b*c^
4)^2*(a + 5*b*c^4))/2 + 3*b^2*c*d^10*x^11*(a + 91*b*c^4) + 3*b*c^3*d^4*x^5
*(7*a^2 + 91*b^2*c^8 + 66*a*b*c^4) + (7*b*c^2*d^5*x^6*(3*a^2 + 143*b^2*c^8
 + 66*a*b*c^4))/2 + 55*b^2*c^3*d^8*x^9*(a + 13*b*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 523, normalized size of antiderivative = 22.74

$$\int (c+dx)^3 \left(a+b(c+dx)^4\right)^3 \, dx$$
$$= \frac{x(b^3d^{15}x^{15} + 16b^3cd^{14}x^{14} + 120b^3c^2d^{13}x^{13} + 560b^3c^3d^{12}x^{12} + 1820b^3c^4d^{11}x^{11} + 4368b^3c^5d^{10}x^{10} + 8008b^3$$

input

```
int((d*x+c)^3*(a+b*(d*x+c)^4)^3,x)
```

output

```
(x*(16*a**3*c**3 + 24*a**3*c**2*d*x + 16*a**3*c*d**2*x**2 + 4*a**3*d**3*x*
*3 + 48*a**2*b*c**7 + 168*a**2*b*c**6*d*x + 336*a**2*b*c**5*d**2*x**2 + 42
0*a**2*b*c**4*d**3*x**3 + 336*a**2*b*c**3*d**4*x**4 + 168*a**2*b*c**2*d**5
*x**5 + 48*a**2*b*c*d**6*x**6 + 6*a**2*b*d**7*x**7 + 48*a*b**2*c**11 + 264
*a*b**2*c**10*d*x + 880*a*b**2*c**9*d**2*x**2 + 1980*a*b**2*c**8*d**3*x**3
 + 3168*a*b**2*c**7*d**4*x**4 + 3696*a*b**2*c**6*d**5*x**5 + 3168*a*b**2*c
**5*d**6*x**6 + 1980*a*b**2*c**4*d**7*x**7 + 880*a*b**2*c**3*d**8*x**8 + 2
64*a*b**2*c**2*d**9*x**9 + 48*a*b**2*c*d**10*x**10 + 4*a*b**2*d**11*x**11
+ 16*b**3*c**15 + 120*b**3*c**14*d*x + 560*b**3*c**13*d**2*x**2 + 1820*b**
3*c**12*d**3*x**3 + 4368*b**3*c**11*d**4*x**4 + 8008*b**3*c**10*d**5*x**5
+ 11440*b**3*c**9*d**6*x**6 + 12870*b**3*c**8*d**7*x**7 + 11440*b**3*c**7*
d**8*x**8 + 8008*b**3*c**6*d**9*x**9 + 4368*b**3*c**5*d**10*x**10 + 1820*b
**3*c**4*d**11*x**11 + 560*b**3*c**3*d**12*x**12 + 120*b**3*c**2*d**13*x**
13 + 16*b**3*c*d**14*x**14 + b**3*d**15*x**15))/16
```

3.113 $\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx$

Optimal result . . . 953
Mathematica [A] (verified) . . . 953
Rubi [A] (verified) . . . 954
Maple [A] (verified) . . . 955
Fricas [B] (verification not implemented) . . . 955
Sympy [B] (verification not implemented) . . . 956
Maxima [A] (verification not implemented) . . . 956
Giac [A] (verification not implemented) . . . 956
Mupad [B] (verification not implemented) . . . 957
Reduce [B] (verification not implemented) . . . 957

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

output
```
1/4*ln(a+b*(d*x+c)^4)/b/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

input
```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^4),x]
```

output
```
Log[a + b*(c + d*x)^4]/(4*b*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^3}{b(c+dx)^4+a} d(c+dx)}{d}$$
$$\downarrow 792$$
$$\frac{\log\left(a+b(c+dx)^4\right)}{4bd}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^4),x]
```

output
```
Log[a + b*(c + d*x)^4]/(4*b*d)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(a+b(xd+c)^4\right)}{4bd}$	21
default	$\frac{\ln\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}{4bd}$	55
norman	$\frac{\ln\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}{4bd}$	55
risch	$\frac{\ln\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}{4bd}$	55
parallelrisch	$\frac{\ln\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}{4bd}$	55

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(a+b*(d*x+c)^4)/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a\right)}{4\,bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*log(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4
+ a)/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left(a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4\right)}{4bd}$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**4),x)
```

output
```
log(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d
**4*x**4)/(4*b*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left((dx+c)^4b+a\right)}{4\,bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
1/4*log((d*x + c)^4*b + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log\left((dx+c)^4b+a\right)}{4\,bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
1/4*log((d*x + c)^4*b + a)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\ln\left(b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a\right)}{4\,b\,d}$$

input
```
int((c + d*x)^3/(a + b*(c + d*x)^4),x)
```

output
```
log(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^3)
/(4*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(c+dx)^3}{a+b(c+dx)^4}\,dx = \frac{\log(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)}{4bd}$$

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4),x)
```

output
```
log(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d
**4*x**4)/(4*b*d)
```

3.114 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx$

Optimal result . . . 958
Mathematica [A] (verified) . . . 958
Rubi [A] (verified) . . . 959
Maple [A] (verified) . . . 960
Fricas [B] (verification not implemented) . . . 960
Sympy [B] (verification not implemented) . . . 961
Maxima [A] (verification not implemented) . . . 961
Giac [A] (verification not implemented) . . . 961
Mupad [B] (verification not implemented) . . . 962
Reduce [B] (verification not implemented) . . . 962

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4bd\left(a+b(c+dx)^4\right)}$$

output
```
-1/4/b/d/(a+b*(d*x+c)^4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4bd\left(a+b(c+dx)^4\right)}$$

input
```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^4)^2,x]
```

output
```
-1/4*1/(b*d*(a + b*(c + d*x)^4))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b(c+dx)^4)^2}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^3}{(b(c+dx)^4+a)^2}\,d(c+dx)}{d}$$
$$\downarrow 793$$
$$-\frac{1}{4bd\,(a+b(c+dx)^4)}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^4)^2,x]
```

output
```
-1/4*1/(b*d*(a + b*(c + d*x)^4))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{4bd\left(a+b(xd+c)^4\right)}$	22
gosper	$-\frac{1}{4(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)bd}$	56
default	$-\frac{1}{4(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)bd}$	56
norman	$-\frac{1}{4(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)bd}$	56
risch	$-\frac{1}{4(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)bd}$	56
parallelrisch	$-\frac{1}{4(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)bd}$	56
orering	$-\frac{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}{4bd\left(a+b(xd+c)^4\right)^2}$	67

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/d/(a+b*(d*x+c)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(21) = 42.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx)^3}{(a+b(c+dx)^4)^2}\,dx = -\frac{1}{4\left(b^2d^5x^4+4\,b^2cd^4x^3+6\,b^2c^2d^3x^2+4\,b^2c^3d^2x+(b^2c^4+ab)d\right)}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="fricas")
```

output
```
-1/4/(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x
+ (b^2*c^4 + a*b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.85 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4abd+4b^2c^4d+16b^2c^3d^2x+24b^2c^2d^3x^2+16b^2cd^4x^3+4b^2d^5x^4}$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**4)**2,x)
```

output
```
-1/(4*a*b*d + 4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2
 + 16*b**2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4\left((dx+c)^4b+a\right)bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="maxima")
```

output
```
-1/4/(((d*x + c)^4*b + a)*b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4\left((dx+c)^4b+a\right)bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^2,x, algorithm="giac")
```

output
```
-1/4/(((d*x + c)^4*b + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4\,b\,d\,\left(b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a\right)}$$

input
```
int((c + d*x)^3/(a + b*(c + d*x)^4)^2,x)
```

output
```
-1/(4*b*d*(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d
^3*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^2}\,dx = -\frac{1}{4bd\,\left(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a\right)}$$

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4)^2,x)
```

output
```
( - 1)/(4*b*d*(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4))
```

3.115 $\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx$

Optimal result 963
Mathematica [A] (verified) 963
Rubi [A] (verified) 964
Maple [A] (verified) 965
Fricas [B] (verification not implemented) 965
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Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx = -\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}$$

output

```
-1/8/b/d/(a+b*(d*x+c)^4)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx = -\frac{1}{8bd\left(a+b(c+dx)^4\right)^2}$$

input

```
Integrate[(c + d*x)^3/(a + b*(c + d*x)^4)^3,x]
```

output

```
-1/8*1/(b*d*(a + b*(c + d*x)^4)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {895, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b(c+dx)^4)^3}\,dx$$
$$\downarrow 895$$
$$\frac{\int \frac{(c+dx)^3}{(b(c+dx)^4+a)^3}\,d(c+dx)}{d}$$
$$\downarrow 793$$
$$-\frac{1}{8bd\,(a+b(c+dx)^4)^2}$$

input
```
Int[(c + d*x)^3/(a + b*(c + d*x)^4)^3,x]
```

output
```
-1/8*1/(b*d*(a + b*(c + d*x)^4)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{8bd\left(a+b(xd+c)^4\right)^2}$	22
gosper	$-\frac{1}{8(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^2bd}$	56
default	$-\frac{1}{8(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^2bd}$	56
norman	$-\frac{1}{8(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^2bd}$	56
risch	$-\frac{1}{8(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^2bd}$	56
parallelrisch	$-\frac{1}{8(b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a)^2bd}$	56
orering	$-\frac{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}{8bd\left(a+b(xd+c)^4\right)^3}$	67

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/b/d/(a+b*(d*x+c)^4)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(21) = 42.

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 7.48

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx =$$

$$-\frac{1}{8\left(b^3d^9x^8+8\,b^3cd^8x^7+28\,b^3c^2d^7x^6+56\,b^3c^3d^6x^5+2\left(35\,b^3c^4+ab^2\right)d^5x^4+8\left(7\,b^3c^5+ab^2c\right)d^4x^3+4\right.}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="fricas")
```

output
```
-1/8/(b^3*d^9*x^8 + 8*b^3*c*d^8*x^7 + 28*b^3*c^2*d^7*x^6 + 56*b^3*c^3*d^6*
x^5 + 2*(35*b^3*c^4 + a*b^2)*d^5*x^4 + 8*(7*b^3*c^5 + a*b^2*c)*d^4*x^3 + 4
*(7*b^3*c^6 + 3*a*b^2*c^2)*d^3*x^2 + 8*(b^3*c^7 + a*b^2*c^3)*d^2*x + (b^3*
c^8 + 2*a*b^2*c^4 + a^2*b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(19) = 38$.

Time = 2.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 8.57

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx =$$

$$-\frac{1}{8a^2bd + 16ab^2c^4d + 8b^3c^8d + 448b^3c^3d^6x^5 + 224b^3c^2d^7x^6 + 64b^3cd^8x^7 + 8b^3d^9x^8 + x^4 \cdot (16ab^2d^5 + 560$$

input
```
integrate((d*x+c)**3/(a+b*(d*x+c)**4)**3,x)
```

output
```
-1/(8*a**2*b*d + 16*a*b**2*c**4*d + 8*b**3*c**8*d + 448*b**3*c**3*d**6*x**
5 + 224*b**3*c**2*d**7*x**6 + 64*b**3*c*d**8*x**7 + 8*b**3*d**9*x**8 + x**
4*(16*a*b**2*d**5 + 560*b**3*c**4*d**5) + x**3*(64*a*b**2*c*d**4 + 448*b**
3*c**5*d**4) + x**2*(96*a*b**2*c**2*d**3 + 224*b**3*c**6*d**3) + x*(64*a*b
**2*c**3*d**2 + 64*b**3*c**7*d**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx = -\frac{1}{8\left((dx+c)^4b+a\right)^2bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="maxima")
```

output
```
-1/8/(((d*x + c)^4*b + a)^2*b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx = -\frac{1}{8\left((dx+c)^4 b+a\right)^2 bd}$$

input
```
integrate((d*x+c)^3/(a+b*(d*x+c)^4)^3,x, algorithm="giac")
```

output
```
-1/8/(((d*x + c)^4*b + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 7.43

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx =$$

$$-\frac{1}{8\,b\,d\,\left(x^4\,\left(70\,b^2\,c^4\,d^4+2\,a\,b\,d^4\right)+x^3\,\left(56\,b^2\,c^5\,d^3+8\,a\,b\,c\,d^3\right)+a^2+x\,\left(8\,d\,b^2\,c^7+8\,a\,d\,b\,c^3\right)+b^2\,c^8+x\right.}$$

input
```
int((c + d*x)^3/(a + b*(c + d*x)^4)^3,x)
```

output
```
-1/(8*b*d*(x^4*(70*b^2*c^4*d^4 + 2*a*b*d^4) + x^3*(56*b^2*c^5*d^3 + 8*a*b*
c*d^3) + a^2 + x*(8*b^2*c^7*d + 8*a*b*c^3*d) + b^2*c^8 + x^2*(28*b^2*c^6*d
^2 + 12*a*b*c^2*d^2) + b^2*d^8*x^8 + 2*a*b*c^4 + 8*b^2*c*d^7*x^7 + 56*b^2*
c^3*d^5*x^5 + 28*b^2*c^2*d^6*x^6))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 7.52

$$\int \frac{(c+dx)^3}{\left(a+b(c+dx)^4\right)^3}\,dx =$$

$$-\frac{1}{8bd\left(b^2d^8x^8+8b^2c\,d^7x^7+28b^2c^2d^6x^6+56b^2c^3d^5x^5+70b^2c^4d^4x^4+56b^2c^5d^3x^3+28b^2c^6d^2x^2+8b^2c^7d\right.}$$

input
```
int((d*x+c)^3/(a+b*(d*x+c)^4)^3,x)
```

output
```
( - 1)/(8*b*d*(a**2 + 2*a*b*c**4 + 8*a*b*c**3*d*x + 12*a*b*c**2*d**2*x**2
+ 8*a*b*c*d**3*x**3 + 2*a*b*d**4*x**4 + b**2*c**8 + 8*b**2*c**7*d*x + 28*b
**2*c**6*d**2*x**2 + 56*b**2*c**5*d**3*x**3 + 70*b**2*c**4*d**4*x**4 + 56*
b**2*c**3*d**5*x**5 + 28*b**2*c**2*d**6*x**6 + 8*b**2*c*d**7*x**7 + b**2*d
**8*x**8))
```

3.116 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal result . . . 969
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Optimal result

Integrand size = 17, antiderivative size = 234

$$\begin{aligned}
&\int \frac{x^3}{a+b(c+dx)^3}\,dx \\
&= \frac{x}{bd^3} + \frac{\left(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} \\
&\quad - \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} \\
&\quad + \frac{\left(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4} \\
&\quad - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}
\end{aligned}$$

output

```
x/b/d^3+1/3*(a-3*a^(1/3)*b^(2/3)*c^2+b*c^3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(4/3)/d^4-1/3*(a+3*a^(1/3)*b^(
2/3)*c^2+b*c^3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(4/3)/d^4+1/6*(a+3*a
^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(2/3)/b^(4/3)/d^4-c*ln(a+b*(d*x+c)^3)/b/d^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = -\frac{-3bdx + \text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\ \frac{a\log(x-\#1)+bc^3\log(x-\#1)+3bc^2d\log(x-\#1}{c^2+2cd\#1+d^2\#1}\right.}{3b^2d^4}$$

input
```
Integrate[x^3/(a + b*(c + d*x)^3),x]
```

output
```
-1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 +
3*b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$
$$\downarrow 896$$
$$\frac{\int \frac{d^3x^3}{b(c+dx)^3+a}\,d(c+dx)}{d^4}$$
$$\downarrow 25$$
$$-\frac{\int -\frac{d^3x^3}{b(c+dx)^3+a}\,d(c+dx)}{d^4}$$

$$\downarrow 2426$$

$$-\frac{\int \left(\frac{bc^3-3b(c+dx)c^2+3b(c+dx)^2c+a}{b(b(c+dx)^3+a)}-\frac{1}{b}\right) d(c+dx)}{d^4}$$

$$\downarrow 2009$$

$$\frac{\left(-3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}-\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{4/3}}+\frac{\left(3\sqrt[3]{a}b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-}{6a^{2/3}}$$

$$d^4$$

input

```
Int[x^3/(a + b*(c + d*x)^3),x]
```

output

```
((c + d*x)/b + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)) - ((a + 3*a
^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b
^(4/3)) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1/
3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)) - (c*Log[a + b*(c
 + d*x)^3])/b)/d^4
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{x}{b\,d^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)}\frac{\left(-3\_R^2{}_{bc\,d^2}-3\_R_{b\,c^2d-c^3b-a}\right)\ln\left(x-\_R\right)}{d^2\_R^2{}_{+2cd}\_R_{+c^2}}}{3b^2d^4}$	108
risch	$\frac{x}{b\,d^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)}\frac{\left(-3\_R^2{}_{bc\,d^2}-3\_R_{b\,c^2d-c^3b-a}\right)\ln\left(x-\_R\right)}{d^2\_R^2{}_{+2cd}\_R_{+c^2}}}{3b^2d^4}$	108

input
```
int(x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2
*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c
^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.89 (sec) , antiderivative size = 6315, normalized size of antiderivative = 26.99

