Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.3/50-1.1.3.3-a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**190**]. This is test number [50].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (190)	0.00 (0)
Rubi	99.47 (189)	0.53 (1)
Maple	58.42 (111)	41.58 (79)
Fricas	46.32 (88)	53.68 (102)
Sympy	37.89 (72)	62.11 (118)
Maxima	36.84 (70)	63.16 (120)
Mupad	27.37 (52)	72.63 (138)
Giac	16.84 (32)	83.16 (158)
Reduce	16.84 (32)	83.16 (158)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

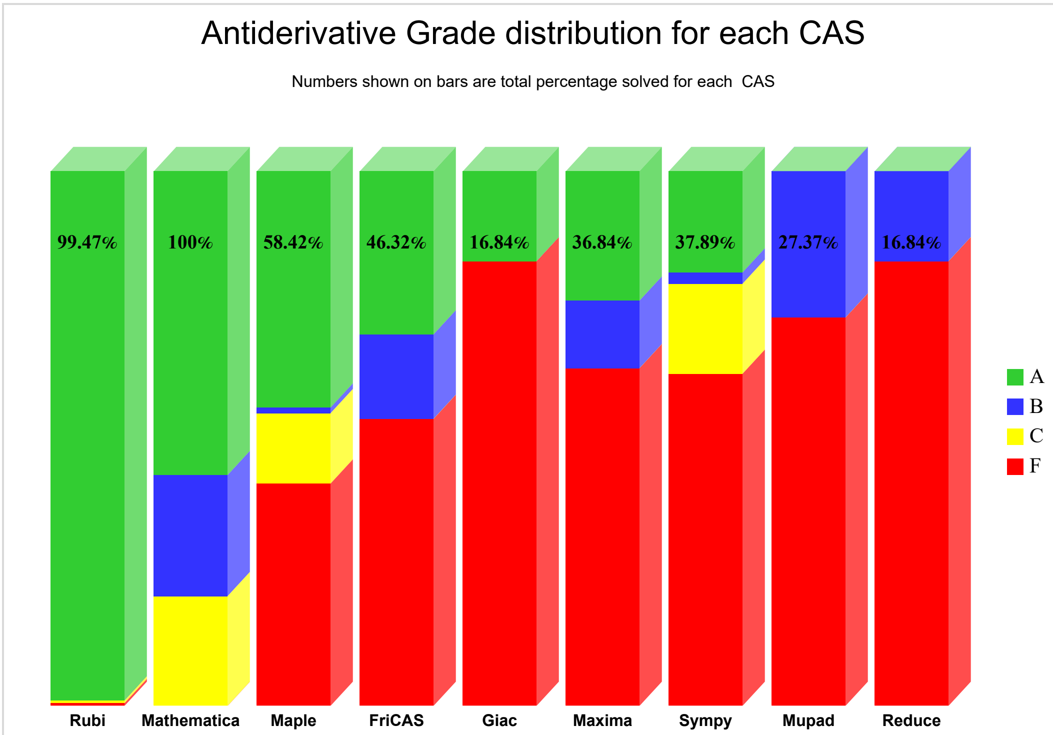
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

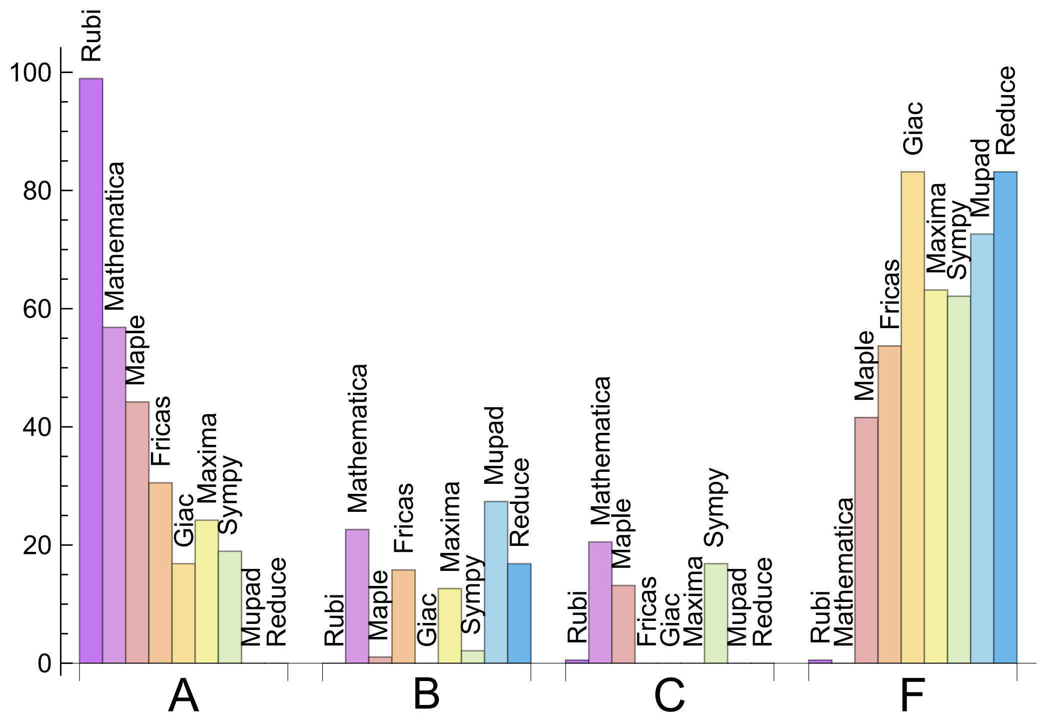
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.947	0.000	0.526	0.526
Mathematica	56.842	22.632	20.526	0.000
Maple	44.211	1.053	13.158	41.579
Fricas	30.526	15.789	0.000	53.684
Maxima	24.211	12.632	0.000	63.158
Sympy	18.947	2.105	16.842	62.105
Giac	16.842	0.000	0.000	83.158
Mupad	0.000	27.368	0.000	72.632
Reduce	0.000	16.842	0.000	83.158

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	79	100.00	0.00	0.00
Fricas	102	53.92	45.10	0.98
Sympy	118	62.71	37.29	0.00
Maxima	120	100.00	0.00	0.00
Mupad	138	0.00	100.00	0.00
Giac	158	100.00	0.00	0.00
Reduce	158	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.13
Reduce	0.23
Rubi	0.51
Maple	1.23
Mupad	1.89
Fricas	2.27
Mathematica	4.68
Sympy	8.91

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	160.86	1.03	122.00	1.00
Sympy	168.24	1.37	126.00	1.00
Maple	224.59	1.61	171.00	1.03
Mathematica	234.52	1.98	173.50	1.10
Giac	236.38	1.18	211.00	1.11
Maxima	243.53	1.57	199.00	1.32
Mupad	322.25	1.40	137.50	0.94
Fricas	441.38	2.33	399.00	2.12
Reduce	468.56	2.09	349.00	1.94

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

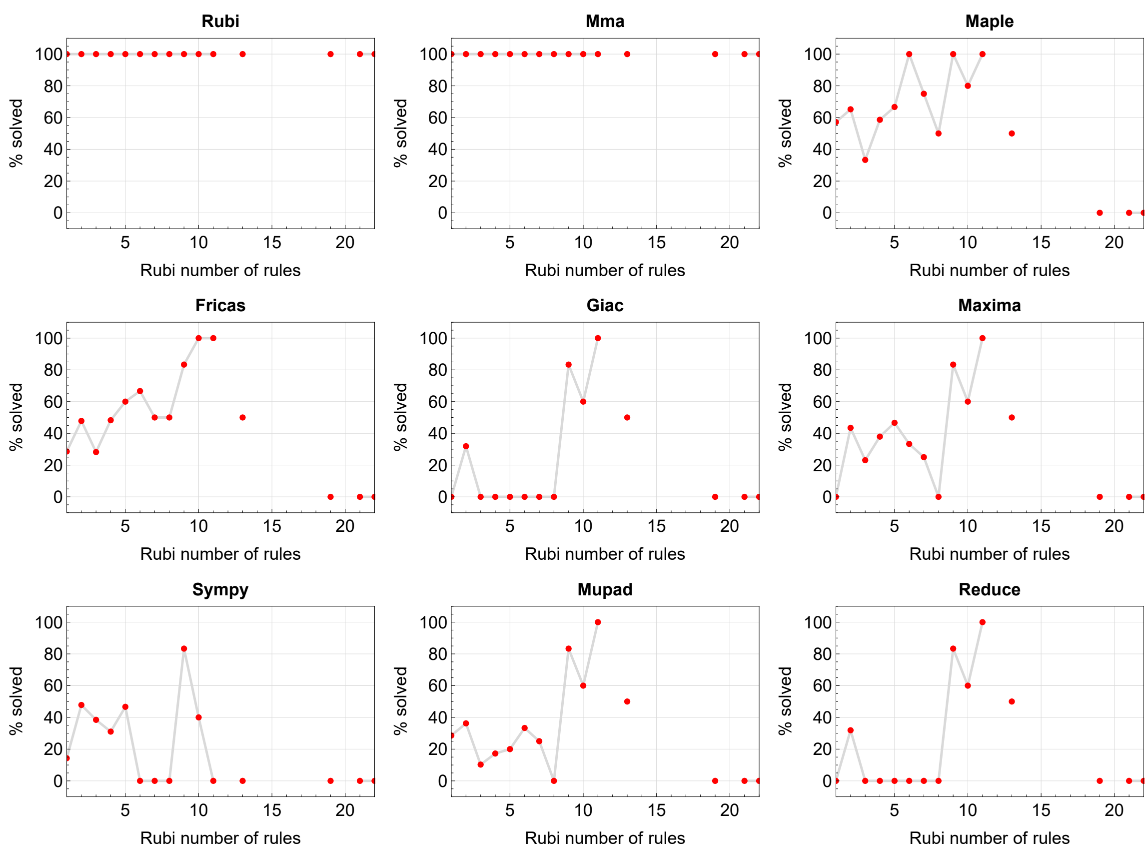


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

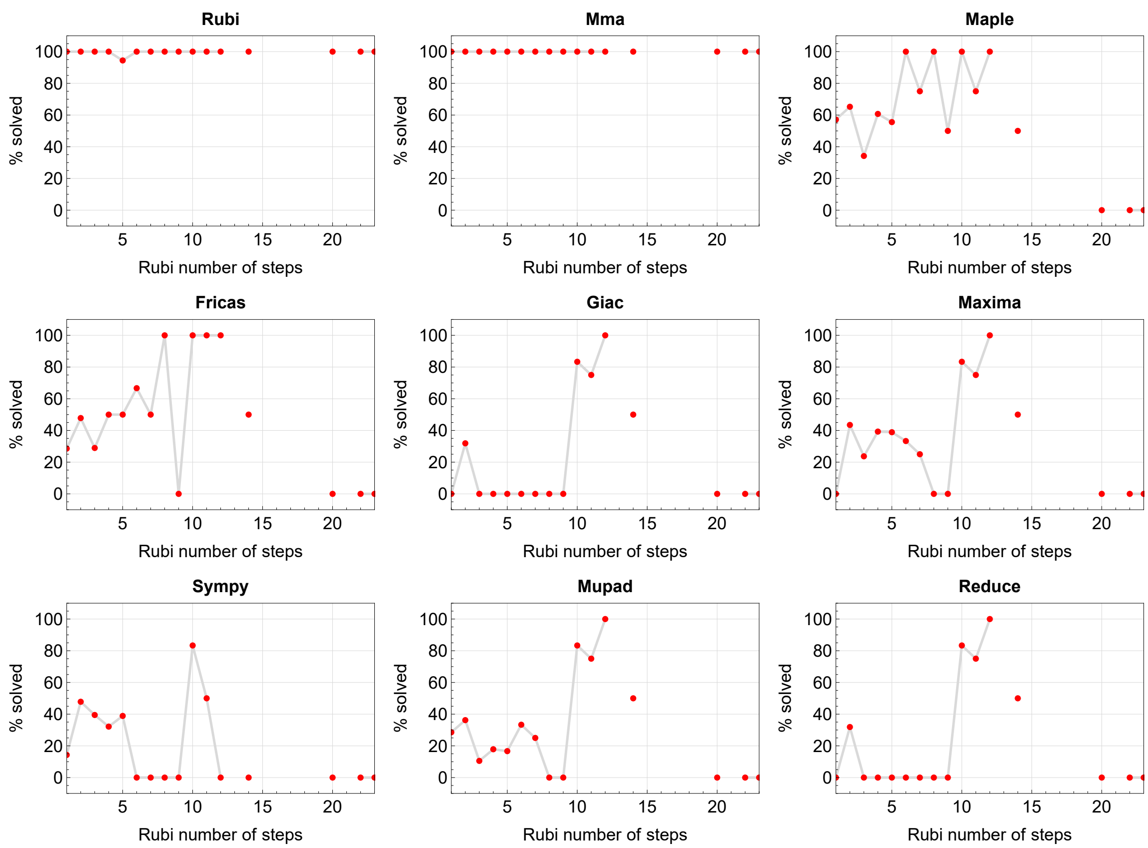


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

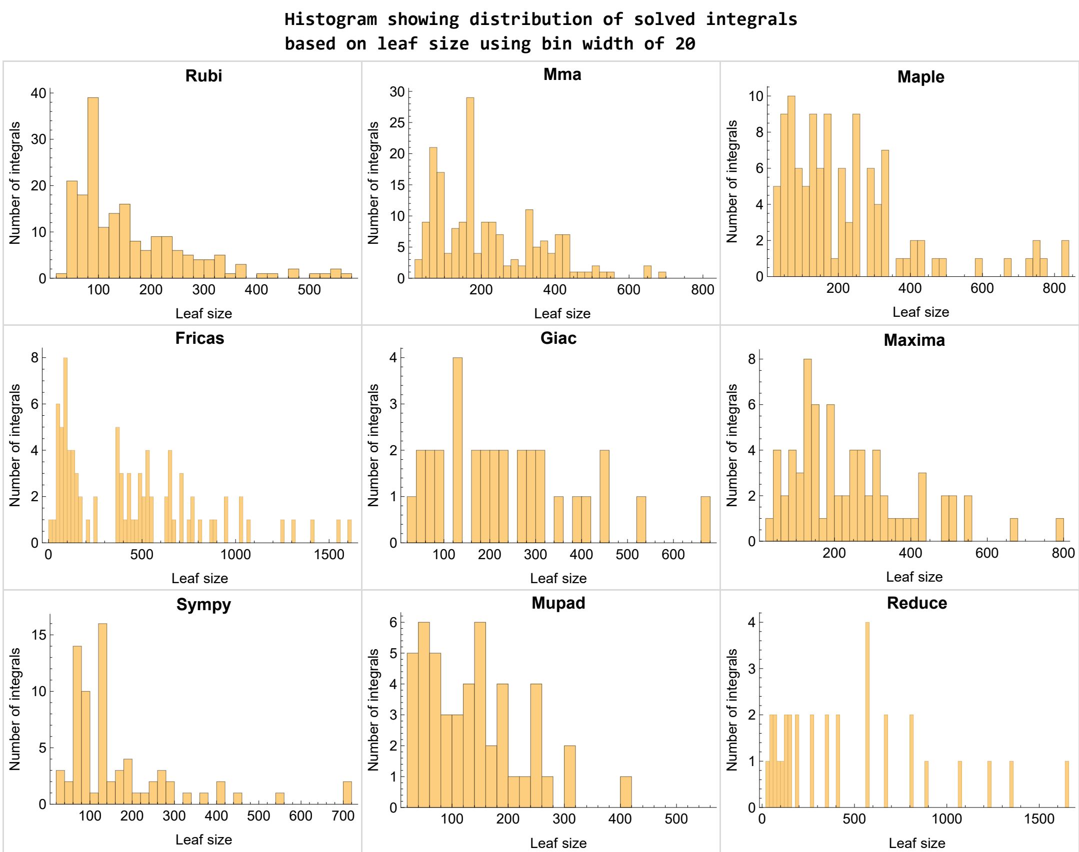


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

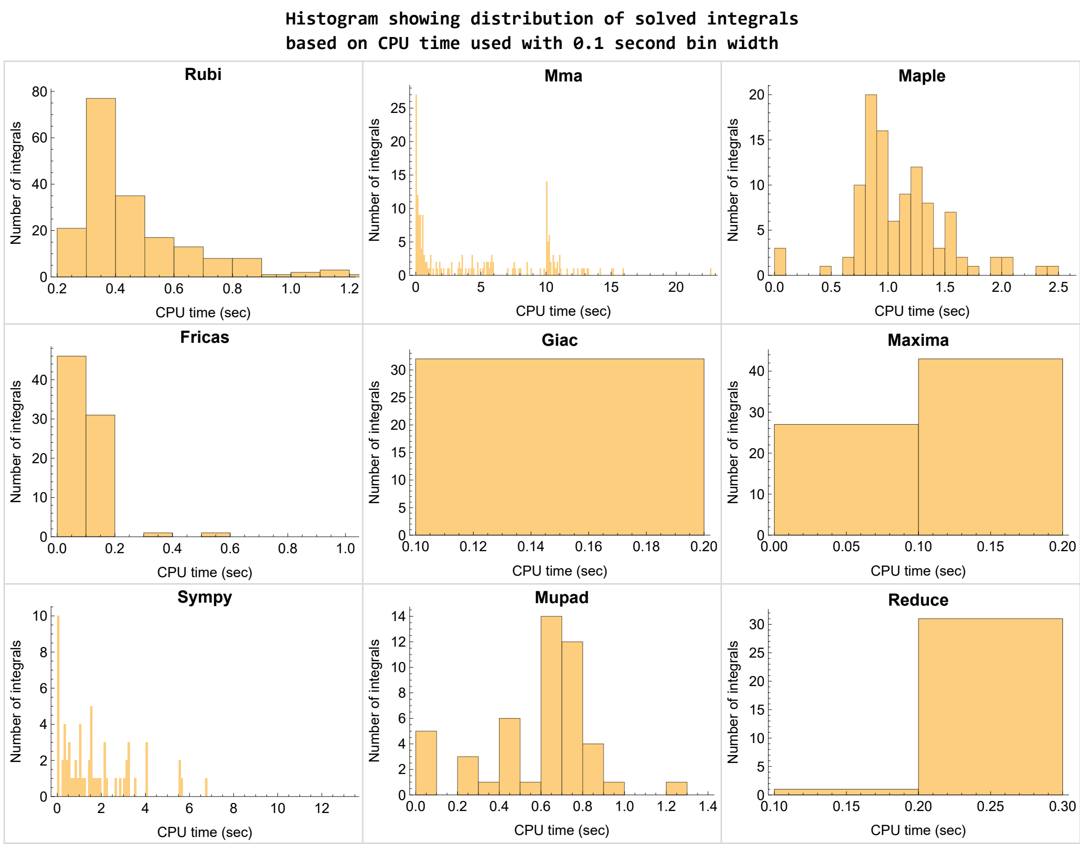


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

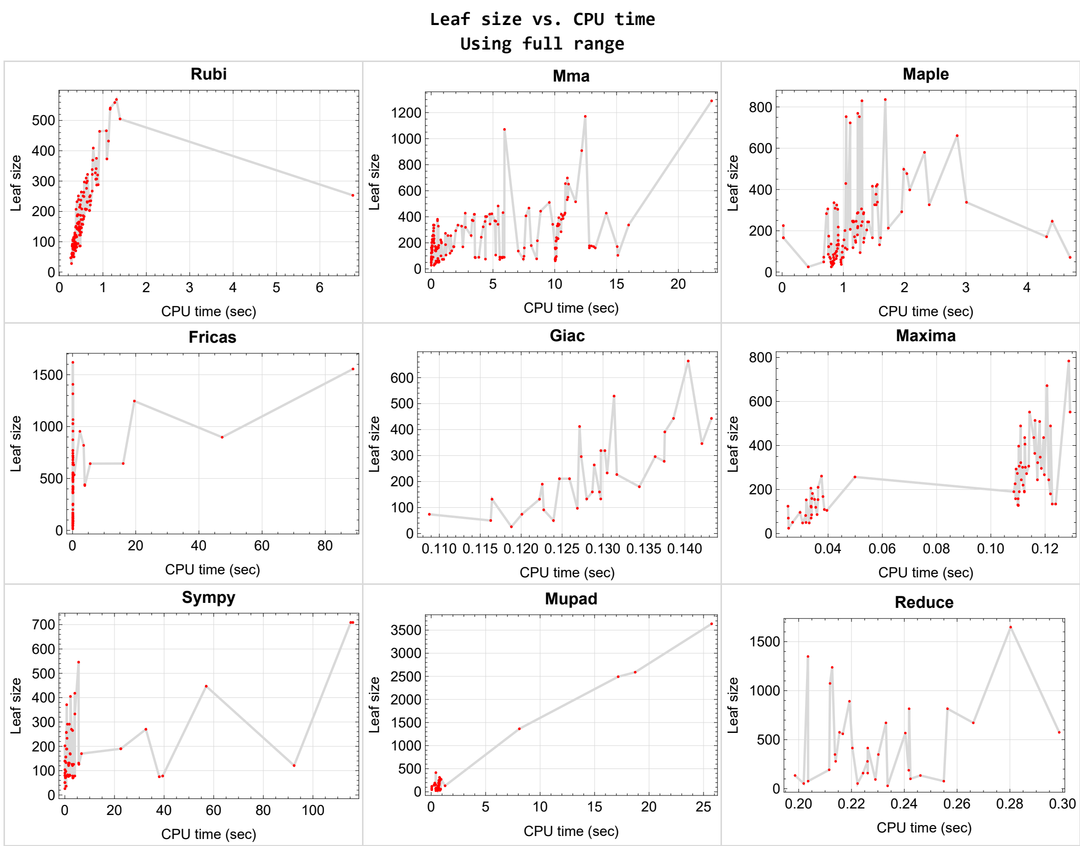


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {6, 7, 49}

Mathematica {1, 2, 3, 5, 8, 9, 44, 45, 46, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 77, 78, 79, 97, 98, 131, 132, 135, 136, 138, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 188, 189}

Maple {44, 45, 49, 50, 51, 55, 56}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

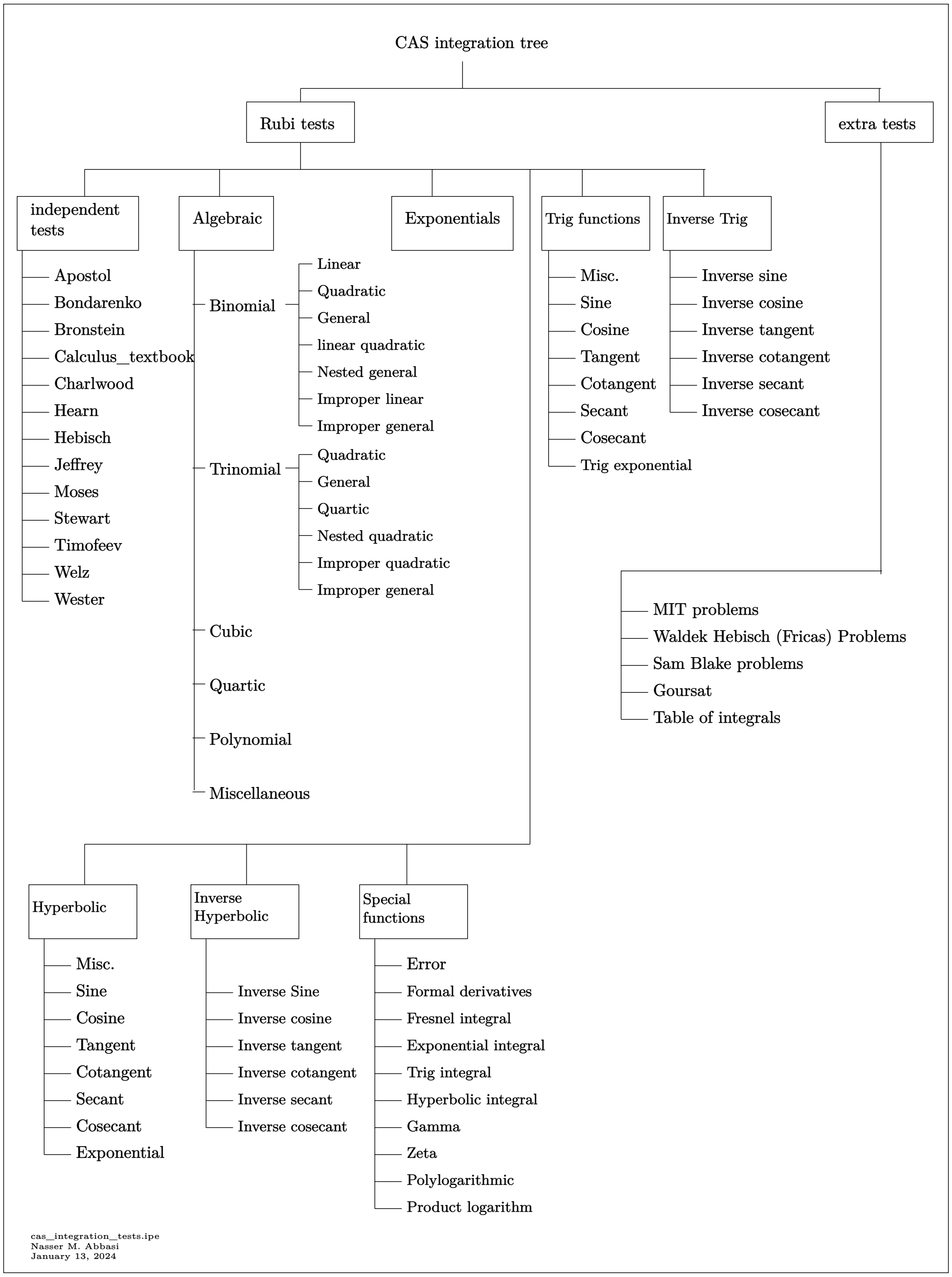
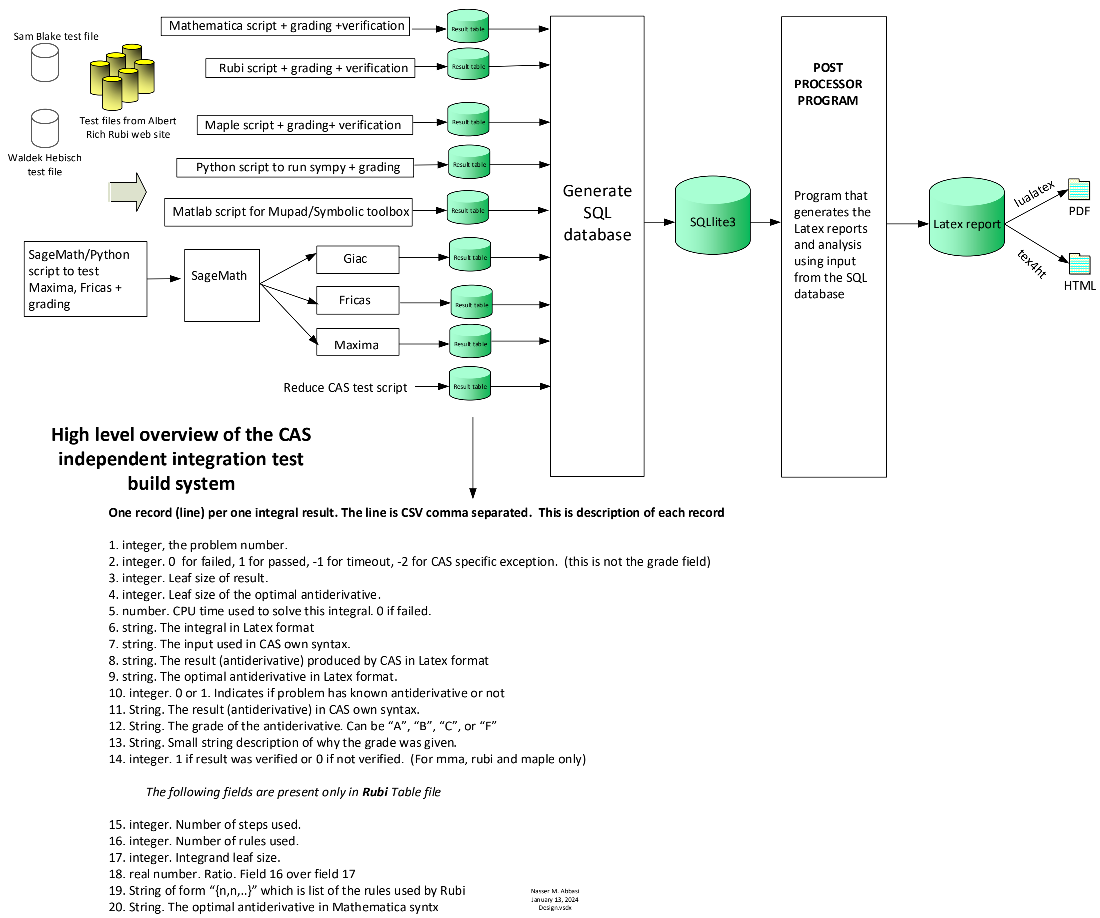


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

B grade { }

C grade { 49 }

F normal fail { 4 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 4, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 67, 68, 69, 70, 71, 72, 73, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 168, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 190 }

B grade { 1, 2, 3, 5, 8, 9, 44, 45, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 97, 98, 145, 146, 147, 148, 149, 156, 157, 158, 159, 160, 169, 170, 171, 172, 173, 183, 187, 188, 189 }

C grade { 6, 42, 43, 46, 47, 48, 49, 52, 53, 54, 74, 75, 77, 78, 79, 101, 102, 106, 107, 138, 139, 140, 141, 142, 143, 144, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 10, 11, 12, 13, 17, 18, 19, 23, 24, 25, 33, 34, 40, 41, 42, 43, 46, 47, 48, 52, 53, 54, 67, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 122, 123, 124, 125, 126, 127, 128, 129, 130, 137, 138, 139, 140, 141, 142, 143, 144, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 168, 190 }

B grade { 100, 110 }

C grade { 14, 15, 16, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 44, 45, 49, 50, 51, 55, 56 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 116, 117, 118, 119, 120, 121, 131, 132, 133, 134, 135, 136, 145, 146, 147, 148, 149, 156, 157, 158, 159, 160, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 29, 30, 31, 32, 33, 34, 39, 40, 42, 43, 46, 47, 48, 52, 53, 54, 70, 71, 72, 73, 80, 81, 84, 85, 86, 87, 99, 100, 108, 109, 110, 112, 113, 114, 115, 122, 123, 124, 127, 128, 129, 130, 137, 190 }

B grade { 16, 21, 22, 27, 28, 35, 36, 37, 38, 41, 67, 68, 69, 76, 82, 83, 103, 104, 105, 111, 125, 126, 138, 139, 140, 150, 151, 161, 162, 163 }

C grade { }

F normal fail { 2, 3, 4, 6, 7, 8, 9, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 75, 88, 89, 90, 91, 92, 93, 94, 95, 96, 116, 117, 118, 119, 120, 121, 131, 132, 133, 134, 135, 136, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189 }

F(-1) timedout fail { 5, 44, 45, 49, 50, 51, 55, 56, 74, 77, 78, 79, 97, 98, 101, 102, 106, 107, 141, 142, 143, 144, 145, 146, 147, 148, 149, 152, 153, 154, 155, 156, 157, 158, 159, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173 }

F(-2) exception fail { 1 }

Maxima

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 69, 70, 71, 105, 111, 112, 113, 114, 115, 126, 127, 128, 129, 130, 137 }

B grade { 41, 67, 68, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 99, 100, 103, 104, 108, 109, 110, 122, 123, 124, 125 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 101, 102, 106, 107, 116, 117, 118, 119, 120, 121, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 48, 54, 70, 71, 72, 73, 84, 85, 86, 87, 112, 113, 114, 115, 127, 128, 129, 130, 137, 190 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189 }

F(-2) exception fail { }

Sympy

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 42, 43, 46, 47, 48, 53, 54 }

B grade { 70, 71, 112, 113 }

C grade { 67, 68, 69, 80, 81, 88, 89, 90, 95, 96, 99, 100, 105, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 185, 186 }

F normal fail { 6, 7, 8, 9, 44, 45, 49, 50, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 82, 83, 91, 92, 97, 98, 101, 102, 103, 104, 106, 107, 125, 126, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 165, 168, 170, 173, 176, 177, 178, 179, 180 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 34, 40, 41, 51, 72, 73, 84, 85, 86, 87, 93, 94, 114, 115, 127, 128, 129, 130, 137, 150, 161, 162, 163, 164, 166, 167, 169, 171, 172, 174, 175, 181, 182, 183, 184, 187, 188, 189, 190 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	1071	0	0	0	0	0	84	0
N.S.	1	1.00	12.17	0.00	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.383	5.933	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	342	0	0	0	0	0	83	0
N.S.	1	1.00	3.98	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.464	5.409	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	342	0	0	0	0	0	91	0
N.S.	1	1.00	3.98	0.00	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.478	9.838	0.000	0.000	0.000	0.000	0.000	1.474	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	687	0	243	0	0	0	0	0	83	0
N.S.	1	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	4.053	0.000	0.000	0.000	0.000	0.000	0.422	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	1290	0	0	0	0	0	21	0
N.S.	1	1.01	13.72	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.382	22.691	0.000	0.000	0.000	0.000	0.000	200.041	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	230	73	0	0	0	0	0	68	0
N.S.	1	1.17	0.37	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.640	0.038	0.000	0.000	0.000	0.000	0.000	0.344	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	253	216	0	0	0	0	0	80	0
N.S.	1	1.18	1.01	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	6.760	8.594	0.000	0.000	0.000	0.000	0.000	0.446	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	402	0	0	0	0	0	80	0
N.S.	1	1.00	4.67	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.374	4.397	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	401	0	0	0	0	0	80	0
N.S.	1	1.00	4.66	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.370	4.490	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	94	96	96	104	97	101	87
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.03	1.07	0.93
time (sec)	N/A	0.426	0.021	1.273	0.030	0.072	0.034	0.127	0.242	0.465

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	80	74	77	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.14	1.06	1.10	0.94
time (sec)	N/A	0.374	0.013	0.776	0.034	0.063	0.029	0.120	0.204	0.435

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	53	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	1.06	0.96
time (sec)	N/A	0.334	0.008	0.796	0.031	0.063	0.027	0.116	0.202	0.056

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	29	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	1.04	0.89
time (sec)	N/A	0.276	0.008	0.427	0.025	0.065	0.024	0.119	0.234	0.432

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	131	128	42	128	369	71	133	159	123
N.S.	1	0.91	0.89	0.29	0.89	2.56	0.49	0.92	1.10	0.85
time (sec)	N/A	0.504	0.072	0.845	0.110	0.088	0.239	0.130	0.224	0.663

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	349	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	2.07	0.85
time (sec)	N/A	0.514	0.076	0.883	0.110	0.088	0.331	0.129	0.230	0.286

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	183	175	84	192	743	133	180	560	173
N.S.	1	0.93	0.89	0.43	0.97	3.77	0.68	0.91	2.84	0.88
time (sec)	N/A	0.567	0.109	0.836	0.112	0.092	0.434	0.134	0.217	0.716

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	135	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	1.11	0.95
time (sec)	N/A	0.483	0.015	0.759	0.034	0.066	0.038	0.116	0.246	0.068

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	90	91	94	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.10	1.11	1.15	0.91
time (sec)	N/A	0.398	0.010	0.753	0.032	0.086	0.030	0.123	0.229	0.050

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	53	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	1.06	0.96
time (sec)	N/A	0.323	0.008	0.679	0.033	0.067	0.025	0.124	0.222	0.054

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	189	505	156	211	279	152
N.S.	1	1.00	0.97	0.45	1.09	2.92	0.90	1.22	1.61	0.88
time (sec)	N/A	0.530	0.069	0.888	0.112	0.092	0.370	0.126	0.214	0.292

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	99	226	771	189	233	575	191
N.S.	1	1.00	1.03	0.49	1.11	3.80	0.93	1.15	2.83	0.94
time (sec)	N/A	0.693	0.163	0.885	0.109	0.093	0.634	0.131	0.299	0.700

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	221	217	131	267	1067	233	264	892	249
N.S.	1	0.87	0.86	0.52	1.06	4.22	0.92	1.04	3.53	0.98
time (sec)	N/A	0.710	0.200	0.901	0.120	0.146	0.868	0.129	0.219	0.726

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	164	174	168	168	202	190	193	153
N.S.	1	1.00	1.04	1.11	1.07	1.07	1.29	1.21	1.23	0.97
time (sec)	N/A	0.537	0.020	0.761	0.038	0.065	0.039	0.123	0.212	0.433

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	135	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	1.11	0.95
time (sec)	N/A	0.460	0.017	0.760	0.025	0.068	0.036	0.122	0.199	0.061

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	80	74	77	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.14	1.06	1.10	0.94
time (sec)	N/A	0.369	0.009	0.680	0.025	0.081	0.028	0.109	0.255	0.407

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	202	131	272	679	257	296	415	193
N.S.	1	1.00	0.98	0.64	1.32	3.30	1.25	1.44	2.01	0.94
time (sec)	N/A	0.605	0.045	0.918	0.113	0.101	0.599	0.127	0.226	0.678

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	227	153	306	1029	291	319	816	239
N.S.	1	1.00	0.97	0.65	1.30	4.38	1.24	1.36	3.47	1.02
time (sec)	N/A	0.721	0.132	0.917	0.110	0.098	1.804	0.130	0.256	0.717

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	291	179	347	1407	333	346	1238	313
N.S.	1	1.00	1.01	0.62	1.21	4.90	1.16	1.21	4.31	1.09
time (sec)	N/A	0.856	0.188	0.915	0.118	0.116	4.035	0.142	0.213	0.762

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	253	201	364	873	371	391	568	250
N.S.	1	1.00	1.00	0.80	1.44	3.46	1.47	1.55	2.25	0.99
time (sec)	N/A	0.708	0.111	1.026	0.116	0.096	0.727	0.138	0.240	0.709

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	203	131	273	700	257	296	415	192
N.S.	1	1.00	0.98	0.63	1.31	3.37	1.24	1.42	2.00	0.92
time (sec)	N/A	0.592	0.065	0.894	0.110	0.151	0.593	0.136	0.220	0.680

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	78	190	507	156	211	279	152
N.S.	1	1.00	0.97	0.45	1.10	2.93	0.90	1.22	1.61	0.88
time (sec)	N/A	0.496	0.079	0.894	0.110	0.090	0.373	0.125	0.226	0.277

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	390	71	133	159	123
N.S.	1	0.90	0.89	0.29	0.88	2.69	0.49	0.92	1.10	0.85
time (sec)	N/A	0.482	0.050	0.804	0.110	0.097	0.234	0.128	0.226	0.631

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	248	224	207	293	254	447	278	188	1364
N.S.	1	0.86	0.78	0.72	1.02	0.88	1.55	0.97	0.65	4.74
time (sec)	N/A	0.694	0.082	1.112	0.109	0.133	56.982	0.137	0.242	8.098

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	307	336	246	489	432	0	443	672	2589
N.S.	1	0.89	0.97	0.71	1.41	1.25	0.00	1.28	1.94	7.48
time (sec)	N/A	0.857	0.187	1.166	0.111	3.886	0.000	0.139	0.266	18.728

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	313	304	509	1619	546	529	1349	416
N.S.	1	1.00	0.98	0.95	1.59	5.06	1.71	1.65	4.22	1.30
time (sec)	N/A	0.878	0.211	0.908	0.118	0.109	5.540	0.131	0.204	0.433

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	260	218	397	1316	405	412	1074	302
N.S.	1	1.00	0.97	0.82	1.49	4.93	1.52	1.54	4.02	1.13
time (sec)	N/A	0.753	0.167	0.916	0.110	0.102	2.258	0.127	0.212	0.794

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	227	153	306	1027	291	319	816	240
N.S.	1	1.00	0.97	0.65	1.31	4.39	1.24	1.36	3.49	1.03
time (sec)	N/A	0.693	0.115	0.913	0.114	0.097	1.032	0.130	0.242	0.765

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	205	101	220	768	189	227	575	191
N.S.	1	1.00	1.01	0.50	1.08	3.78	0.93	1.12	2.83	0.94
time (sec)	N/A	0.690	0.166	0.888	0.112	0.093	0.806	0.132	0.215	0.334

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	158	145	65	158	537	97	160	349	143
N.S.	1	0.93	0.86	0.38	0.93	3.18	0.57	0.95	2.07	0.85
time (sec)	N/A	0.513	0.069	0.790	0.109	0.089	0.385	0.130	0.214	0.703

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	306	337	247	489	440	0	443	672	2492
N.S.	1	0.88	0.97	0.71	1.41	1.27	0.00	1.28	1.94	7.20
time (sec)	N/A	0.834	0.192	1.148	0.122	3.862	0.000	0.143	0.233	17.165

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	373	381	285	784	897	0	664	1648	3637
N.S.	1	0.89	0.91	0.68	1.87	2.14	0.00	1.58	3.93	8.68
time (sec)	N/A	1.092	0.513	1.227	0.129	47.358	0.000	0.140	0.280	25.744

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	322	161	377	0	118	131	0	195	0
N.S.	1	0.99	0.50	1.16	0.00	0.36	0.40	0.00	0.60	0.00
time (sec)	N/A	0.646	7.540	1.533	0.000	0.095	1.532	0.000	0.260	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	264	75	325	0	66	82	0	98	0
N.S.	1	0.99	0.28	1.21	0.00	0.25	0.31	0.00	0.37	0.00
time (sec)	N/A	0.502	4.394	0.891	0.000	0.076	1.082	0.000	0.263	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	723	0	0	0	0	20	0
N.S.	1	1.00	2.73	12.25	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.302	10.030	1.111	0.000	0.000	0.000	0.000	0.268	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	31	0
N.S.	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.301	10.263	1.261	0.000	0.000	0.000	0.000	0.670	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	297	161	339	0	82	126	0	134	0
N.S.	1	1.04	0.56	1.19	0.00	0.29	0.44	0.00	0.47	0.00
time (sec)	N/A	0.572	12.798	1.556	0.000	0.074	1.520	0.000	0.301	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	74	309	0	41	78	0	66	0
N.S.	1	1.00	0.31	1.29	0.00	0.17	0.33	0.00	0.28	0.00
time (sec)	N/A	0.440	10.033	0.865	0.000	0.075	0.943	0.000	0.268	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	46	283	0	14	36	0	20	37
N.S.	1	1.00	0.22	1.37	0.00	0.07	0.17	0.00	0.10	0.18
time (sec)	N/A	0.381	0.003	0.724	0.000	0.093	0.442	0.000	0.218	0.825

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	773	59	161	429	0	0	0	0	35	0
N.S.	1	0.08	0.21	0.55	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.310	10.056	1.039	0.000	0.000	0.000	0.000	0.245	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	335	769	0	0	0	0	59	0
N.S.	1	1.00	5.68	13.03	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.311	10.285	1.235	0.000	0.000	0.000	0.000	0.250	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	420	836	0	0	0	0	83	0
N.S.	1	1.00	7.12	14.17	0.00	0.00	0.00	0.00	1.41	0.00
time (sec)	N/A	0.298	10.529	1.684	0.000	0.000	0.000	0.000	1.687	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	310	161	398	0	141	0	0	306	0
N.S.	1	1.07	0.56	1.37	0.00	0.49	0.00	0.00	1.06	0.00
time (sec)	N/A	0.614	13.277	2.083	0.000	0.092	0.000	0.000	0.323	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	73	336	0	82	78	0	176	0
N.S.	1	1.00	0.29	1.34	0.00	0.33	0.31	0.00	0.70	0.00
time (sec)	N/A	0.435	10.035	0.847	0.000	0.086	3.177	0.000	0.269	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	55	306	0	51	36	0	31	37
N.S.	1	1.00	0.24	1.32	0.00	0.22	0.16	0.00	0.13	0.16
time (sec)	N/A	0.403	0.009	0.746	0.000	0.076	0.544	0.000	0.255	0.617

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	330	753	0	0	0	0	59	0
N.S.	1	1.00	5.32	12.15	0.00	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.310	10.198	1.045	0.000	0.000	0.000	0.000	0.280	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	380	830	0	0	0	0	100	0
N.S.	1	1.00	6.13	13.39	0.00	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.302	10.529	1.303	0.000	0.000	0.000	0.000	1.981	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	323	0	0	0	0	0	393	0
N.S.	1	1.00	3.76	0.00	0.00	0.00	0.00	0.00	4.57	0.00
time (sec)	N/A	0.359	4.210	0.000	0.000	0.000	0.000	0.000	0.729	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	374	0	0	0	0	0	226	0
N.S.	1	1.00	4.45	0.00	0.00	0.00	0.00	0.00	2.69	0.00
time (sec)	N/A	0.351	3.317	0.000	0.000	0.000	0.000	0.000	0.523	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	171	0	0	0	0	0	28	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.351	1.810	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	43	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.362	1.844	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	419	0	0	0	0	0	67	0
N.S.	1	1.00	4.87	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.358	3.482	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	432	0	0	0	0	0	0	0
N.S.	1	1.00	4.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	5.813	0.000	0.000	0.000	0.000	0.000	2.061	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	369	0	0	0	0	0	0	0
N.S.	1	1.00	4.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	5.155	0.000	0.000	0.000	0.000	0.000	1.400	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	256	0	0	0	0	0	39	0
N.S.	1	1.00	2.98	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.346	3.234	0.000	0.000	0.000	0.000	0.000	0.669	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	419	0	0	0	0	0	67	0
N.S.	1	1.00	4.87	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.347	3.507	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	483	0	0	0	0	0	108	0
N.S.	1	1.00	5.43	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.352	5.418	0.000	0.000	0.000	0.000	0.000	1.853	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	156	144	322	399	80	0	45	0
N.S.	1	1.02	1.39	1.29	2.88	3.56	0.71	0.00	0.40	0.00
time (sec)	N/A	0.323	0.398	1.202	0.117	0.114	2.113	0.000	0.259	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	140	118	244	363	76	0	32	0
N.S.	1	1.01	1.54	1.30	2.68	3.99	0.84	0.00	0.35	0.00
time (sec)	N/A	0.296	0.332	1.157	0.111	0.122	1.470	0.000	0.247	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	142	114	130	372	70	0	70	0
N.S.	1	1.00	1.67	1.34	1.53	4.38	0.82	0.00	0.82	0.00
time (sec)	N/A	0.293	0.329	0.905	0.110	0.090	3.227	0.000	0.242	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	47	28	25	50	44	190	0	110	27
N.S.	1	1.31	0.78	0.69	1.39	1.22	5.28	0.00	3.06	0.75
time (sec)	N/A	0.264	0.232	0.803	0.032	0.098	22.568	0.000	0.297	0.590

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	40	37	85	69	709	0	150	44
N.S.	1	1.09	0.73	0.67	1.55	1.25	12.89	0.00	2.73	0.80
time (sec)	N/A	0.286	0.299	0.827	0.034	0.078	115.245	0.000	0.287	0.609

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	87	51	48	119	91	0	0	190	58
N.S.	1	1.18	0.69	0.65	1.61	1.23	0.00	0.00	2.57	0.78
time (sec)	N/A	0.309	0.592	0.830	0.035	0.082	0.000	0.000	0.322	0.874

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	114	62	59	153	113	0	0	230	73
N.S.	1	1.23	0.67	0.63	1.65	1.22	0.00	0.00	2.47	0.78
time (sec)	N/A	0.349	0.529	0.851	0.032	0.088	0.000	0.000	0.337	0.718

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	559	232	0	0	0	0	0	95	0
N.S.	1	1.20	0.50	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.273	10.246	0.000	0.000	0.000	0.000	0.000	0.311	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	537	217	0	0	0	0	0	77	0
N.S.	1	1.16	0.47	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.166	10.117	0.000	0.000	0.000	0.000	0.000	0.310	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	398	409	428	0	0	644	0	0	22	0
N.S.	1	1.03	1.08	0.00	0.00	1.62	0.00	0.00	0.06	0.00
time (sec)	N/A	0.778	2.757	0.000	0.000	16.029	0.000	0.000	0.245	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	464	153	0	0	0	0	0	31	0
N.S.	1	1.03	0.34	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.923	10.050	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	473	541	213	0	0	0	0	0	35	0
N.S.	1	1.14	0.45	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.170	10.099	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	569	240	0	0	0	0	0	70	0
N.S.	1	1.16	0.49	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.312	10.138	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	144	167	164	552	421	126	0	65	0
N.S.	1	1.07	1.25	1.22	4.12	3.14	0.94	0.00	0.49	0.00
time (sec)	N/A	0.407	0.556	1.338	0.129	0.129	5.649	0.000	0.252	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	122	165	144	436	399	121	0	54	0
N.S.	1	1.06	1.43	1.25	3.79	3.47	1.05	0.00	0.47	0.00
time (sec)	N/A	0.371	0.578	1.330	0.119	0.135	3.081	0.000	0.235	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	114	154	132	296	412	0	0	111	0
N.S.	1	1.08	1.45	1.25	2.79	3.89	0.00	0.00	1.05	0.00
time (sec)	N/A	0.364	0.485	1.590	0.119	0.160	0.000	0.000	0.250	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	116	137	117	180	521	0	0	171	0
N.S.	1	1.18	1.40	1.19	1.84	5.32	0.00	0.00	1.74	0.00
time (sec)	N/A	0.372	0.380	1.030	0.122	0.141	0.000	0.000	0.313	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	81	40	37	105	67	0	0	231	44
N.S.	1	1.53	0.75	0.70	1.98	1.26	0.00	0.00	4.36	0.83
time (sec)	N/A	0.318	0.337	0.967	0.040	0.121	0.000	0.000	0.307	0.643

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	118	51	48	155	91	0	0	291	56
N.S.	1	1.64	0.71	0.67	2.15	1.26	0.00	0.00	4.04	0.78
time (sec)	N/A	0.388	0.469	0.960	0.035	0.124	0.000	0.000	0.317	0.664

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	116	62	59	206	113	0	0	351	71
N.S.	1	1.27	0.68	0.65	2.26	1.24	0.00	0.00	3.86	0.78
time (sec)	N/A	0.396	0.622	0.959	0.034	0.105	0.000	0.000	0.370	0.659

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	143	73	70	257	135	0	0	411	86
N.S.	1	1.30	0.66	0.64	2.34	1.23	0.00	0.00	3.74	0.78
time (sec)	N/A	0.430	1.015	0.981	0.050	0.131	0.000	0.000	0.438	0.675

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	99	97	0	0	0	168	0	85	0
N.S.	1	1.11	1.09	0.00	0.00	0.00	1.89	0.00	0.96	0.00
time (sec)	N/A	0.363	7.512	0.000	0.000	0.000	2.182	0.000	0.252	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	85	0	0	0	126	0	65	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	1.42	0.00	0.73	0.00
time (sec)	N/A	0.357	5.749	0.000	0.000	0.000	1.453	0.000	0.261	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	75	0	0	0	121	0	54	0
N.S.	1	1.04	0.84	0.00	0.00	0.00	1.36	0.00	0.61	0.00
time (sec)	N/A	0.385	10.034	0.000	0.000	0.000	1.512	0.000	0.216	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	74	62	0	0	0	0	0	111	0
N.S.	1	0.90	0.76	0.00	0.00	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	0.343	10.036	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	74	70	0	0	0	0	0	171	0
N.S.	1	0.91	0.86	0.00	0.00	0.00	0.00	0.00	2.11	0.00
time (sec)	N/A	0.317	10.052	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	77	85	0	0	0	0	0	231	0
N.S.	1	0.90	0.99	0.00	0.00	0.00	0.00	0.00	2.69	0.00
time (sec)	N/A	0.308	10.055	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	98	95	0	0	0	0	0	291	0
N.S.	1	1.14	1.10	0.00	0.00	0.00	0.00	0.00	3.38	0.00
time (sec)	N/A	0.353	10.130	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	139	179	0	0	0	131	0	173	0
N.S.	1	1.09	1.40	0.00	0.00	0.00	1.02	0.00	1.35	0.00
time (sec)	N/A	0.438	8.096	0.000	0.000	0.000	1.795	0.000	0.263	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	82	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	1.00	0.00
time (sec)	N/A	0.323	5.552	0.000	0.000	0.000	1.092	0.000	0.241	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	21	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.308	10.134	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	32	0
N.S.	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.302	10.261	0.000	0.000	0.000	0.000	0.000	0.527	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	165	223	166	436	554	126	0	54	0
N.S.	1	0.96	1.30	0.97	2.53	3.22	0.73	0.00	0.31	0.00
time (sec)	N/A	0.458	0.777	1.474	0.116	0.139	3.549	0.000	0.259	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	107	163	225	244	362	78	0	31	0
N.S.	1	0.96	1.47	2.03	2.20	3.26	0.70	0.00	0.28	0.00
time (sec)	N/A	0.334	0.447	1.291	0.121	0.098	1.527	0.000	0.235	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	255	171	0	0	0	0	30	0
N.S.	1	1.00	1.72	1.16	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.387	1.589	4.316	0.000	0.000	0.000	0.000	0.238	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	336	246	0	0	0	0	50	0
N.S.	1	0.99	1.55	1.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.471	2.166	4.408	0.000	0.000	0.000	0.000	0.230	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	232	273	213	514	956	0	0	153	0
N.S.	1	1.13	1.33	1.04	2.51	4.66	0.00	0.00	0.75	0.00
time (sec)	N/A	0.636	1.158	1.733	0.116	0.135	0.000	0.000	0.255	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	159	199	166	301	652	0	0	111	0
N.S.	1	1.10	1.38	1.15	2.09	4.53	0.00	0.00	0.77	0.00
time (sec)	N/A	0.444	0.838	1.608	0.113	0.139	0.000	0.000	0.298	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	150	136	134	488	71	0	69	0
N.S.	1	1.04	1.52	1.37	1.35	4.93	0.72	0.00	0.70	0.00
time (sec)	N/A	0.326	0.395	0.918	0.123	0.140	3.237	0.000	0.256	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	189	328	243	0	0	0	0	62	0
N.S.	1	1.06	1.83	1.36	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.415	2.466	1.168	0.000	0.000	0.000	0.000	0.224	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	263	370	288	0	0	0	0	104	0
N.S.	1	1.01	1.42	1.10	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.555	3.395	1.332	0.000	0.000	0.000	0.000	0.235	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	151	208	180	406	482	170	0	125	0
N.S.	1	0.87	1.20	1.03	2.33	2.77	0.98	0.00	0.72	0.00
time (sec)	N/A	0.380	0.829	1.230	0.113	0.101	6.705	0.000	0.287	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	129	180	152	322	424	82	0	83	0
N.S.	1	0.91	1.28	1.08	2.28	3.01	0.58	0.00	0.59	0.00
time (sec)	N/A	0.352	0.610	1.198	0.111	0.099	1.949	0.000	0.243	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	107	163	225	244	362	78	0	31	0
N.S.	1	0.96	1.47	2.03	2.20	3.26	0.70	0.00	0.28	0.00
time (sec)	N/A	0.306	0.034	0.020	0.117	0.089	1.262	0.000	0.228	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	150	136	134	488	71	0	69	0
N.S.	1	1.04	1.52	1.37	1.35	4.93	0.72	0.00	0.70	0.00
time (sec)	N/A	0.307	0.057	0.799	0.124	0.094	3.297	0.000	0.270	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	47	37	34	51	54	190	0	109	33
N.S.	1	0.77	0.61	0.56	0.84	0.89	3.11	0.00	1.79	0.54
time (sec)	N/A	0.258	0.281	0.823	0.027	0.084	22.507	0.000	0.259	0.804

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	86	87	709	0	149	87
N.S.	1	0.97	0.66	0.57	0.95	0.96	7.79	0.00	1.64	0.96
time (sec)	N/A	0.317	0.394	0.826	0.036	0.107	116.091	0.000	0.271	0.696

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	80	71	120	121	0	0	189	105
N.S.	1	0.95	0.66	0.59	0.99	1.00	0.00	0.00	1.56	0.87
time (sec)	N/A	0.357	0.542	0.840	0.034	0.089	0.000	0.000	0.331	0.675

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	142	100	90	154	155	0	0	229	132
N.S.	1	0.94	0.66	0.60	1.02	1.03	0.00	0.00	1.52	0.87
time (sec)	N/A	0.407	0.793	0.865	0.036	0.095	0.000	0.000	0.347	0.690

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	167	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	1.96	0.00
time (sec)	N/A	0.311	8.537	0.000	0.000	0.000	3.122	0.000	0.263	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	125	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	1.51	0.00
time (sec)	N/A	0.317	7.459	0.000	0.000	0.000	2.106	0.000	0.297	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	82	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	1.00	0.00
time (sec)	N/A	0.315	0.065	0.000	0.000	0.000	1.118	0.000	0.227	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	31	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	0.38	0.00
time (sec)	N/A	0.318	10.051	0.000	0.000	0.000	1.004	0.000	0.221	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	93	66	0	0	0	78	0	69	0
N.S.	1	1.19	0.85	0.00	0.00	0.00	1.00	0.00	0.88	0.00
time (sec)	N/A	0.337	10.046	0.000	0.000	0.000	4.019	0.000	0.259	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	94	75	0	0	0	78	0	109	0
N.S.	1	1.12	0.89	0.00	0.00	0.00	0.93	0.00	1.30	0.00
time (sec)	N/A	0.334	10.054	0.000	0.000	0.000	39.442	0.000	0.272	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	208	293	243	672	717	270	0	242	0
N.S.	1	0.80	1.13	0.94	2.59	2.77	1.04	0.00	0.93	0.00
time (sec)	N/A	0.513	1.989	1.413	0.121	0.108	32.661	0.000	0.274	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	187	256	204	552	634	131	0	174	0
N.S.	1	0.88	1.21	0.96	2.60	2.99	0.62	0.00	0.82	0.00
time (sec)	N/A	0.486	1.358	1.381	0.114	0.095	5.560	0.000	0.250	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	165	223	166	436	554	126	0	54	0
N.S.	1	0.96	1.30	0.97	2.53	3.22	0.73	0.00	0.31	0.00
time (sec)	N/A	0.446	0.104	0.021	0.112	0.104	2.874	0.000	0.240	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	159	199	166	301	652	0	0	111	0
N.S.	1	1.10	1.38	1.15	2.09	4.53	0.00	0.00	0.77	0.00
time (sec)	N/A	0.431	0.238	0.021	0.112	0.124	0.000	0.000	0.254	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	164	186	221	190	719	0	0	171	0
N.S.	1	1.12	1.27	1.50	1.29	4.89	0.00	0.00	1.16	0.00
time (sec)	N/A	0.437	1.168	1.235	0.108	0.107	0.000	0.000	0.283	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	84	73	71	109	103	0	0	231	148
N.S.	1	0.72	0.62	0.61	0.93	0.88	0.00	0.00	1.97	1.26
time (sec)	N/A	0.332	0.902	0.999	0.039	0.082	0.000	0.000	0.342	0.736

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	150	106	96	159	152	0	0	291	176
N.S.	1	0.93	0.65	0.59	0.98	0.94	0.00	0.00	1.80	1.09
time (sec)	N/A	0.453	1.199	0.980	0.034	0.090	0.000	0.000	0.323	0.759

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	186	138	126	210	200	0	0	351	217
N.S.	1	0.90	0.67	0.61	1.01	0.97	0.00	0.00	1.70	1.05
time (sec)	N/A	0.544	1.661	0.987	0.036	0.097	0.000	0.000	0.366	0.769

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	207	169	156	261	246	0	0	411	257
N.S.	1	0.83	0.68	0.63	1.05	0.99	0.00	0.00	1.66	1.04
time (sec)	N/A	0.624	2.566	1.010	0.038	0.114	0.000	0.000	0.392	0.842

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	143	177	0	0	0	418	0	310	0
N.S.	1	1.09	1.35	0.00	0.00	0.00	3.19	0.00	2.37	0.00
time (sec)	N/A	0.459	12.804	0.000	0.000	0.000	4.012	0.000	0.296	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	141	176	0	0	0	270	0	242	0
N.S.	1	1.08	1.35	0.00	0.00	0.00	2.08	0.00	1.86	0.00
time (sec)	N/A	0.468	12.966	0.000	0.000	0.000	2.655	0.000	0.311	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	139	179	0	0	0	131	0	173	0
N.S.	1	1.09	1.40	0.00	0.00	0.00	1.02	0.00	1.35	0.00
time (sec)	N/A	0.443	0.077	0.000	0.000	0.000	1.649	0.000	0.256	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	104	0	0	0	126	0	54	0
N.S.	1	1.08	0.83	0.00	0.00	0.00	1.01	0.00	0.43	0.00
time (sec)	N/A	0.461	15.104	0.000	0.000	0.000	1.528	0.000	0.217	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	146	171	0	0	0	0	0	111	0
N.S.	1	1.11	1.30	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.506	13.144	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	149	171	0	0	0	0	0	171	0
N.S.	1	1.15	1.32	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.484	15.051	0.000	0.000	0.000	0.000	0.000	0.245	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	121	120	110	182	166	0	0	393	271
N.S.	1	0.65	0.65	0.59	0.98	0.89	0.00	0.00	2.11	1.46
time (sec)	N/A	0.382	1.529	1.061	0.034	0.105	0.000	0.000	0.388	0.916

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	327	655	338	0	643	0	0	330	0
N.S.	1	0.99	1.98	1.02	0.00	1.94	0.00	0.00	1.00	0.00
time (sec)	N/A	0.825	10.861	3.009	0.000	5.591	0.000	0.000	0.491	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	276	467	326	0	535	0	0	190	0
N.S.	1	1.01	1.71	1.19	0.00	1.96	0.00	0.00	0.70	0.00
time (sec)	N/A	0.623	7.913	2.401	0.000	0.525	0.000	0.000	0.384	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	237	423	326	0	469	0	0	21	0
N.S.	1	1.02	1.82	1.40	0.00	2.01	0.00	0.00	0.09	0.00
time (sec)	N/A	0.492	4.912	1.535	0.000	0.125	0.000	0.000	0.239	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	255	171	0	0	0	0	30	0
N.S.	1	1.00	1.72	1.16	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.371	0.229	1.181	0.000	0.000	0.000	0.000	0.214	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	189	328	243	0	0	0	0	62	0
N.S.	1	1.06	1.83	1.36	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.433	0.585	1.157	0.000	0.000	0.000	0.000	0.246	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	249	364	243	0	0	0	0	104	0
N.S.	1	1.10	1.61	1.08	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.563	4.361	1.395	0.000	0.000	0.000	0.000	0.276	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	322	420	306	0	0	0	0	146	0
N.S.	1	1.15	1.50	1.09	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.756	4.737	1.296	0.000	0.000	0.000	0.000	0.281	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	190	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	3.17	0.00
time (sec)	N/A	0.300	10.259	0.000	0.000	0.000	0.000	0.000	0.378	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	21	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.291	0.044	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	30	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.318	10.070	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	62	0
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.313	10.286	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	104	0
N.S.	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.320	10.830	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	341	698	477	0	819	0	0	0	0
N.S.	1	0.97	1.99	1.36	0.00	2.33	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	11.021	2.036	0.000	3.521	0.000	0.000	0.939	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	300	510	416	0	631	0	0	1010	0
N.S.	1	1.00	1.69	1.38	0.00	2.10	0.00	0.00	3.36	0.00
time (sec)	N/A	0.640	9.559	1.474	0.000	0.376	0.000	0.000	0.491	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	185	319	214	0	0	0	0	831	0
N.S.	1	1.02	1.75	1.18	0.00	0.00	0.00	0.00	4.57	0.00
time (sec)	N/A	0.406	2.779	1.232	0.000	0.000	0.000	0.000	0.445	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	336	246	0	0	0	0	50	0
N.S.	1	0.99	1.55	1.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.462	0.244	1.208	0.000	0.000	0.000	0.000	0.232	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	263	370	288	0	0	0	0	104	0
N.S.	1	1.01	1.42	1.10	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.559	0.513	1.239	0.000	0.000	0.000	0.000	0.244	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	338	443	326	0	0	0	0	172	0
N.S.	1	1.04	1.37	1.01	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.754	8.855	1.511	0.000	0.000	0.000	0.000	0.265	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	1075	0
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	17.92	0.00
time (sec)	N/A	0.294	10.364	0.000	0.000	0.000	0.000	0.000	0.704	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	32	0
N.S.	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.292	0.239	0.000	0.000	0.000	0.000	0.000	0.478	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	50	0
N.S.	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.292	10.328	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	104	0
N.S.	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.290	10.670	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	172	0
N.S.	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	2.77	0.00
time (sec)	N/A	0.290	11.062	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	505	1171	661	0	1555	0	0	0	0
N.S.	1	0.93	2.16	1.22	0.00	2.87	0.00	0.00	0.00	0.00
time (sec)	N/A	1.395	12.470	2.857	0.000	88.771	0.000	0.000	3.620	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	432	908	580	0	1246	0	0	0	0
N.S.	1	0.94	1.98	1.27	0.00	2.72	0.00	0.00	0.00	0.00
time (sec)	N/A	1.129	12.183	2.324	0.000	19.586	0.000	0.000	2.875	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	375	651	498	0	954	0	0	0	0
N.S.	1	0.96	1.66	1.27	0.00	2.44	0.00	0.00	0.00	0.00
time (sec)	N/A	0.851	11.116	1.985	0.000	2.337	0.000	0.000	0.904	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	222	344	244	0	0	0	0	0	0
N.S.	1	1.02	1.59	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	4.761	1.309	0.000	0.000	0.000	0.000	1.548	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	251	366	282	0	0	0	0	0	0
N.S.	1	0.98	1.43	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	5.230	1.327	0.000	0.000	0.000	0.000	1.276	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	294	407	292	0	0	0	0	70	0
N.S.	1	0.96	1.33	0.95	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.622	7.668	1.952	0.000	0.000	0.000	0.000	0.269	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	369	428	416	0	0	0	0	146	0
N.S.	1	1.01	1.17	1.14	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.766	14.166	1.536	0.000	0.000	0.000	0.000	0.260	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	466	337	424	0	0	0	0	240	0
N.S.	1	1.05	0.76	0.96	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	1.080	15.975	1.554	0.000	0.000	0.000	0.000	0.293	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	285	0	0	0	0	0	0	0
N.S.	1	1.00	4.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	10.508	0.000	0.000	0.000	0.000	0.000	1.423	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	407	0	0	0	0	0	43	0
N.S.	1	1.00	6.90	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.290	10.652	0.000	0.000	0.000	0.000	0.000	0.814	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	418	0	0	0	0	0	70	0
N.S.	1	1.00	7.08	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.300	10.789	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	146	0
N.S.	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	2.35	0.00
time (sec)	N/A	0.295	11.003	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	240	0
N.S.	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	3.87	0.00
time (sec)	N/A	0.292	11.687	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	161	90	0	0	0	0	0	0	0
N.S.	1	1.85	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	5.557	0.000	0.000	0.000	0.000	0.000	1.582	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	122	89	0	0	0	0	0	0	0
N.S.	1	1.40	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	5.825	0.000	0.000	0.000	0.000	0.000	0.554	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	122	89	0	0	0	0	0	40	0
N.S.	1	1.40	1.02	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.356	3.861	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	48	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.309	3.568	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	86	0	0	0	0	0	21	0
N.S.	1	1.01	1.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.292	5.738	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	87	89	0	0	0	0	0	48	0
N.S.	1	1.01	1.03	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.295	3.547	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	121	89	0	0	0	0	0	40	0
N.S.	1	1.41	1.03	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.353	5.673	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	121	89	0	0	0	0	0	60	0
N.S.	1	1.41	1.03	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.357	5.917	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	160	90	0	0	0	0	0	128	0
N.S.	1	1.86	1.05	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.413	5.804	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	1249	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	15.81	0.00
time (sec)	N/A	0.361	0.332	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	283	288	137	0	0	0	0	0	0	0
N.S.	1	1.02	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.890	7.047	0.000	0.000	0.000	0.000	0.000	0.313	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	175	106	0	0	0	121	0	1442	0
N.S.	1	1.01	0.61	0.00	0.00	0.00	0.70	0.00	8.34	0.00
time (sec)	N/A	0.554	5.222	0.000	0.000	0.000	92.412	0.000	0.239	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	0	0	0	75	0	498	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.88	0.00	5.86	0.00
time (sec)	N/A	0.357	0.156	0.000	0.000	0.000	38.046	0.000	0.225	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	21	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.304	0.397	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	32	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.300	0.459	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	43	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.301	0.591	0.000	0.000	0.000	0.000	0.000	0.394	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	137	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.58	2.47
time (sec)	N/A	0.296	0.640	4.700	0.000	0.119	0.000	0.000	0.271	1.278

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [79] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	23	0.130
2	A	5	4	1.00	23	0.174
3	A	5	4	1.00	23	0.174
4	F	0	0	N/A	0.000	N/A
5	A	4	3	1.01	23	0.130
6	A	9	8	1.17	19	0.421
7	A	3	3	1.18	23	0.130
8	A	3	3	1.00	23	0.130
9	A	3	3	1.00	23	0.130
10	A	2	2	1.00	17	0.118
11	A	2	2	1.00	17	0.118
12	A	2	2	1.00	17	0.118
13	A	2	2	1.00	15	0.133
14	A	10	9	0.91	17	0.529
15	A	10	9	0.93	17	0.529
16	A	11	10	0.93	17	0.588
17	A	2	2	1.00	19	0.105
18	A	2	2	1.00	19	0.105
19	A	2	2	1.00	17	0.118
20	A	2	2	1.00	19	0.105
21	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	11	10	0.87	19	0.526
23	A	2	2	1.00	19	0.105
24	A	2	2	1.00	19	0.105
25	A	2	2	1.00	17	0.118
26	A	2	2	1.00	19	0.105
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	19	0.105
29	A	2	2	1.00	19	0.105
30	A	2	2	1.00	19	0.105
31	A	2	2	1.00	19	0.105
32	A	10	9	0.90	17	0.529
33	A	10	9	0.86	19	0.474
34	A	11	10	0.89	19	0.526
35	A	2	2	1.00	19	0.105
36	A	2	2	1.00	19	0.105
37	A	2	2	1.00	19	0.105
38	A	2	2	1.00	19	0.105
39	A	10	9	0.93	17	0.529
40	A	12	11	0.88	19	0.579
41	A	14	13	0.89	19	0.684
42	A	5	5	0.99	21	0.238
43	A	3	3	0.99	19	0.158
44	A	2	2	1.00	21	0.095
45	A	2	2	1.00	21	0.095
46	A	4	4	1.04	21	0.190
47	A	2	2	1.00	19	0.105
48	A	1	1	1.00	11	0.091
49	C	2	2	0.08	21	0.095
50	A	2	2	1.00	21	0.095
51	A	2	2	1.00	21	0.095
52	A	4	4	1.07	21	0.190
53	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	11	0.182
55	A	2	2	1.00	21	0.095
56	A	2	2	1.00	21	0.095
57	A	3	3	1.00	23	0.130
58	A	3	3	1.00	23	0.130
59	A	3	3	1.00	23	0.130
60	A	3	3	1.00	23	0.130
61	A	3	3	1.00	23	0.130
62	A	3	3	1.00	23	0.130
63	A	3	3	1.00	23	0.130
64	A	3	3	1.00	23	0.130
65	A	3	3	1.00	23	0.130
66	A	3	3	1.00	23	0.130
67	A	3	3	1.02	20	0.150
68	A	2	2	1.01	20	0.100
69	A	2	2	1.00	20	0.100
70	A	2	2	1.31	20	0.100
71	A	3	3	1.09	20	0.150
72	A	4	4	1.18	20	0.200
73	A	5	5	1.23	20	0.250
74	A	22	21	1.20	22	0.955
75	A	20	19	1.16	22	0.864
76	A	11	10	1.03	22	0.455
77	A	14	13	1.03	22	0.591
78	A	20	19	1.14	22	0.864
79	A	23	22	1.16	22	1.000
80	A	5	5	1.07	22	0.227
81	A	4	4	1.06	22	0.182
82	A	5	5	1.08	22	0.227
83	A	4	4	1.18	22	0.182
84	A	3	3	1.53	22	0.136
85	A	4	4	1.64	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	6	1.27	22	0.273
87	A	7	7	1.30	22	0.318
88	A	5	5	1.11	22	0.227
89	A	5	5	1.04	22	0.227
90	A	5	5	1.04	22	0.227
91	A	4	4	0.90	22	0.182
92	A	4	4	0.91	22	0.182
93	A	4	4	0.90	22	0.182
94	A	5	5	1.14	22	0.227
95	A	4	4	1.09	21	0.190
96	A	3	3	1.00	19	0.158
97	A	2	2	1.00	21	0.095
98	A	2	2	1.00	21	0.095
99	A	3	3	0.96	21	0.143
100	A	2	2	0.96	19	0.105
101	A	1	1	1.00	21	0.048
102	A	2	2	0.99	21	0.095
103	A	5	5	1.13	21	0.238
104	A	4	4	1.10	21	0.190
105	A	2	2	1.04	19	0.105
106	A	2	2	1.06	21	0.095
107	A	4	4	1.01	21	0.190
108	A	4	4	0.87	19	0.211
109	A	3	3	0.91	19	0.158
110	A	2	2	0.96	19	0.105
111	A	2	2	1.04	19	0.105
112	A	2	2	0.77	19	0.105
113	A	3	3	0.97	19	0.158
114	A	4	4	0.95	19	0.211
115	A	5	5	0.94	19	0.263
116	A	3	3	1.00	19	0.158
117	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	3	3	1.00	19	0.158
119	A	3	3	1.00	19	0.158
120	A	3	3	1.19	19	0.158
121	A	3	3	1.12	19	0.158
122	A	5	5	0.80	21	0.238
123	A	4	4	0.88	21	0.190
124	A	3	3	0.96	21	0.143
125	A	4	4	1.10	21	0.190
126	A	3	3	1.12	21	0.143
127	A	3	3	0.72	21	0.143
128	A	4	4	0.93	21	0.190
129	A	5	5	0.90	21	0.238
130	A	6	6	0.83	21	0.286
131	A	4	4	1.09	21	0.190
132	A	4	4	1.08	21	0.190
133	A	4	4	1.09	21	0.190
134	A	4	4	1.08	21	0.190
135	A	4	4	1.11	21	0.190
136	A	4	4	1.15	21	0.190
137	A	4	4	0.65	21	0.190
138	A	7	7	0.99	21	0.333
139	A	5	5	1.01	21	0.238
140	A	3	3	1.02	21	0.143
141	A	1	1	1.00	21	0.048
142	A	2	2	1.06	21	0.095
143	A	5	5	1.10	21	0.238
144	A	7	7	1.15	21	0.333
145	A	2	2	1.00	21	0.095
146	A	2	2	1.00	21	0.095
147	A	2	2	1.00	21	0.095
148	A	2	2	1.00	21	0.095
149	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	6	0.97	21	0.286
151	A	4	4	1.00	21	0.190
152	A	2	2	1.02	21	0.095
153	A	2	2	0.99	21	0.095
154	A	4	4	1.01	21	0.190
155	A	6	6	1.04	21	0.286
156	A	2	2	1.00	21	0.095
157	A	2	2	1.00	21	0.095
158	A	2	2	1.00	21	0.095
159	A	2	2	1.00	21	0.095
160	A	2	2	1.00	21	0.095
161	A	10	10	0.93	21	0.476
162	A	8	8	0.94	21	0.381
163	A	6	6	0.96	21	0.286
164	A	3	3	1.02	21	0.143
165	A	3	3	0.98	21	0.143
166	A	4	4	0.96	21	0.190
167	A	6	6	1.01	21	0.286
168	A	9	9	1.05	21	0.429
169	A	2	2	1.00	21	0.095
170	A	2	2	1.00	21	0.095
171	A	2	2	1.00	21	0.095
172	A	2	2	1.00	21	0.095
173	A	2	2	1.00	21	0.095
174	A	3	3	1.85	23	0.130
175	A	2	2	1.40	23	0.087
176	A	2	2	1.40	23	0.087
177	A	1	1	1.00	23	0.043
178	A	1	1	1.01	23	0.043
179	A	1	1	1.01	23	0.043
180	A	2	2	1.41	23	0.087
181	A	2	2	1.41	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	3	3	1.86	23	0.130
183	A	3	3	1.00	19	0.158
184	A	7	7	1.02	19	0.368
185	A	5	5	1.01	19	0.263
186	A	3	3	1.00	17	0.176
187	A	2	2	1.00	19	0.105
188	A	2	2	1.00	19	0.105
189	A	2	2	1.00	19	0.105
190	A	1	1	1.00	50	0.020

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx=\frac{x\left(1+\frac{bx^n}{a}\right)^{3/4}\sqrt[4]{c+dx^n}\,\text{AppellF1}\left(\frac{1}{n},\frac{15}{4},-\frac{1}{4},1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{a^3(a+bx^n)^{3/4}\sqrt[4]{1+\frac{dx^n}{c}}}$$

output
```
x*(1+b*x^n/a)^(3/4)*(c+d*x^n)^(1/4)*AppellF1(1/n,15/4,-1/4,1+1/n,-b*x^n/a,
-d*x^n/c)/a^3/(a+b*x^n)^(3/4)/(1+d*x^n/c)^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1071 vs. 2(88) = 176.

Time = 5.93 (sec) , antiderivative size = 1071, normalized size of antiderivative = 12.17

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx=\text{Too large to display}$$

input
```
Integrate[(c + d*x^n)^(1/4)/(a + b*x^n)^(15/4),x]
```

output
```
(2*x*((d*(-2 + n)*(a*b*c*d*(-32 + 136*n - 143*n^2) + 4*a^2*d^2*(4 - 16*n +
 15*n^2) + b^2*c^2*(16 - 72*n + 77*n^2))*x^n*(1 + (b*x^n)/a)^(3/4)*(1 + (d
*x^n)/c)^(3/4)*AppellF1[1 + n^(-1), 3/4, 3/4, 2 + n^(-1), -((b*x^n)/a), -(
(d*x^n)/c)])/(1 + n) + (6*a*d*n*x^n*(c + d*x^n)*(a^4*d^2*(16 - 76*n + 111*
n^2) + b^4*c^2*(16 - 72*n + 77*n^2)*x^(2*n) + 2*a^3*b*d*(-4*c*(4 - 20*n +
31*n^2) + d*(16 - 70*n + 75*n^2)*x^n) - a*b^3*c*x^n*(c*(-32 + 156*n - 187*
n^2) + d*(32 - 136*n + 143*n^2)*x^n) + a^2*b^2*(c^2*(16 - 84*n + 131*n^2)
- c*d*(64 - 296*n + 349*n^2)*x^n + 4*d^2*(4 - 16*n + 15*n^2)*x^(2*n)))*App
ellF1[1 + n^(-1), 3/4, 7/4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 6*b*
c*n*x^n*(c + d*x^n)*(a^4*d^2*(16 - 76*n + 111*n^2) + b^4*c^2*(16 - 72*n +
77*n^2)*x^(2*n) + 2*a^3*b*d*(-4*c*(4 - 20*n + 31*n^2) + d*(16 - 70*n + 75*
n^2)*x^n) - a*b^3*c*x^n*(c*(-32 + 156*n - 187*n^2) + d*(32 - 136*n + 143*n
^2)*x^n) + a^2*b^2*(c^2*(16 - 84*n + 131*n^2) - c*d*(64 - 296*n + 349*n^2)
*x^n + 4*d^2*(4 - 16*n + 15*n^2)*x^(2*n)))*AppellF1[1 + n^(-1), 7/4, 3/4,
2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*c*(1 + n)*(b^4*c^2*(16 - 72*
n + 77*n^2)*x^(2*n)*(3*c*n + 4*d*x^n) + a^4*d^2*(231*c*n^3 + 4*d*(16 - 76*
n + 111*n^2)*x^n) + 2*a^3*b*d*(-231*c^2*n^3 + c*d*(-64 + 344*n - 640*n^2 +
 231*n^3)*x^n + 4*d^2*(16 - 70*n + 75*n^2)*x^(2*n)) - 2*a*b^3*c*x^n*(-3*c^
2*n*(8 - 50*n + 77*n^2) + c*d*(-64 + 360*n - 584*n^2 + 231*n^3)*x^n + 2*d^
2*(32 - 136*n + 143*n^2)*x^(2*n)) + a^2*b^2*(231*c^3*n^3 - 4*c^2*d*(-16...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx$$

$$\Big\downarrow \ 937$$

$$\frac{\left(\frac{bx^n}{a}+1\right)^{3/4}\int \frac{\sqrt[4]{dx^n+c}}{\left(\frac{bx^n}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^n\right)^{3/4}}$$

$$\Big\downarrow \ 937$$

$$\frac{\left(\frac{bx^n}{a}+1\right)^{3/4}\sqrt[4]{c+dx^n}\int \frac{\sqrt[4]{\frac{dx^n}{c}+1}}{\left(\frac{bx^n}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^n\right)^{3/4}\sqrt[4]{\frac{dx^n}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^n}{a}+1\right)^{3/4}\sqrt[4]{c+dx^n}\,\text{AppellF1}\left(\frac{1}{n},\frac{15}{4},-\frac{1}{4},1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{a^3\left(a+bx^n\right)^{3/4}\sqrt[4]{\frac{dx^n}{c}+1}}$$

input
```
Int[(c + d*x^n)^(1/4)/(a + b*x^n)^(15/4),x]
```

output
```
(x*(1 + (b*x^n)/a)^(3/4)*(c + d*x^n)^(1/4)*AppellF1[n^(-1), 15/4, -1/4, 1
+ n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(a^3*(a + b*x^n)^(3/4)*(1 + (d*x^n)
/c)^(1/4))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(c+d\,x^{n})^{\frac{1}{4}}}{(a+b\,x^{n})^{\frac{15}{4}}}dx$$

input
```
int((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x)
```

output
```
int((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*x**n)**(1/4)/(a+b*x**n)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx=\int \frac{(dx^n+c)^{\frac{1}{4}}}{(bx^n+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(1/4)/(b*x^n + a)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx=\int \frac{(dx^n+c)^{\frac{1}{4}}}{(bx^n+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(1/4)/(b*x^n + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx=\int \frac{(c+d\,x^n)^{1/4}}{(a+b\,x^n)^{15/4}}\,dx$$

input
```
int((c + d*x^n)^(1/4)/(a + b*x^n)^(15/4),x)
```

output
```
int((c + d*x^n)^(1/4)/(a + b*x^n)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx^n}}{(a+bx^n)^{15/4}}\,dx = \int \frac{(x^n d+c)^{\frac{1}{4}}}{x^{3n}\left(x^n b+a\right)^{\frac{3}{4}} b^3+3x^{2n}\left(x^n b+a\right)^{\frac{3}{4}} a\,b^2+3x^n\left(x^n b+a\right)^{\frac{3}{4}} a^2 b+\left(x^n b+a\right)^{\frac{3}{4}} a^3}dx$$

input
```
int((c+d*x^n)^(1/4)/(a+b*x^n)^(15/4),x)
```

output
```
int((x**n*d + c)**(1/4)/(x**(3*n)*(x**n*b + a)**(3/4)*b**3 + 3*x**(2*n)*(x
**n*b + a)**(3/4)*a*b**2 + 3*x**n*(x**n*b + a)**(3/4)*a**2*b + (x**n*b + a
)**(3/4)*a**3),x)
```

3.2 $\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx=\frac{\left(1+\frac{b}{ax^4}\right)^{3/4}\sqrt[4]{c+\frac{d}{x^4}}\,x\,\text{AppellF1}\left(-\frac{1}{4},\frac{15}{4},-\frac{1}{4},\frac{3}{4},-\frac{b}{ax^4},-\frac{d}{cx^4}\right)}{a^3\left(a+\frac{b}{x^4}\right)^{3/4}\sqrt[4]{1+\frac{d}{cx^4}}}$$

output
```
(1+b/a/x^4)^(3/4)*(c+d/x^4)^(1/4)*x*AppellF1(-1/4,15/4,-1/4,3/4,-b/a/x^4,-
d/c/x^4)/a^3/(a+b/x^4)^(3/4)/(1+d/c/x^4)^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(86) = 172.