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$
$$= \text{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto \right.\right.$$
$$+ \frac{x}{bd^3}$$

input
```
integrate(x**3/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2
*b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c
**6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*
_t*a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**
3*c - 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3
*d - 24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \int \frac{x^3}{(dx+c)^3 b + a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \int \frac{x^3}{(dx+c)^3 b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.60

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \left(\sum_{k=1}^{3} \ln\left(\frac{3\,(b\,c^5+a\,c^2)}{d^2}\right.\right.$$
$$-\operatorname{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)$$
$$\left.\left.-\frac{3\,x\,(a\,c-2\,b\,c^4)}{d}\right)\operatorname{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z\right.\right.$$
$$\left.\left.-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\right)+\frac{x}{b\,d^3}$$

input
```
int(x^3/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3
*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*
x*(b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d
^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b
*c^3 + b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*
a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*
c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x
/(b*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.97

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$

$$= \frac{2b^{\frac{1}{3}}a^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)+2b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)c^3-6\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)abc^2+}$$

input

```
int(x^3/(a+b*(d*x+c)^3),x)
```

output

```
(2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*a + 2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*
b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*c**3 - 6*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c**2 + b
**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a + b**(1/3)*a*
*(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/
3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**3 - 2*b**(1/3)*a**(2
/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a - 2*b**(1/3)*a**(2/3)*log(
a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*b*c**3 - 6*b**(2/3)*a**(1/3)*log(a**
(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b*
*(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*c - 6*b**(2/3)*a**(1/3)*log(a**(1/3)
+ b**(1/3)*c + b**(1/3)*d*x)*a*c + 6*b**(2/3)*a**(1/3)*a*d*x + 3*log(a**(2
/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(
2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c**2 - 6*log(a**(1/3) + b**(1/3)*c +
b**(1/3)*d*x)*a*b*c**2)/(6*b**(2/3)*a**(1/3)*a*b*d**4)
```

3.117 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

Optimal result . . . 976
Mathematica [C] (verified) . . . 977
Rubi [A] (verified) . . . 977
Maple [C] (verified) . . . 982
Fricas [C] (verification not implemented) . . . 982
Sympy [A] (verification not implemented) . . . 983
Maxima [F] . . . 983
Giac [F] . . . 983
Mupad [B] (verification not implemented) . . . 984
Reduce [B] (verification not implemented) . . . 984

Optimal result

Integrand size = 17, antiderivative size = 210

$$\begin{aligned}\int \frac{x^2}{a+b(c+dx)^3}\,dx = &\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3}\\ &+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}\\ &-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}\\ &+\frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}\end{aligned}$$

output

```
1/3*c*(2*a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)
/a^(1/3))*3^(1/2)/a^(2/3)/b^(2/3)/d^3+1/3*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(1/
3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^3-1/6*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(2/3)/d^3+1/3*ln
(a+b*(d*x+c)^3)/b/d^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\ \frac{\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

input
```
Integrate[x^2/(a + b*(c + d*x)^3),x]
```

output
```
RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x
- #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {896, 2410, 792, 2399, 16, 25, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$
$$\downarrow 896$$
$$\frac{\int \frac{d^2x^2}{b(c+dx)^3+a}\,d(c+dx)}{d^3}$$
$$\downarrow 2410$$
$$\frac{\int \frac{c^2-2c(c+dx)}{b(c+dx)^3+a}\,d(c+dx)+\int \frac{(c+dx)^2}{b(c+dx)^3+a}\,d(c+dx)}{d^3}$$
$$\downarrow 792$$

$$\frac{\int \frac{c^2-2c(c+dx)}{b(c+dx)^3+a}d(c+dx)+\frac{\log(a+b(c+dx)^3)}{3b}}{d^3}$$

$$\downarrow 2399$$

$$\frac{\int -\frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{\log(a+b(c+dx)^3)}{3b}}{d^3}$$

$$\downarrow 16$$

$$\frac{\int -\frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log(a+b(c+dx)^3)}{3b}}{d^3}$$

$$\downarrow 25$$

$$\frac{-\frac{\int\frac{c\left(2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log(a+b(c+dx)^3)}{3b}}{d^3}$$

$$\downarrow 27$$

$$\frac{-\frac{c\int\frac{2\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+2\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}+\frac{\log(a+b(c+dx)^3)}{3b}}{d^3}$$

$$\downarrow 1142$$

$$\frac{-\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+}{d^3}$$

$$\downarrow 25$$

$$-\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c(}{}}{d^3}$$

$$\downarrow 27$$

$$-\frac{c\left(\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}-}{d^3}$$

$$\downarrow 1082$$

$$-\frac{c\left(\frac{3\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2}{}}{d^3}$$

$$\downarrow 217$$

$$-\frac{c\left(-\frac{1}{2}\sqrt[3]{b}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log}{3a^{2/}}}{d^3}$$

$$\downarrow 1103$$

$$\frac{-\left(\frac{c\left(\frac{1}{2}\left(\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{\sqrt{3}\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{d^3}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\right.}{3a^{2/3}b^{2/3}}$$

input
```
Int[x^2/(a + b*(c + d*x)^3),x]
```

output
```
((c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b
^(2/3)) - (c*(-((Sqrt[3]*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(1 - (2*b^(1/3)*(c
 + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) + (((2*a^(1/3))/b^(1/3) + c)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/2))/(3*a^(2/3)*b^(
1/3)) + Log[a + b*(c + d*x)^3]/(3*b))/d^3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\_R^2 \ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\_R^2 \ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74

input
```
int(x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_
Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 4759, normalized size of antiderivative = 22.66

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$
$$= \text{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2}{}\right.\right.\right.$$

input
```
integrate(x**2/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d
**3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*
log(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d*
*3 + 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3 b + a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(x^2/((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \int \frac{x^2}{(dx+c)^3 b + a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x^2/((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.08

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \sum_{k=1}^{3} \ln\Big(a+b\,c^3-\text{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2 \\ -18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)\,a\,b\,d^3\,6+3\,b\,c^2\,d\,x \\ +\text{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)^2\,a\,b^2\,d^6\,9 \\ +\text{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6 \\ -a^2,z,k\big)\,b^2\,c^3\,d^3\,3+\text{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z \\ -2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)\,b^2\,c^2\,d^4\,x\,3\Big)\,\text{root}\big(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2 \\ -18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\big)$$

input
```
int(x^2/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18
*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^
3 + 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^
2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6
 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9
*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^
2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z -
2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 -
27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*
c^6 - a^2, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.56

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx$$
$$= \frac{-2b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)c^2+4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)abc-b^{\frac{4}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}c-b^{\frac{1}{3}}a^{\frac{1}{3}}dx+\right.}$$

input
```
int(x^2/(a+b*(d*x+c)^3),x)
```

output
```
( - 2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)
*d*x)/(a**(1/3)*sqrt(3)))*b*c**2 + 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c
 - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b*c - b**(1/3)*a**(2/3)*log(a**(2
/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(
2/3)*c*d*x + b**(2/3)*d**2*x**2)*b*c**2 + 2*b**(1/3)*a**(2/3)*log(a**(1/3)
 + b**(1/3)*c + b**(1/3)*d*x)*b*c**2 + 2*b**(2/3)*a**(1/3)*log(a**(2/3) -
b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c
*d*x + b**(2/3)*d**2*x**2)*a + 2*b**(2/3)*a**(1/3)*log(a**(1/3) + b**(1/3)
*c + b**(1/3)*d*x)*a - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**
(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a*b*c +
 4*log(a**(1/3) + b**(1/3)*c + b**(1/3)*d*x)*a*b*c)/(6*b**(2/3)*a**(1/3)*a
*b*d**3)
```

3.118 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal result . . . 986
Mathematica [C] (verified) . . . 987
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Optimal result

Integrand size = 15, antiderivative size = 180

$$\int \frac{x}{a+b(c+dx)^3}\,dx = -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}$$

output

```
-1/3*(a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^
(1/3))*3^(1/2)/a^(2/3)/b^(2/3)/d^2-1/3*(a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1
/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^2+1/6*(a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(2/3)/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.44

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\ \frac{\log(x-\#1)\#1}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

input
```
Integrate[x/(a + b*(c + d*x)^3),x]
```

output
```
RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x
- #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {896, 25, 2399, 16, 25, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$
$$\downarrow 896$$
$$\frac{\int \frac{dx}{b(c+dx)^3+a}\,d(c+dx)}{d^2}$$
$$\downarrow 25$$
$$-\frac{\int -\frac{dx}{b(c+dx)^3+a}\,d(c+dx)}{d^2}$$
$$\downarrow 2399$$

$$\frac{-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\int\frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}}{d^2}$$

$$\downarrow 16$$

$$\frac{-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}}{d^2}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{b}c+\sqrt[3]{a}\right)(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}}}{d^2}$$

$$\downarrow 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\sqrt[3]{}\right.$$

$$\downarrow 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\sqrt[3]{b}\right.$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\right.$$

$$\downarrow 1082$$

$$\frac{3\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)$$

$$\downarrow 217$$

$$\frac{-\frac{1}{2}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right)$$

$$\downarrow 1103$$

$$\frac{\frac{1}{2}\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)-\frac{\sqrt{3}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}-\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)$$

input

```
Int[x/(a + b*(c + d*x)^3),x]
```

output

```
(-1/3*((a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(a^(2/3)*b^
(2/3)) + (-((Sqrt[3]*(a^(1/3) - b^(1/3)*c)*ArcTan[(1 - (2*b^(1/3)*(c + d*x
))/a^(1/3))/Sqrt[3]])/b^(1/3)) + ((a^(1/3)/b^(1/3) + c)*Log[a^(2/3) - a^(1
/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/2)/(3*a^(2/3)*b^(1/3)))/d^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !MatchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;  FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)   Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\_R \ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\_R \ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72

input

```
int(x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output

```
1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^
2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 1950, normalized size of antiderivative = 10.83

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output

```
1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a
)/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt
(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2
*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b
*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1
/3)))*log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*
(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^
4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)
/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b
*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/
54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(
-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 -
a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3)
+ 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1
/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*
(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(...
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.46

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$
$$= \text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

input

```
integrate(x/(a+b*(d*x+c)**3),x)
```

output

```
RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
 _t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d
 - b*c**3*d))))
```

Maxima [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \int \frac{x}{(dx+c)^3 b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(x/((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \int \frac{x}{(dx+c)^3 b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(x/((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \sum_{k=1}^{3} \ln\left(-\operatorname{root}\left(27\,a^2\,b^2\,d^6\,z^3 - 9\,a\,b\,c\,d^2\,z + b\,c^3 + a, z, k\right)\left(3\,b^2\,c^2\,d^4 - \operatorname{root}\left(27\,a^2\,b^2\,d^6\,z^3 - 9\,a\,b\,c\,d^2\,z + b\,c^3 + a, z, k\right) a\,b^2\,d^6\,9 + 3\,b^2\,c\,d^5\,x\right) + b\,d^3\,x\right) \operatorname{root}\left(27\,a^2\,b^2\,d^6\,z^3 - 9\,a\,b\,c\,d^2\,z + b\,c^3 + a, z, k\right)$$

input
```
int(x/(a + b*(c + d*x)^3),x)
```

output
```
symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a,
z, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 +
 a, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^
2*z + b*c^3 + a, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.29

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$
$$= \frac{2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)c - 2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right)a + b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^{2}\right)c - 2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}c + b^{\frac{1}{3}}dx\right)c + b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^{2} + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^{2}x^{2}\right)c - 2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}c + b^{\frac{1}{3}}dx\right)a + b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^{2} + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^{2}x^{2}\right)a}{6b^{\frac{2}{3}}a^{\frac{1}{3}}ad^{2}}$$

input
```
int(x/(a+b*(d*x+c)^3),x)
```

output
```
(2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*
x)/(a**(1/3)*sqrt(3)))*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**
(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a + b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/
3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x +
 b**(2/3)*d**2*x**2)*c - 2*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*c + b
**(1/3)*d*x)*c + log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*
x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2)*a - 2*log(a**(1
/3) + b**(1/3)*c + b**(1/3)*d*x)*a)/(6*b**(2/3)*a**(1/3)*a*d**2)
```

3.119 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal result . . . 995
Mathematica [A] (verified) . . . 995
Rubi [A] (verified) . . . 996
Maple [C] (verified) . . . 999
Fricas [A] (verification not implemented) . . . 1000
Sympy [A] (verification not implemented) . . . 1001
Maxima [F] . . . 1001
Giac [A] (verification not implemented) . . . 1001
Mupad [B] (verification not implemented) . . . 1002
Reduce [B] (verification not implemented) . . . 1003

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{a+b(c+dx)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

output

```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/b^(1/3)/d+1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2
/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right) + 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right) - \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)\right.}{6a^{2/3}\sqrt[3]{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*
Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x)
 + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {239, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{b(c+dx)^3+a} d(c+dx)}{d}$$

$$\downarrow 750$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{b}(c+dx)+\sqrt[3]{a}} d(c+dx)}{3a^{2/3}}}{d}$$

$$\downarrow 16$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}} d(c+dx)}{3a^{2/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)\right)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1082$$

$$\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 217$$

$$\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{b^{2/3}(c+dx)^2-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+a^{2/3}}d(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

$$\downarrow 1103$$

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}}}{d}$$

input
```
Int[(a + b*(c + d*x)^3)^(-1),x]
```

output
```
(Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan
[(1 - (2*b^(1/3)*(c + d*x))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^
(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(2*b^(1/3)))/(3*a^(2/3)))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Z b\,c^2d+c^3b+a\right)} \frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

input
```
int(1/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.16

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abd^3x^3+6\,abcd^2x^2+6\,abc^2dx+2\,abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2\,abd^2x^2+4\,abcdx+2\,abc^2+(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{}\right.}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\right.}{}\right.$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*
d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4
*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-
(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x
^2 + 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d
*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3
)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)
^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\text{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**3),x)
```

output
```
RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

Maxima [F]

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \int \frac{1}{(dx+c)^3b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^3*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^3}\,dx = {} & \frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+(ab^2)^{\frac{1}{3}}}{\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right)\\ & -\frac{1}{6}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}bdx+\sqrt{3}bc-\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}\right)^2\right.\\ & \left.+4\left(bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right)^2\right)\\ & +\frac{1}{3}\left(\frac{1}{a^2bd^3}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+\left(ab^2\right)^{\frac{1}{3}}\right|\right)\end{aligned}$$

input
```
integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(s
qrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))
^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(
b*d*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x
+ b*c + (a*b^2)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\ln\left(b^{1/3}c + a^{1/3} + b^{1/3}d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d} + \frac{\ln\left(3\,b^2\,c\,d^5 + 3\,b^2\,d^6\,x + \frac{3\,a^{1/3}\,b^{5/3}\,d^5\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d} - \frac{\ln\left(3\,b^2\,c\,d^5 + 3\,b^2\,d^6\,x - \frac{3\,a^{1/3}\,b^{5/3}\,d^5\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

input
```
int(1/(a + b*(c + d*x)^3),x)
```

output
```
log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*
c*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1
/3)*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*
d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+b(c+dx)^3}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}c-2b^{\frac{1}{3}}dx}{a^{\frac{1}{3}}\sqrt{3}}\right) - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}c - b^{\frac{1}{3}}a^{\frac{1}{3}}dx + b^{\frac{2}{3}}c^2 + 2b^{\frac{2}{3}}cdx + b^{\frac{2}{3}}d^2x^2\right) + 2\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}c$$
$$6a^{\frac{2}{3}}b^{\frac{1}{3}}d$$

input
```
int(1/(a+b*(d*x+c)^3),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a
**(1/3)*sqrt(3))) - log(a**(2/3) - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)
*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**(2/3)*d**2*x**2) + 2*log(a**(
1/3) + b**(1/3)*c + b**(1/3)*d*x)))/(6*b**(1/3)*a*d)
```