Time = 5.41 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.98

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx=\frac{\sqrt[4]{c+\frac{d}{x^4}}\,x\left(-7(d+cx^4)\left(96b^4c^2+131a^4d^2x^8+a^3bdx^4(187d-290cx^4)+ab^3c(-179\right.\right.}{}$$

input
```
Integrate[(c + d/x^4)^(1/4)/(a + b/x^4)^(15/4),x]
```

output
```
((c + d/x^4)^(1/4)*x*(-7*(d + c*x^4)*(96*b^4*c^2 + 131*a^4*d^2*x^8 + a^3*b
*d*x^4*(187*d - 290*c*x^4) + a*b^3*c*(-179*d + 228*c*x^4) + a^2*b^2*(77*d^
2 - 427*c*d*x^4 + 153*c^2*x^8)) + 7*d*(96*b^2*c^2 - 179*a*b*c*d + 77*a^2*d
^2)*(b + a*x^4)^2*(1 + (a*x^4)/b)^(3/4)*(1 + (c*x^4)/d)^(3/4)*AppellF1[3/4
, 3/4, 3/4, 7/4, -((a*x^4)/b), -((c*x^4)/d)] + 2*c*(192*b^2*c^2 - 376*a*b*
c*d + 181*a^2*d^2)*x^4*(b + a*x^4)^2*(1 + (a*x^4)/b)^(3/4)*(1 + (c*x^4)/d)
^(3/4)*AppellF1[7/4, 3/4, 3/4, 11/4, -((a*x^4)/b), -((c*x^4)/d)]))/(1617*a
^3*(b*c - a*d)^2*(a + b/x^4)^(3/4)*(b + a*x^4)^2*(d + c*x^4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {899, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt[4]{c+\frac{d}{x^4}}x^2}{\left(a+\frac{b}{x^4}\right)^{15/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{\left(\frac{b}{ax^4}+1\right)^{3/4}\int \frac{\sqrt[4]{c+\frac{d}{x^4}}x^2}{\left(\frac{b}{ax^4}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^4}\right)^{3/4}}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{\left(\frac{b}{ax^4}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^4}}\int \frac{\sqrt[4]{\frac{d}{cx^4}+1}x^2}{\left(\frac{b}{ax^4}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^4}\right)^{3/4}\sqrt[4]{\frac{d}{cx^4}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x\left(\frac{b}{ax^4}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^4}}\,\text{AppellF1}\left(-\frac{1}{4},\frac{15}{4},-\frac{1}{4},\frac{3}{4},-\frac{b}{ax^4},-\frac{d}{cx^4}\right)}{a^3\left(a+\frac{b}{x^4}\right)^{3/4}\sqrt[4]{\frac{d}{cx^4}+1}}$$

input
```
Int[(c + d/x^4)^(1/4)/(a + b/x^4)^(15/4),x]
```

output
```
((1 + b/(a*x^4))^(3/4)*(c + d/x^4)^(1/4)*x*AppellF1[-1/4, 15/4, -1/4, 3/4,
 -(b/(a*x^4)), -(d/(c*x^4))])/(a^3*(a + b/x^4)^(3/4)*(1 + d/(c*x^4))^(1/4)
)
```

Defintions of rubi rules used

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x^4}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^4}\right)^{\frac{15}{4}}}dx$$

input
```
int((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x)
```

output
```
int((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx = \int \frac{\left(c+\frac{d}{x^4}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^4}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x, algorithm="fricas")
```

output
```
integral(x^16*((a*x^4 + b)/x^4)^(1/4)*((c*x^4 + d)/x^4)^(1/4)/(a^4*x^16 +
4*a^3*b*x^12 + 6*a^2*b^2*x^8 + 4*a*b^3*x^4 + b^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx = \text{Timed out}$$

input
```
integrate((c+d/x**4)**(1/4)/(a+b/x**4)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^4}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^4}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x, algorithm="maxima")
```

output
```
integrate((c + d/x^4)^(1/4)/(a + b/x^4)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^4}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^4}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x, algorithm="giac")
```

output
```
integrate((c + d/x^4)^(1/4)/(a + b/x^4)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^4}\right)^{1/4}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx$$

input
```
int((c + d/x^4)^(1/4)/(a + b/x^4)^(15/4),x)
```

output
```
int((c + d/x^4)^(1/4)/(a + b/x^4)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^4}}}{\left(a+\frac{b}{x^4}\right)^{15/4}}\,dx = \int \frac{(c\,x^4+d)^{\frac{1}{4}}\,x^{14}}{(a\,x^4+b)^{\frac{3}{4}}\,a^3x^{12}+3\,(a\,x^4+b)^{\frac{3}{4}}\,a^2b\,x^8+3\,(a\,x^4+b)^{\frac{3}{4}}\,a\,b^2x^4+(a\,x^4+b)^{\frac{3}{4}}\,b^3}dx$$

input
```
int((c+d/x^4)^(1/4)/(a+b/x^4)^(15/4),x)
```

output
```
int(((c*x**4 + d)**(1/4)*x**14)/((a*x**4 + b)**(3/4)*a**3*x**12 + 3*(a*x**
4 + b)**(3/4)*a**2*b*x**8 + 3*(a*x**4 + b)**(3/4)*a*b**2*x**4 + (a*x**4 +
b)**(3/4)*b**3),x)
```

3.3 $\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx=\frac{\left(1+\frac{b}{ax^3}\right)^{3/4}\sqrt[4]{c+\frac{d}{x^3}}\,x\,\text{AppellF1}\left(-\frac{1}{3},\frac{15}{4},-\frac{1}{4},\frac{2}{3},-\frac{b}{ax^3},-\frac{d}{cx^3}\right)}{a^3\left(a+\frac{b}{x^3}\right)^{3/4}\sqrt[4]{1+\frac{d}{cx^3}}}$$

output
```
(1+b/a/x^3)^(3/4)*(c+d/x^3)^(1/4)*x*AppellF1(-1/3,15/4,-1/4,2/3,-b/a/x^3,-
d/c/x^3)/a^3/(a+b/x^3)^(3/4)/(1+d/c/x^3)^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(86) = 172.

Time = 9.84 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.98

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx=\frac{2\sqrt[4]{c+\frac{d}{x^3}}\,x\left(-22(d+cx^3)\left(925b^4c^2+1243a^4d^2x^6+2a^3bdx^3(901d-1372cx^3)+ab^3\right.\right.}{}$$

input
```
Integrate[(c + d/x^3)^(1/4)/(a + b/x^3)^(15/4),x]
```

output
```
(2*(c + d/x^3)^(1/4)*x*(-22*(d + c*x^3)*(925*b^4*c^2 + 1243*a^4*d^2*x^6 +
2*a^3*b*d*x^3*(901*d - 1372*c*x^3) + a*b^3*c*(-1727*d + 2183*c*x^3) + a^2*
b^2*(748*d^2 - 4093*c*d*x^3 + 1447*c^2*x^6)) + 22*d*(925*b^2*c^2 - 1727*a*
b*c*d + 748*a^2*d^2)*(b + a*x^3)^2*(1 + (a*x^3)/b)^(3/4)*(1 + (c*x^3)/d)^(
3/4)*AppellF1[5/6, 3/4, 3/4, 11/6, -((a*x^3)/b), -((c*x^3)/d)] + c*(12025*
b^2*c^2 - 23450*a*b*c*d + 11209*a^2*d^2)*x^3*(b + a*x^3)^2*(1 + (a*x^3)/b)
^(3/4)*(1 + (c*x^3)/d)^(3/4)*AppellF1[11/6, 3/4, 3/4, 17/6, -((a*x^3)/b),
-((c*x^3)/d)]))/(68607*a^3*(b*c - a*d)^2*(a + b/x^3)^(3/4)*(b + a*x^3)^2*(
d + c*x^3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {899, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt[4]{c+\frac{d}{x^3}}x^2}{\left(a+\frac{b}{x^3}\right)^{15/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{\left(\frac{b}{ax^3}+1\right)^{3/4}\int \frac{\sqrt[4]{c+\frac{d}{x^3}}x^2}{\left(\frac{b}{ax^3}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^3}\right)^{3/4}}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{\left(\frac{b}{ax^3}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^3}}\int \frac{\sqrt[4]{\frac{d}{cx^3}+1}x^2}{\left(\frac{b}{ax^3}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^3}\right)^{3/4}\sqrt[4]{\frac{d}{cx^3}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x\left(\frac{b}{ax^3}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^3}}\operatorname{AppellF1}\left(-\frac{1}{3},\frac{15}{4},-\frac{1}{4},\frac{2}{3},-\frac{b}{ax^3},-\frac{d}{cx^3}\right)}{a^3\left(a+\frac{b}{x^3}\right)^{3/4}\sqrt[4]{\frac{d}{cx^3}+1}}$$

input
```
Int[(c + d/x^3)^(1/4)/(a + b/x^3)^(15/4),x]
```

output
```
((1 + b/(a*x^3))^(3/4)*(c + d/x^3)^(1/4)*x*AppellF1[-1/3, 15/4, -1/4, 2/3,
 -(b/(a*x^3)), -(d/(c*x^3))])/(a^3*(a + b/x^3)^(3/4)*(1 + d/(c*x^3))^(1/4)
)
```

Defintions of rubi rules used

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^3}\right)^{\frac{15}{4}}}dx$$

input
```
int((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x)
```

output
```
int((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx = \int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^3}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x, algorithm="fricas")
```

output
```
integral(x^12*((a*x^3 + b)/x^3)^(1/4)*((c*x^3 + d)/x^3)^(1/4)/(a^4*x^12 +
4*a^3*b*x^9 + 6*a^2*b^2*x^6 + 4*a*b^3*x^3 + b^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx = \text{Timed out}$$

input
```
integrate((c+d/x**3)**(1/4)/(a+b/x**3)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^3}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x, algorithm="maxima")
```

output
```
integrate((c + d/x^3)^(1/4)/(a + b/x^3)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^3}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x, algorithm="giac")
```

output
```
integrate((c + d/x^3)^(1/4)/(a + b/x^3)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^3}\right)^{1/4}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx$$

input
```
int((c + d/x^3)^(1/4)/(a + b/x^3)^(15/4),x)
```

output
```
int((c + d/x^3)^(1/4)/(a + b/x^3)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^3}}}{\left(a+\frac{b}{x^3}\right)^{15/4}}\,dx = \int \frac{(c\,x^3+d)^{\frac{1}{4}}\,x^{11}}{\sqrt{x}\,\left(a\,x^3+b\right)^{\frac{3}{4}}a^3x^9+3\sqrt{x}\,\left(a\,x^3+b\right)^{\frac{3}{4}}a^2b\,x^6+3\sqrt{x}\,\left(a\,x^3+b\right)^{\frac{3}{4}}a\,b^2x^3+\sqrt{x}\,\left(a\,x\right.}$$

input
```
int((c+d/x^3)^(1/4)/(a+b/x^3)^(15/4),x)
```

output
```
int(((c*x**3 + d)**(1/4)*x**11)/(sqrt(x)*(a*x**3 + b)**(3/4)*a**3*x**9 + 3
*sqrt(x)*(a*x**3 + b)**(3/4)*a**2*b*x**6 + 3*sqrt(x)*(a*x**3 + b)**(3/4)*a
*b**2*x**3 + sqrt(x)*(a*x**3 + b)**(3/4)*b**3),x)
```

3.4 $\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 687

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx = \frac{2bd(41bc-35ad)\sqrt[4]{c+\frac{d}{x^2}}}{33a^3c(bc-ad)\left(a+\frac{b}{x^2}\right)^{7/4}x^3} - \frac{d\sqrt[4]{c+\frac{d}{x^2}}}{ac\left(a+\frac{b}{x^2}\right)^{11/4}x}$$

$$+ \frac{b(9bc-11ad)(13bc-ad)\sqrt[4]{c+\frac{d}{x^2}}}{77a^3(bc-ad)^2\left(a+\frac{b}{x^2}\right)^{7/4}x} + \frac{b(5bc-11ad)(9bc-11ad)(13bc-ad)\sqrt[4]{c+\frac{d}{x^2}}}{231a^4(bc-ad)^3\left(a+\frac{b}{x^2}\right)^{3/4}x}$$

$$+ \frac{b\sqrt[4]{c+\frac{d}{x^2}}\left(13bc-ad+\frac{12bd}{x^2}\right)}{11a^2(bc-ad)\left(a+\frac{b}{x^2}\right)^{11/4}x} + \frac{\left(c+\frac{d}{x^2}\right)^{5/4}x}{ac\left(a+\frac{b}{x^2}\right)^{11/4}}$$

$$+ \frac{(9bc-11ad)(13bc-ad)\left(5b^2c^2-14abcd+21a^2d^2\right)\left(\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}\right)^{3/4}\sqrt[4]{c+\frac{d}{x^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\right.}{462a^4c(bc-ad)^3\left(a+\frac{b}{x^2}\right)^{3/4}x}$$

$$+ \frac{24b^2d^2\left(\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},\frac{bc-ad}{c\left(a+\frac{b}{x^2}\right)x^2}\right)}{55a^3(bc-ad)\left(a+\frac{b}{x^2}\right)^{7/4}\left(c+\frac{d}{x^2}\right)^{3/4}x^5}$$

$$+ \frac{bd(41bc-35ad)(bc-7ad)\left(\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},\frac{bc-ad}{c\left(a+\frac{b}{x^2}\right)x^2}\right)}{165a^4c(bc-ad)\left(a+\frac{b}{x^2}\right)^{7/4}\left(c+\frac{d}{x^2}\right)^{3/4}x^5}$$

output

```
2/33*b*d*(-35*a*d+41*b*c)*(c+d/x^2)^(1/4)/a^3/c/(-a*d+b*c)/(a+b/x^2)^(7/4)
/x^3-d*(c+d/x^2)^(1/4)/a/c/(a+b/x^2)^(11/4)/x+1/77*b*(-11*a*d+9*b*c)*(-a*d
+13*b*c)*(c+d/x^2)^(1/4)/a^3/(-a*d+b*c)^2/(a+b/x^2)^(7/4)/x+1/231*b*(-11*a
*d+5*b*c)*(-11*a*d+9*b*c)*(-a*d+13*b*c)*(c+d/x^2)^(1/4)/a^4/(-a*d+b*c)^3/(
a+b/x^2)^(3/4)/x+1/11*b*(c+d/x^2)^(1/4)*(13*b*c-a*d+12*b*d/x^2)/a^2/(-a*d+
b*c)/(a+b/x^2)^(11/4)/x+(c+d/x^2)^(5/4)*x/a/c/(a+b/x^2)^(11/4)+1/462*(-11*
a*d+9*b*c)*(-a*d+13*b*c)*(21*a^2*d^2-14*a*b*c*d+5*b^2*c^2)*(c*(a+b/x^2)/a/
(c+d/x^2))^(3/4)*(c+d/x^2)^(1/4)*hypergeom([1/2, 3/4],[3/2],-(-a*d+b*c)/a/
(c+d/x^2)/x^2)/a^4/c/(-a*d+b*c)^3/(a+b/x^2)^(3/4)/x+24/55*b^2*d^2*(a*(c+d/
x^2)/c/(a+b/x^2))^(3/4)*hypergeom([3/4, 5/2],[7/2],(-a*d+b*c)/c/(a+b/x^2)/
x^2)/a^3/(-a*d+b*c)/(a+b/x^2)^(7/4)/(c+d/x^2)^(3/4)/x^5+1/165*b*d*(-35*a*d
+41*b*c)*(-7*a*d+b*c)*(a*(c+d/x^2)/c/(a+b/x^2))^(3/4)*hypergeom([3/4, 5/2]
,[7/2],(-a*d+b*c)/c/(a+b/x^2)/x^2)/a^4/c/(-a*d+b*c)/(a+b/x^2)^(7/4)/(c+d/x
^2)^(3/4)/x^5
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx = \frac{\sqrt[4]{c+\frac{d}{x^2}}(b+ax^2)\left(77x^4(d+cx^2)+\frac{b(d+cx^2)(117b^3c^2+154a^3d^2x^2+11a^2bd(7d-34cx^2)+2ab^2c(-10}{a^2(bc-ad)^2}\right.}{77ac\,(a+}$$

input
```
Integrate[(c + d/x^2)^(1/4)/(a + b/x^2)^(15/4),x]
```

output
```
((c + d/x^2)^(1/4)*(b + a*x^2)*(77*x^4*(d + c*x^2) + (b*(d + c*x^2)*(117*b
^3*c^2 + 154*a^3*d^2*x^2 + 11*a^2*b*d*(7*d - 34*c*x^2) + 2*a*b^2*c*(-103*d
 + 104*c*x^2)))/(a^2*(b*c - a*d)^2) + ((195*b^3*c^3 - 495*a*b^2*c^2*d + 38
5*a^2*b*c*d^2 - 77*a^3*d^3)*(b + a*x^2)^2*Hypergeometric2F1[-3/4, -1/4, 1/
4, (c*(b + a*x^2))/(b*c - a*d)])/(a^3*(b*c - a*d)^2*((a*(d + c*x^2))/(-(b*
c) + a*d))^(1/4))))/(77*a*c*(a + b/x^2)^(15/4)*x^7)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt[4]{c+\frac{d}{x^2}}x^2}{\left(a+\frac{b}{x^2}\right)^{15/4}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\frac{\left(\frac{b}{ax^2}+1\right)^{3/4}\int \frac{\sqrt[4]{c+\frac{d}{x^2}}x^2}{\left(\frac{b}{ax^2}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^2}\right)^{3/4}}$$

$$\Big\downarrow \; 395$$

$$-\frac{\left(\frac{b}{ax^2}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^2}}\int \frac{\sqrt[4]{\frac{d}{cx^2}+1}x^2}{\left(\frac{b}{ax^2}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^2}\right)^{3/4}\sqrt[4]{\frac{d}{cx^2}+1}}$$

$$\Big\downarrow \; 7299$$

$$-\frac{\left(\frac{b}{ax^2}+1\right)^{3/4}\sqrt[4]{c+\frac{d}{x^2}}\int \frac{\sqrt[4]{\frac{d}{cx^2}+1}x^2}{\left(\frac{b}{ax^2}+1\right)^{15/4}}d\frac{1}{x}}{a^3\left(a+\frac{b}{x^2}\right)^{3/4}\sqrt[4]{\frac{d}{cx^2}+1}}$$

input
```
Int[(c + d/x^2)^(1/4)/(a + b/x^2)^(15/4),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x^2}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^2}\right)^{\frac{15}{4}}}dx$$

input
```
int((c+1/x^2*d)^(1/4)/(a+b/x^2)^(15/4),x)
```

output
```
int((c+1/x^2*d)^(1/4)/(a+b/x^2)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^2}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^2}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^2)^(1/4)/(a+b/x^2)^(15/4),x, algorithm="fricas")
```

output
```
integral(x^8*((a*x^2 + b)/x^2)^(1/4)*((c*x^2 + d)/x^2)^(1/4)/(a^4*x^8 + 4*
a^3*b*x^6 + 6*a^2*b^2*x^4 + 4*a*b^3*x^2 + b^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\text{Timed out}$$

input
```
integrate((c+d/x**2)**(1/4)/(a+b/x**2)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^2}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^2}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^2)^(1/4)/(a+b/x^2)^(15/4),x, algorithm="maxima")
```

output
```
integrate((c + d/x^2)^(1/4)/(a + b/x^2)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^2}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x^2}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x^2)^(1/4)/(a+b/x^2)^(15/4),x, algorithm="giac")
```

output
```
integrate((c + d/x^2)^(1/4)/(a + b/x^2)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x^2}\right)^{1/4}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx$$

input
```
int((c + d/x^2)^(1/4)/(a + b/x^2)^(15/4),x)
```

output
```
int((c + d/x^2)^(1/4)/(a + b/x^2)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x^2}}}{\left(a+\frac{b}{x^2}\right)^{15/4}}\,dx=\int \frac{\left(c\,x^2+d\right)^{\frac{1}{4}} x^7}{\left(a\,x^2+b\right)^{\frac{3}{4}} a^3x^6+3\left(a\,x^2+b\right)^{\frac{3}{4}} a^2b\,x^4+3\left(a\,x^2+b\right)^{\frac{3}{4}} a\,b^2x^2+\left(a\,x^2+b\right)^{\frac{3}{4}} b^3}dx$$

input
```
int((c+d/x^2)^(1/4)/(a+b/x^2)^(15/4),x)
```

output
```
int(((c*x**2 + d)**(1/4)*x**7)/((a*x**2 + b)**(3/4)*a**3*x**6 + 3*(a*x**2
+ b)**(3/4)*a**2*b*x**4 + 3*(a*x**2 + b)**(3/4)*a*b**2*x**2 + (a*x**2 + b)
**(3/4)*b**3),x)
```

3.5 $\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx=\frac{4b\sqrt[4]{c+\frac{d}{x}}\,\text{AppellF1}\left(-\frac{11}{4},2,-\frac{1}{4},-\frac{7}{4},1+\frac{b}{ax},-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad}\right)}{11a^2\left(a+\frac{b}{x}\right)^{11/4}\sqrt[4]{\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}}}$$

output
```
4/11*b*(c+d/x)^(1/4)*AppellF1(-11/4,-1/4,2,-7/4,-d*(a+b/x)/(-a*d+b*c),1+b/
a/x)/a^2/(a+b/x)^(11/4)/(b*(c+d/x)/(-a*d+b*c))^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1290 vs. $2(94) = 188$.

Time = 22.69 (sec) , antiderivative size = 1290, normalized size of antiderivative = 13.72

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx=\text{Too large to display}$$

input
```
Integrate[(c + d/x)^(1/4)/(a + b/x)^(15/4),x]
```

output
```
((a + b/x)^(1/4)*(c + d/x)^(1/4)*x*((5*a*(1155*b^5*c^2 + 231*a^5*d^2*x^3 +
 7*a^4*b*d*x^2*(263*d - 66*c*x) + 11*a*b^4*c*(-202*d + 255*c*x) + a^3*b^2*
x*(2569*d^2 - 3830*c*d*x + 231*c^2*x^2) + a^2*b^3*(1043*d^2 - 5422*c*d*x +
 1965*c^2*x^2)))/(b + a*x)^3 - (x*(236775*a^3*b^2*c^2*d*(b*c - a*d)*Appell
F1[1/4, 3/4, 1, 5/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)] + 5792
5*a^5*d^3*(b*c - a*d)*AppellF1[1/4, 3/4, 1, 5/4, (d*(b + a*x))/((-(b*c) +
a*d)*x), 1 + b/(a*x)] + 86625*a^2*b^3*c^3*(-(b*c) + a*d)*AppellF1[1/4, 3/4
, 1, 5/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)] + 209275*a^4*b*c*
d^2*(-(b*c) + a*d)*AppellF1[1/4, 3/4, 1, 5/4, (d*(b + a*x))/((-(b*c) + a*d
)*x), 1 + b/(a*x)] + (2310*b^3*c^2*d*(b + a*x)*((b*d + b*c*x)/(b*c*x - a*d
*x))^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1
+ b/(a*x)]*(5*a*(b*c - a*d)*x*AppellF1[1/4, 3/4, 1, 5/4, (d*(b + a*x))/((-
(b*c) + a*d)*x), 1 + b/(a*x)] + (b + a*x)*(4*(b*c - a*d)*AppellF1[5/4, 3/4
, 2, 9/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)] - 3*a*d*AppellF1[
5/4, 7/4, 1, 9/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)])))/x^3 -
(4444*a*b^2*c*d^2*(b + a*x)*((b*d + b*c*x)/(b*c*x - a*d*x))^(3/4)*AppellF1
[5/4, 3/4, 1, 9/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)]*(5*a*(b*
c - a*d)*x*AppellF1[1/4, 3/4, 1, 5/4, (d*(b + a*x))/((-(b*c) + a*d)*x), 1
+ b/(a*x)] + (b + a*x)*(4*(b*c - a*d)*AppellF1[5/4, 3/4, 2, 9/4, (d*(b + a
*x))/((-(b*c) + a*d)*x), 1 + b/(a*x)] - 3*a*d*AppellF1[5/4, 7/4, 1, 9/4...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {899, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt[4]{c+\frac{d}{x}}x^2}{\left(a+\frac{b}{x}\right)^{15/4}}d\frac{1}{x}$$

$\Big\downarrow$ 154

$$-\frac{\sqrt[4]{c+\frac{d}{x}}\int \frac{\sqrt[4]{\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x}}x^2}{\left(a+\frac{b}{x}\right)^{15/4}}d\frac{1}{x}}{\sqrt[4]{\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}}}$$

$\Big\downarrow$ 153

$$\frac{4b\sqrt[4]{c+\frac{d}{x}}\,\text{AppellF1}\left(-\frac{11}{4},-\frac{1}{4},2,-\frac{7}{4},-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{11a^2\left(a+\frac{b}{x}\right)^{11/4}\sqrt[4]{\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}}}$$

input
```
Int[(c + d/x)^(1/4)/(a + b/x)^(15/4),x]
```

output
```
(4*b*(c + d/x)^(1/4)*AppellF1[-11/4, -1/4, 2, -7/4, -((d*(a + b/x))/(b*c -
 a*d)), (a + b/x)/a])/(11*a^2*(a + b/x)^(11/4)*((b*(c + d/x))/(b*c - a*d))
^(1/4))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x}\right)^{\frac{15}{4}}}dx$$

input
```
int((c+1/x*d)^(1/4)/(a+b/x)^(15/4),x)
```

output
```
int((c+1/x*d)^(1/4)/(a+b/x)^(15/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx=\text{Timed out}$$

input
```
integrate((c+d/x)^(1/4)/(a+b/x)^(15/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx = \text{Timed out}$$

input
```
integrate((c+d/x)**(1/4)/(a+b/x)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx = \int \frac{\left(c+\frac{d}{x}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x)^(1/4)/(a+b/x)^(15/4),x, algorithm="maxima")
```

output
```
integrate((c + d/x)^(1/4)/(a + b/x)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx = \int \frac{\left(c+\frac{d}{x}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x}\right)^{\frac{15}{4}}}\,dx$$

input
```
integrate((c+d/x)^(1/4)/(a+b/x)^(15/4),x, algorithm="giac")
```

output
```
integrate((c + d/x)^(1/4)/(a + b/x)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x}\right)^{1/4}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx$$

input
```
int((c + d/x)^(1/4)/(a + b/x)^(15/4),x)
```

output
```
int((c + d/x)^(1/4)/(a + b/x)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+\frac{d}{x}}}{\left(a+\frac{b}{x}\right)^{15/4}}\,dx=\int \frac{\left(c+\frac{d}{x}\right)^{\frac{1}{4}}}{\left(a+\frac{b}{x}\right)^{\frac{15}{4}}}dx$$

input
```
int((c+d/x)^(1/4)/(a+b/x)^(15/4),x)
```

output
```
int((c+d/x)^(1/4)/(a+b/x)^(15/4),x)
```

3.6 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}} - \frac{4d\sqrt[4]{c+dx}}{77b(bc-ad)(a+bx)^{7/4}} + \frac{8d^2\sqrt[4]{c+dx}}{77b(bc-ad)^2(a+bx)^{3/4}} - \frac{16d^{7/2}(a+bx)^{3/4}\left(\frac{b(c+dx)}{d(a+bx)}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{bc-ad}}{\sqrt{d}\sqrt{a+bx}}\right),2\right)}{77b^2(bc-ad)^{5/2}(c+dx)^{3/4}}$$

output
```
-4/11*(d*x+c)^(1/4)/b/(b*x+a)^(11/4)-4/77*d*(d*x+c)^(1/4)/b/(-a*d+b*c)/(b*
x+a)^(7/4)+8/77*d^2*(d*x+c)^(1/4)/b/(-a*d+b*c)^2/(b*x+a)^(3/4)-16/77*d^(7/
2)*(b*x+a)^(3/4)*(b*(d*x+c)/d/(b*x+a))^(3/4)*InverseJacobiAM(1/2*arctan(1/
d^(1/2)/(b*x+a)^(1/2)*(-a*d+b*c)^(1/2)),2^(1/2))/b^2/(-a*d+b*c)^(5/2)/(d*x
+c)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx = -\frac{4\sqrt[4]{c+dx}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},\frac{d(a+bx)}{-bc+ad}\right)}{11b(a+bx)^{11/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

input
```
Integrate[(c + d*x)^(1/4)/(a + b*x)^(15/4),x]
```

output
```
(-4*(c + d*x)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (d*(a + b*x))/(-(
b*c) + a*d)])/(11*b*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {57, 61, 61, 73, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx$$

$$\Big\downarrow \text{57}$$

$$\frac{d\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}dx}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{d\left(-\frac{6d\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}dx}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$$\Big\downarrow \text{61}$$

$$\frac{d\left(-\frac{6d\left(-\frac{2d\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}dx}{3(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{d\left(-\frac{6d\left(-\frac{8d\int \frac{1}{\left(c-\frac{ad}{b}+\frac{d(a+bx)}{b}\right)^{3/4}}d\sqrt[4]{a+bx}}{3b(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{d\left(-\frac{6d\left(-\frac{8d(a+bx)^{3/4}\left(\frac{bc-ad}{d(a+bx)}+1\right)^{3/4}\int \frac{1}{(a+bx)^{3/4}\left(\frac{bc-ad}{d(a+bx)}+1\right)^{3/4}}d\sqrt[4]{a+bx}}{3b(bc-ad)\left(\frac{d(a+bx)}{b}-\frac{ad}{b}+c\right)^{3/4}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{d\left(-\frac{6d\left(\frac{8d(a+bx)^{3/4}\left(\frac{bc-ad}{d(a+bx)}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{a+bx}\left(\frac{(bc-ad)(a+bx)}{d}+1\right)^{3/4}}d\frac{1}{\sqrt[4]{a+bx}}}{3b(bc-ad)\left(\frac{d(a+bx)}{b}-\frac{ad}{b}+c\right)^{3/4}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$\Big\downarrow$ 807

$$\frac{d\left(-\frac{6d\left(\frac{4d(a+bx)^{3/4}\left(\frac{bc-ad}{d(a+bx)}+1\right)^{3/4}\int \frac{1}{\left(\frac{\sqrt{a+bx}(bc-ad)}{d}+1\right)^{3/4}}d\sqrt{a+bx}}{3b(bc-ad)\left(\frac{d(a+bx)}{b}-\frac{ad}{b}+c\right)^{3/4}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

$\Big\downarrow$ 229

$$\frac{d\left(-\frac{6d\left(\frac{8d^{3/2}(a+bx)^{3/4}\left(\frac{bc-ad}{d(a+bx)}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{bc-ad}\sqrt{a+bx}}{\sqrt{d}}\right),2\right)}{3b(bc-ad)^{3/2}\left(\frac{d(a+bx)}{b}-\frac{ad}{b}+c\right)^{3/4}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}\right)}{7(bc-ad)}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}\right)}{11b}-\frac{4\sqrt[4]{c+dx}}{11b(a+bx)^{11/4}}$$

input
```
Int[(c + d*x)^(1/4)/(a + b*x)^(15/4),x]
```

output
```
(-4*(c + d*x)^(1/4))/(11*b*(a + b*x)^(11/4)) + (d*((-4*(c + d*x)^(1/4))/(7
*(b*c - a*d)*(a + b*x)^(7/4)) - (6*d*((-4*(c + d*x)^(1/4))/(3*(b*c - a*d)*
(a + b*x)^(3/4)) + (8*d^(3/2)*(a + b*x)^(3/4)*(1 + (b*c - a*d)/(d*(a + b*x
)))^(3/4)*EllipticF[ArcTan[(Sqrt[b*c - a*d]*Sqrt[a + b*x])/Sqrt[d]]/2, 2])
/(3*b*(b*c - a*d)^(3/2)*(c - (a*d)/b + (d*(a + b*x))/b)^(3/4))))/(7*(b*c -
 a*d))))/(11*b)
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(xd + c)^{\frac{1}{4}}}{(bx + a)^{\frac{15}{4}}}dx$$

input
```
int((d*x+c)^(1/4)/(b*x+a)^(15/4),x)
```

output
```
int((d*x+c)^(1/4)/(b*x+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c + dx}}{(a + bx)^{15/4}}\, dx = \int \frac{(dx + c)^{\frac{1}{4}}}{(bx + a)^{\frac{15}{4}}}\, dx$$

input
```
integrate((d*x+c)^(1/4)/(b*x+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^
2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx = \int \frac{\sqrt[4]{c+dx}}{(a+bx)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x+c)**(1/4)/(b*x+a)**(15/4),x)
```

output
```
Integral((c + d*x)**(1/4)/(a + b*x)**(15/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx = \int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x+c)^(1/4)/(b*x+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(1/4)/(b*x + a)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx = \int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x+c)^(1/4)/(b*x+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(1/4)/(b*x + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx=\int \frac{{\left(c+d\,x\right)}^{1/4}}{{\left(a+b\,x\right)}^{15/4}} \,dx$$

input
```
int((c + d*x)^(1/4)/(a + b*x)^(15/4),x)
```

output
```
int((c + d*x)^(1/4)/(a + b*x)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx=\int \frac{\left(d x +c \right)^{\frac{1}{4}}}{\left(b x +a \right)^{\frac{3}{4}} a^{3}+3 \left(b x +a \right)^{\frac{3}{4}} a^{2} b x +3 \left(b x +a \right)^{\frac{3}{4}} a \,b^{2} x^{2}+\left(b x +a \right)^{\frac{3}{4}} b^{3} x^{3}}d x$$

input
```
int((d*x+c)^(1/4)/(b*x+a)^(15/4),x)
```

output
```
int((c + d*x)**(1/4)/((a + b*x)**(3/4)*a**3 + 3*(a + b*x)**(3/4)*a**2*b*x
+ 3*(a + b*x)**(3/4)*a*b**2*x**2 + (a + b*x)**(3/4)*b**3*x**3),x)
```

3.7 $\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 214

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \frac{2bx(c+dx^2)^{5/4}}{11a(bc-ad)\,(a+bx^2)^{11/4}} + \frac{6b(3bc-5ad)x(c+dx^2)^{5/4}}{77a^2(bc-ad)^2\,(a+bx^2)^{7/4}} + \frac{(45b^2c^2-110abcd+77a^2d^2)\,x\sqrt[4]{c+dx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{(bc-ad)x^2}{c(a+bx^2)}\right)}{77a^3(bc-ad)^2\,(a+bx^2)^{3/4}\sqrt[4]{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
2/11*b*x*(d*x^2+c)^(5/4)/a/(-a*d+b*c)/(b*x^2+a)^(11/4)+6/77*b*(-5*a*d+3*b*
c)*x*(d*x^2+c)^(5/4)/a^2/(-a*d+b*c)^2/(b*x^2+a)^(7/4)+1/77*(77*a^2*d^2-110
*a*b*c*d+45*b^2*c^2)*x*(d*x^2+c)^(1/4)*hypergeom([-1/4, 1/2],[3/2],(-a*d+b
*c)*x^2/c/(b*x^2+a))/a^3/(-a*d+b*c)^2/(b*x^2+a)^(3/4)/(a*(d*x^2+c)/c/(b*x^
2+a))^(1/4)
```

Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \frac{x\sqrt[4]{c+dx^2}\left(2c\left(21a^2(bc-ad)^2+3a(-bc+ad)(-9bc+8ad)\left(a+bx^2\right)+\left(45b^2c^2-8\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x^2)^(1/4)/(a + b*x^2)^(15/4),x]
```

output
```
(x*(c + d*x^2)^(1/4)*(2*c*(21*a^2*(b*c - a*d)^2 + 3*a*(-(b*c) + a*d)*(-9*b
*c + 8*a*d)*(a + b*x^2) + (45*b^2*c^2 - 83*a*b*c*d + 32*a^2*d^2)*(a + b*x^
2)^2) + c*(45*b^2*c^2 - 110*a*b*c*d + 77*a^2*d^2)*(a + b*x^2)^2*((c*(a + b
*x^2))/(a*(c + d*x^2)))^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, ((-(b*c) +
a*d)*x^2)/(a*(c + d*x^2))]))/(231*a^3*c*(b*c - a*d)^2*(a + b*x^2)^(11/4))
```

Rubi [A] (warning: unable to verify)

Time = 6.76 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx$$

$$\Big\downarrow \; 334$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{\sqrt[4]{dx^2+c}}{\left(\frac{bx^2}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 334$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\sqrt[4]{c+dx^2}\int \frac{\sqrt[4]{\frac{dx^2}{c}+1}}{\left(\frac{bx^2}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^2\right)^{3/4}\sqrt[4]{\frac{dx^2}{c}+1}}$$

$\big\downarrow$ 333

$$\frac{11x\,\text{Gamma}\left(\frac{3}{4}\right)\sqrt[4]{c+dx^2}\left(\frac{dx^2}{c}+1\right)\left(30x^2\left(c+dx^2\right)^2\left(bc-ad\right){}_3F_2\left(2,2,\frac{19}{4};1,\frac{9}{2};\frac{(bc-ad)x^2}{c(bx^2+a)}\right)+30x^2\left(4c^2+7cdx^2\right.\right.}{320a^3c^3}$$

input
```
Int[(c + d*x^2)^(1/4)/(a + b*x^2)^(15/4),x]
```

output
```
(11*x*(c + d*x^2)^(1/4)*(1 + (d*x^2)/c)*Gamma[3/4]*(7*c*(a + b*x^2)*(15*c^
2 + 20*c*d*x^2 + 8*d^2*x^4)*Hypergeometric2F1[1, 15/4, 7/2, ((b*c - a*d)*x
^2)/(c*(a + b*x^2))] + 30*(b*c - a*d)*x^2*(4*c^2 + 7*c*d*x^2 + 3*d^2*x^4)*
Hypergeometric2F1[2, 19/4, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 30*(b
*c - a*d)*x^2*(c + d*x^2)^2*HypergeometricPFQ[{2, 2, 19/4}, {1, 9/2}, ((b*
c - a*d)*x^2)/(c*(a + b*x^2))]))/(320*a^3*c^3*(a + b*x^2)^(7/4)*(1 + (b*x^
2)/a)^3*Gamma[15/4])
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(x^2 d+c)^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{15}{4}}}dx$$

input
```
int((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x)
```

output
```
int((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{(dx^2+c)^{\frac{1}{4}}}{(bx^2+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)^(1/4)/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^
2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x**2+c)**(1/4)/(b*x**2+a)**(15/4),x)
```

output
```
Integral((c + d*x**2)**(1/4)/(a + b*x**2)**(15/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{(dx^2+c)^{\frac{1}{4}}}{(bx^2+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(1/4)/(b*x^2 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{(dx^2+c)^{\frac{1}{4}}}{(bx^2+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^(1/4)/(b*x^2 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{(d\,x^2+c)^{1/4}}{(b\,x^2+a)^{15/4}}\,dx$$

input
```
int((c + d*x^2)^(1/4)/(a + b*x^2)^(15/4),x)
```

output
```
int((c + d*x^2)^(1/4)/(a + b*x^2)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx^2}}{(a+bx^2)^{15/4}}\,dx = \int \frac{(d\,x^2+c)^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^2+a)^{\frac{3}{4}}\,a^2b\,x^2+3\,(b\,x^2+a)^{\frac{3}{4}}\,a\,b^2x^4+(b\,x^2+a)^{\frac{3}{4}}\,b^3x^6}dx$$

input
```
int((d*x^2+c)^(1/4)/(b*x^2+a)^(15/4),x)
```

output
```
int((c + d*x**2)**(1/4)/((a + b*x**2)**(3/4)*a**3 + 3*(a + b*x**2)**(3/4)*
a**2*b*x**2 + 3*(a + b*x**2)**(3/4)*a*b**2*x**4 + (a + b*x**2)**(3/4)*b**3
*x**6),x)
```

3.8 $\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[4]{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{15}{4},-\frac{1}{4},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^3\,(a+bx^3)^{3/4}\sqrt[4]{1+\frac{dx^3}{c}}}$$

output
```
x*(1+b*x^3/a)^(3/4)*(d*x^3+c)^(1/4)*AppellF1(1/3,15/4,-1/4,4/3,-b*x^3/a,-d
*x^3/c)/a^3/(b*x^3+a)^(3/4)/(1+d*x^3/c)^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 402 vs. 2(86) = 172.

Time = 4.40 (sec) , antiderivative size = 402, normalized size of antiderivative = 4.67

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx = \frac{x\left(\frac{8(c+dx^3)\left(787a^4d^2+493b^4c^2x^6+ab^3cx^3(1247c-911dx^3)+2a^3bd(-892c+481dx^3)+a^2b^2(943c^2-2317cdx^3+36\right.}{(a+bx^3)^2}\right.}{}$$

input
```
Integrate[(c + d*x^3)^(1/4)/(a + b*x^3)^(15/4),x]
```

output
```
(x*((8*(c + d*x^3)*(787*a^4*d^2 + 493*b^4*c^2*x^6 + a*b^3*c*x^3*(1247*c -
911*d*x^3) + 2*a^3*b*d*(-892*c + 481*d*x^3) + a^2*b^2*(943*c^2 - 2317*c*d*
x^3 + 364*d^2*x^6)))/(a + b*x^3)^2 + d*(493*b^2*c^2 - 911*a*b*c*d + 364*a^
2*d^2)*x^3*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(3/4)*AppellF1[4/3, 3/4,
3/4, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + (32*a*c^2*(2465*b^2*c^2 - 5338*a*b
*c*d + 3089*a^2*d^2)*AppellF1[1/3, 3/4, 3/4, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(16*a*c*AppellF1[1/3, 3/4, 3/4, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 9*
x^3*(a*d*AppellF1[4/3, 3/4, 7/4, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Ap
pellF1[4/3, 7/4, 3/4, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(12474*a^3*(b*c
 - a*d)^2*(a + b*x^3)^(3/4)*(c + d*x^3)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx^3}}{\left(a+bx^3\right)^{15/4}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{3/4}\int \frac{\sqrt[4]{dx^3+c}}{\left(\frac{bx^3}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[4]{c+dx^3}\int \frac{\sqrt[4]{\frac{dx^3}{c}+1}}{\left(\frac{bx^3}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^3\right)^{3/4}\sqrt[4]{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[4]{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{15}{4},-\frac{1}{4},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^3\left(a+bx^3\right)^{3/4}\sqrt[4]{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(1/4)/(a + b*x^3)^(15/4),x]
```

output
```
(x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/4)*AppellF1[1/3, 15/4, -1/4, 4/3,
-((b*x^3)/a), -((d*x^3)/c)])/(a^3*(a + b*x^3)^(3/4)*(1 + (d*x^3)/c)^(1/4))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(d\,x^3+c\right)^{\frac{1}{4}}}{\left(b\,x^3+a\right)^{\frac{15}{4}}}dx$$

input
```
int((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x)
```

output
```
int((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx = \int \frac{(dx^3+c)^{\frac{1}{4}}}{(bx^3+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(1/4)/(b^4*x^12 + 4*a*b^3*x^9 + 6*a
^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx = \int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x**3+c)**(1/4)/(b*x**3+a)**(15/4),x)
```

output
```
Integral((c + d*x**3)**(1/4)/(a + b*x**3)**(15/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx = \int \frac{(dx^3+c)^{\frac{1}{4}}}{(bx^3+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(1/4)/(b*x^3 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx=\int \frac{(dx^3+c)^{\frac{1}{4}}}{(bx^3+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(1/4)/(b*x^3 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx=\int \frac{(d\,x^3+c)^{1/4}}{(b\,x^3+a)^{15/4}}\,dx$$

input
```
int((c + d*x^3)^(1/4)/(a + b*x^3)^(15/4),x)
```

output
```
int((c + d*x^3)^(1/4)/(a + b*x^3)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx^3}}{(a+bx^3)^{15/4}}\,dx=\int \frac{(d\,x^3+c)^{\frac{1}{4}}}{(b\,x^3+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^3+a)^{\frac{3}{4}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{3}{4}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{3}{4}}\,b^3x^9}dx$$

input
```
int((d*x^3+c)^(1/4)/(b*x^3+a)^(15/4),x)
```

output
```
int((c + d*x**3)**(1/4)/((a + b*x**3)**(3/4)*a**3 + 3*(a + b*x**3)**(3/4)*
a**2*b*x**3 + 3*(a + b*x**3)**(3/4)*a*b**2*x**6 + (a + b*x**3)**(3/4)*b**3
*x**9),x)
```

3.9 $\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx=\frac{x\left(1+\frac{bx^4}{a}\right)^{3/4}\sqrt[4]{c+dx^4}\,\text{AppellF1}\left(\frac{1}{4},\frac{15}{4},-\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3\left(a+bx^4\right)^{3/4}\sqrt[4]{1+\frac{dx^4}{c}}}$$

output
```
x*(1+b*x^4/a)^(3/4)*(d*x^4+c)^(1/4)*AppellF1(1/4,15/4,-1/4,5/4,-b*x^4/a,-d
*x^4/c)/a^3/(b*x^4+a)^(3/4)/(1+d*x^4/c)^(1/4)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(86) = 172.

Time = 4.49 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.66

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx=\frac{x\left(\frac{5(c+dx^4)\left(31a^4d^2+20b^4c^2x^8+ab^3cx^4(50c-37dx^4)+a^3bd(-70c+39dx^4)+a^2b^2(37c^2-93cdx^4+15d^2x^8)\right)}{(a+bx^4)^2}+d\right.}{}$$

input
```
Integrate[(c + d*x^4)^(1/4)/(a + b*x^4)^(15/4),x]
```

output
```
(x*((5*(c + d*x^4)*(31*a^4*d^2 + 20*b^4*c^2*x^8 + a*b^3*c*x^4*(50*c - 37*d
*x^4) + a^3*b*d*(-70*c + 39*d*x^4) + a^2*b^2*(37*c^2 - 93*c*d*x^4 + 15*d^2
*x^8)))/(a + b*x^4)^2 + d*(20*b^2*c^2 - 37*a*b*c*d + 15*a^2*d^2)*x^4*(1 +
(b*x^4)/a)^(3/4)*(1 + (d*x^4)/c)^(3/4)*AppellF1[5/4, 3/4, 3/4, 9/4, -((b*x
^4)/a), -((d*x^4)/c)] + (50*a*c^2*(20*b^2*c^2 - 42*a*b*c*d + 23*a^2*d^2)*A
ppellF1[1/4, 3/4, 3/4, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(5*a*c*AppellF1[1
/4, 3/4, 3/4, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 3*x^4*(a*d*AppellF1[5/4,
3/4, 7/4, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 7/4, 3/4, 9
/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(385*a^3*(b*c - a*d)^2*(a + b*x^4)^(3/
4)*(c + d*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{\sqrt[4]{dx^4+c}}{\left(\frac{bx^4}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\sqrt[4]{c+dx^4}\int \frac{\sqrt[4]{\frac{dx^4}{c}+1}}{\left(\frac{bx^4}{a}+1\right)^{15/4}}dx}{a^3\left(a+bx^4\right)^{3/4}\sqrt[4]{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^4}{a}+1\right)^{3/4}\sqrt[4]{c+dx^4}\,\text{AppellF1}\left(\frac{1}{4},\frac{15}{4},-\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3\left(a+bx^4\right)^{3/4}\sqrt[4]{\frac{dx^4}{c}+1}}$$

input
```
Int[(c + d*x^4)^(1/4)/(a + b*x^4)^(15/4),x]
```

output
```
(x*(1 + (b*x^4)/a)^(3/4)*(c + d*x^4)^(1/4)*AppellF1[1/4, 15/4, -1/4, 5/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^3*(a + b*x^4)^(3/4)*(1 + (d*x^4)/c)^(1/4))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(d\,x^4+c)^{\frac{1}{4}}}{(b\,x^4+a)^{\frac{15}{4}}}dx$$

input
```
int((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x)
```

output
```
int((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx=\int \frac{(dx^4+c)^{\frac{1}{4}}}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)^(1/4)/(b^4*x^16 + 4*a*b^3*x^12 + 6*
a^2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx=\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x**4+c)**(1/4)/(b*x**4+a)**(15/4),x)
```

output
```
Integral((c + d*x**4)**(1/4)/(a + b*x**4)**(15/4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx=\int \frac{(dx^4+c)^{\frac{1}{4}}}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^(1/4)/(b*x^4 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx = \int \frac{(dx^4+c)^{\frac{1}{4}}}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^(1/4)/(b*x^4 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx = \int \frac{(d\,x^4+c)^{1/4}}{(b\,x^4+a)^{15/4}}\,dx$$

input
```
int((c + d*x^4)^(1/4)/(a + b*x^4)^(15/4),x)
```

output
```
int((c + d*x^4)^(1/4)/(a + b*x^4)^(15/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c+dx^4}}{(a+bx^4)^{15/4}}\,dx = \int \frac{(d\,x^4+c)^{\frac{1}{4}}}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}\,dx$$

input
```
int((d*x^4+c)^(1/4)/(b*x^4+a)^(15/4),x)
```

output
```
int((c + d*x**4)**(1/4)/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*
a**2*b*x**4 + 3*(a + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3
*x**12),x)
```

3.10 $\int (a+bx^3)(c+dx^3)^4\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7 +\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

output
```
a*c^4*x+1/4*c^3*(4*a*d+b*c)*x^4+2/7*c^2*d*(3*a*d+2*b*c)*x^7+1/5*c*d^2*(2*a
*d+3*b*c)*x^10+1/13*d^3*(a*d+4*b*c)*x^13+1/16*b*d^4*x^16
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7 +\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{13}+\frac{1}{16}bd^4x^{16}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c
*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/
16
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4 dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^3x^3(4ad+bc)+2c^2dx^6(3ad+2bc)+d^3x^{12}(ad+4bc)+2cd^2x^9(2ad+3bc)+ac^4+bd^4x^{15}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

output

```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c
*d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/
16
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
norman	$a\,c^4x+\left(a\,c^3d+\frac{1}{4}b\,c^4\right)x^4+\left(\frac{6}{7}a\,c^2d^2+\frac{4}{7}b\,c^3d\right)x^7+\left(\frac{2}{5}ac\,d^3+\frac{3}{5}b\,c^2d^2\right)x^{10}+\left(\frac{1}{13}a\,d^4+\frac{4}{13}b\right.$
default	$\frac{b\,d^4x^{16}}{16}+\frac{(a\,d^4+4bc\,d^3)x^{13}}{13}+\frac{(4ac\,d^3+6b\,c^2d^2)x^{10}}{10}+\frac{(6a\,c^2d^2+4b\,c^3d)x^7}{7}+\frac{(4a\,c^3d+b\,c^4)x^4}{4}+a\,c^4x$
gosper	$a\,c^4x+x^4a\,c^3d+\frac{1}{4}x^4b\,c^4+\frac{6}{7}x^7a\,c^2d^2+\frac{4}{7}x^7b\,c^3d+\frac{2}{5}x^{10}ac\,d^3+\frac{3}{5}x^{10}b\,c^2d^2+\frac{1}{13}x^{13}a\,d^4+\frac{4}{13}$
risch	$a\,c^4x+x^4a\,c^3d+\frac{1}{4}x^4b\,c^4+\frac{6}{7}x^7a\,c^2d^2+\frac{4}{7}x^7b\,c^3d+\frac{2}{5}x^{10}ac\,d^3+\frac{3}{5}x^{10}b\,c^2d^2+\frac{1}{13}x^{13}a\,d^4+\frac{4}{13}$
parallelrisch	$a\,c^4x+x^4a\,c^3d+\frac{1}{4}x^4b\,c^4+\frac{6}{7}x^7a\,c^2d^2+\frac{4}{7}x^7b\,c^3d+\frac{2}{5}x^{10}ac\,d^3+\frac{3}{5}x^{10}b\,c^2d^2+\frac{1}{13}x^{13}a\,d^4+\frac{4}{13}$
orering	$\frac{x\left(455b\,d^4x^{15}+560a\,d^4x^{12}+2240bc\,d^3x^{12}+2912ac\,d^3x^9+4368b\,c^2d^2x^9+6240a\,c^2d^2x^6+4160b\,c^3d\,x^6+7280a\,c^3d\,x^3+1820b\,c^4\right.}{7280}$

input
```
int((b*x^3+a)*(d*x^3+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+(a*c^3*d+1/4*b*c^4)*x^4+(6/7*a*c^2*d^2+4/7*b*c^3*d)*x^7+(2/5*a*c*d
^3+3/5*b*c^2*d^2)*x^10+(1/13*a*d^4+4/13*b*c*d^3)*x^13+1/16*b*d^4*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^3)\left(c+dx^3\right)^4\,dx=\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\left(bc^4+4\,ac^3d\right)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

output
```
1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*
a*c^3*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int (a+bx^3)(c+dx^3)^4\,dx = ac^4x + \frac{bd^4x^{16}}{16} + x^{13}\left(\frac{ad^4}{13} + \frac{4bcd^3}{13}\right) + x^{10}\cdot\left(\frac{2acd^3}{5} + \frac{3bc^2d^2}{5}\right) + x^7\cdot\left(\frac{6ac^2d^2}{7} + \frac{4bc^3d}{7}\right) + x^4\left(ac^3d + \frac{bc^4}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

output
```
a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*
a*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x*
*4*(a*c**3*d + b*c**4/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^3)(c+dx^3)^4\,dx = \frac{1}{16}\,bd^4x^{16} + \frac{1}{13}\,(4\,bcd^3 + ad^4)x^{13} + \frac{1}{5}\,(3\,bc^2d^2 + 2\,acd^3)x^{10} + \frac{2}{7}\,(2\,bc^3d + 3\,ac^2d^2)x^7 + ac^4x + \frac{1}{4}\,(bc^4 + 4\,ac^3d)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

output
```
1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*
a*c^3*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int (a+bx^3)(c+dx^3)^4 \, dx = \frac{1}{16}bd^4x^{16} + \frac{4}{13}bcd^3x^{13} + \frac{1}{13}ad^4x^{13} + \frac{3}{5}bc^2d^2x^{10} + \frac{2}{5}acd^3x^{10} + \frac{4}{7}bc^3dx^7 + \frac{6}{7}ac^2d^2x^7 + \frac{1}{4}bc^4x^4 + ac^3dx^4 + ac^4x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

output
```
1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
 + 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4
+ a*c^3*d*x^4 + a*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int (a+bx^3)(c+dx^3)^4 \, dx = x^4\left(\frac{b\,c^4}{4} + a\,d\,c^3\right) + x^{13}\left(\frac{a\,d^4}{13} + \frac{4\,b\,c\,d^3}{13}\right) + \frac{b\,d^4\,x^{16}}{16} + a\,c^4\,x + \frac{2\,c^2\,d\,x^7\,(3\,a\,d+2\,b\,c)}{7} + \frac{c\,d^2\,x^{10}\,(2\,a\,d+3\,b\,c)}{5}$$

input
```
int((a + b*x^3)*(c + d*x^3)^4,x)
```

output
```
x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^
16)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d +
3*b*c))/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int (a+bx^3)(c+dx^3)^4\,dx$$
$$= \frac{x(455b\,d^4x^{15}+560a\,d^4x^{12}+2240bc\,d^3x^{12}+2912ac\,d^3x^9+4368b\,c^2d^2x^9+6240a\,c^2d^2x^6+4160b\,c^3d\,x^6 -}{7280}$$

input
```
int((b*x^3+a)*(d*x^3+c)^4,x)
```

output
```
(x*(7280*a*c**4 + 7280*a*c**3*d*x**3 + 6240*a*c**2*d**2*x**6 + 2912*a*c*d*
*3*x**9 + 560*a*d**4*x**12 + 1820*b*c**4*x**3 + 4160*b*c**3*d*x**6 + 4368*
b*c**2*d**2*x**9 + 2240*b*c*d**3*x**12 + 455*b*d**4*x**15))/7280
```

3.11 $\int (a+bx^3) (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^3) (c+dx^3)^3 \, dx = ac^3x + \frac{1}{4}c^2(bc+3ad)x^4 + \frac{3}{7}cd(bc+ad)x^7 + \frac{1}{10}d^2(3bc+ad)x^{10} + \frac{1}{13}bd^3x^{13}$$

output
```
a*c^3*x+1/4*c^2*(3*a*d+b*c)*x^4+3/7*c*d*(a*d+b*c)*x^7+1/10*d^2*(a*d+3*b*c)
*x^10+1/13*b*d^3*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3) (c+dx^3)^3 \, dx = ac^3x + \frac{1}{4}c^2(bc+3ad)x^4 + \frac{3}{7}cd(bc+ad)x^7 + \frac{1}{10}d^2(3bc+ad)x^{10} + \frac{1}{13}bd^3x^{13}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*
b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)\left(c+dx^3\right)^3 dx$$

$$\Big\downarrow \ 897$$

$$\int \left(c^2x^3(3ad+bc)+d^2x^9(ad+3bc)+3cdx^6(ad+bc)+ac^3+bd^3x^{12}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

output

```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*
b*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```


Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	siz
norman	$\frac{b\,d^3 x^{13}}{13}+\left(\frac{1}{10}a\,d^3+\frac{3}{10}bc\,d^2\right)x^{10}+\left(\frac{3}{7}ac\,d^2+\frac{3}{7}b\,c^2 d\right)x^7+\left(\frac{3}{4}a\,c^2 d+\frac{1}{4}c^3 b\right)x^4+a\,c^3 x$	72
default	$\frac{b\,d^3 x^{13}}{13}+\frac{(a\,d^3+3bc\,d^2)x^{10}}{10}+\frac{(3ac\,d^2+3b\,c^2 d)x^7}{7}+\frac{(3a\,c^2 d+c^3 b)x^4}{4}+a\,c^3 x$	73
gosper	$\frac{1}{13}b\,d^3 x^{13}+\frac{1}{10}x^{10}a\,d^3+\frac{3}{10}x^{10}bc\,d^2+\frac{3}{7}x^7 ac\,d^2+\frac{3}{7}x^7 b\,c^2 d+\frac{3}{4}x^4 a\,c^2 d+\frac{1}{4}x^4 c^3 b+a\,c^3 x$	75
risch	$\frac{1}{13}b\,d^3 x^{13}+\frac{1}{10}x^{10}a\,d^3+\frac{3}{10}x^{10}bc\,d^2+\frac{3}{7}x^7 ac\,d^2+\frac{3}{7}x^7 b\,c^2 d+\frac{3}{4}x^4 a\,c^2 d+\frac{1}{4}x^4 c^3 b+a\,c^3 x$	75
parallelrisch	$\frac{1}{13}b\,d^3 x^{13}+\frac{1}{10}x^{10}a\,d^3+\frac{3}{10}x^{10}bc\,d^2+\frac{3}{7}x^7 ac\,d^2+\frac{3}{7}x^7 b\,c^2 d+\frac{3}{4}x^4 a\,c^2 d+\frac{1}{4}x^4 c^3 b+a\,c^3 x$	75
orering	$\frac{x\left(140b\,d^3 x^{12}+182a\,d^3 x^9+546bc\,d^2 x^9+780ac\,d^2 x^6+780b\,c^2 d\,x^6+1365a\,c^2 d\,x^3+455b\,c^3 x^3+1820c^3 a\right)}{1820}$	78

input
```
int((b*x^3+a)*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^3*x^13+(1/10*a*d^3+3/10*b*c*d^2)*x^10+(3/7*a*c*d^2+3/7*b*c^2*d)*x
^7+(3/4*a*c^2*d+1/4*c^3*b)*x^4+a*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^3\,dx=\frac{1}{13}\,bd^3x^{13}+\frac{1}{10}\left(3\,bcd^2+ad^3\right)x^{10}+\frac{3}{7}\left(bc^2d+acd^2\right)x^7+ac^3x+\frac{1}{4}\left(bc^3+3\,ac^2d\right)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*
x^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int (a+bx^3)\left(c+dx^3\right)^3\,dx = ac^3x + \frac{bd^3x^{13}}{13} + x^{10}\left(\frac{ad^3}{10} + \frac{3bcd^2}{10}\right) + x^7 \cdot \left(\frac{3acd^2}{7} + \frac{3bc^2d}{7}\right) + x^4 \cdot \left(\frac{3ac^2d}{4} + \frac{bc^3}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

output
```
a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a
*c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3)\left(c+dx^3\right)^3\,dx = \frac{1}{13}\,bd^3x^{13} + \frac{1}{10}\,(3\,bcd^2 + ad^3)x^{10} + \frac{3}{7}\,(bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4}\,(bc^3 + 3\,ac^2d)x^4$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*
x^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^3)(c+dx^3)^3\,dx = \frac{1}{13}bd^3x^{13} + \frac{3}{10}bcd^2x^{10} + \frac{1}{10}ad^3x^{10} + \frac{3}{7}bc^2dx^7 + \frac{3}{7}acd^2x^7 + \frac{1}{4}bc^3x^4 + \frac{3}{4}ac^2dx^4 + ac^3x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 +
3/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^3)(c+dx^3)^3\,dx = x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right) + x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right) + \frac{b\,d^3\,x^{13}}{13} + a\,c^3\,x + \frac{3\,c\,d\,x^7\,(a\,d+b\,c)}{7}$$

input
```
int((a + b*x^3)*(c + d*x^3)^3,x)
```

output
```
x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*
d^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (a+bx^3)(c+dx^3)^3\,dx$$
$$=\frac{x(140b\,d^3x^{12}+182a\,d^3x^9+546bc\,d^2x^9+780ac\,d^2x^6+780b\,c^2d\,x^6+1365a\,c^2d\,x^3+455b\,c^3x^3+1820a\,c}{1820}$$

input
```
int((b*x^3+a)*(d*x^3+c)^3,x)
```

output
```
(x*(1820*a*c**3 + 1365*a*c**2*d*x**3 + 780*a*c*d**2*x**6 + 182*a*d**3*x**9
 + 455*b*c**3*x**3 + 780*b*c**2*d*x**6 + 546*b*c*d**2*x**9 + 140*b*d**3*x*
*12))/1820
```

3.12 $\int (a+bx^3)(c+dx^3)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)(c+dx^3)^2 \, dx = ac^2x + \frac{1}{4}c(bc+2ad)x^4 + \frac{1}{7}d(2bc+ad)x^7 + \frac{1}{10}bd^2x^{10}$$

output
```
a*c^2*x+1/4*c*(2*a*d+b*c)*x^4+1/7*d*(a*d+2*b*c)*x^7+1/10*b*d^2*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)^2 \, dx = ac^2x + \frac{1}{4}c(bc+2ad)x^4 + \frac{1}{7}d(2bc+ad)x^7 + \frac{1}{10}bd^2x^{10}$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)
/10
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3) \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^6(ad+2bc)+cx^3(2ad+bc)+ac^2+bd^2x^9\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

input
```
Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)
/10
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^{2} x^{10}}{10}+\frac{\left(a\,d^{2}+2 b c d \right) x^{7}}{7}+\frac{\left(2 a c d +b\,c^{2}\right) x^{4}}{4}+a\,c^{2} x$	49
norman	$\frac{b\,d^{2} x^{10}}{10}+\left(\frac{1}{7} a\,d^{2}+\frac{2}{7} b c d \right) x^{7}+\left(\frac{1}{2} a c d +\frac{1}{4} b\,c^{2}\right) x^{4}+a\,c^{2} x$	49
gosper	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
risch	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
parallelrisch	$\frac{1}{10} b\,d^{2} x^{10}+\frac{1}{7} x^{7} a\,d^{2}+\frac{2}{7} x^{7} b c d +\frac{1}{2} x^{4} a c d +\frac{1}{4} x^{4} b\,c^{2}+a\,c^{2} x$	51
orering	$\frac{x \left(14 b\,d^{2} x^{9}+20 a\,d^{2} x^{6}+40 b c d\,x^{6}+70 a c d\,x^{3}+35 b\,c^{2} x^{3}+140 a\,c^{2}\right)}{140}$	54

input
```
int((b*x^3+a)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx=\frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\left(2\,bcd+ad^2\right)x^7+\frac{1}{4}\left(bc^2+2\,acd\right)x^4+ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 +
a*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx=ac^2x+\frac{bd^2x^{10}}{10}+x^7\left(\frac{ad^2}{7}+\frac{2bcd}{7}\right)+x^4\left(\frac{acd}{2}+\frac{bc^2}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

output
```
a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
 b*c**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx=\frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\left(2\,bcd+ad^2\right)x^7+\frac{1}{4}\left(bc^2+2\,acd\right)x^4+ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 +
a*c^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx=\frac{1}{10}\,bd^2x^{10}+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,ad^2x^7+\frac{1}{4}\,bc^2x^4+\frac{1}{2}\,acdx^4+ac^2x$$

input
```
integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*
d*x^4 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3) (c+dx^3)^2 \, dx = x^4 \left(\frac{b\,c^2}{4}+\frac{a\,d\,c}{2}\right)+x^7 \left(\frac{a\,d^2}{7}+\frac{2\,b\,c\,d}{7}\right)+\frac{b\,d^2\,x^{10}}{10}+a\,c^2\,x$$

input
```
int((a + b*x^3)*(c + d*x^3)^2,x)
```

output
```
x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)
/10 + a*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (a+bx^3) (c+dx^3)^2 \, dx = \frac{x(14b\,d^2x^9+20a\,d^2x^6+40bcd\,x^6+70acd\,x^3+35b\,c^2x^3+140a\,c^2)}{140}$$

input
```
int((b*x^3+a)*(d*x^3+c)^2,x)
```

output
```
(x*(140*a*c**2 + 70*a*c*d*x**3 + 20*a*d**2*x**6 + 35*b*c**2*x**3 + 40*b*c*
d*x**6 + 14*b*d**2*x**9))/140
```

3.13 $\int (a+bx^3)(c+dx^3)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^3)(c+dx^3)\,dx = acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7$$

output
```
a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(c+dx^3)\,dx = acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7$$

input
```
Integrate[(a + b*x^3)*(c + d*x^3),x]
```

output
```
a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^3(ad+bc)+ac+bdx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

input
```
Int[(a + b*x^3)*(c + d*x^3),x]
```

output
```
a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^4}{4}+\frac{bd\,x^7}{7}$	25
norman	$\frac{bd\,x^7}{7}+\left(\frac{ad}{4}+\frac{bc}{4}\right)x^4+acx$	26
gosper	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}bc\,x^4+acx$	27
risch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}bc\,x^4+acx$	27
parallelrisch	$\frac{1}{7}bd\,x^7+\frac{1}{4}ad\,x^4+\frac{1}{4}bc\,x^4+acx$	27
orering	$\frac{x(4bd\,x^6+7ad\,x^3+7x^3bc+28ac)}{28}$	30

input
```
int((b*x^3+a)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx=\frac{1}{7}\,bdx^7+\frac{1}{4}\,(bc+ad)x^4+acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx = acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

input
```
integrate((b*x**3+a)*(d*x**3+c),x)
```

output
```
a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx = \frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc+ad)x^4 + acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx = \frac{1}{7}\,bdx^7 + \frac{1}{4}\,bcx^4 + \frac{1}{4}\,adx^4 + acx$$

input
```
integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
```

output
```
1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^3)(c+dx^3)\,dx = \frac{b\,d\,x^7}{7} + \left(\frac{a\,d}{4} + \frac{b\,c}{4}\right)x^4 + a\,c\,x$$

input
```
int((a + b*x^3)*(c + d*x^3),x)
```

output
```
x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int (a+bx^3)(c+dx^3)\,dx = \frac{x(4bd\,x^6 + 7ad\,x^3 + 7bc\,x^3 + 28ac)}{28}$$

input
```
int((b*x^3+a)*(d*x^3+c),x)
```

output
```
(x*(28*a*c + 7*a*d*x**3 + 7*b*c*x**3 + 4*b*d*x**6))/28
```

3.14 $\int \frac{a+bx^3}{c+dx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 144

$$\int \frac{a+bx^3}{c+dx^3}\,dx=\frac{bx}{d}+\frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}$$

output
```
b*x/d+1/3*(-a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(
1/2)/c^(2/3)/d^(4/3)-1/3*(-a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+
1/6*(-a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(4/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3}\,dx$$

$$=\frac{6bc^{2/3}\sqrt[3]{d}x+2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+(bc-ad)\log\left(c^{2/3}-\sqrt[3]{\ }\right.}{6c^{2/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3),x]
```

output
```
(6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Lo
g[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{c+dx^3}dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^3+c}dx}{d}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{d}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$\big\downarrow$ 25

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$\big\downarrow$ 27

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$\big\downarrow$ 1082

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$\big\downarrow$ 217

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{d}$$

input
```
Int[(a + b*x^3)/(c + d*x^3),x]
```

output
```
(b*x)/d - ((b*c - a*d)*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$	42
default	$\frac{bx}{d}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)(ad-bc)}{d}$	110

input
```
int((b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+1/3/d^2*sum((a*d-b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.56

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \left[\frac{6\,bc^2dx-3\,\sqrt{\frac{1}{3}}(bc^2d-acd^2)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+}{6\,c^2d^2}\right.$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^
2*d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) -
2*(c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*
b*c^2*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(s
qrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2)
+ (c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*
c) - 2*(c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t\log\left(\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c),x)
```

output
```
b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**
2*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{bx}{d} - \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output
```
b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/
d)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c
/d)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(
c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} + \frac{bx}{d} + \frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/
3))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/
3))/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/
d)^(1/3)))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{b\,x}{d} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}$$

input
```
int((a + b*x^3)/(c + d*x^3),x)
```

output
```
(b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (lo
g(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
- b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^3}{c+dx^3}\,dx = \frac{-2c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)ad+2c^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)b-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}x^{2}\right)ad+c^{\frac{4}{3}}\log\left(c^{\frac{2}{3}}-d\right.}{6d^{\frac{4}{3}}c}$$

input
```
int((b*x^3+a)/(d*x^3+c),x)
```

output
```
( - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*
a*d + 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3))
)*b*c - c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*d +
 c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c + 2*c**(
1/3)*log(c**(1/3) + d**(1/3)*x)*a*d - 2*c**(1/3)*log(c**(1/3) + d**(1/3)*x
)*b*c + 6*d**(1/3)*b*c*x)/(6*d**(1/3)*c*d)
```

3.15 $\int \frac{a+bx^3}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{a+bx^3}{(c+dx^3)^2} dx = -\frac{(bc-ad)x}{3cd(c+dx^3)} - \frac{(bc+2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}$$

output
```
-1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)-1/9*(2*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1
/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(5/3)/d^(4/3)+1/9*(2*a*d+b*c)*ln(c^(1/3)
+d^(1/3)*x)/c^(5/3)/d^(4/3)-1/18*(2*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/c^(5/3)/d^(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$
$$= \frac{-\frac{6c^{2/3}\sqrt[3]{d}(bc-ad)x}{c+dx^3} - 2\sqrt{3}(bc+2ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) - (bc+2ad)\log}{18c^{5/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

output
```
((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*
ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3)
+ d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/(18*c^(5/3)*d^(4/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(2ad+bc)\int \frac{1}{dx^3+c}dx}{3cd} - \frac{x(bc-ad)}{3cd\,(c+dx^3)}$$

$$\Big\downarrow \; 750$$

$$\frac{(2ad+bc)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$\Big\downarrow$ 16

$$\frac{(2ad+bc)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$\Big\downarrow$ 1142

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{(2ad+bc)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(2ad+bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(2ad+bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(2ad+bc)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3cd}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

input
```
Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

output
```
-1/3*((b*c - a*d)*x)/(c*d*(c + d*x^3)) + ((b*c + 2*a*d)*(Log[c^(1/3) + d^(
1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*
d^(1/3)))/(3*c^(2/3))))/(3*c*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(ad-bc)x}{3dc(d\,x^3+c)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\,\_Z^3+c\right)}\frac{(2ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{9c\,d^2}$	65
default	$\frac{(ad-bc)x}{3dc(d\,x^3+c)}+\frac{(2ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3cd}$	134

input
```
int((b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*d-b*c)/d/c*x/(d*x^3+c)+1/9/c/d^2*sum((2*a*d+b*c)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(bc^3d+2\,ac^2d^2+(bc^2d^2+2\,acd^3)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)^{\frac{1}{3}}c\right)}{dx^3+c}\right.}{} \right.$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*s
qrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(
1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d
))/(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*
log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^
3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d
 - a*c^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a
*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(
1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((
b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d
)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)
*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^
3*d^3*x^3 + c^4*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{x(ad-bc)}{3c^2d+3cd^2x^3} + \operatorname{RootSum}\left(729t^3c^5d^4 - 8a^3d^3 - 12a^2bcd^2 - 6ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

output
```
x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8
*a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t
*log(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{(bc-ad)x}{3\,(cd^2x^3+c^2d)} + \frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+2\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
-1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(
1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b
*c + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1
/9*(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = -\frac{\sqrt{3}(bc+2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(-cd^2\right)^{\frac{2}{3}}c} - \frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d} - \frac{bcx-adx}{3\,(dx^3+c)cd}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) +
(-c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(
x - (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}} + \frac{x\,\left(a\,d-b\,c\right)}{3\,c\,d\,\left(d\,x^3+c\right)}$$

input
```
int((a + b*x^3)/(c + d*x^3)^2,x)
```

output
```
(log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2
)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c)
)/(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))
/(3*c*d*(c + d*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.07

$$\int \frac{a+bx^3}{(c+dx^3)^2}\,dx$$
$$= \frac{-4c^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)ad-4c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a\,d^2x^3-2c^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)b-2c^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c}{}\right.}{}$$

input
```
int((b*x^3+a)/(d*x^3+c)^2,x)
```

output
```
( - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*
a*c*d - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3
)))*a*d**2*x**3 - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1
/3)*sqrt(3)))*b*c**2 - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(
c**(1/3)*sqrt(3)))*b*c*d*x**3 - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3
)*x + d**(2/3)*x**2)*a*c*d - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x
 + d**(2/3)*x**2)*a*d**2*x**3 - c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*
x + d**(2/3)*x**2)*b*c**2 - c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x +
d**(2/3)*x**2)*b*c*d*x**3 + 4*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*c*d +
4*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*d**2*x**3 + 2*c**(1/3)*log(c**(1/3
) + d**(1/3)*x)*b*c**2 + 2*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*b*c*d*x**3
+ 6*d**(1/3)*a*c*d*x - 6*d**(1/3)*b*c**2*x)/(18*d**(1/3)*c**2*d*(c + d*x**
3))
```

3.16 $\int \frac{a+bx^3}{(c+dx^3)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 197

$$\int \frac{a+bx^3}{(c+dx^3)^3} dx = -\frac{(bc-ad)x}{6cd(c+dx^3)^2} + \frac{(bc+5ad)x}{18c^2d(c+dx^3)} - \frac{(bc+5ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}} - \frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}$$

output
```
-1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)-1/27*
(5*a*d+b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(8
/3)/d^(4/3)+1/27*(5*a*d+b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(4/3)-1/54*(5
*a*d+b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(4/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx$$
$$= \frac{-\frac{9c^{5/3}\sqrt[3]{d}(bc-ad)x}{(c+dx^3)^2}+\frac{3c^{2/3}\sqrt[3]{d}(bc+5ad)x}{c+dx^3}-2\sqrt{3}(bc+5ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+2(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{54c^{8/3}d^{4/3}}$$

input
```
Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

output
```
((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*
c + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)
*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c +
5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3)
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx$$
$$\Big\downarrow \; 910$$
$$\frac{(5ad+bc)\int \frac{1}{(dx^3+c)^2}dx}{6cd}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$
$$\Big\downarrow \; 749$$

$$\frac{(5ad+bc)\left(\frac{2\int \frac{1}{dx^3+c}dx}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

$$\Big\downarrow \; 750$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\,(c+dx^3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{(5ad+bc)\left(\frac{2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{3c}+\frac{x}{3c(c+dx^3)}\right)}{6cd}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x)/(c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*(x/(3*c*(c + d*x
^3)) + (2*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcT
an[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/(3*c)))/(6*c*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(d\,x^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{(5ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{27c^2d^2}$	84
default	$\frac{\frac{(5ad+bc)x^4}{18c^2}+\frac{(4ad-bc)x}{9cd}}{(d\,x^3+c)^2}+\frac{(5ad+bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{9c^2d}$	153

input
```
int((b*x^3+a)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+1/27/c^2/d^2*
sum((5*a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(156) = 312$.

Time = 0.09 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
[1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d
^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(
c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(
2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*
x^3 + c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*
a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*
c) + 2*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d
^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c
^2*d^3)*x^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3
*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(
1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((
b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*
(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d
^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*
d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^
4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5ad^2+bcd)+x(8acd-2bc^2)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6} + \operatorname{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3, \left(t\mapsto t\log\left(\frac{27tc^3d}{5ad+bc}+x\right)\right)\right)$$

input
```
integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

output
```
(x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d*
*2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d*
*3 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27
*_t*c**3*d/(5*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{(bcd+5\,ad^2)x^4-2\,(bc^2-4\,acd)x}{18\,(c^2d^3x^6+2\,c^3d^2x^3+c^4d)} + \frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+5\,ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+5\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*
d^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c
/d)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)
*log(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^2\right)^{\frac{2}{3}}c^2} - \frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^2\right)^{\frac{2}{3}}c^2} - \frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d} + \frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\left(dx^3+c\right)^2c^2d}$$

input
```
integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3)
 + (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*lo
g(abs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2
*x + 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{(c+dx^3)^3}\,dx = \frac{\frac{x^4\,(5\,a\,d+b\,c)}{18\,c^2}+\frac{x\,(4\,a\,d-b\,c)}{9\,c\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}$$

input
```
int((a + b*x^3)/(c + d*x^3)^3,x)
```

output
```
((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6
+ 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3
)) - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/
2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/
3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.84

$$\int \frac{a+bx^3}{(c+dx^3)^3} \, dx = \text{Too large to display}$$

input
```
int((b*x^3+a)/(d*x^3+c)^3,x)
```

output
```
( - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))
*a*c**2*d - 20*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*s
qrt(3)))*a*c*d**2*x**3 - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x
)/(c**(1/3)*sqrt(3)))*a*d**3*x**6 - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*
d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b*c**3 - 4*c**(1/3)*sqrt(3)*atan((c**(1/3)
 - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b*c**2*d*x**3 - 2*c**(1/3)*sqrt(3)*at
an((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b*c*d**2*x**6 - 5*c**(1/3
)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*c**2*d - 10*c**(1/
3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*c*d**2*x**3 - 5*c
**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*d**3*x**6 -
c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c**3 - 2*c*
*(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c**2*d*x**3 -
 c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*c*d**2*x**
6 + 10*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*c**2*d + 20*c**(1/3)*log(c**(
1/3) + d**(1/3)*x)*a*c*d**2*x**3 + 10*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*
a*d**3*x**6 + 2*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*b*c**3 + 4*c**(1/3)*lo
g(c**(1/3) + d**(1/3)*x)*b*c**2*d*x**3 + 2*c**(1/3)*log(c**(1/3) + d**(1/3
)*x)*b*c*d**2*x**6 + 24*d**(1/3)*a*c**2*d*x + 15*d**(1/3)*a*c*d**2*x**4 -
6*d**(1/3)*b*c**3*x + 3*d**(1/3)*b*c**2*d*x**4)/(54*d**(1/3)*c**3*d*(c**2
+ 2*c*d*x**3 + d**2*x**6))
```

3.17 $\int (a+bx^3)^2 (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{1}{4}ac^2(2bc+3ad)x^4 + \frac{1}{7}c(b^2c^2+6abcd+3a^2d^2)\,x^7 + \frac{1}{10}d(3b^2c^2+6abcd+a^2d^2)\,x^{10} + \frac{1}{13}bd^2(3bc+2ad)x^{13} + \frac{1}{16}b^2d^3x^{16}$$

output
```
a^2*c^3*x+1/4*a*c^2*(3*a*d+2*b*c)*x^4+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^7+1/10*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^10+1/13*b*d^2*(2*a*d+3*b*c)*x^1
3+1/16*b^2*d^3*x^16
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{1}{4}ac^2(2bc+3ad)x^4 + \frac{1}{7}c(b^2c^2+6abcd+3a^2d^2)\,x^7 + \frac{1}{10}d(3b^2c^2+6abcd+a^2d^2)\,x^{10} + \frac{1}{13}bd^2(3bc+2ad)x^{13} + \frac{1}{16}b^2d^3x^{16}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*
b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+cx^6\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^3(3ad+2bc)+bd^2x^{12}(2ad+3bc)+b$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*
b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x + \left(\frac{3}{4}a^2c^2d + \frac{1}{2}ab\,c^3\right)x^4 + \left(\frac{3}{7}c\,a^2d^2 + \frac{6}{7}ab\,c^2d + \frac{1}{7}b^2c^3\right)x^7 + \left(\frac{1}{10}a^2d^3 + \frac{3}{5}ac\,d^2b + \frac{3}{10}b^2\right.$
default	$\frac{b^2d^3x^{16}}{16} + \frac{(2a\,d^3b+3b^2c\,d^2)x^{13}}{13} + \frac{(a^2d^3+6ac\,d^2b+3b^2c^2d)x^{10}}{10} + \frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^7}{7} + \frac{(3a^2c^2d+2ab\,c^3)x^4}{4}$
gosper	$a^2c^3x + \frac{3}{4}x^4a^2c^2d + \frac{1}{2}x^4ab\,c^3 + \frac{3}{7}x^7c\,a^2d^2 + \frac{6}{7}x^7ab\,c^2d + \frac{1}{7}x^7b^2c^3 + \frac{1}{10}x^{10}a^2d^3 + \frac{3}{5}x^{10}ac\,d^2b$
risch	$a^2c^3x + \frac{3}{4}x^4a^2c^2d + \frac{1}{2}x^4ab\,c^3 + \frac{3}{7}x^7c\,a^2d^2 + \frac{6}{7}x^7ab\,c^2d + \frac{1}{7}x^7b^2c^3 + \frac{1}{10}x^{10}a^2d^3 + \frac{3}{5}x^{10}ac\,d^2b$
parallelrisch	$a^2c^3x + \frac{3}{4}x^4a^2c^2d + \frac{1}{2}x^4ab\,c^3 + \frac{3}{7}x^7c\,a^2d^2 + \frac{6}{7}x^7ab\,c^2d + \frac{1}{7}x^7b^2c^3 + \frac{1}{10}x^{10}a^2d^3 + \frac{3}{5}x^{10}ac\,d^2b$
orering	$\frac{x(455b^2d^3x^{15}+1120ab\,d^3x^{12}+1680b^2c\,d^2x^{12}+728a^2d^3x^9+4368abc\,d^2x^9+2184b^2c^2d\,x^9+3120a^2c\,d^2x^6+6240ab\,c^2d\,x^6+104}{7280}$

input
```
int((b*x^3+a)^2*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x+(3/4*a^2*c^2*d+1/2*a*b*c^3)*x^4+(3/7*c*a^2*d^2+6/7*a*b*c^2*d+1/7
*b^2*c^3)*x^7+(1/10*a^2*d^3+3/5*a*c*d^2*b+3/10*b^2*c^2*d)*x^10+(2/13*a*d^3
*b+3/13*b^2*c*d^2)*x^13+1/16*b^2*d^3*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = \frac{1}{16} b^2 d^3 x^{16} + \frac{1}{13} (3 b^2 c d^2 + 2 a b d^3) x^{13} + \frac{1}{10} (3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3) x^{10} + \frac{1}{7} (b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2) x^7 + a^2 c^3 x + \frac{1}{4} (2 a b c^3 + 3 a^2 c^2 d) x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{16}}{16} + x^{13} \cdot \left(\frac{2abd^3}{13} + \frac{3b^2cd^2}{13}\right) + x^{10}\left(\frac{a^2d^3}{10} + \frac{3abcd^2}{5} + \frac{3b^2c^2d}{10}\right) + x^7 \cdot \left(\frac{3a^2cd^2}{7} + \frac{6abc^2d}{7} + \frac{b^2c^3}{7}\right) + x^4 \cdot \left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

output
```
a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13
) + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**
2*c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c
**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = \frac{1}{16} b^2 d^3 x^{16} + \frac{1}{13} (3 b^2 c d^2 + 2 abd^3) x^{13} + \frac{1}{10} (3 b^2 c^2 d + 6 abcd^2 + a^2 d^3) x^{10} + \frac{1}{7} (b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2) x^7 + a^2 c^3 x + \frac{1}{4} (2 abc^3 + 3 a^2 c^2 d) x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int (a+bx^3)^2 (c+dx^3)^3 \, dx = \frac{1}{16} b^2 d^3 x^{16} + \frac{3}{13} b^2 c d^2 x^{13} + \frac{2}{13} abd^3 x^{13} + \frac{3}{10} b^2 c^2 d x^{10} + \frac{3}{5} abcd^2 x^{10} + \frac{1}{10} a^2 d^3 x^{10} + \frac{1}{7} b^2 c^3 x^7 + \frac{6}{7} abc^2 d x^7 + \frac{3}{7} a^2 c d^2 x^7 + \frac{1}{2} abc^3 x^4 + \frac{3}{4} a^2 c^2 d x^4 + a^2 c^3 x$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2
*d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a
*b*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a
^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = x^7 \left(\frac{3\,a^2\,c\,d^2}{7}+\frac{6\,a\,b\,c^2\,d}{7}+\frac{b^2\,c^3}{7}\right) + x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{16}}{16} + \frac{a\,c^2\,x^4\,(3\,a\,d+2\,b\,c)}{4}+\frac{b\,d^2\,x^{13}\,(2\,a\,d+3\,b\,c)}{13}$$

input
```
int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

output
```
x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
 + (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (
a*c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \frac{x(455b^2d^3x^{15}+1120ab\,d^3x^{12}+1680b^2c\,d^2x^{12}+728a^2d^3x^9+4368abc\,d^2x^9+2184b^2c^2d\,x^9+3120a^2c\,d^2x}{7280}$$

input
```
int((b*x^3+a)^2*(d*x^3+c)^3,x)
```

output
```
(x*(7280*a**2*c**3 + 5460*a**2*c**2*d*x**3 + 3120*a**2*c*d**2*x**6 + 728*a
**2*d**3*x**9 + 3640*a*b*c**3*x**3 + 6240*a*b*c**2*d*x**6 + 4368*a*b*c*d**
2*x**9 + 1120*a*b*d**3*x**12 + 1040*b**2*c**3*x**6 + 2184*b**2*c**2*d*x**9
 + 1680*b**2*c*d**2*x**12 + 455*b**2*d**3*x**15))/7280
```

3.18 $\int (a+bx^3)^2 (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

output
```
a^2*c^2*x+1/2*a*c*(a*d+b*c)*x^4+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+1/5*b*
d*(a*d+b*c)*x^10+1/13*b^2*d^2*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = a^2c^2x + \frac{1}{2}ac(bc+ad)x^4 + \frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7 + \frac{1}{5}bd(bc+ad)x^{10} + \frac{1}{13}b^2d^2x^{13}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

output
```
a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(x^6\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^9(ad+bc)+2acx^3(ad+bc)+b^2d^2x^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

output
```
a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^2 d^2 x^{13}}{13}+\left(\frac{1}{5} ab\, d^2+\frac{1}{5} b^2 cd\right) x^{10}+\left(\frac{1}{7} a^2 d^2+\frac{4}{7} abcd+\frac{1}{7} b^2 c^2\right) x^7+\left(\frac{1}{2} a^2 cd+\frac{1}{2} b\, c^2 a\right) x^4+a^2 c^2$
default	$\frac{b^2 d^2 x^{13}}{13}+\frac{(2ab\, d^2+2b^2 cd) x^{10}}{10}+\frac{(a^2 d^2+4abcd+b^2 c^2) x^7}{7}+\frac{(2a^2 cd+2b\, c^2 a) x^4}{4}+a^2 c^2 x$
gosper	$\frac{1}{13} b^2 d^2 x^{13}+\frac{1}{5} x^{10} ab\, d^2+\frac{1}{5} x^{10} b^2 cd+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 abcd+\frac{1}{7} x^7 b^2 c^2+\frac{1}{2} x^4 a^2 cd+\frac{1}{2} x^4 b\, c^2 a+$
risch	$\frac{1}{13} b^2 d^2 x^{13}+\frac{1}{5} x^{10} ab\, d^2+\frac{1}{5} x^{10} b^2 cd+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 abcd+\frac{1}{7} x^7 b^2 c^2+\frac{1}{2} x^4 a^2 cd+\frac{1}{2} x^4 b\, c^2 a+$
parallelrisch	$\frac{1}{13} b^2 d^2 x^{13}+\frac{1}{5} x^{10} ab\, d^2+\frac{1}{5} x^{10} b^2 cd+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 abcd+\frac{1}{7} x^7 b^2 c^2+\frac{1}{2} x^4 a^2 cd+\frac{1}{2} x^4 b\, c^2 a+$
orering	$\frac{x(70b^2 d^2 x^{12}+182ab\, d^2 x^9+182b^2 cd\, x^9+130a^2 d^2 x^6+520abcd\, x^6+130b^2 c^2 x^6+455a^2 cd\, x^3+455ab\, c^2 x^3+910a^2 c^2)}{910}$

input
```
int((b*x^3+a)^2*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d^2*x^13+(1/5*a*b*d^2+1/5*b^2*c*d)*x^10+(1/7*a^2*d^2+4/7*a*b*c*d+
1/7*b^2*c^2)*x^7+(1/2*a^2*c*d+1/2*b*c^2*a)*x^4+a^2*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)^2 \, dx = \frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} (b^2 cd + abd^2) x^{10} + \frac{1}{7} (b^2 c^2 + 4\, abcd + a^2 d^2) x^7 + a^2 c^2 x + \frac{1}{2} (abc^2 + a^2 cd) x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^{13}}{13} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^4\left(\frac{a^2cd}{2} + \frac{abc^2}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*
(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\,\left(b^2cd + abd^2\right)x^{10} + \frac{1}{7}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + a^2c^2x + \frac{1}{2}\,\left(abc^2 + a^2cd\right)x^4$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = \frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} b^2 c d x^{10} + \frac{1}{5} a b d^2 x^{10} + \frac{1}{7} b^2 c^2 x^7 + \frac{4}{7} a b c d x^7 + \frac{1}{7} a^2 d^2 x^7 + \frac{1}{2} a b c^2 x^4 + \frac{1}{2} a^2 c d x^4 + a^2 c^2 x$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7
+ 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 +
a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx = x^7 \left(\frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7}\right) + a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + \frac{a c x^4 (a d + b c)}{2} + \frac{b d x^{10} (a d + b c)}{5}$$

input
```
int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

output
```
x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^1
3)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$$
$$= \frac{x(70b^2d^2x^{12}+182ab\,d^2x^9+182b^2cd\,x^9+130a^2d^2x^6+520abcd\,x^6+130b^2c^2x^6+455a^2cd\,x^3+455ab\,c^2x}{910}$$

input
```
int((b*x^3+a)^2*(d*x^3+c)^2,x)
```

output
```
(x*(910*a**2*c**2 + 455*a**2*c*d*x**3 + 130*a**2*d**2*x**6 + 455*a*b*c**2*
x**3 + 520*a*b*c*d*x**6 + 182*a*b*d**2*x**9 + 130*b**2*c**2*x**6 + 182*b**
2*c*d*x**9 + 70*b**2*d**2*x**12))/910
```

3.19 $\int (a+bx^3)^2 (c+dx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)^2 (c+dx^3)\, dx = a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10}$$

output
```
a^2*c*x+1/4*a*(a*d+2*b*c)*x^4+1/7*b*(2*a*d+b*c)*x^7+1/10*b^2*d*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)\, dx = a^2cx + \frac{1}{4}a(2bc+ad)x^4 + \frac{1}{7}b(bc+2ad)x^7 + \frac{1}{10}b^2dx^{10}$$

input
```
Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)
/10
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2c+bx^6(2ad+bc)+ax^3(ad+2bc)+b^2dx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

input
```
Int[(a + b*x^3)^2*(c + d*x^3),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)
/10
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^{10}}{10}+\frac{(2abd+b^2c)x^7}{7}+\frac{(d\,a^2+2abc)x^4}{4}+a^2cx$	49
norman	$\frac{b^2 d\,x^{10}}{10}+\left(\frac{2}{7}abd+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{4}d\,a^2+\frac{1}{2}abc\right)x^4+a^2cx$	49
gosper	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4d\,a^2+\frac{1}{2}x^4abc+a^2cx$	51
risch	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4d\,a^2+\frac{1}{2}x^4abc+a^2cx$	51
parallelrisch	$\frac{1}{10}b^2d\,x^{10}+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{4}x^4d\,a^2+\frac{1}{2}x^4abc+a^2cx$	51
orering	$\frac{x(14b^2d\,x^9+40abd\,x^6+20b^2c\,x^6+35a^2d\,x^3+70abc\,x^3+140a^2c)}{140}$	54

input
```
int((b*x^3+a)^2*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx=\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,(b^2c+2\,abd)x^7+\frac{1}{4}\,(2\,abc+a^2d)x^4+a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 +
a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (a+bx^3)^2 (c+dx^3)\,dx = a^2cx + \frac{b^2dx^{10}}{10} + x^7\cdot\left(\frac{2abd}{7}+\frac{b^2c}{7}\right) + x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

input
```
integrate((b*x**3+a)**2*(d*x**3+c),x)
```

output
```
a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4
+ a*b*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3)^2 (c+dx^3)\,dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,(b^2c+2\,abd)x^7 + \frac{1}{4}\,(2\,abc+a^2d)x^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 +
a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (c+dx^3)\,dx = \frac{1}{10}\,b^2dx^{10} + \frac{1}{7}\,b^2cx^7 + \frac{2}{7}\,abdx^7 + \frac{1}{2}\,abcx^4 + \frac{1}{4}\,a^2dx^4 + a^2cx$$

input
```
integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

output
```
1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*
d*x^4 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{b^2\,d\,x^{10}}{10}+a^2\,c\,x$$

input
```
int((a + b*x^3)^2*(c + d*x^3),x)
```

output
```
x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)
/10 + a^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx = \frac{x(14b^2d\,x^9+40abd\,x^6+20b^2c\,x^6+35a^2d\,x^3+70abc\,x^3+140a^2c)}{140}$$

input
```
int((b*x^3+a)^2*(d*x^3+c),x)
```

output
```
(x*(140*a**2*c + 35*a**2*d*x**3 + 70*a*b*c*x**3 + 40*a*b*d*x**6 + 20*b**2*
c*x**6 + 14*b**2*d*x**9))/140
```

3.20 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(a+bx^3)^2}{c+dx^3} \, dx = -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d-1/3*(-a*d+b*c)^2*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(2/3)/d^(7/3)+1/3*(-a*d+b*c)^2*ln(c
^(1/3)+d^(1/3)*x)/c^(2/3)/d^(7/3)-1/6*(-a*d+b*c)^2*ln(c^(2/3)-c^(1/3)*d^(1
/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(7/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{-12bc^{2/3}\sqrt[3]{d}(bc-2ad)x+3b^2c^{2/3}d^{4/3}x^4+4\sqrt{3}(bc-ad)^2\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{c}\right.}{12c^{2/3}d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

output
```
(-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*
(b*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^3)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^3}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1
/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c -
 a*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{b^2x^4}{4d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^3}$	78
default	$\frac{b\left(\frac{1}{4}bd\,x^4+2adx-bcx\right)}{d^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)\left(a^2d^2-2abcd+b^2c^2\right)}{d^2}$	140

input
```
int((b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*c*x+1/3/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.92

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$= \left[\frac{3\,b^2c^2d^2x^4+6\sqrt{\frac{1}{3}}(b^2c^3d-2\,abc^2d^2+a^2cd^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\,(c^2d)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+(c^2d)^{\frac{2}{3}}x-(c^2d)\right.}{dx^3+c}\right.}{} \right.$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3
*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(
1/3)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*lo
g(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*
c^2*d^2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d -
 2*a*b*c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^
(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^
2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2x^4}{4d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right)$$
$$+\mathrm{RootSum}\left(27t^3c^2d^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto\right.\right.$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c),x)
```

output
```
b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
 a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*
d**3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t
*log(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2dx^4-4\,(b^2c-2\,abd)x}{4\,d^2} + \frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(
c/d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(x + (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d} - \frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^4} + \frac{b^2d^3x^4-4\,b^2cd^2x+8\,abd^3x}{4\,d^4}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c
/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3
)))/(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \frac{b^2\,x^4}{4\,d} - x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right) + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)^2}{3\,c^{2/3}\,d^{7/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^2}{c^{2/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^2}{3\,c^{2/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3),x)
```

output
```
(b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*
(a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2)/(3*c^(2/3)*d^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

$$= \frac{-4c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^2d^2+8c^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)abd-4c^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)b^2-2c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}-d\right.}{}$$

input
```
int((b*x^3+a)^2/(d*x^3+c),x)
```

output
```
( - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*
a**2*d**2 + 8*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sq
rt(3)))*a*b*c*d - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1
/3)*sqrt(3)))*b**2*c**2 - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x +
d**(2/3)*x**2)*a**2*d**2 + 4*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x +
 d**(2/3)*x**2)*a*b*c*d - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x +
d**(2/3)*x**2)*b**2*c**2 + 4*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**2*d**2
 - 8*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b*c*d + 4*c**(1/3)*log(c**(1/3)
 + d**(1/3)*x)*b**2*c**2 + 24*d**(1/3)*a*b*c*d*x - 12*d**(1/3)*b**2*c**2*x
 + 3*d**(1/3)*b**2*c*d*x**4)/(12*d**(1/3)*c*d**2)
```

3.21 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{3cd^2(c+dx^3)} + \frac{2(bc-ad)(2bc+ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} - \frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}} + \frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}$$

output
```
b^2*x/d^2+1/3*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)+2/9*(-a*d+b*c)*(a*d+2*b*c)*ar
ctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(5/3)/d^(7/3)-2/
9*(-a*d+b*c)*(a*d+2*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/d^(7/3)+1/9*(-a*d+b
*c)*(a*d+2*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(7/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$$

$$= \frac{9b^2\sqrt[3]{d}x + \frac{3\sqrt[3]{d}(bc-ad)^2x}{c(c+dx^3)} + \frac{2\sqrt{3}(2b^2c^2-abcd-a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} - \frac{2(2b^2c^2-abcd-a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}} + \frac{(2b^2c^2-\cdots}{\cdots}}{9d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

output
```
(9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3
]*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[
3]])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x]
)/c^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{ 915}$$

$$\int \left(\frac{b^2}{d^2} - \frac{-a^2d^2+2bdx^3(bc-ad)+b^2c^2}{d^2(c+dx^3)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(bc-ad)(ad+2bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}}+\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2(c+dx^3)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*
b*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c
^(5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(
9*c^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.49

method	result	s
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3c\,d^2(d\,x^3+c)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^2d^2+abcd-2b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{9d^3c}$	9
default	$\frac{b^2x}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{3c\left(d\,x^3+c\right)}+\frac{2\left(a^2d^2+abcd-2b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}}{d^2}$	1

input
```
int((b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^3+c)+2/9/d^3/c*sum(
(a^2*d^2+a*b*c*d-2*b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(164) = 328$.

Time = 0.09 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.80

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2
*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/
d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (
c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (
2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^
3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b
^2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*
(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2
+ a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt
(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*
d^3 - a^2*c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2
/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2
*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*lo
g(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d
- a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c
*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(
c^3*d^4*x^3 + c^4*d^3)]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{3c^2d^2+3cd^3x^3}$$
$$+\text{RootSum}\Big(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 96ab^5c^5d + 64b^6c$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

output
```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**
3*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 2
4*a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 -
96*a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*
d**2 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(cd^3x^3+c^2d^2)} + \frac{b^2x}{d^2} - \frac{2\,\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,cd^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^3\,\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 -
2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2
*d^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*
b^2*c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2x}{d^2} + \frac{2\sqrt{3}(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}cd} + \frac{(2\,b^2c^2-abcd-a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}cd} + \frac{2\,(2\,b^2c^2-abcd-a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(dx^3+c)cd^2}$$

input

```
integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

output

```
b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)
*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2
- a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2
/3)*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-
c/d)^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3
 + c)*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{3\,c\,(d^3\,x^3+c\,d^2)} + \frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}} + \frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}} - \frac{2\,\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(a\,d+2\,b\,c)}{9\,c^{5/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3))
+ (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3
)) + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/
3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2
*b*c))/(9*c^(5/3)*d^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 575, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^2/(d*x^3+c)^2,x)
```

output
```
( - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*
a**2*c*d**2 - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*
sqrt(3)))*a**2*d**3*x**3 - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*
x)/(c**(1/3)*sqrt(3)))*a*b*c**2*d - 2*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*
d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b*c*d**2*x**3 + 4*c**(1/3)*sqrt(3)*atan(
(c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b**2*c**3 + 4*c**(1/3)*sqrt(
3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b**2*c**2*d*x**3 - c
**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*c*d**2 -
c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*d**3*x**
3 - c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b*c**2*
d - c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b*c*d**
2*x**3 + 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b*
*2*c**3 + 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b
**2*c**2*d*x**3 + 2*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**2*c*d**2 + 2*c*
*(1/3)*log(c**(1/3) + d**(1/3)*x)*a**2*d**3*x**3 + 2*c**(1/3)*log(c**(1/3)
 + d**(1/3)*x)*a*b*c**2*d + 2*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b*c*d*
*2*x**3 - 4*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*b**2*c**3 - 4*c**(1/3)*log
(c**(1/3) + d**(1/3)*x)*b**2*c**2*d*x**3 + 3*d**(1/3)*a**2*c*d**2*x - 6*d*
*(1/3)*a*b*c**2*d*x + 12*d**(1/3)*b**2*c**3*x + 9*d**(1/3)*b**2*c**2*d*x**
4)/(9*d**(1/3)*c**2*d**2*(c + d*x**3))
```

3.22 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 253

$$\begin{aligned}\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx &= \frac{(bc-ad)^2x}{6cd^2(c+dx^3)^2} - \frac{(bc-ad)(7bc+5ad)x}{18c^2d^2(c+dx^3)} \\ &\quad - \frac{(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} \\ &\quad + \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} \\ &\quad - \frac{(2b^2c^2+2abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}\end{aligned}$$

output
```
1/6*(-a*d+b*c)^2*x/c/d^2/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+7*b*c)*x/c^2/d
^2/(d*x^3+c)-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*arctan(1/3*(c^(1/3)-2*d^
(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(8/3)/d^(7/3)+1/27*(5*a^2*d^2+2*a*b*c*
d+2*b^2*c^2)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/d^(7/3)-1/54*(5*a^2*d^2+2*a*b*c
*d+2*b^2*c^2)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(7/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$$

$$= \frac{-\frac{3c^{2/3}\sqrt[3]{d}(bc-ad)x(ad(8c+5dx^3)+bc(4c+7dx^3))}{(c+dx^3)^2} - 2\sqrt{3}(2b^2c^2+2abcd+5a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 2(2b^2c^2+2}{54c^{8/3}d^{7/3}}$$

input
```
Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

output
```
((-3*c^(2/3)*d^(1/3)*(b*c - a*d)*x*(a*d*(8*c + 5*d*x^3) + b*c*(4*c + 7*d*x
^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTan
[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d
^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {930, 910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{2b(2bc+ad)x^3+a(bc+5ad)}{(dx^3+c)^2}dx}{6cd} - \frac{x(a+bx^3)(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \; 910$$

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\int\frac{1}{dx^3+c}dx-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 750

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+4\ldots\right.}{3(c+dx^3)}}{6cd}$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\ldots\right.}{3(c+dx^3\ldots}}{6cd}$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+a\ldots\right.}{3(c+dx\ldots}}{6cd}$$

$$\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{\frac{2}{3}\left(\frac{5a^2d}{c}+2ab+\frac{2b^2c}{d}\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)-\frac{x\left(-\frac{5a^2d}{c}+ab+\frac{4b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3))/(c*d*(c + d*x^3)^2) + (-1/3*((a*b + (4*b^
2*c)/d - (5*a^2*d)/c)*x)/(c + d*x^3) + (2*(2*a*b + (2*b^2*c)/d + (5*a^2*d)
/c)*(Log[c^(1/3) + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1
- (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2]/(2*d^(1/3)))/(3*c^(2/3))))/3)/(6*c*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{\left(d\,x^3+c\right)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}}{27c^2d^3}$
default	$\frac{\frac{\left(5a^2d^2+2abcd-7b^2c^2\right)x^4}{18c^2d}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)x}{9c\,d^2}}{\left(d\,x^3+c\right)^2}+\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan(}{}\right.}{9c^2d^2}$

input
```
int((b*x^3+a)^2/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*b*c*d-b^2
*c^2)/c/d^2*x)/(d*x^3+c)^2+1/27/c^2/d^3*sum((5*a^2*d^2+2*a*b*c*d+2*b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(212) = 424$.

Time = 0.15 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.22

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
[-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3
)*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^
2*d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^
4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2
+ 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d
)^(1/3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6
+ 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2
 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)
^(1/3)*c) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^
3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4
*d^2 - 2*a^2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3
*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*
c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5
*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*s
qrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c
)*sqrt((c^2*d)^(1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*
x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2
*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^
2*d)^(1/3)*c) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^...
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{x^4\cdot(5a^2d^3+2abcd^2-7b^2c^2d)+x(8a^2cd^2-4abc^2d-4b^2c^3)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}$$
$$+\mathrm{RootSum}\left(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210a^4b^2c^2d^4-128a^3b^3c^3d^3-84a^2b^4c^4d^2-24ab^5c^5d\right.$$

input
```
integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

output
```
(x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*
a*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**
4*x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**
5 - 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*
d**2 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(
5*a**2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3)x^4+4\,(b^2c^3+abc^2d-2\,a^2cd^2)x}{18\,(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2)}$$
$$+\frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$
$$-\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$
$$+\frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
-1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*
d - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)
*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2
*d^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*
(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2
/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{\sqrt{3}(2\,b^2c^2+2\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2d} - \frac{(2\,b^2c^2+2\,abcd+5\,a^2d^2)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^2} - \frac{7\,b^2c^2dx^4-2\,abcd^2x^4-5\,a^2d^3x^4+4\,b^2c^3x+4\,abc^2dx-8\,a^2cd^2x}{18\,(dx^3+c)^2c^2d^2}$$

input
```
integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x
+ (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2
*a*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(
2/3)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(ab
s(x - (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 -
 5*a^2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)
^2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\frac{2\,x\,\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}$$

input
```
int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

output
```
(log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)
*d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2
*d^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (
log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*
a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1
/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*
c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.53

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

output
```
( - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))
*a**2*c**2*d**2 - 20*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(
1/3)*sqrt(3)))*a**2*c*d**3*x**3 - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d
**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*d**4*x**6 - 4*c**(1/3)*sqrt(3)*atan((c
**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b*c**3*d - 8*c**(1/3)*sqrt(3
)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b*c**2*d**2*x**3 -
4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b*
c*d**3*x**6 - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*
sqrt(3)))*b**2*c**4 - 8*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c
**(1/3)*sqrt(3)))*b**2*c**3*d*x**3 - 4*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2
*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b**2*c**2*d**2*x**6 - 5*c**(1/3)*log(c**(
2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*c**2*d**2 - 10*c**(1/3)*l
og(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*c*d**3*x**3 - 5*c*
*(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*d**4*x**6
- 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b*c**3*
d - 4*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b*c**
2*d**2*x**3 - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**
2)*a*b*c*d**3*x**6 - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2
/3)*x**2)*b**2*c**4 - 4*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(
2/3)*x**2)*b**2*c**3*d*x**3 - 2*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/...
```

3.23 $\int (a+bx^3)^3 (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\begin{aligned}\int (a+bx^3)^3 (c+dx^3)^3 \, dx = {} & a^3c^3x + \frac{3}{4}a^2c^2(bc+ad)x^4 + \frac{3}{7}ac\left(b^2c^2+3abcd+a^2d^2\right)x^7 \\ & + \frac{1}{10}(bc+ad)\left(b^2c^2+8abcd+a^2d^2\right)x^{10} \\ & + \frac{3}{13}bd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} \\ & + \frac{3}{16}b^2d^2(bc+ad)x^{16} + \frac{1}{19}b^3d^3x^{19}\end{aligned}$$

output
```
a^3*c^3*x+3/4*a^2*c^2*(a*d+b*c)*x^4+3/7*a*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^
7+1/10*(a*d+b*c)*(a^2*d^2+8*a*b*c*d+b^2*c^2)*x^10+3/13*b*d*(a^2*d^2+3*a*b*
c*d+b^2*c^2)*x^13+3/16*b^2*d^2*(a*d+b*c)*x^16+1/19*b^3*d^3*x^19
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int (a+bx^3)^3 (c+dx^3)^3 \, dx = a^3c^3x + \frac{3}{4}a^2c^2(bc+ad)x^4 + \frac{3}{7}ac(b^2c^2+3abcd+a^2d^2)\,x^7 + \frac{1}{10}(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)\,x^{10} + \frac{3}{13}bd(b^2c^2+3abcd+a^2d^2)\,x^{13} + \frac{3}{16}b^2d^2(bc+ad)x^{16} + \frac{1}{19}b^3d^3x^{19}$$

input
```
Integrate[(a + b*x^3)^3*(c + d*x^3)^3,x]
```

output
```
a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^4)/4 + (3*a*c*(b^2*c^2 + 3*a*b*c*d +
a^2*d^2)*x^7)/7 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^1
0)/10 + (3*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (3*b^2*d^2*(b*c
+ a*d)*x^16)/16 + (b^3*d^3*x^19)/19
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^3 (c+dx^3)^3 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^3c^3 + 3bdx^{12}(a^2d^2+3abcd+b^2c^2) + x^9(ad+bc)\,(a^2d^2+8abcd+b^2c^2) + 3acx^6(a^2d^2+3abcd+b^2c^2) + 3a\right.$$

$$\Big\downarrow \text{2009}$$

$$a^3c^3x + \frac{3}{13}bdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{10}x^{10}(ad + bc)(a^2d^2 + 8abcd + b^2c^2) + \frac{3}{7}acx^7(a^2d^2 + 3abcd + b^2c^2) + \frac{3}{4}a^2c^2x^4(ad + bc) + \frac{3}{16}b^2d^2x^{16}(ad + bc) + \frac{1}{19}b^3d^3x^{19}$$

input
```
Int[(a + b*x^3)^3*(c + d*x^3)^3,x]
```

output
```
a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^4)/4 + (3*a*c*(b^2*c^2 + 3*a*b*c*d +
a^2*d^2)*x^7)/7 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^10)/10 +
(3*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (3*b^2*d^2*(b*c + a*d)*x
^16)/16 + (b^3*d^3*x^19)/19
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.11

method	result
norman	$a^3c^3x + \left(\frac{3}{4}a^3c^2d + \frac{3}{4}a^2b\,c^3\right)x^4 + \left(\frac{3}{7}a^3c\,d^2 + \frac{9}{7}a^2b\,c^2d + \frac{3}{7}b^2c^3a\right)x^7 + \left(\frac{1}{10}a^3d^3 + \frac{9}{10}a^2bc\,d^2 +\right.$
default	$\frac{b^3d^3x^{19}}{19} + \frac{(3a\,b^2d^3+3b^3c\,d^2)x^{16}}{16} + \frac{(3a^2b\,d^3+9a\,b^2c\,d^2+3b^3c^2d)x^{13}}{13} + \frac{(a^3d^3+9a^2bc\,d^2+9a\,b^2c^2d+b^3c^3)x^{10}}{10} + \frac{(3a^3c}{}$
gosper	$a^3c^3x + \frac{3}{4}x^4a^3c^2d + \frac{3}{4}a^2b\,c^3x^4 + \frac{3}{7}a^3c\,d^2x^7 + \frac{9}{7}a^2b\,c^2d\,x^7 + \frac{3}{7}x^7b^2c^3a + \frac{1}{10}x^{10}a^3d^3 + \frac{9}{10}x^{10}a^2$
risch	$a^3c^3x + \frac{3}{4}x^4a^3c^2d + \frac{3}{4}a^2b\,c^3x^4 + \frac{3}{7}a^3c\,d^2x^7 + \frac{9}{7}a^2b\,c^2d\,x^7 + \frac{3}{7}x^7b^2c^3a + \frac{1}{10}x^{10}a^3d^3 + \frac{9}{10}x^{10}a^2$
parallelrisch	$a^3c^3x + \frac{3}{4}x^4a^3c^2d + \frac{3}{4}a^2b\,c^3x^4 + \frac{3}{7}a^3c\,d^2x^7 + \frac{9}{7}a^2b\,c^2d\,x^7 + \frac{3}{7}x^7b^2c^3a + \frac{1}{10}x^{10}a^3d^3 + \frac{9}{10}x^{10}a^2$
orering	$\frac{x(7280b^3d^3x^{18}+25935a\,b^2d^3x^{15}+25935b^3c\,d^2x^{15}+31920a^2b\,d^3x^{12}+95760a\,b^2c\,d^2x^{12}+31920b^3c^2d\,x^{12}+13832a^3d^3x^9+124}{}$

input
```
int((b*x^3+a)^3*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^3*x+(3/4*a^3*c^2*d+3/4*a^2*b*c^3)*x^4+(3/7*a^3*c*d^2+9/7*a^2*b*c^2*d
+3/7*b^2*c^3*a)*x^7+(1/10*a^3*d^3+9/10*a^2*b*c*d^2+9/10*a*b^2*c^2*d+1/10*b
^3*c^3)*x^10+(3/13*a^2*b*d^3+9/13*a*b^2*c*d^2+3/13*b^3*c^2*d)*x^13+(3/16*a
*b^2*d^3+3/16*b^3*c*d^2)*x^16+1/19*b^3*d^3*x^19
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int (a+bx^3)^3 (c+dx^3)^3 \, dx = \frac{1}{19} b^3 d^3 x^{19} + \frac{3}{16} (b^3 c d^2 + a b^2 d^3) x^{16} + \frac{3}{13} (b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3) x^{13} + \frac{1}{10} (b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3) x^{10} + \frac{3}{7} (a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2) x^7 + a^3 c^3 x + \frac{3}{4} (a^2 b c^3 + a^3 c^2 d) x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/19*b^3*d^3*x^19 + 3/16*(b^3*c*d^2 + a*b^2*d^3)*x^16 + 3/13*(b^3*c^2*d +
3*a*b^2*c*d^2 + a^2*b*d^3)*x^13 + 1/10*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*x^10 + 3/7*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^7 +
a^3*c^3*x + 3/4*(a^2*b*c^3 + a^3*c^2*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int (a+bx^3)^3 (c+dx^3)^3 \, dx = a^3 c^3 x + \frac{b^3 d^3 x^{19}}{19} + x^{16} \cdot \left(\frac{3ab^2d^3}{16} + \frac{3b^3cd^2}{16}\right) + x^{13} \cdot \left(\frac{3a^2bd^3}{13} + \frac{9ab^2cd^2}{13} + \frac{3b^3c^2d}{13}\right) + x^{10}\left(\frac{a^3d^3}{10} + \frac{9a^2bcd^2}{10} + \frac{9ab^2c^2d}{10} + \frac{b^3c^3}{10}\right) + x^7 \cdot \left(\frac{3a^3cd^2}{7} + \frac{9a^2bc^2d}{7} + \frac{3ab^2c^3}{7}\right) + x^4 \cdot \left(\frac{3a^3c^2d}{4} + \frac{3a^2bc^3}{4}\right)$$

input
```
integrate((b*x**3+a)**3*(d*x**3+c)**3,x)
```

output
```
a**3*c**3*x + b**3*d**3*x**19/19 + x**16*(3*a*b**2*d**3/16 + 3*b**3*c*d**2
/16) + x**13*(3*a**2*b*d**3/13 + 9*a*b**2*c*d**2/13 + 3*b**3*c**2*d/13) +
x**10*(a**3*d**3/10 + 9*a**2*b*c*d**2/10 + 9*a*b**2*c**2*d/10 + b**3*c**3/
10) + x**7*(3*a**3*c*d**2/7 + 9*a**2*b*c**2*d/7 + 3*a*b**2*c**3/7) + x**4*
(3*a**3*c**2*d/4 + 3*a**2*b*c**3/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int (a+bx^3)^3 (c+dx^3)^3 \, dx = \frac{1}{19} b^3 d^3 x^{19} + \frac{3}{16} (b^3 c d^2 + a b^2 d^3) x^{16} + \frac{3}{13} (b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3) x^{13} + \frac{1}{10} (b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3) x^{10} + \frac{3}{7} (a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2) x^7 + a^3 c^3 x + \frac{3}{4} (a^2 b c^3 + a^3 c^2 d) x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
1/19*b^3*d^3*x^19 + 3/16*(b^3*c*d^2 + a*b^2*d^3)*x^16 + 3/13*(b^3*c^2*d +
3*a*b^2*c*d^2 + a^2*b*d^3)*x^13 + 1/10*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*x^10 + 3/7*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^7 +
a^3*c^3*x + 3/4*(a^2*b*c^3 + a^3*c^2*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^3 \, dx = \frac{1}{19} b^3 d^3 x^{19} + \frac{3}{16} b^3 c d^2 x^{16} + \frac{3}{16} a b^2 d^3 x^{16} + \frac{3}{13} b^3 c^2 d x^{13} + \frac{9}{13} a b^2 c d^2 x^{13} + \frac{3}{13} a^2 b d^3 x^{13} + \frac{1}{10} b^3 c^3 x^{10} + \frac{9}{10} a b^2 c^2 d x^{10} + \frac{9}{10} a^2 b c d^2 x^{10} + \frac{1}{10} a^3 d^3 x^{10} + \frac{3}{7} a b^2 c^3 x^7 + \frac{9}{7} a^2 b c^2 d x^7 + \frac{3}{7} a^3 c d^2 x^7 + \frac{3}{4} a^2 b c^3 x^4 + \frac{3}{4} a^3 c^2 d x^4 + a^3 c^3 x$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
1/19*b^3*d^3*x^19 + 3/16*b^3*c*d^2*x^16 + 3/16*a*b^2*d^3*x^16 + 3/13*b^3*c
^2*d*x^13 + 9/13*a*b^2*c*d^2*x^13 + 3/13*a^2*b*d^3*x^13 + 1/10*b^3*c^3*x^1
0 + 9/10*a*b^2*c^2*d*x^10 + 9/10*a^2*b*c*d^2*x^10 + 1/10*a^3*d^3*x^10 + 3/
7*a*b^2*c^3*x^7 + 9/7*a^2*b*c^2*d*x^7 + 3/7*a^3*c*d^2*x^7 + 3/4*a^2*b*c^3*
x^4 + 3/4*a^3*c^2*d*x^4 + a^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^3 \, dx = x^{10} \left(\frac{a^3\,d^3}{10} + \frac{9\,a^2\,b\,c\,d^2}{10} + \frac{9\,a\,b^2\,c^2\,d}{10} + \frac{b^3\,c^3}{10}\right) + a^3\,c^3\,x + \frac{b^3\,d^3\,x^{19}}{19} + \frac{3\,a\,c\,x^7\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{7} + \frac{3\,b\,d\,x^{13}\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{13} + \frac{3\,a^2\,c^2\,x^4\,\left(a\,d+b\,c\right)}{4} + \frac{3\,b^2\,d^2\,x^{16}\,\left(a\,d+b\,c\right)}{16}$$

input
```
int((a + b*x^3)^3*(c + d*x^3)^3,x)
```

output
```
x^10*((a^3*d^3)/10 + (b^3*c^3)/10 + (9*a*b^2*c^2*d)/10 + (9*a^2*b*c*d^2)/1
0) + a^3*c^3*x + (b^3*d^3*x^19)/19 + (3*a*c*x^7*(a^2*d^2 + b^2*c^2 + 3*a*b
*c*d))/7 + (3*b*d*x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13 + (3*a^2*c^2*x^
4*(a*d + b*c))/4 + (3*b^2*d^2*x^16*(a*d + b*c))/16
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.23

$$\int (a + bx^3)^3 (c + dx^3)^3 dx$$
$$= \frac{x(7280b^3d^3x^{18} + 25935a\,b^2d^3x^{15} + 25935b^3c\,d^2x^{15} + 31920a^2b\,d^3x^{12} + 95760a\,b^2c\,d^2x^{12} + 31920b^3c^2d\,x^{12}}{}$$

input
```
int((b*x^3+a)^3*(d*x^3+c)^3,x)
```

output
```
(x*(138320*a**3*c**3 + 103740*a**3*c**2*d*x**3 + 59280*a**3*c*d**2*x**6 +
13832*a**3*d**3*x**9 + 103740*a**2*b*c**3*x**3 + 177840*a**2*b*c**2*d*x**6
 + 124488*a**2*b*c*d**2*x**9 + 31920*a**2*b*d**3*x**12 + 59280*a*b**2*c**3
*x**6 + 124488*a*b**2*c**2*d*x**9 + 95760*a*b**2*c*d**2*x**12 + 25935*a*b*
*2*d**3*x**15 + 13832*b**3*c**3*x**9 + 31920*b**3*c**2*d*x**12 + 25935*b**
3*c*d**2*x**15 + 7280*b**3*d**3*x**18))/138320
```

3.24 $\int (a+bx^3)^3 (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^3)^3 (c+dx^3)^2 \, dx = a^3c^2x + \frac{1}{4}a^2c(3bc+2ad)x^4 + \frac{1}{7}a\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{10}b\left(b^2c^2+6abcd+3a^2d^2\right)x^{10} + \frac{1}{13}b^2d(2bc+3ad)x^{13} + \frac{1}{16}b^3d^2x^{16}$$

output
```
a^3*c^2*x+1/4*a^2*c*(2*a*d+3*b*c)*x^4+1/7*a*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*
x^7+1/10*b*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^10+1/13*b^2*d*(3*a*d+2*b*c)*x^1
3+1/16*b^3*d^2*x^16
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^3 (c+dx^3)^2 \, dx = a^3c^2x + \frac{1}{4}a^2c(3bc+2ad)x^4 + \frac{1}{7}a\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{10}b\left(b^2c^2+6abcd+3a^2d^2\right)x^{10} + \frac{1}{13}b^2d(2bc+3ad)x^{13} + \frac{1}{16}b^3d^2x^{16}$$

input
```
Integrate[(a + b*x^3)^3*(c + d*x^3)^2,x]
```

output
```
a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^4)/4 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^7)/7 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^10)/10 + (b^2*d*(2*
b*c + 3*a*d)*x^13)/13 + (b^3*d^2*x^16)/16
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^3c^2+bx^9\left(3a^2d^2+6abcd+b^2c^2\right)+ax^6\left(a^2d^2+6abcd+3b^2c^2\right)+a^2cx^3(2ad+3bc)+b^2dx^{12}(3ad+2bc)+b$$

$$\Big\downarrow \text{2009}$$

$$a^3c^2x+\frac{1}{10}bx^{10}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{7}ax^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}a^2cx^4(2ad+3bc)+\frac{1}{13}b^2dx^{13}(3ad+2bc)+\frac{1}{16}b^3d^2x^{16}$$

input
```
Int[(a + b*x^3)^3*(c + d*x^3)^2,x]
```

output
```
a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^4)/4 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^7)/7 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^10)/10 + (b^2*d*(2*
b*c + 3*a*d)*x^13)/13 + (b^3*d^2*x^16)/16
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^3c^2x+\left(\frac{1}{2}a^3cd+\frac{3}{4}a^2b\,c^2\right)x^4+\left(\frac{1}{7}a^3d^2+\frac{6}{7}a^2bcd+\frac{3}{7}a\,b^2c^2\right)x^7+\left(\frac{3}{10}a^2b\,d^2+\frac{3}{5}a\,b^2cd+\frac{1}{10}c\right.$
default	$\frac{b^3d^2x^{16}}{16}+\frac{(3a\,b^2d^2+2d\,b^3c)x^{13}}{13}+\frac{(3a^2b\,d^2+6a\,b^2cd+c^2b^3)x^{10}}{10}+\frac{(a^3d^2+6a^2bcd+3a\,b^2c^2)x^7}{7}+\frac{(2a^3cd+3a^2b\,c^2)x^4}{4}$
gosper	$a^3c^2x+\frac{1}{2}x^4a^3cd+\frac{3}{4}x^4a^2b\,c^2+\frac{1}{7}x^7a^3d^2+\frac{6}{7}x^7a^2bcd+\frac{3}{7}x^7a\,b^2c^2+\frac{3}{10}x^{10}a^2b\,d^2+\frac{3}{5}x^{10}a\,b^2c$
risch	$a^3c^2x+\frac{1}{2}x^4a^3cd+\frac{3}{4}x^4a^2b\,c^2+\frac{1}{7}x^7a^3d^2+\frac{6}{7}x^7a^2bcd+\frac{3}{7}x^7a\,b^2c^2+\frac{3}{10}x^{10}a^2b\,d^2+\frac{3}{5}x^{10}a\,b^2c$
parallelrisch	$a^3c^2x+\frac{1}{2}x^4a^3cd+\frac{3}{4}x^4a^2b\,c^2+\frac{1}{7}x^7a^3d^2+\frac{6}{7}x^7a^2bcd+\frac{3}{7}x^7a\,b^2c^2+\frac{3}{10}x^{10}a^2b\,d^2+\frac{3}{5}x^{10}a\,b^2c$
orering	$\frac{x(455b^3d^2x^{15}+1680a\,b^2d^2x^{12}+1120b^3cd\,x^{12}+2184a^2b\,d^2x^9+4368a\,b^2cd\,x^9+728b^3c^2x^9+1040a^3d^2x^6+6240a^2bcd\,x^6+3120}{7280}$

input
```
int((b*x^3+a)^3*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*c^2*x+(1/2*a^3*c*d+3/4*a^2*b*c^2)*x^4+(1/7*a^3*d^2+6/7*a^2*b*c*d+3/7*a
*b^2*c^2)*x^7+(3/10*a^2*b*d^2+3/5*a*b^2*c*d+1/10*c^2*b^3)*x^10+(3/13*a*b^2
*d^2+2/13*d*b^3*c)*x^13+1/16*b^3*d^2*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^2 \, dx = \frac{1}{16} b^3 d^2 x^{16} + \frac{1}{13} \left(2 b^3 cd + 3 ab^2 d^2\right) x^{13} + \frac{1}{10} \left(b^3 c^2 + 6 ab^2 cd + 3 a^2 bd^2\right) x^{10} + \frac{1}{7} \left(3 ab^2 c^2 + 6 a^2 bcd + a^3 d^2\right) x^7 + a^3 c^2 x + \frac{1}{4} \left(3 a^2 bc^2 + 2 a^3 cd\right) x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/16*b^3*d^2*x^16 + 1/13*(2*b^3*c*d + 3*a*b^2*d^2)*x^13 + 1/10*(b^3*c^2 +
6*a*b^2*c*d + 3*a^2*b*d^2)*x^10 + 1/7*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2
)*x^7 + a^3*c^2*x + 1/4*(3*a^2*b*c^2 + 2*a^3*c*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^2 \, dx = a^3 c^2 x + \frac{b^3 d^2 x^{16}}{16} + x^{13} \cdot \left(\frac{3ab^2 d^2}{13} + \frac{2b^3 cd}{13}\right) + x^{10} \cdot \left(\frac{3a^2 bd^2}{10} + \frac{3ab^2 cd}{5} + \frac{b^3 c^2}{10}\right) + x^7 \left(\frac{a^3 d^2}{7} + \frac{6a^2 bcd}{7} + \frac{3ab^2 c^2}{7}\right) + x^4 \left(\frac{a^3 cd}{2} + \frac{3a^2 bc^2}{4}\right)$$

input
```
integrate((b*x**3+a)**3*(d*x**3+c)**2,x)
```

output
```
a**3*c**2*x + b**3*d**2*x**16/16 + x**13*(3*a*b**2*d**2/13 + 2*b**3*c*d/13
) + x**10*(3*a**2*b*d**2/10 + 3*a*b**2*c*d/5 + b**3*c**2/10) + x**7*(a**3*
d**2/7 + 6*a**2*b*c*d/7 + 3*a*b**2*c**2/7) + x**4*(a**3*c*d/2 + 3*a**2*b*c
**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^3)^3 (c+dx^3)^2 \, dx = \frac{1}{16} b^3 d^2 x^{16} + \frac{1}{13} (2 b^3 cd + 3 ab^2 d^2) x^{13} + \frac{1}{10} (b^3 c^2 + 6 ab^2 cd + 3 a^2 bd^2) x^{10} + \frac{1}{7} (3 ab^2 c^2 + 6 a^2 bcd + a^3 d^2) x^7 + a^3 c^2 x + \frac{1}{4} (3 a^2 bc^2 + 2 a^3 cd) x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/16*b^3*d^2*x^16 + 1/13*(2*b^3*c*d + 3*a*b^2*d^2)*x^13 + 1/10*(b^3*c^2 +
6*a*b^2*c*d + 3*a^2*b*d^2)*x^10 + 1/7*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2
)*x^7 + a^3*c^2*x + 1/4*(3*a^2*b*c^2 + 2*a^3*c*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int (a+bx^3)^3 (c+dx^3)^2 \, dx = \frac{1}{16} b^3 d^2 x^{16} + \frac{2}{13} b^3 cdx^{13} + \frac{3}{13} ab^2 d^2 x^{13} + \frac{1}{10} b^3 c^2 x^{10} + \frac{3}{5} ab^2 cdx^{10} + \frac{3}{10} a^2 bd^2 x^{10} + \frac{3}{7} ab^2 c^2 x^7 + \frac{6}{7} a^2 bcdx^7 + \frac{1}{7} a^3 d^2 x^7 + \frac{3}{4} a^2 bc^2 x^4 + \frac{1}{2} a^3 cdx^4 + a^3 c^2 x$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
1/16*b^3*d^2*x^16 + 2/13*b^3*c*d*x^13 + 3/13*a*b^2*d^2*x^13 + 1/10*b^3*c^2
*x^10 + 3/5*a*b^2*c*d*x^10 + 3/10*a^2*b*d^2*x^10 + 3/7*a*b^2*c^2*x^7 + 6/7
*a^2*b*c*d*x^7 + 1/7*a^3*d^2*x^7 + 3/4*a^2*b*c^2*x^4 + 1/2*a^3*c*d*x^4 + a
^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^2 \, dx = x^7 \left(\frac{a^3 d^2}{7}+\frac{6 a^2 b c d}{7}+\frac{3 a b^2 c^2}{7}\right) + x^{10} \left(\frac{3 a^2 b d^2}{10}+\frac{3 a b^2 c d}{5}+\frac{b^3 c^2}{10}\right)+a^3 c^2 x+\frac{b^3 d^2 x^{16}}{16} + \frac{a^2 c x^4 (2 a d+3 b c)}{4}+\frac{b^2 d x^{13} (3 a d+2 b c)}{13}$$

input
```
int((a + b*x^3)^3*(c + d*x^3)^2,x)
```

output
```
x^7*((a^3*d^2)/7 + (3*a*b^2*c^2)/7 + (6*a^2*b*c*d)/7) + x^10*((b^3*c^2)/10
 + (3*a^2*b*d^2)/10 + (3*a*b^2*c*d)/5) + a^3*c^2*x + (b^3*d^2*x^16)/16 + (
a^2*c*x^4*(2*a*d + 3*b*c))/4 + (b^2*d*x^13*(3*a*d + 2*b*c))/13
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)^2 \, dx = \frac{x(455b^3d^2x^{15}+1680a\,b^2d^2x^{12}+1120b^3cd\,x^{12}+2184a^2b\,d^2x^9+4368a\,b^2cd\,x^9+728b^3c^2x^9+1040a^3d^2x^6}{7280}$$

input
```
int((b*x^3+a)^3*(d*x^3+c)^2,x)
```

output
```
(x*(7280*a**3*c**2 + 3640*a**3*c*d*x**3 + 1040*a**3*d**2*x**6 + 5460*a**2*
b*c**2*x**3 + 6240*a**2*b*c*d*x**6 + 2184*a**2*b*d**2*x**9 + 3120*a*b**2*c
**2*x**6 + 4368*a*b**2*c*d*x**9 + 1680*a*b**2*d**2*x**12 + 728*b**3*c**2*x
**9 + 1120*b**3*c*d*x**12 + 455*b**3*d**2*x**15))/7280
```

3.25 $\int (a+bx^3)^3 (c+dx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^3)^3 (c+dx^3)\, dx = a^3cx + \frac{1}{4}a^2(3bc+ad)x^4 + \frac{3}{7}ab(bc+ad)x^7 + \frac{1}{10}b^2(bc+3ad)x^{10} + \frac{1}{13}b^3dx^{13}$$

output
```
a^3*c*x+1/4*a^2*(a*d+3*b*c)*x^4+3/7*a*b*(a*d+b*c)*x^7+1/10*b^2*(3*a*d+b*c)
*x^10+1/13*b^3*d*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^3 (c+dx^3)\, dx = a^3cx + \frac{1}{4}a^2(3bc+ad)x^4 + \frac{3}{7}ab(bc+ad)x^7 + \frac{1}{10}b^2(bc+3ad)x^{10} + \frac{1}{13}b^3dx^{13}$$

input
```
Integrate[(a + b*x^3)^3*(c + d*x^3),x]
```

output
```
a^3*c*x + (a^2*(3*b*c + a*d)*x^4)/4 + (3*a*b*(b*c + a*d)*x^7)/7 + (b^2*(b*
c + 3*a*d)*x^10)/10 + (b^3*d*x^13)/13
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(a^3c+a^2x^3(ad+3bc)+b^2x^9(3ad+bc)+3abx^6(ad+bc)+b^3dx^{12}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3cx+\frac{1}{4}a^2x^4(ad+3bc)+\frac{1}{10}b^2x^{10}(3ad+bc)+\frac{3}{7}abx^7(ad+bc)+\frac{1}{13}b^3dx^{13}$$

input
```
Int[(a + b*x^3)^3*(c + d*x^3),x]
```

output
```
a^3*c*x + (a^2*(3*b*c + a*d)*x^4)/4 + (3*a*b*(b*c + a*d)*x^7)/7 + (b^2*(b*
c + 3*a*d)*x^10)/10 + (b^3*d*x^13)/13
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
norman	$\frac{b^3 d\,x^{13}}{13}+\left(\frac{3}{10}a\,b^2 d+\frac{1}{10}b^3 c\right)x^{10}+\left(\frac{3}{7}a^2 bd+\frac{3}{7}a\,b^2 c\right)x^7+\left(\frac{1}{4}a^3 d+\frac{3}{4}a^2 bc\right)x^4+a^3 cx$	72
default	$\frac{b^3 d\,x^{13}}{13}+\frac{(3a\,b^2 d+b^3 c)x^{10}}{10}+\frac{(3a^2 bd+3a\,b^2 c)x^7}{7}+\frac{(a^3 d+3a^2 bc)x^4}{4}+a^3 cx$	73
gosper	$\frac{1}{13}b^3 d\,x^{13}+\frac{3}{10}x^{10}a\,b^2 d+\frac{1}{10}x^{10}b^3 c+\frac{3}{7}x^7 a^2 bd+\frac{3}{7}x^7 a\,b^2 c+\frac{1}{4}x^4 a^3 d+\frac{3}{4}a^2 bc\,x^4+a^3 cx$	75
risch	$\frac{1}{13}b^3 d\,x^{13}+\frac{3}{10}x^{10}a\,b^2 d+\frac{1}{10}x^{10}b^3 c+\frac{3}{7}x^7 a^2 bd+\frac{3}{7}x^7 a\,b^2 c+\frac{1}{4}x^4 a^3 d+\frac{3}{4}a^2 bc\,x^4+a^3 cx$	75
parallelrisch	$\frac{1}{13}b^3 d\,x^{13}+\frac{3}{10}x^{10}a\,b^2 d+\frac{1}{10}x^{10}b^3 c+\frac{3}{7}x^7 a^2 bd+\frac{3}{7}x^7 a\,b^2 c+\frac{1}{4}x^4 a^3 d+\frac{3}{4}a^2 bc\,x^4+a^3 cx$	75
orering	$\frac{x\left(140b^3 d\,x^{12}+546a\,b^2 d\,x^9+182b^3 c\,x^9+780a^2 bd\,x^6+780a\,b^2 c\,x^6+455a^3 d\,x^3+1365a^2 bc\,x^3+1820a^3 c\right)}{1820}$	78

input
```
int((b*x^3+a)^3*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*d*x^13+(3/10*a*b^2*d+1/10*b^3*c)*x^10+(3/7*a^2*b*d+3/7*a*b^2*c)*x
^7+(1/4*a^3*d+3/4*a^2*b*c)*x^4+a^3*c*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^3\left(c+dx^3\right)\,dx=\frac{1}{13}\,b^3dx^{13}+\frac{1}{10}\left(b^3c+3\,ab^2d\right)x^{10}+\frac{3}{7}\left(ab^2c+a^2bd\right)x^7+a^3cx+\frac{1}{4}\left(3\,a^2bc+a^3d\right)x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c),x, algorithm="fricas")
```

output
```
1/13*b^3*d*x^13 + 1/10*(b^3*c + 3*a*b^2*d)*x^10 + 3/7*(a*b^2*c + a^2*b*d)*
x^7 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*d)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)\,dx = a^3cx+\frac{b^3dx^{13}}{13}+x^{10}\cdot\left(\frac{3ab^2d}{10}+\frac{b^3c}{10}\right)+x^7\cdot\left(\frac{3a^2bd}{7}+\frac{3ab^2c}{7}\right)+x^4\left(\frac{a^3d}{4}+\frac{3a^2bc}{4}\right)$$

input
```
integrate((b*x**3+a)**3*(d*x**3+c),x)
```

output
```
a**3*c*x + b**3*d*x**13/13 + x**10*(3*a*b**2*d/10 + b**3*c/10) + x**7*(3*a
**2*b*d/7 + 3*a*b**2*c/7) + x**4*(a**3*d/4 + 3*a**2*b*c/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)\,dx = \frac{1}{13}\,b^3dx^{13}+\frac{1}{10}\,\left(b^3c+3\,ab^2d\right)x^{10}+\frac{3}{7}\,\left(ab^2c+a^2bd\right)x^7+a^3cx+\frac{1}{4}\,\left(3\,a^2bc+a^3d\right)x^4$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c),x, algorithm="maxima")
```

output
```
1/13*b^3*d*x^13 + 1/10*(b^3*c + 3*a*b^2*d)*x^10 + 3/7*(a*b^2*c + a^2*b*d)*
x^7 + a^3*c*x + 1/4*(3*a^2*b*c + a^3*d)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^3)^3 (c+dx^3)\ dx = \frac{1}{13}b^3dx^{13} + \frac{1}{10}b^3cx^{10} + \frac{3}{10}ab^2dx^{10} + \frac{3}{7}ab^2cx^7 + \frac{3}{7}a^2bdx^7 + \frac{3}{4}a^2bcx^4 + \frac{1}{4}a^3dx^4 + a^3cx$$

input
```
integrate((b*x^3+a)^3*(d*x^3+c),x, algorithm="giac")
```

output
```
1/13*b^3*d*x^13 + 1/10*b^3*c*x^10 + 3/10*a*b^2*d*x^10 + 3/7*a*b^2*c*x^7 +
3/7*a^2*b*d*x^7 + 3/4*a^2*b*c*x^4 + 1/4*a^3*d*x^4 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^3)^3 (c+dx^3)\ dx = x^{10}\left(\frac{c\,b^3}{10} + \frac{3\,a\,d\,b^2}{10}\right) + x^4\left(\frac{d\,a^3}{4} + \frac{3\,b\,c\,a^2}{4}\right) + \frac{b^3\,d\,x^{13}}{13} + a^3\,c\,x + \frac{3\,a\,b\,x^7\,(a\,d+b\,c)}{7}$$

input
```
int((a + b*x^3)^3*(c + d*x^3),x)
```

output
```
x^10*((b^3*c)/10 + (3*a*b^2*d)/10) + x^4*((a^3*d)/4 + (3*a^2*b*c)/4) + (b^
3*d*x^13)/13 + a^3*c*x + (3*a*b*x^7*(a*d + b*c))/7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \left(a+bx^3\right)^3 \left(c+dx^3\right)\,dx$$
$$=\frac{x(140b^3d\,x^{12}+546a\,b^2d\,x^9+182b^3c\,x^9+780a^2bd\,x^6+780a\,b^2c\,x^6+455a^3d\,x^3+1365a^2bc\,x^3+1820a^3c}{1820}$$

input
```
int((b*x^3+a)^3*(d*x^3+c),x)
```

output
```
(x*(1820*a**3*c + 455*a**3*d*x**3 + 1365*a**2*b*c*x**3 + 780*a**2*b*d*x**6
 + 780*a*b**2*c*x**6 + 546*a*b**2*d*x**9 + 182*b**3*c*x**9 + 140*b**3*d*x*
*12))/1820
```

3.26 $\int \frac{(a+bx^3)^3}{c+dx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\int \frac{(a+bx^3)^3}{c+dx^3} \, dx = \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3} - \frac{b^2(bc-3ad)x^4}{4d^2} + \frac{b^3x^7}{7d} + \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{10/3}} - \frac{(bc-ad)^3 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{10/3}} + \frac{(bc-ad)^3 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{10/3}}$$

output
```
b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x/d^3-1/4*b^2*(-3*a*d+b*c)*x^4/d^2+1/7*b^3
*x^7/d+1/3*(-a*d+b*c)^3*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*
3^(1/2)/c^(2/3)/d^(10/3)-1/3*(-a*d+b*c)^3*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^
(10/3)+1/6*(-a*d+b*c)^3*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/
d^(10/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx$$

$$= \frac{84b\sqrt[3]{d}(b^2c^2-3abcd+3a^2d^2)\,x-21b^2d^{4/3}(bc-3ad)x^4+12b^3d^{7/3}x^7-\frac{28\sqrt{3}(bc-ad)^3\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{c^{2/3}}-}{84d^{10/3}}$$

input
```
Integrate[(a + b*x^3)^3/(c + d*x^3),x]
```

output
```
(84*b*d^(1/3)*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x - 21*b^2*d^(4/3)*(b*c -
3*a*d)*x^4 + 12*b^3*d^(7/3)*x^7 - (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-c^(1/
3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/c^(2/3) - (28*(b*c - a*d)^3*Log[c^(1
/3) + d^(1/3)*x])/c^(2/3) + (14*(b*c - a*d)^3*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/c^(2/3))/(84*d^(10/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{b\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{d^3\left(c+dx^3\right)}-\frac{b^2x^3(bc-3ad)}{d^2}+\frac{b^3x^6}{d}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx(3a^2d^2 - 3abcd + b^2c^2)}{d^3} + \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{10/3}} - \frac{b^2x^4(bc-3ad)}{4d^2} + \frac{(bc-ad)^3 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6c^{2/3}d^{10/3}} - \frac{(bc-ad)^3 \log\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{3c^{2/3}d^{10/3}} + \frac{b^3x^7}{7d}$$

input