3.120 $\int \frac{1}{x(a+b(c+dx)^3)}\,dx$

Optimal result . . . 1004
Mathematica [C] (verified) . . . 1005
Rubi [A] (verified) . . . 1005
Maple [C] (verified) . . . 1007
Fricas [C] (verification not implemented) . . . 1007
Sympy [F(-1)] . . . 1008
Maxima [F] . . . 1008
Giac [F(-2)] . . . 1008
Mupad [B] (verification not implemented) . . . 1009
Reduce [B] (verification not implemented) . . . 1010

Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \frac{\sqrt[3]{b}c\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)} + \frac{\log(x)}{a+bc^3} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)} - \frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}$$

output

```
1/3*b^(1/3)*c*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1
/2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+ln(x)/(b*c^3+a)-1/3*ln
(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c)-1/6*(2*a^(1/3)-b^(1/
3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(a^(2/
3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = -\frac{-3\log(x)+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\ \frac{3c^2\log(x-\#1)+3cd\log(x-\#1)\#1+d^2\log}{c^2+2cd\#1+d^2\#1^2}\right.}{3\,(a+bc^3)}$$

input
```
Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

output
```
-1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^
2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx$$
$$\downarrow 896$$
$$\int \frac{1}{dx\,(a+b(c+dx)^3)}\,d(c+dx)$$
$$\downarrow 25$$
$$-\int -\frac{1}{dx\,(b(c+dx)^3+a)}\,d(c+dx)$$
$$\downarrow 7276$$

$$-\int \left(\frac{b\left(c^2+(c+dx)c+(c+dx)^2\right)}{\left(bc^3+a\right)\left(b(c+dx)^3+a\right)} - \frac{1}{\left(bc^3+a\right)dx} \right) d(c+dx)$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)} -$$

$$\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)} +$$

$$\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} + \frac{\log(-dx)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}$$

input
```
Int[1/(x*(a + b*(c + d*x)^3)),x]
```

output
```
(b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[-(d*x)]/(a + b*c^3)
 + (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^
(2/3)*(a + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[
a + b*(c + d*x)^3]/(3*(a + b*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.47

method	result
default	$\frac{\ln(x)}{c^3b+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Zb\,c^2d+c^3b+a\right)}\frac{\left(d^2\_R^2+3cd\_R+3c^2\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3(c^3b+a)}$
risch	$\frac{\ln(-x)}{c^3b+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^2b\,c^3+a^3\right)\_Z^3+3\_Z^2a^2+3a\_Z\right)}\_R\ln\left(\left((2ab\,c^3d-4d\,a^2)\_R^2+(b\,c^3d-8ad)\_R-4d\right)x+(ab\,c^4+\right.\right.}{3}$

input
```
int(1/x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/(b*c^3+a)-1/3*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*
ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3
+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 4370, normalized size of antiderivative = 19.51

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.47

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx$$

$$= \frac{\ln(x)}{b\,c^3+a} + \left(\sum_{k=1}^{3} \ln\Big(\operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^2\,b^4\,c^4\,d^8\,3 - \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)\,b^3\,c\,d^8\,3 - \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)\,b^3\,d^9\,x\,4 - \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^2\,a\,b^3\,c\,d^8\,6 - \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^2\,a\,b^3\,d^9\,x\,24 + \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^3\,a^2\,b^3\,c\,d^8\,9 + \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^3\,a\,b^4\,c^4\,d^8\,9 - \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^3\,a^2\,b^3\,d^9\,x\,36 + \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^2\,b^4\,c^3\,d^9\,x\,3 + \operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)^3\,a\,b^4\,c^3\,d^9\,x\,18\Big)\,\operatorname{root}\big(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\big)\Big)$$

input

```
int(1/(x*(a + b*(c + d*x)^3)),x)
```

output

```
log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*
a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^
3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3
 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c
^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*roo
t(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^
9*x + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)
^3*a^2*b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a
*z + 1, z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27
*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27
*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*
root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1,
3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1347, normalized size of antiderivative = 6.01

$$\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input

```
int(1/x/(a+b*(d*x+c)^3),x)
```

output

```
(16*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a**2*b*c**2 + 140*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1
/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**5 + 16*a**(2/3)*sqrt
(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b*
*3*c**8 + 56*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b
**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**2*b*c**3 + 112*b**(1/3)*a**(1/3)*sqrt(
3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b
**2*c**6 + 2*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b
**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**3*c**9 + 2*b**(2/3)*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**3*c + 112*b**
(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sq
rt(3)))*a**2*b*c**4 + 56*b**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c -
2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**7 - 48*a**(2/3)*log(a**(2/3)
 - b**(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3
)*c*d*x + b**(2/3)*d**2*x**2)*a**2*b*c**2 - 42*a**(2/3)*log(a**(2/3) - b**
(1/3)*a**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*
x + b**(2/3)*d**2*x**2)*a*b**2*c**5 + 6*a**(2/3)*log(a**(2/3) - b**(1/3)*a
**(1/3)*c - b**(1/3)*a**(1/3)*d*x + b**(2/3)*c**2 + 2*b**(2/3)*c*d*x + b**
(2/3)*d**2*x**2)*b**3*c**8 - 30*a**(2/3)*log(a**(1/3) + b**(1/3)*c + b**(1
/3)*d*x)*a**2*b*c**2 - 42*a**(2/3)*log(a**(1/3) + b**(1/3)*c + b**(1/3)...
```

3.121 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$

Optimal result . . . 1011
Mathematica [C] (verified) . . . 1012
Rubi [A] (verified) . . . 1012
Maple [C] (verified) . . . 1014
Fricas [C] (verification not implemented) . . . 1014
Sympy [F(-1)] . . . 1015
Maxima [F] . . . 1015
Giac [F] . . . 1015
Mupad [B] (verification not implemented) . . . 1016
Reduce [B] (verification not implemented) . . . 1016

Optimal result

Integrand size = 17, antiderivative size = 312

$$\begin{aligned}
&\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx \\
&= -\frac{1}{(a+bc^3)\,x} + \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2} \\
&\quad - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} + \frac{\sqrt[3]{b}\left(\sqrt[3]{a}(a-2bc^3)-\sqrt[3]{b}c(2a-bc^3)\right) d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2} \\
&\quad + \frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2} \\
&\quad + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}
\end{aligned}$$

output

```
-1/(b*c^3+a)/x+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3)*c)^3*d*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/(b*c^
3+a)^2-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/
3)*c*(-b*c^3+2*a))*d*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^2+1/6*b
^(2/3)*(2*a*c-b*c^4-a^(1/3)*(-2*b*c^3+a)/b^(1/3))*d*ln(a^(2/3)-a^(1/3)*b^(
1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*ln(a+b*(d*x+c)
^3)/(b*c^3+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$$

$$= \frac{-3(a+bc^3+3bc^2dx\log(x))+dx\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\ \frac{-3ac\log(x-\#1)+}{}\right.}{3\left(a+bc^3\right)^2x}$$

input
```
Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

output
```
(-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#
1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x -
#1] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x -
#1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$$

$$\downarrow 896$$

$$d\int \frac{1}{d^2x^2\left(b(c+dx)^3+a\right)}\,d(c+dx)$$

$$\downarrow 7276$$

$$d\int\left(-\frac{3bc^2}{\left(bc^3+a\right)^2dx}+\frac{b\left(3bc^2(c+dx)^2-\left(a-2bc^3\right)(c+dx)-c\left(2a-bc^3\right)\right)}{\left(bc^3+a\right)^2\left(b(c+dx)^3+a\right)}+\frac{1}{\left(bc^3+a\right)d^2x^2}\right)d(c+dx)$$

$$\downarrow 2009$$

$$d\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^2}+\frac{b^{2/3}\left(-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.\right.$$

input
```
Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

output
```
d*(-(1/((a + b*c^3)*d*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1
/3)*c)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[
3]*a^(2/3)*(a + b*c^3)^2) - (3*b*c^2*Log[-(d*x)])/(a + b*c^3)^2 + (b^(1/3)
*(a^(1/3)*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*Log[a^(1/3) + b^(1/3)*(
c + d*x)])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*
(a - 2*b*c^3))/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*
(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*Log[a + b*(c + d*x)^3])/(
a + b*c^3)^2)
```

Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.46

method	result
default	$-\frac{1}{(c^3b+a)x}-\frac{3bc^2d\ln(x)}{(c^3b+a)^2}-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Zb\,c^2d+c^3b+a\right)}\frac{\left(-3\_R^2{}_{b\,c^2d^2}-8\_R_{b\,c^3d}-6b\,c^4+\_R_{ad}\right.}{d^2\_R^2{}_{+2cd}\_R_{+c^2}}\right.}{3(c^3b+a)^2}$
risch	$-\frac{1}{(c^3b+a)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b^2c^6+2a^3c^3b+a^4\right)\_Z^3-9a^2b\,c^2d\_Z^2+6abc\,d^2\_Z-b\,d^3\right)}\_R\ln\left(\left((2a\,b^3c^9d-6a^3b\,c^3d-4a^4d\right)\_R\right.\right.}{}$

input
```
int(1/x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2-1/3*d*sum((-3*_R^2*b*c^2*d^2-8*
_R*b*c^3*d-6*b*c^4+_R*a*d+3*a*c)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootO
f(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 8919, normalized size of antiderivative = 28.59

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d
^2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)
*x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1588, normalized size of antiderivative = 5.09

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

output
```
symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27
*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^
9 - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c
^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2
*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d
^2*z - b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c
^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2
*a^2*b^4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*
z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8
 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c
^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b
*c*d^2*z - b*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^
3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z,
 k)*b^5*c^4*d^11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8
+ 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^
2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^
2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*
d^2*z - b*d^3, z, k)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3657, normalized size of antiderivative = 11.72

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(a+b*(d*x+c)^3),x)
```

output

```
(234*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**
(1/3)*sqrt(3)))*a**6*b*c**2*d*x + 5472*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**5*b**2*c**5*d*x - 116
28*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*a**4*b**3*c**8*d*x + 11628*a**(2/3)*sqrt(3)*atan((a**(1/3) -
 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**2*b**5*c**14*d*x -
5472*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**
(1/3)*sqrt(3)))*a*b**6*c**17*d*x - 234*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**7*c**20*d*x + 2*b**(1
/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a**7*d*x + 1024*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) -
2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**6*b*c**3*d*x + 5814*
b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/
(a**(1/3)*sqrt(3)))*a**5*b**2*c**6*d*x - 20672*b**(1/3)*a**(1/3)*sqrt(3)*a
tan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**4*b*
*3*c**9*d*x + 16796*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**3*b**4*c**12*d*x - 2926*b**(1/3
)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/
3)*sqrt(3)))*a*b**6*c**18*d*x - 32*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3
) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*b**7*c**21*d*x +...
```

3.122 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

Optimal result . . . 1018
Mathematica [C] (verified) . . . 1019
Rubi [A] (verified) . . . 1020
Maple [C] (verified) . . . 1022
Fricas [C] (verification not implemented) . . . 1022
Sympy [F(-1)] . . . 1023
Maxima [F] . . . 1023
Giac [F] . . . 1023
Mupad [B] (verification not implemented) . . . 1024
Reduce [B] (verification not implemented) . . . 1024

Optimal result

Integrand size = 17, antiderivative size = 388

$$\begin{aligned}\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx &= -\frac{1}{2\left(a+bc^3\right)x^2}+\frac{3bc^2d}{\left(a+bc^3\right)^2x}\\ &+\frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}bc\right)^3\left(a-3a^{2/3}\sqrt[3]{b}bc+bc^3\right)d^2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}\\ &-\frac{3bc(a-2bc^3)\,d^2\log(x)}{\left(a+bc^3\right)^3}\\ &-\frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7abc^3-3\sqrt[3]{a}b^{5/3}c^5+b^2c^6\right)d^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}\\ &+\frac{b^{2/3}\left(a^2-7abc^3+b^2c^6+3\sqrt[3]{a}b^{2/3}c^2(2a-bc^3)\right)d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}\\ &+\frac{bc(a-2bc^3)\,d^2\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}\end{aligned}$$

output

```
-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+1/3*b^(2/3)*(a^(1/3)+b^(1/3)*c)
^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c)
)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/(b*c^3+a)^3-3*b*c*(-2*b*c^3+a)*d^2*ln(x
)/(b*c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b
^(5/3)*c^5+b^2*c^6)*d^2*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/
6*b^(2/3)*(a^2-7*a*b*c^3+b^2*c^6+3*a^(1/3)*b^(2/3)*c^2*(-b*c^3+2*a))*d^2*l
n(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^3+b
*c*(-2*b*c^3+a)*d^2*ln(a+b*(d*x+c)^3)/(b*c^3+a)^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx =$$

$$-\frac{3(a+bc^3)\left(a+bc^2(c-6dx)\right)+18bc(a-2bc^3)\,d^2x^2\log(x)+2d^2x^2\text{RootSum}\left[a+bc^3+3bc^2d\#1+3b\right.}{}$$

input

```
Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

output

```
-1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2
*Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*
d^3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x
- #1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*
c*d^2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 +
 d^2*#1^2) & ])/((a + b*c^3)^3*x^2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx$$

$$\downarrow 896$$

$$d^2\int \frac{1}{d^3x^3\left(b(c+dx)^3+a\right)}\,d(c+dx)$$

$$\downarrow 25$$

$$-d^2\int -\frac{1}{d^3x^3\left(b(c+dx)^3+a\right)}\,d(c+dx)$$

$$\downarrow 7276$$

$$-d^2\int \left(\frac{3bc^2}{\left(bc^3+a\right)^2d^2x^2}-\frac{3b\left(2bc^3-a\right)c}{\left(bc^3+a\right)^3dx}+\frac{b\left(b^2c^6-7abc^3-3b\left(2a-bc^3\right)(c+dx)c^2-3b\left(a-2bc^3\right)(c+dx)^2c\right.}{\left(bc^3+a\right)^3\left(b(c+dx)^3+a\right)}\right.$$
$$dx)$$

$$\downarrow 2009$$

$$d^2\left(\frac{b^{2/3}\left(-3a^{2/3}\sqrt[3]{b}c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}\left(a^2+3\sqrt[3]{a}b^{2/3}c^2\left(2a-bc^3\right)-7c\right.}{3a^{2/3}(a\right.}\right.$$

input
```
Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

output
```
d^2*(-1/2*1/((a + b*c^3)*d^2*x^2) + (3*b*c^2)/((a + b*c^3)^2*d*x) + (b^(2/
3)*(a^(1/3) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)
^3) - (3*b*c*(a - 2*b*c^3)*Log[-(d*x)])/(a + b*c^3)^3 - (b^(2/3)*(a^2 - 7*
a*b*c^3 + b^2*c^6 + 3*a^(1/3)*b^(2/3)*c^2*(2*a - b*c^3))*Log[a^(1/3) + b^(
1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 - 7*a*b*c^3 + b
^2*c^6 + 3*a^(1/3)*b^(2/3)*c^2*(2*a - b*c^3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3) + (b*c*(a -
2*b*c^3)*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.52

method	result
default	$-\frac{1}{2(c^3b+a)x^2}+\frac{3bc^2d}{(c^3b+a)^2x}-\frac{3bc(-2c^3b+a)d^2\ln(x)}{(c^3b+a)^3}-\frac{d^2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^3\_Z^3+3bc\,d^2\_Z^2+3\_Zb\,c^2d+c^3b+a\right)}\frac{\left(6\_R^2{}_{b^2c^4}\right.}{}\right.$
risch	$\frac{\frac{3b\,c^2dx}{b^2c^6+2ab\,c^3+a^2}-\frac{1}{2\left(c^3b+a\right)}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b^3c^9+3a^3b^2c^6+3a^4b\,c^3+a^5\right)\_Z^3+\left(18a^2b^2c^4d^2-9a^3bc\,d^2\right)\_Z^2+9a\,b^2c^2d^4\_Z+b^2d^6\right)}\right.}{}$

input
```
int(1/x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b
*c^3+a)^3-1/3*d^2*sum((6*_R^2*b^2*c^4*d^2+15*_R*b^2*c^5*d+10*b^2*c^6-3*_R^
2*a*b*c*d^2-12*_R*a*b*c^2*d-16*a*b*c^3+a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_
R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 14765, normalized size of antiderivative = 38.05

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+b(c+dx)^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2
+ 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*
b^2*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) +
1/2*(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+b(c+dx)^3\right)}\,dx = \int \frac{1}{\left((dx+c)^3 b+a\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.42

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input

```
int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

output

```
symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9
*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^
4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^
9*d^12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b
*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3
*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^
2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z
^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z
^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6
*b^3*c*d^8 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 +
90*a^3*b^6*c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 +
 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 1
26*a^5*b^4*c^3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7
*c^12*d^9))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6))
+ (3*b^8*c^14*d^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5
*c^5*d^10 - 63*a^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c
^9 + 6*a^2*b^2*c^6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c
^10*d^11 + 39*a^3*b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 +
4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6603, normalized size of antiderivative = 17.02

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = \text{Too large to display}$$

input

```
int(1/x^3/(a+b*(d*x+c)^3),x)
```

output

```
(30*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(
1/3)*sqrt(3)))*a**10*b*c*d**2*x**2 - 1464*a**(2/3)*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**9*b**2*c**4*d**2*x
**2 - 3186*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x
)/(a**(1/3)*sqrt(3)))*a**8*b**3*c**7*d**2*x**2 + 967014*a**(2/3)*sqrt(3)*a
tan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**7*b*
*4*c**10*d**2*x**2 - 10710594*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**6*b**5*c**13*d**2*x**2 - 96043
86*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1
/3)*sqrt(3)))*a**5*b**6*c**16*d**2*x**2 + 10111536*a**(2/3)*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**4*b**7*c*
*19*d**2*x**2 - 1788066*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2
*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**3*b**8*c**22*d**2*x**2 - 207792*a**(
2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqr
t(3)))*a**2*b**9*c**25*d**2*x**2 + 51492*a**(2/3)*sqrt(3)*atan((a**(1/3) -
 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a*b**10*c**28*d**2*x**
2 + 534*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(
a**(1/3)*sqrt(3)))*b**11*c**31*d**2*x**2 + 174*b**(1/3)*a**(1/3)*sqrt(3)*a
tan((a**(1/3) - 2*b**(1/3)*c - 2*b**(1/3)*d*x)/(a**(1/3)*sqrt(3)))*a**10*b
*c**2*d**2*x**2 - 11022*b**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b*...
```

3.123 $\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1026
Mathematica [C] (warning: unable to verify) . . . 1027
Rubi [A] (verified) . . . 1028
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Fricas [F] . . . 1031
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Maxima [F] . . . 1032
Giac [F] . . . 1032
Mupad [F(-1)] . . . 1033
Reduce [F] . . . 1033

Optimal result

Integrand size = 19, antiderivative size = 361

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$= \frac{\sqrt{a+b(c+dx)^4}}{2bd^4} - \frac{3c(c+dx)\sqrt{a+b(c+dx)^4}}{\sqrt{b}d^4\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)} + \frac{3c^2\operatorname{arctanh}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}d^4}$$

$$+ \frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}d^4\sqrt{a+b(c+dx)^4}}$$

$$- \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}d^4\sqrt{a+b(c+dx)^4}}$$

output

```
1/2*(a+b*(d*x+c)^4)^(1/2)/b/d^4-3*c*(d*x+c)*(a+b*(d*x+c)^4)^(1/2)/b^(1/2)/
d^4/(a^(1/2)+b^(1/2)*(d*x+c)^2)+3/2*c^2*arctanh(b^(1/2)*(d*x+c)^2/(a+b*(d*
x+c)^4)^(1/2))/b^(1/2)/d^4+3*a^(1/4)*c*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(
d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1
/4)*(d*x+c)/a^(1/4))),1/2*2^(1/2))/b^(3/4)/d^4/(a+b*(d*x+c)^4)^(1/2)-1/2*c
*(3*a^(1/2)+b^(1/2)*c^2)*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(
1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(d*x+c)/
a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(3/4)/d^4/(a+b*(d*x+c)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.65 (sec) , antiderivative size = 1258, normalized size of antiderivative = 3.48

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \text{Too large to display}$$

input

```
Integrate[x^3/Sqrt[a + b*(c + d*x)^4],x]
```

output

```
(a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^4*x^4 + (
-6*(-1)^(3/4)*a*b^(1/4)*c - (6*I)*a^(3/4)*Sqrt[b]*c*(c + d*x) - 6*(-1)^(1/
4)*Sqrt[a]*b^(3/4)*c*(c + d*x)^2 - 6*a^(1/4)*b*c*(c + d*x)^3 - (1 + I)*Sqr
t[2]*b^(3/4)*c*Sqrt[((1 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-
1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[-(((-1)^(3/4)*a^(1/4) + I*b^(1
/4)*(c + d*x))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x)))]*Sqrt[((-1 - I)*(
(-1)^(3/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c
+ d*x))]*((-1)^(1/4)*a^(1/4)*c - b^(1/4)*c*(c + d*x))^2*EllipticF[ArcSin[S
qrt[-(((-1)^(3/4)*a^(1/4) + I*b^(1/4)*(c + d*x))/((-1)^(1/4)*a^(1/4) - b^(
1/4)*(c + d*x)))]], -1] - 6*(-1)^(1/4)*Sqrt[b]*c^2*((-1)^(1/4)*a^(1/4) - b
^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x))
)/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4)
 + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-1
 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1
/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((
-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4
)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt[((-I
)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*
(c + d*x))]], -1]) + (3*I)*b^(1/4)*c*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*
x))^2*Sqrt[((1 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 896$$

$$\frac{\int \frac{d^3x^3}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{d^4}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{d^3x^3}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{d^4}$$

$$\downarrow 2424$$

$$-\frac{\int \left(\frac{(c+dx)(-3c^2-(c+dx)^2)}{\sqrt{b(c+dx)^4+a}}+\frac{c^3+3(c+dx)^2c}{\sqrt{b(c+dx)^4+a}}\right)d(c+dx)}{d^4}$$

$$\downarrow 2009$$

$$-\frac{\sqrt[4]{a}c\left(\frac{\sqrt{b}c^2}{\sqrt{a}}+3\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+b(c+dx)^4}}+\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}}{b^{3/4}\sqrt{a+b(c}}$$

$$d^4$$

input
```
Int[x^3/Sqrt[a + b*(c + d*x)^4],x]
```

output
```
(Sqrt[a + b*(c + d*x)^4]/(2*b) - (3*c*(c + d*x)*Sqrt[a + b*(c + d*x)^4])/(
Sqrt[b]*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)) + (3*c^2*ArcTanh[(Sqrt[b]*(c + d*
x)^2)/Sqrt[a + b*(c + d*x)^4]])/(2*Sqrt[b]) + (3*a^(1/4)*c*(Sqrt[a] + Sqrt
[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b]*(c + d*x)^2)^
2]*EllipticE[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a
+ b*(c + d*x)^4]) - (a^(1/4)*c*(3 + (Sqrt[b]*c^2)/Sqrt[a])*(Sqrt[a] + Sqrt
[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b]*(c + d*x)^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[
a + b*(c + d*x)^4]))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 18.93 (sec) , antiderivative size = 4787, normalized size of antiderivative = 13.26

method	result	size
risch	Expression too large to display	4787
default	Expression too large to display	4788
elliptic	Expression too large to display	4788

input
```
int(x^3/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/b/d^4*(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1
/2)-c/d^3*(2*c^2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((
-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-
c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3
)^(1/4)-c)/d))^(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-
c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(
1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c
)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-
a*b^3)^(1/4)-c)/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*EllipticF(((-I/b*(-a*b^3)^(1/4)-
c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3
)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)
,(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1
/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3
)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
+3*d^2*((x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/
b*(-a*b^3)^(1/4)-c)/d)+(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-...
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^3}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input

```
integrate(x^3/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output

```
integral(x^3/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*
x + b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^3}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(x**3/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(x**3/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*
d**3*x**3 + b*d**4*x**4), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^3}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^3}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt((d*x + c)^4*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^3}{\sqrt{a+b\,(c+d\,x)^4}}\,dx$$

input
```
int(x^3/(a + b*(c + d*x)^4)^(1/2),x)
```

output
```
int(x^3/(a + b*(c + d*x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+b(c+dx)^4}}\,dx$$
$$= \frac{3\sqrt{b}\log\left(-\sqrt{b}\sqrt{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}-b\,c^2-2bcdx-b\,d^2x^2\right)c^2-6\left(\int \frac{\sqrt{b}}{b\,a}\right.$$

input
```
int(x^3/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
(3*sqrt(b)*log( - sqrt(b)*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x
**2 + 4*b*c*d**3*x**3 + b*d**4*x**4) - b*c**2 - 2*b*c*d*x - b*d**2*x**2)*c
**2 - 6*int(sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d*
*3*x**3 + b*d**4*x**4)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4
*b*c*d**3*x**3 + b*d**4*x**4),x)*b*c**3*d + 2*int((sqrt(a + b*c**4 + 4*b*c
**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4)*x**3)/(a + b
*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4)
,x)*b*d**4 - 6*int((sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 +
4*b*c*d**3*x**3 + b*d**4*x**4)*x)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d*
*2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4),x)*b*c**2*d**2)/(2*b*d**4)
```

3.124 $\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1034
Mathematica [C] (warning: unable to verify) . . . 1035
Rubi [A] (verified) . . . 1036
Maple [C] (verified) . . . 1037
Fricas [F] . . . 1038
Sympy [F] . . . 1039
Maxima [F] . . . 1039
Giac [F] . . . 1039
Mupad [F(-1)] . . . 1040
Reduce [F] . . . 1040

Optimal result

Integrand size = 19, antiderivative size = 326

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{(c+dx)\sqrt{a+b(c+dx)^4}}{\sqrt{b}d^3\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{\sqrt{b}d^3}$$
$$- \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}d^3\sqrt{a+b(c+dx)^4}}$$
$$+ \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}d^3\sqrt{a+b(c+dx)^4}}$$

output

```
(d*x+c)*(a+b*(d*x+c)^4)^(1/2)/b^(1/2)/d^3/(a^(1/2)+b^(1/2)*(d*x+c)^2)-c*ar
ctanh(b^(1/2)*(d*x+c)^2/(a+b*(d*x+c)^4)^(1/2))/b^(1/2)/d^3-a^(1/4)*(a^(1/2
)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/2))/b^(3/4)/d^3
/(a+b*(d*x+c)^4)^(1/2)+1/2*(b^(1/2)*c^2+a^(1/2))*(a^(1/2)+b^(1/2)*(d*x+c)^
2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(b^(1/4)*(d*x+c)/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(3/4)/d^3/(a+b*(d*
x+c)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.67 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$= \frac{2\left((-1)^{3/4}a^{3/4}+i\sqrt{a}\sqrt[4]{b}(c+dx)+\sqrt[4]{-1}\sqrt[4]{a}\sqrt{b}(c+dx)^2+b^{3/4}(c+dx)^3\right) - \frac{2i\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2\sqrt{\frac{(1}{-}}}{}$$

input

```
Integrate[x^2/Sqrt[a + b*(c + d*x)^4],x]
```

output

```
(2*((-1)^(3/4)*a^(3/4) + I*Sqrt[a]*b^(1/4)*(c + d*x) + (-1)^(1/4)*a^(1/4)*
Sqrt[b]*(c + d*x)^2 + b^(3/4)*(c + d*x)^3) - ((2*I)*((-1)^(1/4)*a^(1/4) -
b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)
))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-
1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(
1/4)*(c + d*x))]*((-1)^(3/4)*Sqrt[a]*EllipticE[ArcSin[Sqrt[((-I)*((-1)^(1/
4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]
], -1] + (-((-1)^(3/4)*Sqrt[a]) + 2*a^(1/4)*b^(1/4)*c + (-1)^(3/4)*Sqrt[b]
*c^2)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x))
)/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 4*a^(1/4)*b^(1/4)*c*El
lipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/(
(-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/a^(1/4))/(2*b^(3/4)*d^3*S
qrt[a + b*(c + d*x)^4])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {896, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 896$$

$$\frac{\int \frac{d^2x^2}{\sqrt{b(c+dx)^4+a}}\,d(c+dx)}{d^3}$$

$$\downarrow 2424$$

$$\frac{\int \left(\frac{c^2+(c+dx)^2}{\sqrt{b(c+dx)^4+a}} - \frac{2c(c+dx)}{\sqrt{b(c+dx)^4+a}}\right) d(c+dx)}{d^3}$$

$$\downarrow 2009$$

$$\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+b(c+dx)^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}E\left(\right.}{b^{3/4}\sqrt{a+b(c+dx)^4}}$$

$$d^3$$

input

```
Int[x^2/Sqrt[a + b*(c + d*x)^4],x]
```

output

```
(((c + d*x)*Sqrt[a + b*(c + d*x)^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*(c + d*x)
^2)) - (c*ArcTanh[(Sqrt[b]*(c + d*x)^2)/Sqrt[a + b*(c + d*x)^4]])/Sqrt[b]
- (a^(1/4)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[
a] + Sqrt[b]*(c + d*x)^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(b^(3/4)*Sqrt[a + b*(c + d*x)^4]) + ((Sqrt[a] + Sqrt[b]*c^2)*(Sq
rt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b]*(
c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2*a
^(1/4)*b^(3/4)*Sqrt[a + b*(c + d*x)^4]))/d^3
```

Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.14 (sec) , antiderivative size = 2153, normalized size of antiderivative = 6.60

method	result	size
default	Expression too large to display	2153
elliptic	Expression too large to display	2153

input
```
int(x^2/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
((x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b
^3)^(1/4)-c)/d)+(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-
I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c
)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)
^(1/4)-c)/d))^(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c
)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3
)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)
*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1
/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a
*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4)-c)/d^2*(1/b*(-a*b^3)^(1/4)-
c)-(-I/b*(-a*b^3)^(1/4)-c)/d^2*(1/b*(-a*b^3)^(1/4)-c)+(-I/b*(-a*b^3)^(1/4)
-c)/d^2*(I/b*(-a*b^3)^(1/4)-c)+(I/b*(-a*b^3)^(1/4)-c)^2/d^2)/((-I/b*(-a*b^
3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a
*b^3)^(1/4)-c)/d)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4
)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-
c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^
(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a
*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+((1/b*(-a*b^3)^(1/4)-c
)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*EllipticE((((-I/b*(-a*b^3)^(1/4)-c)/d-(I...
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^2}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input

```
integrate(x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output

```
integral(x^2/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*
x + b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^2}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(x**2/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*
d**3*x**3 + b*d**4*x**4), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^2}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^2}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((d*x + c)^4*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x^2}{\sqrt{a+b\,(c+d\,x)^4}}\,dx$$

input
```
int(x^2/(a + b*(c + d*x)^4)^(1/2),x)
```

output
```
int(x^2/(a + b*(c + d*x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+b(c+dx)^4}}\,dx$$
$$= \frac{\sqrt{b}\log\left(-\sqrt{b}\sqrt{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}-b\,c^2-2bcdx-b\,d^2x^2\right)c+2\sqrt{b}\log\left($$

input
```
int(x^2/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
(sqrt(b)*log( - sqrt(b)*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**
2 + 4*b*c*d**3*x**3 + b*d**4*x**4) - b*c**2 - 2*b*c*d*x - b*d**2*x**2)*c +
 2*sqrt(b)*log(sqrt(b)*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2
 + 4*b*c*d**3*x**3 + b*d**4*x**4) - b*c**2 - 2*b*c*d*x - b*d**2*x**2)*c +
2*int(sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**
3 + b*d**4*x**4)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d
**3*x**3 + b*d**4*x**4),x)*b*c**2*d + int((sqrt(a + b*c**4 + 4*b*c**3*d*x
+ 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4)*x**2)/(a + b*c**4 +
4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4),x)*b*d*
*3 + 2*int((sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d*
*3*x**3 + b*d**4*x**4)*x)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2
+ 4*b*c*d**3*x**3 + b*d**4*x**4),x)*b*c*d**2)/(b*d**3)
```

3.125 $\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1041
Mathematica [C] (warning: unable to verify) . . . 1042
Rubi [A] (verified) . . . 1042
Maple [C] (verified) . . . 1044
Fricas [F] . . . 1045
Sympy [F] . . . 1046
Maxima [F] . . . 1046
Giac [F] . . . 1046
Mupad [F(-1)] . . . 1047
Reduce [F] . . . 1047