```
Int[(a + b*x^3)^3/(c + d*x^3),x]
```

output

```
(b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^4)/(4*d
^2) + (b^3*x^7)/(7*d) + ((b*c - a*d)^3*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(10/3)) - ((b*c - a*d)^3*Log[c^(1/3) +
d^(1/3)*x]])/(3*c^(2/3)*d^(10/3)) + ((b*c - a*d)^3*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2]])/(6*c^(2/3)*d^(10/3))
```

Defintions of rubi rules used

rule 915

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

method	result
risch	$\frac{b^3x^7}{7d}+\frac{3b^2a\,x^4}{4d}-\frac{b^3c\,x^4}{4d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3c^2x}{d^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(x-\_R\right.}{\_R^2}}{3d^4}$
default	$\frac{b\left(\frac{1}{7}b^2d^2x^7+\frac{3}{4}ab\,d^2x^4-\frac{1}{4}b^2cd\,x^4+3a^2d^2x-3abcdx+b^2c^2x\right)}{d^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right.}{d^3}$

input
```
int((b*x^3+a)^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*x^7/d+3/4*b^2/d*a*x^4-1/4*b^3/d^2*c*x^4+3*b/d*a^2*x-3*b^2/d^2*a*c*
x+b^3/d^3*c^2*x+1/3/d^4*sum((a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 679, normalized size of antiderivative = 3.30

$$\int\frac{\left(a+bx^3\right)^3}{c+dx^3}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
[1/84*(12*b^3*c^2*d^3*x^7 - 21*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3)*x^4 - 42*sq
rt(1/3)*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*sqrt(-
(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*
(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d
*x^3 + c)) + 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(c^2*d
)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 84*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3)*x)/(c^2*d^4),
1/84*(12*b^3*c^2*d^3*x^7 - 21*(b^3*c^3*d^2 - 3*a*b^2*c^2*d^3)*x^4 - 84*sqr
t(1/3)*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*sqrt((c
^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt
((c^2*d)^(1/3)/d)/c^2) + 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 28*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(c^2*d)^(2/3)*log(c*d*
x + (c^2*d)^(2/3)) + 84*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3)*x)
/(c^2*d^4)]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx = \frac{b^3x^7}{7d} + x^4\cdot\left(\frac{3ab^2}{4d} - \frac{b^3c}{4d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right)$$
$$+\text{RootSum}\left(27t^3c^2d^{10} - a^9d^9 + 9a^8bcd^8 - 36a^7b^2c^2d^7 + 84a^6b^3c^3d^6 - 126a^5b^4c^4d^5 + 126a^4b^5c^5d^4 - 84a\right.$$

input
```
integrate((b*x**3+a)**3/(d*x**3+c),x)
```

output
```
b**3*x**7/(7*d) + x**4*(3*a*b**2/(4*d) - b**3*c/(4*d**2)) + x*(3*a**2*b/d
- 3*a*b**2*c/d**2 + b**3*c**2/d**3) + RootSum(27*_t**3*c**2*d**10 - a**9*d
**9 + 9*a**8*b*c*d**8 - 36*a**7*b**2*c**2*d**7 + 84*a**6*b**3*c**3*d**6 -
126*a**5*b**4*c**4*d**5 + 126*a**4*b**5*c**5*d**4 - 84*a**3*b**6*c**6*d**3
 + 36*a**2*b**7*c**7*d**2 - 9*a*b**8*c**8*d + b**9*c**9, Lambda(_t, _t*log
(3*_t*c*d**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
 x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx = \frac{4\,b^3d^2x^7 - 7\,(b^3cd - 3\,ab^2d^2)x^4 + 28\,(b^3c^2 - 3\,ab^2cd + 3\,a^2bd^2)x}{28\,d^3} - \frac{\sqrt{3}(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input

```
integrate((b*x^3+a)^3/(d*x^3+c),x, algorithm="maxima")
```

output

```
1/28*(4*b^3*d^2*x^7 - 7*(b^3*c*d - 3*a*b^2*d^2)*x^4 + 28*(b^3*c^2 - 3*a*b^
2*c*d + 3*a^2*b*d^2)*x)/d^3 - 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d
^4*(c/d)^(2/3)) + 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^4*(c/d)^(2/3)) - 1/3*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (c/d)^(1/3))/(d^4*(c/d)^(
2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx$$
$$= \frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d^2} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d^2} + \frac{(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^7} + \frac{4\,b^3d^6x^7-7\,b^3cd^5x^4+21\,ab^2d^6x^4+28\,b^3c^2d^4x-84\,ab^2cd^5x+84\,a^2bd^6x}{28\,d^7}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3
*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d^2) + 1/6*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-c/d)^(1/3)
+ (-c/d)^(2/3))/((-c*d^2)^(2/3)*d^2) + 1/3*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5
+ 3*a^2*b*c*d^6 - a^3*d^7)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^7)
 + 1/28*(4*b^3*d^6*x^7 - 7*b^3*c*d^5*x^4 + 21*a*b^2*d^6*x^4 + 28*b^3*c^2*d
^4*x - 84*a*b^2*c*d^5*x + 84*a^2*b*d^6*x)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx = x^4\left(\frac{3\,a\,b^2}{4\,d}-\frac{b^3\,c}{4\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right) + \frac{b^3\,x^7}{7\,d}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)^3}{3\,c^{2/3}\,d^{10/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^3}{c^{2/3}\,d^{10/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^3}{3\,c^{2/3}\,d^{10/3}}$$

input
```
int((a + b*x^3)^3/(c + d*x^3),x)
```

output
```
x^4*((3*a*b^2)/(4*d) - (b^3*c)/(4*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
 - (b^3*c)/d^2))/d) + (b^3*x^7)/(7*d) + (log(d^(1/3)*x + c^(1/3))*(a*d - b
*c)^3)/(3*c^(2/3)*d^(10/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1
/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^3)/(c^(2/3)*d^(10/3)) - (log(3^(1/
2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^
3)/(3*c^(2/3)*d^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^3}{c+dx^3}\,dx = \frac{-28c^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^3d^3+84c^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^2b\,d^2-84c^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a\,b^2d+28c^{\frac{10}{3}}}{}$$

input
```
int((b*x^3+a)^3/(d*x^3+c),x)
```

output
```
( - 28*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))
*a**3*d**3 + 84*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*
sqrt(3)))*a**2*b*c*d**2 - 84*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*
x)/(c**(1/3)*sqrt(3)))*a*b**2*c**2*d + 28*c**(1/3)*sqrt(3)*atan((c**(1/3)
- 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*b**3*c**3 - 14*c**(1/3)*log(c**(2/3) -
 d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*d**3 + 42*c**(1/3)*log(c**(2/3)
 - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*b*c*d**2 - 42*c**(1/3)*log(c*
*(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b**2*c**2*d + 14*c**(1/3)*
log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b**3*c**3 + 28*c**(1/3
)*log(c**(1/3) + d**(1/3)*x)*a**3*d**3 - 84*c**(1/3)*log(c**(1/3) + d**(1/
3)*x)*a**2*b*c*d**2 + 84*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b**2*c**2*d
 - 28*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*b**3*c**3 + 252*d**(1/3)*a**2*b*
c*d**2*x - 252*d**(1/3)*a*b**2*c**2*d*x + 63*d**(1/3)*a*b**2*c*d**2*x**4 +
 84*d**(1/3)*b**3*c**3*x - 21*d**(1/3)*b**3*c**2*d*x**4 + 12*d**(1/3)*b**3
*c*d**2*x**7)/(84*d**(1/3)*c*d**3)
```

3.27 $\int \frac{(a+bx^3)^3}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 235

$$\begin{aligned}\int \frac{(a+bx^3)^3}{(c+dx^3)^2} dx = & -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b^3x^4}{4d^2} - \frac{(bc-ad)^3x}{3cd^3(c+dx^3)} \\ & - \frac{(bc-ad)^2(7bc+2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{10/3}} \\ & + \frac{(bc-ad)^2(7bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{10/3}} \\ & - \frac{(bc-ad)^2(7bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{10/3}}\end{aligned}$$

output

```
-b^2*(-3*a*d+2*b*c)*x/d^3+1/4*b^3*x^4/d^2-1/3*(-a*d+b*c)^3*x/c/d^3/(d*x^3+
c)-1/9*(-a*d+b*c)^2*(2*a*d+7*b*c)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)
/c^(1/3))*3^(1/2)/c^(5/3)/d^(10/3)+1/9*(-a*d+b*c)^2*(2*a*d+7*b*c)*ln(c^(1/
3)+d^(1/3)*x)/c^(5/3)/d^(10/3)-1/18*(-a*d+b*c)^2*(2*a*d+7*b*c)*ln(c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/d^(10/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx$$

$$= \frac{-36b^2\sqrt[3]{d}(2bc-3ad)x+9b^3d^{4/3}x^4+\frac{12\sqrt[3]{d}(-bc+ad)^3x}{c(c+dx^3)}+\frac{4\sqrt{3}(bc-ad)^2(7bc+2ad)\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{c^{5/3}}+\frac{4(bc-ad)^2(\cdots}{\cdots}}{36d^{10/3}}$$

input
```
Integrate[(a + b*x^3)^3/(c + d*x^3)^2,x]
```

output
```
(-36*b^2*d^(1/3)*(2*b*c - 3*a*d)*x + 9*b^3*d^(4/3)*x^4 + (12*d^(1/3)*(-(b*
c) + a*d)^3*x)/(c*(c + d*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(7*b*c + 2*a*d)*
ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/c^(5/3) + (4*(b*c - a*
d)^2*(7*b*c + 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (2*(b*c - a*d)^2*
(7*b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(
36*d^(10/3))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{3bdx^3(bc-ad)^2+(bc-ad)^2(ad+2bc)}{d^3\left(c+dx^3\right)^2}+\frac{b^3x^3}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2(2ad+7bc)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{10/3}}-\frac{b^2x(2bc-3ad)}{d^3}-\frac{(bc-ad)^2(2ad+7bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{10/3}}+\frac{(bc-ad)^2(2ad+7bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{10/3}}-\frac{x(bc-ad)^3}{3cd^3(c+dx^3)}+\frac{b^3x^4}{4d^2}$$

input
```
Int[(a + b*x^3)^3/(c + d*x^3)^2,x]
```

output
```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^4)/(4*d^2) - ((b*c - a*d)^3*x)/(3*
c*d^3*(c + d*x^3)) - ((b*c - a*d)^2*(7*b*c + 2*a*d)*ArcTan[(c^(1/3) - 2*d^
(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(10/3)) + ((b*c - a*d)^2
*(7*b*c + 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(10/3)) - ((b*c -
a*d)^2*(7*b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*c^(5/3)*d^(10/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
risch	$\frac{b^3x^4}{4d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{3c\,d^3(d\,x^3+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(2a^3d^3+3a^2bc\,d^2-12a\,b^2c^2d+7b^3c^3\right)\ln(}{\_R^2}}{9d^4c}$
default	$\frac{b^2\left(\frac{1}{4}bd\,x^4+3adx-2bcx\right)}{d^3}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{3c\left(d\,x^3+c\right)}+\frac{\left(2a^3d^3+3a^2bc\,d^2-12a\,b^2c^2d+7b^3c^3\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x\right.}{6d\left(\frac{c}{d}\right)}\right.}{3c}}{d^3}$

input
```
int((b*x^3+a)^3/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^3*x^4/d^2+3*b^2/d^2*a*x-2*b^3/d^3*c*x+1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/c*x/d^3/(d*x^3+c)+1/9/d^4/c*sum((2*a^3*d^3+3*a^2*b*c*d
^2-12*a*b^2*c^2*d+7*b^3*c^3)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(194) = 388$.

Time = 0.10 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.38

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/36*(9*b^3*c^3*d^3*x^7 - 9*(7*b^3*c^4*d^2 - 12*a*b^2*c^3*d^3)*x^4 + 6*sq
rt(1/3)*(7*b^3*c^5*d - 12*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + 2*a^3*c^2*d^4
+ (7*b^3*c^4*d^2 - 12*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 + 2*a^3*c*d^5)*x^3)*
sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt
(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/
d))/(d*x^3 + c)) - 2*(7*b^3*c^4 - 12*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + 2*a^3
*c*d^3 + (7*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^3)
*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(7*b^3
*c^4 - 12*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + 2*a^3*c*d^3 + (7*b^3*c^3*d - 12*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^3)*(c^2*d)^(2/3)*log(c*d*x +
(c^2*d)^(2/3)) - 12*(7*b^3*c^5*d - 12*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^
3*c^2*d^4)*x)/(c^3*d^5*x^3 + c^4*d^4), 1/36*(9*b^3*c^3*d^3*x^7 - 9*(7*b^3*
c^4*d^2 - 12*a*b^2*c^3*d^3)*x^4 + 12*sqrt(1/3)*(7*b^3*c^5*d - 12*a*b^2*c^4
*d^2 + 3*a^2*b*c^3*d^3 + 2*a^3*c^2*d^4 + (7*b^3*c^4*d^2 - 12*a*b^2*c^3*d^3
 + 3*a^2*b*c^2*d^4 + 2*a^3*c*d^5)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1
/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(
7*b^3*c^4 - 12*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + 2*a^3*c*d^3 + (7*b^3*c^3*d
- 12*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^3)*(c^2*d)^(2/3)*log(c*d
*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(7*b^3*c^4 - 12*a*b^2*c^3*d
+ 3*a^2*b*c^2*d^2 + 2*a^3*c*d^3 + (7*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 3*a...
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx = \frac{b^3x^4}{4d^2} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{x(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{3c^2d^3 + 3cd^4x^3}$$
$$+\mathrm{RootSum}\Big(729t^3c^5d^{10} - 8a^9d^9 - 36a^8bcd^8 + 90a^7b^2c^2d^7 + 321a^6b^3c^3d^6 - 792a^5b^4c^4d^5 - 477a^4b^5c^5d^4$$

input
```
integrate((b*x**3+a)**3/(d*x**3+c)**2,x)
```

output
```
b**3*x**4/(4*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*
a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(3*c**2*d**3 + 3*c*d**4*x**3)
 + RootSum(729*_t**3*c**5*d**10 - 8*a**9*d**9 - 36*a**8*b*c*d**8 + 90*a**7
*b**2*c**2*d**7 + 321*a**6*b**3*c**3*d**6 - 792*a**5*b**4*c**4*d**5 - 477*
a**4*b**5*c**5*d**4 + 2946*a**3*b**6*c**6*d**3 - 3465*a**2*b**7*c**7*d**2
+ 1764*a*b**8*c**8*d - 343*b**9*c**9, Lambda(_t, _t*log(9*_t*c**2*d**3/(2*
a**3*d**3 + 3*a**2*b*c*d**2 - 12*a*b**2*c**2*d + 7*b**3*c**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{3\,(cd^4x^3+c^2d^3)} + \frac{b^3dx^4-4\,(2\,b^3c-3\,ab^2d)x}{4\,d^3} + \frac{\sqrt{3}(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^4\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
-1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^3 + c^
2*d^3) + 1/4*(b^3*d*x^4 - 4*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 1/9*sqrt(3)*(7*
b^3*c^3 - 12*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3)*arctan(1/3*sqrt(3)*(
2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^4*(c/d)^(2/3)) - 1/18*(7*b^3*c^3 - 12
*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^
(2/3))/(c*d^4*(c/d)^(2/3)) + 1/9*(7*b^3*c^3 - 12*a*b^2*c^2*d + 3*a^2*b*c*d
^2 + 2*a^3*d^3)*log(x + (c/d)^(1/3))/(c*d^4*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,\left(-cd^2\right)^{\frac{2}{3}}cd^2}$$
$$-\frac{(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,\left(-cd^2\right)^{\frac{2}{3}}cd^2}$$
$$-\frac{(7\,b^3c^3-12\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d^3}$$
$$-\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(dx^3+c)cd^3}+\frac{b^3d^6x^4-8\,b^3cd^5x+12\,ab^2d^6x}{4\,d^8}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(7*b^3*c^3 - 12*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3)*arct
an(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d^2) -
 1/18*(7*b^3*c^3 - 12*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3)*log(x^2 + x
*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*c*d^2) - 1/9*(7*b^3*c^3 - 12
*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^
(1/3)))/(c^2*d^3) - 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a
^3*d^3*x)/((d*x^3 + c)*c*d^3) + 1/4*(b^3*d^6*x^4 - 8*b^3*c*d^5*x + 12*a*b^
2*d^6*x)/d^8
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx = x\left(\frac{3\,a\,b^2}{d^2}-\frac{2\,b^3\,c}{d^3}\right)+\frac{b^3\,x^4}{4\,d^2}+\frac{x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{3\,c\,(d^4\,x^3+c\,d^3)} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)^2\,(2\,a\,d+7\,b\,c)}{9\,c^{5/3}\,d^{10/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(2\,a\,d+7\,b\,c)}{9\,c^{5/3}\,d^{10/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^2\,(2\,a\,d+7\,b\,c)}{9\,c^{5/3}\,d^{10/3}}$$

input
```
int((a + b*x^3)^3/(c + d*x^3)^2,x)
```

output
```
x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^4)/(4*d^2) + (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*c*(c*d^3 + d^4*x^3)) + (log(d^(1/
3)*x + c^(1/3))*(a*d - b*c)^2*(2*a*d + 7*b*c))/(9*c^(5/3)*d^(10/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2*(2*a*d + 7*b*c))/(9*c^(5/3)*d^(10/3)) + (log(3^(1/2)*c^(1/3)*1i +
2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(2*a*d + 7*b*c
))/(9*c^(5/3)*d^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.47

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^3/(d*x^3+c)^2,x)
```

output

```
( - 8*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*
a**3*c*d**3 - 8*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*
sqrt(3)))*a**3*d**4*x**3 - 12*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)
*x)/(c**(1/3)*sqrt(3)))*a**2*b*c**2*d**2 - 12*c**(1/3)*sqrt(3)*atan((c**(1
/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*b*c*d**3*x**3 + 48*c**(1/3)*s
qrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b**2*c**3*d +
48*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b
**2*c**2*d**2*x**3 - 28*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c
**(1/3)*sqrt(3)))*b**3*c**4 - 28*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1
/3)*x)/(c**(1/3)*sqrt(3)))*b**3*c**3*d*x**3 - 4*c**(1/3)*log(c**(2/3) - d*
*(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*c*d**3 - 4*c**(1/3)*log(c**(2/3) -
 d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*d**4*x**3 - 6*c**(1/3)*log(c**(
2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*b*c**2*d**2 - 6*c**(1/3)*
log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*b*c*d**3*x**3 + 2
4*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b**2*c**3
*d + 24*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b**
2*c**2*d**2*x**3 - 14*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/
3)*x**2)*b**3*c**4 - 14*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(
2/3)*x**2)*b**3*c**3*d*x**3 + 8*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**3*c
*d**3 + 8*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**3*d**4*x**3 + 12*c**(1...
```

3.28 $\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 287

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{6cd^3(c+dx^3)^2} + \frac{(bc-ad)^2(13bc+5ad)x}{18c^2d^3(c+dx^3)} + \frac{(bc-ad)(14b^2c^2+8abcd+5a^2d^2)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{10/3}} - \frac{(bc-ad)(14b^2c^2+8abcd+5a^2d^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{10/3}} + \frac{(bc-ad)(14b^2c^2+8abcd+5a^2d^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{10/3}}$$

output
```
b^3*x/d^3-1/6*(-a*d+b*c)^3*x/c/d^3/(d*x^3+c)^2+1/18*(-a*d+b*c)^2*(5*a*d+13
*b*c)*x/c^2/d^3/(d*x^3+c)+1/27*(-a*d+b*c)*(5*a^2*d^2+8*a*b*c*d+14*b^2*c^2)
*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(8/3)/d^(10/3
)-1/27*(-a*d+b*c)*(5*a^2*d^2+8*a*b*c*d+14*b^2*c^2)*ln(c^(1/3)+d^(1/3)*x)/c
^(8/3)/d^(10/3)+1/54*(-a*d+b*c)*(5*a^2*d^2+8*a*b*c*d+14*b^2*c^2)*ln(c^(2/3
)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/d^(10/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx$$

$$= \frac{54b^3\sqrt[3]{d}x + \frac{9\sqrt[3]{d}(-bc+ad)^3x}{c(c+dx^3)^2} + \frac{3\sqrt[3]{d}(bc-ad)^2(13bc+5ad)x}{c^2(c+dx^3)} + \frac{2\sqrt{3}(14b^3c^3-6ab^2c^2d-3a^2bcd^2-5a^3d^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}} - \frac{2(}{}}{54d^{10/3}}$$

input
```
Integrate[(a + b*x^3)^3/(c + d*x^3)^3,x]
```

output
```
(54*b^3*d^(1/3)*x + (9*d^(1/3)*(-(b*c) + a*d)^3*x)/(c*(c + d*x^3)^2) + (3*
d^(1/3)*(b*c - a*d)^2*(13*b*c + 5*a*d)*x)/(c^2*(c + d*x^3)) + (2*Sqrt[3]*(
14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(1 - (2*d^(
1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (2*(14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^
2*b*c*d^2 - 5*a^3*d^3)*Log[c^(1/3) + d^(1/3)*x])/c^(8/3) + ((14*b^3*c^3 -
6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/c^(8/3))/(54*d^(10/3))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{b^3}{d^3} - \frac{-a^3d^3+3b^2d^2x^6(bc-ad)+3bdx^3(bc-ad)(ad+bc)+b^3c^3}{d^3(c+dx^3)^3}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{(bc-ad)\left(5a^2d^2+8abcd+14b^2c^2\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{10/3}}+\frac{(bc-ad)\left(5a^2d^2+8abcd+14b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{10/3}}-\frac{(bc-ad)\left(5a^2d^2+8abcd+14b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{10/3}}+\frac{x(bc-ad)^2(5ad+13bc)}{18c^2d^3\left(c+dx^3\right)}-\frac{x(bc-ad)^3}{6cd^3\left(c+dx^3\right)^2}+\frac{b^3x}{d^3}$$

input
```
Int[(a + b*x^3)^3/(c + d*x^3)^3,x]
```

output
```
(b^3*x)/d^3 - ((b*c - a*d)^3*x)/(6*c*d^3*(c + d*x^3)^2) + ((b*c - a*d)^2*(
13*b*c + 5*a*d)*x)/(18*c^2*d^3*(c + d*x^3)) + ((b*c - a*d)*(14*b^2*c^2 + 8
*a*b*c*d + 5*a^2*d^2)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
9*Sqrt[3]*c^(8/3)*d^(10/3)) - ((b*c - a*d)*(14*b^2*c^2 + 8*a*b*c*d + 5*a^2
*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(8/3)*d^(10/3)) + ((b*c - a*d)*(14*b
^2*c^2 + 8*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(54*c^(8/3)*d^(10/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.62

method	result
risch	$\frac{b^3x}{d^3}+\frac{\frac{d(5a^3d^3+3a^2bcd^2-21ab^2c^2d+13b^3c^3)x^4}{18c^2}+\frac{(4a^3d^3-3a^2bcd^2-6ab^2c^2d+5b^3c^3)x}{9c}}{d^3(dx^3+c)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^3+c)}\frac{(5a^3d^3+3a^2bcd^2+6\cdots}{\cdots}}{27d^4c^2}$
default	$\frac{b^3x}{d^3}+\frac{\frac{\frac{d(5a^3d^3+3a^2bcd^2-21ab^2c^2d+13b^3c^3)x^4}{18c^2}+\frac{(4a^3d^3-3a^2bcd^2-6ab^2c^2d+5b^3c^3)x}{9c}}{(dx^3+c)^2}+(5a^3d^3+3a^2bcd^2+6ab^2c^2d-14b^3c^3)\left(\frac{\ln\left(x+\left(\cdots\right.\right.}{3d\left(\frac{c}{d}\cdots\right.}\cdots\right.}{d^3}$

input
```
int((b*x^3+a)^3/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d^3+(1/18*d*(5*a^3*d^3+3*a^2*b*c*d^2-21*a*b^2*c^2*d+13*b^3*c^3)/c^2*
x^4+1/9*(4*a^3*d^3-3*a^2*b*c*d^2-6*a*b^2*c^2*d+5*b^3*c^3)/c*x)/d^3/(d*x^3+
c)^2+1/27/d^4/c^2*sum((5*a^3*d^3+3*a^2*b*c*d^2+6*a*b^2*c^2*d-14*b^3*c^3)/_
R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(246) = 492.

Time = 0.12 (sec) , antiderivative size = 1407, normalized size of antiderivative = 4.90

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
[1/54*(54*b^3*c^4*d^3*x^7 + 3*(49*b^3*c^5*d^2 - 21*a*b^2*c^4*d^3 + 3*a^2*b
*c^3*d^4 + 5*a^3*c^2*d^5)*x^4 - 3*sqrt(1/3)*(14*b^3*c^6*d - 6*a*b^2*c^5*d^
2 - 3*a^2*b*c^4*d^3 - 5*a^3*c^3*d^4 + (14*b^3*c^4*d^3 - 6*a*b^2*c^3*d^4 -
3*a^2*b*c^2*d^5 - 5*a^3*c*d^6)*x^6 + 2*(14*b^3*c^5*d^2 - 6*a*b^2*c^4*d^3 -
 3*a^2*b*c^3*d^4 - 5*a^3*c^2*d^5)*x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x
^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x
- (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (14*b^3*c^5 - 6*
a*b^2*c^4*d - 3*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (14*b^3*c^3*d^2 - 6*a*b^2*
c^2*d^3 - 3*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(14*b^3*c^4*d - 6*a*b^2*c^3*d
^2 - 3*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*
d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(14*b^3*c^5 - 6*a*b^2*c^4*d - 3*a^2*b*c^
3*d^2 - 5*a^3*c^2*d^3 + (14*b^3*c^3*d^2 - 6*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4
- 5*a^3*d^5)*x^6 + 2*(14*b^3*c^4*d - 6*a*b^2*c^3*d^2 - 3*a^2*b*c^2*d^3 - 5
*a^3*c*d^4)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 6*(14*b^3*c^6*
d - 6*a*b^2*c^5*d^2 - 3*a^2*b*c^4*d^3 + 4*a^3*c^3*d^4)*x)/(c^4*d^6*x^6 + 2
*c^5*d^5*x^3 + c^6*d^4), 1/54*(54*b^3*c^4*d^3*x^7 + 3*(49*b^3*c^5*d^2 - 21
*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 + 5*a^3*c^2*d^5)*x^4 - 6*sqrt(1/3)*(14*b^
3*c^6*d - 6*a*b^2*c^5*d^2 - 3*a^2*b*c^4*d^3 - 5*a^3*c^3*d^4 + (14*b^3*c^4*
d^3 - 6*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5 - 5*a^3*c*d^6)*x^6 + 2*(14*b^3*c^5
*d^2 - 6*a*b^2*c^4*d^3 - 3*a^2*b*c^3*d^4 - 5*a^3*c^2*d^5)*x^3)*sqrt((c^...
```

Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \frac{b^3x}{d^3} + \frac{x^4\cdot(5a^3d^4+3a^2bcd^3-21ab^2c^2d^2+13b^3c^3d)+x(8a^3cd^3-6a^2bc^2d^2-12ab^2c^3d+10b^3c^4)}{18c^4d^3+36c^3d^4x^3+18c^2d^5x^6} + \mathrm{RootSum}\left(19683t^3c^8d^{10}-125a^9d^9-225a^8bcd^8-585a^7b^2c^2d^7+483a^6b^3c^3d^6+558a^5b^4c^4d^5+2574a\right.$$

input
```
integrate((b*x**3+a)**3/(d*x**3+c)**3,x)
```

output
```
b**3*x/d**3 + (x**4*(5*a**3*d**4 + 3*a**2*b*c*d**3 - 21*a*b**2*c**2*d**2 +
 13*b**3*c**3*d) + x*(8*a**3*c*d**3 - 6*a**2*b*c**2*d**2 - 12*a*b**2*c**3*
d + 10*b**3*c**4))/(18*c**4*d**3 + 36*c**3*d**4*x**3 + 18*c**2*d**5*x**6)
+ RootSum(19683*_t**3*c**8*d**10 - 125*a**9*d**9 - 225*a**8*b*c*d**8 - 585
*a**7*b**2*c**2*d**7 + 483*a**6*b**3*c**3*d**6 + 558*a**5*b**4*c**4*d**5 +
 2574*a**4*b**5*c**5*d**4 - 1644*a**3*b**6*c**6*d**3 - 252*a**2*b**7*c**7*
d**2 - 3528*a*b**8*c**8*d + 2744*b**9*c**9, Lambda(_t, _t*log(27*_t*c**3*d
**3/(5*a**3*d**3 + 3*a**2*b*c*d**2 + 6*a*b**2*c**2*d - 14*b**3*c**3) + x))
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \frac{b^3x}{d^3} + \frac{(13\,b^3c^3d-21\,ab^2c^2d^2+3\,a^2bcd^3+5\,a^3d^4)x^4+2\,(5\,b^3c^4-6\,ab^2c^3d-3\,a^2bc^2d^2+4\,a^3cd^3)x}{18\,(c^2d^5x^6+2\,c^3d^4x^3+c^4d^3)} - \frac{\sqrt{3}(14\,b^3c^3-6\,ab^2c^2d-3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(14\,b^3c^3-6\,ab^2c^2d-3\,a^2bcd^2-5\,a^3d^3)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(14\,b^3c^3-6\,ab^2c^2d-3\,a^2bcd^2-5\,a^3d^3)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^4\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
b^3*x/d^3 + 1/18*((13*b^3*c^3*d - 21*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + 5*a^3
*d^4)*x^4 + 2*(5*b^3*c^4 - 6*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 4*a^3*c*d^3)*
x)/(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3) - 1/27*sqrt(3)*(14*b^3*c^3 - 6*
a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(
1/3))/(c/d)^(1/3))/(c^2*d^4*(c/d)^(2/3)) + 1/54*(14*b^3*c^3 - 6*a*b^2*c^2*
d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2
*d^4*(c/d)^(2/3)) - 1/27*(14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a
^3*d^3)*log(x + (c/d)^(1/3))/(c^2*d^4*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx$$

$$= \frac{b^3x}{d^3} + \frac{\sqrt{3}(14\,b^3c^3 - 6\,ab^2c^2d - 3\,a^2bcd^2 - 5\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,(-cd^2)^{\frac{2}{3}}\,c^2d^2}$$
$$+ \frac{(14\,b^3c^3 - 6\,ab^2c^2d - 3\,a^2bcd^2 - 5\,a^3d^3)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,(-cd^2)^{\frac{2}{3}}\,c^2d^2}$$
$$+ \frac{(14\,b^3c^3 - 6\,ab^2c^2d - 3\,a^2bcd^2 - 5\,a^3d^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^3}$$
$$+ \frac{13\,b^3c^3dx^4 - 21\,ab^2c^2d^2x^4 + 3\,a^2bcd^3x^4 + 5\,a^3d^4x^4 + 10\,b^3c^4x - 12\,ab^2c^3dx - 6\,a^2bc^2d^2x + 8\,a^3cd^3x}{18\,(dx^3+c)^2c^2d^3}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
b^3*x/d^3 + 1/27*sqrt(3)*(14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a
^3*d^3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2
/3)*c^2*d^2) + 1/54*(14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^
3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2*d^2) + 1/2
7*(14*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*(-c/d)^(1/3)*lo
g(abs(x - (-c/d)^(1/3)))/(c^3*d^3) + 1/18*(13*b^3*c^3*d*x^4 - 21*a*b^2*c^2
*d^2*x^4 + 3*a^2*b*c*d^3*x^4 + 5*a^3*d^4*x^4 + 10*b^3*c^4*x - 12*a*b^2*c^3
*d*x - 6*a^2*b*c^2*d^2*x + 8*a^3*c*d^3*x)/((d*x^3 + c)^2*c^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \frac{\frac{x\,(4\,a^3\,d^3-3\,a^2\,b\,c\,d^2-6\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{9\,c}+\frac{x^4\,(5\,a^3\,d^4+3\,a^2\,b\,c\,d^3-21\,a\,b^2\,c^2\,d^2+13\,b^3\,c^3\,d)}{18\,c^2}}{c^2\,d^3+2\,c\,d^4\,x^3+d^5\,x^6} + \frac{b^3\,x}{d^3} + \frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,(a\,d-b\,c)\,(5\,a^2\,d^2+8\,a\,b\,c\,d+14\,b^2\,c^2)}{27\,c^{8/3}\,d^{10/3}} - \frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(5\,a^2\,d^2+8\,a\,b\,c\,d+14\,b^2\,c^2)}{27\,c^{8/3}\,d^{10/3}} + \frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(5\,a^2\,d^2+8\,a\,b\,c\,d+14\,b^2\,c^2)}{27\,c^{8/3}\,d^{10/3}}$$

input
```
int((a + b*x^3)^3/(c + d*x^3)^3,x)
```

output
```
((x*(4*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(9*c) + (x^4*
(5*a^3*d^4 + 13*b^3*c^3*d - 21*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3))/(18*c^2))/(
c^2*d^3 + d^5*x^6 + 2*c*d^4*x^3) + (b^3*x)/d^3 + (log(d^(1/3)*x + c^(1/3))
*(a*d - b*c)*(5*a^2*d^2 + 14*b^2*c^2 + 8*a*b*c*d))/(27*c^(8/3)*d^(10/3)) -
 (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)*(5*a^2*d^2 + 14*b^2*c^2 + 8*a*b*c*d))/(27*c^(8/3)*d^(10/3)) + (
log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*
d - b*c)*(5*a^2*d^2 + 14*b^2*c^2 + 8*a*b*c*d))/(27*c^(8/3)*d^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1238, normalized size of antiderivative = 4.31

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^3/(d*x^3+c)^3,x)
```

output

```
( - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))
*a**3*c**2*d**3 - 20*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(
1/3)*sqrt(3)))*a**3*c*d**4*x**3 - 10*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d
**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**3*d**5*x**6 - 6*c**(1/3)*sqrt(3)*atan((c
**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*b*c**3*d**2 - 12*c**(1/3)
*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*b*c**2*d*
*3*x**3 - 6*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt
(3)))*a**2*b*c*d**4*x**6 - 12*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)
*x)/(c**(1/3)*sqrt(3)))*a*b**2*c**4*d - 24*c**(1/3)*sqrt(3)*atan((c**(1/3)
 - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b**2*c**3*d**2*x**3 - 12*c**(1/3)*s
qrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b**2*c**2*d**3
*x**6 + 28*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(
3)))*b**3*c**5 + 56*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1
/3)*sqrt(3)))*b**3*c**4*d*x**3 + 28*c**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d*
*(1/3)*x)/(c**(1/3)*sqrt(3)))*b**3*c**3*d**2*x**6 - 5*c**(1/3)*log(c**(2/3
) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*c**2*d**3 - 10*c**(1/3)*log(
c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*c*d**4*x**3 - 5*c**(1
/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*d**5*x**6 - 3
*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*b*c**3*
d**2 - 6*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a...
```

3.29 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 252

$$\begin{aligned}\int \frac{(c+dx^3)^4}{a+bx^3}\,dx &= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} \\ &\quad + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3} + \frac{d^3(4bc-ad)x^7}{7b^2} + \frac{d^4x^{10}}{10b} \\ &\quad - \frac{(bc-ad)^4\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}} + \frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}} \\ &\quad - \frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}\end{aligned}$$

output
```
d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/4*d^2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)*x^4/b^3+1/7*d^3*(-a*d+4*b*c)*x^7/b^2+1/10*d^4*x^10/b-1/3*(-
a*d+b*c)^4*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)/b^(13/3)+1/3*(-a*d+b*c)^4*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(13/3)-1/6*(-
a*d+b*c)^4*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(13/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$= \frac{420\sqrt[3]{b}d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+105b^{4/3}d^2(6b^2c^2-4abcd+a^2d^2)\,x^4+60b^{7/3}d^3(4bc-ad}{}$$

input
```
Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

output
```
(420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 1
05*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b
*c - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(
-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*L
og[a^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{b^3}+\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3}{b^4\left(a+bx^3\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4}{\sqrt{3}a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

input
```
Int[(c + d*x^3)^4/(a + b*x^3),x]
```

output
```
(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^
2 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) +
(d^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1
/3)*x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

method	result
risch	$\frac{d^4x^{10}}{10b}-\frac{d^4x^7a}{7b^2}+\frac{4d^3c\,x^7}{7b}-\frac{d^3ac\,x^4}{b^2}+\frac{3d^2c^2x^4}{2b}+\frac{d^4a^2x^4}{4b^3}-\frac{d^4a^3x}{b^4}+\frac{4d^3a^2cx}{b^3}-\frac{6d^2a\,c^2x}{b^2}+\frac{4d\,c^3x}{b}+$
default	$-\frac{d\left(-\frac{d^3x^{10}b^3}{10}+\frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^7}{7}+\frac{\left(2(ad-2bc)b^2cd-db\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^4}{4}+(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4}+$

input
```
int((d*x^3+c)^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*d^4*x^10/b-1/7*d^4/b^2*x^7*a+4/7*d^3/b*c*x^7-d^3/b^2*a*c*x^4+3/2*d^2/
b*c^2*x^4+1/4*d^4/b^3*a^2*x^4-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*
c^2*x+4*d/b*c^3*x+1/3/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105
*(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(
a*b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*
d^4)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3
*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^
2*b)^(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^
4*c^3*d - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1
/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(
6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^
4)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/
3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2
/3)*x + (a^2*b)^(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 42
0*(4*a^2*b^4*c^3*d - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(
a^2*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{13}-a^{12}d^{12}+12a^{11}bcd^{11}-66a^{10}b^2c^2d^{10}+220a^9b^3c^3d^9-495a^8b^4c^4d^8+792a^7b^5c^5\right.$$
$$+\frac{d^4x^{10}}{10b}$$

input
```
integrate((d*x**3+c)**4/(b*x**3+a),x)
```

output
```
x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*
d**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 -
 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d*
*12 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d
**9 - 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c*
*6*d**6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**
9*c**9*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12,
 Lambda(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*
c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$
$$= \frac{14\,b^3d^4x^{10} + 20\,(4\,b^3cd^3 - ab^2d^4)x^7 + 35\,(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4)x^4 + 140\,(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4\,a}{140\,b^4}$$
$$+ \frac{\sqrt{3}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2
+ 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$$

$$= -\frac{\sqrt{3}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(-ab^2)^{\frac{2}{3}}\,b^3}$$
$$- \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(-ab^2)^{\frac{2}{3}}\,b^3}$$
$$- \frac{(b^{10}c^4 - 4\,ab^9c^3d + 6\,a^2b^8c^2d^2 - 4\,a^3b^7cd^3 + a^4b^6d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{10}}$$
$$+ \frac{14\,b^9d^4x^{10} + 80\,b^9cd^3x^7 - 20\,ab^8d^4x^7 + 210\,b^9c^2d^2x^4 - 140\,ab^8cd^3x^4 + 35\,a^2b^7d^4x^4 + 560\,b^9c^3dx - 840}{140\,b^{10}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)
^(2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b
^3) - 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(1
4*b^9*d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4
 - 140*a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*
c^2*d^2*x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = x\left(\frac{4\,c^3\,d}{b}-\frac{a\,\left(\frac{a\,\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right) - x^7\,\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)+x^4\,\left(\frac{a\,\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right) + \frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^4}{3\,a^{2/3}\,b^{13/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^4}{a^{2/3}\,b^{13/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^4}{3\,a^{2/3}\,b^{13/3}}$$

input
```
int((c + d*x^3)^4/(a + b*x^3),x)
```

output
```
x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b
) - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*
d^3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x +
 a^(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i +
2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(1
3/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 +
 1/2)*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.25

$$\int \frac{(c+dx^3)^4}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^4/(b*x^3+a),x)
```

output
```
( - 140*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))
)*a**4*d**4 + 560*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3
)*sqrt(3)))*a**3*b*c*d**3 - 840*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2*c**2*d**2 + 560*a**(1/3)*sqrt(3)*atan(
(a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*c**3*d - 140*a**(1/3)
*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*c**4 - 70
*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4*d**4 +
280*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*c*
d**3 - 420*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*
*2*b**2*c**2*d**2 + 280*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*a*b**3*c**3*d - 70*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*b**4*c**4 + 140*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**4*
d**4 - 560*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**3*b*c*d**3 + 840*a**(1/3
)*log(a**(1/3) + b**(1/3)*x)*a**2*b**2*c**2*d**2 - 560*a**(1/3)*log(a**(1/
3) + b**(1/3)*x)*a*b**3*c**3*d + 140*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b
**4*c**4 - 420*b**(1/3)*a**4*d**4*x + 1680*b**(1/3)*a**3*b*c*d**3*x + 105*
b**(1/3)*a**3*b*d**4*x**4 - 2520*b**(1/3)*a**2*b**2*c**2*d**2*x - 420*b**(
1/3)*a**2*b**2*c*d**3*x**4 - 60*b**(1/3)*a**2*b**2*d**4*x**7 + 1680*b**(1/
3)*a*b**3*c**3*d*x + 630*b**(1/3)*a*b**3*c**2*d**2*x**4 + 240*b**(1/3)*a*b
**3*c*d**3*x**7 + 42*b**(1/3)*a*b**3*d**4*x**10)/(420*b**(1/3)*a*b**4)
```

3.30 $\int \frac{(c+dx^3)^3}{a+bx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 208

$$\begin{aligned}\int \frac{(c+dx^3)^3}{a+bx^3} dx =& \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b} \\ &-\frac{(bc-ad)^3\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}} \\ &-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}\end{aligned}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/4*d^2*(-a*d+3*b*c)*x^4/b^2+1/7*d^3
*x^7/b-1/3*(-a*d+b*c)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*
3^(1/2)/a^(2/3)/b^(10/3)+1/3*(-a*d+b*c)^3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^
(10/3)-1/6*(-a*d+b*c)^3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/
b^(10/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$

$$= \frac{84\sqrt[3]{b}d(3b^2c^2-3abcd+a^2d^2)\,x+21b^{4/3}d^2(3bc-ad)x^4+12b^{7/3}d^3x^7+\frac{28\sqrt{3}(bc-ad)^3\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+}{84b^{10/3}}$$

input
```
Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

output
```
(84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c
- a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/
3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1
/3) + b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3(a+bx^3)}+\frac{d^2x^3(3bc-ad)}{b^2}+\frac{d^3x^6}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3}{\sqrt{3}a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

input
```
Int[(c + d*x^3)^3/(a + b*x^3),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b
^2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) +
b^(1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.63

method	result
risch	$\frac{d^3x^7}{7b}-\frac{d^3a\,x^4}{4b^2}+\frac{3d^2c\,x^4}{4b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_\right.}{\_R^2}}{3b^4}$
default	$\frac{d\left(\frac{1}{7}b^2d^2x^7-\frac{1}{4}ab\,d^2x^4+\frac{3}{4}b^2cd\,x^4+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{b^3}$

input
```
int((d*x^3+c)^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*d^3*x^7/b-1/4*d^3/b^2*a*x^4+3/4*d^2/b*c*x^4+d^3/b^3*a^2*x-3*d^2/b^2*a*
c*x+3*d/b*c^2*x+1/3/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)
/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.37

$$\int \frac{\left(c+dx^3\right)^3}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sq
rt(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((
-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)
*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))
/(b*x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a
^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (
-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^
2*b^4), 1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4
+ 84*sqrt(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)
*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(
1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2
*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*
d^2 + a^4*b*d^3)*x)/(a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(27t^3a^2b^{10}+a^9d^9-9a^8bcd^8+36a^7b^2c^2d^7-84a^6b^3c^3d^6+126a^5b^4c^4d^5-126a^4b^5c^5d^4+84a\right.$$
$$+\frac{d^3x^7}{7b}$$

input
```
integrate((d*x**3+c)**3/(b*x**3+a),x)
```

output
```
x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/
b**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*
d**8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**
4*d**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c
**7*d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a*
*3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**
7/(7*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \frac{4\,b^2d^3x^7+7\,(3\,b^2cd^2-abd^3)x^4+28\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{28\,b^3} + \frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b
^4*(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(
2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx$$
$$= -\frac{\sqrt{3}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^7} + \frac{4\,b^6d^3x^7+21\,b^6cd^2x^4-7\,ab^5d^3x^4+84\,b^6c^2dx-84\,ab^5cd^2x+28\,a^2b^4d^3x}{28\,b^7}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/
3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a
^2*b^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7
) + 1/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*
d*x - 84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)+\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^3}{a^{2/3}\,b^{10/3}}$$

input
```
int((c + d*x^3)^3/(a + b*x^3),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
 - (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b
*c)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(
1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c
)^3)/(a^(2/3)*b^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx^3)^3}{a+bx^3}\,dx = \frac{28a^{\frac{10}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)d^3-84a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bc\,d^2+84a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2c^2d-28a^{\frac{1}{3}}\sqrt{3}\,a}{}$$

input
```
int((d*x^3+c)^3/(b*x^3+a),x)
```

output

```
(28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
*3*d**3 - 84*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a**2*b*c*d**2 + 84*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/
(a**(1/3)*sqrt(3)))*a*b**2*c**2*d - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*c**3 + 14*a**(1/3)*log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*d**3 - 42*a**(1/3)*log(a**(2/3) -
b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*c*d**2 + 42*a**(1/3)*log(a**(2
/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*c**2*d - 14*a**(1/3)*log
(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*c**3 - 28*a**(1/3)*l
og(a**(1/3) + b**(1/3)*x)*a**3*d**3 + 84*a**(1/3)*log(a**(1/3) + b**(1/3)*
x)*a**2*b*c*d**2 - 84*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c**2*d +
28*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**3*c**3 + 84*b**(1/3)*a**3*d**3*x
 - 252*b**(1/3)*a**2*b*c*d**2*x - 21*b**(1/3)*a**2*b*d**3*x**4 + 252*b**(1
/3)*a*b**2*c**2*d*x + 63*b**(1/3)*a*b**2*c*d**2*x**4 + 12*b**(1/3)*a*b**2*
d**3*x**7)/(84*b**(1/3)*a*b**3)
```

3.31 $\int \frac{(c+dx^3)^2}{a+bx^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int \frac{(c+dx^3)^2}{a+bx^3} \, dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^4}{4b} - \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} + \frac{(bc-ad)^2 \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}} - \frac{(bc-ad)^2 \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/4*d^2*x^4/b-1/3*(-a*d+b*c)^2*arctan(1/3*(a^(1/3)-2*
b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(7/3)+1/3*(-a*d+b*c)^2*ln(a^
(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/6*(-a*d+b*c)^2*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$$
$$= \frac{-12a^{2/3}\sqrt[3]{b}d(-2bc+ad)x+3a^{2/3}b^{4/3}d^2x^4+4\sqrt{3}(bc-ad)^2\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)+4(bc-ad)^2\log\left(\sqrt[3]{\cdots}\right.}{12a^{2/3}b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

output
```
(-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sq
rt[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4
*(b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$$
$$\Big\downarrow \; 915$$
$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^3)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2}{\sqrt{3}a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*
d)^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.45

method	result
risch	$\frac{d^{2}x^{4}}{4b}-\frac{d^{2}ax}{b^{2}}+\frac{2dcx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{3}}$
default	$-\frac{d\left(-\frac{1}{4}bd\,x^{4}+adx-2bcx\right)}{b^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{b^{2}}$

input
```
int((d*x^3+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*c*x+1/3/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.93

$$\int\frac{\left(c+dx^{3}\right)^{2}}{a+bx^{3}}\,dx$$

$$=\left[\frac{3\,a^{2}b^{2}d^{2}x^{4}+6\,\sqrt{\frac{1}{3}}(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2})\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)\right.}{bx^{3}+a}\right.}{}\right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*
d^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3
*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(
1/3)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^
3*b*d^2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 -
 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2
*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^
2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)$$
$$+\text{RootSum}\Bigg(27t^3a^2b^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \Bigg(t \mapsto$$
$$+\frac{d^2x^4}{4b}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a*
*5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b*
*4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/
(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{bd^2x^4+4\,(2\,bcd-ad^2)x}{4\,b^2} + \frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(
a/b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = -\frac{\sqrt{3}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b} - \frac{(b^4c^2-2\,ab^3cd+a^2b^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{b^3d^2x^4+8\,b^3cdx-4\,ab^2d^2x}{4\,b^4}$$

input

```
integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

output

```
-1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*
(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3
)))/(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^3)^2}{a+bx^3}\,dx = \frac{d^2\,x^4}{4\,b} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right) + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)^2}{3\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,\left(a\,d-b\,c\right)^2}{a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^2}{3\,a^{2/3}\,b^{7/3}}$$

input

```
int((c + d*x^3)^2/(a + b*x^3),x)
```

output
```
(d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*
(a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2)/(3*a^(2/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^3)^2}{a+bx^3} \, dx = \frac{-4a^{\frac{7}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) d^2+8a^{\frac{4}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) bcd-4a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2c^2-2a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-\right.}{}$$

input
```
int((d*x^3+c)^2/(b*x^3+a),x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2*d**2 + 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sq
rt(3)))*a*b*c*d - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*b**2*c**2 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x +
b**(2/3)*x**2)*a**2*d**2 + 4*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x +
 b**(2/3)*x**2)*a*b*c*d - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x +
b**(2/3)*x**2)*b**2*c**2 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2*d**2
 - 8*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*c*d + 4*a**(1/3)*log(a**(1/3)
 + b**(1/3)*x)*b**2*c**2 - 12*b**(1/3)*a**2*d**2*x + 24*b**(1/3)*a*b*c*d*x
 + 3*b**(1/3)*a*b*d**2*x**4)/(12*b**(1/3)*a*b**2)
```

3.32 $\int \frac{c+dx^3}{a+bx^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{c+dx^3}{a+bx^3} \, dx = \frac{dx}{b} - \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

output
```
d*x/b-1/3*(-a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(
1/2)/a^(2/3)/b^(4/3)+1/3*(-a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-
1/6*(-a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{a+bx^3} \, dx$$
$$= \frac{6a^{2/3}\sqrt[3]{b}dx - 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (bc-ad)\log\left(a^{2/3}-\sqrt[3]{\right.}{6a^{2/3}b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3),x]
```

output
```
(6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{a+bx^3}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{bx^3+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{750}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(bc-ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}+\frac{dx}{b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3),x]
```

output
```
(d*x)/b + ((b*c - a*d)*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^{2}}}{3b^{2}}$	42
default	$\frac{dx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{b}$	110

input
```
int((d*x^3+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+1/3/b^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int\frac{c+dx^{3}}{a+bx^{3}}\,dx$$

$$=\left[\frac{6\,a^{2}bdx-3\sqrt{\frac{1}{3}}(ab^{2}c-a^{2}bd)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}+3\,(-a^{2}b)^{\frac{1}{3}}ax-a^{2}-3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(-a^{2}b)^{\frac{2}{3}}x+(-a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right.}{6}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-
a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) -
(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2),
 1/6*(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^
(1/3)/b)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2
/3)))/(a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \text{RootSum}\left(27t^3a^2b^4+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(-\frac{3tab}{ad-bc}+x\right)\right)\right) + \frac{dx}{b}$$

input
```
integrate((d*x**3+c)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
 - b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^3}{a+bx^3}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{dx}{b} - \frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab}$$

input
```
integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a
/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{d\,x}{b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3),x)
```

output
```
(d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (lo
g(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
- b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)d-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bc+a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)d-a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a\right.}{6b^{\frac{4}{3}}a}$$

input
```
int((d*x^3+c)/(b*x^3+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*d
 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b
*c + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*d - a*
*(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*c - 2*a**(1/3
)*log(a**(1/3) + b**(1/3)*x)*a*d + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b
*c + 6*b**(1/3)*a*d*x)/(6*b**(1/3)*a*b)
```

3.33 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = {} & -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} \\ & +\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} \\ & -\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} \\ & +\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}\end{aligned}$$

output
```
-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(
2/3)/(-a*d+b*c)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/
3))*3^(1/2)/c^(2/3)/(-a*d+b*c)+1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(
-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*
ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/\cdots\right.}{}}{-6bc+6ad}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) -
 (2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d
^(1/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(
2/3))/(-6*b*c + 6*a*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {917, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$\Big\downarrow \text{ 917}$$
$$\frac{b\int \frac{1}{bx^3+a}dx}{bc-ad} - \frac{d\int \frac{1}{dx^3+c}dx}{bc-ad}$$
$$\Big\downarrow \text{ 750}$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}$$

$$\Big\downarrow \text{16}$$

$$\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1142}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{\int\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1082$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1103}$$

$$\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{2\sqrt[3]{d}}}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

output
```
(b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(Log[c^(1/3)
 + d^(1/3)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/
c^(1/3))/Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2]/(2*d^(1/3)))/(3*c^(2/3))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\sqrt{3}\,\text{ar}\ldots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^{3}c^{2}d^{3}-3a^{2}b\,c^{3}d^{2}+3a\,b^{2}c^{4}d-c^{5}b^{3}\right)\_Z^{3}-d^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-c^{5}b^{5}\right)\_\ldots\right.\right.\right.}{3}$

input
```
int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2\ldots\right.}{\ldots}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3
) - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d
^2/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2
/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*
(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-
b^2/a^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a
*d)
```

Sympy [A] (verification not implemented)

Time = 56.98 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \text{RootSum}\left(t^3\cdot\left(27a^5d^3-81a^4bcd^2+81a^3b^2c^2d-27a^2b^3c^3\right)+b^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t^4a^6\right.\right.\right.$$
$$+\text{RootSum}\left(t^3\cdot\left(27a^3c^2d^3-81a^2bc^3d^2+81ab^2c^4d-27b^3c^5\right)-d^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t\right.\right.\right.$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

output
```
RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*
a**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 2
43*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4
*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3
*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/
(a**2*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**
2*b*c**3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log
(x + (81*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**
5*b**2*c**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**
6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t
*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

input

```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

output

```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b
)^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(
2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)
^(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*
d*(a/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/
3) - a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)}+\frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)}-\frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

input

```
integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

output

```
-1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c
/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)
*a^2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*
(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)),x)
```

output
```
log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*
a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d +
b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b
^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3
) + log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 -
18*a*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d
+ b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-
d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3) + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)
*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*
c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3)
)^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c
^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b
^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(
81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*
(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5
))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d
^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.65

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{2d^{\frac{1}{3}}a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bc-2c^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)ad+d^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)bc-2d^{\frac{1}{3}}a^{\frac{1}{3}}\cdots}{6d^{\frac{1}{3}}b^{\frac{1}{3}}ac\,(ad-bc)}$$

input

```
int(1/(b*x^3+a)/(d*x^3+c),x)
```

output

```
(2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b*c - 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c*
*(1/3)*sqrt(3)))*a*d + d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*b*c - 2*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b*
c - c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*
a*d + 2*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*d)/(6*d**(1/3)*b**(
1/3)*a*c*(a*d - b*c))
```

3.34 $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 346

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = -\frac{dx}{3c(bc-ad)(c+dx^3)} - \frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}$$

output
```
-1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)-1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/(-a*d+b*c)^2+1/9*d^(2/3)*(-2*a*d+5*b*c
)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(5/3)/(-a*d+
b*c)^2+1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)^2-1/9*d^(2/3)*
(-2*a*d+5*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^2-1/6*b^(5/3)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)^2+1/18*d^(2/3)*(-
2*a*d+5*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)^
2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$

$$= \frac{6a^{2/3}c^{2/3}d(-bc+ad)x - 6\sqrt{3}b^{5/3}c^{5/3}(c+dx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\sqrt{3}a^{2/3}d^{2/3}(-5bc+2ad)(c+dx^3)}{}$$

input
```
Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

output
```
(6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x
^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3
)*(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]]
 + 6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2
/3)*(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5
/3)*(c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d
^(2/3)*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2])/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {931, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-2bdx^3+3bc-2ad}{(bx^3+a)(dx^3+c)}dx}{3c(bc-ad)}-\frac{dx}{3c(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 1020}$$

$$\frac{\frac{3b^2c\int \frac{1}{bx^3+a}dx}{bc-ad}-\frac{d(5bc-2ad)\int \frac{1}{dx^3+c}dx}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{\frac{3b^2c\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{3b^2c\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{\cdots}\right)}{\cdots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 25

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{3\cdots}\right)}{\cdots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{3b^2c\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2}\cdots}\cdots}{\cdots}\right)}{\cdots}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1082

$$\frac{\frac{3b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\right.}{}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 217

$$\frac{\frac{3b^2c\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan}{}}{3c^{2/3}}\right.}{bc-ad}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1103

$$\frac{\frac{3b^2c\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}-\frac{d(5bc-2ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{c}\cdots\right.}{2\cdots}}{3c^{2/3}}\right.}{bc-ad}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b^2*c*(Log[a^(1/3) + b^(1/3)*
x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/
3)))/(3*a^(2/3))))/(b*c - a*d) - (d*(5*b*c - 2*a*d)*(Log[c^(1/3) + d^(1/3)
*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/S
qrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1
/3)))/(3*c^(2/3))))/(b*c - a*d))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.71

method	result
default	$\frac{d\left(\frac{(ad-bc)x}{3c\left(d\,x^{3}+c\right)}+\frac{(2ad-5bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d/(a*d-b*c)^2*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+1/3*(2*a*d-5*b*c)/c*(1/3/d/(c/d
)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/
3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+(1
/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*b^2/(a*d-b*c)^2
```

Fricas [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$
$$= \frac{6\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-2\sqrt{3}((5\,bcd-2\,ad^2)x^3+5\,bc^2-2\,acd)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\operatorname{arct}}{}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*
a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 +
 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 -
a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 +
5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b
^2/a^2)^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^
2*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3))
- 1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)
^(1/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^
(1/3))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3)
- 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3
) + a^2*c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/
3) - 2*a*b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*lo
g(x + (c/d)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*
d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$$

$$= -\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)} + \frac{(-ab^2)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\,\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}$$
$$+ \frac{(-ab^2)^{\frac{1}{3}}\,b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)}$$
$$+ \frac{\left(5\,bcd-2\,ad^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)}$$
$$- \frac{\left(5\,(-cd^2)^{\frac{1}{3}}\,bc-2\,(-cd^2)^{\frac{1}{3}}\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4-2\,\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2\right)}$$
$$- \frac{\left(5\,(-cd^2)^{\frac{1}{3}}\,bc-2\,(-cd^2)^{\frac{1}{3}}\,ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)}$$
$$- \frac{dx}{3\,(dx^3+c)(bc^2-acd)}$$

input
```
integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d
+ a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/
6*(-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2
*a^2*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (
-c/d)^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3
)*b*c - 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) -
1/18*(5*(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3
) + (-c/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3
+ c)*(b*c^2 - a*c*d))
```

Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 2589, normalized size of antiderivative = 7.48

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

output
```
log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
 + (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c
^5*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c
^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c
^5*(a*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*
b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*
c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*
d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 +
 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x
*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*
(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*
d))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3
+ 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d
- b*c)^6))^(1/3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*
d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*
c^6 - 162*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*
a^6*b^2*c^2*d^4 - 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1
/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*
c*d))/c + (27*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^3+a)/(d*x^3+c)^2,x)
```

output
```
( - 6*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*b**2*c**3 - 6*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*x)/(a**(1/3)*sqrt(3)))*b**2*c**2*d*x**3 - 4*c**(1/3)*b**(1/3)*sqrt(3)*a
tan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*c*d**2 - 4*c**(1/3)
*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*
d**3*x**3 + 10*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c
**(1/3)*sqrt(3)))*a*b*c**2*d + 10*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3)
 - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a*b*c*d**2*x**3 - 3*d**(1/3)*a**(1/3)
*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*c**3 - 3*d**(1/3
)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*c**2*d
*x**3 + 6*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*c**3 + 6*d**(1
/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*c**2*d*x**3 - 2*c**(1/3)*b**(
1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*c*d**2 - 2*c
**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2*
d**3*x**3 + 5*c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2
/3)*x**2)*a*b*c**2*d + 5*c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3
)*x + d**(2/3)*x**2)*a*b*c*d**2*x**3 + 4*c**(1/3)*b**(1/3)*log(c**(1/3) +
d**(1/3)*x)*a**2*c*d**2 + 4*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a
**2*d**3*x**3 - 10*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b*c**2*d
 - 10*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b*c*d**2*x**3 + 6*...
```

3.35 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 320

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4}{4b^4} + \frac{d^4(5bc-2ad)x^7}{7b^3} + \frac{d^5x^{10}}{10b^2} + \frac{(bc-ad)^5x}{3ab^5\,(a+bx^3)} - \frac{(bc-ad)^4(2bc+13ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{16/3}} + \frac{(bc-ad)^4(2bc+13ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} - \frac{(bc-ad)^4(2bc+13ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}$$

output
```
d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3
*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/1
0*d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)-1/9*(-a*d+b*c)^4*(13*a*d
+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/
b^(16/3)+1/9*(-a*d+b*c)^4*(13*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(
16/3)-1/18*(-a*d+b*c)^4*(13*a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/a^(5/3)/b^(16/3)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$$

$$= \frac{1260\sqrt[3]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+315b^{4/3}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^4+180b^{7/3}}{}$$

input
```
Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

output
```
(1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(
7/3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c -
 a*d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*A
rcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a
*d)^4*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)
^4*(2*b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3
))/(1260*b^(16/3))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d^3x^3(3a^2d^2-10abcd+10b^2c^2)}{b^4}+\frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{5bdx^3(bc-ad)^4+(4ad}{b^5(a+bx^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^4(13ad+2bc)}{3\sqrt{3}a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}}+\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{3ab^5(a+bx^3)}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{d^5x^{10}}{10b^2}$$

input
```
Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a
*d)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b
*x^3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 1
3*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*
b^(16/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.95

method	result
risch	$\frac{d^5x^{10}}{10b^2}-\frac{2d^5a\,x^7}{7b^3}+\frac{5d^4c\,x^7}{7b^2}+\frac{3d^5a^2x^4}{4b^4}-\frac{5d^4ac\,x^4}{2b^3}+\frac{5d^3c^2x^4}{2b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-\frac{(c}{}$
default	$-\frac{d^2\left(-\frac{1}{10}d^3x^{10}b^3+\frac{2}{7}a\,b^2d^3x^7-\frac{5}{7}b^3c\,d^2x^7-\frac{3}{4}a^2b\,d^3x^4+\frac{5}{2}a\,b^2c\,d^2x^4-\frac{5}{2}b^3c^2d\,x^4+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+$

input
```
int((d*x^3+c)^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*d^5*x^10/b^2-2/7*d^5/b^3*a*x^7+5/7*d^4/b^2*c*x^7+3/4*d^5/b^4*a^2*x^4-
5/2*d^4/b^3*a*c*x^4+5/2*d^3/b^2*c^2*x^4-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x
-20*d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/3*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2
*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^3+a)+1/9/b
^6/a*sum((13*a^5*d^5-50*a^4*b*c*d^4+70*a^3*b^2*c^2*d^3-40*a^2*b^3*c^3*d^2+
5*a*b^4*c^4*d+2*b^5*c^5)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(275) = 550.

Time = 0.11 (sec) , antiderivative size = 1619, normalized size of antiderivative = 5.06

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^1
0 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*
(40*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d
^5)*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*
d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5
+ 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c
*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2
*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(
1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4
*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6
*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^
3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2
*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a
^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6
*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^
2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 42
0*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3
 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(12
6*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*
a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*...
```

Sympy [A] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = x^7\left(-\frac{2ad^5}{7b^3}+\frac{5cd^4}{7b^2}\right)+x^4\cdot\left(\frac{3a^2d^5}{4b^4}-\frac{5acd^4}{2b^3}+\frac{5c^2d^3}{2b^2}\right)$$
$$+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)$$
$$+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{3a^2b^5+3ab^6x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^2c^2d^{13}+418280a^{12}b^3c^3d^{12}-87463\right.$$
$$+\frac{d^5x^{10}}{10b^2}$$

input
```
integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

output
```
x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
 - 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15
*a**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d*
*5 + 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5
*a*b**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t
**3*a**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b
**2*c**2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**
11 + 1271886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*
a**8*b**7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d*
*6 + 7194*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*
b**12*c**12*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**1
5*c**15, Lambda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4
 + 70*a**3*b**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b
**5*c**5) + x))) + d**5*x**10/(10*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{3\,(ab^6x^3 + a^2b^5)}$$

$$+ \frac{14\,b^3d^5x^{10} + 20\,(5\,b^3cd^4 - 2\,ab^2d^5)x^7 + 35\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^4 + 140\,(10\,b^3c^3d^2 - 20\,ab}{140\,b^5}$$

$$+ \frac{\sqrt{3}(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$- \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$$+ \frac{(2\,b^5c^5 + 5\,ab^4c^4d - 40\,a^2b^3c^3d^2 + 70\,a^3b^2c^2d^3 - 50\,a^4bcd^4 + 13\,a^5d^5)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^6\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 +
 20*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4
+ 3*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d
^4 - 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b
^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5
*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*
c*d^4 + 13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2
/3)) + 1/9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^
2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/
3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx =$$

$$-\frac{\sqrt{3}(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^4}$$
$$-\frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab^4}$$
$$-\frac{(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^5}$$
$$+\frac{b^5c^5x-5\,ab^4c^4dx+10\,a^2b^3c^3d^2x-10\,a^3b^2c^2d^3x+5\,a^4bcd^4x-a^5d^5x}{3\,(bx^3+a)ab^5}$$
$$+\frac{14\,b^{18}d^5x^{10}+100\,b^{18}cd^4x^7-40\,ab^{17}d^5x^7+350\,b^{18}c^2d^3x^4-350\,ab^{17}cd^4x^4+105\,a^2b^{16}d^5x^4+1400\,b^{18}c^3}{140\,b^{20}}$$

input
```
integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*
c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*
d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*
b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4
*b*c*d^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) +
 1/3*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*
d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*
d^5*x^10 + 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 -
 350*a*b^17*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*
a*b^17*c^2*d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right) - x^7\left(\frac{2\,a\,d^5}{7\,b^3}-\frac{5\,c\,d^4}{7\,b^2}\right)+x^4\left(\frac{a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{2\,b}-\frac{a^2\,d^5}{4\,b^4}+\frac{5\,c^2\,d^3}{2\,b^2}\right)+\frac{d^5\,x^{10}}{10\,b^2} - \frac{x\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{3\,a\left(b^6\,x^3+a\,b^5\right)} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^4\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$

input
```
int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

output
```
x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2
*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b
^2) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3
 - (5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5
*x^10)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*
c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b
^(1/3)*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) -
 (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)
*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d
+ 2*b*c))/(9*a^(5/3)*b^(16/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1349, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^5/(b*x^3+a)^2,x)
```

output

```
( - 1820*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a**6*d**5 + 7000*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*a**5*b*c*d**4 - 1820*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**
(1/3)*x)/(a**(1/3)*sqrt(3)))*a**5*b*d**5*x**3 - 9800*a**(1/3)*sqrt(3)*atan
((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4*b**2*c**2*d**3 + 7000*
a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**4*b
**2*c*d**4*x**3 + 5600*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a*
*(1/3)*sqrt(3)))*a**3*b**3*c**3*d**2 - 9800*a**(1/3)*sqrt(3)*atan((a**(1/3
) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b**3*c**2*d**3*x**3 - 700*a**(1
/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**4*c
**4*d + 5600*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a**2*b**4*c**3*d**2*x**3 - 280*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*
b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**5*c**5 - 700*a**(1/3)*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**5*c**4*d*x**3 - 280*a**(1
/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**6*c**5*x
**3 - 910*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**
6*d**5 + 3500*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)
*a**5*b*c*d**4 - 910*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3
)*x**2)*a**5*b*d**5*x**3 - 4900*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*a**4*b**2*c**2*d**3 + 3500*a**(1/3)*log(a**(2/3) - b...
```

3.36 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 267

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4\,(a+bx^3)} - \frac{2(bc-ad)^3(bc+5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(bc+5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}} - \frac{(bc-ad)^3(bc+5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}$$

output
```
d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7
*d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)-2/9*(-a*d+b*c)^3*(5*a*d+b*
c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(13
/3)+2/9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(13/3)-1/
9*(-a*d+b*c)^3*(5*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/
3)/b^(13/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$

$$= \frac{126\sqrt[3]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+63b^{4/3}d^3(2bc-ad)x^4+18b^{7/3}d^4x^7+\frac{42\sqrt[3]{b}(bc-ad)^4x}{a(a+bx^3)}+\frac{28\sqrt{3}(bc-ad)^3(bc}{}}{126b^1}$$

input
```
Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

output
```
(126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2
*b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a
+ b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Lo
g[a^(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^2(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{4bdx^3(bc-ad)^3+(3ad+bc)(bc-ad)^3}{b^4(a+bx^3)^2}+\frac{2d^3x^3(2bc-ad)}{b^3}+\frac{d^4x^6}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^3(5ad+bc)}{3\sqrt{3}a^{5/3}b^{13/3}}-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{3ab^4(a+bx^3)}+\frac{d^3x^4(2bc-ad)}{2b^3}+\frac{d^4x^7}{7b^2}$$

input
```
Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

output
```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/
(2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2
*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.82

method	result
risch	$\frac{d^4x^7}{7b^2} - \frac{d^4a\,x^4}{2b^3} + \frac{d^3c\,x^4}{b^2} + \frac{3d^4a^2x}{b^4} - \frac{8d^3acx}{b^3} + \frac{6d^2c^2x}{b^2} + \frac{(d^4a^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4)x}{3a\,b^4(b\,x^3+a)} - \frac{2\left(\_R\text{=Ro}\right.}{}$ $2\left(5d^4a^4-14a^3bc\,d\right.$
default	$\frac{d^2\left(\frac{1}{7}b^2d^2x^7-\frac{1}{2}ab\,d^2x^4+b^2cd\,x^4+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4} - \frac{-\frac{\left(d^4a^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)x}{3a\left(b\,x^3+a\right)}+}{}$

input
```
int((d*x^3+c)^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*d^4*x^7/b^2-1/2*d^4/b^3*a*x^4+d^3/b^2*c*x^4+3*d^4/b^4*a^2*x-8*d^3/b^3*
a*c*x+6*d^2/b^2*c^2*x+1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)/a*x/b^4/(b*x^3+a)-2/9/b^5/a*sum((5*a^4*d^4-14*a^3*b*c*d^3+
12*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d-b^4*c^4)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(224) = 448.

Time = 0.10 (sec) , antiderivative size = 1316, normalized size of antiderivative = 4.93

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 6
3*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/
3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3
- 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4
*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3
*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-
a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a
^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4
+ 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3
)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(
a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*
d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5
*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*
c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d
^4)*x)/(a^3*b^6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4
*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 +
5*a^5*b^2*d^4)*x^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*
b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*
d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^
2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*...
```

Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)$$
$$+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{3a^2b^4+3ab^5x^3}$$
$$+\mathrm{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c\right.$$
$$+\frac{d^4x^7}{7b^2}$$

input
```
integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

output
```
x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b
**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + Root
Sum(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720
*a**10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*
d**8 - 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b*
*7*c**7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a*
*2*b**10*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*lo
g(-9*_t*a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.49

$$\begin{aligned}
\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx &= \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)x}{3\,(ab^5x^3+a^2b^4)} \\
&+ \frac{2\,b^2d^4x^7+7\,(2\,b^2cd^3-abd^4)x^4+14\,(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4)x}{14\,b^4} \\
&+ \frac{2\,\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&- \frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\
&+ \frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}
\end{aligned}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4
)*x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*
(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
- 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a
^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(
b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)
*log(x + (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = -\frac{2\sqrt{3}(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^3} - \frac{(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab^3} - \frac{2\,(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4} + \frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{3\,(bx^3+a)ab^4} + \frac{2\,b^{12}d^4x^7+14\,b^{12}cd^3x^4-7\,ab^{11}d^4x^4+84\,b^{12}c^2d^2x-112\,ab^{11}cd^3x+42\,a^2b^{10}d^4x}{14\,b^{14}}$$

input
```
integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^
3 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*
b^2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14
*a^3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^
2)^(2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a
^3*b*c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4)
+ 1/3*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x
 + a^4*d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x
^4 - 7*a*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^
10*d^4*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx$$
$$= x\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)$$
$$+\frac{d^4\,x^7}{7\,b^2}+\frac{x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{3\,a\,(b^5\,x^3+a\,b^4)}$$
$$-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$
$$+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$
$$-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^3\,(5\,a\,d+b\,c)}{9\,a^{5/3}\,b^{13/3}}$$

input
```
int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

output
```
x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b
^2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*
b^4 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))
/(9*a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
- (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1074, normalized size of antiderivative = 4.02

$$\int \frac{(c + dx^3)^4}{(a + bx^3)^2} \, dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

output
```
(140*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a
**5*d**4 - 392*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a**4*b*c*d**3 + 140*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
x)/(a**(1/3)*sqrt(3)))*a**4*b*d**4*x**3 + 336*a**(1/3)*sqrt(3)*atan((a**(1
/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b**2*c**2*d**2 - 392*a**(1/3)
*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*b**2*c*d*
*3*x**3 - 56*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a**2*b**3*c**3*d + 336*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)
*x)/(a**(1/3)*sqrt(3)))*a**2*b**3*c**2*d**2*x**3 - 28*a**(1/3)*sqrt(3)*ata
n((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**4*c**4 - 56*a**(1/3)*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**4*c**3*d*x
**3 - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*b**5*c**4*x**3 + 70*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2
/3)*x**2)*a**5*d**4 - 196*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b*
*(2/3)*x**2)*a**4*b*c*d**3 + 70*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*a**4*b*d**4*x**3 + 168*a**(1/3)*log(a**(2/3) - b**(1/3)
*a**(1/3)*x + b**(2/3)*x**2)*a**3*b**2*c**2*d**2 - 196*a**(1/3)*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b**2*c*d**3*x**3 - 28*a**(1
/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**3*c**3*d +
 168*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*...
```

3.37 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc-ad)^3x}{3ab^3(a+bx^3)} - \frac{(bc-ad)^2(2bc+7ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(2bc+7ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a
)-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/
a^(1/3))*3^(1/2)/a^(5/3)/b^(10/3)+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(1/3
)+b^(1/3)*x)/a^(5/3)/b^(10/3)-1/18*(-a*d+b*c)^2*(7*a*d+2*b*c)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(10/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}d^2(3bc-2ad)x+9b^{4/3}d^3x^4+\frac{12\sqrt[3]{b}(bc-ad)^3x}{a(a+bx^3)}+\frac{4\sqrt{3}(bc-ad)^2(2bc+7ad)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+\frac{4(bc-ad)^2(2bc+\cdots}{\cdots}}{36b^{10/3}}$$

input
```
Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

output
```
(36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
 a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2
*(2*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b
*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b
^(10/3))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^3(bc-ad)^2+(bc-ad)^2(2ad+bc)}{b^3(a+bx^3)^2}+\frac{d^3x^3}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)^2(7ad+2bc)}{3\sqrt{3}a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3(a+bx^3)}+\frac{d^3x^4}{4b^2}
$$

input
```
Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b
^3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d
)^2*(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^
(5/3)*b^(10/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
risch	$\frac{d^3x^4}{4b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{3a\,b^3(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(7a^3d^3-12a^2bc\,d^2+3a\,b^2c^2d+2b^3c^3)\ln}{\_R^2}}{9b^4a}$
default	$-\frac{d^2(-\frac{1}{4}bd\,x^4+2adx-3bcx)}{b^3}+\frac{-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{3a(b\,x^3+a)}+\frac{(7a^3d^3-12a^2bc\,d^2+3a\,b^2c^2d+2b^3c^3)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\right.}{}\right.}{3a}}{b^3}$

input
```
int((d*x^3+c)^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^3+a)+1/9/b^4/a*sum((7*a^3*d^3-12*a^2*b*c*
d^2+3*a*b^2*c^2*d+2*b^3*c^3)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(193) = 386.

Time = 0.10 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.39

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sq
rt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3
+ (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*
sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt
(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a
^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b
^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x +
(a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*
a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3
*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*
c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(
2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 +
3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b
*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2
*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b...
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{3a^2b^3+3ab^4x^3}$$
$$+\mathrm{RootSum}\Big(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a$$
$$+\frac{d^3x^4}{4b^2}$$

input
```
integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**
3*a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d*
*7 + 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c*
*5*d**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**
8*d - 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2
*b*c*d**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{3\,(ab^4x^3+a^2b^3)} + \frac{bd^3x^4+4\,(3\,bcd^2-2\,ad^3)x}{4\,b^3} + \frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2
*b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b
^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2
*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a
*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^
2 + 7*a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$$
$$= -\frac{\sqrt{3}(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{3\,(bx^3+a)ab^3} + \frac{b^6d^3x^4+12\,b^6cd^2x-8\,ab^5d^3x}{4\,b^8}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) -
 1/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*
a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a
^3*d^3*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^
5*d^3*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \frac{d^3\,x^4}{4\,b^2} - x\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{3\,a\,(b^4\,x^3 + a\,b^3)} + \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(a\,d - b\,c)^2\,(7\,a\,d + 2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} - \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d - b\,c)^2\,(7\,a\,d + 2\,b\,c)}{9\,a^{5/3}\,b^{10/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d - b\,c)^2\,(7\,a\,d + 2\,b\,c)}{9\,a^{5/3}\,b^{10/3}}$$

input
```
int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

output
```
(d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/
3)*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i +
2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c
))/(9*a^(5/3)*b^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.49

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^3/(b*x^3+a)^2,x)
```

output
```
( - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**4*d**3 + 48*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*a**3*b*c*d**2 - 28*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*
x)/(a**(1/3)*sqrt(3)))*a**3*b*d**3*x**3 - 12*a**(1/3)*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2*c**2*d + 48*a**(1/3)*sqrt
(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2*c*d**2*x*
*3 - 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a*b**3*c**3 - 12*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3
)*sqrt(3)))*a*b**3*c**2*d*x**3 - 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**
(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*c**3*x**3 - 14*a**(1/3)*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**4*d**3 + 24*a**(1/3)*log(a**(2/3) -
 b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*c*d**2 - 14*a**(1/3)*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*b*d**3*x**3 - 6*a**(1/3)*
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*c**2*d + 24*
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*c*d
**2*x**3 - 4*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*
a*b**3*c**3 - 6*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*a*b**3*c**2*d*x**3 - 4*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b*
*(2/3)*x**2)*b**4*c**3*x**3 + 28*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**4*
d**3 - 48*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**3*b*c*d**2 + 28*a**(1/...
```

3.38 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{3ab^2(a+bx^3)} - \frac{2(bc-ad)(bc+2ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}$$

output
```
d^2*x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)-2/9*(-a*d+b*c)*(2*a*d+b*c)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(7/3)+2/
9*(-a*d+b*c)*(2*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/9*(-a*d+b
*c)*(2*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$= \frac{9\sqrt[3]{b}d^2x + \frac{3\sqrt[3]{b}(bc-ad)^2x}{a(a+bx^3)} - \frac{2\sqrt{3}(b^2c^2+abcd-2a^2d^2)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2(b^2c^2+abcd-2a^2d^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{(b^2c^2+ab}{}}{9b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

output
```
(9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3
]*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x]
)/a^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \ 915$$

$$\int \left(\frac{-a^2d^2+2bdx^3(bc-ad)+b^2c^2}{b^2(a+bx^3)^2}+\frac{d^2}{b^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)(bc-ad)(2ad+bc)}{3\sqrt{3}a^{5/3}b^{7/3}}-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2(a+bx^3)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*
c + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a
^(5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(
9*a^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{3a\,b^2(b\,x^3+a)}-\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(2a^2d^2-abcd-b^2c^2\right)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^3a}$
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{3a\left(b\,x^3+a\right)}+\frac{2\left(2a^2d^2-abcd-b^2c^2\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3a}}{b^2}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^3+a)-2/9/b^3/a*sum(
(2*a^2*d^2-a*b*c*d-b^2*c^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(164) = 328.

Time = 0.09 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b
*d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 +
(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a))
- (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)
*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
2*(a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)
*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*
b^2*c*d + 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4
+ 6*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*
b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2
*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-
a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2
*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-
a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3a^2b^2 + 3ab^3x^3}$$
$$+\mathrm{RootSum}\Big(729t^3a^5b^7 + 64a^6d^6 - 96a^5bcd^5 - 48a^4b^2c^2d^4 + 88a^3b^3c^3d^3 + 24a^2b^4c^4d^2 - 24ab^5c^5d - 8b^6$$
$$+\frac{d^2x}{b^2}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + Root
Sum(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c
**2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**
5*d - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*
c*d - 2*b**2*c**2) + x))) + d**2*x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{3\,(ab^3x^3+a^2b^2)} + \frac{d^2x}{b^2} + \frac{2\,\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output

```
1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 +
2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2
*d^2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^
2*c^2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2x}{b^2} - \frac{2\sqrt{3}(b^2c^2+abcd-2\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{(b^2c^2+abcd-2\,a^2d^2)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{2\,(b^2c^2+abcd-2\,a^2d^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{3\,(bx^3+a)ab^2}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)
*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 +
a*b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2
/3)*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3
 + a)*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{3\,a\,(b^3\,x^3+a\,b^2)} - \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d-b\,c)\,(2\,a\,d+b\,c)}{9\,a^{5/3}\,b^{7/3}} - \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(2\,a\,d+b\,c)}{9\,a^{5/3}\,b^{7/3}} + \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)\,(2\,a\,d+b\,c)}{9\,a^{5/3}\,b^{7/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3))
- (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3
)) - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 -
1/2)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/
3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d +
 b*c))/(9*a^(5/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 575, normalized size of antiderivative = 2.83

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^2,x)
```

output

```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**
3*d**2 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a**2*b*c*d + 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*a**2*b*d**2*x**3 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**
(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2 - 2*a**(1/3)*sqrt(3)*atan((a**(1/
3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*d*x**3 - 2*a**(1/3)*sqrt(3
)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*c**2*x**3 + 2*a*
*(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3*d**2 - a**
(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*c*d + 2*a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*d**2*x*
*3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*c
**2 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*
c*d*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*
*3*c**2*x**3 - 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**3*d**2 + 2*a**(1/3
)*log(a**(1/3) + b**(1/3)*x)*a**2*b*c*d - 4*a**(1/3)*log(a**(1/3) + b**(1/
3)*x)*a**2*b*d**2*x**3 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c**2
 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c*d*x**3 + 2*a**(1/3)*log(
a**(1/3) + b**(1/3)*x)*b**3*c**2*x**3 + 12*b**(1/3)*a**3*d**2*x - 6*b**(1/
3)*a**2*b*c*d*x + 9*b**(1/3)*a**2*b*d**2*x**4 + 3*b**(1/3)*a*b**2*c**2*x)/
(9*b**(1/3)*a**2*b**2*(a + b*x**3))
```

3.39 $\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3ab\,(a+bx^3)} - \frac{(2bc+ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

output
```
1/3*(-a*d+b*c)*x/a/b/(b*x^3+a)-1/9*(a*d+2*b*c)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(4/3)+1/9*(a*d+2*b*c)*ln(a^(1/3)+
b^(1/3)*x)/a^(5/3)/b^(4/3)-1/18*(a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(5/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$
$$=\frac{-\frac{6a^{2/3}\sqrt[3]{b}(-bc+ad)x}{a+bx^3}-2\sqrt{3}(2bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(2bc+ad)\log}{18a^{5/3}b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

output
```
((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*
d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/
3) + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(18*a^(5/3)*b^(4/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {910, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+2bc)\int \frac{1}{bx^3+a}dx}{3ab}+\frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \; 750$$

$$\frac{(ad+2bc)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 16

$$\frac{(ad+2bc)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 1142

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 25

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{(ad+2bc)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(ad+2bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(ad+2bc)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(ad+2bc)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3ab}+\frac{x(bc-ad)}{3ab\,(a+bx^3)}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

output
```
((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) + ((2*b*c + a*d)*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^3+a\right)}\frac{(ad+2bc)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$-\frac{(ad-bc)x}{3ba(b\,x^3+a)}+\frac{(ad+2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

input
```
int((d*x^3+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d-b*c)/b/a*x/(b*x^3+a)+1/9/a/b^2*sum((a*d+2*b*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(2a^2b^2c+a^3bd+(2ab^3c+a^2b^2d)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right.}{bx^3+a}\right.}{}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*s
qrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(
1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b
))/(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*
log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^
3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2
*c - a^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c +
 a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((
2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b
)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^
3*b^3*x^3 + a^4*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{x(-ad+bc)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4 - a^3d^3 - 6a^2bcd^2 - 12ab^2c^2d - 8b^3c^3, \left(t \mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

output
```
x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 -
a**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _
t*log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{(bc-ad)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,bc+ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,bc+ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*
b*c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/
9*(2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(2\,bc+ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(2\,bc+ad)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} + \frac{bcx-adx}{3\,(bx^3+a)ab}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(
x - (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d+2\,b\,c)}{9\,a^{5/3}\,b^{4/3}} - \frac{x\,(a\,d-b\,c)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

input
```
int((c + d*x^3)/(a + b*x^3)^2,x)
```

output
```
(log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2
)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c)
)/(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*(
(3^(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))
/(3*a*b*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.07

$$\int \frac{c + dx^3}{(a + bx^3)^2}\,dx$$
$$= \frac{-2a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)d - 4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bc - 2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)bd\,x^3 - 4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\right.}{}$$

input
```
int((d*x^3+c)/(b*x^3+a)^2,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2*d - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a*b*c - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a*b*d*x**3 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*b**2*c*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*
x + b**(2/3)*x**2)*a**2*d - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*a*b*c - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**
(2/3)*x**2)*a*b*d*x**3 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b
**(2/3)*x**2)*b**2*c*x**3 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2*d +
 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*c + 2*a**(1/3)*log(a**(1/3) + b
**(1/3)*x)*a*b*d*x**3 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*c*x**3
- 6*b**(1/3)*a**2*d*x + 6*b**(1/3)*a*b*c*x)/(18*b**(1/3)*a**2*b*(a + b*x**
3))
```

3.40 $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 346

$$\begin{aligned}\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^3)} \\ &- \frac{b^{2/3}(2bc-5ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} \\ &- \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} \\ &+ \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} \\ &- \frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2} \\ &- \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}\end{aligned}$$

output
```
1/3*b*x/a/(-a*d+b*c)/(b*x^3+a)-1/9*b^(2/3)*(-5*a*d+2*b*c)*arctan(1/3*(a^(1
/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/(-a*d+b*c)^2-1/3*d^(5/3)
*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(2/3)/(-a*d+b
*c)^2+1/9*b^(2/3)*(-5*a*d+2*b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)^
2+1/3*d^(5/3)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)^2-1/18*b^(2/3)*(-5*
a*d+2*b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)^2-
1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$$

$$= \frac{6a^{2/3}bc^{2/3}(bc-ad)x - 2\sqrt{3}b^{2/3}c^{2/3}(2bc-5ad)(a+bx^3)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 6\sqrt{3}a^{5/3}d^{5/3}(a+bx^3)\,\mathrm{arc}}{}$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

output
```
(6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*
a*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a
^(5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2
*b^(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*
a^(5/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*
b*c - 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] -
3*a^(5/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2])/(18*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {931, 25, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{3a(a+bx^3)(bc-ad)} - \frac{\int -\frac{2bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)}dx}{3a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)}dx}{3a(bc-ad)} + \frac{bx}{3a(a+bx^3)(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{3ad^2\int \frac{1}{dx^3+c}dx}{bc-ad} + \frac{b(2bc-5ad)\int \frac{1}{bx^3+a}dx}{bc-ad}}{3a(bc-ad)} + \frac{bx}{3a(a+bx^3)(bc-ad)}$$

$$\Big\downarrow \; 750$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad} + \frac{3ad^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}}{3a(bc-ad)} + \frac{bx}{3a(a+bx^3)(bc-ad)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c \cdots}\cdots\right)}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/}\cdots}\cdots\right)}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{3}{2}\sqrt[3]{c}\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{\cdots}\right)}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}\cdots}{\cdots}\right)}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\arctan\cdots}{\cdots}}{3c^{2/3}}\cdots\right)}{bc-ad}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{b(2bc-5ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{3ad^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^{2/3}-\sqrt[3]{\cdots}\right)}{2\cdots}}{3c^{2/3}}\cdots\right)}{bc-ad}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(bc-ad\right)}$$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) + ((b*(2*b*c - 5*a*d)*(Log[a^(1/3) + b
^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(2/3))))/(b*c - a*d) + (3*a*d^2*(Log[c^(1/3) + d^(1/3)*x]
/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt
[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(1/3)
))/(3*c^(2/3))))/(b*c - a*d))/(3*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{2}}{(ad-bc)^{2}}-\frac{b\left(\frac{(ad-bc)x}{3a\left(b\,x^{3}+a\right)}+\frac{(5ad-2bc)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{(a\cdots}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d^2/(a*d-b*c)^2-b/(a*d-b*c)^2*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+1/3*(5*
a*d-2*b*c)/a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}((2b^2c-5abd)x^3+2abc-5a^2d)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}(abdx^3+a^2d)\left(\frac{d^2}{c^2}\right.}{}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^
(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(
3)*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)
^(2/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b
^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3))
 + 3*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/
3) + c^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d
)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(
d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^
2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(
1/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1
/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^
(1/3))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3)
- 1/18*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(
a/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^
2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2
/3) + a^2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b
^2*c^2*(a/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*
d*log(x + (c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*
d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$$

$$= -\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^3-2\,abc^2d+a^2cd^2)} + \frac{(-cd^2)^{\frac{1}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\,\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}$$
$$+ \frac{(-cd^2)^{\frac{1}{3}}\,d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^3-2\,abc^2d+a^2cd^2)}$$
$$- \frac{(2\,b^2c-5\,abd)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)}$$
$$+ \frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^2c^2-2\,\sqrt{3}a^3bcd+\sqrt{3}a^4d^2\right)}$$
$$+ \frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,bc-5\,(-ab^2)^{\frac{1}{3}}\,ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)}$$
$$+ \frac{bx}{3\,(bx^3+a)(abc-a^2d)}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
-1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/
6*(-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (
-a/b)^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3
)*b*c - 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) +
1/18*(2*(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3
) + (-a/b)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3
+ a)*(a*b*c - a^2*d))
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.20

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

output
```
log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5
*(a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/
3))/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^
2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^
(1/3))/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b
^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*
c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*
d^4 - 4374*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a
^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c
)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 -
(b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^
4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^
3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4
))*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6
*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3)
 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^
3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)} dx = \text{Too large to display}$$

input
```
int(1/(b*x^3+a)^2/(d*x^3+c),x)
```

output
```
(10*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a**2*b*c*d - 4*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c**2 + 10*d**(1/3)*a**(1/3)*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*d*x**3 - 4*d**(1/3)*
a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*c
**2*x**3 - 6*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**
(1/3)*sqrt(3)))*a**3*d**2 - 6*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2
*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*b*d**2*x**3 + 5*d**(1/3)*a**(1/3)*lo
g(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*c*d - 2*d**(1/3)*
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*c**2 +
 5*d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a
*b**2*c*d*x**3 - 2*d**(1/3)*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x +
b**(2/3)*x**2)*b**3*c**2*x**3 - 10*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/
3)*x)*a**2*b*c*d + 4*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c
**2 - 10*d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b**2*c*d*x**3 + 4*
d**(1/3)*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b**3*c**2*x**3 - 3*c**(1/3)*b
**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*d**2 - 3*
c**(1/3)*b**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**2
*b*d**2*x**3 + 6*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**3*d**2 +
6*c**(1/3)*b**(1/3)*log(c**(1/3) + d**(1/3)*x)*a**2*b*d**2*x**3 - 6*d**...
```

3.41 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 419

$$\begin{aligned}\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx &= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} \\
&\quad - \frac{2b^{5/3}(bc-4ad)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3} \\
&\quad - \frac{2d^{5/3}(4bc-ad)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^3} \\
&\quad + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3} \\
&\quad + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^3} \\
&\quad - \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3} \\
&\quad - \frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}\end{aligned}$$

output
```
1/3*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+1/3*b*x/a/(-a*d+b*c)/(b*x^3+a
)/(d*x^3+c)-2/9*b^(5/3)*(-4*a*d+b*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1
/2)/a^(1/3))*3^(1/2)/a^(5/3)/(-a*d+b*c)^3-2/9*d^(5/3)*(-a*d+4*b*c)*arctan(
1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(5/3)/(-a*d+b*c)^3+2/
9*b^(5/3)*(-4*a*d+b*c)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)^3+2/9*d^(5
/3)*(-a*d+4*b*c)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^3-1/9*b^(5/3)*(-
4*a*d+b*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)^3-
1/9*d^(5/3)*(-a*d+4*b*c)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)
/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^2}\,dx = \frac{1}{9}\left(\frac{3b^2x}{a(bc-ad)^2(a+bx^3)}+\frac{3d^2x}{c(bc-ad)^2(c+dx^3)} + \frac{2\sqrt{3}b^{5/3}(bc-4ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(-bc+ad)^3} + \frac{2\sqrt{3}d^{5/3}(-4bc+ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(-bc+4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(-bc+ad)^3} + \frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}(bc-ad)^3} + \frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(-bc+ad)^3} + \frac{d^{5/3}(-4bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

output
```
((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
 d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a
*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) +
 (2*b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x]])/(a^(5/3)*(-(b*c) +
a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x]])/(c^(5/3)*(b*c
 - a*d)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2]])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/(c^(5/3)*(b*c - a*d)^3))/9
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {931, 25, 1024, 27, 1020, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} dx$$

$$\Big\downarrow \; 931$$

$$\frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)} - \frac{\int -\frac{5bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)^2}dx}{3a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5bdx^3+2bc-3ad}{(bx^3+a)(dx^3+c)^2}dx}{3a(bc-ad)} + \frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{6\left(bd(bc+ad)x^3+b^2c^2+a^2d^2-3abcd\right)}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{3c(bc-ad)} + \frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)} + \frac{bx}{3a(a+bx^3)(c+dx^3)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{bd(bc+ad)x^3+b^2c^2+a^2d^2-3abcd}{\left(bx^3+a\right)\left(dx^3+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\int\frac{1}{bx^3+a}dx}{bc-ad}+\frac{ad^2(4bc-ad)\int\frac{1}{dx^3+c}dx}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 750$$

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}}\right)}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}}+\frac{\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}\sqrt[3]{d}}\right)}{bc-ad}\right)}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^3)(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \; 1142$$

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{c(bc-ad)} \cdots$$

$$3a(bc-ad)$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 25

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots}{\cdots}\right)}{\cdots}\right)}{c(bc-ad)} \cdots$$

$$3a(bc-ad)$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{c}\int\frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots\right.}{\cdots}\cdots\right)}{c(bc-ad)}\cdots}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}\cdots}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}\cdots}\cdots\right.}{\cdots}\cdots\right)}{c(bc-ad)}\cdots}{3a(bc-ad)}\cdots$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\cdots}{3c^{2/3}}\cdots\right)}{bc-ad}\cdots\right)}{c(bc-ad)}\cdots$$

$$3a(bc-ad)$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{b^2c(bc-4ad)\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{bc-ad}+\frac{ad^2(4bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{\log\left(c^2\cdots\right.}{\cdots}}{3c^{2/3}}\cdots\right.}{bc-\cdots}\cdots\right.}{c(bc-ad)}\cdots}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)\left(bc-ad\right)}$$

input
```
Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^3)*(c + d*x^3)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^3)) + (2*((b^2*c*(b*c - 4*a*d)*(Log[a^(1/3) + b^(1/3)*x
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(2/3))))/(b*c - a*d) + (a*d^2*(4*b*c - a*d)*(Log[c^(1/3) + d^(1/3
)*x]/(3*c^(2/3)*d^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/
Sqrt[3]])/d^(1/3)) - Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(2*d^(
1/3)))/(3*c^(2/3))))/(b*c - a*d)))/(c*(b*c - a*d)))/(3*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.68

method	result
default	$\frac{d^2\left(\frac{(ad-bc)x}{3c\left(d\,x^3+c\right)}+\frac{2(ad-4bc)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)}{3c}\right)}{(ad-bc)^3}+\frac{b^2\left(\frac{(ad-bc)x}{3a\left(b\,x^3+a\right)}+\frac{2(4aa}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(a*d-b*c)^3*(1/3*(a*d-b*c)/c*x/(d*x^3+c)+2/3*(a*d-4*b*c)/c*(1/3/d/(c/d
)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/
3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))))+b^
2/(a*d-b*c)^3*(1/3*(a*d-b*c)/a*x/(b*x^3+a)+2/3*(4*a*d-b*c)/a*(1/3/b/(a/b)^
(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(341) = 682$.

Time = 47.36 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.14

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/9*(3*(b^3*c^2*d - a^2*b*d^3)*x^4 + 2*sqrt(3)*((b^3*c^2*d - 4*a*b^2*c*d^2
)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d
^2)*x^3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(
3)*b)/b) + 2*sqrt(3)*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^
3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*a
rctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - ((b^3*c^2*d - 4
*a*b^2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c^2*d -
 4*a^2*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) +
 a^2*(b^2/a^2)^(2/3)) - ((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d -
 a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3
)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*((b^3*c^2
*d - 4*a*b^2*c*d^2)*x^6 + a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^3 - 3*a*b^2*c
^2*d - 4*a^2*b*c*d^2)*x^3)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) +
2*((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2
)^(1/3)) + 3*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c
^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^
2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^6 + (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(341) = 682$.

Time = 0.13 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.87

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*
d^2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d
 - a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c
/d)^(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*
c*d^3*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3)
+ 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)
*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3
*d*(c/d)^(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/
9*(b^2*c - 4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^
2*c^2*d*(a/b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2
/9*(4*b*c*d - a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c
^3*d*(c/d)^(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) +
1/3*((b^2*c*d + a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3
*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x
^6 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
-2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*
c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*
b*c^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*
b*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^
3*c^3 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3)
 + 2/3*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*
d + 3*sqrt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b
^2*c - 4*(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^
(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3
))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*
d*x^4 + a*b*d^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3
 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

Mupad [B] (verification not implemented)

Time = 25.74 (sec) , antiderivative size = 3637, normalized size of antiderivative = 8.68

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
4*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d +
b*c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c
^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a
*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*
d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6
 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5
*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3
)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 2
68*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*
c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^
3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^
9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3
 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6
 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^
3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(
c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/
3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c
^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1648, normalized size of antiderivative = 3.93

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)
```

output
```
( - 8*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a**2*b**2*c**3*d - 8*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2
*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b**2*c**2*d**2*x**3 + 2*d**(1/3)*a**
(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*c*
*4 - 6*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*a*b**3*c**3*d*x**3 - 8*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3)
- 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**3*c**2*d**2*x**6 + 2*d**(1/3)*a**
(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**4*c**4
*x**3 + 2*d**(1/3)*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/
3)*sqrt(3)))*b**4*c**3*d*x**6 - 2*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3)
 - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**4*c*d**3 - 2*c**(1/3)*b**(1/3)*sqr
t(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**4*d**4*x**3 + 8
*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3
)))*a**3*b*c**2*d**2 + 6*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(
1/3)*x)/(c**(1/3)*sqrt(3)))*a**3*b*c*d**3*x**3 - 2*c**(1/3)*b**(1/3)*sqrt(
3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**3*b*d**4*x**6 + 8
*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(3
)))*a**2*b**2*c**2*d**2*x**3 + 8*c**(1/3)*b**(1/3)*sqrt(3)*atan((c**(1/3)
- 2*d**(1/3)*x)/(c**(1/3)*sqrt(3)))*a**2*b**2*c*d**3*x**6 - 4*d**(1/3)*a**
(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b**2*c**...
```

3.42 $\int \sqrt{a+bx^3}(c+dx^3)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 324

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{2}{935}\left(187c^2-\frac{4ad(17bc-4ad)}{b^2}\right)x\sqrt{a+bx^3} + \frac{4d(17bc-4ad)x\left(a+bx^3\right)^{3/2}}{187b^2} + \frac{2d^2x^4\left(a+bx^3\right)^{3/2}}{17b}$$
$$+ \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(187b^2c^2-4ad(17bc-4ad)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\right.\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/935*(187*c^2-4*a*d*(-4*a*d+17*b*c)/b^2)*x*(b*x^3+a)^(1/2)+4/187*d*(-4*a*
d+17*b*c)*x*(b*x^3+a)^(3/2)/b^2+2/17*d^2*x^4*(b*x^3+a)^(3/2)/b+2/935*3^(3/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(187*b^2*c^2-4*a*d*(-4*a*d+17*b*c))*(a^(1/3
)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.50

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx$$
$$= \frac{x\sqrt{a+bx^3}\left(40a(14c^2+7cdx^3+2d^2x^6)\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)+3bx^3(11c^2+16cdx^3+ \right.}{560a\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]*(c + d*x^3)^2,x]
```

output
```
(x*Sqrt[a + b*x^3]*(40*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Hypergeometric2F
1[-1/2, 1/3, 10/3, -((b*x^3)/a)] + 3*b*x^3*(11*c^2 + 16*c*d*x^3 + 5*d^2*x^
6)*Hypergeometric2F1[1/2, 4/3, 13/3, -((b*x^3)/a)] + 9*b*x^3*(c + d*x^3)^2
*HypergeometricPFQ[{1/2, 4/3, 2}, {1, 13/3}, -((b*x^3)/a)]))/(560*a*Sqrt[1
 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{2\int \frac{1}{2}\sqrt{bx^3+a}\left(d(23bc-8ad)x^3+c(17bc-2ad)\right)dx}{17b}+\frac{2dx\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)}{17b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{bx^3+a}\left(d(23bc-8ad)x^3+c(17bc-2ad)\right)dx}{17b}+\frac{2dx\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)}{17b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(16a^2d^2-68abcd+187b^2c^2\right)\int \sqrt{bx^3+a}dx}{11b}+\frac{2dx\left(a+bx^3\right)^{3/2}(23bc-8ad)}{11b}}{17b}+\frac{2dx\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)}{17b}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{\left(16a^2d^2-68abcd+187b^2c^2\right)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{11b}+\frac{2dx\left(a+bx^3\right)^{3/2}(23bc-8ad)}{11b}}{17b}+\frac{2dx\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)}{17b}$$

$$\Big\downarrow \ 759$$

$$\frac{\frac{\left(16a^2d^2-68abcd+187b^2c^2\right)\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a+\cdots}\right)}{11b}}{17b}$$

$$\frac{2dx\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)}{17b}$$

input
```
Int[Sqrt[a + b*x^3]*(c + d*x^3)^2,x]
```

output
```
(2*d*x*(a + b*x^3)^(3/2)*(c + d*x^3))/(17*b) + ((2*d*(23*b*c - 8*a*d)*x*(a
 + b*x^3)^(3/2))/(11*b) + ((187*b^2*c^2 - 68*a*b*c*d + 16*a^2*d^2)*((2*x*S
qrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])))/(11*b))/(17*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2x\left(-55b^{2}d^{2}x^{6}-15x^{3}ab\,d^{2}-170x^{3}b^{2}cd+24a^{2}d^{2}-102abcd-187b^{2}c^{2}\right)\sqrt{b\,x^{3}+a}}{935b^{2}}-\frac{2ia\left(16a^{2}d^{2}-68abcd+187b^{2}c^{2}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{}}{}$
elliptic	$\frac{2d^{2}x^{7}\sqrt{b\,x^{3}+a}}{17}+\frac{2\left(\frac{3}{17}a\,d^{2}+2bcd\right)x^{4}\sqrt{b\,x^{3}+a}}{11b}+\frac{2\left(2acd+b\,c^{2}-\frac{8a\left(\frac{3}{17}a\,d^{2}+2bcd\right)}{11b}\right)x\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(a\,c^{2}-\frac{2a\left(2acd+b\,c^{2}-\frac{8a\left(\frac{3}{1}\right.}{}\right)}{5b}\right.}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/935*x*(-55*b^2*d^2*x^6-15*a*b*d^2*x^3-170*b^2*c*d*x^3+24*a^2*d^2-102*a*
b*c*d-187*b^2*c^2)/b^2*(b*x^3+a)^(1/2)-2/935*I*a*(16*a^2*d^2-68*a*b*c*d+18
7*b^2*c^2)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/
2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.36

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{2\left(3\left(187\,ab^2c^2-68\,a^2bcd+16\,a^3d^2\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(55\,b^3d^2x^7+5\left(34\,b^3cd+3\,ab^2\right.\right.\right.}{935\,b^3}$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
2/935*(3*(187*a*b^2*c^2 - 68*a^2*b*c*d + 16*a^3*d^2)*sqrt(b)*weierstrassPI
nverse(0, -4*a/b, x) + (55*b^3*d^2*x^7 + 5*(34*b^3*c*d + 3*a*b^2*d^2)*x^4
+ (187*b^3*c^2 + 102*a*b^2*c*d - 24*a^2*b*d^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.40

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{\sqrt{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(d*x**3+c)**2,x)
```

output
```
sqrt(a)*c**2*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*sqrt(a)*c*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)*d**2*x**7*gamma(7/
3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \sqrt{bx^3+a}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \sqrt{bx^3+a}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \sqrt{b\,x^3+a}\,\left(d\,x^3+c\right)^2\,dx$$

input
```
int((a + b*x^3)^(1/2)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/2)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)^2\,dx$$
$$= \frac{-48\sqrt{b\,x^3+a}\,a^2d^2x+204\sqrt{b\,x^3+a}\,abcdx+30\sqrt{b\,x^3+a}\,ab\,d^2x^4+374\sqrt{b\,x^3+a}\,b^2c^2x+340\sqrt{b\,x^3+a}}{9}$$

input
```
int((b*x^3+a)^(1/2)*(d*x^3+c)^2,x)
```

output
```
( - 48*sqrt(a + b*x**3)*a**2*d**2*x + 204*sqrt(a + b*x**3)*a*b*c*d*x + 30*
sqrt(a + b*x**3)*a*b*d**2*x**4 + 374*sqrt(a + b*x**3)*b**2*c**2*x + 340*sq
rt(a + b*x**3)*b**2*c*d*x**4 + 110*sqrt(a + b*x**3)*b**2*d**2*x**7 + 48*in
t(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3*d**2 - 204*int(sqrt(a + b*x**3)/(a
 + b*x**3),x)*a**2*b*c*d + 561*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a*b**2
*c**2)/(935*b**2)
```

3.43 $\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx=\frac{2(11bc-2ad)x\sqrt{a+bx^3}}{55b}+\frac{2dx\left(a+bx^3\right)^{3/2}}{11b}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a(11bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/55*(-2*a*d+11*b*c)*x*(b*x^3+a)^(1/2)/b+2/11*d*x*(b*x^3+a)^(3/2)/b+2/55*3
^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(-2*a*d+11*b*c)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx = \frac{2x\sqrt{a+bx^3}\left(d\left(a+bx^3\right)+\frac{(11bc-2ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

input
```
Integrate[Sqrt[a + b*x^3]*(c + d*x^3),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(d*(a + b*x^3) + ((11*b*c - 2*a*d)*Hypergeometric2F1[
-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(11bc-2ad)\int \sqrt{bx^3+a}dx}{11b}+\frac{2dx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(11bc-2ad)\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)}{11b}+\frac{2dx\left(a+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{(11bc-2ad)\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a}\right.}{11b}$$

$$\frac{2dx\left(a+bx^3\right)^{3/2}}{11b}$$

input
```
Int[Sqrt[a + b*x^3]*(c + d*x^3),x]
```

output
```
(2*d*x*(a + b*x^3)^(3/2))/(11*b) + ((11*b*c - 2*a*d)*((2*x*Sqrt[a + b*x^3]
)/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.21

method	result
risch	$\frac{2x\left(5bd\,x^{3}+3ad+11bc\right)\sqrt{b\,x^{3}+a}}{55b}+\frac{2ia(2ad-11bc)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b\cdots\right.}{2b}}}\cdots}{\cdots}$
elliptic	$\frac{2d\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{2\left(\frac{3ad}{11}+bc\right)x\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(ac-\frac{2a\left(\frac{3ad}{11}+bc\right)}{5b}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3\left(-a\cdots\right.}{2\cdots}}}\cdots}{\cdots}$
default	$c\left(\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\cdots\right.}}\cdots}{5b\sqrt{b\cdots}}\cdots\right.$

input
```
int((b*x^3+a)^(1/2)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*b*d*x^3+3*a*d+11*b*c)/b*(b*x^3+a)^(1/2)+2/55*I*a*(2*a*d-11*b*c)/
b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.25

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx = \frac{2\left(3\left(11\,abc-2\,a^2d\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(5\,b^2dx^4+\left(11\,b^2c+3\,abd\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c),x, algorithm="fricas")
```

output
```
2/55*(3*(11*a*b*c - 2*a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (
5*b^2*d*x^4 + (11*b^2*c + 3*a*b*d)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(d*x**3+c),x)
```

output
```
sqrt(a)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + sqrt(a)*d*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx = \int \sqrt{bx^3+a}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(d*x^3 + c), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx=\int \sqrt{bx^3+a}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/2)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(1/2)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/2)*(c + d*x^3), x)
```

Reduce [F]

$$\int \sqrt{a+bx^3}\left(c+dx^3\right)\,dx=\frac{6\sqrt{b\,x^3+a}\,adx+22\sqrt{b\,x^3+a}\,bcx+10\sqrt{b\,x^3+a}\,bd\,x^4-6\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2d+33\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)abc}{55b}$$

input
```
int((b*x^3+a)^(1/2)*(d*x^3+c),x)
```

output
```
(6*sqrt(a + b*x**3)*a*d*x + 22*sqrt(a + b*x**3)*b*c*x + 10*sqrt(a + b*x**3
)*b*d*x**4 - 6*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**2*d + 33*int(sqrt(a
 + b*x**3)/(a + b*x**3),x)*a*b*c)/(55*b)
```

3.44 $\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \frac{x\sqrt{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/2)*AppellF1(1/3,-1/2,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{8acx\sqrt{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(8ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(-2ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\text{A}\right.\right.}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c + d*x^3),x]
```

output
```
(8*a*c*x*Sqrt[a + b*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/((c + d*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + 3*x^3*(-2*a*d*AppellF1[4/3, -1/2, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{a+bx^3} \int \frac{\sqrt{\frac{bx^3}{a}+1}}{dx^3+c} dx}{\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^3} \, \text{AppellF1}\left(\frac{1}{3}, -\frac{1}{2}, 1, \frac{4}{3}, -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^3]/(c + d*x^3),x]
```

output
```
(x*Sqrt[a + b*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(c*Sqrt[1 + (b*x^3)/a])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.11 (sec) , antiderivative size = 723, normalized size of antiderivative = 12.25

method	result	size
default	Expression too large to display	723
elliptic	Expression too large to display	723

input
```
int((b*x^3+a)^(1/2)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-1/3*I/d/b^2*2^(1/2)*sum((-a*d+b*c)/_alpha^2/(a*d-b*c)*(-a*b^2)^(1/3)*(1
/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))
^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/
3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I*
3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3
^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)
^(2/3)*_alpha-3*a*b)/(a*d-b*c),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/2)/(d*x**3+c),x)
```

output
```
Integral(sqrt(a + b*x**3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt{bx^3+a}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt{bx^3+a}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt{b\,x^3+a}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/2)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt{b\,x^3+a}}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^(1/2)/(d*x^3+c),x)
```

output
```
int(sqrt(a + b*x**3)/(c + d*x**3),x)
```

3.45 $\int \frac{\sqrt{a+bx^3}}{\left(c+dx^3\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{a+bx^3}}{\left(c+dx^3\right)^2} \, dx = \frac{x\sqrt{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/2)*AppellF1(1/3,-1/2,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/
a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{a+bx^3}}{\left(c+dx^3\right)^2} \, dx$$
$$= \frac{x\left(\frac{bx^3\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{8\left(\frac{a+bx^3}{c}+\frac{16a^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(2ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.}\right.}{c+dx^3}\right.}{24\sqrt{a+bx^3}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c + d*x^3)^2,x]
```

output
```
(x*((b*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -(
(d*x^3)/c)])/c^2 + (8*((a + b*x^3)/c + (16*a^2*AppellF1[1/3, 1/2, 1, 4/3,
-((b*x^3)/a), -((d*x^3)/c)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a
), -((d*x^3)/c)] - 3*x^3*(2*a*d*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -
((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])
)))/(c + d*x^3)))/(24*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2} \, dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{a+bx^3} \int \frac{\sqrt{\frac{bx^3}{a}+1}}{(dx^3+c)^2}dx}{\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^3]/(c + d*x^3)^2,x]
```

output
```
(x*Sqrt[a + b*x^3]*AppellF1[1/3, -1/2, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(c^2*Sqrt[1 + (b*x^3)/a])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.26 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.76

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int((b*x^3+a)^(1/2)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x/c*(b*x^3+a)^(1/2)/(d*x^3+c)-1/9*I/c/d*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/18*I/c/d/b^2*2^(1/2)*sum((-4*a*d+b
*c)/_alpha^2/(a*d-b*c)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*
3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(
-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a
*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*
b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/(a*d-b*c),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{\left(c+dx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/2)/(d*x**3+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt{bx^3+a}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt{bx^3+a}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx=\int \frac{\sqrt{b\,x^3+a}}{{\left(d\,x^3+c\right)}^2}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/2)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(c+dx^3)^2}\,dx=\int \frac{\sqrt{b\,x^3+a}}{d^2x^6+2cd\,x^3+c^2}dx$$

input
```
int((b*x^3+a)^(1/2)/(d*x^3+c)^2,x)
```

output
```
int(sqrt(a + b*x**3)/(c**2 + 2*c*d*x**3 + d**2*x**6),x)
```

3.46 $\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 285

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \frac{4d(11bc-4ad)x\sqrt{a+bx^3}}{55b^2} + \frac{2d^2x^4\sqrt{a+bx^3}}{11b}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(55b^2c^2-4ad(11bc-4ad))\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
4/55*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(1/2)/b^2+2/11*d^2*x^4*(b*x^3+a)^(1/2)/
b+2/165*(1/2*6^(1/2)+1/2*2^(1/2))*(55*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(a^(1
/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.80 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(40a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{10}{3},-\frac{bx^3}{a}\right)-3bx^3(11c^2+16cdx^3+5d\right.}{560a\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x^3)^2/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(40*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Hypergeometr
ic2F1[1/3, 1/2, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^3 + 5*d^2
*x^6)*Hypergeometric2F1[4/3, 3/2, 13/3, -((b*x^3)/a)] - 9*b*x^3*(c + d*x^3
)^2*HypergeometricPFQ[{4/3, 3/2, 2}, {1, 13/3}, -((b*x^3)/a)]))/(560*a*Sqr
t[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 27, 913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{2\int \frac{d(17bc-8ad)x^3+c(11bc-2ad)}{2\sqrt{bx^3+a}}dx}{11b}+\frac{2dx\sqrt{a+bx^3}(c+dx^3)}{11b}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(17bc-8ad)x^3+c(11bc-2ad)}{\sqrt{bx^3+a}}dx}{11b}+\frac{2dx\sqrt{a+bx^3}(c+dx^3)}{11b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(16a^2d^2-44abcd+55b^2c^2)\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}+\frac{2dx\sqrt{a+bx^3}(17bc-8ad)}{5b}}{11b}+\frac{2dx\sqrt{a+bx^3}(c+dx^3)}{11b}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(16a^2d^2-44abcd+55b^2c^2)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2dx\sqrt{a-}}{}}{11b}$$

$$\frac{2dx\sqrt{a+bx^3}(c+dx^3)}{11b}$$

input
```
Int[(c + d*x^3)^2/Sqrt[a + b*x^3],x]
```

output
```
(2*d*x*Sqrt[a + b*x^3]*(c + d*x^3))/(11*b) + ((2*d*(17*b*c - 8*a*d)*x*Sqrt
[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(55*b^2*c^2 - 44*a*b*c*d + 16*a^
2*d^2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]
])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(11*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{2xd(-5bd\,x^3+8ad-22bc)\sqrt{b\,x^3+a}}{55b^2}-\frac{2i(16a^2d^2-44abcd+55b^2c^2)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(}{}}}}{}$
elliptic	$\frac{2d^2x^4\sqrt{b\,x^3+a}}{11b}+\frac{2\left(2cd-\frac{8d^2a}{11b}\right)x\sqrt{b\,x^3+a}}{5b}-\frac{2i\left(c^2-\frac{2a\left(2cd-\frac{8d^2a}{11b}\right)}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{}}{}$
default	Expression too large to display

input

```
int((d*x^3+c)^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/55*x*d*(-5*b*d*x^3+8*a*d-22*b*c)/b^2*(b*x^3+a)^(1/2)-2/165*I*(16*a^2*d^
2-44*a*b*c*d+55*b^2*c^2)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.29

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \frac{2\left((55\,b^2c^2-44\,abcd+16\,a^2d^2)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(5\,b^2d^2x^4+2\,(11\,b^2cd-4\,abd^2)x)\sqrt{}\right.}{55\,b^3}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/55*((55*b^2*c^2 - 44*a*b*c*d + 16*a^2*d^2)*sqrt(b)*weierstrassPInverse(0
, -4*a/b, x) + (5*b^2*d^2*x^4 + 2*(11*b^2*c*d - 4*a*b*d^2)*x)*sqrt(b*x^3 +
 a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(1/2),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((
1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^2}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{-16\sqrt{b\,x^3+a}\,a\,d^2x + 44\sqrt{b\,x^3+a}\,bcdx + 10\sqrt{b\,x^3+a}\,b\,d^2x^4 + 16\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2d^2 - 44\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right.}{55b^2}$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/2),x)
```

output

```
( - 16*sqrt(a + b*x**3)*a*d**2*x + 44*sqrt(a + b*x**3)*b*c*d*x + 10*sqrt(a
 + b*x**3)*b*d**2*x**4 + 16*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**2*d**2
 - 44*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a*b*c*d + 55*int(sqrt(a + b*x**
3)/(a + b*x**3),x)*b**2*c**2)/(55*b**2)
```

3.47 $\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \frac{2dx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/5*d*x*(b*x^3+a)^(1/2)/b+2/15*(1/2*6^(1/2)+1/2*2^(1/2))*(-2*a*d+5*b*c)*(a
^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \frac{2dx(a+bx^3)+(5bc-2ad)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*d*x*(a + b*x^3) + (5*b*c - 2*a*d)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 913$$

$$\frac{(5bc-2ad)\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}+\frac{2dx\sqrt{a+bx^3}}{5b}$$

$$\Big\downarrow \ 759$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5bc-2ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2dx\sqrt{a+bx^3}}{5b}$$

input
```
Int[(c + d*x^3)/Sqrt[a + b*x^3],x]
```

output
```
(2*d*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*b*c - 2*a*d)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4
)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

method	result
risch	$\frac{2dx\sqrt{b\,x^3+a}}{5b}+\frac{2i(2ad-5bc)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2}{2b}\right.}{}}}{15b^2}$
elliptic	$\frac{2dx\sqrt{b\,x^3+a}}{5b}-\frac{2i\left(c-\frac{2ad}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)}{2b}\right.}{}}}{3b\sqrt{}}$
default	$-\frac{2ic\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^3+a}}$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*d*x*(b*x^3+a)^(1/2)/b+2/15*I*(2*a*d-5*b*c)/b^2*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.17

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}bdx + (5\,bc-2\,ad)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a)*b*d*x + (5*b*c - 2*a*d)*sqrt(b)*weierstrassPInverse(0
, -4*a/b, x))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{dx^3+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{dx^3+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{d\,x^3+c}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,dx-2\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)ad+5\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)bc}{5b}$$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*d*x - 2*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a*d + 5*i
nt(sqrt(a + b*x**3)/(a + b*x**3),x)*b*c)/(5*b)
```

3.48 $\int \frac{1}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 207

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3
^(3/4)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

input
```
Integrate[1/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqr
t[a + b*x^3]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 759$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[1/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.37

method	result
default	$-\frac{2 i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\, b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\, b^{2}\right)^{\frac{1}{3}}}}}{3 b \sqrt{b\, x^{3}+a}}$
elliptic	$-\frac{2 i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\, b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\, b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\, b^{2}\right)^{\frac{1}{3}}}}}{3 b \sqrt{b\, x^{3}+a}}$

input
```
int(1/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

input
```
int(1/(a + b*x^3)^(1/2),x)
```

output
```
(x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx$$

input
```
int(1/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)
```

3.49 $\int \frac{1}{\sqrt{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 773

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{2(-1)^{2/3}\left(-\sqrt[3]{a}b^{2/3}c^{2/3}-a^{2/3}\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{d}-ad^{2/3}\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\ \text{EllipticPi}\left(\frac{(1+\sqrt[3]{}}{-\sqrt[3]{b}}\right.}{\sqrt{3}\left(1+\sqrt[3]{-1}\right)c^{2/3}(bc-ad)\sqrt{a+bx^3}}$$

$$+\frac{2(-1)^{2/3}\sqrt[3]{a}\left(-\sqrt[3]{b}\sqrt[3]{c}+\sqrt[3]{a}\sqrt[3]{d}\right)\left(\sqrt[3]{b}\sqrt[3]{c}+\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\ \text{Elliptic}}{\sqrt{3}\left(1+\sqrt[3]{-1}\right)c^{2/3}(bc-ad)\sqrt{a+bx^3}}$$

$$+\frac{2(-1)^{2/3}\sqrt[3]{a}\left(-\sqrt[3]{b}\sqrt[3]{c}+\sqrt[3]{a}\sqrt[3]{d}\right)\left(\sqrt[3]{b}\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{a}\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\ \text{Ellipt}}{\sqrt{3}\left(1+\sqrt[3]{-1}\right)c^{2/3}(bc-ad)\sqrt{a+bx}}$$

output
```
2/3*(-1)^(2/3)*(-a^(1/3)*b^(2/3)*c^(2/3)-a^(2/3)*b^(1/3)*c^(1/3)*d^(1/3)-a
*d^(2/3))*((a^(1/3)+b^(1/3)*x)/(1+(-1)^(1/3))/a^(1/3))^(1/2)*(1-b^(1/3)*x/
a^(1/3)+b^(2/3)*x^2/a^(2/3))^(1/2)*EllipticPi(((a^(1/3)+(-1)^(2/3)*b^(1/3)
*x)/(1+(-1)^(1/3))/a^(1/3))^(1/2),(1+(-1)^(1/3))*a^(1/3)*d^(1/3)/(-b^(1/3)
*c^(1/3)+a^(1/3)*d^(1/3)),(-1)^(1/6))*3^(1/2)/(1+(-1)^(1/3))/c^(2/3)/(-a*d
+b*c)/(b*x^3+a)^(1/2)+2/3*(-1)^(2/3)*a^(1/3)*(-b^(1/3)*c^(1/3)+a^(1/3)*d^(
1/3))*(b^(1/3)*c^(1/3)+(-1)^(1/3)*a^(1/3)*d^(1/3))*((a^(1/3)+b^(1/3)*x)/(1
+(-1)^(1/3))/a^(1/3))^(1/2)*(1-b^(1/3)*x/a^(1/3)+b^(2/3)*x^2/a^(2/3))^(1/2
)*EllipticPi(((a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(1+(-1)^(1/3))/a^(1/3))^(1/2)
,(1+(-1)^(1/3))*a^(1/3)*d^(1/3)/((-1)^(1/3)*b^(1/3)*c^(1/3)+a^(1/3)*d^(1/3
)),(-1)^(1/6))*3^(1/2)/(1+(-1)^(1/3))/c^(2/3)/(-a*d+b*c)/(b*x^3+a)^(1/2)+2
/3*(-1)^(2/3)*a^(1/3)*(-b^(1/3)*c^(1/3)+a^(1/3)*d^(1/3))*(b^(1/3)*c^(1/3)-
(-1)^(2/3)*a^(1/3)*d^(1/3))*((a^(1/3)+b^(1/3)*x)/(1+(-1)^(1/3))/a^(1/3))^(
1/2)*(1-b^(1/3)*x/a^(1/3)+b^(2/3)*x^2/a^(2/3))^(1/2)*EllipticPi(((a^(1/3)+
(-1)^(2/3)*b^(1/3)*x)/(1+(-1)^(1/3))/a^(1/3))^(1/2),I*3^(1/2)*a^(1/3)*d^(1
/3)/(b^(1/3)*c^(1/3)+(-1)^(1/3)*a^(1/3)*d^(1/3)),(-1)^(1/6))*3^(1/2)/(1+(-
1)^(1/3))/c^(2/3)/(-a*d+b*c)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx =$$

$$-\frac{8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\,(c+dx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(2ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx}{c}\right.\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*(c + d*x^3)),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a
+ b*x^3]*(c + d*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + 3*x^3*(2*a*d*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + b*c*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}(dx^3+c)}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt{a+bx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*(c + d*x^3)),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.55

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}+i\cdots}{\cdots}\right)}{2\left(-a\,b^2\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+c\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}+i\cdots}{\cdots}\right)}{2\left(-a\,b^2\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*I/b^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/
b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1
/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1
/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I*3^(1/2)*(-a*b^
2)^(2/3)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^
2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-
3*a*b)/(a*d-b*c),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,\left(d\,x^3+c\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{\sqrt{b\,x^3+a}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c),x)
```

output
```
int(sqrt(a + b*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)
```

3.50 $\int \frac{1}{\sqrt{a+bx^3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt{a+bx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*AppellF1(1/3,1/2,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(59) = 118.

Time = 10.29 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.68

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx$$
$$=\frac{x\left(\frac{bdx^3\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{-bc+ad}+\frac{c\left(64ac(-3bc+3ad+bdx^3)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-24dx^3(a+bx^3)\left(2ad\,\text{App}\right.\right.}{(bc-ad)(c+dx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(2ad\,\text{AppellF1}\left(\right.\right.\right.}\right.}{24c^2\sqrt{a+bx^3}}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*(c + d*x^3)^2),x]
```

output
```
(x*((b*d*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/(-(b*c) + a*d) + (c*(64*a*c*(-3*b*c + 3*a*d + b*d*x^3)*Appe
llF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 24*d*x^3*(a + b*x^3)*
(2*a*d*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appell
F1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)*(c + d*x^
3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*
(2*a*d*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appell
F1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(24*c^2*Sqrt[a + b*x
^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\left(c+dx^3\right)^2}dx$$

$$\Big\downarrow \ 937$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}(dx^3+c)^2}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 936$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt{a+bx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*(c + d*x^3)^2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.24 (sec) , antiderivative size = 769, normalized size of antiderivative = 13.03

method	result	size
default	Expression too large to display	769
elliptic	Expression too large to display	769

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*d/(a*d-b*c)*x*(b*x^3+a)^(1/2)/(d*x^3+c)-1/9*I/c/(a*d-b*c)*3^(1/2)*(-
a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/18*I/b^2/c*2^(
1/2)*sum((-4*a*d+7*b*c)/(a*d-b*c)^2/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x
-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(
-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I*3^(1/2)*(-a*
b^2)^(2/3)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alph
a^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alph
a-3*a*b)/(a*d-b*c),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^2,x)
```

output
```
int(sqrt(a + b*x**3)/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x**6 + b*c**2*x**3 +
2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.51 $\int \frac{1}{\sqrt{a+bx^3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt{a+bx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*AppellF1(1/3,1/2,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 420 vs. $2(59) = 118$.

Time = 10.53 (sec) , antiderivative size = 420, normalized size of antiderivative = 7.12

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx$$
$$=\frac{x\left(bd(-19bc+10ad)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{8c\left(-8ac(6a^2d^2(6c+5dx^3)+b^2c(36c^2+11cdx^3\right.}{}\right.}{}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*(c + d*x^3)^3),x]
```

output
```
(x*(b*d*(-19*b*c + 10*a*d)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + (8*c*(-8*a*c*(6*a^2*d^2*(6*c + 5*d*x^3)
+ b^2*c*(36*c^2 + 11*c*d*x^3 - 19*d^2*x^6) + 2*a*b*d*(-36*c^2 - 25*c*d*x^3
 + 5*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*
d*x^3*(a + b*x^3)*(2*a*d*(8*c + 5*d*x^3) - b*c*(25*c + 19*d*x^3))*(2*a*d*A
ppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3,
3/2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-8*a*c*AppellF
1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(2*a*d*AppellF1[4/
3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])))))/(288*c^3*(b*c - a*d)^2*Sqrt[a + b*x^3]
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\left(dx^3+c\right)^3}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt{a+bx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*(c + d*x^3)^3),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.68 (sec) , antiderivative size = 836, normalized size of antiderivative = 14.17

method	result	size
default	Expression too large to display	836
elliptic	Expression too large to display	836

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/c*d/(a*d-b*c)*x*(b*x^3+a)^(1/2)/(d*x^3+c)^2+1/36*d*(10*a*d-19*b*c)/(a*
d-b*c)^2/c^2*x*(b*x^3+a)^(1/2)/(d*x^3+c)-1/108*I*(10*a*d-19*b*c)/(a*d-b*c)
^2/c^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
-1/216*I/b^2/c^2*2^(1/2)*sum((-40*a^2*d^2+104*a*b*c*d-91*b^2*c^2)/(a*d-b*c
)^3/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-
a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3
)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*
b^2)^(1/3)*_alpha*3^(1/2)*b-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*b^2-(-a*b^
2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/(a*d-b*c),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^3}\,dx$$
$$= \int \frac{\sqrt{b\,x^3+a}}{b\,d^3x^{12}+a\,d^3x^9+3bc\,d^2x^9+3ac\,d^2x^6+3b\,c^2d\,x^6+3a\,c^2d\,x^3+b\,c^3x^3+a\,c^3}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^3,x)
```

output
```
int(sqrt(a + b*x**3)/(a*c**3 + 3*a*c**2*d*x**3 + 3*a*c*d**2*x**6 + a*d**3*
x**9 + b*c**3*x**3 + 3*b*c**2*d*x**6 + 3*b*c*d**2*x**9 + b*d**3*x**12),x)
```

3.52 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 290

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \frac{2(bc-ad)^2x}{3ab^2\sqrt{a+bx^3}} + \frac{2d^2x\sqrt{a+bx^3}}{5b^2}$$

$$+\frac{2\sqrt{2+\sqrt{3}}(5b^2c^2+20abcd-16a^2d^2)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{15\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
2/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(1/2)+2/5*d^2*x*(b*x^3+a)^(1/2)/b^2+2/4
5*(1/2*6^(1/2)+1/2*2^(1/2))*(-16*a^2*d^2+20*a*b*c*d+5*b^2*c^2)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(40a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{10}{3},-\frac{bx^3}{a}\right)-9bx^3(\right.}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(40*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Hypergeometr
ic2F1[1/3, 3/2, 10/3, -((b*x^3)/a)] - 9*b*x^3*(11*c^2 + 16*c*d*x^3 + 5*d^2
*x^6)*Hypergeometric2F1[4/3, 5/2, 13/3, -((b*x^3)/a)] - 27*b*x^3*(c + d*x^
3)^2*HypergeometricPFQ[{4/3, 2, 5/2}, {1, 13/3}, -((b*x^3)/a)]))/(560*a^2*
Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 27, 913, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{2\int \frac{c(bc+2ad)-d(5bc-8ad)x^3}{2\sqrt{bx^3+a}}dx}{3ab}+\frac{2x(c+dx^3)(bc-ad)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{c(bc+2ad)-d(5bc-8ad)x^3}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2x(c+dx^3)(bc-ad)}{3ab\sqrt{a+bx^3}}$$

$$\downarrow 913$$

$$\frac{\frac{(-16a^2d^2+20abcd+5b^2c^2)\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}-\frac{2dx\sqrt{a+bx^3}(5bc-8ad)}{5b}}{3ab}+\frac{2x(c+dx^3)(bc-ad)}{3ab\sqrt{a+bx^3}}$$

$$\downarrow 759$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(-16a^2d^2+20abcd+5b^2c^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2dx\sqrt{a}\cdots}{}}{3ab}$$

$$\frac{2x(c+dx^3)(bc-ad)}{3ab\sqrt{a+bx^3}}$$

input

```
Int[(c + d*x^3)^2/(a + b*x^3)^(3/2),x]
```

output

```
(2*(b*c - a*d)*x*(c + d*x^3))/(3*a*b*Sqrt[a + b*x^3]) + ((-2*d*(5*b*c - 8*
a*d)*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*b^2*c^2 + 20*a*b*c
*d - 16*a^2*d^2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a*b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.37

method	result
elliptic	$\frac{2x\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{3b^{2}a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2d^{2}x\sqrt{b\,x^{3}+a}}{5b^{2}}-\frac{2i\left(-\frac{d(ad-2bc)}{b^{2}}+\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{3a\,b^{2}}-\frac{2a\,d^{2}}{5b^{2}}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\right.}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

input
```
int((d*x^3+c)^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^2*x/a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((x^3+a/b)*b)^(1/2)+2/5*d^2*x*(b*x
^3+a)^(1/2)/b^2-2/3*I*(-d*(a*d-2*b*c)/b^2+1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
a/b^2-2/5*a/b^2*d^2)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\left(5\,ab^2c^2+20\,a^2bcd-16\,a^3d^2+\left(5\,b^3c^2+20\,ab^2cd-16\,a^2bd^2\right)x^3\right)\sqrt{b}\text{weierstrassPIn}}{15\,(ab^4x^3+a^2}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/15*((5*a*b^2*c^2 + 20*a^2*b*c*d - 16*a^3*d^2 + (5*b^3*c^2 + 20*a*b^2*c*d
 - 16*a^2*b*d^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (3*a*b^2
*d^2*x^4 + (5*b^3*c^2 - 10*a*b^2*c*d + 8*a^2*b*d^2)*x)*sqrt(b*x^3 + a))/(a
*b^4*x^3 + a^2*b^3)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(3/2),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{3/2}}\,dx = \frac{16\sqrt{b\,x^3+a}\,a\,d^2x-20\sqrt{b\,x^3+a}\,bcdx+2\sqrt{b\,x^3+a}\,b\,d^2x^4-16\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(3/2),x)
```

output
```
(16*sqrt(a + b*x**3)*a*d**2*x - 20*sqrt(a + b*x**3)*b*c*d*x + 2*sqrt(a + b
*x**3)*b*d**2*x**4 - 16*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**
6),x)*a**3*d**2 + 20*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),
x)*a**2*b*c*d - 16*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
*a**2*b*d**2*x**3 + 5*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6)
,x)*a*b**2*c**2 + 20*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),
x)*a*b**2*c*d*x**3 + 5*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6
),x)*b**3*c**2*x**3)/(5*b**2*(a + b*x**3))
```

3.53 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{3/2}} \, dx = \frac{2(bc-ad)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(bc+2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(-a*d+b*c)*x/a/b/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(2*a*d+
b*c)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(4/3)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(2bc-2ad+(bc+2ad)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(x*(2*b*c - 2*a*d + (b*c + 2*a*d)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
3, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {910, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(2ad+bc)\int \frac{1}{\sqrt{bx^3+a}}dx}{3ab}+\frac{2x(bc-ad)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(bc-ad)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(b*c + 2*
a*d)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
elliptic	$-\frac{2x(ad-bc)}{3ba\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\left(\frac{d}{b}-\frac{ad-bc}{3ba}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x-\right.}{}}}{}$
default	$c\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\right.}{(-}}}{9ab}\right.$

input
```
int((d*x^3+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*x/a*(a*d-b*c)/((x^3+a/b)*b)^(1/2)-2/3*I*(d/b-1/3*(a*d-b*c)/b/a)*3^(
1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{bx^3+a}(b^2c-abd)x+((b^2c+2\,abd)x^3+abc+2\,a^2d)\sqrt{b}\text{weierstrassPInverse}(0,\right.}{3\,(ab^3x^3+a^2b^2)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*x^3 + a)*(b^2*c - a*b*d)*x + ((b^2*c + 2*a*b*d)*x^3 + a*b*c +
2*a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(3/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,dx + 2\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2d + \left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)abc + 2\left(\int \frac{\sqrt{b\,x}}{b^2x^6+2a}\right.}{b\,(b\,x^3+a)}$$

input
```
int((d*x^3+c)/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*d*x + 2*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b
**2*x**6),x)*a**2*d + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6)
,x)*a*b*c + 2*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*
d*x**3 + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*b**2*c*x*
*3)/(b*(a + b*x**3))
```

3.54 $\int \frac{1}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 232

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{2x}{3a\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x/a/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2
)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.24

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(2+\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3a\sqrt{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(-3/2),x]
```

output
```
(x*(2 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
))/(3*a*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 749$$

$$\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 759$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}$$

input
```
Int[(a + b*x^3)^(-3/2),x]
```

output
```
(2*x)/(3*a*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32

method	result
default	$\frac{2x}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^{2}\right)}}}{9ab\sqrt{b\,x}}$
elliptic	$\frac{2x}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^{2}\right)}}}{9ab\sqrt{b\,x}}$

input
```
int(1/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x/a/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{bx^3+a}bx + (bx^3+a)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{3\,(ab^2x^3+a^2b)}$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*x^3 + a)*b*x + (b*x^3 + a)*sqrt(b)*weierstrassPInverse(0, -4*a
/b, x))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(3/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(\frac{b\,x^3}{a}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{(b\,x^3+a)^{3/2}}$$

input
```
int(1/(a + b*x^3)^(3/2),x)
```

output
```
(x*((b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(1/(b*x^3+a)^(3/2),x)
```

output

```
int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.55 $\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)} \, dx = \frac{x\sqrt{1+\frac{bx^3}{a}} \operatorname{AppellF1}\left(\frac{1}{3},\frac{3}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{a+bx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*AppellF1(1/3,3/2,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 330 vs. 2(62) = 124.

Time = 10.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.32

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)} \, dx = \frac{x\left(-\frac{bdx^3\sqrt{1+\frac{bx^3}{a}} \operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + \frac{8\left(4ac(-3bc+3ad-2bdx^3) \operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\right.\right.}{(c+dx^3)\left(8ac \operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3}\right.\right.}\right.}{12a(-bc}$$

input
```
Integrate[1/((a + b*x^3)^(3/2)*(c + d*x^3)),x]
```

output
```
(x*(-((b*d*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])/c) + (8*(4*a*c*(-3*b*c + 3*a*d - 2*b*d*x^3)*AppellF1[1/3,
 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*b*x^3*(c + d*x^3)*(2*a*d*App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/
2, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(8*a*c*AppellF1[1/3
, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(2*a*d*AppellF1[4/3, 1/
2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(12*a*(-(b*c) + a*d)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}(dx^3+c)}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{a+bx^3}}$$

input
```
Int[1/((a + b*x^3)^(3/2)*(c + d*x^3)),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 3/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.04 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.15

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b*x/a/(a*d-b*c)/((x^3+a/b)*b)^(1/2)+2/9*I/(a*d-b*c)/a*3^(1/2)*(-a*b^2
)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/b^2*d*2^(1/2)*s
um(1/(a*d-b*c)^2/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3)
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*(
(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha
^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/(a*d-b*c),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{2}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/2)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(3/2)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{3/2}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*c + a**2*d*x**3 + 2*a*b*c*x**3 + 2*a*b*d*x**6 +
 b**2*c*x**6 + b**2*d*x**9),x)
```

3.56 $\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\sqrt{a+bx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*AppellF1(1/3,3/2,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3
+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(62) = 124.

Time = 10.53 (sec) , antiderivative size = 380, normalized size of antiderivative = 6.13

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \frac{x\left(bd(2bc+ad)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(64ac(3a^2d^2\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(3/2)*(c + d*x^3)^2),x]
```

output
```
(x*(b*d*(2*b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + (c*(64*a*c*(3*a^2*d^2 + a*b*d*(-6*c + d*x^3)
+ b^2*c*(3*c + 2*d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 24*x^3*(a^2*d^2 + a*b*d^2*x^3 + 2*b^2*c*(c + d*x^3))*(2*a*d*Appell
F1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(8*a*c*AppellF1[1/3, 1
/2, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(2*a*d*AppellF1[4/3, 1/2,
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1, 7/3, -((b*
x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}(dx^3+c)^2}dx}{a\sqrt{a+bx^3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\sqrt{a+bx^3}}$$

input
```
Int[1/((a + b*x^3)^(3/2)*(c + d*x^3)^2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 3/2, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c^2*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 1.30 (sec) , antiderivative size = 830, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	830
elliptic	Expression too large to display	830

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*d^2/(a*d-b*c)^2*x*(b*x^3+a)^(1/2)/(d*x^3+c)+2/3*b^2*x/a/(a*d-b*c)^2/
((x^3+a/b)*b)^(1/2)-2/3*I*(1/6*b*d/(a*d-b*c)^2/c+1/3*b^2/(a*d-b*c)^2/a)*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/18*I
/c/b^2*d*2^(1/2)*sum((-4*a*d+13*b*c)/(a*d-b*c)^3/_alpha^2*(-a*b^2)^(1/3)*(
1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3)
)^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1
/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b-I
*3^(1/2)*(-a*b^2)^(2/3)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*d*(2*I*(-a*b^2)^(1/3)*
3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2
)^(2/3)*_alpha-3*a*b)/(a*d-b*c),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{2}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/2)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(3/2)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{3/2}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^2}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x^3}$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^2,x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*c**2 + 2*a**2*c*d*x**3 + a**2*d**2*x**6 + 2*a*b
*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c**2*x**6 + 2*b**2*c*
d*x**9 + b**2*d**2*x**12),x)
```

3.57 $\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^2x\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{5}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c^2*x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,1/2,-5/2,4/3,-b*x^3/a
,-d*x^3/c)/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 323 vs. 2(86) = 172.

Time = 4.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.76

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \frac{x\Big(8d(a+bx^3)(c+dx^3)(31bc-11ad+8bdx^3)+d(181b^2c^2-164abcd+55a^2d^2)x^3 \cdots}{\cdots}$$

input
```
Integrate[(c + d*x^3)^(5/2)/Sqrt[a + b*x^3],x]
```

output
```
(x*(8*d*(a + b*x^3)*(c + d*x^3)*(31*b*c - 11*a*d + 8*b*d*x^3) + d*(181*b^2
*c^2 - 164*a*b*c*d + 55*a^2*d^2)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/
c]*AppellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + (64*a*c^2*(5
6*b^2*c^2 - 31*a*b*c*d + 11*a^2*d^2)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3
)/a), -((d*x^3)/c)])/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((
d*x^3)/c)] - 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^
3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/
(448*b^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{5/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{\left(dx^3+c\right)^{5/2}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{c^2\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{5/2}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{c^2x\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{5}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(5/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 1/2, -5/2, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{5}{2}}}{\sqrt{b\,x^{3}+a}}dx$$

input
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x)
```

output
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*sqrt(d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(c+dx^3)^{\frac{5}{2}}}{\sqrt{a+bx^3}}\,dx$$

input
```
integrate((d*x**3+c)**(5/2)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((c + d*x**3)**(5/2)/sqrt(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(5/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(5/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^{5/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x^3)^(5/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x^3)^(5/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{5/2}}{\sqrt{a+bx^3}}\,dx = \frac{-22\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,a\,d^2x+62\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,bcdx+16\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{}$$

input
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(1/2),x)
```

output
```
( - 22*sqrt(c + d*x**3)*sqrt(a + b*x**3)*a*d**2*x + 62*sqrt(c + d*x**3)*sq
rt(a + b*x**3)*b*c*d*x + 16*sqrt(c + d*x**3)*sqrt(a + b*x**3)*b*d**2*x**4
+ 55*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x*
*3 + b*d*x**6),x)*a**2*d**3 - 164*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x
**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c*d**2 + 181*int((sqrt(
c + d*x**3)*sqrt(a + b*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),
x)*b**2*c**2*d + 22*int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c + a*d*x**
3 + b*c*x**3 + b*d*x**6),x)*a**2*c*d**2 - 62*int((sqrt(c + d*x**3)*sqrt(a
+ b*x**3))/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*c**2*d + 112*int(
(sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6)
,x)*b**2*c**3)/(112*b**2)
```

3.58 $\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}} \, dx = \frac{cx\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,1/2,-3/2,4/3,-b*x^3/a,-
d*x^3/c)/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 374 vs. 2(84) = 168.

Time = 3.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 4.45

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}} \, dx = \frac{x\left(-d(-11bc+5ad)x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{8\left(-8ac\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x^3)^(3/2)/Sqrt[a + b*x^3],x]
```

output
```
(x*(-(d*(-11*b*c + 5*a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*Appe
llF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) + (8*(-8*a*c*(a*d^2*
x^3 + b*(4*c^2 + c*d*x^3 + d^2*x^6))*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3
)/a), -((d*x^3)/c)] + 3*d*x^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1
/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])))/(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^
3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d
*x^3)/c)]))))/(32*b*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{(dx^3+c)^{3/2}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{c\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/Sqrt[a + b*x^3],x]
```

output
```
(c*x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 1/2, -3/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{3}{2}}}{\sqrt{b\,x^{3}+a}}dx$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x)
```

output
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)^(3/2)/sqrt(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{\sqrt{a+bx^3}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((c + d*x**3)**(3/2)/sqrt(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,dx - 5\left(\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a\,d^2 + 11\left(\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}\,x^3}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right.}{8b}$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(c + d*x**3)*sqrt(a + b*x**3)*d*x - 5*int((sqrt(c + d*x**3)*sqrt(a
+ b*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*d**2 + 11*int(
(sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*
x**6),x)*b*c*d - 2*int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c + a*d*x**3
 + b*c*x**3 + b*d*x**6),x)*a*c*d + 8*int((sqrt(c + d*x**3)*sqrt(a + b*x**3
))/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b*c**2)/(8*b)
```

3.59 $\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,1/2,-1/2,4/3,-b*x^3/a,-d*
x^3/c)/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. 2(83) = 166.