Optimal result

Integrand size = 17, antiderivative size = 154

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+b(c+dx)^4}}$$

output

```
1/2*arctanh(b^(1/2)*(d*x+c)^2/(a+b*(d*x+c)^4)^(1/2))/b^(1/2)/d^2-1/2*c*(a^
(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)),1/2*2^(1/2))/a^(1/4
)/b^(1/4)/d^2/(a+b*(d*x+c)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.53

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$= \frac{\sqrt[4]{-1}\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2\sqrt{\frac{(1-i)\left((-1)^{3/4}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{(1+i)\left((-1)^{3/4}\right.}{\sqrt[4]{-1}\sqrt[4]{}}}$$

input
```
Integrate[x/Sqrt[a + b*(c + d*x)^4],x]
```

output
```
((-1)^(1/4)*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)
^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*
x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))]*Sqrt[((-1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c
+ d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4) -
b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c +
d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1
/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*
x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(a^(1/4)*Sqrt[b]*d^
2*Sqrt[a + b*(c + d*x)^4])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 896$$

$$\frac{\int \frac{dx}{\sqrt{b(c+dx)^4+a}} d(c+dx)}{d^2}$$

$$\downarrow 25$$

$$-\frac{\int -\frac{dx}{\sqrt{b(c+dx)^4+a}} d(c+dx)}{d^2}$$

$$\downarrow 2424$$

$$-\frac{\int \left(\frac{c}{\sqrt{b(c+dx)^4+a}} - \frac{c+dx}{\sqrt{b(c+dx)^4+a}}\right) d(c+dx)}{d^2}$$

$$\downarrow 2009$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}} - \frac{c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b(c+dx)^4}}}{d^2}$$

input
```
Int[x/Sqrt[a + b*(c + d*x)^4],x]
```

output
```
(ArcTanh[(Sqrt[b]*(c + d*x)^2)/Sqrt[a + b*(c + d*x)^4]]/(2*Sqrt[b]) - (c*(
Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b]
*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2
*a^(1/4)*b^(1/4)*Sqrt[a + b*(c + d*x)^4]))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 1528, normalized size of antiderivative = 9.92

method	result	size
default	Expression too large to display	1528
elliptic	Expression too large to display	1528

input
```
int(x/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(
1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-
a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(
1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*
b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-
I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c
)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*
(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-
1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2))+(-(I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*EllipticP
i((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(
1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b...
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input

```
integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output

```
integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**
3*x**3 + b*d**4*x**4), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x}{\sqrt{(dx+c)^4b+a}}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt((d*x + c)^4*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{x}{\sqrt{a+b\,(c+d\,x)^4}}\,dx$$

input
```
int(x/(a + b*(c + d*x)^4)^(1/2),x)
```

output
```
int(x/(a + b*(c + d*x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+b(c+dx)^4}}\,dx$$
$$= \frac{\sqrt{b}\log\left(-\sqrt{b}\,\sqrt{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}-b\,c^2-2bcdx-b\,d^2x^2\right)-2\left(\int \frac{\sqrt{b\,d^4x^4}}{b\,d^4x^4+}\right.}{2b\,d^2}$$

input
```
int(x/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
(sqrt(b)*log( - sqrt(b)*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**
2 + 4*b*c*d**3*x**3 + b*d**4*x**4) - b*c**2 - 2*b*c*d*x - b*d**2*x**2) - 2
*int(sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3
 + b*d**4*x**4)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d*
*3*x**3 + b*d**4*x**4),x)*b*c*d)/(2*b*d**2)
```

3.126 $\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1048
Mathematica [C] (verified) . . . 1048
Rubi [A] (verified) . . . 1049
Maple [C] (verified) . . . 1050
Fricas [F] . . . 1051
Sympy [F] . . . 1052
Maxima [F] . . . 1052
Giac [F] . . . 1052
Mupad [B] (verification not implemented) . . . 1053
Reduce [F] . . . 1053

Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+b(c+dx)^4}}$$

output

```
1/2*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^
2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)),1/2*2^(1/2))
/a^(1/4)/b^(1/4)/d/(a+b*(d*x+c)^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = -\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right),-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{a+b(c+dx)^4}}$$

input
```
Integrate[1/Sqrt[a + b*(c + d*x)^4],x]
```

output
```
((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqr
t[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4
])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {239, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 239$$

$$\frac{\int \frac{1}{\sqrt{b(c+dx)^4+a}}\,d(c+dx)}{d}$$

$$\downarrow 761$$

$$\frac{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+b(c+dx)^4}}$$

input
```
Int[1/Sqrt[a + b*(c + d*x)^4],x]
```

output
```
((Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[
b]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/
(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*(c + d*x)^4])
```

Defintions of rubi rules used

rule 239

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 1036, normalized size of antiderivative = 9.33

method	result	size
default	Expression too large to display	1036
elliptic	Expression too large to display	1036

input

```
int(1/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(
1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-
a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(
1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b
^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1
/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3
)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(
-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/(
(I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{a+b\,(c+dx)^4}}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*(c + d*x)**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+a}}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \frac{\sqrt{\frac{b(c+dx)^4}{a}+1}\,(c+dx)\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;-\frac{b(c+dx)^4}{a}\right)}{d\sqrt{a+b(c+dx)^4}}$$

input
```
int(1/(a + b*(c + d*x)^4)^(1/2),x)
```

output
```
(((b*(c + d*x)^4)/a + 1)^(1/2)*(c + d*x)*hypergeom([1/4, 1/2], 5/4, -(b*(c
 + d*x)^4)/a))/(d*(a + b*(c + d*x)^4)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{\sqrt{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx$$

input
```
int(1/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
int(sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3
 + b*d**4*x**4)/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**
3*x**3 + b*d**4*x**4),x)
```

3.127 $\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1054
Mathematica [C] (warning: unable to verify) . . . 1055
Rubi [A] (verified) . . . 1055
Maple [C] (verified) . . . 1060
Fricas [F] . . . 1061
Sympy [F] . . . 1061
Maxima [F] . . . 1061
Giac [F] . . . 1062
Mupad [F(-1)] . . . 1062
Reduce [F] . . . 1062

Optimal result

Integrand size = 19, antiderivative size = 413

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bc^4}(c+dx)}{c\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{a+bc^4}} - \frac{\operatorname{arctanh}\left(\frac{a+bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{a+bc^4}}$$
$$- \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}$$
$$- \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)^2}{4\sqrt{a}\sqrt{b}c^2}, 2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right), \frac{1}{2}\right.}{4\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}$$

output

```
-1/2*arctanh((b*c^4+a)^(1/2)*(d*x+c)/c/(a+b*(d*x+c)^4)^(1/2))/(b*c^4+a)^(1
/2)-1/2*arctanh((a+b*c^2*(d*x+c)^2)/(b*c^4+a)^(1/2)/(a+b*(d*x+c)^4)^(1/2))
/(b*c^4+a)^(1/2)-1/2*b^(1/4)*c*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4
)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(d
*x+c)/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(b^(1/2)*c^2+a^(1/2))/(a+b*(d*x+c)^4)^
(1/2)-1/4*(a^(1/2)-b^(1/2)*c^2)*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^
4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*(d
*x+c)/a^(1/4))),1/4*(b^(1/2)*c^2+a^(1/2))^2/a^(1/2)/b^(1/2)/c^2,1/2*2^(1/2
))/a^(1/4)/b^(1/4)/c/(b^(1/2)*c^2+a^(1/2))/(a+b*(d*x+c)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx =$$

$$-\frac{(-1)^{3/4}\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2\sqrt{\frac{(1-i)\left((-1)^{3/4}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{(1+i)\left(\right.}{\sqrt[4]{}}}$$

input

```
Integrate[1/(x*Sqrt[a + b*(c + d*x)^4]),x]
```

output

```
-(((-1)^(3/4)*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-
1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c +
d*x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^
(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(
c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4)
+ b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c
+ d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^
(1/4)*EllipticPi[((-1)^(3/4)*a^(1/4) - I*b^(1/4)*c)/((-1)^(1/4)*a^(1/4) +
b^(1/4)*c), ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((
-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(a^(1/4)*(Sqrt[a] + I*Sqrt
[b]*c^2)*Sqrt[a + b*(c + d*x)^4]))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {896, 25, 2263, 1541, 27, 761, 1577, 488, 219, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 896$$

$$\int \frac{1}{dx\sqrt{a+b(c+dx)^4}}d(c+dx)$$

$$\downarrow 25$$

$$-\int -\frac{1}{dx\sqrt{b(c+dx)^4+a}}d(c+dx)$$

$$\downarrow 2263$$

$$-c\int \frac{1}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)-\int \frac{c+dx}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)$$

$$\downarrow 1541$$

$$-\int \frac{c+dx}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)-c\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\sqrt{a}\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{\sqrt{a}(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}\right)$$

$$\downarrow 27$$

$$-\int \frac{c+dx}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)-c\left(\frac{\sqrt{b}\int \frac{1}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}\right)$$

$$\downarrow 761$$

$$-c\left(\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.\right.}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}$$

$$\int \frac{c+dx}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)$$

$$\downarrow 1577$$

$$-c\left(\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(}{\sqrt[4]{}}\right.\right.\right.}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}$$

$$\frac{1}{2}\int\frac{1}{(c^2-c-dx)\sqrt{b(c+dx)^4+a}}d(c+dx)^2$$

$$\downarrow 488$$

$$\frac{1}{2}\int\frac{1}{bc^4-(c+dx)^4+a}d\frac{-bc^2(c+dx)^2-a}{\sqrt{b(c+dx)^4+a}}-$$

$$c\left(\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+}{\sqrt[4]{}}\right.\right.\right.}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}\right)$$

$$\downarrow 219$$

$$\frac{\text{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{a+bc^4}}-$$

$$c\left(\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+}{\sqrt[4]{}}\right.\right.\right.}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}\right)$$

$$\downarrow 2223$$

$$\frac{\text{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{a+bc^4}}-$$

$$c\left(\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\text{arctanh}\left(\frac{\sqrt{a+bc^4}(c+dx)}{c\sqrt{a+b(c+dx)^4}}\right)}{2c\sqrt{a+bc^4}}-\frac{\left(\sqrt{b}-\frac{\sqrt{a}}{c^2}\right)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}c^2+\sqrt{a}\right)^2}{4\sqrt{a}\sqrt{b}c^2},2\arctan\left(\frac{\sqrt[4]{b}(c+}{\sqrt[4]{}}\right.\right.\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+b(c+dx)^4}}}{\sqrt{a}+\sqrt{b}c^2}\right)$$

input
```
Int[1/(x*Sqrt[a + b*(c + d*x)^4]),x]
```

output
```
ArcTanh[(-a - b*c^2*(c + d*x)^2)/(Sqrt[a + b*c^4]*Sqrt[a + b*(c + d*x)^4])
]/(2*Sqrt[a + b*c^4]) - c*((b^(1/4)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(
a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt[b]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[a] + Sqrt[b]*c^2)*Sqrt
[a + b*(c + d*x)^4]) + (((Sqrt[a] + Sqrt[b]*c^2)*ArcTanh[(Sqrt[a + b*c^4]*
(c + d*x))/(c*Sqrt[a + b*(c + d*x)^4])])/(2*c*Sqrt[a + b*c^4]) - ((Sqrt[b]
 - Sqrt[a]/c^2)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(
Sqrt[a] + Sqrt[b]*(c + d*x)^2)^2]*EllipticPi[(Sqrt[a] + Sqrt[b]*c^2)^2/(4*
Sqrt[a]*Sqrt[b]*c^2), 2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(4*a^(1
/4)*b^(1/4)*Sqrt[a + b*(c + d*x)^4]))/(Sqrt[a] + Sqrt[b]*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 896

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1541

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2263

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[d
  Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Simp[e   Int[x/((d^2 -
e^2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.07 (sec) , antiderivative size = 1582, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	1582
elliptic	Expression too large to display	1582

input
```
int(1/x/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(
1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-
a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(
1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)/(I/b*(-a*b^3)^(1/4)-c)*d*(EllipticF((((-I/b*(-a*
b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-
I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c
)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*
(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-
1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2))+((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(1/b*(-a*b
^3)^(1/4)-c)*d*EllipticPi((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-
c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*...
```

Fricas [F]

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
b*c^4 + a)/(b*d^4*x^5 + 4*b*c*d^3*x^4 + 6*b*c^2*d^2*x^3 + 4*b*c^3*d*x^2 +
(b*c^4 + a)*x), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*
d**3*x**3 + b*d**4*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((d*x + c)^4*b + a)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((d*x + c)^4*b + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x\sqrt{a+b\,(c+d\,x)^4}}\,dx$$

input
```
int(1/(x*(a + b*(c + d*x)^4)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c + d*x)^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x\sqrt{a+b\,(dx+c)^4}}dx$$

input
```
int(1/x/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
int(1/x/(a+b*(d*x+c)^4)^(1/2),x)
```

3.128 $\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx$

Optimal result . . . 1063
Mathematica [C] (warning: unable to verify) . . . 1064
Rubi [A] (verified) . . . 1065
Maple [C] (warning: unable to verify) . . . 1072
Fricas [F(-1)] . . . 1073
Sympy [F] . . . 1073
Maxima [F] . . . 1073
Giac [F] . . . 1074
Mupad [F(-1)] . . . 1074
Reduce [F] . . . 1074

Optimal result

Integrand size = 19, antiderivative size = 629

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = -\frac{\sqrt{a+b(c+dx)^4}}{(a+bc^4)\,x} + \frac{\sqrt{b}d(c+dx)\sqrt{a+b(c+dx)^4}}{(a+bc^4)\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)}$$
$$+ \frac{bc^3d\operatorname{arctanh}\left(\frac{\sqrt{a+bc^4}(c+dx)}{c\sqrt{a+b(c+dx)^4}}\right)}{(a+bc^4)^{3/2}} + \frac{bc^3d\operatorname{arctanh}\left(\frac{a+bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{(a+bc^4)^{3/2}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(a+bc^4)\sqrt{a+b(c+dx)^4}}$$
$$+ \frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)\sqrt{a+b(c+dx)^4}}$$
$$+ \frac{b^{3/4}c^2\left(\sqrt{a}-\sqrt{b}c^2\right)d\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)^2}{4\sqrt{a}\sqrt{b}c^2},2\arctan\left(\frac{\sqrt[4]{b}(c+}{\sqrt[4]{a}}\right.\right.}{2\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}c^2\right)(a+bc^4)\sqrt{a+b(c+dx)^4}}$$

output

```
-(a+b*(d*x+c)^4)^(1/2)/(b*c^4+a)/x+b^(1/2)*d*(d*x+c)*(a+b*(d*x+c)^4)^(1/2)
/(b*c^4+a)/(a^(1/2)+b^(1/2)*(d*x+c)^2)+b*c^3*d*arctanh((b*c^4+a)^(1/2)*(d*
x+c)/c/(a+b*(d*x+c)^4)^(1/2))/(b*c^4+a)^(3/2)+b*c^3*d*arctanh((a+b*c^2*(d*
x+c)^2)/(b*c^4+a)^(1/2)/(a+b*(d*x+c)^4)^(1/2))/(b*c^4+a)^(3/2)-a^(1/4)*b^(
1/4)*d*(a^(1/2)+b^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+
c)^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/2
))/(b*c^4+a)/(a+b*(d*x+c)^4)^(1/2)+1/2*b^(1/4)*d*(a^(1/2)+b^(1/2)*(d*x+c)^
2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(b^(1/4)*(d*x+c)/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(b^(1/2)*c^2+a^(1/2)
)/(a+b*(d*x+c)^4)^(1/2)+1/2*b^(3/4)*c^2*(a^(1/2)-b^(1/2)*c^2)*d*(a^(1/2)+b
^(1/2)*(d*x+c)^2)*((a+b*(d*x+c)^4)/(a^(1/2)+b^(1/2)*(d*x+c)^2)^2)^(1/2)*El
lipticPi(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/4*(b^(1/2)*c^2+a^(1/2))^
2/a^(1/2)/b^(1/2)/c^2,1/2*2^(1/2))/a^(1/4)/(b^(1/2)*c^2+a^(1/2))/(b*c^4+a)
/(a+b*(d*x+c)^4)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.87 (sec) , antiderivative size = 1385, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \text{Too large to display}$$