Time = 1.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx$$

$$=\frac{8acx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\left(8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-bc\,\text{Appe}\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^3]/Sqrt[a + b*x^3],x]
```

output
```
(8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1/2, -1/2, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/(Sqrt[a + b*x^3]*(8*a*c*AppellF1[1/3, 1/2, -1/2, 4/3, -((b*x^3)/
a), -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] - b*c*AppellF1[4/3, 3/2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{\sqrt{dx^3+c}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 1/2, -1/2, 4/3, -((b*
x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{3}+c}}{\sqrt{b\,x^{3}+a}}dx$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{dx^3+c}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**3)/sqrt(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{dx^3+c}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{dx^3+c}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{b\,x^3+a}dx$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a + b*x**3),x)
```

3.60 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d
*x^3/c)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 1.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[
a + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/
a), -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{dx^3+c}}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}}dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c)
, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c + a*d*x**3 + b*c*x**3 + b*d*x
**6),x)
```

3.61 $\int \frac{1}{\sqrt{a+bx^3}\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{1}{\sqrt{a+bx^3}\left(c+dx^3\right)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(1/3,1/2,3/2,4/3,-b*x^3/a,-d
*x^3/c)/c/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 419 vs. 2(86) = 172.

Time = 3.48 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.87

$$\int \frac{1}{\sqrt{a+bx^3}\left(c+dx^3\right)^{3/2}}\,dx=\frac{-8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(6bc-6ad-4bdx^3+bdx^3\sqrt{1+\right.}{6c(bc-ad}$$

input
```
Integrate[1/(Sqrt[a + b*x^3]*(c + d*x^3)^(3/2)),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(6*b*c
- 6*a*d - 4*b*d*x^3 + b*d*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*Appe
llF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - 3*d*x^4*(4*(a + b*
x^3) - b*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((
b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)]))/(6*c*(b*c - a*d)*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*A
ppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(a*d*Appel
lF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/
2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}(dx^3+c)^{3/2}}dx}{\sqrt{a+bx^3}}$$
$$\Big\downarrow \text{937}$$
$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{c\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$\Big\downarrow \text{936}$$
$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/(Sqrt[a + b*x^3]*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 3/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(c*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\left(d\,x^{3}+c\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x)
```

output
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d^2*x^9 + (2*b*c*d + a*d^2)*x^
6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^3+a}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a}\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}\,(c+dx^3)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{b\,d^2x^9+a\,d^2x^6+2bcd\,x^6+2acd\,x^3+b\,c^2x^3+a\,c^2}dx$$

input
```
int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a*c**2 + 2*a*c*d*x**3 + a*d**2*x*
*6 + b*c**2*x**3 + 2*b*c*d*x**6 + b*d**2*x**9),x)
```

3.62 $\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^2x\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{5}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c^2*x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,3/2,-5/2,4/3,-b*x^3/a
,-d*x^3/c)/a/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 432 vs. 2(89) = 178.

Time = 5.81 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.85

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \frac{-d(16b^2c^2-89abcd+55a^2d^2)\,x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.}{}$$

input
```
Integrate[(c + d*x^3)^(5/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-(d*(16*b^2*c^2 - 89*a*b*c*d + 55*a^2*d^2)*x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1
 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) +
(64*a*c*x*(11*a^2*d^3*x^3 + 4*b^2*c^2*(3*c + 2*d*x^3) + a*b*d^2*x^3*(-13*c
 + 3*d*x^3))*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 24
*x^4*(c + d*x^3)*(8*b^2*c^2 + 11*a^2*d^2 + a*b*d*(-16*c + 3*d*x^3))*(a*d*A
ppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3
, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(8*a*c*AppellF1[1/3, 1/2, 1
/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2,
7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)]))))/(96*a*b^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{(dx^3+c)^{5/2}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{c^2\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{5/2}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{c^2x\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{5}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(5/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^2*x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 3/2, -5/2, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(a*Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(d\,x^3+c)^{\frac{5}{2}}}{(b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x)
```

output
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b^2*
x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c+dx^3)^{\frac{5}{2}}}{(a+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**3+c)**(5/2)/(b*x**3+a)**(3/2),x)
```

output
```
Integral((c + d*x**3)**(5/2)/(a + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(d\,x^3+c)^{5/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x^3)^(5/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x^3)^(5/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x^3+c)^(5/2)/(b*x^3+a)^(3/2),x)
```

output
```
( - 16*sqrt(c + d*x**3)*sqrt(a + b*x**3)*a*c*d**2*x + 10*sqrt(c + d*x**3)*
sqrt(a + b*x**3)*a*d**3*x**4 + 48*sqrt(c + d*x**3)*sqrt(a + b*x**3)*b*c**2
*d*x - 2*sqrt(c + d*x**3)*sqrt(a + b*x**3)*b*c*d**2*x**4 - 275*int((sqrt(c
 + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*
c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*
b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d*x**9),x)*a*
*4*d**5 + 665*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5
*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 -
2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x*
*6 - b**3*c*d*x**9),x)*a**3*b*c*d**4 - 275*int((sqrt(c + d*x**3)*sqrt(a +
b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*d*
x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 5*a*
b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d*x**9),x)*a**3*b*d**5*x**3 - 457
*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x*
*3 - a**2*b*c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2
*x**3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d
*x**9),x)*a**2*b**2*c**2*d**3 + 665*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)
*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*d*x**3 +
10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d*
*2*x**9 - b**3*c**2*x**6 - b**3*c*d*x**9),x)*a**2*b**2*c*d**4*x**3 + 67...
```

3.63 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\frac{cx\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\operatorname{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
c*x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,3/2,-3/2,4/3,-b*x^3/a,-
d*x^3/c)/a/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(87) = 174.

Time = 5.16 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.24

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\frac{x\left(d(-2bc+5ad)x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{8\left(4ac(-2ac\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^(3/2),x]
```

output
```
(x*(d*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF
1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + (8*(4*a*c*(-2*a*d^2*x^
3 + b*c*(3*c + 2*d*x^3))*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] + 3*(-(b*c) + a*d)*x^3*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3
)/a), -((d*x^3)/c)]))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -
((d*x^3)/c)] - 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
)/(12*a*b*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{\left(dx^3+c\right)^{3/2}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{c\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\left(\frac{dx^3}{c}+1\right)^{3/2}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[(c + d*x^3)^(3/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c*x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 3/2, -3/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(a*Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{3}{2}}}{(b\,x^{3}+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x)
```

output
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*(d*x^3 + c)^(3/2)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**3+c)**(3/2)/(b*x**3+a)**(3/2),x)
```

output
```
Integral((c + d*x**3)**(3/2)/(a + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(d\,x^3+c)^{3/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x^3)^(3/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((d*x^3+c)^(3/2)/(b*x^3+a)^(3/2),x)
```

output
```
(4*sqrt(c + d*x**3)*sqrt(a + b*x**3)*c*d*x + 25*int((sqrt(c + d*x**3)*sqrt
(a + b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b
*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 +
 5*a*b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d*x**9),x)*a**3*d**4 - 30*in
t((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3
- a**2*b*c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x*
*3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d*x*
*9),x)*a**2*b*c*d**3 + 25*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*
a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*
d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 -
b**3*c**2*x**6 - b**3*c*d*x**9),x)*a**2*b*d**4*x**3 + 5*int((sqrt(c + d*x*
*3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 +
9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*
d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x**6 - b**3*c*d*x**9),x)*a*b**2*c*
*2*d**2 - 30*int((sqrt(c + d*x**3)*sqrt(a + b*x**3)*x**6)/(5*a**3*c*d + 5*
a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*d*x**3 + 10*a**2*b*d**2*x**6 - 2
*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 + 5*a*b**2*d**2*x**9 - b**3*c**2*x**
6 - b**3*c*d*x**9),x)*a*b**2*c*d**3*x**3 + 5*int((sqrt(c + d*x**3)*sqrt(a
+ b*x**3)*x**6)/(5*a**3*c*d + 5*a**3*d**2*x**3 - a**2*b*c**2 + 9*a**2*b*c*
d*x**3 + 10*a**2*b*d**2*x**6 - 2*a*b**2*c**2*x**3 + 3*a*b**2*c*d*x**6 +...
```

3.64 $\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{1+\frac{dx^3}{c}}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(d*x^3+c)^(1/2)*AppellF1(1/3,3/2,-1/2,4/3,-b*x^3/a,-d*
x^3/c)/a/(b*x^3+a)^(1/2)/(1+d*x^3/c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 256 vs. $2(86) = 172$.

Time = 3.23 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\left(\frac{4(c+dx^3)}{a}-\frac{dx^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}+\frac{\cdots}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\cdots}\right.\cdots}{6\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^(3/2),x]
```

output
```
(x*((4*(c + d*x^3))/a - (d*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*App
ellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/a + (16*c^2*AppellF
1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(8*a*c*AppellF1[1/3, 1/
2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^3*(a*d*AppellF1[4/3, 1/2, 3
/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)]))))/(6*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{\sqrt{dx^3+c}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\int \frac{\sqrt{\frac{dx^3}{c}+1}}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{\frac{dx^3}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x^3)^(3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[c + d*x^3]*AppellF1[1/3, 3/2, -1/2, 4/3, -((b*
x^3)/a), -((d*x^3)/c)])/(a*Sqrt[a + b*x^3]*Sqrt[1 + (d*x^3)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{3}+c}}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x)
```

output
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x**3+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^3+c}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int((d*x^3+c)^(1/2)/(b*x^3+a)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.65 $\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(1/3,3/2,1/2,4/3,-b*x^3/a,-d
*x^3/c)/a/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 419 vs. 2(86) = 172.

Time = 3.51 (sec) , antiderivative size = 419, normalized size of antiderivative = 4.87

$$\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx = \frac{-8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(-6bc+6ad-4bdx^3+bdx^3\sqrt{1-\cdots}\right.}{6a(-bc+a\cdots}$$

input
```
Integrate[1/((a + b*x^3)^(3/2)*Sqrt[c + d*x^3]),x]
```

output
```
(-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(-6*b*c
 + 6*a*d - 4*b*d*x^3 + b*d*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*App
ellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - 3*b*x^4*(4*(c + d
*x^3) - d*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1
/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)]))/(6*a*(-(b*c) + a*d)*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]*(-8*a
*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x^3*(a*d*A
ppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3
, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}\sqrt{c+dx^3}}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}\sqrt{dx^3+c}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}\sqrt{\frac{dx^3}{c}+1}}dx}{a\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^(3/2)*Sqrt[c + d*x^3]),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 3/2, 1/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(a*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}\sqrt{d\,x^{3}+c}}dx$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x)
```

output
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^
6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(a+bx^3\right)^{\frac{3}{2}}\sqrt{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/2)/(d*x**3+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**3)**(3/2)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\sqrt{dx^3+c}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{3/2}\,\sqrt{d\,x^3+c}}\,dx$$

input
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^(1/2)),x)
```

output
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}\sqrt{c+dx^3}}\,dx = \int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{b^2d\,x^9+2abd\,x^6+b^2c\,x^6+a^2d\,x^3+2abc\,x^3+a^2c}dx$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a**2*c + a**2*d*x**3 + 2*a*b*c*x*
*3 + 2*a*b*d*x**6 + b**2*c*x**6 + b**2*d*x**9),x)
```

3.66 $\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

output
```
x*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)^(1/2)*AppellF1(1/3,3/2,3/2,4/3,-b*x^3/a,-d
*x^3/c)/a/c/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 483 vs. 2(89) = 178.

Time = 5.42 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.43

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx = \frac{8acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(-2(3a^2d^2+2abd(-3c+dx^3)+\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(3/2)*(c + d*x^3)^(3/2)),x]
```

output
```
(8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(-2*(3*a
^2*d^2 + 2*a*b*d*(-3*c + d*x^3) + b^2*c*(3*c + 2*d*x^3)) + b*d*(b*c + a*d)
*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]) + 3*x^4*(4*(a^2*d^2 + a*b*d^2*x^3 + b^2*c*(c
+ d*x^3)) - b*d*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[4/3, 1/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])*(a*d*AppellF1[4/3,
 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(6*a*c*(b*c - a*d)^2*Sqrt[a + b*x^3]*Sq
rt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{3/2}\left(c+dx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}\left(dx^3+c\right)^{3/2}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{3/2}\left(\frac{dx^3}{c}+1\right)^{3/2}}dx}{ac\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{3}{2},\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^(3/2)*(c + d*x^3)^(3/2)),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 3/2, 3/2, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(a*c*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{3}{2}} \left(d\,x^{3}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x)
```

output
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b^2*d^2*x^12 + 2*(b^2*c*d + a*b*
d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^
2*c*d)*x^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{2}}(c+dx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/2)/(d*x**3+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**3)**(3/2)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{3}{2}}(dx^3+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(b\,x^3+a)^{3/2}\,(d\,x^3+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^(3/2)),x)
```

output
```
int(1/((a + b*x^3)^(3/2)*(c + d*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}(c+dx^3)^{3/2}}\,dx=\int \frac{\sqrt{d\,x^3+c}\,\sqrt{b\,x^3+a}}{b^2d^2x^{12}+2ab\,d^2x^9+2b^2cd\,x^9+a^2d^2x^6+4abcd\,x^6+b^2c^2x^6+2a^2cd\,x}$$

input
```
int(1/(b*x^3+a)^(3/2)/(d*x^3+c)^(3/2),x)
```

output
```
int((sqrt(c + d*x**3)*sqrt(a + b*x**3))/(a**2*c**2 + 2*a**2*c*d*x**3 + a**
2*d**2*x**6 + 2*a*b*c**2*x**3 + 4*a*b*c*d*x**6 + 2*a*b*d**2*x**9 + b**2*c*
*2*x**6 + 2*b**2*c*d*x**9 + b**2*d**2*x**12),x)
```

3.67 $\int (a-bx^3)(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (a-bx^3)(a+bx^3)^{2/3}\,dx=\frac{7}{18}ax(a+bx^3)^{2/3}-\frac{1}{6}x(a+bx^3)^{5/3}+\frac{7a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

output
```
7/18*a*x*(b*x^3+a)^(2/3)-1/6*x*(b*x^3+a)^(5/3)+7/27*a^2*arctan(1/3*(1+2*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-7/18*a^2*ln(-b^(1/3)*x+(
b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int (a-bx^3)(a+bx^3)^{2/3}\,dx=\frac{3\sqrt[3]{b}(a+bx^3)^{2/3}(4ax-3bx^4)+14\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-14a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{\ldots}\right.}{54\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(1/3)*(a + b*x^3)^(2/3)*(4*a*x - 3*b*x^4) + 14*Sqrt[3]*a^2*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 14*a^2*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)] + 7*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}dx$$

$$\Big\downarrow \ 913$$

$$\frac{7}{6}a\int \left(bx^3+a\right)^{2/3}dx-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \ 748$$

$$\frac{7}{6}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \ 769$$

$$\frac{7}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

input
```
Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

output
```
-1/6*(x*(a + b*x^3)^(5/3)) + (7*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b
^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{-9\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}} x^{4}+12 a x \left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}-14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-14\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}+7\ln}{54 b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-9*(b*x^3+a)^(2/3)*b^(4/3)*x^4+12*a*x*(b*x^3+a)^(2/3)*b^(1/3)-14*3^(
1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a^2-14*ln
((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+7*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.

Time = 0.11 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.56

$$\int (a-bx^3)\,(a+bx^3)^{2/3}\,dx = \left[\frac{21\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+\right.\right.}{}\right.$$

$$-\frac{42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+14\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-7\,a}{54\,b}$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(
1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x
^2) - 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^
2*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1
/3))*sqrt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3
 + a)^(1/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 +
 a)^(2/3))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx = \frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

output
```
a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.88

$$\int (a - bx^3) (a + bx^3)^{2/3} \, dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*ar
ctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*
log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3)
 + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3
)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 +
(b*x^3 + a)^2*b/x^6))*b
```

Giac [F]

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx=\int -\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)dx$$

input
```
integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx=\int \left(b\,x^3+a\right)^{2/3}\left(a-b\,x^3\right)dx$$

input
```
int((a + b*x^3)^(2/3)*(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

Reduce [F]

$$\int (a-bx^3)\left(a+bx^3\right)^{2/3}\,dx=\frac{2(b\,x^3+a)^{\frac{2}{3}}\,ax}{9}-\frac{(b\,x^3+a)^{\frac{2}{3}}\,b\,x^4}{6}+\frac{7\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2}{9}$$

input
```
int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

output
```
(4*(a + b*x**3)**(2/3)*a*x - 3*(a + b*x**3)**(2/3)*b*x**4 + 14*int((a + b*
x**3)**(2/3)/(a + b*x**3),x)*a**2)/18
```

3.68 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{3}x\left(a+bx^3\right)^{2/3} + \frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} - \frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

output
```
-1/3*x*(b*x^3+a)^(2/3)+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))*3^(1/2)/b^(1/3)-2/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{-3\sqrt[3]{b}x(a+bx^3)^{2/3} + 4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 4a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + 2a\log\left(b^{2/3}x^2\right.}{9\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 4*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + 2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)])/(9*b^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{4}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{1}{3}x\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{4}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{1}{3}x\left(a+bx^3\right)^{2/3}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
-1/3*(x*(a + b*x^3)^(2/3)) + (4*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(
2*b^(1/3))))/3
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{4\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a+\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}}x\,b^{\frac{1}{3}}}{4}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+(b}{x^{2}}\right.}{2}\right.}{9b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-4/9*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*
a+3/4*(b*x^3+a)^(2/3)*x*b^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-1/2*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.99

$$\int \frac{a - bx^3}{\sqrt[3]{a + bx^3}}\,dx$$

$$= \left[\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3 - 3\,(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 + 2\,(bx^3 + a)^{\frac{2}{3}}(\right.\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x - 2\,(bx^3 + a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) + 3\,(bx^3 + a)^{\frac{2}{3}}bx + 4\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}}{}\right.}{9\,b}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)
*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2
+ 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 +
 a)^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) +
 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*
x^3 + a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan
(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) +
3*(b*x^3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(
1/3))/x) - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

output
```
a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.68

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$-\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx\ldots}{}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

Giac [F]

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=\int -\frac{bx^3-a}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{a-b\,x^3}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((a - b*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx=-\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

output
```
- int(x**3/(a + b*x**3)**(1/3),x)*b + int(1/(a + b*x**3)**(1/3),x)*a
```

3.69 $\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
2*x/(b*x^3+a)^(1/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2)
)*3^(1/2)/b^(1/3)+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}-\frac{\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(2*x)/(a + b*x^3)^(1/3) - ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b
*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3
*b^(1/3)) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2
/3)]/(6*b^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{2x}{\sqrt[3]{a+bx^3}}-\int \frac{1}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{ 769}$$

$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{2x}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}+\frac{\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6b^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)}{3b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
2*x/(b*x^3+a)^(1/3)+1/3/b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/6/b^(
1/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+1/3*3
^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.

Time = 0.09 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.38

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(b^2x^3+ab)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx\right.\right.}{}\right.$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3
+ a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^
2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a
)^(2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2
*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2) + 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x
^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

output
```
x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(
4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \frac{1}{6}\,b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3}{x}\right.}{b^{\frac{4}{3}}}\right.$$

$$+\frac{x}{(bx^3+a)^{\frac{1}{3}}}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)
*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)
^(1/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{a-b\,x^3}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((a - b*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{4/3}}\,dx = -\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)a$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

output
```
- int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*b + in
t(1/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*a
```

3.70 $\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{x}{2\left(a+bx^3\right)^{4/3}}+\frac{x}{2a\sqrt[3]{a+bx^3}}$$

output
```
1/2*x/(b*x^3+a)^(4/3)+1/2*x/a/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx=\frac{2ax+bx^4}{2a\left(a+bx^3\right)^{4/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(2*a*x + b*x^4)/(2*a*(a + b*x^3)^(4/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}} a}$	25
trager	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}} a}$	25
pseudoelliptic	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}} a}$	25
orering	$\frac{x\left(b\,x^{3}+2a\right)}{2\left(b\,x^{3}+a\right)^{\frac{4}{3}} a}$	25

input
```
int((-b*x^3+a)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{(bx^4+2\,ax)(bx^3+a)^{\frac{2}{3}}}{2\,(ab^2x^6+2\,a^2bx^3+a^3)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(27) = 54.

Time = 22.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 5.28

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) - \frac{bx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

output
```
a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)
*gamma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}a} - \frac{bx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
 + a)^(4/3)*a)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = \frac{x\,(b\,x^3+a)+a\,x}{2\,a\,(b\,x^3+a)^{4/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(7/3),x)
```

output
```
(x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

Reduce [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx = -\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)a$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(7/3),x)
```

output

```
 - int(x**3/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (
a + b*x**3)**(1/3)*b**2*x**6),x)*b + int(1/((a + b*x**3)**(1/3)*a**2 + 2*(
a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*a
```

3.71 $\int \frac{a-bx^3}{(a+bx^3)^{10/3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}} \, dx = \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

output
```
2/7*x/(b*x^3+a)^(7/3)+5/28*x/a/(b*x^3+a)^(4/3)+15/28*x/a^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}} \, dx = \frac{28a^2x + 35abx^4 + 15b^2x^7}{28a^2(a+bx^3)^{7/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(28*a^2*x + 35*a*b*x^4 + 15*b^2*x^7)/(28*a^2*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{5}{7}\int \frac{1}{(bx^3+a)^{7/3}}dx+\frac{2x}{7(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{5}{7}\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)+\frac{2x}{7(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{5}{7}\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)+\frac{2x}{7(a+bx^3)^{7/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(2*x)/(7*(a + b*x^3)^(7/3)) + (5*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2
*(a + b*x^3)^(1/3))))/7
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37
trager	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37
pseudoelliptic	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37
orering	$\frac{x\left(15b^{2}x^{6}+35ab\,x^{3}+28a^{2}\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{2}}$	37

input
```
int((-b*x^3+a)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(15\,b^2x^7+35\,abx^4+28\,a^2x)(bx^3+a)^{\frac{2}{3}}}{28\,(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
 3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(49) = 98.

Time = 115.25 (sec) , antiderivative size = 709, normalized size of antiderivative = 12.89

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

output
```
a*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) - b*(7*a*x**4*gamma(4/3)/(9*
a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gam
ma(10/3)) + 3*b*x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(1
0/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*
b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = \frac{15\,x\,(b\,x^3+a)^2+8\,a^2\,x+5\,a\,x\,(b\,x^3+a)}{28\,a^2\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(10/3),x)
```

output
```
(15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/
3))
```

Reduce [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{10/3}}\,dx = -\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)a$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(10/3),x)
```

output

```
 - int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3
+ 3*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*b
+ int(1/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*
(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*a
```

3.72 $\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 74

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx=\frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{6x}{35a^2(a+bx^3)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

output
```
1/5*x/(b*x^3+a)^(10/3)+4/35*x/a/(b*x^3+a)^(7/3)+6/35*x/a^2/(b*x^3+a)^(4/3)
+18/35*x/a^3/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx=\frac{35a^3x+70a^2bx^4+60ab^2x^7+18b^3x^{10}}{35a^3(a+bx^3)^{10/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
(35*a^3*x + 70*a^2*b*x^4 + 60*a*b^2*x^7 + 18*b^3*x^10)/(35*a^3*(a + b*x^3)
^(10/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{4}{5}\int \frac{1}{\left(bx^3+a\right)^{10/3}}dx+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{4}{5}\left(\frac{6\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{4}{5}\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{4}{5}\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

input
```
Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
x/(5*(a + b*x^3)^(10/3)) + (4*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a +
 b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/5
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x\left(18b^{3}x^{9}+60a\,b^{2}x^{6}+70a^{2}b\,x^{3}+35a^{3}\right)}{35\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{3}}$	48
trager	$\frac{x\left(18b^{3}x^{9}+60a\,b^{2}x^{6}+70a^{2}b\,x^{3}+35a^{3}\right)}{35\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{3}}$	48
pseudoelliptic	$\frac{x\left(18b^{3}x^{9}+60a\,b^{2}x^{6}+70a^{2}b\,x^{3}+35a^{3}\right)}{35\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{3}}$	48
orering	$\frac{x\left(18b^{3}x^{9}+60a\,b^{2}x^{6}+70a^{2}b\,x^{3}+35a^{3}\right)}{35\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{3}}$	48

input
```
int((-b*x^3+a)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x)(bx^3+a)^{\frac{2}{3}}}{35\,(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2
/3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx = -\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x
^3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3
+ a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

Giac [F]

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x}{5\,(b\,x^3+a)^{10/3}} + \frac{18\,x}{35\,a^3\,(b\,x^3+a)^{1/3}} + \frac{6\,x}{35\,a^2\,(b\,x^3+a)^{4/3}} + \frac{4\,x}{35\,a\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

output
```
x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a
^2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

Reduce [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = -\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}} \right. + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}} \right.$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(13/3),x)
```

output
```
 - int(x**3/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3
+ 6*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9
 + (a + b*x**3)**(1/3)*b**4*x**12),x)*b + int(1/((a + b*x**3)**(1/3)*a**4
+ 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2*x**6
 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 + (a + b*x**3)**(1/3)*b**4*x**12),x)*
a
```

3.73 $\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2x}{13(a+bx^3)^{13/3}}+\frac{11x}{130a(a+bx^3)^{10/3}}+\frac{99x}{910a^2(a+bx^3)^{7/3}}+\frac{297x}{1820a^3(a+bx^3)^{4/3}}+\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}$$

output
```
2/13*x/(b*x^3+a)^(13/3)+11/130*x/a/(b*x^3+a)^(10/3)+99/910*x/a^2/(b*x^3+a)
^(7/3)+297/1820*x/a^3/(b*x^3+a)^(4/3)+891/1820*x/a^4/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{x(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12})}{1820a^4(a+bx^3)^{13/3}}$$

input
```
Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

output
```
(x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^
4*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}} \, dx$$

$$\Big\downarrow \text{910}$$

$$\frac{11}{13} \int \frac{1}{(bx^3+a)^{13/3}} dx + \frac{2x}{13 (a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{11}{13} \left(\frac{9 \int \frac{1}{(bx^3+a)^{10/3}} dx}{10a} + \frac{x}{10a (a+bx^3)^{10/3}} \right) + \frac{2x}{13 (a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{11}{13} \left(\frac{9 \left(\frac{6 \int \frac{1}{(bx^3+a)^{7/3}} dx}{7a} + \frac{x}{7a(a+bx^3)^{7/3}} \right)}{10a} + \frac{x}{10a (a+bx^3)^{10/3}} \right) + \frac{2x}{13 (a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{11}{13}\left(\frac{9\left(\frac{6\left(\frac{3\int\frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

$\big\downarrow$ 746

$$\frac{11}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

input

```
Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

output

```
(2*x)/(13*(a + b*x^3)^(13/3)) + (11*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(
7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a +
 b*x^3)^(1/3))))/(7*a)))/(10*a)))/13
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{x\left(891b^{4}x^{12}+3861a\,b^{3}x^{9}+6435a^{2}b^{2}x^{6}+5005a^{3}b\,x^{3}+1820a^{4}\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{4}}$	59
trager	$\frac{x\left(891b^{4}x^{12}+3861a\,b^{3}x^{9}+6435a^{2}b^{2}x^{6}+5005a^{3}b\,x^{3}+1820a^{4}\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{4}}$	59
pseudoelliptic	$\frac{x\left(891b^{4}x^{12}+3861a\,b^{3}x^{9}+6435a^{2}b^{2}x^{6}+5005a^{3}b\,x^{3}+1820a^{4}\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{4}}$	59
orering	$\frac{x\left(891b^{4}x^{12}+3861a\,b^{3}x^{9}+6435a^{2}b^{2}x^{6}+5005a^{3}b\,x^{3}+1820a^{4}\right)}{1820\left(b\,x^{3}+a\right)^{\frac{13}{3}}a^{4}}$	59

input
```
int((-b*x^3+a)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820
*a^4)/(b*x^3+a)^(13/3)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x)(bx^3+a)^{\frac{2}{3}}}{1820\,(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9)}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
 + 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b
^3*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(73) = 146.

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.65

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^4}$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(
b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 +
 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

Giac [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \int -\frac{bx^3-a}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx = \frac{2\,x}{13\,\left(b\,x^3+a\right)^{13/3}}+\frac{891\,x}{1820\,a^4\,\left(b\,x^3+a\right)^{1/3}} + \frac{297\,x}{1820\,a^3\,\left(b\,x^3+a\right)^{4/3}}+\frac{99\,x}{910\,a^2\,\left(b\,x^3+a\right)^{7/3}}+\frac{11\,x}{130\,a\,\left(b\,x^3+a\right)^{10/3}}$$

input
```
int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

output
```
(2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (29
7*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (
11*x)/(130*a*(a + b*x^3)^(10/3))
```

Reduce [F]

$$\int \frac{a-bx^3}{(a+bx^3)^{16/3}}\,dx =$$
$$-\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\,a$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\,a$$

input
```
int((-b*x^3+a)/(b*x^3+a)^(16/3),x)
```

output
```
- int(x**3/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x**3
+ 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*b**3
*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*x**1
5),x)*b + int(1/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x
**3 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*
b**3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*
x**15),x)*a
```

3.74 $\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 464

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\frac{1}{5}x(a+bx^3)^{4/3} - \frac{4\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{14ax\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{2\sqrt[3]{2}a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2}a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

output
```
-1/5*x*(b*x^3+a)^(4/3)-4/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-2/3*2^(1/3)*a^(5/3)
*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/
2)/b^(1/3)-14/5*a*x*(b*x^3+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/
(1+b*x^3/a)^(1/3)-2/3*2^(1/3)*a^(5/3)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))/b^(1/3)+2/3*2^(1/3)*a^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2
/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)-4/3*
2^(1/3)*a^(5/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+
1/3*2^(1/3)*a^(5/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(
2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \frac{27abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+4x\left(-8a^2-9abx^3-b^2x^6+\frac{\ldots}{(a-\ldots}\right.}{20\,(a\ldots}$$

input
```
Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

output
```
(27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3,
1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)
/a])))))/(20*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.20, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {933, 27, 1025, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}dx$$

$$\Big\downarrow \text{933}$$

$$-\frac{\int -\frac{2ab\sqrt[3]{bx^3+a}(7bx^3+3a)}{a-bx^3}dx}{5b}-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}a\int \frac{\sqrt[3]{bx^3+a}\left(7bx^3+3a\right)}{a-bx^3}dx-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{1025}$$

$$\frac{2}{5}a\left(-\frac{\int -\frac{ab(27bx^3+13a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}a\left(\frac{\int \frac{ab(27bx^3+13a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\int \frac{27bx^3+13a}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{1026}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-27\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx\right)-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{27\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}\right)-\frac{7}{2}x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\int \frac{1}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\frac{7}{2}x\sqrt[3]{a+\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{928}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}\right)-\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\right.}{\left(a+bx^3\right)^{2/3}}\right.\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)-\frac{27x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hyp}}{(}\right.\right.$$
$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{927}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hyperge}}{2a\,(a}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 982

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\right.}{2a^{2/3}\sqrt[3]{b}}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 821

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\,\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{2a\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 16

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \ 1142$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}}{}\right.\right.}{}\right.\right.\right.$$

$$\frac{1}{5}x\left(bx^3+a\right)^{4/3}$$

$$\downarrow 25$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\cdots}}{\cdots}\right.\right.}{\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(bx^3+a\right)^{4/3}$$

$$\Big\downarrow 27$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right.}{\sqrt[3]{bx^3+a}}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\cdots}{\sqrt[3]{a}\sqrt[3]{b\cdots}}}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.}{\cdots}\right.\right.\right)$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 217

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\,\frac{9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \text{lo}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

$\downarrow$ 1103

$$\frac{2}{5}a\left(\frac{1}{2}a\left(40a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\right.\right.}{}\right.\right.\right.$$

$$\frac{1}{5}x\left(a+bx^3\right)^{4/3}$$

input
```
Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

output

```
-1/5*(x*(a + b*x^3)^(4/3)) + (2*a*((-7*x*(a + b*x^3)^(1/3))/2 + (a*((-27*x
*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a
+ b*x^3)^(2/3) + 40*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3
, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan
[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1
/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3
) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))
)/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3
)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(
a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3)
 + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)
*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b
^(1/3)))))/2))/5
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{7}{3}}}{-b\,x^3+a}dx$$

input
```
int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = -\int \frac{a^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{b^2x^6\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{2abx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
```

output
```
-Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*
(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**
(1/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{7}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx=\int \frac{(b\,x^3+a)^{7/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(7/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(7/3)/(a - b*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx=-\frac{8(b\,x^3+a)^{\frac{1}{3}}\,ax}{5}-\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^4}{5}+\frac{13\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^6+a^2}dx\right)a^3}{5}+\frac{27\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^3}{-b^2x^6+a^2}dx\right)a^2b}{5}$$

input
```
int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

output
```
( - 8*(a + b*x**3)**(1/3)*a*x - (a + b*x**3)**(1/3)*b*x**4 + 13*int((a + b
*x**3)**(1/3)/(a**2 - b**2*x**6),x)*a**3 + 27*int(((a + b*x**3)**(1/3)*x**
3)/(a**2 - b**2*x**6),x)*a**2*b)/5
```

3.75 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 464

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = -\frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{2\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

$$- \frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

$$- \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}}$$

$$- \frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$- \frac{2\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

$$+ \frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{b}}$$

output
```
-1/2*x*(b*x^3+a)^(1/3)-2/3*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-1/3*2^(1/3)*a^(2/3)
*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/
2)/b^(1/3)-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/
(b*x^3+a)^(2/3)-1/3*2^(1/3)*a^(2/3)*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))/b^(1/3)+1/3*2^(1/3)*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(
b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)-2/3*2^
(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(1/3)+1/
6*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1
/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \frac{x\left(-4(a+bx^3)+5bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{\cdots}{(a-bx^3)\left(4a\operatorname{AppellF}\cdots\right.}\right.}{8\left(a+bx^3\right)^{2\cdots}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

output
```
(x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7
/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3
)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/
a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*(a + b*
x^3)^(2/3))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {933, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3} \, dx$$

$$\Big\downarrow \text{933}$$

$$-\frac{\int -\frac{ab(5bx^3+3a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ab(5bx^3+3a)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{2b}-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a\int \frac{5bx^3+3a}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{1026}$$

$$\frac{1}{2}a\left(8a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-5\int \frac{1}{(bx^3+a)^{2/3}}dx\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{779}$$

$$\frac{1}{2}a\left(8a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx-\frac{5\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{778}$$

$$\frac{1}{2}a\left(8a\int \frac{1}{(a-bx^3)\,(bx^3+a)^{2/3}}dx-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 928

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 779

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\,(a+bx^3)^{2/3}}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\right)-\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 778

$$\frac{1}{2}a\left(8a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\,(a+bx^3)^{2/3}}\right)-\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeome}}{(a+bx^3}\right.$$
$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 927

$$\frac{1}{2}a\left(8a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometr}}{2a\,(a+bx^3}\right.\right.$$
$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{982}$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x\cdots}{\sqrt[3]{a}\sqrt[3]{\cdots}}\cdots}{2a^{2/3}\sqrt[3]{b}}\cdots\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\cdots}{2a^{2/3}\sqrt[3]{b}\cdots}\cdots\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int\ldots}{\ldots}\right.\right.\ldots}{2a^{2/3}\sqrt[3]{b}\ldots}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\downarrow 1142$$

$$\frac{1}{2}a\left(8a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\cdots}{\sqrt[3]{\cdots}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}a\left(8a\,\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\cdots}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.}{\cdots}\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\downarrow 27$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\cdots}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{\cdots}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 217

$$\frac{1}{2}a\left(8a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}} - \frac{\log\left(\frac{\sqrt[3]{2}\ldots}{\ldots}\right.}{\ldots}\right.\right.\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

$\downarrow$ 1103

$$\frac{1}{2}a\left(8a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right.\right.}{2}\right.\right.$$

$$\frac{1}{2}x\sqrt[3]{a+bx^3}$$

input

```
Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

output
```
-1/2*(x*(a + b*x^3)^(1/3)) + (a*((-5*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometri
c2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 8*a*((x*(1 + (b*x^3
)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3
)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x
))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(
1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-
1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3
)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/
3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3
)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/
(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3)))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{-b\,x^3+a}dx$$

input
```
int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
integral(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = -\left(\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx\right) - \int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
```

output
```
-Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b
*x**3)**(1/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{4}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)/(a - b*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{a-bx^3}\,dx = -\frac{(b\,x^3+a)^{\frac{1}{3}}\,x}{2} + \frac{3\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b^2x^6+a^2}dx\right)a^2}{2} + \frac{5\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}x^3}{-b^2x^6+a^2}dx\right)ab}{2}$$

input
```
int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

output
```
( - (a + b*x**3)**(1/3)*x + 3*int((a + b*x**3)**(1/3)/(a**2 - b**2*x**6),x
)*a**2 + 5*int(((a + b*x**3)**(1/3)*x**3)/(a**2 - b**2*x**6),x)*a*b)/2
```

3.76 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 398

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}$$

output
```
-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*
3^(1/2))*3^(1/2)/a^(1/3)/b^(1/3)-1/6*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(1/3)/b^(1/3)-1/6*ln(2^(2
/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)+1/6*ln(1+
2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)-1/3*2^(1/3)*ln(1+2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)+1/12*ln(2*2^(1/3)+(a^(1/3)+b^
(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^
(1/3)/a^(1/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$$
$$= \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right)-4\log\left(\sqrt[3]{2}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

output
```
(4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1
/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3
)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Lo
g[2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/
3)*a^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/
3) + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a +
b*x^3)^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))]
+ 2*Log[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x
^3)^(1/3) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a
+ b*x^3)^(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$$
$$\Big\downarrow \; 927$$

$$\frac{9\sqrt[3]{a}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 982$$

$$\frac{9\sqrt[3]{a}\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}\right.}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 821$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int -}{2^{2}}\right.\right.}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 16$$

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2}}{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}}{\left(bx^3+a\right)^{2/3}}\right.\right.}{\sqrt[3]{b}}$$

$\Big\downarrow$ 1142

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+}{\sqrt[3]{a}\sqrt[3]{bx}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\Big\downarrow$ 25

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$\Big\downarrow$ 27

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\cdots}{\sqrt[3]{a}\sqrt[3]{b\cdots}}\cdots}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.$$

$\downarrow$ 1082

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\cdots}{\cdots}\cdots\right.\right.$$

$\downarrow$ 217

$$9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\cdots\right.}{\sqrt[3]{a+\cdots}}\right.}{3\ 2^{2/3}a\cdots}\cdots\right.\right.$$

$\downarrow$ 1103

$$\frac{9\sqrt[3]{a}\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\cdots\right.}{\sqrt[3]{b}\cdots}$$

input
```
Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

output
```
(9*a^(1/3)*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3
) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a +
 b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3)))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)
*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3
*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3))
- ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(
2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*
2^(2/3)*a^(1/3)))/9))/b^(1/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^3+a}dx$$

input
```
int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

output
```
int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(284) = 568.

Time = 16.03 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^
6*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2
*x^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a
*b^5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b
*x^3 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417
*a^2*b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(
b^6*x^18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b
^2*x^6 + 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(
2/3)*(a*b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^
(2/3) - 2^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 +
 a^4)*(-1/(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x
)*(b*x^3 + a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3
 + a^4)) + 1/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 +
2^(2/3)*(b^2*x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 +
a*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = -\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
```

output
```
-Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
```

output
```
integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{a-b\,x^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(a - b*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{-b\,x^3+a}dx$$

input
```
int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

output
```
int((a + b*x**3)**(1/3)/(a - b*x**3),x)
```

3.77 $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 452

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{4/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{4/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{4/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{4/3}\sqrt[3]{b}}$$

output

```
-1/6*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))
*2^(1/3)*3^(1/2)/a^(4/3)/b^(1/3)-1/12*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(4/3)/b^(1/3)+1/2*x*(1+b
*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)-1/12*
ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/
12*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)-1/6*ln(1+2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/b^(1/3)+1/24*ln(2*2^(1/3)+(a^
(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(
1/3))*2^(1/3)/a^(4/3)/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \frac{4ax\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.}{(a-bx^3)(a+bx^3)^{2/3}\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\text{AppellF1}\right.\right.}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

output
```
(4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(
a + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a]
+ b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \ 928$$

$$\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}$$

$$\Big\downarrow \ 779$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 778$$

$$\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 927$$

$$\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 982$$

$$\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}$$
$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 821$$

$$\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \frac{\cdots}{2^{2/3}-\cdots}}{\cdots}\cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 16

$$\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\cdots}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\cdots}\cdots}{\cdots}\right.\right.}{2a^{2/3}\sqrt[3]{b}\cdots}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 1142

$$\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{\int -\frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+\cdots}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\cdots\right.\right.}{\cdots}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 25

$$9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{a}\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+}}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 27

$$9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3-}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 1082

$$9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{}}{}\right.}{}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 217$$

$$9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\right.\right.$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(\cdots\right.}{2a^{2/3}\sqrt[3]{b}}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}$$

input
```
Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 +
 (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) -
 Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(
2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(
2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x
)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{1}{(-b\,x^3+a)\,(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{2/3}}\,dx = -\int \frac{1}{-a\left(a+bx^3\right)^{\frac{2}{3}}+bx^3\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
```

output
```
-Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(a - b*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a-(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a - (a + b*x**3)**(2/3)*b*x**3),x)
```

3.78 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 473

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{x}{4a^2(a+bx^3)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{7/3}\sqrt[3]{b}} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2(a+bx^3)^{2/3}} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{7/3}\sqrt[3]{b}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\,2^{2/3}a^{7/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{7/3}\sqrt[3]{b}}$$

output
```
1/4*x/a^2/(b*x^3+a)^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(7/3)/b^(1/3)-1/24*arctan(1/3
*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/
a^(7/3)/b^(1/3)+1/2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/
a)/a^2/(b*x^3+a)^(2/3)-1/24*ln(2^(2/3)-(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3)
)*2^(1/3)/a^(7/3)/b^(1/3)+1/24*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a
)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/
3)-1/12*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/
b^(1/3)+1/48*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{x\left(\frac{4}{a^2}-\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{\cdots}{(a-bx^3)\left(4a\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\cdots\right.}\right.}{16\,(a+bx^3)^{2/3}}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

output
```
(x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x
^3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3
)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/
3))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.14, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {931, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{x}{4a^2(a+bx^3)^{2/3}}-\frac{\int -\frac{b(3a-bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b(3a-bx^3)}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2b}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a-bx^3}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{\int \frac{1}{(bx^3+a)^{2/3}}dx+2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{2a\int \frac{1}{(a-bx^3)(bx^3+a)^{2/3}}dx+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}}{4a^2}+\frac{x}{4a^2(a+bx^3)^{2/3}}$$

$$\downarrow 928$$

$$\frac{2a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 779$$

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 778$$

$$\frac{2a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a^2}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 927$$

$$\frac{2a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{b\ldots}{\ldots}\right.}{2a\left(a+bx^3\right)^{2/3}}\right.}{4a^2}\ldots$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 982$$

$$\frac{2a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}\cdots\right)}{4a^2}$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\big\downarrow$ 821

$$\frac{2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int \frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\frac{\int \cdots}{\cdots}\cdots\right)\right)}{2a^{2/3}\sqrt[3]{b}}\cdots\right)}{\cdots}$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\big\downarrow$ 16

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\frac{2^{2/3}\cdots}{\cdots}}\cdots}{\cdots}\cdots\right.\right.}{2a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{x}{4a^2\left(bx^3+a\right)^{2/3}}+$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(bx^3+a)^{2/3}}+2a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right.}}{\left(bx^3+a\right)^{2/3}}\right.\right.}{}\right.$$

$\downarrow$ 25

$$\frac{x}{4a^2\left(bx^3+a\right)^{2/3}}+$$

$$\frac{3}{2}\int \frac{}{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right.}$$

$$\left(bx^3+a\right)^{2/3}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(bx^3+a)^{2/3}}+2a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(bx^3+a)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\right.\right.}{}\right.$$

$$\downarrow 27$$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int \frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+}{\sqrt[3]{a}\sqrt[3]{bx}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.}{}\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$$\downarrow 1082$$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{3\int \frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\cdots\right.}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\big\downarrow$ 217

$$2a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}{\sqrt[3]{a+bx^3}}\right.}{3\ 2^{2/3}a^{2/3}}\ldots\right.\right.\right.$$

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}$$

$\downarrow$ 1103

$$\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}+$$

$$2a\left(\frac{9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\ldots\right.}{2a^{2/3}\sqrt[3]{b}}\right.$$

input
```
Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

output
```
x/(4*a^2*(a + b*x^3)^(2/3)) + ((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + 2*a*((x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/
3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3)
+ b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(
a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*L
og[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) -
((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/S
qrt[3]])/a^(1/3) - Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/
3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^
(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1/3))))/(4*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{1}{(-b\,x^3+a)\,(b\,x^3+a)^{\frac{5}{3}}}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{5/3}}\,dx = -\int \frac{1}{-a^2\,(a+bx^3)^{\frac{2}{3}}+b^2x^6\,(a+bx^3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
```

output
```
-Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x
)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2-(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**2 - (a + b*x**3)**(2/3)*b**2*x**6),x)
```

3.79 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 492

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{x}{10a^2(a+bx^3)^{5/3}} + \frac{13x}{40a^3(a+bx^3)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{4\,2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}a^{10/3}\sqrt[3]{b}}$$
$$+ \frac{9x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{20a^3(a+bx^3)^{2/3}}$$
$$- \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}}$$
$$- \frac{\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{12\,2^{2/3}a^{10/3}\sqrt[3]{b}} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{48\,2^{2/3}a^{10/3}\sqrt[3]{b}}$$

output
```
1/10*x/a^2/(b*x^3+a)^(5/3)+13/40*x/a^3/(b*x^3+a)^(2/3)-1/24*arctan(1/3*(1-
2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^
(10/3)/b^(1/3)-1/48*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*3^(1/2))*2^(1/3)*3^(1/2)/a^(10/3)/b^(1/3)+9/20*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^3/(b*x^3+a)^(2/3)-1/48*ln(2^(2/3)-(a^
(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/48*ln(1+2^(2/
3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)-1/24*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b*x^3+a)^(1/3))*2^(1/3)/a^(10/3)/b^(1/3)+1/96*ln(2*2^(1/3)+(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1
/3)/a^(10/3)/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{x\left(16a^2+52a(a+bx^3)-13bx^3(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},\right.\right.}{}$$

input
```
Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

output
```
(x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)
*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3
)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*A
ppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3
, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.16, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {931, 25, 27, 1024, 25, 27, 1026, 779, 778, 928, 779, 778, 927, 982, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^3)\,(a+bx^3)^{8/3}}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{x}{10a^2\,(a+bx^3)^{5/3}}-\frac{\int -\frac{b(9a-4bx^3)}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b(9a-4bx^3)}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2b}+\frac{x}{10a^2\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9a-4bx^3}{(a-bx^3)(bx^3+a)^{5/3}}dx}{10a^2}+\frac{x}{10a^2\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{13x}{4a(a+bx^3)^{2/3}}-\frac{\int -\frac{ab\left(23a-13bx^3\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2b}}{10a^2}+\frac{x}{10a^2\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ab\left(23a-13bx^3\right)}{\left(a-bx^3\right)\left(bx^3+a\right)^{2/3}}dx}{4a^2b}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{23a-13bx^3}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 1026$$

$$\frac{\frac{13\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx+10a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{10a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx+\frac{13\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{10a\int \frac{1}{(a-bx^3)\left(bx^3+a\right)^{2/3}}dx+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 928$$

$$\frac{\frac{10a\left(\frac{\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a\left(a+bx^3\right)^{2/3}}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{10a\left(\frac{\int \frac{\sqrt[3]{bx^3+a}}{a-bx^3}dx}{2a}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)+\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}}{4a}+\frac{13x}{4a(a+bx^3)^{2/3}}}{10a^2}$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 927$$

$$\frac{\frac{10a\left(\frac{9\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{2a^{2/3}\sqrt[3]{b}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(a+bx^3\right)^{2/3}}\right)}{4a}}{10a^2}$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 982$$

$$\frac{\frac{10a\left(\frac{9\left(\frac{1}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(4-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}+\frac{2}{9}\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}\left(\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^3}{bx^3+a}+1\right)}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}\right)}{2a^{2/3}\sqrt[3]{b}}+\ldots\right)}{4a}}{10a^2}$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 821$$

$$10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\int\frac{1}{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right)+\frac{1}{9}\left(\int\frac{1}{2^{2/3}-\frac{\sqrt[3]{\ldots}}{\sqrt[3]{\ldots}}}\ldots\right.}{2a^{2/3}\sqrt[3]{b}}\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 16

$$10a\left(\frac{9\left(\frac{2}{9}\left(\frac{\int \frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{2/3}}\right)+\frac{1}{9}\left(-\frac{\int \frac{2^{2/3}-\frac{\sqrt[3]{b}\cdots}{\sqrt[3]{b\cdots}}}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}+\frac{2^{2/3}\left(\cdots\right.}{\sqrt[3]{\cdots}}}\cdots}{\cdots}\right.\cdots}{2a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 1142

$$\frac{x}{10a^2\left(bx^3+a\right)^{5/3}}+$$

$$\frac{\frac{13x}{4a(bx^3+a)^{2/3}}+\frac{\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(bx^3+a\right)^{2/3}}+10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{\frac{2^{2/3}\left(\sqrt[3]{\cdots}\right.}{\left(bx^3\cdots\right.}}\cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{x}{10a^2\left(bx^3+a\right)^{5/3}}+$$

$$\frac{13x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\left(bx^3+a\right)^{2/3}}+10a\left(\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a\left(bx^3+a\right)^{2/3}}+\frac{9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{}{\frac{2^{2/3}\left(\sqrt[3]{\cdots}\right)}{\left(bx^3\cdots\right)}}}{}\right.\right.}{}\right.$$

$$\frac{13x}{4a\left(bx^3+a\right)^{2/3}}+\frac{}{}$$

$\downarrow$ 27

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3}{2}\int\frac{1}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{}}{\sqrt[3]{a}\sqrt[3]{bx^3}}}{3\sqrt[3]{2}\sqrt[3]{a}}\right.\right.\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \; 1082$$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{3\int\frac{1}{-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\left(bx^3+a\right)^{2/3}}-3}d\left(1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{1}{2}\int\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{\cdots}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 217

$$10a\left(9\left(\frac{2}{9}\left(\frac{-\frac{1}{2}\int \frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}}{\frac{2^{2/3}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{\left(bx^3+a\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\sqrt[3]{bx^3+a}}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt[3]{bx^3+a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\sqrt[3]{a+bx^3}\cdots}\right.}{3\,2^{2/3}a^{2/3}\cdots}\cdots\right.\right.\right.$$

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}$$

$\downarrow$ 1103

$$\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}+$$

$$10a\left(9\left(\frac{2}{9}\left(\frac{\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}}{3\sqrt[3]{2}\sqrt[3]{a}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{2/3}}\right)}{2a^{2/3}\sqrt[3]{b}}+\frac{1}{9}\left(-\frac{\log \cdots}{\cdots}\right.\right.\right.$$

input

```
Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

output
```
x/(10*a^2*(a + b*x^3)^(5/3)) + ((13*x)/(4*a*(a + b*x^3)^(2/3)) + ((13*x*(1
 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b
*x^3)^(2/3) + 10*a*((x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4
/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)) + (9*((2*((-((Sqrt[3]*ArcTan[(1
 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)
*a^(1/3))) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) -
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(2*2^(1/3)*a^(1/3))))/(
3*2^(1/3)*a^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(
1/3)]/(3*2^(2/3)*a^(2/3))))/9 + (-1/3*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/
(a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/a^(1/3) - Log[2*2^(1/3) +
(a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)]/(2*a^(1/3)))/(3*2^(2/3)*a^(1/3)))/9))/(2*a^(2/3)*b^(1
/3))))/(4*a))/(10*a^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 927
```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[9*(a/(c*q))   Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
 x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 928
```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^3)^(2/3), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0] && EqQ[b*c + a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)^{\frac{8}{3}}}d x$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

output
```
int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{8/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{8/3}}\,dx = -\int \frac{1}{-a^3\left(a+bx^3\right)^{\frac{2}{3}}-a^2bx^3\left(a+bx^3\right)^{\frac{2}{3}}+ab^2x^6\left(a+bx^3\right)^{\frac{2}{3}}+b^3x^9\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
```

output
```
-Integral(1/(-a**3*(a + b*x**3)**(2/3) - a**2*b*x**3*(a + b*x**3)**(2/3) +
 a*b**2*x**6*(a + b*x**3)**(2/3) + b**3*x**9*(a + b*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^3)\left(a+bx^3\right)^{8/3}}\,dx=\int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int -\frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

input
```
integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(a-b\,x^3)}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^3+(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3-(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6-(b\,x^3+a)^{\frac{2}{3}}\,b^3x^9}$$

input
```
int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**3 + (a + b*x**3)**(2/3)*a**2*b*x**3 - (a + b
*x**3)**(2/3)*a*b**2*x**6 - (a + b*x**3)**(2/3)*b**3*x**9),x)
```

3.80 $\int (a-bx^3)^2 (a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int (a-bx^3)^2 (a+bx^3)^{2/3}\,dx = \frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{11}{27}ax(a+bx^3)^{5/3} + \frac{1}{9}bx^4(a+bx^3)^{5/3} + \frac{76a^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{38a^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81\sqrt[3]{b}}$$

output

```
38/81*a^2*x*(b*x^3+a)^(2/3)-11/27*a*x*(b*x^3+a)^(5/3)+1/9*b*x^4*(b*x^3+a)^
(5/3)+76/243*a^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/
2)/b^(1/3)-38/81*a^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.25

$$\int (a-bx^3)^2 (a+bx^3)^{2/3}\,dx = \frac{3\sqrt[3]{b}(a+bx^3)^{2/3}(5a^2x-24abx^4+9b^2x^7)+76\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-76a^3\log}{243\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(1/3)*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + 76*Sqrt[
3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 76*
a^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 38*a^3*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(1/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {933, 27, 913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int 2ab\left(5a-8bx^3\right)\left(bx^3+a\right)^{2/3}dx}{9b}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{9}a\int \left(5a-8bx^3\right)\left(bx^3+a\right)^{2/3}dx-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\int \left(bx^3+a\right)^{2/3}dx-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{2}{9}a\left(\frac{19}{3}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{4}{3}x\left(a+bx^3\right)^{5/3}\right)-$$

$$\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

output
```
-1/9*(x*(a - b*x^3)*(a + b*x^3)^(5/3)) + (2*a*((-4*x*(a + b*x^3)^(5/3))/3
+ (19*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1
/3)]/(2*b^(1/3))))/3))/3))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{27 b^{\frac{7}{3}} \left(b \,x^{3}+a \right)^{\frac{2}{3}} x^{7}-72 a \,b^{\frac{4}{3}} x^{4} \left(b \,x^{3}+a \right)^{\frac{2}{3}}+15 a^{2} x \,b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{2}{3}}-76 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(b^{\frac{1}{3}} x +2 \left(b \,x^{3}+a \right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a^{3}-76 \ln \left(\frac{-b^{\frac{1}{3}} \cdots}{\cdots}\right) \cdots}{243 b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/243*(27*b^(7/3)*(b*x^3+a)^(2/3)*x^7-72*a*b^(4/3)*x^4*(b*x^3+a)^(2/3)+15*
a^2*x*b^(1/3)*(b*x^3+a)^(2/3)-76*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(
b*x^3+a)^(1/3))/b^(1/3)/x)*a^3-76*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^3+3
8*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^3)/b^(
1/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.14

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3}\,dx = \left[\frac{114\sqrt{\frac{1}{3}}a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2\right.\right.}{}\right.$$

$$-\frac{228\,\sqrt{\frac{1}{3}}a^3b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+76\,a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-3}{243\,b}$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)
^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*
b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a
^3*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3
)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1
/243*(228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3
)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(
((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*
x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*
x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.94

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx = \frac{a^{\frac{8}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{5}{3}}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

output
```
a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*h
yper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(103) = 206.

Time = 0.13 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.12

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx = \text{Too large to display}$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*
arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^
2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/
3) + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2
/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3
+ (b*x^3 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(
1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^
3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/
3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 1
1*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x
^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

Giac [F]

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \int (bx^3+a)^{\frac{2}{3}} (bx^3-a)^2 \, dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^3)^2 (a+bx^3)^{2/3} \, dx = \int (b\,x^3+a)^{2/3}\,(a-b\,x^3)^2 \, dx$$

input
```
int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

Reduce [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3}\,dx = \frac{5(b\,x^3+a)^{\frac{2}{3}}\,a^2x}{81} - \frac{8(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^4}{27} + \frac{(b\,x^3+a)^{\frac{2}{3}}\,b^2x^7}{9} + \frac{76\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^3}{81}$$

input

```
int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

output

```
(5*(a + b*x**3)**(2/3)*a**2*x - 24*(a + b*x**3)**(2/3)*a*b*x**4 + 9*(a + b
*x**3)**(2/3)*b**2*x**7 + 76*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**3)
/81
```

3.81 $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{8}{9}ax\left(a+bx^3\right)^{2/3}+\frac{1}{6}bx^4\left(a+bx^3\right)^{2/3} + \frac{17a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}$$

output
```
-8/9*a*x*(b*x^3+a)^(2/3)+1/6*b*x^4*(b*x^3+a)^(2/3)+17/27*a^2*arctan(1/3*(1
+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-17/18*a^2*ln(-b^(1/
3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(a+bx^3\right)^{2/3}\left(-16ax+3bx^4\right) + \frac{17a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27\sqrt[3]{b}} + \frac{17a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{54\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-16*a*x + 3*b*x^4))/18 + (17*a^2*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (17*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (17*a^2*Log[b^(2/3)*x^
2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {933, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \frac{ab(7a-13bx^3)}{\sqrt[3]{bx^3+a}}dx}{6b} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{7a-13bx^3}{\sqrt[3]{bx^3+a}}dx-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$\Big\downarrow$ 913

$$\frac{1}{6}a\left(\frac{34}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{13}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

$\Big\downarrow$ 769

$$\frac{1}{6}a\left(\frac{34}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{13}{3}x\left(a+bx^3\right)^{2/3}\right)-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
-1/6*(x*(a - b*x^3)*(a + b*x^3)^(2/3)) + (a*((-13*x*(a + b*x^3)^(2/3))/3 +
 (34*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(
1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$\frac{9\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}} x^{4}-48 a x \left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}-34\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-34\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}+17\ln\left(\right.}{54 b^{\frac{1}{3}}}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*(b*x^3+a)^(2/3)*b^(4/3)*x^4-48*a*x*(b*x^3+a)^(2/3)*b^(1/3)-34*3^(1
/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a^2-34*ln(
(-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+17*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.47

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)\right.\right.}{}\right.$$

$$-\frac{102\,\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+34\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)}{54\,b}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(
1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*
log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/
x^2) + 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)
*a^2*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*
x^3 + a)^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*
x^3 + a)^(2/3))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

output
```
a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(88) = 176.

Time = 0.12 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.79

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*
(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a
^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)
^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x
^3 + (b*x^3 + a)^2*b^2/x^6))*b^2
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b^2 - 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)ab + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)*b**2 - 2*int(x**3/(a + b*x**3)**(1/3),x)*a
*b + int(1/(a + b*x**3)**(1/3),x)*a**2
```

3.82 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{4ax}{\sqrt[3]{a+bx^3}} + \frac{1}{3}x(a+bx^3)^{2/3} - \frac{10a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

output
```
4*a*x/(b*x^3+a)^(1/3)+1/3*x*(b*x^3+a)^(2/3)-10/9*a*arctan(1/3*(1+2*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)+5/3*a*ln(-b^(1/3)*x+(b*x^3+a)
^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\left(\frac{3(13ax+bx^4)}{\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{10a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{b}} - \frac{5a\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{\sqrt[3]{b}}\right)$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((3*(13*a*x + b*x^4))/(a + b*x^3)^(1/3) - (10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(1/3) + (10*a*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)]])/b^(1/3) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/b^(1/3))/9
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {930, 25, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int -\frac{ab(a-7bx^3)}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}-\frac{\int \frac{ab(a-7bx^3)}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}-\int \frac{a-7bx^3}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{913}$$

$$-\frac{10}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{769}$$

$$-\frac{10}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
(2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*(A
rcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

method	result
pseudoelliptic	$-\frac{a\left(-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x}{a}-\frac{10\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}+\frac{5\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{3b^{\frac{1}{3}}}-\frac{10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}} \cdots\right.}{3x}\right.}{3b^{\frac{1}{3}}} \cdots\right)}{3}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*(-(b*x^3+a)^(2/3)/a*x-10/3/b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/
x)+5/3/b^(1/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/
x^2)-10/3*3^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)
/x)-12*x/(b*x^3+a)^(1/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.89

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left[\frac{15\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)\right.\right.}{}\right.$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b
*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)
*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*
x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 +
a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^
(2/3))/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1
/9*(10*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5
*(a*b*x^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3
)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13
*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.79

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,b^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} + \frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}} - \frac{2\,a\log\left(b^{\frac{2}{3}}+\cdots\right.}{\cdots}\right.$$

$$+\frac{1}{3}\,ab\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(-\cdots\right.}{\cdots}\right.$$

$$+\frac{ax}{(bx^3+a)^{\frac{1}{3}}}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/
x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3)
+ (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^
(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)b^2 - 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)ab + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*b**2 - 2*
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*a*b + int
(1/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*a**2
```

3.83 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{ax}{(a+bx^3)^{4/3}} - \frac{x}{\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output

```
a*x/(b*x^3+a)^(4/3)-x/(b*x^3+a)^(1/3)+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(
1/3)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}} + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + \log\left(b^2\right.}{6\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)
/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(6*b^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {930, 27, 910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{2ab(2bx^3+a)}{(bx^3+a)^{4/3}}dx}{4ab} + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2bx^3+a}{(bx^3+a)^{4/3}}dx + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{2}\left(2\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{x}{\sqrt[3]{a+bx^3}}\right)+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{2}\left(2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{x}{\sqrt[3]{a+bx^3}}\right)+\frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
(x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) + (-(x/(a + b*x^3)^(1/3)) + 2*(ArcTa
n[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-
(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}+\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6b^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)}{3b^{\frac{1}{3}}}-\frac{b}{(b\,x^{3}}$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/6/b^(1/3)*ln((b^(2/3)*x^
2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-1/3*3^(1/2)/b^(1/3)*arct
an(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)-b*x^4/(b*x^3+a)^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(78) = 156.

Time = 0.14 (sec) , antiderivative size = 521, normalized size of antiderivative = 5.32

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx=\text{Too large to display}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
[-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2
 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)
^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 +
a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a
*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3
)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b
*x^3 + a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqr
t(-(-b)^(1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(
1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)]
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.84

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}} - \frac{bx^4}{2\,(bx^3+a)^{\frac{4}{3}}} - \frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \right.$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)
^(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*s
qrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/
3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)b^2 - 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)ab + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a +
 b*x**3)**(1/3)*b**2*x**6),x)*b**2 - 2*int(x**3/((a + b*x**3)**(1/3)*a**2
+ 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*a*b +
 int(1/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b
*x**3)**(1/3)*b**2*x**6),x)*a**2
```

3.84 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{4ax}{7(a+bx^3)^{7/3}} - \frac{x}{7(a+bx^3)^{4/3}} + \frac{4x}{7a\sqrt[3]{a+bx^3}}$$

output
```
4/7*a*x/(b*x^3+a)^(7/3)-1/7*x/(b*x^3+a)^(4/3)+4/7*x/a/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{7a^2x + 7abx^4 + 4b^2x^7}{7a(a+bx^3)^{7/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(7*a^2*x + 7*a*b*x^4 + 4*b^2*x^7)/(7*a*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.53, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{6}{7}\int \frac{a-bx^3}{(bx^3+a)^{7/3}}dx+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{6}{7}\left(\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{6}{7}\left(\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*((x*(a - b*x^3))/(4*a*(a +
b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))))/7
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37
trager	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37
pseudoelliptic	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37
orering	$\frac{x\left(4b^{2}x^{6}+7ab\,x^{3}+7a^{2}\right)}{7\left(b\,x^{3}+a\right)^{\frac{7}{3}}a}$	37

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{(4\,b^2x^7+7\,abx^4+7\,a^2x)(bx^3+a)^{\frac{2}{3}}}{7\,(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2
*b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.98

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)bx^7}{14\,(bx^3+a)^{\frac{7}{3}}a} + \frac{b^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)x^7}{14\,(bx^3+a)^{\frac{7}{3}}a}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/(
(b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^
2/x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{4\,x\,(b\,x^3+a)^2+4\,a^2\,x-a\,x\,(b\,x^3+a)}{7\,a\,(b\,x^3+a)^{7/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
```

output
```
(4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)ab$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*b**2
- 2*int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3
 + 3*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*a
*b + int(1/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 +
 3*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*a**
2
```

3.85 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{2ax}{5(a+bx^3)^{10/3}} - \frac{2x}{35(a+bx^3)^{7/3}} + \frac{23x}{140a(a+bx^3)^{4/3}} + \frac{69x}{140a^2\sqrt[3]{a+bx^3}}$$

output
```
2/5*a*x/(b*x^3+a)^(10/3)-2/35*x/(b*x^3+a)^(7/3)+23/140*x/a/(b*x^3+a)^(4/3)
+69/140*x/a^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{140a^3x + 245a^2bx^4 + 230ab^2x^7 + 69b^3x^{10}}{140a^2(a+bx^3)^{10/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(140*a^3*x + 245*a^2*b*x^4 + 230*a*b^2*x^7 + 69*b^3*x^10)/(140*a^2*(a + b*
x^3)^(10/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {907, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{19\int \frac{(a-bx^3)^2}{(bx^3+a)^{10/3}}dx}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 903}$$

$$\frac{19\left(\frac{6}{7}\int \frac{a-bx^3}{(bx^3+a)^{7/3}}dx+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 903}$$

$$\frac{19\left(\frac{6}{7}\left(\frac{3}{4}\int \frac{1}{(bx^3+a)^{4/3}}dx+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)+\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{20a}+\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19\left(\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{6}{7}\left(\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}\right)\right)}{20a}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*((x*(a - b*x^3)^2)/(7*
a*(a + b*x^3)^(7/3)) + (6*((x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)
/(4*a*(a + b*x^3)^(1/3))))/7))/(20*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(69 b^{3} x^{9}+230 a\,b^{2} x^{6}+245 a^{2} b\,x^{3}+140 a^{3}\right)}{140 \left(b\,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48
trager	$\frac{x\left(69 b^{3} x^{9}+230 a\,b^{2} x^{6}+245 a^{2} b\,x^{3}+140 a^{3}\right)}{140 \left(b\,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48
pseudoelliptic	$\frac{x\left(69 b^{3} x^{9}+230 a\,b^{2} x^{6}+245 a^{2} b\,x^{3}+140 a^{3}\right)}{140 \left(b\,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48
orering	$\frac{x\left(69 b^{3} x^{9}+230 a\,b^{2} x^{6}+245 a^{2} b\,x^{3}+140 a^{3}\right)}{140 \left(b\,x^{3}+a \right)^{\frac{10}{3}} a^{2}}$	48

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/
a^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x)(bx^3+a)^{\frac{2}{3}}}{140\,(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a
)^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(56) = 112$.

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)b^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^2}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*
(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)
^(10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*
b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{69\,x}{140\,a^2\,(b\,x^3+a)^{1/3}} - \frac{2\,x}{35\,(b\,x^3+a)^{7/3}} + \frac{23\,x}{140\,a\,(b\,x^3+a)^{4/3}} + \frac{2\,a\,x}{5\,(b\,x^3+a)^{10/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
```

output
```
(69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)
/(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9 \cdots\right.$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{1} \cdots\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}} \cdots\right.$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6
*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 +
(a + b*x**3)**(1/3)*b**4*x**12),x)*b**2 - 2*int(x**3/((a + b*x**3)**(1/3)*
a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2
*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 + (a + b*x**3)**(1/3)*b**4*x**12
),x)*a*b + int(1/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*
x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3
*x**9 + (a + b*x**3)**(1/3)*b**4*x**12),x)*a**2
```

3.86 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}} dx = \frac{4ax}{13(a+bx^3)^{13/3}} - \frac{2x}{65(a+bx^3)^{10/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{423x}{910a^3\sqrt[3]{a+bx^3}}$$

output
```
4/13*a*x/(b*x^3+a)^(13/3)-2/65*x/(b*x^3+a)^(10/3)+47/455*x/a/(b*x^3+a)^(7/
3)+141/910*x/a^2/(b*x^3+a)^(4/3)+423/910*x/a^3/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}} dx = \frac{910a^4x + 2275a^3bx^4 + 3055a^2b^2x^7 + 1833ab^3x^{10} + 423b^4x^{13}}{910a^3(a+bx^3)^{13/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(910*a^4*x + 2275*a^3*b*x^4 + 3055*a^2*b^2*x^7 + 1833*a*b^3*x^10 + 423*b^4
*x^13)/(910*a^3*(a + b*x^3)^(13/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {930, 27, 910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{ab(11a-5bx^3)}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{13}\int \frac{11a-5bx^3}{(bx^3+a)^{13/3}}dx+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{13}\left(\frac{47}{5}\int \frac{1}{(bx^3+a)^{10/3}}dx+\frac{8x}{5(a+bx^3)^{10/3}}\right)+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)+\frac{8x}{5(a+bx^3)^{10/3}}\right)+\frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{8x}{5\left(a+bx^3\right)^{10/3}}\right)+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

$\downarrow$ 746

$$\frac{1}{13}\left(\frac{47}{5}\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)+\frac{8x}{5\left(a+bx^3\right)^{10/3}}\right)+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + ((8*x)/(5*(a + b*x^3)^(10/3))
+ (47*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(
4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/5)/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x\left(423 b^{4} x^{12}+1833 a \,b^{3} x^{9}+3055 a^{2} b^{2} x^{6}+2275 a^{3} b \,x^{3}+910 a^{4}\right)}{910 \left(b \,x^{3}+a \right)^{\frac{13}{3}} a^{3}}$	59
trager	$\frac{x\left(423 b^{4} x^{12}+1833 a \,b^{3} x^{9}+3055 a^{2} b^{2} x^{6}+2275 a^{3} b \,x^{3}+910 a^{4}\right)}{910 \left(b \,x^{3}+a \right)^{\frac{13}{3}} a^{3}}$	59
pseudoelliptic	$\frac{x\left(423 b^{4} x^{12}+1833 a \,b^{3} x^{9}+3055 a^{2} b^{2} x^{6}+2275 a^{3} b \,x^{3}+910 a^{4}\right)}{910 \left(b \,x^{3}+a \right)^{\frac{13}{3}} a^{3}}$	59
orering	$\frac{x\left(423 b^{4} x^{12}+1833 a \,b^{3} x^{9}+3055 a^{2} b^{2} x^{6}+2275 a^{3} b \,x^{3}+910 a^{4}\right)}{910 \left(b \,x^{3}+a \right)^{\frac{13}{3}} a^{3}}$	59

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a
^4)/(b*x^3+a)^(13/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.24

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(423\,b^4x^{13}+1833\,ab^3x^{10}+3055\,a^2b^2x^7+2275\,a^3bx^4+910\,a^4x)(bx^3+a)^{\frac{2}{3}}}{910\,(a^3b^5x^{15}+5\,a^4b^4x^{12}+10\,a^5b^3x^9+10\,a^6b^2x^6+5\,a^7bx^3+a^8)}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4
+ 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3
*x^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(71) = 142$.

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.26

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{\left(35\,b^2 - \frac{91\,(bx^3+a)b}{x^3} + \frac{65\,(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} + \frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^3} + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*
x^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x
^3 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) +
 1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455
*(b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^
3)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{423\,x}{910\,a^3\,(b\,x^3+a)^{1/3}} - \frac{2\,x}{65\,(b\,x^3+a)^{10/3}} + \frac{141\,x}{910\,a^2\,(b\,x^3+a)^{4/3}} + \frac{47\,x}{455\,a\,(b\,x^3+a)^{7/3}} + \frac{4\,a\,x}{13\,(b\,x^3+a)^{13/3}}$$

input

```
int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
```

output

```
(423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141
*x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*
x)/(13*(a + b*x^3)^(13/3))
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\right.$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\,a\right.$$

input

```
int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x)
```

output

```
int(x**6/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x**3 + 1
0*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*b**3*x*
*9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*x**15),
x)*b**2 - 2*int(x**3/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**
4*b*x**3 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*
a**2*b**3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*
b**5*x**15),x)*a*b + int(1/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/
3)*a**4*b*x**3 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**
(1/3)*a**2*b**3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**
(1/3)*b**5*x**15),x)*a**2
```

3.87 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx=\frac{ax}{4(a+bx^3)^{16/3}}-\frac{x}{52(a+bx^3)^{13/3}}+\frac{x}{13a(a+bx^3)^{10/3}}+\frac{9x}{91a^2(a+bx^3)^{7/3}}+\frac{27x}{182a^3(a+bx^3)^{4/3}}+\frac{81x}{182a^4\sqrt[3]{a+bx^3}}$$

output
```
1/4*a*x/(b*x^3+a)^(16/3)-1/52*x/(b*x^3+a)^(13/3)+1/13*x/a/(b*x^3+a)^(10/3)
+9/91*x/a^2/(b*x^3+a)^(7/3)+27/182*x/a^3/(b*x^3+a)^(4/3)+81/182*x/a^4/(b*x
^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx=\frac{364a^5x+1183a^4bx^4+2080a^3b^2x^7+1872a^2b^3x^{10}+864ab^4x^{13}+162b^5x^{16}}{364a^4(a+bx^3)^{16/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(364*a^5*x + 1183*a^4*b*x^4 + 2080*a^3*b^2*x^7 + 1872*a^2*b^3*x^10 + 864*a
*b^4*x^13 + 162*b^5*x^16)/(364*a^4*(a + b*x^3)^(16/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {930, 27, 910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{2ab(7a-4bx^3)}{(bx^3+a)^{16/3}}dx}{16ab}+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{7a-4bx^3}{(bx^3+a)^{16/3}}dx+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{1}{8}\left(\frac{80}{13}\int \frac{1}{(bx^3+a)^{13/3}}dx+\frac{11x}{13(a+bx^3)^{13/3}}\right)+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\int \frac{1}{(bx^3+a)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{11x}{13(a+bx^3)^{13/3}}\right)+\frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\int\frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{11x}{13\left(a+bx^3\right)^{13/3}}\right)+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

$\downarrow$ 749

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\left(\frac{3\int\frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{11x}{13\left(a+bx^3\right)^{13/3}}\right)+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

$\downarrow$ 746

$$\frac{1}{8}\left(\frac{80}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a\left(a+bx^3\right)^{10/3}}\right)+\frac{11x}{13\left(a+bx^3\right)^{13/3}}\right)+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + ((11*x)/(13*(a + b*x^3)^(13/3)) +
 (80*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/
(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(10*a)
))/13)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x\left(162 b^{5} x^{15}+864 a\,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b\,x^{3}+364 a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}} a^{4}}$	70
trager	$\frac{x\left(162 b^{5} x^{15}+864 a\,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b\,x^{3}+364 a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}} a^{4}}$	70
pseudoelliptic	$\frac{x\left(162 b^{5} x^{15}+864 a\,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b\,x^{3}+364 a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}} a^{4}}$	70
orering	$\frac{x\left(162 b^{5} x^{15}+864 a\,b^{4} x^{12}+1872 a^{2} b^{3} x^{9}+2080 a^{3} b^{2} x^{6}+1183 a^{4} b\,x^{3}+364 a^{5}\right)}{364\left(b\,x^{3}+a\right)^{\frac{16}{3}} a^{4}}$	70

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+118
3*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(162\,b^5x^{16}+864\,ab^4x^{13}+1872\,a^2b^3x^{10}+2080\,a^3b^2x^7+1183\,a^4bx^4+364\,a^5x)(bx^3+}{364\,(a^4b^6x^{18}+6\,a^5b^5x^{15}+15\,a^6b^4x^{12}+20\,a^7b^3x^9+15\,a^8b^2x^6+6\,a^9bx^3+a^{10})}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

output
```
1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^
7 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^
5*x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 +
 a^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(86) = 172.

Time = 0.05 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.34

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = -\frac{\left(455\,b^3 - \frac{1680\,(bx^3+a)b^2}{x^3} + \frac{2184\,(bx^3+a)^2 b}{x^6} - \frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4} - \frac{\left(455\,b^4 - \frac{2240\,(bx^3+a)b^3}{x^3} + \frac{4368\,(bx^3+a)^2b^2}{x^6} - \frac{4160\,(bx^3+a)^3b}{x^9} + \frac{1820\,(bx^3+a)^4}{x^{12}}\right)bx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^4} - \frac{\left(91\,b^5 - \frac{560\,(bx^3+a)b^4}{x^3} + \frac{1456\,(bx^3+a)^2b^3}{x^6} - \frac{2080\,(bx^3+a)^3b^2}{x^9} + \frac{1820\,(bx^3+a)^4b}{x^{12}} - \frac{1456\,(bx^3+a)^5}{x^{15}}\right)x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^4}$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

output
```
-1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 1
040*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4
 - 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a
)^3*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1
456*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*
(b*x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^1
5)*x^16/((b*x^3 + a)^(16/3)*a^4)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{81\,x}{182\,a^4\,(b\,x^3+a)^{1/3}} - \frac{x}{52\,(b\,x^3+a)^{13/3}} + \frac{27\,x}{182\,a^3\,(b\,x^3+a)^{4/3}} + \frac{9\,x}{91\,a^2\,(b\,x^3+a)^{7/3}} + \frac{x}{13\,a\,(b\,x^3+a)^{10/3}} + \frac{a\,x}{4\,(b\,x^3+a)^{16/3}}$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
```

output
```
(81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(1
82*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a
+ b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\right.$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^3x^9+15\,(b\,x^3+a)\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^3x^9+15\,(b\,x^3+a)^{\frac{1}{3}}\right.$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**6 + 6*(a + b*x**3)**(1/3)*a**5*b*x**3 + 1
5*(a + b*x**3)**(1/3)*a**4*b**2*x**6 + 20*(a + b*x**3)**(1/3)*a**3*b**3*x*
*9 + 15*(a + b*x**3)**(1/3)*a**2*b**4*x**12 + 6*(a + b*x**3)**(1/3)*a*b**5
*x**15 + (a + b*x**3)**(1/3)*b**6*x**18),x)*b**2 - 2*int(x**3/((a + b*x**3
)**(1/3)*a**6 + 6*(a + b*x**3)**(1/3)*a**5*b*x**3 + 15*(a + b*x**3)**(1/3)
*a**4*b**2*x**6 + 20*(a + b*x**3)**(1/3)*a**3*b**3*x**9 + 15*(a + b*x**3)*
*(1/3)*a**2*b**4*x**12 + 6*(a + b*x**3)**(1/3)*a*b**5*x**15 + (a + b*x**3)
**(1/3)*b**6*x**18),x)*a*b + int(1/((a + b*x**3)**(1/3)*a**6 + 6*(a + b*x*
*3)**(1/3)*a**5*b*x**3 + 15*(a + b*x**3)**(1/3)*a**4*b**2*x**6 + 20*(a + b
*x**3)**(1/3)*a**3*b**3*x**9 + 15*(a + b*x**3)**(1/3)*a**2*b**4*x**12 + 6*
(a + b*x**3)**(1/3)*a*b**5*x**15 + (a + b*x**3)**(1/3)*b**6*x**18),x)*a**2
```

3.88 $\int (a-bx^3)^2 (a+bx^3)^{4/3}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int (a-bx^3)^2 (a+bx^3)^{4/3}\, dx = -\frac{13}{44}ax(a+bx^3)^{7/3} + \frac{1}{11}bx^4(a+bx^3)^{7/3} + \frac{57a^3x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-13/44*a*x*(b*x^3+a)^(7/3)+1/11*b*x^4*(b*x^3+a)^(7/3)+57/44*a^3*x*(b*x^3+a
)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int (a-bx^3)^2 (a+bx^3)^{4/3}\, dx = \frac{x\left(106a^4+53a^3bx^3-78a^2b^2x^6-5ab^3x^9+20b^4x^{12}+114a^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F}\right.}{220(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

output
```
(x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 +
114*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)]))/(220*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int 6ab\left(2a-3bx^3\right)\left(bx^3+a\right)^{4/3}dx}{11b}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{11}a\int \left(2a-3bx^3\right)\left(bx^3+a\right)^{4/3}dx-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{913}$$

$$\frac{6}{11}a\left(\frac{19}{8}a\int \left(bx^3+a\right)^{4/3}dx-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{6}{11}a\left(\frac{19a^2\sqrt[3]{a+bx^3}\int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{8\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{6}{11}a\left(\frac{19a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}x\left(a+bx^3\right)^{7/3}\right)-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

output
```
-1/11*(x*(a - b*x^3)*(a + b*x^3)^(7/3)) + (6*a*((-3*x*(a + b*x^3)^(7/3))/8
 + (19*a^2*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)
/a)])/(8*(1 + (b*x^3)/a)^(1/3))))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}d x$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

output
```
int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} \, dx=\int \left(bx^{3}+a\right)^{\frac{4}{3}} \left(bx^{3}-a\right)^{2} \,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (a-bx^3)^2 (a+bx^3)^{4/3}\, dx = \frac{a^{\frac{10}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{7}{3}} b x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{a^{\frac{4}{3}} b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a} b^3 x^{10} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
```

output
```
a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hy
per((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(
1/3)*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(
I*pi)/a)/(3*gamma(13/3))
```

Maxima [F]

$$\int (a-bx^3)^2 (a+bx^3)^{4/3}\, dx = \int (bx^3+a)^{\frac{4}{3}} (bx^3-a)^2\, dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Giac [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} dx = \int \left(bx^3+a\right)^{\frac{4}{3}} \left(bx^3-a\right)^2 dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} dx = \int \left(b\,x^3+a\right)^{4/3} \left(a-b\,x^3\right)^2 dx$$

input
```
int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)
```

Reduce [F]

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} dx = \frac{53(b\,x^3+a)^{\frac{1}{3}}\,a^3x}{110} - \frac{53(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^4}{220} - \frac{5(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^7}{44} + \frac{(b\,x^3+a)^{\frac{1}{3}}\,b^3x^{10}}{11} + \frac{57\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^4}{110}$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

output
```
(106*(a + b*x**3)**(1/3)*a**3*x - 53*(a + b*x**3)**(1/3)*a**2*b*x**4 - 25*
(a + b*x**3)**(1/3)*a*b**2*x**7 + 20*(a + b*x**3)**(1/3)*b**3*x**10 + 114*
int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**4)/220
```

3.89 $\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = -\frac{1}{2}ax\left(a+bx^3\right)^{4/3}+\frac{1}{8}bx^4\left(a+bx^3\right)^{4/3} + \frac{3a^2x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*a*x*(b*x^3+a)^(4/3)+1/8*b*x^4*(b*x^3+a)^(4/3)+3/2*a^2*x*(b*x^3+a)^(1/
3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 5.75 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \frac{x\left(2a^3-a^2bx^3-2ab^2x^6+b^3x^9+6a^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{8\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

output
```
(x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int 3ab\left(3a-5bx^3\right)\sqrt[3]{bx^3+a}dx}{8b}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}a\int \left(3a-5bx^3\right)\sqrt[3]{bx^3+a}dx-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{913}$$

$$\frac{3}{8}a\left(4a\int \sqrt[3]{bx^3+a}dx-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{779}$$

$$\frac{3}{8}a\left(\frac{4a\sqrt[3]{a+bx^3}\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

$$\Big\downarrow \text{778}$$

$$\frac{3}{8}a\left(\frac{4ax\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}-x\left(a+bx^3\right)^{4/3}\right)-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

input
```
Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

output
```
-1/8*(x*(a - b*x^3)*(a + b*x^3)^(4/3)) + (3*a*(-(x*(a + b*x^3)^(4/3)) + (4
*a*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(1
 + (b*x^3)/a)^(1/3)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(-b\,x^{3}+a\right)^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}d x$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

output
```
int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(bx^3-a\right)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.42

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{4}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
```

output
```
a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*h
yper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int (a-bx^3)^2 \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(bx^3-a)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Giac [F]

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}} \left(bx^3-a\right)^2\,dx$$

input
```
integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\int \left(b\,x^3+a\right)^{1/3} \left(a-b\,x^3\right)^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(a - b*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(a - b*x^3)^2, x)
```

Reduce [F]

$$\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx=\frac{\left(b\,x^3+a\right)^{\frac{1}{3}} a^2 x}{4}-\frac{3\left(b\,x^3+a\right)^{\frac{1}{3}} ab\,x^4}{8}+\frac{\left(b\,x^3+a\right)^{\frac{1}{3}} b^2 x^7}{8}+\frac{3\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right) a^3}{4}$$

input
```
int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*x - 3*(a + b*x**3)**(1/3)*a*b*x**4 + (a + b*x*
*3)**(1/3)*b**2*x**7 + 6*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3)/8
```

3.90 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = -\frac{7}{5}ax\sqrt[3]{a+bx^3}+\frac{1}{5}bx^4\sqrt[3]{a+bx^3} + \frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

output
```
-7/5*a*x*(b*x^3+a)^(1/3)+1/5*b*x^4*(b*x^3+a)^(1/3)+12/5*a^2*x*(1+b*x^3/a)^
(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{-7a^2x-6abx^4+b^2x^7+12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \frac{6ab(a-2bx^3)}{(bx^3+a)^{2/3}}dx}{5b}-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{5}a\int \frac{a-2bx^3}{(bx^3+a)^{2/3}}dx-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{913}$$

$$\frac{6}{5}a\left(2a\int \frac{1}{(bx^3+a)^{2/3}}dx-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{779}$$

$$\frac{6}{5}a\left(\frac{2a\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{778}$$

$$\frac{6}{5}a\left(\frac{2ax\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-x\sqrt[3]{a+bx^3}\right)-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
-1/5*(x*(a - b*x^3)*(a + b*x^3)^(1/3)) + (6*a*(-(x*(a + b*x^3)^(1/3)) + (2
*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])
/(a + b*x^3)^(2/3)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}d x$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt[3]{a}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

output
```
a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx=\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}}\,dx=\left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b^2-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)ab+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

output
```
int(x**6/(a + b*x**3)**(2/3),x)*b**2 - 2*int(x**3/(a + b*x**3)**(2/3),x)*a
*b + int(1/(a + b*x**3)**(2/3),x)*a**2
```

3.91 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx = \frac{2ax}{(a+bx^3)^{2/3}} + \frac{1}{2}x\sqrt[3]{a+bx^3} - \frac{3ax\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}}$$

output
```
2*a*x/(b*x^3+a)^(2/3)+1/2*x*(b*x^3+a)^(1/3)-3/2*a*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} \, dx = \frac{5ax + bx^4 - 3ax\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2(a+bx^3)^{2/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4
/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {930, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{6ab^2x^3}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 27$$

$$3b\int \frac{x^3}{(bx^3+a)^{2/3}}dx+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 889$$

$$\frac{3b\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 888$$

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^
3 + a^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx=&\left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)b^2\\&-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)ab\\&+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)a^2\end{aligned}$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*b**2 - 2*
int(x**3/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*a*b + int
(1/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*a**2
```

3.92 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\frac{4ax}{5\,(a+bx^3)^{5/3}}-\frac{x}{(a+bx^3)^{2/3}}+\frac{6x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{2/3}}$$

output
```
4/5*a*x/(b*x^3+a)^(5/3)-x/(b*x^3+a)^(2/3)+6/5*x*(1+b*x^3/a)^(2/3)*hypergeo
m([1/3, 5/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\frac{2x(a-bx^3)+3x(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\,(a+bx^3)^{5/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {930, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{3ab}{(bx^3+a)^{2/3}}dx}{5ab}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{5}\int \frac{1}{(bx^3+a)^{2/3}}dx+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{5(a+bx^3)^{2/3}}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}}+\frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*
x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(-a+bx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
```

output
```
Integral((-a + b*x**3)**2/(a + b*x**3)**(8/3), x)
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx=\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}}\,dx &= \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)b^2\\
&\quad-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)ab\\
&\quad+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)a^2\end{aligned}$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a +
 b*x**3)**(2/3)*b**2*x**6),x)*b**2 - 2*int(x**3/((a + b*x**3)**(2/3)*a**2
+ 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*a*b +
 int(1/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b
*x**3)**(2/3)*b**2*x**6),x)*a**2
```

3.93 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \frac{ax}{2(a+bx^3)^{8/3}} - \frac{x}{4(a+bx^3)^{5/3}} + \frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}$$

output
```
1/2*a*x/(b*x^3+a)^(8/3)-1/4*x/(b*x^3+a)^(5/3)+3/4*x*(1+b*x^3/a)^(2/3)*hype
rgeom([1/3, 8/3],[4/3],-b*x^3/a)/a/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \frac{7a^2x+5abx^4+3b^2x^7+3x(a+bx^3)^2\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10a(a+bx^3)^{8/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

output
```
(7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {930, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{6a^2b}{(bx^3+a)^{8/3}}dx}{8ab}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}a\int \frac{1}{(bx^3+a)^{8/3}}dx+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{3\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}}dx}{4a(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}}+\frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

output
```
(x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperge
ometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{11}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3
*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{11}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{11/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{2}{3}}\,b^3x^9}dx\right)$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{2}{3}}\,b^3x^9}dx\right)ab$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{2}{3}}\,b^3x^9}dx\right)a^2$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

output

```
int(x**6/((a + b*x**3)**(2/3)*a**3 + 3*(a + b*x**3)**(2/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(2/3)*a*b**2*x**6 + (a + b*x**3)**(2/3)*b**3*x**9),x)*b**2
- 2*int(x**3/((a + b*x**3)**(2/3)*a**3 + 3*(a + b*x**3)**(2/3)*a**2*b*x**3
 + 3*(a + b*x**3)**(2/3)*a*b**2*x**6 + (a + b*x**3)**(2/3)*b**3*x**9),x)*a
*b + int(1/((a + b*x**3)**(2/3)*a**3 + 3*(a + b*x**3)**(2/3)*a**2*b*x**3 +
 3*(a + b*x**3)**(2/3)*a*b**2*x**6 + (a + b*x**3)**(2/3)*b**3*x**9),x)*a**
2
```

3.94 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \frac{4ax}{11\left(a+bx^3\right)^{11/3}} - \frac{x}{7\left(a+bx^3\right)^{8/3}} + \frac{60x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77a^2\left(a+bx^3\right)^{2/3}}$$

output
```
4/11*a*x/(b*x^3+a)^(11/3)-1/7*x/(b*x^3+a)^(8/3)+60/77*x*(1+b*x^3/a)^(2/3)*
hypergeom([1/3, 11/3],[4/3],-b*x^3/a)/a^2/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \frac{x\left(16a^3+23a^2bx^3+21ab^2x^6+6b^3x^9+6(a+bx^3)^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F}\right.}{22a^2\left(a+bx^3\right)^{11/3}}$$

input
```
Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

output
```
(x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1
+ (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^
2*(a + b*x^3)^(11/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {930, 27, 910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{3ab(3a-bx^3)}{(bx^3+a)^{11/3}}dx}{11ab}+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{11}\int \frac{3a-bx^3}{(bx^3+a)^{11/3}}dx+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{3}{11}\left(\frac{5}{2}\int \frac{1}{(bx^3+a)^{8/3}}dx+\frac{x}{2(a+bx^3)^{8/3}}\right)+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{3}{11}\left(\frac{5\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}}dx}{2a^2(a+bx^3)^{2/3}}+\frac{x}{2(a+bx^3)^{8/3}}\right)+\frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{3}{11}\left(\frac{5x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}+\frac{x}{2\left(a+bx^3\right)^{8/3}}\right)+\frac{2x(a-bx^3)}{11\left(a+bx^3\right)^{11/3}}$$

input
```
Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

output
```
(2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*(x/(2*(a + b*x^3)^(8/3)) +
(5*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(2*a^2*(a + b*x^3)^(2/3))))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(-b\,x^{3}+a\right)^{2}}{\left(b\,x^{3}+a\right)^{\frac{14}{3}}}dx$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

output
```
int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

Fricas [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4
*x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Giac [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx = \int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

input
```
integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx=\int \frac{(a-b\,x^3)^2}{(b\,x^3+a)^{14/3}}\,dx$$

input
```
int((a - b*x^3)^2/(a + b*x^3)^(14/3),x)
```

output
```
int((a - b*x^3)^2/(a + b*x^3)^(14/3), x)
```

Reduce [F]

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx=\left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{2}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^3x^9\right.$$
$$-2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{2}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{2}{3}}\,b^4x^{1}\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{2}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{2}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{2}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{2}{3}}\,b^4x^{12}}\right.$$

input
```
int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*a**4 + 4*(a + b*x**3)**(2/3)*a**3*b*x**3 + 6
*(a + b*x**3)**(2/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(2/3)*a*b**3*x**9 +
(a + b*x**3)**(2/3)*b**4*x**12),x)*b**2 - 2*int(x**3/((a + b*x**3)**(2/3)*
a**4 + 4*(a + b*x**3)**(2/3)*a**3*b*x**3 + 6*(a + b*x**3)**(2/3)*a**2*b**2
*x**6 + 4*(a + b*x**3)**(2/3)*a*b**3*x**9 + (a + b*x**3)**(2/3)*b**4*x**12
),x)*a*b + int(1/((a + b*x**3)**(2/3)*a**4 + 4*(a + b*x**3)**(2/3)*a**3*b*
x**3 + 6*(a + b*x**3)**(2/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(2/3)*a*b**3
*x**9 + (a + b*x**3)**(2/3)*b**4*x**12),x)*a**2
```

3.95 $\int \sqrt[3]{a+bx^3}(c+dx^3)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{d(4bc-ad)x(a+bx^3)^{4/3}}{10b^2}+\frac{d^2x^4(a+bx^3)^{4/3}}{8b} + \frac{(10b^2c^2-4abcd+a^2d^2)\,x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/10*d*(-a*d+4*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d^2*x^4*(b*x^3+a)^(4/3)/b+1/
10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3, 1/3],
[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 8.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{x\sqrt[3]{a+bx^3}\Big(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{1}{3}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)-3bx^3(11c}{280}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hy
pergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)
] - 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int \sqrt[3]{bx^3+a}\left(d(11bc-4ad)x^3+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{4(a^2d^2-4abcd+10b^2c^2)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \ 779$$

$$\frac{\frac{4\sqrt[3]{a+bx^3}(a^2d^2-4abcd+10b^2c^2)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{4x\sqrt[3]{a+bx^3}(a^2d^2-4abcd+10b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(4/3)*(c + d*x^3))/(8*b) + ((d*(11*b*c - 4*a*d)*x*(a + b*
x^3)^(4/3))/(5*b) + (4*(10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1
/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^
(1/3)))/(8*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{2}d x$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}(c+dx^3)^2\, dx = \int (bx^3+a)^{\frac{1}{3}}(dx^3+c)^2\, dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

output
```
a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
)
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$$
$$=\frac{-2(b\,x^3+a)^{\frac{1}{3}}\,a^2d^2x+8(b\,x^3+a)^{\frac{1}{3}}\,abcdx+(b\,x^3+a)^{\frac{1}{3}}\,ab\,d^2x^4+20(b\,x^3+a)^{\frac{1}{3}}\,b^2c^2x+16(b\,x^3+a)^{\frac{1}{3}}\,b^2}{4}$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

output
```
( - 2*(a + b*x**3)**(1/3)*a**2*d**2*x + 8*(a + b*x**3)**(1/3)*a*b*c*d*x +
(a + b*x**3)**(1/3)*a*b*d**2*x**4 + 20*(a + b*x**3)**(1/3)*b**2*c**2*x + 1
6*(a + b*x**3)**(1/3)*b**2*c*d*x**4 + 5*(a + b*x**3)**(1/3)*b**2*d**2*x**7
 + 2*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3*d**2 - 8*int((a + b*x**3
)**(1/3)/(a + b*x**3),x)*a**2*b*c*d + 20*int((a + b*x**3)**(1/3)/(a + b*x*
*3),x)*a*b**2*c**2)/(40*b**2)
```

3.96 $\int \sqrt[3]{a+bx^3}(c+dx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{dx(a+bx^3)^{4/3}}{5b} + \frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1
/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 5.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)+\frac{(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/
3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(5bc-ad)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^3}(5bc-ad)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperge
ometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)d x$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}(c+dx^3)\ dx=\int (bx^3+a)^{\frac{1}{3}}(dx^3+c)\ dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

output
```
a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx$$
$$=\frac{(b\,x^3+a)^{\frac{1}{3}}\,adx+5(b\,x^3+a)^{\frac{1}{3}}\,bcx+2(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^4-\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2d+5\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)abc}{10b}$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*a*d*x + 5*(a + b*x**3)**(1/3)*b*c*x + 2*(a + b*x**3)*
*(1/3)*b*d*x**4 - int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*d + 5*int((
a + b*x**3)**(1/3)/(a + b*x**3),x)*a*b*c)/(10*b)
```

3.97 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 10.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$=\frac{4acx\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\text{Ap}\right.\right.}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{a+bx^3} \,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c + d*x**3),x)
```

3.98 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx=\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$
$$=\frac{x\left(\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d\right.\right.}{-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\right.\right.}}{c+dx^3}\right.}{12\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -
((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
]))))/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{(dx^3+c)^2}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^3}\operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{(d\,x^{3}+c)^{2}}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d^2x^6+2cd\,x^3+c^2}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int((a + b*x**3)**(1/3)/(c**2 + 2*c*d*x**3 + d**2*x**6),x)
```

3.99 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{2d(3bc-ad)x(a+bx^3)^{2/3}}{9b^2}+\frac{d^2x^4(a+bx^3)^{2/3}}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

output

```
2/9*d*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d^2*x^4*(b*x^3+a)^(2/3)/b+1/2
7*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))*3^(1/2)/b^(7/3)-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/
3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{3\sqrt[3]{b}dx(a+bx^3)^{2/3}(-4ad+3b(4c+dx^3))+2\sqrt{3}(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
(3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + 2*Sqrt[3]*
(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
+ 2*(a + b*x^3)^(1/3))] - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(7/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {933, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \frac{d(9bc-4ad)x^3+c(6bc-ad)}{\sqrt[3]{bx^3+a}}dx}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d*x*(a + b*x^3)^(2/3)*(c + d*x^3))/(6*b) + ((d*(9*b*c - 4*a*d)*x*(a + b*x
^3)^(2/3))/(3*b) + (2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(ArcTan[(1 + (2*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b))/(6*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(-3\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{d\,x^{3}}{4}+c\right)dx\,b^{\frac{4}{3}}+a\,d^{2}x\,b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\frac{\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right.}{}\right.}{9b^{\frac{7}{3}}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-3*(b*x^3+a)^(2/3)*(1/4*d*x^3+c)*d*x*b^(4/3)+a*d^2*x*b^(1/3)*(b*x^3+
a)^(2/3)+1/3*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(
1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-3*a*b*c*d+9/2*b^2*c^2))/b^(
7/3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)
/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(
1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*s
qrt((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3
)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a
^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b
*x^3 + a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*
b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b
)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d
 + 2*a^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3
)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)
*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper
((1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(145) = 290$.

Time = 0.12 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*
(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a
^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)
^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x
^3 + (b*x^3 + a)^2*b^2/x^6))*d^2
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)d^2+2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)*d**2 + 2*int(x**3/(a + b*x**3)**(1/3),x)*c
*d + int(1/(a + b*x**3)**(1/3),x)*c**2
```

3.100 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}}{3b} + \frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
1/3*d*x*(b*x^3+a)^(2/3)/b+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3
+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{6\sqrt[3]{b}dx(a+bx^3)^{2/3}+2\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2(-3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx}\right.}{18b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(1/3)*d*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*b*c + a*d)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + (3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$\Big\downarrow \text{913}$$
$$\frac{(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{dx(a+bx^3)^{2/3}}{3b}$$
$$\Big\downarrow \text{769}$$

$$\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)]/(2*b^(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 1.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$6dx\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)ad-6\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)bc+2\ln\left(\frac{-b^{\frac{1}{3}}x+(b}{x}\right.$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*d*x*(b*x^3+a)^(2/3)*b^(1/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a*d-6*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*b*c+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a
*d-6*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*b*c-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d+3*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b*c)/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx - 3\sqrt{\frac{1}{3}}\left(3\,b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}\right.\right.}{} \right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b
^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/
3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(
3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*
d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2
/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*
(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input

```
integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

output

```
c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.20

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

Giac [F]

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{d\,x^3+c}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)d+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)*d + int(1/(a + b*x**3)**(1/3),x)*c
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(2/3)/(-a*d+b*c)^(1/3)+1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1
/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+(1+i\sqrt{3})\left(2\log\left(2\sqrt[3]{bc-ad}x+(1+\right.\right.}{12}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt
[3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(
12*c^(2/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$\Big\downarrow \; 901$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/
(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d
)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/
3)*(b*c - a*d)^(1/3))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3)
)/((a*d-b*c)/c)^(1/3)/x)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-l
n((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))/((a*d-b*c)/c)^(1/3)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.102 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)\,(c+dx^3)} + \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}$$

output
```
-1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/9*(-2*a*d+3*b*c)*arctan(
1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5
/3)/(-a*d+b*c)^(4/3)+1/18*(-2*a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4
/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5
/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$= \frac{-12c^{2/3}d\sqrt[3]{bc-ad}x(a+bx^3)^{2/3}+2\left(3-i\sqrt{3}\right)(3bc-2ad)\left(c+dx^3\right)\operatorname{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
(-12*c^(2/3)*d*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) + 2*(3 - I*Sqrt[3])*(
3*b*c - 2*a*d)*(c + d*x^3)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]] + 2*(1 + I*Sqrt[3])*(3*b*c - 2*a*
d)*(c + d*x^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3)] - I*(-I + Sqrt[3])*(3*b*c - 2*a*d)*(c + d*x^3)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1
/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^
(4/3)*(c + d*x^3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d
)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c -
a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-\frac{\left(d\,x^3+c\right)\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9}+\frac{2\left(d\,x^3+c\right)\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right.}{x}\right.}{9}}{(ad-bc)c^2(d\,x^3+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input

```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output

```
2/9/((a*d-b*c)/c)^(1/3)*(-1/2*(d*x^3+c)*(a*d-3/2*b*c)*ln((((a*d-b*c)/c)^(2
/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(d*x^3
+c)*(a*d-3/2*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*d*(b*x
^3+a)^(2/3)*x*c*((a*d-b*c)/c)^(1/3)+(d*x^3+c)*arctan(1/3*3^(1/2)*(-2/((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)*(a*d-3/2*b*c))/(a*d-b*c)/c^2/
(d*x^3+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx=\text{Timed out}$$

input

```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,cd\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,d^2x^6}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c**2 + 2*(a + b*x**3)**(1/3)*c*d*x**3 + (a + b*
x**3)**(1/3)*d**2*x**6),x)
```

3.103 $\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)^3x}{ab^3\sqrt[3]{a+bx^3}} + \frac{d^2(9bc-5ad)x(a+bx^3)^{2/3}}{9b^3} + \frac{d^3x^4(a+bx^3)^{2/3}}{6b^2} + \frac{d(27b^2c^2-36abcd+14a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{10/3}} - \frac{d(27b^2c^2-36abcd+14a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{10/3}}$$

output
```
(-a*d+b*c)^3*x/a/b^3/(b*x^3+a)^(1/3)+1/9*d^2*(-5*a*d+9*b*c)*x*(b*x^3+a)^(2
/3)/b^3+1/6*d^3*x^4*(b*x^3+a)^(2/3)/b^2+1/27*d*(14*a^2*d^2-36*a*b*c*d+27*b
^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(10/
3)-1/18*d*(14*a^2*d^2-36*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3)
)/b^(10/3)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{b}x(18b^3c^3-28a^3d^3+a^2bd^2(72c-7dx^3)+3ab^2d(-18c^2+6cdx^3+d^2x^6))}{a\sqrt[3]{a+bx^3}}+2\sqrt{3}d(27b^2c^2-36abcd+14}{}$$

input
```
Integrate[(c + d*x^3)^3/(a + b*x^3)^(4/3),x]
```

output
```
((3*b^(1/3)*x*(18*b^3*c^3 - 28*a^3*d^3 + a^2*b*d^2*(72*c - 7*d*x^3) + 3*a*
b^2*d*(-18*c^2 + 6*c*d*x^3 + d^2*x^6)))/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*
d*(27*b^2*c^2 - 36*a*b*c*d + 14*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/
3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*(27*b^2*c^2 - 36*a*b*c*d + 14*a^2*d^2)*
Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + d*(27*b^2*c^2 - 36*a*b*c*d + 14*a^
2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(54*b^(10/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {930, 27, 1025, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(dx^3+c)(ac-(6bc-7ad)x^3)}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x(c+dx^3)^2(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{(dx^3+c)(ac-(6bc-7ad)x^3)}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x(c+dx^3)^2(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$\big\downarrow$ 1025

$$\frac{d\left(\frac{\int \frac{ac(12bc-7ad)-\left(18b^2c^2-51abdc+28a^2d^2\right)x^3}{\sqrt[3]{bx^3+a}}dx}{6b}-\frac{x(a+bx^3)^{2/3}(c+dx^3)(6bc-7ad)}{6b}\right)}{ab}+\frac{x(c+dx^3)^2(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$\big\downarrow$ 913

$$\frac{d\left(\frac{\frac{2a\left(14a^2d^2-36abcd+27b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}-\frac{x\left(a+bx^3\right)^{2/3}\left(28a^2d^2-51abcd+18b^2c^2\right)}{3b}}{6b}-\frac{x(a+bx^3)^{2/3}(c+dx^3)(6bc-7ad)}{6b}\right)}{ab}+\frac{x(c+dx^3)^2(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$\big\downarrow$ 769

$$\frac{d\left(\frac{\frac{2a\left(14a^2d^2-36abcd+27b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}-\frac{x\left(a+bx^3\right)^{2/3}\left(28a^2d^2-51abcd+18b^2c^2\right)}{3b}}{6b}-\frac{x(a+bx^3)^{2}\cdots}{}\right)}{ab}$$

$$\frac{x(c+dx^3)^2(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)^3/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3)^2)/(a*b*(a + b*x^3)^(1/3)) + (d*(-1/6*((6*b*c -
 7*a*d)*x*(a + b*x^3)^(2/3)*(c + d*x^3))/b + (-1/3*((18*b^2*c^2 - 51*a*b*c
*d + 28*a^2*d^2)*x*(a + b*x^3)^(2/3))/b + (2*a*(27*b^2*c^2 - 36*a*b*c*d +
14*a^2*d^2)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3
]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b))/(6
*b)))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{14\left(ad\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right)(a}{27(b\,x^{3}+a}$

input
```
int((d*x^3+c)^3/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-14/27/(b*x^3+a)^(1/3)/b^(10/3)*(a*d*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-18/
7*a*b*c*d+27/14*b^2*c^2)*(b*x^3+a)^(1/3)+3*x*(27/14*a*d*(-1/18*d^2*x^6-1/3
*c*d*x^3+c^2)*b^(7/3)-18/7*a^2*d^2*(-7/72*d*x^3+c)*b^(4/3)+a^3*d^3*b^(1/3)
-9/14*b^(10/3)*c^3))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(176) = 352.

Time = 0.14 (sec) , antiderivative size = 956, normalized size of antiderivative = 4.66

$$\int \frac{\left(c+dx^3\right)^3}{\left(a+bx^3\right)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*(27*a^2*b^3*c^2*d - 36*a^3*b^2*c*d^2 + 14*a^4*b*d^3 + (
27*a*b^4*c^2*d - 36*a^2*b^3*c*d^2 + 14*a^3*b^2*d^3)*x^3)*sqrt((-b)^(1/3)/b
)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqr
t((-b)^(1/3)/b) + 2*a) - 2*(27*a^2*b^2*c^2*d - 36*a^3*b*c*d^2 + 14*a^4*d^3
 + (27*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 14*a^3*b*d^3)*x^3)*(-b)^(2/3)*log(
((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a^2*b^2*c^2*d - 36*a^3*b*c*d^2
 + 14*a^4*d^3 + (27*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 14*a^3*b*d^3)*x^3)*(-
b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2) + 3*(3*a*b^3*d^3*x^7 + (18*a*b^3*c*d^2 - 7*a^2*b^2*d^3)*x^4
+ 2*(9*b^4*c^3 - 27*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 14*a^3*b*d^3)*x)*(b*x
^3 + a)^(2/3))/(a*b^5*x^3 + a^2*b^4), -1/54*(6*sqrt(1/3)*(27*a^2*b^3*c^2*d
 - 36*a^3*b^2*c*d^2 + 14*a^4*b*d^3 + (27*a*b^4*c^2*d - 36*a^2*b^3*c*d^2 +
14*a^3*b^2*d^3)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x -
 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(27*a^2*b^2*c^2*d - 36*a^
3*b*c*d^2 + 14*a^4*d^3 + (27*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 14*a^3*b*d^3
)*x^3)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a^2*b^2*
c^2*d - 36*a^3*b*c*d^2 + 14*a^4*d^3 + (27*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 +
 14*a^3*b*d^3)*x^3)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b
)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*a*b^3*d^3*x^7 + (18*a*b^3*c*...
```

Sympy [F]

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{(c+dx^3)^3}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**3/(b*x**3+a)**(4/3),x)
```

output
```
Integral((c + d*x**3)**3/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(176) = 352$.

Time = 0.12 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.51

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/54*d^3*(28*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3
)/x)/b^(1/3))/b^(10/3) + 3*(18*a^2*b^2 - 49*(b*x^3 + a)*a^2*b/x^3 + 28*(b*
x^3 + a)^2*a^2/x^6)/((b*x^3 + a)^(1/3)*b^5/x - 2*(b*x^3 + a)^(4/3)*b^4/x^4
 + (b*x^3 + a)^(7/3)*b^3/x^7) - 14*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(
1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(10/3) + 28*a^2*log(-b^(1/3) + (b*x^3 +
a)^(1/3)/x)/b^(10/3)) + 1/3*c*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3)
 + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^
3)/((b*x^3 + a)^(1/3)*b^3/x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3)
 + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(
-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3)) - 1/2*c^2*d*(2*sqrt(3)*arctan(1/3
*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3
+ a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^3*x/
((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{(dx^3+c)^3}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^3/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^3/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{(d\,x^3+c)^3}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)^3/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)^3/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^9}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d^3 + 3\left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c\,d^2 + 3\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c^2d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c^3$$

input
```
int((d*x^3+c)^3/(b*x^3+a)^(4/3),x)
```

output
```
int(x**9/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d**3 + 3*
int(x**6/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c*d**2 +
3*int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c**2*d
+ int(1/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c**3
```

3.104 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)^2x}{ab^2\sqrt[3]{a+bx^3}} + \frac{d^2x(a+bx^3)^{2/3}}{3b^2} + \frac{2d(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{7/3}}$$

output

```
(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(1/3)+1/3*d^2*x*(b*x^3+a)^(2/3)/b^2+2/9*d*(
-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/
b^(7/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{b}x(3b^2c^2+4a^2d^2+abd(-6c+dx^3))}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}d(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2d(-$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((3*b^(1/3)*x*(3*b^2*c^2 + 4*a^2*d^2 + a*b*d*(-6*c + d*x^3)))/(a*(a + b*x^
3)^(1/3)) + 2*Sqrt[3]*d*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3
)*x + 2*(a + b*x^3)^(1/3))] + 2*d*(-3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a +
 b*x^3)^(1/3)] + d*(3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)
^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{d(ac-(3bc-4ad)x^3)}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 27$$

$$\frac{d\int \frac{ac-(3bc-4ad)x^3}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{913}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}-\frac{x(a+bx^3)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{769}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}-\frac{x(a+bx^3)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(a*b*(a + b*x^3)^(1/3)) + (d*(-1/3*((3*b*c - 4
*a*d)*x*(a + b*x^3)^(2/3))/b + (2*a*(3*b*c - 2*a*d)*(ArcTan[(1 + (2*b^(1/3
)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a
 + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{\frac{4 a d b^2 \left(\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b x^3+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}} x^2+b^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}\right)\left(a d-\frac{3bc}{2}\right)\left(b x\right.}{9}}{b^{\frac{13}{3}}\left(b x^3+a\right)^{\frac{1}{3}} a}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
4/9*(a*d*b^2*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+
ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-3/2*b*c)*(b*x^3+a)^(1/3)+9/4*(b^2*c^2-2
*a*d*(-1/6*d*x^3+c)*b+4/3*a^2*d^2)*x*b^(7/3))/(b*x^3+a)^(1/3)/b^(13/3)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(119) = 238$.

Time = 0.14 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.53

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2
*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2
- 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)
*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^
2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
- (3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(
(b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(
a*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/
3))/(a*b^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d
 - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2
/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1
/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a
*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3
))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(119) = 238.

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,d^2\left(\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\cdots\right.}{}\right.$$

$$-\frac{1}{3}\,cd\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-\cdots\right.}{}\right.$$

$$+\frac{c^2x}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/
x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3)
+ (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^
(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d**2 + 2*
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c*d + int
(1/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c**2
```

3.105 $\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx=\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output
```
(-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4
/3)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx=\frac{\frac{6\sqrt[3]{b}(bc-ad)x}{a\sqrt[3]{a+bx^3}}+2\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+d\,\mathrm{lo}}{6b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((6*b^(1/3)*(b*c - a*d)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*d*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)] + d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/(6*b^(4/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \; 769$$

$$\frac{d\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*(ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)]/(2*b^(1/3))))/b
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$-\frac{xd}{b(b\,x^3+a)^{\frac{1}{3}}}+\frac{xc}{a(b\,x^3+a)^{\frac{1}{3}}}-\frac{d\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{4}{3}}}+\frac{d\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6b^{\frac{4}{3}}}-\frac{d\sqrt{3}\ \text{arct}}{}$

input
```
int((d*x^3+c)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x/(b*x^3+a)^(1/3)*d+1/a*x/(b*x^3+a)^(1/3)*c-1/3*d/b^(4/3)*ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x)+1/6*d/b^(4/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)-1/3*d/b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(81) = 162$.

Time = 0.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.93

$$\int \frac{c + dx^3}{(a + bx^3)^{4/3}} \, dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^2dx^3 + a^2bd)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3\,bx^3 - 3\,(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3\right.\right.}{} \right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(ab^2dx^3 + a^2bd)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}} \arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x - 2\,(bx^3 + a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right) - 6\,(bx^3 + a)^{\frac{2}{3}}(b^2c - abd)x +}{6\,(ab^3x^3}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x
^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b)
 + 2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^
2*b*d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2
*(a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

output
```
c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamm
a(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = -\frac{1}{6}d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\log\left(-\right.}{} \right. + \frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3
)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d + int(1
/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c
```

3.106 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/(-a*d+b*c)^(4/3)-1/6*d*l
n(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b
*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})\,d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]
]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*
(-I + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)]])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(b*c - a*d
)^(4/3)))/12
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{901}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)
^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c
 - a*d)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right)ca}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d
-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(
1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2
*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/
c/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*x**
3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.107 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{2d(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}$$

output
```
1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/
(b*x^3+a)^(1/3)/(d*x^3+c)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b*c)^(7/3)-1
/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(7/3)+1/3*d*(-a*d+3*b*c)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a^2d^2+abd^2x^3+3b^2c(c+dx^3))}{a(bc-ad)^2\sqrt[3]{a+bx^3}(c+dx^3)} + \frac{2i(3i+\sqrt{3})d(3bc-ad)\text{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right.}{(bc-ad)^{7/3}}}{}$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output
```
((6*c^(2/3)*x*(a^2*d^2 + a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3)))/(a*(b*c - a*d
)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((2*I)*(3*I + Sqrt[3])*d*(3*b*c - a*d
)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/
3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*d*(-3*b*c + a*d)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(7/3) + ((1 + I*Sqrt[3])*d*(3*b*c - a*d)*Log[2*(b*c - a*d)^(2/3)*
x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(
I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(18*c^(5/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {931, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^3+3bc-2ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\int \frac{2ad(3bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{2d(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{2d(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

input

```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output

```
-1/3*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((b*(3*b*c + a*
d)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (2*d*(3*b*c - a*d)*(ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(
2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - L
og[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*
d)^(1/3))))/(b*c - a*d))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}} a d \left(d\,x^{3}+c\right)\left(a d-3 b c\right)\ln\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{9}+\frac{c\left(a\left(b\,x^{3}+a\right)d^{2}+3x^{3}b^{2}cd+3b^{2}c^{2}\right)x\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}}}{3}+\frac{2ad\left(d\,x^{3}+c\right)\left(\cdots\right.}{\cdots}}{c^{2}\left(d\,x^{3}+c\right)\left(a d-b c\right)^{2}\left(\frac{a d-b\cdots}{c}\right.\cdots}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)*((b*x^3+a)^(1/3)*a*d*(d*x^3+c)*(a*d-3*b*c)*ln((((a
*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*c*(a*(b*x^3+a)*d^2+3*x^3*b^2*c*
d+3*b^2*c^2)*x*((a*d-b*c)/c)^(1/3)+a*d*(d*x^3+c)*(arctan(1/3*3^(1/2)*(-2/(
(a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/
3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-3
*b*c)*(b*x^3+a)^(1/3))/(b*x^3+a)^(1/3)/c^2/(d*x^3+c)/(a*d-b*c)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx=\int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a\,c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,acd\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,a\,d^2x^6+(b\,x^3+a)^{\frac{1}{3}}$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c**2 + 2*(a + b*x**3)**(1/3)*a*c*d*x**3 + (a
+ b*x**3)**(1/3)*a*d**2*x**6 + (a + b*x**3)**(1/3)*b*c**2*x**3 + 2*(a + b*
x**3)**(1/3)*b*c*d*x**6 + (a + b*x**3)**(1/3)*b*d**2*x**9),x)
```

3.108 $\int (a+bx^3)^{5/3} (c+dx^3) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int (a+bx^3)^{5/3} (c+dx^3) \, dx = \frac{5a(9bc-ad)x(a+bx^3)^{2/3}}{162b} + \frac{(9bc-ad)x(a+bx^3)^{5/3}}{54b} + \frac{dx(a+bx^3)^{8/3}}{9b} + \frac{5a^2(9bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}} - \frac{5a^2(9bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{162b^{4/3}}$$

output
```
5/162*a*(-a*d+9*b*c)*x*(b*x^3+a)^(2/3)/b+1/54*(-a*d+9*b*c)*x*(b*x^3+a)^(5/
3)/b+1/9*d*x*(b*x^3+a)^(8/3)/b+5/243*a^2*(-a*d+9*b*c)*arctan(1/3*(1+2*b^(1
/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-5/162*a^2*(-a*d+9*b*c)*ln(
-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(10a^2d+9b^2x^3(3c+2dx^3)+ab(72c+33dx^3)\right)-10\sqrt{3}a^2(-9bc+ad)\arctan}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(10*a^2*d + 9*b^2*x^3*(3*c + 2*d*x^3) + a*b
*(72*c + 33*d*x^3)) - 10*Sqrt[3]*a^2*(-9*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 10*a^2*(-9*b*c + a*d)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] - 5*a^2*(-9*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/
3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(486*b^(4/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {913, 748, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(9bc-ad)\int \left(bx^3+a\right)^{5/3}dx}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(9bc-ad)\left(\frac{5}{6}a\int \left(bx^3+a\right)^{2/3}dx+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(9bc-ad)\left(\frac{5}{6}a\left(\frac{2}{3}a\int\frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(9bc-ad)\left(\frac{5}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)+\frac{1}{6}x\left(a+bx^3\right)^{5/3}\right)}{9b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

input
```
Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(8/3))/(9*b) + ((9*b*c - a*d)*((x*(a + b*x^3)^(5/3))/6 +
(5*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
]/(2*b^(1/3))))/3))/6))/(9*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\frac{4a\left(b\,x^3+a\right)^{\frac{2}{3}}x\left(\frac{11d\,x^3}{24}+c\right)b^{\frac{4}{3}}}{9}+\frac{\left(\frac{2d\,x^3}{3}+c\right)\left(b\,x^3+a\right)^{\frac{2}{3}}x^4b^{\frac{7}{3}}}{6}+\frac{5a^2\left(dx\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}-\frac{\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)\right.}{}\right.}{}}{b^{\frac{4}{3}}}$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
5/81*(36/5*a*(b*x^3+a)^(2/3)*x*(11/24*d*x^3+c)*b^(4/3)+27/10*(2/3*d*x^3+c)
*(b*x^3+a)^(2/3)*x^4*b^(7/3)+a^2*(d*x*(b*x^3+a)^(2/3)*b^(1/3)-1/6*(-2*3^(1
/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3
)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*
x^3+a)^(1/3))/x))*(a*d-9*b*c)))/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.77

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx = \left[-\frac{15\sqrt{\frac{1}{3}}(9\,a^2b^2c-a^3bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2 \cdots\right.\right.}{\cdots}\right.$$

$$-\frac{10\,(9\,a^2bc-a^3d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-5\,(9\,a^2bc-a^3d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{30\sqrt{\frac{1}{3}}(9 \cdots}{\cdots}}{4 \cdots}$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a
)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 1
0*(9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*
(9*a^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^
4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^
2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*
b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3
)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7
 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a
)^(2/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx = \frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

output
```
a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hy
per((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/
3)*b*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(143) = 286$.

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.33

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)\,dx =$$

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{486}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10\,a^3\log\left(-b^{\frac{1}{3}}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)
/b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x
^3 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(1/3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b
^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a
^3*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4
/3) + 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a
)^(2/3)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3
)*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3
 + a)^3*b/x^9))*d
```

Giac [F]

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx = \int (bx^3+a)^{\frac{5}{3}} (dx^3+c) \, dx$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx = \int \left(b\,x^3+a\right)^{5/3} \left(d\,x^3+c\right) \, dx$$

input
```
int((a + b*x^3)^(5/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) \, dx = \frac{10(b\,x^3+a)^{\frac{2}{3}} a^2 dx + 72(b\,x^3+a)^{\frac{2}{3}} abcx + 33(b\,x^3+a)^{\frac{2}{3}} abd\,x^4 + 27(b\,x^3+a)^{\frac{2}{3}} b^2 c\,x^4 + 18(b\,x}{162b}$$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

output
```
(10*(a + b*x**3)**(2/3)*a**2*d*x + 72*(a + b*x**3)**(2/3)*a*b*c*x + 33*(a
+ b*x**3)**(2/3)*a*b*d*x**4 + 27*(a + b*x**3)**(2/3)*b**2*c*x**4 + 18*(a +
 b*x**3)**(2/3)*b**2*d*x**7 - 10*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a
**3*d + 90*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**2*b*c)/(162*b)
```

3.109 $\int (a+bx^3)^{2/3} (c+dx^3)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int (a+bx^3)^{2/3} (c+dx^3)\, dx = \frac{(6bc-ad)x(a+bx^3)^{2/3}}{18b} + \frac{dx(a+bx^3)^{5/3}}{6b} + \frac{a(6bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

output
```
1/18*(-a*d+6*b*c)*x*(b*x^3+a)^(2/3)/b+1/6*d*x*(b*x^3+a)^(5/3)/b+1/27*a*(-a
*d+6*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4
/3)-1/18*a*(-a*d+6*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.28

$$\int (a+bx^3)^{2/3} (c+dx^3)\, dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}(6bc+2ad+3bdx^3) - 2\sqrt{3}a(-6bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2a(-}{54}$$

input
```
Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(6*b*c + 2*a*d + 3*b*d*x^3) - 2*Sqrt[3]*a*(
-6*b*c + a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
] + 2*a*(-6*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - a*(-6*b*c +
 a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/
(54*b^(4/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(6bc-ad)\int \left(bx^3+a\right)^{2/3}dx}{6b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(6bc-ad)\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

$$\Big\downarrow \text{769}$$

$$\frac{(6bc-ad)\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}\right)}{6b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

input
```
Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(5/3))/(6*b) + ((6*b*c - a*d)*((x*(a + b*x^3)^(2/3))/3 +
(2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/
3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(6*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{3\left(\frac{d x^{3}}{2}+c\right)\left(b x^{3}+a\right)^{\frac{2}{3}} x b^{\frac{4}{3}}+a\left(d x\left(b x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}-\frac{\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}} x+\left(b x\right.}{x^{2}}\right.\right.}{6}\right.}{9 b^{\frac{4}{3}}}$

input

```
int((b*x^3+a)^(2/3)*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output

```
1/9/b^(4/3)*(3*(1/2*d*x^3+c)*(b*x^3+a)^(2/3)*x*b^(4/3)+a*(d*x*(b*x^3+a)^(2
/3)*b^(1/3)-1/6*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3
))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x
^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a*d-6*b*c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.01

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(6\,ab^2c-a^2bd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-\right.\right.}{}\right.$$

$$-\frac{2\,(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right)-(6\,abc-a^2d)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}x+(bx^3+a)^{\frac{2}{3}}}{x^2}\right)+\frac{6\sqrt{\frac{1}{3}}(6\,ab^2c-a}{}}{54\,b^2}$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
[-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3
*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1
/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*
a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c
- a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3
))/b^2, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(
1/3))/x) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*
b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*ar
ctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*
(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx = \frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

output
```
a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(114) = 228$.

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.28

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\,dx =$$

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{54}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3\right.}{}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*ar
ctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*
log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3)
 + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3
)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 +
(b*x^3 + a)^2*b/x^6))*d
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx = \int \left(bx^3+a\right)^{\frac{2}{3}} \left(dx^3+c\right) dx$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx = \int \left(b\,x^3+a\right)^{2/3} \left(d\,x^3+c\right) \, dx$$

input
```
int((a + b*x^3)^(2/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) \, dx = \frac{2(b\,x^3+a)^{\frac{2}{3}}\,adx+6(b\,x^3+a)^{\frac{2}{3}}\,bcx+3(b\,x^3+a)^{\frac{2}{3}}\,bd\,x^4-2\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2d+12\left(\int \frac{1}{(b\,x^3+}\right.}{18b}$$

input
```
int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(2/3)*a*d*x + 6*(a + b*x**3)**(2/3)*b*c*x + 3*(a + b*x**3
)**(2/3)*b*d*x**4 - 2*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**2*d + 12*
int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a*b*c)/(18*b)
```

3.110 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}}{3b} + \frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
1/3*d*x*(b*x^3+a)^(2/3)/b+1/9*(-a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/6*(-a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3
+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$= \frac{6\sqrt[3]{b}dx(a+bx^3)^{2/3} + 2\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2(-3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx}\right.}{18b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(1/3)*d*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*(-3*b*c + a*d)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)] + (3*b*c - a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$
$$\Big\downarrow \ 913$$
$$\frac{(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b} + \frac{dx(a+bx^3)^{2/3}}{3b}$$
$$\Big\downarrow \ 769$$

$$\frac{(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)]/(2*b^(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(88) = 176$.

Time = 0.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.03

method	result
pseudoelliptic	$6dx\left(b\,x^3+a\right)^{\frac{2}{3}}b^{\frac{1}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)ad-6\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)bc+2\ln\left(\frac{-b^{\frac{1}{3}}x+(b}{x}\right.$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*d*x*(b*x^3+a)^(2/3)*b^(1/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a*d-6*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*b*c+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a
*d-6*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*b*c-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d+3*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)*b*c)/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.26

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{6\,(bx^3+a)^{\frac{2}{3}}bdx - 3\,\sqrt{\frac{1}{3}}(3\,b^2c - abd)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\,\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3+a)^{\frac{1}{3}}\right.\right.}{} \right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b
^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/
3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(
3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*
d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2
/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*
(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.20

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx =$$

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right.}{b^{\frac{1}{3}}}\right.$$

$$+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3}{}\right.}{b^{\frac{4}{3}}}\right.$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(
2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
+ a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

Giac [F]

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)*d + int(1/(a + b*x**3)**(1/3),x)*c
```

3.111 $\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} - \frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output
```
(-a*d+b*c)*x/a/b/(b*x^3+a)^(1/3)+1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)-1/2*d*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4
/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.52

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{b}(bc-ad)x}{a\sqrt[3]{a+bx^3}} + 2\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + d\log}{6b^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((6*b^(1/3)*(b*c - a*d)*x)/(a*(a + b*x^3)^(1/3)) + 2*Sqrt[3]*d*ArcTan[(Sqr
t[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*d*Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)] + d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/(6*b^(4/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \ 910$$

$$\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 769$$

$$\frac{d\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*(ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)]/(2*b^(1/3))))/b
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$-\frac{xd}{b\left(b\,x^{3}+a\right)^{\frac{1}{3}}}+\frac{xc}{a\left(b\,x^{3}+a\right)^{\frac{1}{3}}}-\frac{d\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{4}{3}}}+\frac{d\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6b^{\frac{4}{3}}}-\frac{d\sqrt{3}\,\text{arct}}{}$

input
```
int((d*x^3+c)/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/b*x/(b*x^3+a)^(1/3)*d+1/a*x/(b*x^3+a)^(1/3)*c-1/3*d/b^(4/3)*ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x)+1/6*d/b^(4/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)-1/3*d/b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 488, normalized size of antiderivative = 4.93

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}(ab^2dx^3+a^2bd)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-6\,(bx^3+a)^{\frac{2}{3}}(b^2c-abd)x+}{6\,(ab^3x}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x
^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b)
 + 2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^
2*b*d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2
*(a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
- (a*b*d*x^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

output
```
c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamm
a(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx =$$

$$-\frac{1}{6}d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\log\left(-\right.}{}\right.$$

$$+\frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
-1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3
)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a
)^(1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d + int(1
/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c
```

3.112 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx=\frac{(bc-ad)x}{4ab\left(a+bx^3\right)^{4/3}}+\frac{(3bc+ad)x}{4a^2b\sqrt[3]{a+bx^3}}$$

output
```
1/4*(-a*d+b*c)*x/a/b/(b*x^3+a)^(4/3)+1/4*(a*d+3*b*c)*x/a^2/b/(b*x^3+a)^(1/
3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx=\frac{x(4ac+3bcx^3+adx^3)}{4a^2\left(a+bx^3\right)^{4/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3c\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3)
)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x\left(ad\,x^{3}+3x^{3}bc+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34
trager	$\frac{x\left(ad\,x^{3}+3x^{3}bc+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34
pseudoelliptic	$\frac{x\left(ad\,x^{3}+3x^{3}bc+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34
orering	$\frac{x\left(ad\,x^{3}+3x^{3}bc+4ac\right)}{4\left(b\,x^{3}+a\right)^{\frac{4}{3}}a^{2}}$	34

input
```
int((d*x^3+c)/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{\left((3\,bc+ad)x^4+4\,acx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{4\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b
*x^3 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(53) = 106$.

Time = 22.51 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.11

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) + \frac{dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

output
```
c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)
*gamma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)cx^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{dx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b
*x^3 + a)^(4/3)*a)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{4\,a\,c\,x+a\,d\,x^4+3\,b\,c\,x^4}{4\,a^2\,(b\,x^3+a)^{4/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(7/3),x)
```

output
```
(4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(7/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a +
 b*x**3)**(1/3)*b**2*x**6),x)*d + int(1/((a + b*x**3)**(1/3)*a**2 + 2*(a +
 b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*c
```

3.113 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx=\frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}}+\frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

output
```
1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^(7/3)+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^(4
/3)+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx=\frac{28a^2cx+42abcx^4+7a^2dx^4+18b^2cx^7+3abdx^7}{28a^3\left(a+bx^3\right)^{7/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(28*a^2*c*x + 42*a*b*c*x^4 + 7*a^2*d*x^4 + 18*b^2*c*x^7 + 3*a*b*d*x^7)/(28
*a^3*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+6bc)\int \frac{1}{(bx^3+a)^{7/3}}dx}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+6bc)\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)(ad+6bc)}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^3)^{7/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*(x/(4*a*(a + b*
x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{2}+\frac{3\left(\frac{d\,x^{3}}{14}+c\right)b\,x^{3}a}{2}+\frac{9b^{2}c\,x^{6}}{14}\right)}{\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	52
gosper	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57
trager	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57
orering	$\frac{x\left(3abd\,x^{6}+18b^{2}c\,x^{6}+7a^{2}d\,x^{3}+42abc\,x^{3}+28a^{2}c\right)}{28\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	57

input
```
int((d*x^3+c)/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^3+a)^(7/3)*x*((1/4*d*x^3+c)*a^2+3/2*(1/14*d*x^3+c)*b*x^3*a+9/14*b^2
*c*x^6)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(3\,(6\,b^2c+abd)x^7+7\,(6\,abc+a^2d)x^4+28\,a^2cx)(bx^3+a)^{\frac{2}{3}}}{28\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x
^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(83) = 166$.

Time = 116.09 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

output
```
c*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) + d*(7*a*x**4*gamma(4/3)/(9*
a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gam
ma(10/3)) + 3*b*x**7*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(1
0/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*
b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)dx^7}{28\,(bx^3+a)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)cx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
-1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^
2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a
^3)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{3\,a\,d\,x\,(b\,x^3+a)^2-4\,a^3\,d\,x+18\,b\,c\,x\,(b\,x^3+a)^2+a^2\,d\,x\,(b\,x^3+a)+4\,a^2\,b\,c\,x+6\,a}{28\,a^3\,b\,(b\,x^3+a)^{7/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

output
```
(3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
 b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3)
)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{10/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(10/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*d + i
nt(1/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*(a
+ b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*c
```

3.114 $\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx=\frac{(bc-ad)x}{10ab\,(a+bx^3)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\,(a+bx^3)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\,(a+bx^3)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

output
```
1/10*(-a*d+b*c)*x/a/b/(b*x^3+a)^(10/3)+1/70*(a*d+9*b*c)*x/a^2/b/(b*x^3+a)^
(7/3)+3/140*(a*d+9*b*c)*x/a^3/b/(b*x^3+a)^(4/3)+9/140*(a*d+9*b*c)*x/a^4/b/
(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx=\frac{x(81b^3cx^9+35a^3(4c+dx^3)+9ab^2x^6(30c+dx^3)+15a^2bx^3(21c+2dx^3))}{140a^4\,(a+bx^3)^{10/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
(x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*
a^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+9bc)\int \frac{1}{(bx^3+a)^{10/3}}dx}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(ad+9bc)\left(\frac{6\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(ad+9bc)\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)(ad+9bc)}{10ab}+\frac{x(bc-ad)}{10ab\,(a+bx^3)^{10/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

output
```
((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*(x/(7*a*(a +
b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(
1/3))))/(7*a)))/(10*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{d\,x^{3}}{4}+c\right)a^{3}+\frac{9b\,x^{3}\left(\frac{2d\,x^{3}}{21}+c\right)a^{2}}{4}+\frac{27b^{2}x^{6}\left(\frac{d\,x^{3}}{30}+c\right)a}{14}+\frac{81b^{3}c\,x^{9}}{140}\right)x}{\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	71
gosper	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}bc\,x^{3}+140a^{3}c\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81
trager	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}bc\,x^{3}+140a^{3}c\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81
orering	$\frac{x\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}bc\,x^{3}+140a^{3}c\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	81

input
```
int((d*x^3+c)/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
((1/4*d*x^3+c)*a^3+9/4*b*x^3*(2/21*d*x^3+c)*a^2+27/14*b^2*x^6*(1/30*d*x^3+
c)*a+81/140*b^3*c*x^9)/(b*x^3+a)^(10/3)*x/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{(9\,(9\,b^3c+ab^2d)x^{10}+30\,(9\,ab^2c+a^2bd)x^7+140\,a^3cx+35\,(9\,a^2bc+a^3d)x^4)(bx^3+}{140\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3
*c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5
*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{\left(14\,b^2 - \frac{40\,(bx^3+a)b}{x^3} + \frac{35\,(bx^3+a)^2}{x^6}\right)dx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}\,a^3} - \frac{\left(14\,b^3 - \frac{60\,(bx^3+a)b^2}{x^3} + \frac{105\,(bx^3+a)^2 b}{x^6} - \frac{140\,(bx^3+a)^3}{x^9}\right)cx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}\,a^4}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
 a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x\left(\frac{c}{10\,a}-\frac{d}{10\,b}\right)}{(b\,x^3+a)^{10/3}} + \frac{x\,(a\,d+9\,b\,c)}{70\,a^2\,b\,(b\,x^3+a)^{7/3}} + \frac{x\,(3\,a\,d+27\,b\,c)}{140\,a^3\,b\,(b\,x^3+a)^{4/3}} + \frac{x\,(9\,a\,d+81\,b\,c)}{140\,a^4\,b\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(13/3),x)
```

output
```
(x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b
*(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) +
 (x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{13/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9\right. + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}}\right.$$

input
```
int((d*x^3+c)/(b*x^3+a)^(13/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6
*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 +
(a + b*x**3)**(1/3)*b**4*x**12),x)*d + int(1/((a + b*x**3)**(1/3)*a**4 + 4
*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2*x**6 +
4*(a + b*x**3)**(1/3)*a*b**3*x**9 + (a + b*x**3)**(1/3)*b**4*x**12),x)*c
```

3.115 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx=\frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}}+\frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{81(12bc+ad)x}{1820a^5b\sqrt[3]{a+bx^3}}$$

output
```
1/13*(-a*d+b*c)*x/a/b/(b*x^3+a)^(13/3)+1/130*(a*d+12*b*c)*x/a^2/b/(b*x^3+a
)^(10/3)+9/910*(a*d+12*b*c)*x/a^3/b/(b*x^3+a)^(7/3)+27/1820*(a*d+12*b*c)*x
/a^4/b/(b*x^3+a)^(4/3)+81/1820*(a*d+12*b*c)*x/a^5/b/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx=\frac{x(972b^4cx^{12}+455a^4(4c+dx^3)+351a^2b^2x^6(20c+dx^3)+81ab^3x^9(52c+dx^3)+195}{1820a^5\left(a+bx^3\right)^{13/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

output
```
(x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3
) + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a
^5*(a + b*x^3)^(13/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+12bc)\int \frac{1}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{x(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{9\int \frac{1}{(bx^3+a)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{9\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{9\left(\frac{6\left(\frac{3\int\frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

$\big\downarrow$ 746

$$\frac{\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)(ad+12bc)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

input

```
Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

output

```
((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*(x/(10*a*(a
+ b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^
(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(10*a)))/(13*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{\left(\left(\frac{d\,x^3}{4}+c\right)a^4+3b\,x^3\left(\frac{3d\,x^3}{28}+c\right)a^3+\frac{27b^2x^6\left(\frac{d\,x^3}{20}+c\right)a^2}{7}+\frac{81\left(\frac{d\,x^3}{52}+c\right)b^3x^9a}{35}+\frac{243b^4c\,x^{12}}{455}\right)x}{(b\,x^3+a)^{\frac{13}{3}}a^5}$
gosper	$\frac{x(81a\,b^3d\,x^{12}+972b^4c\,x^{12}+351a^2b^2d\,x^9+4212a\,b^3c\,x^9+585a^3bd\,x^6+7020a^2b^2c\,x^6+455a^4d\,x^3+5460a^3bc\,x^3+1820c\,a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^5}$
trager	$\frac{x(81a\,b^3d\,x^{12}+972b^4c\,x^{12}+351a^2b^2d\,x^9+4212a\,b^3c\,x^9+585a^3bd\,x^6+7020a^2b^2c\,x^6+455a^4d\,x^3+5460a^3bc\,x^3+1820c\,a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^5}$
orering	$\frac{x(81a\,b^3d\,x^{12}+972b^4c\,x^{12}+351a^2b^2d\,x^9+4212a\,b^3c\,x^9+585a^3bd\,x^6+7020a^2b^2c\,x^6+455a^4d\,x^3+5460a^3bc\,x^3+1820c\,a^4)}{1820(b\,x^3+a)^{\frac{13}{3}}a^5}$

input
```
int((d*x^3+c)/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
((1/4*d*x^3+c)*a^4+3*b*x^3*(3/28*d*x^3+c)*a^3+27/7*b^2*x^6*(1/20*d*x^3+c)*
a^2+81/35*(1/52*d*x^3+c)*b^3*x^9*a+243/455*b^4*c*x^12)/(b*x^3+a)^(13/3)*x/
a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \frac{(81\,(12\,b^4c+ab^3d)x^{13}+351\,(12\,ab^3c+a^2b^2d)x^{10}+585\,(12\,a^2b^2c+a^3bd)x^7+1820\,a}{1820\,(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
 585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 +
 10*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = -\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)dx^{13}}{1820\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)cx^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
-1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455
*(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*
(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9
+ 455*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx=\frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{(b\,x^3+a)^{13/3}}+\frac{x\,(a\,d+12\,b\,c)}{130\,a^2\,b\,(b\,x^3+a)^{10/3}}+\frac{x\,(9\,a\,d+108\,b\,c)}{910\,a^3\,b\,(b\,x^3+a)^{7/3}}+\frac{x\,(27\,a\,d+324\,b\,c)}{1820\,a^4\,b\,(b\,x^3+a)^{4/3}}+\frac{x\,(81\,a\,d+972\,b\,c)}{1820\,a^5\,b\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

output
```
(x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2
*b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3
)) + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d +
972*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{16/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b}\right.$$

$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\,a}\right.$$

input

```
int((d*x^3+c)/(b*x^3+a)^(16/3),x)
```

output

```
int(x**3/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x**3 + 1
0*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*b**3*x*
*9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*x**15),
x)*d + int(1/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x**3
 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*b**
3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*x**
15),x)*c
```

3.116 $\int (a+bx^3)^{7/3} (c+dx^3)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (a+bx^3)^{7/3} (c+dx^3)\, dx = \frac{dx(a+bx^3)^{10/3}}{11b} + \frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/11*d*x*(b*x^3+a)^(10/3)/b+1/11*a^2*(-a*d+11*b*c)*x*(b*x^3+a)^(1/3)*hyper
geom([-7/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 8.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int (a+bx^3)^{7/3} (c+dx^3)\, dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^3 - \frac{a^2(-11bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{11b}$$

input
```
Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometri
c2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(11bc-ad)\int \left(bx^3+a\right)^{7/3}dx}{11b}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{a^2\sqrt[3]{a+bx^3}(11bc-ad)\int \left(\frac{bx^3}{a}+1\right)^{7/3}dx}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{a^2x\sqrt[3]{a+bx^3}(11bc-ad)\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

input
```
Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*
Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/
3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{7}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*
c)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.12

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}b^2dx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

output
```
a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*
*(4/3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*
pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3)
, (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**
10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx = \int \left(b\,x^3+a\right)^{7/3} \left(d\,x^3+c\right) \, dx$$

input
```
int((a + b*x^3)^(7/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(7/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) \, dx = \frac{14(b\,x^3+a)^{\frac{1}{3}}\,a^3dx+286(b\,x^3+a)^{\frac{1}{3}}\,a^2bcx+103(b\,x^3+a)^{\frac{1}{3}}\,a^2bd\,x^4+187(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2c\,x^4+}{}$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

output
```
(14*(a + b*x**3)**(1/3)*a**3*d*x + 286*(a + b*x**3)**(1/3)*a**2*b*c*x + 10
3*(a + b*x**3)**(1/3)*a**2*b*d*x**4 + 187*(a + b*x**3)**(1/3)*a*b**2*c*x**
4 + 115*(a + b*x**3)**(1/3)*a*b**2*d*x**7 + 55*(a + b*x**3)**(1/3)*b**3*c*
x**7 + 40*(a + b*x**3)**(1/3)*b**3*d*x**10 - 14*int((a + b*x**3)**(1/3)/(a
 + b*x**3),x)*a**4*d + 154*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3*b*
c)/(440*b)
```

3.117 $\int (a+bx^3)^{4/3}(c+dx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx=\frac{dx(a+bx^3)^{7/3}}{8b}+\frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/8*d*x*(b*x^3+a)^(7/3)/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([
-4/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 7.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx=\frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)^2-\frac{a(-8bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{8b}$$

input
```
Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F
1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) \, dx$$

$$\Big\downarrow \ 913$$

$$\frac{(8bc-ad) \int \left(bx^3+a\right)^{4/3}dx}{8b}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

$$\Big\downarrow \ 779$$

$$\frac{a\sqrt[3]{a+bx^3}(8bc-ad) \int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

$$\Big\downarrow \ 778$$

$$\frac{ax\sqrt[3]{a+bx^3}(8bc-ad) \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

input
```
Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyper
geometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}} \left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx = \frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{4}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
```

output
```
a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hy
per((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(10/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) \, dx=\int \left(bx^3+a\right)^{\frac{4}{3}} \left(dx^3+c\right) dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) \, dx=\int \left(bx^3+a\right)^{\frac{4}{3}} \left(dx^3+c\right) dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) \, dx=\int {\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right) \,dx$$

input
```
int((a + b*x^3)^(4/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int (a+bx^3)^{4/3}(c+dx^3)\,dx = \frac{2(b\,x^3+a)^{\frac{1}{3}}\,a^2dx+24(b\,x^3+a)^{\frac{1}{3}}\,abcx+9(b\,x^3+a)^{\frac{1}{3}}\,abd\,x^4+8(b\,x^3+a)^{\frac{1}{3}}\,b^2c\,x^4+5(b\,x^3+a}{40b}$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*d*x + 24*(a + b*x**3)**(1/3)*a*b*c*x + 9*(a +
b*x**3)**(1/3)*a*b*d*x**4 + 8*(a + b*x**3)**(1/3)*b**2*c*x**4 + 5*(a + b*x
**3)**(1/3)*b**2*d*x**7 - 2*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3*d
 + 16*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*b*c)/(40*b)
```

3.118 $\int \sqrt[3]{a+bx^3}(c+dx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{dx(a+bx^3)^{4/3}}{5b}+\frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/5*d*x*(b*x^3+a)^(4/3)/b+1/5*(-a*d+5*b*c)*x*(b*x^3+a)^(1/3)*hypergeom([-1
/3, 1/3],[4/3],-b*x^3/a)/b/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(d(a+bx^3)+\frac{(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}\right)}{5b}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/
3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(5bc-ad)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^3}(5bc-ad)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperge
ometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

output
```
a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx =\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}a d x+5\left(b\,x^3+a\right)^{\frac{1}{3}}b c x+2\left(b\,x^3+a\right)^{\frac{1}{3}}b d\,x^4-\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)a^2 d+5\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)a b c}{10b}$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*a*d*x + 5*(a + b*x**3)**(1/3)*b*c*x + 2*(a + b*x**3)*
*(1/3)*b*d*x**4 - int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*d + 5*int((
a + b*x**3)**(1/3)/(a + b*x**3),x)*a*b*c)/(10*b)
```

3.119 $\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx = \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

output
```
1/2*d*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d+2*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([
1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}} \, dx = \frac{dx(a+bx^3)+(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{913, 779, 778\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(2bc-ad)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2b}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hyp
ergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{dx^{3}+c}{\left(bx^{3}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**3/(a + b*x**3)**(2/3),x)*d + int(1/(a + b*x**3)**(2/3),x)*c
```

3.120 $\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=-\frac{dx}{b(a+bx^3)^{2/3}}+\frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab(a+bx^3)^{2/3}}$$

output
```
-d*x/b/(b*x^3+a)^(2/3)+(a*d+b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 5/3],
[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=\frac{-adx+(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3
, 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+bc)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 779$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2ab\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

$$\Big\downarrow \; 778$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{2ab\,(a+bx^3)^{2/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)
^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^
(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

input
```
int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{5/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(5/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(5/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\left(\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{2}{3}}a+\left(b\,x^3+a\right)^{\frac{2}{3}}b\,x^3}dx\right)d+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}a+\left(b\,x^3+a\right)^{\frac{2}{3}}b\,x^3}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*d + int(1
/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*c
```

3.121 $\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = -\frac{dx}{4b(a+bx^3)^{5/3}} + \frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a^2b(a+bx^3)^{2/3}}$$

output
```
-1/4*d*x/b/(b*x^3+a)^(5/3)+1/4*(a*d+4*b*c)*x*(1+b*x^3/a)^(2/3)*hypergeom([
1/3, 8/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(-d+\frac{(4bc+ad)(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{a^2}\right)}{4b(a+bx^3)^{5/3}}$$

input
```
Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F
1[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx$$

$$\Big\downarrow \ 910$$

$$\frac{(ad+4bc)\int \frac{1}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \ 779$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}}dx}{5a^2b\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \ 778$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b\,(a+bx^3)^{2/3}}+\frac{x(bc-ad)}{5ab\,(a+bx^3)^{5/3}}$$

input
```
Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/
a)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x
^3)^(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{3}+c}{\left(b\,x^{3}+a\right)^{\frac{8}{3}}}d x$$

input
```
int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

output
```
int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx=\int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^
3 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx=\frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
8/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{dx^3+c}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{d\,x^3+c}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((c + d*x^3)/(a + b*x^3)^(8/3),x)
```

output
```
int((c + d*x^3)/(a + b*x^3)^(8/3), x)
```

Reduce [F]

$$\int \frac{c+dx^3}{(a+bx^3)^{8/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)c$$

input
```
int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a +
 b*x**3)**(2/3)*b**2*x**6),x)*d + int(1/((a + b*x**3)**(2/3)*a**2 + 2*(a +
 b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*c
```

3.122 $\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\int (a+bx^3)^{5/3} (c+dx^3)^2 \, dx = \frac{5a(27b^2c^2-6abcd+a^2d^2)\,x(a+bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2-6abcd+a^2d^2)\,x(a+bx^3)^{5/3}}{162b^2} + \frac{d(6bc-ad)x(a+bx^3)^{8/3}}{27b^2} + \frac{d^2x^4(a+bx^3)^{8/3}}{12b} + \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{243\sqrt{3}b^{7/3}} - \frac{5a^2(27b^2c^2-6abcd+a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{486b^{7/3}}$$

output

```
5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^
2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/27*d*(-a*d+6*b*c)*x*(b*x^3
+a)^(8/3)/b^2+1/12*d^2*x^4*(b*x^3+a)^(8/3)/b+5/729*a^2*(a^2*d^2-6*a*b*c*d+
27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^
(7/3)-5/486*a^2*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/
3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.13

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(-20a^3d^2+15a^2bd(8c+dx^3)+27b^3x^3(6c^2+8cdx^3+3d^2x^6)+18ab^2(24c^2\right.}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 2
7*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3
+ 7*d^2*x^6)) + 20*Sqrt[3]*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 20*a^2*(27*b^2*c^2
 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 10*a^2*(27
*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1
/3) + (a + b*x^3)^(2/3)])/(2916*b^(7/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 913, 748, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int \left(bx^3+a\right)^{5/3} \left(d(15bc-4ad)x^3+c(12bc-ad)\right) dx}{12b}+\frac{dx\left(a+bx^3\right)^{8/3} \left(c+dx^3\right)}{12b}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\int (bx^3+a)^{5/3}dx}{9b}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

$$\Big\downarrow \textbf{748}$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\int (bx^3+a)^{2/3}dx+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

$$\Big\downarrow \textbf{748}$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\left(\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x(a+bx^3)^{2/3}\right)+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx^3)^{8/3}(15bc-4ad)}{9b}}{12b}+\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

$$\Big\downarrow \textbf{769}$$

$$\frac{\frac{4(a^2d^2-6abcd+27b^2c^2)\left(\frac{5}{6}a\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x(a+bx^3)^{2/3}\right)+\frac{1}{6}x(a+bx^3)^{5/3}\right)}{9b}+\frac{dx(a+bx}{}}{12b}$$
$$\frac{dx(a+bx^3)^{8/3}(c+dx^3)}{12b}$$

input
```
Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(8/3)*(c + d*x^3))/(12*b) + ((d*(15*b*c - 4*a*d)*x*(a + b
*x^3)^(8/3))/(9*b) + (4*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*((x*(a + b*x^3)
^(5/3))/6 + (5*a*((x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a +
b*x^3)^(1/3)]/(2*b^(1/3))))/3))/6))/(9*b))/(12*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{5\left(-\frac{243\left(\frac{1}{2}d^{2}x^{6}+\frac{4}{3}cd\,x^{3}+c^{2}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}x^{4}b^{\frac{10}{3}}}{10}+a\left(-18a\left(b\,x^{3}+a\right)^{\frac{2}{3}}d\left(\frac{d\,x^{3}}{8}+c\right)x\,b^{\frac{4}{3}}-\frac{324\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{7}{24}d^{2}x^{6}+\frac{11}{12}cd\,x^{3}+c^{2}\right.}{5}\right.\right.}{}$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/729*(-243/10*(1/2*d^2*x^6+4/3*c*d*x^3+c^2)*(b*x^3+a)^(2/3)*x^4*b^(10/3)
+a*(-18*a*(b*x^3+a)^(2/3)*d*(1/8*d*x^3+c)*x*b^(4/3)-324/5*(b*x^3+a)^(2/3)*
(7/24*d^2*x^6+11/12*c*d*x^3+c^2)*x*b^(7/3)+a*(3*a*d^2*x*b^(1/3)*(b*x^3+a)^
(2/3)+(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln(
(b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))))/b^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 717, normalized size of antiderivative = 2.77

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-
b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2
/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*
d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2
*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)
^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(1
2*b^4*c*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d
^2)*x^4 + 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^
(2/3))/b^3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*
d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(
1/3))*sqrt(-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2
 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^
4*c*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*
x^4 + 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3
))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.04

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{a^{\frac{5}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{5}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{5}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{2}{3}}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

output
```
a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma
(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*
d**2*x**10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(224) = 448$.