input

```
Integrate[1/(x^2*Sqrt[a + b*(c + d*x)^4]),x]
```

output

```
((-2*(a + b*(c + d*x)^4))/x + (4*(-1)^(1/4)*Sqrt[b]*c*d*((-1)^(1/4)*a^(1/4
) - b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c +
d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-I)*((-1)^(1/4)*a^
(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt
[((-1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) -
 b^(1/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sq
rt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b
^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt
[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(
1/4)*(c + d*x))]], -1]))/a^(1/4) + b^(1/4)*d*(2*((-1)^(3/4)*a^(3/4) + I*Sq
rt[a]*b^(1/4)*(c + d*x) + (-1)^(1/4)*a^(1/4)*Sqrt[b]*(c + d*x)^2 + b^(3/4)
*(c + d*x)^3) - ((2*I)*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))^2*Sqrt[((1
 - I)*((-1)^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1
/4)*(c + d*x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)
^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*Sqrt[((-1 - I)*((-1)^(3/4)*a^(1/4) +
b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*((-1)^(3/4)*
Sqrt[a]*EllipticE[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x
)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] + (-((-1)^(3/4)*Sqrt[a
]) + 2*a^(1/4)*b^(1/4)*c + (-1)^(3/4)*Sqrt[b]*c^2)*EllipticF[ArcSin[Sqrt[(
(-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^...
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {896, 2265, 25, 2280, 27, 1577, 488, 219, 2233, 25, 27, 1510, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx$$

$$\downarrow 896$$

$$d\int \frac{1}{d^2x^2\sqrt{b(c+dx)^4+a}}d(c+dx)$$

$$\downarrow 2265$$

$$d\left(-\frac{b\int\frac{c^3+(c+dx)c^2+(c+dx)^2c-(c+dx)^3}{dx\sqrt{b(c+dx)^4+a}}d(c+dx)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 25$$

$$d\left(\frac{b\int-\frac{c^3+(c+dx)c^2+(c+dx)^2c-(c+dx)^3}{dx\sqrt{b(c+dx)^4+a}}d(c+dx)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 2280$$

$$d\left(\frac{b\left(\int\frac{c^4+2(c+dx)^2c^2-(c+dx)^4}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)+\int\frac{2c^3(c+dx)}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)\right)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 27$$

$$d\left(\frac{b\left(\int\frac{c^4+2(c+dx)^2c^2-(c+dx)^4}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)+2c^3\int\frac{c+dx}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)\right)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 1577$$

$$d\left(\frac{b\left(\int\frac{c^4+2(c+dx)^2c^2-(c+dx)^4}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)+c^3\int\frac{1}{(c^2-c-dx)\sqrt{b(c+dx)^4+a}}d(c+dx)^2\right)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 488$$

$$d\left(\frac{b\left(\int\frac{c^4+2(c+dx)^2c^2-(c+dx)^4}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)-c^3\int\frac{1}{bc^4-(c+dx)^4+a}d\frac{-bc^2(c+dx)^2-a}{\sqrt{b(c+dx)^4+a}}\right)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 219$$

$$d\left(\frac{b\left(\int\frac{c^4+2(c+dx)^2c^2-(c+dx)^4}{(c^2-(c+dx)^2)\sqrt{b(c+dx)^4+a}}d(c+dx)-\frac{c^3\operatorname{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{\sqrt{a+bc^4}}\right)}{a+bc^4}-\frac{\sqrt{a+b(c+dx)^4}}{dx\,(a+bc^4)}\right)$$

$$\downarrow 2233$$

$$d\left(\frac{b\left(-\frac{\int -\frac{\sqrt{b}\left(c^2\left(\sqrt{b}c^2+\sqrt{a}\right)-\left(\sqrt{a}-\sqrt{b}c^2\right)(c+dx)^2\right)}{\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{b}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{\sqrt{a}\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{b}}-\frac{c^3\text{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{\sqrt{a+bc^4}}\right)}{a+bc^4}\right)$$

$$\downarrow 25$$

$$d\left(\frac{b\left(\frac{\int\frac{\sqrt{b}\left(c^2\left(\sqrt{b}c^2+\sqrt{a}\right)-\left(\sqrt{a}-\sqrt{b}c^2\right)(c+dx)^2\right)}{\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{b}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{\sqrt{a}\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{b}}-\frac{c^3\text{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{\sqrt{a+bc^4}}\right)}{a+bc^4}\right)$$

$$\downarrow 27$$

$$d\left(\frac{b\left(\frac{\int\frac{c^2\left(\sqrt{b}c^2+\sqrt{a}\right)-\left(\sqrt{a}-\sqrt{b}c^2\right)(c+dx)^2}{\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{b}}-\frac{c^3\text{arctanh}\left(\frac{-a-bc^2(c+dx)^2}{\sqrt{a+bc^4}\sqrt{a+b(c+dx)^4}}\right)}{\sqrt{a+bc^4}}\right)}{a+bc^4}\right)$$

$$\downarrow 1510$$

$$d\left(\frac{b\left(\frac{\int \frac{c^2\left(\sqrt{b}c^2+\sqrt{a}\right)-\left(\sqrt{a}-\sqrt{b}c^2\right)(c+dx)^2}{\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{b}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+b(c+dx)^4}}-\frac{(c+dx)\sqrt{a+b(c}}{\sqrt{a}+\sqrt{b}(c+}}{\sqrt{b}}\right)}{a+bc^4}\right)$$

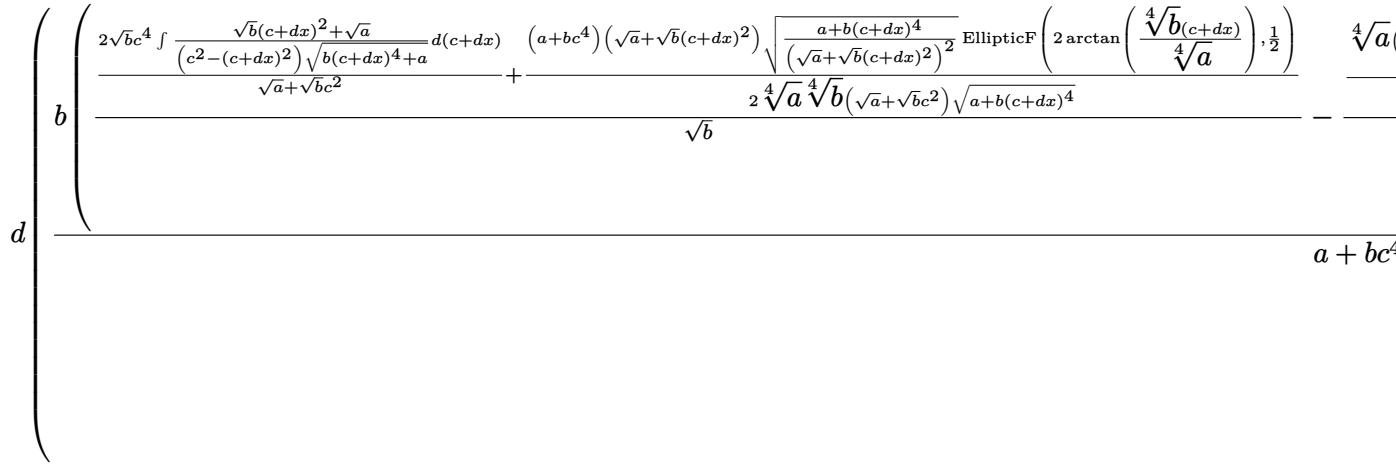
$$\downarrow 2227$$

$$d\left(\frac{b\left(\frac{\frac{2\sqrt{a}\sqrt{b}c^4\int\frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{\sqrt{a}\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\left(a+bc^4\right)\int\frac{1}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}}{\sqrt{b}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}E\left(2\arc\right.}{\sqrt[4]{b}\sqrt{a+b(c+dx)^4}}}{\sqrt{b}}\right)}{a+bc^4}\right)$$

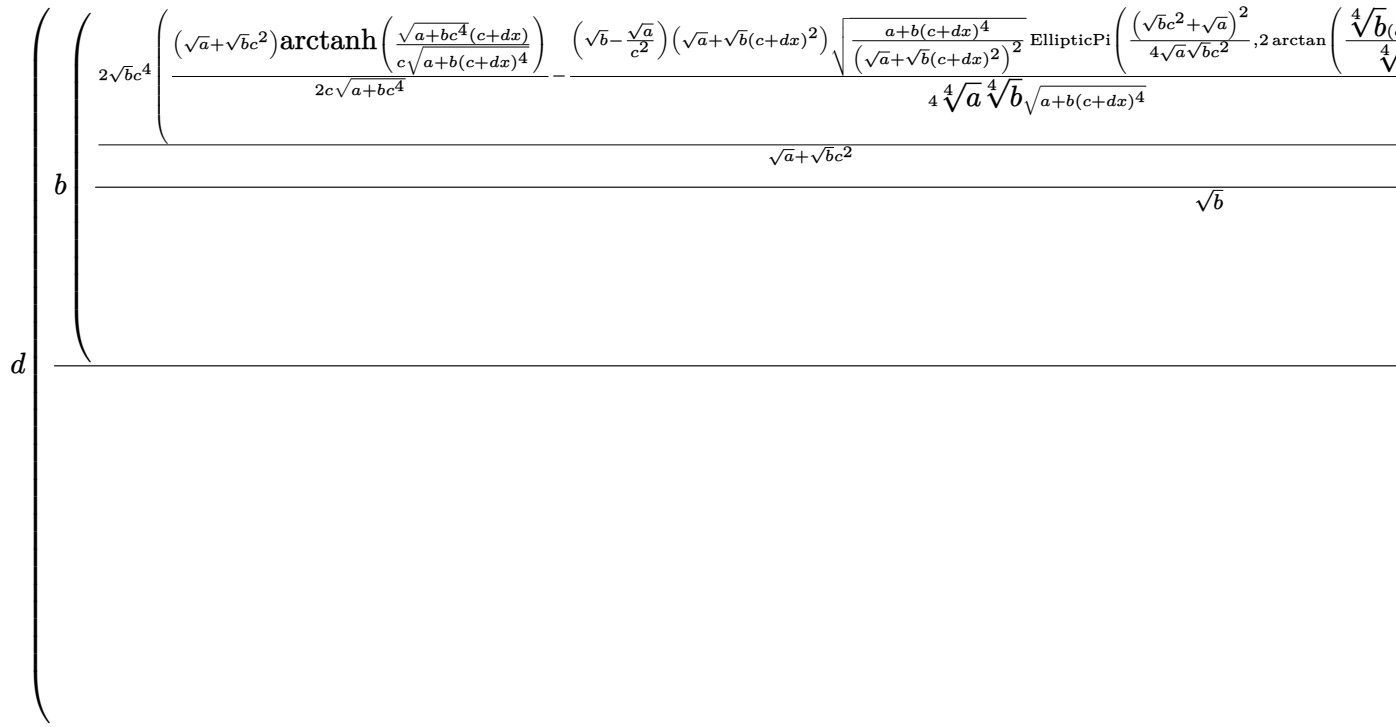
$$\downarrow 27$$

$$d\left(\frac{b\left(\frac{\frac{2\sqrt{b}c^4\int\frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{\left(c^2-(c+dx)^2\right)\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}+\frac{\left(a+bc^4\right)\int\frac{1}{\sqrt{b(c+dx)^4+a}}d(c+dx)}{\sqrt{a}+\sqrt{b}c^2}}{\sqrt{b}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}(c+dx)^2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{b}\sqrt{a+b(c+dx)^4}}}{\sqrt{b}}\right)}{a+bc^4}\right)$$

$$\downarrow 761$$



↓ 2223



input
```
Int[1/(x^2*Sqrt[a + b*(c + d*x)^4]),x]
```

output
```
d*(-(Sqrt[a + b*(c + d*x)^4]/((a + b*c^4)*d*x)) + (b*(-((c^3*ArcTanh[(-a -
 b*c^2*(c + d*x)^2)/(Sqrt[a + b*c^4]*Sqrt[a + b*(c + d*x)^4])])/Sqrt[a + b
*c^4]) - (-(((c + d*x)*Sqrt[a + b*(c + d*x)^4])/(Sqrt[a] + Sqrt[b]*(c + d*
x)^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)
/(Sqrt[a] + Sqrt[b]*(c + d*x)^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*(c + d*x))
/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*(c + d*x)^4]))/Sqrt[b] + (((a + b*c^4
)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + Sqrt
[b]*(c + d*x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])
/(2*a^(1/4)*b^(1/4)*(Sqrt[a] + Sqrt[b]*c^2)*Sqrt[a + b*(c + d*x)^4]) + (2*
Sqrt[b]*c^4*(((Sqrt[a] + Sqrt[b]*c^2)*ArcTanh[(Sqrt[a + b*c^4]*(c + d*x))/
(c*Sqrt[a + b*(c + d*x)^4])])/(2*c*Sqrt[a + b*c^4]) - ((Sqrt[b] - Sqrt[a]/
c^2)*(Sqrt[a] + Sqrt[b]*(c + d*x)^2)*Sqrt[(a + b*(c + d*x)^4)/(Sqrt[a] + S
qrt[b]*(c + d*x)^2)^2]*EllipticPi[(Sqrt[a] + Sqrt[b]*c^2)^2/(4*Sqrt[a]*Sqr
t[b]*c^2), 2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)
*Sqrt[a + b*(c + d*x)^4]))))/(Sqrt[a] + Sqrt[b]*c^2))/Sqrt[b]))/(a + b*c^4)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 896

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1510

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1577

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2265
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp
[c/((q + 1)*(c*d^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp
[d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x
], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.34 (sec) , antiderivative size = 5348, normalized size of antiderivative = 8.50

method	result	size
risch	Expression too large to display	5348
default	Expression too large to display	5365
elliptic	Expression too large to display	5365

input
```
int(1/x^2/(a+b*(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x^2\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**4)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b
*c*d**3*x**3 + b*d**4*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4b+ax^2}}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((d*x + c)^4*b + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{\sqrt{(dx+c)^4 b + ax^2}}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((d*x + c)^4*b + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x^2\sqrt{a+b\,(c+d\,x)^4}}\,dx$$

input
```
int(1/(x^2*(a + b*(c + d*x)^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*(c + d*x)^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+b(c+dx)^4}}\,dx = \int \frac{1}{x^2\sqrt{a+b\,(dx+c)^4}}dx$$

input
```
int(1/x^2/(a+b*(d*x+c)^4)^(1/2),x)
```

output
```
int(1/x^2/(a+b*(d*x+c)^4)^(1/2),x)
```

3.129 $\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx$

Optimal result . . . 1075
Mathematica [A] (verified) . . . 1075
Rubi [A] (warning: unable to verify) . . . 1076
Maple [C] (warning: unable to verify) . . . 1077
Fricas [B] (verification not implemented) . . . 1078
Sympy [F] . . . 1079
Maxima [F] . . . 1080
Giac [B] (verification not implemented) . . . 1080
Mupad [F(-1)] . . . 1081
Reduce [B] (verification not implemented) . . . 1081

Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(e(c+dx))^{3/2}}{e^{3/2}\sqrt{a+b(c+dx)^3}}\right)}{3\sqrt{b}d}$$

output

```
2/3*e^(1/2)*arctanh(b^(1/2)*(e*(d*x+c))^(3/2)/e^(3/2)/(a+b*(d*x+c)^3)^(1/2
))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = \frac{2\sqrt{e(c+dx)}\log\left(\sqrt{b}(c+dx)^{3/2}+\sqrt{a+b(c+dx)^3}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input

```
Integrate[Sqrt[e*(c + d*x)]/Sqrt[a + b*(c + d*x)^3],x]
```

output

```
(2*Sqrt[e*(c + d*x)]*Log[Sqrt[b]*(c + d*x)^(3/2) + Sqrt[a + b*(c + d*x)^3]
])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {895, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx$$

$$\downarrow 895$$

$$\frac{\sqrt{e(c+dx)}\int \frac{\sqrt{c+dx}}{\sqrt{b(c+dx)^3+a}}d(c+dx)}{d\sqrt{c+dx}}$$

$$\downarrow 851$$

$$\frac{2\sqrt{e(c+dx)}\int \frac{c+dx}{\sqrt{b(c+dx)^3+a}}d\sqrt{c+dx}}{d\sqrt{c+dx}}$$

$$\downarrow 807$$

$$\frac{2\sqrt{e(c+dx)}\int \frac{1}{\sqrt{a+b(c+dx)}}d(c+dx)^{3/2}}{3d\sqrt{c+dx}}$$

$$\downarrow 224$$

$$\frac{2\sqrt{e(c+dx)}\int \frac{1}{1-b(c+dx)}d\frac{(c+dx)^{3/2}}{\sqrt{a+b(c+dx)}}}{3d\sqrt{c+dx}}$$

$$\downarrow 219$$

$$\frac{2\sqrt{e(c+dx)}\text{arctanh}\left(\frac{\sqrt{b}(c+dx)^{3/2}}{\sqrt{a+b(c+dx)}}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input
```
Int[Sqrt[e*(c + d*x)]/Sqrt[a + b*(c + d*x)^3],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*ArcTanh[(Sqrt[b]*(c + d*x)^(3/2))/Sqrt[a + b*(c + d*x
)]])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 10.57 (sec) , antiderivative size = 864, normalized size of antiderivative = 15.71

method	result
default	$\frac{2\sqrt{e(xd+c)}\sqrt{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}\,e\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-bdx-bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{}$
elliptic	Expression too large to display

input
```
int((e*(d*x+c))^(1/2)/(a+b*(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*(d*x+c))^(1/2)*(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2)*e*
((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d
*x+2*b*c+(-a*b^2)^(1/3)))^(1/2)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-
a*b^2)^(1/3))^2*((I*3^(1/2)+3)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*d*x-2*b*c-(-a
*b^2)^(1/3))/(I*3^(1/2)-3)/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)+3)*(d*x+c)*b/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x
+2*b*c+(-a*b^2)^(1/3)))^(1/2)/b^2/d*(I*3^(1/2)*EllipticF(((1+I*3^(1/2))*(-
b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)
^(1/3)))^(1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-I*3^(1/2)
*EllipticPi(((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)
/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))+EllipticF(((1+I*3^(1/2))*(-b*d*x-b*c+
(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(
1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-3*EllipticPi(((1+I*
3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b
*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+
I*3^(1/2)))^(1/2)))/((b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*e*(d*x+
c))^(1/2)/(1+I*3^(1/2))/(I*3^(1/2)+3)/(1/b^2*e*(-b*d*x-b*c+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*d*x-2*b*c-(-a*b^2)^(1/3))*(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.

Time = 0.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.89

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx$$

$$= \left[\frac{\sqrt{\frac{e}{b}}\log\left(-8\,b^2d^6ex^6-48\,b^2cd^5ex^5-120\,b^2c^2d^4ex^4-8\,(20\,b^2c^3+ab)d^3ex^3-24\,(5\,b^2c^4+abc)d^2ex^2-\right.\right.$$

$$\left.\left.-\frac{\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}(bdx+bc)\sqrt{dex+ce}\sqrt{-\frac{e}{b}}}{2\,bd^3ex^3+6\,bcd^2ex^2+6\,bc^2dex+(2\,bc^3+a)e}\right)}{3\,d}\right]\right.$$

input
```
integrate((e*(d*x+c))^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(e/b)*log(-8*b^2*d^6*e*x^6 - 48*b^2*c*d^5*e*x^5 - 120*b^2*c^2*d^4
*e*x^4 - 8*(20*b^2*c^3 + a*b)*d^3*e*x^3 - 24*(5*b^2*c^4 + a*b*c)*d^2*e*x^2
 - 24*(2*b^2*c^5 + a*b*c^2)*d*e*x - 4*(2*b^2*d^4*x^4 + 8*b^2*c*d^3*x^3 + 1
2*b^2*c^2*d^2*x^2 + 2*b^2*c^4 + a*b*c + (8*b^2*c^3 + a*b)*d*x)*sqrt(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*sqrt(d*e*x + c*e)*sqrt(e/b)
 - (8*b^2*c^6 + 8*a*b*c^3 + a^2)*e)/d, -1/3*sqrt(-e/b)*arctan(2*sqrt(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(b*d*x + b*c)*sqrt(d*e*x +
 c*e)*sqrt(-e/b)/(2*b*d^3*e*x^3 + 6*b*c*d^2*e*x^2 + 6*b*c^2*d*e*x + (2*b*c
^3 + a)*e))/d]
```

Sympy [F]

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{e\,(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

input
```
integrate((e*(d*x+c))**(1/2)/(a+b*(d*x+c)**3)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))/sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**
2 + b*d**3*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{(dx+c)e}}{\sqrt{(dx+c)^3 b+a}}\,dx$$

input
```
integrate((e*(d*x+c))^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((d*x + c)*e)/sqrt((d*x + c)^3*b + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(41) = 82.

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = -\frac{2\,e^4 \log\left(\left|-2\sqrt{be}\left(3\sqrt{dex+ce}c-\frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right)+2\sqrt{\left(3\sqrt{dex+ce}c-\frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right.}\right.\right.}{3\sqrt{bed}|e|^3}$$

input
```
integrate((e*(d*x+c))^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-2/3*e^4*log(abs(-2*sqrt(b*e)*(3*sqrt(d*e*x + c*e)*c - (3*sqrt(d*e*x + c*e
)*c*e - (d*e*x + c*e)^(3/2))/e) + 2*sqrt((3*sqrt(d*e*x + c*e)*c - (3*sqrt(
d*e*x + c*e)*c*e - (d*e*x + c*e)^(3/2))/e)^2*b*e + a*e^2)))/(sqrt(b*e)*d*a
bs(e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{e\,(c+d\,x)}}{\sqrt{a+b\,(c+d\,x)^3}}\,dx$$

input
```
int((e*(c + d*x))^(1/2)/(a + b*(c + d*x)^3)^(1/2),x)
```

output
```
int((e*(c + d*x))^(1/2)/(a + b*(c + d*x)^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+b(c+dx)^3}}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{b}\left(-\log\left(\sqrt{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}-\sqrt{b}\,\sqrt{dx+c}\,c-\sqrt{b}\,\sqrt{dx+c}\,dx\right)+\log\left(\sqrt{b\,d^3x\right.}\right.}{3bd}$$

input
```
int((e*(d*x+c))^(1/2)/(a+b*(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(b)*( - log(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2
+ b*d**3*x**3) - sqrt(b)*sqrt(c + d*x)*c - sqrt(b)*sqrt(c + d*x)*d*x) + lo
g(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) + sqrt(b
)*sqrt(c + d*x)*c + sqrt(b)*sqrt(c + d*x)*d*x)))/(3*b*d)
```

3.130 $\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx$

Optimal result . . . 1082
Mathematica [A] (verified) . . . 1082
Rubi [A] (warning: unable to verify) . . . 1083
Maple [C] (warning: unable to verify) . . . 1084
Fricas [B] (verification not implemented) . . . 1085
Sympy [F] . . . 1086
Maxima [F] . . . 1087
Giac [B] (verification not implemented) . . . 1087
Mupad [F(-1)] . . . 1088
Reduce [B] (verification not implemented) . . . 1088

Optimal result

Integrand size = 28, antiderivative size = 55

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(e(c+dx))^{3/2}}{e^{3/2}\sqrt{a+b(c+dx)^3}}\right)}{3\sqrt{b}d}$$

output
```
2/3*e^(1/2)*arctanh(b^(1/2)*(e*(d*x+c))^(3/2)/e^(3/2)/(a+b*(d*x+c)^3)^(1/2
))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = \frac{2\sqrt{e(c+dx)}\log\left(\sqrt{b}(c+dx)^{3/2}+\sqrt{a+b(c+dx)^3}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c*e + d*e*x]/Sqrt[a + b*(c + d*x)^3],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*Log[Sqrt[b]*(c + d*x)^(3/2) + Sqrt[a + b*(c + d*x)^3]
])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {895, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx$$

$$\downarrow 895$$

$$\frac{\sqrt{ce+dex}\int \frac{\sqrt{c+dx}}{\sqrt{b(c+dx)^3+a}}d(c+dx)}{d\sqrt{c+dx}}$$

$$\downarrow 851$$

$$\frac{2\sqrt{ce+dex}\int \frac{c+dx}{\sqrt{b(c+dx)^3+a}}d\sqrt{c+dx}}{d\sqrt{c+dx}}$$

$$\downarrow 807$$

$$\frac{2\sqrt{ce+dex}\int \frac{1}{\sqrt{a+b(c+dx)}}d(c+dx)^{3/2}}{3d\sqrt{c+dx}}$$

$$\downarrow 224$$

$$\frac{2\sqrt{ce+dex}\int \frac{1}{1-b(c+dx)}d\frac{(c+dx)^{3/2}}{\sqrt{a+b(c+dx)}}}{3d\sqrt{c+dx}}$$

$$\downarrow 219$$

$$\frac{2\sqrt{ce+dex}\operatorname{arctanh}\left(\frac{\sqrt{b}(c+dx)^{3/2}}{\sqrt{a+b(c+dx)}}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input
```
Int[Sqrt[c*e + d*e*x]/Sqrt[a + b*(c + d*x)^3],x]
```

output
```
(2*Sqrt[c*e + d*e*x]*ArcTanh[(Sqrt[b]*(c + d*x)^(3/2))/Sqrt[a + b*(c + d*x
)]])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.33 (sec) , antiderivative size = 864, normalized size of antiderivative = 15.71

method	result
default	$\frac{2\sqrt{e(xd+c)}\sqrt{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}\,e\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-bdx-bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{}$
elliptic	Expression too large to display

input
```
int((d*e*x+c*e)^(1/2)/(a+b*(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*(d*x+c))^(1/2)*(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2)*e*
((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d
*x+2*b*c+(-a*b^2)^(1/3)))^(1/2)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-
a*b^2)^(1/3))^2*((I*3^(1/2)+3)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*d*x-2*b*c-(-a
*b^2)^(1/3))/(I*3^(1/2)-3)/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)+3)*(d*x+c)*b/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x
+2*b*c+(-a*b^2)^(1/3)))^(1/2)/b^2/d*(I*3^(1/2)*EllipticF(((1+I*3^(1/2))*(-
b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)
^(1/3)))^(1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-I*3^(1/2)
*EllipticPi(((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)
/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))+EllipticF(((1+I*3^(1/2))*(-b*d*x-b*c+
(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(
1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-3*EllipticPi(((1+I*
3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b
*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+
I*3^(1/2)))^(1/2)))/((b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*e*(d*x+
c))^(1/2)/(1+I*3^(1/2))/(I*3^(1/2)+3)/(1/b^2*e*(-b*d*x-b*c+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*d*x-2*b*c-(-a*b^2)^(1/3))*(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.

Time = 0.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 6.89

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx$$

$$= \left[\frac{\sqrt{\frac{e}{b}}\log\left(-8\,b^2d^6ex^6 - 48\,b^2cd^5ex^5 - 120\,b^2c^2d^4ex^4 - 8\,(20\,b^2c^3+ab)d^3ex^3 - 24\,(5\,b^2c^4+abc)d^2ex^2 - \right.}{} \right.$$

$$\left. \left. - \frac{\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}(bdx+bc)\sqrt{dex+ce}\sqrt{-\frac{e}{b}}}{2\,bd^3ex^3+6\,bcd^2ex^2+6\,bc^2dex+(2\,bc^3+a)e}\right)}{3\,d} \right]$$

input
```
integrate((d*e*x+c*e)^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(e/b)*log(-8*b^2*d^6*e*x^6 - 48*b^2*c*d^5*e*x^5 - 120*b^2*c^2*d^4
*e*x^4 - 8*(20*b^2*c^3 + a*b)*d^3*e*x^3 - 24*(5*b^2*c^4 + a*b*c)*d^2*e*x^2
 - 24*(2*b^2*c^5 + a*b*c^2)*d*e*x - 4*(2*b^2*d^4*x^4 + 8*b^2*c*d^3*x^3 + 1
2*b^2*c^2*d^2*x^2 + 2*b^2*c^4 + a*b*c + (8*b^2*c^3 + a*b)*d*x)*sqrt(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*sqrt(d*e*x + c*e)*sqrt(e/b)
 - (8*b^2*c^6 + 8*a*b*c^3 + a^2)*e)/d, -1/3*sqrt(-e/b)*arctan(2*sqrt(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(b*d*x + b*c)*sqrt(d*e*x +
 c*e)*sqrt(-e/b)/(2*b*d^3*e*x^3 + 6*b*c*d^2*e*x^2 + 6*b*c^2*d*e*x + (2*b*c
^3 + a)*e))/d]
```

Sympy [F]

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{e\,(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

input
```
integrate((d*e*x+c*e)**(1/2)/(a+b*(d*x+c)**3)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))/sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**
2 + b*d**3*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{dex+ce}}{\sqrt{(dx+c)^3 b+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*e*x + c*e)/sqrt((d*x + c)^3*b + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(41) = 82$.

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = -\frac{2\,e^4 \log\left(\left|-2\sqrt{be}\left(3\sqrt{dex+ce}c - \frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right) + 2\sqrt{\left(3\sqrt{dex+ce}c - \frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right.}\right.\right.}{3\sqrt{bed}|e|^3}$$

input
```
integrate((d*e*x+c*e)^(1/2)/(a+b*(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-2/3*e^4*log(abs(-2*sqrt(b*e)*(3*sqrt(d*e*x + c*e)*c - (3*sqrt(d*e*x + c*e
)*c*e - (d*e*x + c*e)^(3/2))/e) + 2*sqrt((3*sqrt(d*e*x + c*e)*c - (3*sqrt(
d*e*x + c*e)*c*e - (d*e*x + c*e)^(3/2))/e)^2*b*e + a*e^2)))/(sqrt(b*e)*d*a
bs(e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx = \int \frac{\sqrt{c\,e+d\,e\,x}}{\sqrt{a+b\,(c+d\,x)^3}}\,dx$$

input
```
int((c*e + d*e*x)^(1/2)/(a + b*(c + d*x)^3)^(1/2),x)
```

output
```
int((c*e + d*e*x)^(1/2)/(a + b*(c + d*x)^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+b(c+dx)^3}}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{b}\left(-\log\left(\sqrt{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}-\sqrt{b}\,\sqrt{dx+c}\,c-\sqrt{b}\,\sqrt{dx+c}\,dx\right)+\log\left(\sqrt{b\,d^3x}\right.\right.}{3bd}$$

input
```
int((d*e*x+c*e)^(1/2)/(a+b*(d*x+c)^3)^(1/2),x)
```

output
```
(sqrt(e)*sqrt(b)*( - log(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2
+ b*d**3*x**3) - sqrt(b)*sqrt(c + d*x)*c - sqrt(b)*sqrt(c + d*x)*d*x) + lo
g(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) + sqrt(b
)*sqrt(c + d*x)*c + sqrt(b)*sqrt(c + d*x)*d*x)))/(3*b*d)
```

3.131 $\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$

Optimal result . . . 1089
Mathematica [A] (verified) . . . 1089
Rubi [A] (warning: unable to verify) . . . 1090
Maple [C] (warning: unable to verify) . . . 1092
Fricas [B] (verification not implemented) . . . 1093
Sympy [F] . . . 1094
Maxima [F] . . . 1094
Giac [B] (verification not implemented) . . . 1095
Mupad [F(-1)] . . . 1095
Reduce [B] (verification not implemented) . . . 1096

Optimal result

Integrand size = 49, antiderivative size = 55

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(e(c+dx))^{3/2}}{e^{3/2}\sqrt{a+b(c+dx)^3}}\right)}{3\sqrt{b}d}$$

output
```
2/3*e^(1/2)*arctanh(b^(1/2)*(e*(d*x+c))^(3/2)/e^(3/2)/(a+b*(d*x+c)^3)^(1/2
))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \frac{2\sqrt{e(c+dx)}\log\left(\sqrt{b}(c+dx)^{3/2}+\sqrt{a+b(c+dx)^3}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[e*(c + d*x)]/Sqrt[a + b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 +
 b*d^3*x^3],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*Log[Sqrt[b]*(c + d*x)^(3/2) + Sqrt[a + b*(c + d*x)^3]
])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2510, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

$$\downarrow 2510$$

$$\frac{\int \frac{\sqrt{e(c+dx)}}{\sqrt{b(c+dx)^3+a}} d(e(c+dx))}{de}$$

$$\downarrow 851$$

$$\frac{2\int \frac{e^2(c+dx)^2}{\sqrt{be^3(c+dx)^6+a}} d\sqrt{e(c+dx)}}{de}$$

$$\downarrow 807$$

$$\frac{2\int \frac{1}{\sqrt{\frac{b(c+dx)^2}{e}+a}} d\left(e^3(c+dx)^3\right)}{3de}$$

$$\downarrow 224$$

$$\frac{2\int \frac{1}{1-\frac{b(c+dx)^2}{e}} d\frac{e^3(c+dx)^3}{\sqrt{\frac{b(c+dx)^2}{e}+a}}}{3de}$$

$$\downarrow 219$$

$$\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{e}}\right)}{3\sqrt{b}d}$$

input
```
Int[Sqrt[e*(c + d*x)]/Sqrt[a + b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3
*x^3],x]
```

output
```
(2*Sqrt[e]*ArcTanh[(Sqrt[b]*(c + d*x))/Sqrt[e]])/(3*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2510
```
Int[(Pn_)^(p_.)*(u_)^(m_.), x_Symbol] :> With[{Px = Pn /. x -> (x - Coeff[u
, x, 0])/Coeff[u, x, 1]}, Simp[1/Coeff[u, x, 1]   Subst[Int[x^m*ExpandToSum
[Px, x]^p, x], x, u], x] /; BinomialQ[Px, x]] /; FreeQ[{m, p}, x] && Linear
Q[u, x] && PolyQ[Pn, x] && NeQ[Coeff[u, x, 0], 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.23 (sec) , antiderivative size = 864, normalized size of antiderivative = 15.71

method	result
default	$\frac{2\sqrt{e(xd+c)}\,\sqrt{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}\,e\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-bdx-bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2$
elliptic	Expression too large to display

input
```
int((e*(d*x+c))^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
2*(e*(d*x+c))^(1/2)*(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2)*e*
((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d
*x+2*b*c+(-a*b^2)^(1/3)))^(1/2)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-
a*b^2)^(1/3))^2*((I*3^(1/2)+3)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*d*x-2*b*c-(-a
*b^2)^(1/3))/(I*3^(1/2)-3)/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)+3)*(d*x+c)*b/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x
+2*b*c+(-a*b^2)^(1/3)))^(1/2)/b^2/d*(I*3^(1/2)*EllipticF(((1+I*3^(1/2))*(-
b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)
^(1/3)))^(1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-I*3^(1/2)
*EllipticPi(((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)
/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))+EllipticF(((1+I*3^(1/2))*(-b*d*x-b*c+
(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(
1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-3*EllipticPi(((1+I*
3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b
*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+
I*3^(1/2)))^(1/2)))/((b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*e*(d*x+
c))^(1/2)/(1+I*3^(1/2))/(I*3^(1/2)+3)/(1/b^2*e*(-b*d*x-b*c+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*d*x-2*b*c-(-a*b^2)^(1/3))*(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(41) = 82$.

Time $= 0.36$ (sec) , antiderivative size $= 379$, normalized size of antiderivative $= 6.89$

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

$$= \left[\frac{\sqrt{\frac{e}{b}}\log\left(-8\,b^2d^6ex^6-48\,b^2cd^5ex^5-120\,b^2c^2d^4ex^4-8\,(20\,b^2c^3+ab)d^3ex^3-24\,(5\,b^2c^4+abc)d^2ex^2-\right.\right.$$

$$\left.\left.-\frac{\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}(bdx+bc)\sqrt{dex+ce}\sqrt{-\frac{e}{b}}}{2\,bd^3ex^3+6\,bcd^2ex^2+6\,bc^2dex+(2\,bc^3+a)e}\right)}{3\,d}\right]$$

input
```
integrate((e*(d*x+c))^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(e/b)*log(-8*b^2*d^6*e*x^6 - 48*b^2*c*d^5*e*x^5 - 120*b^2*c^2*d^4
*e*x^4 - 8*(20*b^2*c^3 + a*b)*d^3*e*x^3 - 24*(5*b^2*c^4 + a*b*c)*d^2*e*x^2
 - 24*(2*b^2*c^5 + a*b*c^2)*d*e*x - 4*(2*b^2*d^4*x^4 + 8*b^2*c*d^3*x^3 + 1
2*b^2*c^2*d^2*x^2 + 2*b^2*c^4 + a*b*c + (8*b^2*c^3 + a*b)*d*x)*sqrt(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*sqrt(d*e*x + c*e)*sqrt(e/b)
 - (8*b^2*c^6 + 8*a*b*c^3 + a^2)*e)/d, -1/3*sqrt(-e/b)*arctan(2*sqrt(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(b*d*x + b*c)*sqrt(d*e*x +
 c*e)*sqrt(-e/b)/(2*b*d^3*e*x^3 + 6*b*c*d^2*e*x^2 + 6*b*c^2*d*e*x + (2*b*c
^3 + a)*e))/d]
```

Sympy [F]

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$=\int \frac{\sqrt{e\,(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

input
```
integrate((e*(d*x+c))**(1/2)/(b*d**3*x**3+3*b*c*d**2*x**2+3*b*c**2*d*x+b*c
**3+a)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))/sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**
2 + b*d**3*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$=\int \frac{\sqrt{(dx+c)e}}{\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}}\,dx$$

input
```
integrate((e*(d*x+c))^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((d*x + c)*e)/sqrt(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(41) = 82$.

Time $= 0.16$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 2.44$

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx =$$
$$-\frac{2\,e^4\log\left(\left|-2\sqrt{be}\left(3\sqrt{dex+ce}c-\frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right)+2\sqrt{\left(3\sqrt{dex+ce}c-\frac{3\sqrt{dex+ce}ce-(dex+ce)^{\frac{3}{2}}}{e}\right.}\right.\right.}{3\sqrt{bed}|e|^3}$$

input
```
integrate((e*(d*x+c))^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="giac")
```

output
```
-2/3*e^4*log(abs(-2*sqrt(b*e)*(3*sqrt(d*e*x + c*e)*c - (3*sqrt(d*e*x + c*e
)*c*e - (d*e*x + c*e)^(3/2))/e) + 2*sqrt((3*sqrt(d*e*x + c*e)*c - (3*sqrt(
d*e*x + c*e)*c*e - (d*e*x + c*e)^(3/2))/e)^2*b*e + a*e^2)))/(sqrt(b*e)*d*a
bs(e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$=\int \frac{\sqrt{e\,(c+d\,x)}}{\sqrt{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}}\,dx$$

input
```
int((e*(c + d*x))^(1/2)/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x
^2)^(1/2),x)
```

output
```
int((e*(c + d*x))^(1/2)/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x
^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{e(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{b}\left(-\log\left(\sqrt{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}-\sqrt{b}\,\sqrt{dx+c}\,c-\sqrt{b}\,\sqrt{dx+c}\,dx\right)+\log\left(\sqrt{b\,d^3x\right.}\right.}{3bd}$$

input
```
int((e*(d*x+c))^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2),
x)
```

output
```
(sqrt(e)*sqrt(b)*( - log(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2
+ b*d**3*x**3) - sqrt(b)*sqrt(c + d*x)*c - sqrt(b)*sqrt(c + d*x)*d*x) + lo
g(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) + sqrt(b
)*sqrt(c + d*x)*c + sqrt(b)*sqrt(c + d*x)*d*x)))/(3*b*d)
```

3.132 $\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$

Optimal result 1097
Mathematica [A] (verified) 1097
Rubi [A] (warning: unable to verify) 1098
Maple [C] (warning: unable to verify) 1100
Fricas [B] (verification not implemented) 1101
Sympy [F] 1102
Maxima [F] 1102
Giac [B] (verification not implemented) 1103
Mupad [F(-1)] 1103
Reduce [B] (verification not implemented) 1104

Optimal result

Integrand size = 50, antiderivative size = 56

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ce+dex)^{3/2}}{e^{3/2}\sqrt{a+b(c+dx)^3}}\right)}{3\sqrt{bd}}$$

output
```
2/3*e^(1/2)*arctanh(b^(1/2)*(d*e*x+c*e)^(3/2)/e^(3/2)/(a+b*(d*x+c)^3)^(1/2
))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \frac{2\sqrt{e(c+dx)}\log\left(\sqrt{b}(c+dx)^{3/2}+\sqrt{a+b(c+dx)^3}\right)}{3\sqrt{b}d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c*e + d*e*x]/Sqrt[a + b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 +
 b*d^3*x^3],x]
```

output
```
(2*Sqrt[e*(c + d*x)]*Log[Sqrt[b]*(c + d*x)^(3/2) + Sqrt[a + b*(c + d*x)^3]
])/(3*Sqrt[b]*d*Sqrt[c + d*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2487, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

$$\downarrow 2487$$

$$\int \frac{\sqrt{de\left(\frac{c}{d}+x\right)}}{\sqrt{a+bd^3\left(\frac{c}{d}+x\right)^3}}d\left(\frac{c}{d}+x\right)$$

$$\downarrow 851$$

$$\frac{2\int \frac{de\left(\frac{c}{d}+x\right)}{\sqrt{bd^3\left(\frac{c}{d}+x\right)^3+a}}d\sqrt{de\left(\frac{c}{d}+x\right)}}{de}$$

$$\downarrow 807$$

$$\frac{2\int \frac{1}{\sqrt{a+\frac{bd\left(\frac{c}{d}+x\right)}{e^2}}}d\left(de\left(\frac{c}{d}+x\right)\right)^{3/2}}{3de}$$

$$\downarrow 224$$

$$\frac{2\int \frac{1}{1-\frac{bd\left(\frac{c}{d}+x\right)}{e^2}}d\frac{\left(de\left(\frac{c}{d}+x\right)\right)^{3/2}}{\sqrt{a+\frac{bd\left(\frac{c}{d}+x\right)}{e^2}}}}{3de}$$

$$\downarrow 219$$

$$\frac{2\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}\left(de\left(\frac{c}{d}+x\right)\right)^{3/2}}{e^{3/2}\sqrt{a+\frac{bd\left(\frac{c}{d}+x\right)}{e^2}}}\right)}{3\sqrt{b}d}$$

input
```
Int[Sqrt[c*e + d*e*x]/Sqrt[a + b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3
*x^3],x]
```

output
```
(2*Sqrt[e]*ArcTanh[(Sqrt[b]*(d*e*(c/d + x))^(3/2))/(e^(3/2)*Sqrt[a + (b*d*
(c/d + x))/e^2])])/(3*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2487
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Subst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[a -
b^2/(3*c) + d*x^3, x]^p, x], x, x + c/(3*d)] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && EqQ[c^2 - 3*b*d, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 2.20 (sec) , antiderivative size = 864, normalized size of antiderivative = 15.43

method	result
default	$\frac{2\sqrt{e(xd+c)}\,\sqrt{b\,d^3x^3+3bc\,d^2x^2+3dbx\,c^2+c^3b+a}\,e\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-bdx-bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}}}\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bdx+2bc+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2$
elliptic	Expression too large to display

input
```
int((d*e*x+c*e)^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
2*(e*(d*x+c))^(1/2)*(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2)*e*
((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d
*x+2*b*c+(-a*b^2)^(1/3)))^(1/2)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-
a*b^2)^(1/3))^2*((I*3^(1/2)+3)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*d*x-2*b*c-(-a
*b^2)^(1/3))/(I*3^(1/2)-3)/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)+3)*(d*x+c)*b/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x
+2*b*c+(-a*b^2)^(1/3)))^(1/2)/b^2/d*(I*3^(1/2)*EllipticF(((1+I*3^(1/2))*(-
b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)
^(1/3)))^(1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-I*3^(1/2)
*EllipticPi(((1+I*3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)
/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))+EllipticF(((1+I*3^(1/2))*(-b*d*x-b*c+
(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3)))^(
1/2),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+I*3^(1/2)))^(1/2))-3*EllipticPi(((1+I*
3^(1/2))*(-b*d*x-b*c+(-a*b^2)^(1/3))/(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b
*c+(-a*b^2)^(1/3)))^(1/2),-2/(1+I*3^(1/2)),2*(-I*3^(1/2)/(I*3^(1/2)-3)/(1+
I*3^(1/2)))^(1/2)))/((b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)*e*(d*x+
c))^(1/2)/(1+I*3^(1/2))/(I*3^(1/2)+3)/(1/b^2*e*(-b*d*x-b*c+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*d*x+2*b*c+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*d*x-2*b*c-(-a*b^2)^(1/3))*(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(42) = 84$.

Time $= 0.35$ (sec) , antiderivative size $= 379$, normalized size of antiderivative $= 6.77$

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

$$= \left[\frac{\sqrt{\frac{e}{b}}\log\left(-8\,b^2d^6ex^6-48\,b^2cd^5ex^5-120\,b^2c^2d^4ex^4-8\,(20\,b^2c^3+ab)d^3ex^3-24\,(5\,b^2c^4+abc)d^2ex^2-\right.\right.$$

$$\left.\left.-\frac{\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}(bdx+bc)\sqrt{dex+ce}\sqrt{-\frac{e}{b}}}{2\,bd^3ex^3+6\,bcd^2ex^2+6\,bc^2dex+(2\,bc^3+a)e}\right)}{3\,d}\right]$$

input
```
integrate((d*e*x+c*e)^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(e/b)*log(-8*b^2*d^6*e*x^6 - 48*b^2*c*d^5*e*x^5 - 120*b^2*c^2*d^4
*e*x^4 - 8*(20*b^2*c^3 + a*b)*d^3*e*x^3 - 24*(5*b^2*c^4 + a*b*c)*d^2*e*x^2
 - 24*(2*b^2*c^5 + a*b*c^2)*d*e*x - 4*(2*b^2*d^4*x^4 + 8*b^2*c*d^3*x^3 + 1
2*b^2*c^2*d^2*x^2 + 2*b^2*c^4 + a*b*c + (8*b^2*c^3 + a*b)*d*x)*sqrt(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*sqrt(d*e*x + c*e)*sqrt(e/b)
 - (8*b^2*c^6 + 8*a*b*c^3 + a^2)*e)/d, -1/3*sqrt(-e/b)*arctan(2*sqrt(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*(b*d*x + b*c)*sqrt(d*e*x +
 c*e)*sqrt(-e/b)/(2*b*d^3*e*x^3 + 6*b*c*d^2*e*x^2 + 6*b*c^2*d*e*x + (2*b*c
^3 + a)*e))/d]
```

Sympy [F]

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \int \frac{\sqrt{e\,(c+dx)}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$

input
```
integrate((d*e*x+c*e)**(1/2)/(b*d**3*x**3+3*b*c*d**2*x**2+3*b*c**2*d*x+b*c
**3+a)**(1/2),x)
```

output
```
Integral(sqrt(e*(c + d*x))/sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**
2 + b*d**3*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx = \int \frac{\sqrt{dex+ce}}{\sqrt{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}}\,dx$$

input
```
integrate((d*e*x+c*e)^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*e*x + c*e)/sqrt(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(42) = 84$.

Time $= 0.16$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 2.39$

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx =$$
$$-\frac{2\,e^4\log\left(\left|-2\sqrt{be}\left(3\sqrt{dex+cec}-\frac{3\sqrt{dex+cec}ce-(dex+ce)^{\frac{3}{2}}}{e}\right)+2\sqrt{\left(3\sqrt{dex+cec}-\frac{3\sqrt{dex+cec}ce-(dex+ce)^{\frac{3}{2}}}{e}\right.}\right.\right.}{3\sqrt{bed}|e|^3}$$

input
```
integrate((d*e*x+c*e)^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^
(1/2),x, algorithm="giac")
```

output
```
-2/3*e^4*log(abs(-2*sqrt(b*e)*(3*sqrt(d*e*x + c*e)*c - (3*sqrt(d*e*x + c*e
)*c*e - (d*e*x + c*e)^(3/2))/e) + 2*sqrt((3*sqrt(d*e*x + c*e)*c - (3*sqrt(
d*e*x + c*e)*c*e - (d*e*x + c*e)^(3/2))/e)^2*b*e + a*e^2)))/(sqrt(b*e)*d*a
bs(e)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$=\int \frac{\sqrt{c\,e+d\,e\,x}}{\sqrt{b\,c^3+3\,b\,c^2\,d\,x+3\,b\,c\,d^2\,x^2+b\,d^3\,x^3+a}}\,dx$$

input
```
int((c*e + d*e*x)^(1/2)/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x
^2)^(1/2),x)
```

output
```
int((c*e + d*e*x)^(1/2)/(a + b*c^3 + b*d^3*x^3 + 3*b*c^2*d*x + 3*b*c*d^2*x
^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{ce+dex}}{\sqrt{a+bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3}}\,dx$$
$$= \frac{\sqrt{e}\,\sqrt{b}\left(-\log\left(\sqrt{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}-\sqrt{b}\,\sqrt{dx+c}\,c-\sqrt{b}\,\sqrt{dx+c}\,dx\right)+\log\left(\sqrt{b\,d^3x}\right.\right.}{3bd}$$

input
```
int((d*e*x+c*e)^(1/2)/(b*d^3*x^3+3*b*c*d^2*x^2+3*b*c^2*d*x+b*c^3+a)^(1/2),
x)
```

output
```
(sqrt(e)*sqrt(b)*( - log(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2
+ b*d**3*x**3) - sqrt(b)*sqrt(c + d*x)*c - sqrt(b)*sqrt(c + d*x)*d*x) + lo
g(sqrt(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) + sqrt(b
)*sqrt(c + d*x)*c + sqrt(b)*sqrt(c + d*x)*d*x)))/(3*b*d)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1105
4.2 Links to plain text integration problems used in this report for each CAS . 1123

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file