Time = 0.12 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.59

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
-1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)
/b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x
^3 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(1/3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b
^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3
*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5
*a^3*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^
(4/3) + 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 +
 a)^(2/3)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8
/3)*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x
^3 + a)^3*b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3)
 + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 +
 a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3)
 + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36
*(b*x^3 + a)^(5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^
3 + a)^(11/3)*a^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4
/x^6 - 4*(b*x^3 + a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

Giac [F]

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{5}{3}} \left(dx^3+c\right)^2 \, dx$$

input
```
integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^2 \,dx$$

input
```
int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx = \frac{-20(b\,x^3+a)^{\frac{2}{3}}\,a^3d^2x+120(b\,x^3+a)^{\frac{2}{3}}\,a^2bcdx+15(b\,x^3+a)^{\frac{2}{3}}\,a^2b\,d^2x^4+432(b\,x^3+a)^{\frac{2}{3}}\,a\,b^2c}{}$$

input
```
int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

output
```
( - 20*(a + b*x**3)**(2/3)*a**3*d**2*x + 120*(a + b*x**3)**(2/3)*a**2*b*c*
d*x + 15*(a + b*x**3)**(2/3)*a**2*b*d**2*x**4 + 432*(a + b*x**3)**(2/3)*a*
b**2*c**2*x + 396*(a + b*x**3)**(2/3)*a*b**2*c*d*x**4 + 126*(a + b*x**3)**
(2/3)*a*b**2*d**2*x**7 + 162*(a + b*x**3)**(2/3)*b**3*c**2*x**4 + 216*(a +
 b*x**3)**(2/3)*b**3*c*d*x**7 + 81*(a + b*x**3)**(2/3)*b**3*d**2*x**10 + 2
0*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**4*d**2 - 120*int((a + b*x**3)
**(2/3)/(a + b*x**3),x)*a**3*b*c*d + 540*int((a + b*x**3)**(2/3)/(a + b*x*
*3),x)*a**2*b**2*c**2)/(972*b**2)
```

3.123 $\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 212

$$\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx = \frac{1}{81}\left(27c^2-\frac{ad(9bc-2ad)}{b^2}\right)x(a+bx^3)^{2/3} + \frac{d(9bc-2ad)x(a+bx^3)^{5/3}}{27b^2} + \frac{d^2x^4(a+bx^3)^{5/3}}{9b} + \frac{2a(27b^2c^2-9abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{81\sqrt{3}b^{7/3}} - \frac{a(27b^2c^2-9abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{81b^{7/3}}$$

output
```
1/81*(27*c^2-a*d*(-2*a*d+9*b*c)/b^2)*x*(b*x^3+a)^(2/3)+1/27*d*(-2*a*d+9*b*
c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d^2*x^4*(b*x^3+a)^(5/3)/b+2/243*a*(2*a^2*d^2-
9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*
3^(1/2)/b^(7/3)-1/81*a*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x
^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.21

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \frac{3\sqrt[3]{b}x(a+bx^3)^{2/3}\left(-4a^2d^2+3abd(6c+dx^3)+9b^2(3c^2+3cdx^3+d^2x^6)\right)+2\sqrt{3}a(27b^2c^2-9a}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

output
```
(3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2
*(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + 2*Sqrt[3]*a*(27*b^2*c^2 - 9*a*b*c*d + 2*
a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2
*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/
3)] + a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^(7/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int \left(bx^3+a\right)^{2/3} \left(4d(3bc-ad)x^3+c(9bc-ad)\right)dx}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{\left(2a^2d^2-9abcd+27b^2c^2\right)\int \left(bx^3+a\right)^{2/3}dx}{3b}+\frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

$$\Big\downarrow \text{748}$$

$$\frac{\frac{(2a^2d^2-9abcd+27b^2c^2)\left(\frac{2}{3}a\int\frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x(a+bx^3)^{2/3}\right)}{3b}+\frac{2dx(a+bx^3)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{(2a^2d^2-9abcd+27b^2c^2)\left(\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x(a+bx^3)^{2/3}\right)}{3b}+\frac{2dx(a+bx^3)^{5/3}(3bc-ad)}{3b}}{9b}+\frac{dx(a+bx^3)^{5/3}(c+dx^3)}{9b}$$

input
```
Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(5/3)*(c + d*x^3))/(9*b) + ((2*d*(3*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(3*b) + ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*((x*(a + b*x^3)^(
2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqr
t[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/3))/(3
*b))/(9*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2adx\left(\frac{dx^3}{6}+c\right)\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{4}{3}}}{9}+\frac{\left(\frac{1}{3}d^2x^6+cdx^3+c^2\right)x\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{7}{3}}}{3}+\frac{2a\left(-6ad^2xb^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{2}{3}}+\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx\right.\right.}{3b^{\frac{1}{3}}x}\right.\right.\right.}{b^{\frac{7}{3}}}$

input
```
int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/243/b^(7/3)*(27*a*d*x*(1/6*d*x^3+c)*(b*x^3+a)^(2/3)*b^(4/3)+81/2*(1/3*d^
2*x^6+c*d*x^3+c^2)*x*(b*x^3+a)^(2/3)*b^(7/3)+a*(-6*a*d^2*x*b^(1/3)*(b*x^3+
a)^(2/3)+(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1
/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*l
n((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a^2*d^2-9/2*a*b*c*d+27/2*b^2*c^2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.99

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
[1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)
^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3
)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2
)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9
*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d
+ a*b^2*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a
)^(2/3))/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*
d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(
1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a
^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(
-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d +
a*b^2*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^
(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.62

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \frac{a^{\frac{2}{3}} c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{2}{3}} c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

output
```
a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma
(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(181) = 362$.

Time = 0.11 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.60

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
-1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^
(2/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(
b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*
arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^
2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/
3) + 2*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2
/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3
+ (b*x^3 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(
1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^
3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/
3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 1
1*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x
^3 + a)*b^4/x^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{2}{3}} \left(dx^3+c\right)^2 dx$$

input
```
integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^{2/3} \left(d\,x^3+c\right)^2 dx$$

input
```
int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{2/3} (c + dx^3)^2 \, dx = \frac{-4(b\,x^3+a)^{\frac{2}{3}}\,a^2d^2x+18(b\,x^3+a)^{\frac{2}{3}}\,abcdx+3(b\,x^3+a)^{\frac{2}{3}}\,ab\,d^2x^4+27(b\,x^3+a)^{\frac{2}{3}}\,b^2c^2x+27\cdots}{\cdots}$$

input
```
int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

output
```
( - 4*(a + b*x**3)**(2/3)*a**2*d**2*x + 18*(a + b*x**3)**(2/3)*a*b*c*d*x +
 3*(a + b*x**3)**(2/3)*a*b*d**2*x**4 + 27*(a + b*x**3)**(2/3)*b**2*c**2*x
+ 27*(a + b*x**3)**(2/3)*b**2*c*d*x**4 + 9*(a + b*x**3)**(2/3)*b**2*d**2*x
**7 + 4*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a**3*d**2 - 18*int((a + b*
x**3)**(2/3)/(a + b*x**3),x)*a**2*b*c*d + 54*int((a + b*x**3)**(2/3)/(a +
b*x**3),x)*a*b**2*c**2)/(81*b**2)
```

3.124 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{2d(3bc-ad)x(a+bx^3)^{2/3}}{9b^2} + \frac{d^2x^4(a+bx^3)^{2/3}}{6b} + \frac{(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{(9b^2c^2-6abcd+2a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{7/3}}$$

output

```
2/9*d*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b^2+1/6*d^2*x^4*(b*x^3+a)^(2/3)/b+1/2
7*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))*3^(1/2)/b^(7/3)-1/18*(2*a^2*d^2-6*a*b*c*d+9*b^2*c^2)*ln(-b^(1/
3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{3\sqrt[3]{b}dx(a+bx^3)^{2/3}(-4ad+3b(4c+dx^3))+2\sqrt{3}(9b^2c^2-6abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output
```
(3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + 2*Sqrt[3]*
(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
+ 2*(a + b*x^3)^(1/3))] - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)] + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*Log[b^(2
/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(7/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {933, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int \frac{d(9bc-4ad)x^3+c(6bc-ad)}{\sqrt[3]{bx^3+a}}dx}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{2(2a^2d^2-6abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}+\frac{dx(a+bx^3)^{2/3}(9bc-4ad)}{3b}}{6b}+\frac{dx(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

input

```
Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

output

```
(d*x*(a + b*x^3)^(2/3)*(c + d*x^3))/(6*b) + ((d*(9*b*c - 4*a*d)*x*(a + b*x
^3)^(2/3))/(3*b) + (2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(ArcTan[(1 + (2*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x
) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b))/(6*b)
```

Defintions of rubi rules used

rule 769

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(-3\left(b x^3+a\right)^{\frac{2}{3}}\left(\frac{d x^3}{4}+c\right) d x\, b^{\frac{4}{3}}+a d^2 x\, b^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{2}{3}}+\frac{\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b x^3+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b x^3+a\right)^{\frac{1}{3}}}{x}\right)\right.}{}\right.}{9 b^{\frac{7}{3}}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-3*(b*x^3+a)^(2/3)*(1/4*d*x^3+c)*d*x*b^(4/3)+a*d^2*x*b^(1/3)*(b*x^3+
a)^(2/3)+1/3*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(
1/3)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a^2*d^2-3*a*b*c*d+9/2*b^2*c^2))/b^(
7/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)
/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(
1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*s
qrt((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3
)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a
^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b
*x^3 + a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*
b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)
^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b
)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d
 + 2*a^2*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3
)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)
*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper
((1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(145) = 290$.

Time $= 0.11$ (sec) , antiderivative size $= 436$, normalized size of antiderivative $= 2.53$

$$\int \frac{(c + dx^3)^2}{\sqrt[3]{a + bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*
(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)
/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3
 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*a
rctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a
^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)
^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x
^3 + (b*x^3 + a)^2*b^2/x^6))*d^2
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)*d**2 + 2*int(x**3/(a + b*x**3)**(1/3),x)*c
*d + int(1/(a + b*x**3)**(1/3),x)*c**2
```

3.125 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{(bc-ad)^2x}{ab^2\sqrt[3]{a+bx^3}} + \frac{d^2x(a+bx^3)^{2/3}}{3b^2} + \frac{2d(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{7/3}}$$

output

```
(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(1/3)+1/3*d^2*x*(b*x^3+a)^(2/3)/b^2+2/9*d*(
-2*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/
b^(7/3)-1/3*d*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{\frac{3\sqrt[3]{b}x(3b^2c^2+4a^2d^2+abd(-6c+dx^3))}{a\sqrt[3]{a+bx^3}}+2\sqrt{3}d(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2d(-$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((3*b^(1/3)*x*(3*b^2*c^2 + 4*a^2*d^2 + a*b*d*(-6*c + d*x^3)))/(a*(a + b*x^
3)^(1/3)) + 2*Sqrt[3]*d*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3
)*x + 2*(a + b*x^3)^(1/3))] + 2*d*(-3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a +
 b*x^3)^(1/3)] + d*(3*b*c - 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)
^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 27, 913, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(ac-(3bc-4ad)x^3)}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{ac-(3bc-4ad)x^3}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\downarrow \text{913}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}-\frac{x(a+bx^3)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

$$\downarrow \text{769}$$

$$\frac{d\left(\frac{2a(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}-\frac{x(a+bx^3)^{2/3}(3bc-4ad)}{3b}\right)}{ab}+\frac{x(c+dx^3)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(a*b*(a + b*x^3)^(1/3)) + (d*(-1/3*((3*b*c - 4
*a*d)*x*(a + b*x^3)^(2/3))/b + (2*a*(3*b*c - 2*a*d)*(ArcTan[(1 + (2*b^(1/3
)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a
 + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$\frac{\frac{4 a d b^{2} \left(\arctan \left(\frac{\sqrt{3}\, \left(\frac{2 \left(b \,x^{3}+a \right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x \right)}{3 x}\right) \sqrt{3}+\ln \left(\frac{-b^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{1}{3}}}{x}\right)-\frac{\ln \left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} x +\left(b \,x^{3}+a \right)^{\frac{2}{3}}}{x^{2}}\right)}{2}\right) \left(a d -\frac{3 b c}{2}\right) (b x}{9}}{b^{\frac{13}{3}} \left(b \,x^{3}+a \right)^{\frac{1}{3}} a}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
4/9*(a*d*b^2*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*3^(1/2)+
ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-3/2*b*c)*(b*x^3+a)^(1/3)+9/4*(b^2*c^2-2
*a*d*(-1/6*d*x^3+c)*b+4/3*a^2*d^2)*x*b^(7/3))/(b*x^3+a)^(1/3)/b^(13/3)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(119) = 238$.

Time = 0.12 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.53

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
[-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2
*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2
- 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)
*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^
2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
- (3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(
(b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(
a*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/
3))/(a*b^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d
 - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2
/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1
/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a
*b^2*d^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3
))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(119) = 238.

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \frac{1}{9}\,d^2\left(\frac{4\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} + \frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}} - \frac{2\,a\log\left(b^{\frac{2}{3}}+\cdots\right.}{\cdots}\right.$$
$$-\frac{1}{3}\,cd\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,\log\left(\cdots\right.}{\cdots}\right.$$
$$+\frac{c^2x}{(bx^3+a)^{\frac{1}{3}}a}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/
x - (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/
3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/
3)/x)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3)
+ (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^
(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d**2 + 2*
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c*d + int
(1/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c**2
```

3.126 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{(bc-ad)^2x}{4ab^2(a+bx^3)^{4/3}} + \frac{(bc-ad)(3bc+5ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{7/3}}$$

output

```
1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(4/3)+1/4*(-a*d+b*c)*(5*a*d+3*b*c)*x/a^
2/b^2/(b*x^3+a)^(1/3)+1/3*d^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(7/3)-1/2*d^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \frac{\frac{3\sqrt[3]{b}(bc-ad)x(4a^2d+3b^2cx^3+ab(4c+5dx^3))}{a^2(a+bx^3)^{4/3}}+4\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-4d^2\log\left(-\cdots\right.}{12b^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((3*b^(1/3)*(b*c - a*d)*x*(4*a^2*d + 3*b^2*c*x^3 + a*b*(4*c + 5*d*x^3)))/(
a^2*(a + b*x^3)^(4/3)) + 4*Sqrt[3]*d^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)
*x + 2*(a + b*x^3)^(1/3))] - 4*d^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] +
 2*d^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(12*b^(7/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {930, 910, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{4ad^2x^3+c(3bc+ad)}{(bx^3+a)^{4/3}}dx}{4ab}+\frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{4ad^2\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{b}+\frac{x(bc-ad)(4ad+3bc)}{ab\sqrt[3]{a+bx^3}}}{4ab}+\frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

$$\big\downarrow 769$$

$$\frac{\frac{4ad^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{b}+\frac{x(bc-ad)(4ad+3bc)}{ab\sqrt[3]{a+bx^3}}}{4ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (((b*c - a*d)*(3*b
*c + 4*a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (4*a*d^2*(ArcTan[(1 + (2*b^(1/3)*
x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a +
 b*x^3)^(1/3)]/(2*b^(1/3))))/b)/(4*a*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.50

method	result
pseudoelliptic	$\frac{\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) a^{2} d^{2} b^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b^{\frac{1}{3}}}+x\right)}{3x}\right) a^{2} d^{2} b^{2} \left(b\,x^{3}+a\right)^{\frac{4}{3}}-2\ln\left(\right.}{6\left(b\,x^{3}+a\right)^{\frac{4}{3}} b^{\frac{13}{3}} a^{2}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/6/(b*x^3+a)^(4/3)*(ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(
2/3))/x^2)*a^2*d^2*b^2*(b*x^3+a)^(4/3)-2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*
x^3+a)^(1/3)/b^(1/3)+x)/x)*a^2*d^2*b^2*(b*x^3+a)^(4/3)-2*ln((-b^(1/3)*x+(b
*x^3+a)^(1/3))/x)*a^2*d^2*b^2*(b*x^3+a)^(4/3)-3/2*x*b^(7/3)*(5*a^2*b*d^2*x
^3-2*a*b^2*c*d*x^3-3*b^3*c^2*x^3+4*a^3*d^2-4*a*b^2*c^2))/b^(13/3)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(123) = 246.

Time = 0.11 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.89

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(
(-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/
3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^
(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3
+ a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b
^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (
b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 +
2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a
)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2
*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-
sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*
(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x
 + (b*x^3 + a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
+ a)^(2/3))/x^2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a
*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 +
 a^4*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = -\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)c^2x^4}{4\,(bx^3+a)^{\frac{4}{3}}a^2} + \frac{cdx^4}{2\,(bx^3+a)^{\frac{4}{3}}a} - \frac{1}{12}\left(\frac{3\left(b+\frac{4\,(bx^3+a)}{x^3}\right)x^4}{(bx^3+a)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\,\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \right.$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4
/((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^
(4/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x
)/b^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
+ a)^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d
^2
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a +
 b*x**3)**(1/3)*b**2*x**6),x)*d**2 + 2*int(x**3/((a + b*x**3)**(1/3)*a**2
+ 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*c*d +
 int(1/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b
*x**3)**(1/3)*b**2*x**6),x)*c**2
```

3.127 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{(bc-ad)^2x}{7ab^2(a+bx^3)^{7/3}} + \frac{(bc-ad)(3bc+4ad)x}{14a^2b^2(a+bx^3)^{4/3}} + \frac{(9b^2c^2+3abcd+2a^2d^2)\,x}{14a^3b^2\sqrt[3]{a+bx^3}}$$

output
```
1/7*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(7/3)+1/14*(-a*d+b*c)*(4*a*d+3*b*c)*x/a
^2/b^2/(b*x^3+a)^(4/3)+1/14*(2*a^2*d^2+3*a*b*c*d+9*b^2*c^2)*x/a^3/b^2/(b*x
^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{9b^2c^2x^7+3abcx^4(7c+dx^3)+a^2(14c^2x+7cdx^4+2d^2x^7)}{14a^3(a+bx^3)^{7/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(9*b^2*c^2*x^7 + 3*a*b*c*x^4*(7*c + d*x^3) + a^2*(14*c^2*x + 7*c*d*x^4 + 2
*d^2*x^7))/(14*a^3*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{6c\int \frac{dx^3+c}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \; 903$$

$$\frac{6c\left(\frac{3c\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

$$\Big\downarrow \; 746$$

$$\frac{6c\left(\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

output
```
(x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*c*((3*c*x)/(4*a^2*(a + b*x^
3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{\left(2a^{2}d^{2}+3abcd+9b^{2}c^{2}\right)x^{7}+7\left(a^{2}cd+3b\,c^{2}a\right)x^{4}+14a^{2}c^{2}x}{14\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	71
gosper	$\frac{x\left(2a^{2}d^{2}x^{6}+3abcd\,x^{6}+9b^{2}c^{2}x^{6}+7a^{2}cd\,x^{3}+21ab\,c^{2}x^{3}+14a^{2}c^{2}\right)}{14\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	76
trager	$\frac{x\left(2a^{2}d^{2}x^{6}+3abcd\,x^{6}+9b^{2}c^{2}x^{6}+7a^{2}cd\,x^{3}+21ab\,c^{2}x^{3}+14a^{2}c^{2}\right)}{14\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	76
orering	$\frac{x\left(2a^{2}d^{2}x^{6}+3abcd\,x^{6}+9b^{2}c^{2}x^{6}+7a^{2}cd\,x^{3}+21ab\,c^{2}x^{3}+14a^{2}c^{2}\right)}{14\left(b\,x^{3}+a\right)^{\frac{7}{3}}a^{3}}$	76

input
```
int((d*x^3+c)^2/(b*x^3+a)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
1/14*((2*a^2*d^2+3*a*b*c*d+9*b^2*c^2)*x^7+7*(a^2*c*d+3*a*b*c^2)*x^4+14*a^2
*c^2*x)/(b*x^3+a)^(7/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{((9\,b^2c^2+3\,abcd+2\,a^2d^2)x^7+14\,a^2c^2x+7\,(3\,abc^2+a^2cd)x^4)(bx^3+a)^{\frac{2}{3}}}{14\,(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6)}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^
2 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b
*x^3 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = -\frac{\left(4\,b-\frac{7\,(bx^3+a)}{x^3}\right)cdx^7}{14\,(bx^3+a)^{\frac{7}{3}}a^2} + \frac{d^2x^7}{7\,(bx^3+a)^{\frac{7}{3}}a} + \frac{\left(2\,b^2-\frac{7\,(bx^3+a)b}{x^3}+\frac{14\,(bx^3+a)^2}{x^6}\right)c^2x^7}{14\,(bx^3+a)^{\frac{7}{3}}a^3}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
-1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*
x^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3
+ a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \frac{2\,a^4\,d^2\,x+2\,a^2\,d^2\,x\,(b\,x^3+a)^2+9\,b^2\,c^2\,x\,(b\,x^3+a)^2+2\,a^2\,b^2\,c^2\,x-4\,a^3\,d^2\,x\,(b\,x^3+ \cdots}{14\,a^3\,b^2\,(b\,x \cdots}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

output
```
(2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a
^2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3
*b*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^
2*(a + b*x^3)^(7/3))
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)$$
$$+2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)cd$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(10/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*d**2
+ 2*int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3
 + 3*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*c
*d + int(1/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 +
 3*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*c**
2
```

3.128 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{(bc-ad)^2x}{10ab^2(a+bx^3)^{10/3}}+\frac{(bc-ad)(9bc+11ad)x}{70a^2b^2(a+bx^3)^{7/3}} + \frac{(27b^2c^2+6abcd+2a^2d^2)\,x}{140a^3b^2(a+bx^3)^{4/3}}+\frac{3(27b^2c^2+6abcd+2a^2d^2)\,x}{140a^4b^2\sqrt[3]{a+bx^3}}$$

output
```
1/10*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(10/3)+1/70*(-a*d+b*c)*(11*a*d+9*b*c)*
x/a^2/b^2/(b*x^3+a)^(7/3)+1/140*(2*a^2*d^2+6*a*b*c*d+27*b^2*c^2)*x/a^3/b^2
/(b*x^3+a)^(4/3)+3/140*(2*a^2*d^2+6*a*b*c*d+27*b^2*c^2)*x/a^4/b^2/(b*x^3+a
)^(1/3)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x(81b^3c^2x^9+18ab^2cx^6(15c+dx^3)+10a^3(14c^2+7cdx^3+2d^2x^6)+3a^2bx^3(105c^2+}{140a^4(a+bx^3)^{10/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*
d*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140
*a^4*(a + b*x^3)^(10/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {907, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx$$

$$\Big\downarrow \text{907}$$

$$\frac{(9bc-10ad)\int \frac{(dx^3+c)^2}{(bx^3+a)^{10/3}}dx}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(9bc-10ad)\left(\frac{6c\int \frac{dx^3+c}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(9bc-10ad)\left(\frac{6c\left(\frac{3c\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

$$\Big\downarrow \text{746}$$

$$\frac{(9bc-10ad)\left(\frac{6c\left(\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}\right)}{10a(bc-ad)}+\frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

output
```
(b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3)) + ((9*b*c - 10*a
*d)*((x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (6*c*((3*c*x)/(4*a^2*(a +
 b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))))/(7*a)))/(10*a*(
b*c - a*d))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{1}{7}d^{2}x^{6}+\frac{1}{2}cd\,x^{3}+c^{2}\right)a^{3}+\frac{9\left(\frac{2}{105}d^{2}x^{6}+\frac{4}{21}cd\,x^{3}+c^{2}\right)b\,x^{3}a^{2}}{4}+\frac{27c\left(\frac{d\,x^{3}}{15}+c\right)b^{2}x^{6}a}{14}+\frac{81b^{3}c^{2}x^{9}}{140}\right)x}{\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	96
gosper	$\frac{x\left(6a^{2}b\,d^{2}x^{9}+18a\,b^{2}cd\,x^{9}+81b^{3}c^{2}x^{9}+20a^{3}d^{2}x^{6}+60a^{2}bcd\,x^{6}+270a\,b^{2}c^{2}x^{6}+70a^{3}cd\,x^{3}+315a^{2}b\,c^{2}x^{3}+140a^{3}c^{2}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	115
trager	$\frac{x\left(6a^{2}b\,d^{2}x^{9}+18a\,b^{2}cd\,x^{9}+81b^{3}c^{2}x^{9}+20a^{3}d^{2}x^{6}+60a^{2}bcd\,x^{6}+270a\,b^{2}c^{2}x^{6}+70a^{3}cd\,x^{3}+315a^{2}b\,c^{2}x^{3}+140a^{3}c^{2}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	115
orering	$\frac{x\left(6a^{2}b\,d^{2}x^{9}+18a\,b^{2}cd\,x^{9}+81b^{3}c^{2}x^{9}+20a^{3}d^{2}x^{6}+60a^{2}bcd\,x^{6}+270a\,b^{2}c^{2}x^{6}+70a^{3}cd\,x^{3}+315a^{2}b\,c^{2}x^{3}+140a^{3}c^{2}\right)}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$	115

input
```
int((d*x^3+c)^2/(b*x^3+a)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^3+9/4*(2/105*d^2*x^6+4/21*c*d*x^3+c^2)*b*
x^3*a^2+27/14*c*(1/15*d*x^3+c)*b^2*x^6*a+81/140*b^3*c^2*x^9)/(b*x^3+a)^(10
/3)*x/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{\left(3\left(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2\right)x^{10}+10\left(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2\right)x^7+140\,a^3c^2x -}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right.}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

output
```
1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2
+ 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c
*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 +
 4*a^7*b*x^3 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = -\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)d^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^2} + \frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)cdx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)c^2x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^4}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

output
```
-1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*
(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 +
a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^
2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{(b\,x^3+a)^{10/3}} - \frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{(b\,x^3+a)^{7/3}} + \frac{x\,(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2)}{140\,a^3\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(6\,a^2\,d^2+18\,a\,b\,c\,d+81\,b^2\,c^2)}{140\,a^4\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

output
```
(x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (
x*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x
^3)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x
^3)^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b
*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9}\right.$$

$$+2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}}\right.$$

$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^4+4\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,x^3+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^2x^6+4\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^3x^9+(b\,x^3+a)^{\frac{1}{3}}\,b^4x^{12}}\right.$$

input

```
int((d*x^3+c)^2/(b*x^3+a)^(13/3),x)
```

output

```
int(x**6/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6
*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 +
(a + b*x**3)**(1/3)*b**4*x**12),x)*d**2 + 2*int(x**3/((a + b*x**3)**(1/3)*
a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2
*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3*x**9 + (a + b*x**3)**(1/3)*b**4*x**12
),x)*c*d + int(1/((a + b*x**3)**(1/3)*a**4 + 4*(a + b*x**3)**(1/3)*a**3*b*
x**3 + 6*(a + b*x**3)**(1/3)*a**2*b**2*x**6 + 4*(a + b*x**3)**(1/3)*a*b**3
*x**9 + (a + b*x**3)**(1/3)*b**4*x**12),x)*c**2
```

3.129 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\, dx$

Optimal result

Integrand size = 21, antiderivative size = 207

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\, dx = \frac{(bc-ad)^2x}{13ab^2\,(a+bx^3)^{13/3}} + \frac{(bc-ad)(6bc+7ad)x}{65a^2b^2\,(a+bx^3)^{10/3}} + \frac{(54b^2c^2+9abcd+2a^2d^2)\,x}{455a^3b^2\,(a+bx^3)^{7/3}} + \frac{3(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^4b^2\,(a+bx^3)^{4/3}} + \frac{9(54b^2c^2+9abcd+2a^2d^2)\,x}{910a^5b^2\sqrt[3]{a+bx^3}}$$

output

```
1/13*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(13/3)+1/65*(-a*d+b*c)*(7*a*d+6*b*c)*x
/a^2/b^2/(b*x^3+a)^(10/3)+1/455*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^3/b^2
/(b*x^3+a)^(7/3)+3/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^4/b^2/(b*x^3+a
)^(4/3)+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^5/b^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{x(486b^4c^2x^{12}+81ab^3cx^9(26c+dx^3)+65a^4(14c^2+7cdx^3+2d^2x^6)+39a^3bx^3(70c^2+ \ldots}{910a^5\,(a+bx^3)^{13/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
(x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*
c*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*
a^2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/
3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {930, 910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{d(9bc+4ad)x^3+c(12bc+ad)}{(bx^3+a)^{13/3}}dx}{13ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\,(a+bx^3)^{13/3}}$$

$$\Big\downarrow \; 910$$

$$\frac{\frac{(2a^2d^2+9abcd+54b^2c^2)\int \frac{1}{(bx^3+a)^{10/3}}dx}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\,(a+bx^3)^{13/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{\frac{(2a^2d^2+9abcd+54b^2c^2)\left(\frac{6\int \frac{1}{(bx^3+a)^{7/3}}dx}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \; 749$$

$$\frac{\frac{(2a^2d^2+9abcd+54b^2c^2)\left(\frac{6\left(\frac{3\int \frac{1}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

$$\Big\downarrow \; 746$$

$$\frac{\frac{\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a(a+bx^3)^{4/3}}\right)}{7a}+\frac{x}{7a(a+bx^3)^{7/3}}\right)(2a^2d^2+9abcd+54b^2c^2)}{5ab}+\frac{2x(bc-ad)(ad+3bc)}{5ab(a+bx^3)^{10/3}}}{13ab}+\frac{x(c+dx^3)(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3)) + ((2*(b*c - a*d)*
(3*b*c + a*d)*x)/(5*a*b*(a + b*x^3)^(10/3)) + ((54*b^2*c^2 + 9*a*b*c*d + 2
*a^2*d^2)*(x/(7*a*(a + b*x^3)^(7/3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*
x)/(4*a^2*(a + b*x^3)^(1/3))))/(7*a)))/(5*a*b))/(13*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{\Bigg(\left(\frac{1}{7} d^{2} x^{6}+\frac{1}{2} c d\, x^{3}+c^{2}\right) a^{4}+3\left(\frac{1}{35} d^{2} x^{6}+\frac{3}{14} c d\, x^{3}+c^{2}\right) b\, x^{3} a^{3}+\frac{27 b^{2} x^{6} \left(\frac{1}{195} d^{2} x^{6}+\frac{1}{10} c d\, x^{3}+c^{2}\right) a^{2}}{7}+\frac{81 c \left(\frac{d\, x^{3}}{26}+c \right) b^{3} x^{9} a}{35}+\frac{243 b^{4} c}{45}}{\left(b\, x^{3}+a \right)^{\frac{13}{3}} a^{5}}$
gosper	$\frac{x \big(18 a^{2} b^{2} d^{2} x^{12}+81 a\, b^{3} c d\, x^{12}+486 b^{4} c^{2} x^{12}+78 a^{3} b\, d^{2} x^{9}+351 a^{2} b^{2} c d\, x^{9}+2106 a\, b^{3} c^{2} x^{9}+130 a^{4} d^{2} x^{6}+585 a^{3} b c d\, x^{6}+3510 a^{2} b}{910 \left(b\, x^{3}+a \right)^{\frac{13}{3}} a^{5}}$
trager	$\frac{x \big(18 a^{2} b^{2} d^{2} x^{12}+81 a\, b^{3} c d\, x^{12}+486 b^{4} c^{2} x^{12}+78 a^{3} b\, d^{2} x^{9}+351 a^{2} b^{2} c d\, x^{9}+2106 a\, b^{3} c^{2} x^{9}+130 a^{4} d^{2} x^{6}+585 a^{3} b c d\, x^{6}+3510 a^{2} b}{910 \left(b\, x^{3}+a \right)^{\frac{13}{3}} a^{5}}$
orering	$\frac{x \big(18 a^{2} b^{2} d^{2} x^{12}+81 a\, b^{3} c d\, x^{12}+486 b^{4} c^{2} x^{12}+78 a^{3} b\, d^{2} x^{9}+351 a^{2} b^{2} c d\, x^{9}+2106 a\, b^{3} c^{2} x^{9}+130 a^{4} d^{2} x^{6}+585 a^{3} b c d\, x^{6}+3510 a^{2} b}{910 \left(b\, x^{3}+a \right)^{\frac{13}{3}} a^{5}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^4+3*(1/35*d^2*x^6+3/14*c*d*x^3+c^2)*b*x^3
*a^3+27/7*b^2*x^6*(1/195*d^2*x^6+1/10*c*d*x^3+c^2)*a^2+81/35*c*(1/26*d*x^3
+c)*b^3*x^9*a+243/455*b^4*c^2*x^12)/(b*x^3+a)^(13/3)*x/a^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{(9\,(54\,b^4c^2 + 9\,ab^3cd + 2\,a^2b^2d^2)x^{13} + 39\,(54\,ab^3c^2 + 9\,a^2b^2cd + 2\,a^3bd^2)x^{10} + 65\,(54\,a}{910\,(a^5b^5x^{15} + 5\,a^6b^4x^{12} + 10\,a^7b^3x^9}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

output
```
1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^
2 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d +
 2*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3
+ a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^
6 + 5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{\left(35\,b^2 - \frac{91\,(bx^3+a)b}{x^3} + \frac{65\,(bx^3+a)^2}{x^6}\right)d^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^3} - \frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)cdx^{13}}{910\,(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)c^2x^{13}}{455\,(bx^3+a)^{\frac{13}{3}}a^5}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

output
```
1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*
x^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x
^3 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4)
 + 1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 4
55*(b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13
/3)*a^5)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{16}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{(b\,x^3+a)^{13/3}} - \frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{(b\,x^3+a)^{10/3}} + \frac{x\,(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2)}{455\,a^3\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(6\,a^2\,d^2+27\,a\,b\,c\,d+162\,b^2\,c^2)}{910\,a^4\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(18\,a^2\,d^2+81\,a\,b\,c\,d+486\,b^2\,c^2)}{910\,a^5\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

output
```
(x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3)
- (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a
 + b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(
a + b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^
2*(a + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^
5*b^2*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\right. \\ +2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\right. \\ +\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^5+5\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b\,x^3+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^2x^6+10\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b^3x^9+5\,(b\,x^3+a)^{\frac{1}{3}}\,a\right.$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(16/3),x)
```

output
```
int(x**6/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**4*b*x**3 + 1
0*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*a**2*b**3*x*
*9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*b**5*x**15),
x)*d**2 + 2*int(x**3/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/3)*a**
4*b*x**3 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**(1/3)*
a**2*b**3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**(1/3)*
b**5*x**15),x)*c*d + int(1/((a + b*x**3)**(1/3)*a**5 + 5*(a + b*x**3)**(1/
3)*a**4*b*x**3 + 10*(a + b*x**3)**(1/3)*a**3*b**2*x**6 + 10*(a + b*x**3)**
(1/3)*a**2*b**3*x**9 + 5*(a + b*x**3)**(1/3)*a*b**4*x**12 + (a + b*x**3)**
(1/3)*b**5*x**15),x)*c**2
```

3.130 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 248

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} \, dx = \frac{(bc-ad)^2 x}{16ab^2 (a+bx^3)^{16/3}} + \frac{(bc-ad)(15bc+17ad)x}{208a^2b^2 (a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\, x}{520a^3b^2 (a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)\, x}{3640a^4b^2 (a+bx^3)^{7/3}} + \frac{27(45b^2c^2+6abcd+a^2d^2)\, x}{7280a^5b^2 (a+bx^3)^{4/3}} + \frac{81(45b^2c^2+6abcd+a^2d^2)\, x}{7280a^6b^2 \sqrt[3]{a+bx^3}}$$

output
```
1/16*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(16/3)+1/208*(-a*d+b*c)*(17*a*d+15*b*c
)*x/a^2/b^2/(b*x^3+a)^(13/3)+1/520*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^3/b^
2/(b*x^3+a)^(10/3)+9/3640*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^4/b^2/(b*x^3+
a)^(7/3)+27/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^5/b^2/(b*x^3+a)^(4/3)+
81/7280*(a^2*d^2+6*a*b*c*d+45*b^2*c^2)*x/a^6/b^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(3645b^5c^2x^{15}+486ab^4cx^{12}(40c+dx^3)+81a^2b^3x^9(520c^2+32cdx^3+d^2x^6)+520a^5}{72}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
(x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(
520*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)
 + 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175
*c^2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {930, 910, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{4d(3bc+ad)x^3+c(15bc+ad)}{(bx^3+a)^{16/3}}dx}{16ab}+\frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}$$

$$\Big\downarrow \ 910$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\int \frac{1}{(bx^3+a)^{13/3}}dx+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x(c+dx^3)(bc-ad)}{16ab(a+bx^3)^{16/3}}$$

$$\Big\downarrow \ 749$$

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\int \frac{1}{\left(bx^3+a\right)^{10/3}}dx}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$\Big\downarrow$ 749

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\left(\frac{6\int \frac{1}{\left(bx^3+a\right)^{7/3}}dx}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$\Big\downarrow$ 749

$$\frac{\frac{4}{13}\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)\left(\frac{9\left(\frac{6\left(\frac{3\int \frac{1}{\left(bx^3+a\right)^{4/3}}dx}{4a}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

$\Big\downarrow$ 746

$$\frac{\frac{4}{13}\left(\frac{9\left(\frac{6\left(\frac{3x}{4a^2\sqrt[3]{a+bx^3}}+\frac{x}{4a\left(a+bx^3\right)^{4/3}}\right)}{7a}+\frac{x}{7a\left(a+bx^3\right)^{7/3}}\right)}{10a}+\frac{x}{10a(a+bx^3)^{10/3}}\right)\left(\frac{45bc^2}{a}+\frac{ad^2}{b}+6cd\right)+\frac{x\left(\frac{15bc^2}{a}-\frac{4ad^2}{b}-11cd\right)}{13(a+bx^3)^{13/3}}}{16ab} + \frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3)) + ((((15*b*c^2)/a
 - 11*c*d - (4*a*d^2)/b)*x)/(13*(a + b*x^3)^(13/3)) + (4*((45*b*c^2)/a + 6*
c*d + (a*d^2)/b)*(x/(10*a*(a + b*x^3)^(10/3)) + (9*(x/(7*a*(a + b*x^3)^(7/
3)) + (6*(x/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a^2*(a + b*x^3)^(1/3))))/(7
*a)))/(10*a)))/13)/(16*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{7}d^2x^6+\frac{1}{2}cd\,x^3+c^2\right)a^5+\frac{15b\left(\frac{6}{175}d^2x^6+\frac{8}{35}cd\,x^3+c^2\right)x^3a^4}{4}+\frac{45\left(\frac{3}{325}d^2x^6+\frac{3}{25}cd\,x^3+c^2\right)b^2x^6a^3}{7}+\frac{81\left(\frac{1}{520}d^2x^6+\frac{4}{65}cd\,x^3+c^2\right)b}{14}\right.}{(b\,x^3+a)^{\frac{16}{3}}a^6}$
gosper	$\frac{x\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2c\right.}{7280(b\,x^3+a)^{\frac{16}{3}}}$
trager	$\frac{x\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2c\right.}{7280(b\,x^3+a)^{\frac{16}{3}}}$
orering	$\frac{x\left(81a^2b^3d^2x^{15}+486a\,b^4cd\,x^{15}+3645b^5c^2x^{15}+432a^3b^2d^2x^{12}+2592a^2b^3cd\,x^{12}+19440a\,b^4c^2x^{12}+936a^4b\,d^2x^9+5616a^3b^2c\right.}{7280(b\,x^3+a)^{\frac{16}{3}}}$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
((1/7*d^2*x^6+1/2*c*d*x^3+c^2)*a^5+15/4*b*(6/175*d^2*x^6+8/35*c*d*x^3+c^2)
*x^3*a^4+45/7*(3/325*d^2*x^6+3/25*c*d*x^3+c^2)*b^2*x^6*a^3+81/14*(1/520*d^
2*x^6+4/65*c*d*x^3+c^2)*b^3*x^9*a^2+243/91*c*(1/40*d*x^3+c)*b^4*x^12*a+729
/1456*b^5*c^2*x^15)/(b*x^3+a)^(16/3)*x/a^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(81\,(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2)x^{16}+432\,(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2)x^{13}+936\,(45}{7280\,(a^6b^6x^{18}+6\,a^7b^}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

output
```
1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c
^2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c
*d + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d
 + a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(
a^6*b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10
*b^2*x^6 + 6*a^11*b*x^3 + a^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = -\frac{\left(455\,b^3 - \frac{1680\,(bx^3+a)b^2}{x^3} + \frac{2184\,(bx^3+a)^2 b}{x^6} - \frac{1040\,(bx^3+a)^3}{x^9}\right)d^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4} + \frac{\left(455\,b^4 - \frac{2240\,(bx^3+a)b^3}{x^3} + \frac{4368\,(bx^3+a)^2 b^2}{x^6} - \frac{4160\,(bx^3+a)^3 b}{x^9} + \frac{1820\,(bx^3+a)^4}{x^{12}}\right)cdx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^5} - \frac{\left(91\,b^5 - \frac{560\,(bx^3+a)b^4}{x^3} + \frac{1456\,(bx^3+a)^2 b^3}{x^6} - \frac{2080\,(bx^3+a)^3 b^2}{x^9} + \frac{1820\,(bx^3+a)^4 b}{x^{12}} - \frac{1456\,(bx^3+a)^5}{x^{15}}\right)c^2x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^6}$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

output
```
-1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 1
040*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4
 - 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a
)^3*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1
/1456*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 208
0*(b*x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x
^15)*c^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x\left(\frac{c^2}{16\,a}+\frac{a\left(\frac{d^2}{16\,b}-\frac{c\,d}{8\,a}\right)}{b}\right)}{(b\,x^3+a)^{16/3}} - \frac{x\left(\frac{d^2}{13\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{(b\,x^3+a)^{13/3}} + \frac{x\,(a^2\,d^2+6\,a\,b\,c\,d+45\,b^2\,c^2)}{520\,a^3\,b^2\,(b\,x^3+a)^{10/3}} + \frac{x\,(9\,a^2\,d^2+54\,a\,b\,c\,d+405\,b^2\,c^2)}{3640\,a^4\,b^2\,(b\,x^3+a)^{7/3}} + \frac{x\,(27\,a^2\,d^2+162\,a\,b\,c\,d+1215\,b^2\,c^2)}{7280\,a^5\,b^2\,(b\,x^3+a)^{4/3}} + \frac{x\,(81\,a^2\,d^2+486\,a\,b\,c\,d+3645\,b^2\,c^2)}{7280\,a^6\,b^2\,(b\,x^3+a)^{1/3}}$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

output
```
(x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (
x*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a +
b*x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b
*x^3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(
a + b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^
5*b^2*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(
7280*a^6*b^2*(a + b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b \cdots}\right.$$
$$+2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^3x^9+15\,(b\,x^3+a)^{\frac{1}{3}} \cdots}\right.$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^6+6\,(b\,x^3+a)^{\frac{1}{3}}\,a^5b\,x^3+15\,(b\,x^3+a)^{\frac{1}{3}}\,a^4b^2x^6+20\,(b\,x^3+a)^{\frac{1}{3}}\,a^3b^3x^9+15\,(b\,x^3+a)^{\frac{1}{3}} \cdots}\right.$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(19/3),x)
```

output

```
int(x**6/((a + b*x**3)**(1/3)*a**6 + 6*(a + b*x**3)**(1/3)*a**5*b*x**3 + 1
5*(a + b*x**3)**(1/3)*a**4*b**2*x**6 + 20*(a + b*x**3)**(1/3)*a**3*b**3*x*
*9 + 15*(a + b*x**3)**(1/3)*a**2*b**4*x**12 + 6*(a + b*x**3)**(1/3)*a*b**5
*x**15 + (a + b*x**3)**(1/3)*b**6*x**18),x)*d**2 + 2*int(x**3/((a + b*x**3
)**(1/3)*a**6 + 6*(a + b*x**3)**(1/3)*a**5*b*x**3 + 15*(a + b*x**3)**(1/3)
*a**4*b**2*x**6 + 20*(a + b*x**3)**(1/3)*a**3*b**3*x**9 + 15*(a + b*x**3)*
*(1/3)*a**2*b**4*x**12 + 6*(a + b*x**3)**(1/3)*a*b**5*x**15 + (a + b*x**3)
**(1/3)*b**6*x**18),x)*c*d + int(1/((a + b*x**3)**(1/3)*a**6 + 6*(a + b*x*
*3)**(1/3)*a**5*b*x**3 + 15*(a + b*x**3)**(1/3)*a**4*b**2*x**6 + 20*(a + b
*x**3)**(1/3)*a**3*b**3*x**9 + 15*(a + b*x**3)**(1/3)*a**2*b**4*x**12 + 6*
(a + b*x**3)**(1/3)*a*b**5*x**15 + (a + b*x**3)**(1/3)*b**6*x**18),x)*c**2
```

3.131 $\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{2d(7bc-ad)x(a+bx^3)^{10/3}}{77b^2} + \frac{d^2x^4(a+bx^3)^{10/3}}{14b} + \frac{a^2(77b^2c^2-2ad(7bc-ad))\,x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{77b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
2/77*d*(-a*d+7*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d^2*x^4*(b*x^3+a)^(10/3)/b
+1/77*a^2*(77*b^2*c^2-2*a*d*(-a*d+7*b*c))*x*(b*x^3+a)^(1/3)*hypergeom([-7/
3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 12.80 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

$$\int (a+bx^3)^{7/3} (c+dx^3)^2 \, dx = \frac{ax\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{7}{3}\right)\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)\right.}{}$$

input
```
Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*
Hypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*
d*x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3
)/a)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}
, {1, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \left(bx^3+a\right)^{7/3} \left(d(17bc-4ad)x^3+c(14bc-ad)\right) dx}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{2\left(2a^2d^2-14abcd+77b^2c^2\right) \int \left(bx^3+a\right)^{7/3}dx}{11b}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \ 779$$

$$\frac{\frac{2a^2 \sqrt[3]{a+bx^3}\left(2a^2d^2-14abcd+77b^2c^2\right) \int \left(\frac{bx^3}{a}+1\right)^{7/3}dx}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx\left(a+bx^3\right)^{10/3} \left(c+dx^3\right)}{14b}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{2a^2x\sqrt[3]{a+bx^3}(2a^2d^2-14abcd+77b^2c^2)\,\text{Hypergeometric2F1}\left(-\frac{7}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{10/3}(17bc-4ad)}{11b}}{14b}+\frac{dx(a+bx^3)^{10/3}(c+dx^3)}{14b}$$

input
```
Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(10/3)*(c + d*x^3))/(14*b) + ((d*(17*b*c - 4*a*d)*x*(a +
b*x^3)^(10/3))/(11*b) + (2*a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a
+ b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 +
 (b*x^3)/a)^(1/3)))/(14*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{3}+a \right)^{\frac{7}{3}} \left(d\,x^{3}+c \right)^{2}d x$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^{3}+a\right)^{\frac{7}{3}} \left(dx^{3}+c\right)^{2} \,d x$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.19

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

output
```
a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + 2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((
-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3
)*b*d**2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I
*pi)/a)/(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2
*c*d*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13
/3), (16/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)^2 \, dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(bx^3+a\right)^{\frac{7}{3}} \left(dx^3+c\right)^2 dx$$

input
```
integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \int \left(b\,x^3+a\right)^{7/3} \left(d\,x^3+c\right)^2 dx$$

input
```
int((a + b*x^3)^(7/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(7/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx = \frac{-28(b\,x^3+a)^{\frac{1}{3}}\,a^4d^2x+196(b\,x^3+a)^{\frac{1}{3}}\,a^3bcdx+14(b\,x^3+a)^{\frac{1}{3}}\,a^3b\,d^2x^4+2002(b\,x^3+a)^{\frac{1}{3}}\,a^2b}{}$$

input
```
int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

output
```
( - 28*(a + b*x**3)**(1/3)*a**4*d**2*x + 196*(a + b*x**3)**(1/3)*a**3*b*c*
d*x + 14*(a + b*x**3)**(1/3)*a**3*b*d**2*x**4 + 2002*(a + b*x**3)**(1/3)*a
**2*b**2*c**2*x + 1442*(a + b*x**3)**(1/3)*a**2*b**2*c*d*x**4 + 430*(a + b
*x**3)**(1/3)*a**2*b**2*d**2*x**7 + 1309*(a + b*x**3)**(1/3)*a*b**3*c**2*x
**4 + 1610*(a + b*x**3)**(1/3)*a*b**3*c*d*x**7 + 580*(a + b*x**3)**(1/3)*a
*b**3*d**2*x**10 + 385*(a + b*x**3)**(1/3)*b**4*c**2*x**7 + 560*(a + b*x**
3)**(1/3)*b**4*c*d*x**10 + 220*(a + b*x**3)**(1/3)*b**4*d**2*x**13 + 28*in
t((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**5*d**2 - 196*int((a + b*x**3)**(1
/3)/(a + b*x**3),x)*a**4*b*c*d + 1078*int((a + b*x**3)**(1/3)/(a + b*x**3)
,x)*a**3*b**2*c**2)/(3080*b**2)
```

3.132 $\int (a+bx^3)^{4/3} (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx = \frac{d(11bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2} + \frac{d^2x^4\left(a+bx^3\right)^{7/3}}{11b} + \frac{a\left(44b^2c^2-11abcd+2a^2d^2\right)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/44*d*(-2*a*d+11*b*c)*x*(b*x^3+a)^(7/3)/b^2+1/11*d^2*x^4*(b*x^3+a)^(7/3)/
b+1/44*a*(2*a^2*d^2-11*a*b*c*d+44*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-4
/3, 1/3],[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 12.97 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.35

$$\int \left(a+bx^3\right)^{4/3} (c +dx^3)^2 \, dx = \frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{4}{3}\right)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)\right.}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-4/3]*Hy
pergeometric2F1[-4/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-1/3]*Hypergeometric2F1[-1/3, 4/3, 13/3, -((b*x^3)/
a)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-1/3]*HypergeometricPFQ[{-1/3, 4/3, 2},
{1, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-4/3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int \left(bx^3+a\right)^{4/3} \left(2d(7bc-2ad)x^3+c(11bc-ad)\right)dx}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{\left(2a^2d^2-11abcd+44b^2c^2\right)\int \left(bx^3+a\right)^{4/3}dx}{4b}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{a\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right)\int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{4b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{ax\sqrt[3]{a+bx^3}(2a^2d^2-11abcd+44b^2c^2)\text{ Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx(a+bx^3)^{7/3}(7bc-2ad)}{4b}}{11b}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

input
```
Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(7/3)*(c + d*x^3))/(11*b) + ((d*(7*b*c - 2*a*d)*x*(a + b*
x^3)^(7/3))/(4*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4*b*(1 + (b*x^3)/
a)^(1/3)))/(11*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)^{2}d x$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx=\int \left(bx^{3}+a\right)^{\frac{4}{3}} \left(dx^{3}+c\right)^{2} \,dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^
2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.08

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2\sqrt[3]{a}bcdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt[3]{a}bd^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

output
```
a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
) + a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*
d**2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx=\int \left(b\,x^3+a\right)^{4/3}\left(d\,x^3+c\right)^2\,dx$$

input
```
int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int (a+bx^3)^{4/3} (c+dx^3)^2\, dx = \frac{-4(b\,x^3+a)^{\frac{1}{3}}\,a^3d^2x+22(b\,x^3+a)^{\frac{1}{3}}\,a^2bcdx+2(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,d^2x^4+132(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2c^2x}{}$$

input
```
int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

output
```
( - 4*(a + b*x**3)**(1/3)*a**3*d**2*x + 22*(a + b*x**3)**(1/3)*a**2*b*c*d*
x + 2*(a + b*x**3)**(1/3)*a**2*b*d**2*x**4 + 132*(a + b*x**3)**(1/3)*a*b**
2*c**2*x + 99*(a + b*x**3)**(1/3)*a*b**2*c*d*x**4 + 30*(a + b*x**3)**(1/3)
*a*b**2*d**2*x**7 + 44*(a + b*x**3)**(1/3)*b**3*c**2*x**4 + 55*(a + b*x**3
)**(1/3)*b**3*c*d*x**7 + 20*(a + b*x**3)**(1/3)*b**3*d**2*x**10 + 4*int((a
 + b*x**3)**(1/3)/(a + b*x**3),x)*a**4*d**2 - 22*int((a + b*x**3)**(1/3)/(
a + b*x**3),x)*a**3*b*c*d + 88*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**
2*b**2*c**2)/(220*b**2)
```

3.133 $\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{d(4bc-ad)x\left(a+bx^3\right)^{4/3}}{10b^2}+\frac{d^2x^4\left(a+bx^3\right)^{4/3}}{8b}+\frac{\left(10b^2c^2-4abcd+a^2d^2\right)x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/10*d*(-a*d+4*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d^2*x^4*(b*x^3+a)^(4/3)/b+1/
10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*hypergeom([-1/3, 1/3],
[4/3],-b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{x\sqrt[3]{a+bx^3}\left(20a(14c^2+7cdx^3+2d^2x^6)\,\text{Gamma}\left(-\frac{1}{3}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{10}{3},-\frac{bx^3}{a}\right)-3bx^3(11c}{280}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hy
pergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)
] - 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int \sqrt[3]{bx^3+a}\left(d(11bc-4ad)x^3+c(8bc-ad)\right)dx}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{4\left(a^2d^2-4abcd+10b^2c^2\right)\int \sqrt[3]{bx^3+a}dx}{5b}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{4\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right)\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{4x\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{5b}}{8b}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

input
```
Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(4/3)*(c + d*x^3))/(8*b) + ((d*(11*b*c - 4*a*d)*x*(a + b*
x^3)^(4/3))/(5*b) + (4*(10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1
/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^
(1/3)))/(8*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)^{2}dx$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^{3}+a\right)^{\frac{1}{3}}\left(dx^{3}+c\right)^{2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \sqrt[3]{a+bx^3}\,(c+dx^3)^2\,dx = \frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt[3]{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

output
```
a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (
7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma
(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)
)
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\,(c+dx^3)^2\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx^3+c)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx=\int {\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$$

$$=\frac{-2(b\,x^3+a)^{\frac{1}{3}}\,a^2d^2x+8(b\,x^3+a)^{\frac{1}{3}}\,abcdx+(b\,x^3+a)^{\frac{1}{3}}\,ab\,d^2x^4+20(b\,x^3+a)^{\frac{1}{3}}\,b^2c^2x+16(b\,x^3+a)^{\frac{1}{3}}\,b^2}{4}$$

input
```
int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

output
```
( - 2*(a + b*x**3)**(1/3)*a**2*d**2*x + 8*(a + b*x**3)**(1/3)*a*b*c*d*x +
(a + b*x**3)**(1/3)*a*b*d**2*x**4 + 20*(a + b*x**3)**(1/3)*b**2*c**2*x + 1
6*(a + b*x**3)**(1/3)*b**2*c*d*x**4 + 5*(a + b*x**3)**(1/3)*b**2*d**2*x**7
 + 2*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3*d**2 - 8*int((a + b*x**3
)**(1/3)/(a + b*x**3),x)*a**2*b*c*d + 20*int((a + b*x**3)**(1/3)/(a + b*x*
*3),x)*a*b**2*c**2)/(40*b**2)
```

3.134 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{d(5bc-2ad)x\sqrt[3]{a+bx^3}}{5b^2}+\frac{d^2x^4\sqrt[3]{a+bx^3}}{5b} + \frac{\left(5c^2-\frac{ad(5bc-2ad)}{b^2}\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}$$

output
```
1/5*d*(-2*a*d+5*b*c)*x*(b*x^3+a)^(1/3)/b^2+1/5*d^2*x^4*(b*x^3+a)^(1/3)/b+1
/5*(5*c^2-a*d*(-2*a*d+5*b*c)/b^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3]
,[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 15.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(-d(a+bx^3)\left(2ad-b(5c+dx^3)\right)+(5b^2c^2-5abcd+2a^2d^2)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hyperge}\right.}{5b^2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(x*(-(d*(a + b*x^3)*(2*a*d - b*(5*c + d*x^3))) + (5*b^2*c^2 - 5*a*b*c*d +
2*a^2*d^2)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)]))/(5*b^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int \frac{4d(2bc-ad)x^3+c(5bc-ad)}{(bx^3+a)^{2/3}}dx}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(2a^2d^2-5abcd+5b^2c^2)\int \frac{1}{(bx^3+a)^{2/3}}dx}{b}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{b(a+bx^3)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2a^2d^2-5abcd+5b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{b(a+bx^3)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{b}}{5b}+\frac{dx\sqrt[3]{a+bx^3}(c+dx^3)}{5b}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

output
```
(d*x*(a + b*x^3)^(1/3)*(c + d*x^3))/(5*b) + ((2*d*(2*b*c - a*d)*x*(a + b*x
^3)^(1/3))/b + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(2/3
)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(b*(a + b*x^3)^(2/3)))/(
5*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{2}}{(b\,x^{3}+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper
((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int(x**6/(a + b*x**3)**(2/3),x)*d**2 + 2*int(x**3/(a + b*x**3)**(2/3),x)*c
*d + int(1/(a + b*x**3)**(2/3),x)*c**2
```

3.135 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{(bc-ad)^2x}{2ab^2(a+bx^3)^{2/3}} + \frac{d^2x\sqrt[3]{a+bx^3}}{2b^2} + \frac{(b^2c^2+2abcd-2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab^2(a+bx^3)^{2/3}}$$

output
```
1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(2/3)+1/2*d^2*x*(b*x^3+a)^(1/3)/b^2+1/2
*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[
4/3],-b*x^3/a)/a/b^2/(b*x^3+a)^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 13.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(4a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{10}{3},\right.\right.}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*
Hypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*
(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/
(84*a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(c+dx^3)(bc-ad)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(-2a^2d^2+2abcd+b^2c^2)\int \frac{1}{(bx^3+a)^{2/3}}dx}{b}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^3)(bc-ad)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(-2a^2d^2+2abcd+b^2c^2)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{b(a+bx^3)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^3)(bc-ad)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(-2a^2d^2+2abcd+b^2c^2)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{b(a+bx^3)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^3)(bc-ad)}{2ab(a+bx^3)^{2/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(2*a*b*(a + b*x^3)^(2/3)) + (-((d*(b*c - 2*a*d
)*x*(a + b*x^3)^(1/3))/b) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (b*x
^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(b*(a + b*x^3
)^(2/3)))/(2*a*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{5}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^
3 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(5/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(5/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)d^2 + 2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*d**2 + 2*
int(x**3/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*c*d + int
(1/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*c**2
```

3.136 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{(bc-ad)^2x}{5ab^2\,(a+bx^3)^{5/3}} - \frac{d^2x}{b^2\,(a+bx^3)^{2/3}} + \frac{2(2b^2c^2+abcd+2a^2d^2)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2b^2\,(a+bx^3)^{2/3}}$$

output
```
1/5*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)^(5/3)-d^2*x/b^2/(b*x^3+a)^(2/3)+2/5*(2*
a^2*d^2+a*b*c*d+2*b^2*c^2)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 5/3],[4/3],
-b*x^3/a)/a^2/b^2/(b*x^3+a)^(2/3)
```

Mathematica [A] (warning: unable to verify)

Time = 15.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Gamma}\left(\frac{2}{3}\right)\left(5a(14c^2+7cdx^3+2d^2x^6)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{10}{3},\right.\right.}{}$$

input
```
Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(5*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*
Hypergeometric2F1[1/3, 8/3, 10/3, -((b*x^3)/a)] - 2*b*x^3*(11*c^2 + 16*c*d
*x^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 11/3, 13/3, -((b*x^3)/a)] - 6*b*x
^3*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 11/3}, {1, 13/3}, -((b*x^3)/a)
]))/(63*a^3*(a + b*x^3)^(2/3)*Gamma[8/3])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(bc+4ad)x^3+c(4bc+ad)}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\int \frac{1}{(bx^3+a)^{2/3}}dx+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{2bc^2}{a}+\frac{2ad^2}{b}+cd\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{2x\left(\frac{bc^2}{a}-\frac{ad^2}{b}\right)}{(a+bx^3)^{2/3}}}{5ab}+\frac{x(c+dx^3)(bc-ad)}{5ab(a+bx^3)^{5/3}}$$

input
```
Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

output
```
((b*c - a*d)*x*(c + d*x^3))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*((b*c^2)/a - (
a*d^2)/b)*x)/(a + b*x^3)^(2/3) + (((2*b*c^2)/a + c*d + (2*a*d^2)/b)*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x
^3)^(2/3))/(5*a*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{\frac{8}{3}}}dx$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

output
```
int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*
x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(c+dx^3)^2}{(a+bx^3)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
```

output
```
Integral((c + d*x**3)**2/(a + b*x**3)**(8/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \int \frac{(d\,x^3+c)^2}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((c + d*x^3)^2/(a + b*x^3)^(8/3),x)
```

output
```
int((c + d*x^3)^2/(a + b*x^3)^(8/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}}\,dx = \left(\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)d^2$$
$$+2\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)cd$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)c^2$$

input
```
int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

output
```
int(x**6/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a +
 b*x**3)**(2/3)*b**2*x**6),x)*d**2 + 2*int(x**3/((a + b*x**3)**(2/3)*a**2
+ 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*c*d +
 int(1/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b
*x**3)**(2/3)*b**2*x**6),x)*c**2
```

3.137 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 186

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = -\frac{(bc-ad)^3x}{10cd^3(c+dx^3)^{10/3}} + \frac{3(bc-ad)^2(7bc+3ad)x}{70c^2d^3(c+dx^3)^{7/3}} - \frac{3(bc-ad)(14b^2c^2+12abcd+9a^2d^2)x}{140c^3d^3(c+dx^3)^{4/3}} + \frac{(14b^3c^3+18ab^2c^2d+27a^2bcd^2+81a^3d^3)x}{140c^4d^3\sqrt[3]{c+dx^3}}$$

output
```
-1/10*(-a*d+b*c)^3*x/c/d^3/(d*x^3+c)^(10/3)+3/70*(-a*d+b*c)^2*(3*a*d+7*b*c
)*x/c^2/d^3/(d*x^3+c)^(7/3)-3/140*(-a*d+b*c)*(9*a^2*d^2+12*a*b*c*d+14*b^2*
c^2)*x/c^3/d^3/(d*x^3+c)^(4/3)+1/140*(81*a^3*d^3+27*a^2*b*c*d^2+18*a*b^2*c
^2*d+14*b^3*c^3)*x/c^4/d^3/(d*x^3+c)^(1/3)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{x(14b^3c^3x^9+6ab^2c^2x^6(10c+3dx^3)+3a^2bcx^3(35c^2+30cdx^3+9d^2x^6)+a^3(140c^3+}{140c^4\,(c+dx^3)^{10/3}}$$

input
```
Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

output
```
(x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*
c^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x
^6 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\int \frac{(bx^3+a)^2}{(dx^3+c)^{10/3}}dx}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\left(\frac{6a\int \frac{bx^3+a}{(dx^3+c)^{7/3}}dx}{7c}+\frac{x(a+bx^3)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

$$\Big\downarrow \text{903}$$

$$\frac{9a\left(\frac{6a\left(\frac{3a\int \frac{1}{\left(dx^3+c\right)^{4/3}}dx}{4c}+\frac{x\left(a+bx^3\right)}{4c\left(c+dx^3\right)^{4/3}}\right)}{7c}+\frac{x\left(a+bx^3\right)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

$$\Big\downarrow \text{746}$$

$$\frac{9a\left(\frac{6a\left(\frac{x\left(a+bx^3\right)}{4c\left(c+dx^3\right)^{4/3}}+\frac{3ax}{4c^2\sqrt[3]{c+dx^3}}\right)}{7c}+\frac{x\left(a+bx^3\right)^2}{7c(c+dx^3)^{7/3}}\right)}{10c}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

input
```
Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

output
```
(x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*((x*(a + b*x^3)^2)/(7*c
*(c + d*x^3)^(7/3)) + (6*a*((x*(a + b*x^3))/(4*c*(c + d*x^3)^(4/3)) + (3*a
*x)/(4*c^2*(c + d*x^3)^(1/3))))/(7*c)))/(10*c)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{10}b^3x^9+\frac{3}{7}a\,b^2x^6+\frac{3}{4}a^2b\,x^3+a^3\right)c^3+\frac{9ad\left(\frac{2}{35}b^2x^6+\frac{2}{7}ab\,x^3+a^2\right)x^3c^2}{4}+\frac{27a^2d^2\left(\frac{b\,x^3}{10}+a\right)x^6c}{14}+\frac{81a^3d^3x^9}{140}\right)x}{(d\,x^3+c)^{\frac{10}{3}}c^4}$
gosper	$\frac{x(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+}{140(d\,x^3+c)^{\frac{10}{3}}c^4}$
trager	$\frac{x(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+}{140(d\,x^3+c)^{\frac{10}{3}}c^4}$
orering	$\frac{x(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+}{140(d\,x^3+c)^{\frac{10}{3}}c^4}$

input
```
int((b*x^3+a)^3/(d*x^3+c)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
((1/10*b^3*x^9+3/7*a*b^2*x^6+3/4*a^2*b*x^3+a^3)*c^3+9/4*a*d*(2/35*b^2*x^6+
2/7*a*b*x^3+a^2)*x^3*c^2+27/14*a^2*d^2*(1/10*b*x^3+a)*x^6*c+81/140*a^3*d^3
*x^9)/(d*x^3+c)^(10/3)*x/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{((14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3)x^{10}+30\,(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2)x^7}{140\,(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7d}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

output
```
1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 +
30*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(
a^2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*
x^9 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \frac{b^3x^{10}}{10\,(dx^3+c)^{\frac{10}{3}}c} - \frac{3\,ab^2\left(7\,d - \frac{10\,(dx^3+c)}{x^3}\right)x^{10}}{70\,(dx^3+c)^{\frac{10}{3}}c^2} + \frac{3\left(14\,d^2 - \frac{40\,(dx^3+c)d}{x^3} + \frac{35\,(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^3} - \frac{\left(14\,d^3 - \frac{60\,(dx^3+c)d^2}{x^3} + \frac{105\,(dx^3+c)^2d}{x^6} - \frac{140\,(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140\,(dx^3+c)^{\frac{10}{3}}c^4}$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

output
```
1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^
3)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 +
35*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3
- 60*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9
)*a^3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

Giac [F]

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx=\int \frac{(bx^3+a)^3}{(dx^3+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx=\frac{x\left(\frac{a^3}{10\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{\left(d\,x^3+c\right)^{10/3}}-\frac{x\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{\left(d\,x^3+c\right)^{4/3}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{70\,c^2\,d^3}\right)}{\left(d\,x^3+c\right)^{7/3}}+\frac{x\,\left(81\,a^3\,d^3+27\,a^2\,b\,c\,d^2+18\,a\,b^2\,c^2\,d+14\,b^3\,c^3\right)}{140\,c^4\,d^3\,{\left(d\,x^3+c\right)}^{1/3}}$$

input
```
int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

output
```
(x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10
*c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 +
 6*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c
*(b^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*
a^3*d^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c +
 d*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx = \left(\int \frac{x^9}{(d\,x^3+c)^{\frac{1}{3}}c^4+4\,(d\,x^3+c)^{\frac{1}{3}}c^3d\,x^3+6\,(d\,x^3+c)^{\frac{1}{3}}c^2d^2x^6+4\,(d\,x^3+c)^{\frac{1}{3}}c\,d^3x^9\right.$$
$$+3\left(\int \frac{x^6}{(d\,x^3+c)^{\frac{1}{3}}c^4+4\,(d\,x^3+c)^{\frac{1}{3}}c^3d\,x^3+6\,(d\,x^3+c)^{\frac{1}{3}}c^2d^2x^6+4\,(d\,x^3+c)^{\frac{1}{3}}c\,d^3x^9+(d\,x^3+c)^{\frac{1}{3}}d^4x^{12}}\right.$$
$$+3\left(\int \frac{x^3}{(d\,x^3+c)^{\frac{1}{3}}c^4+4\,(d\,x^3+c)^{\frac{1}{3}}c^3d\,x^3+6\,(d\,x^3+c)^{\frac{1}{3}}c^2d^2x^6+4\,(d\,x^3+c)^{\frac{1}{3}}c\,d^3x^9+(d\,x^3+c)^{\frac{1}{3}}d^4x^{12}}\right.$$
$$+\left(\int \frac{1}{(d\,x^3+c)^{\frac{1}{3}}c^4+4\,(d\,x^3+c)^{\frac{1}{3}}c^3d\,x^3+6\,(d\,x^3+c)^{\frac{1}{3}}c^2d^2x^6+4\,(d\,x^3+c)^{\frac{1}{3}}c\,d^3x^9+(d\,x^3+c)^{\frac{1}{3}}d^4x^{12}}\right.$$

input
```
int((b*x^3+a)^3/(d*x^3+c)^(13/3),x)
```

output
```
int(x**9/((c + d*x**3)**(1/3)*c**4 + 4*(c + d*x**3)**(1/3)*c**3*d*x**3 + 6
*(c + d*x**3)**(1/3)*c**2*d**2*x**6 + 4*(c + d*x**3)**(1/3)*c*d**3*x**9 +
(c + d*x**3)**(1/3)*d**4*x**12),x)*b**3 + 3*int(x**6/((c + d*x**3)**(1/3)*
c**4 + 4*(c + d*x**3)**(1/3)*c**3*d*x**3 + 6*(c + d*x**3)**(1/3)*c**2*d**2
*x**6 + 4*(c + d*x**3)**(1/3)*c*d**3*x**9 + (c + d*x**3)**(1/3)*d**4*x**12
),x)*a*b**2 + 3*int(x**3/((c + d*x**3)**(1/3)*c**4 + 4*(c + d*x**3)**(1/3)
*c**3*d*x**3 + 6*(c + d*x**3)**(1/3)*c**2*d**2*x**6 + 4*(c + d*x**3)**(1/3
)*c*d**3*x**9 + (c + d*x**3)**(1/3)*d**4*x**12),x)*a**2*b + int(1/((c + d*
x**3)**(1/3)*c**4 + 4*(c + d*x**3)**(1/3)*c**3*d*x**3 + 6*(c + d*x**3)**(1
/3)*c**2*d**2*x**6 + 4*(c + d*x**3)**(1/3)*c*d**3*x**9 + (c + d*x**3)**(1/
3)*d**4*x**12),x)*a**3
```

3.138 $\int \frac{(a+bx^3)^{8/3}}{c+dx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 331

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = &-\frac{b(6bc-11ad)x(a+bx^3)^{2/3}}{18d^2}+\frac{bx(a+bx^3)^{5/3}}{6d} \\ &+\frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^3} \\ &-\frac{(bc-ad)^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^3}-\frac{(bc-ad)^{8/3}\log(c+dx^3)}{6c^{2/3}d^3} \\ &+\frac{(bc-ad)^{8/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^3} \\ &-\frac{b^{2/3}(9b^2c^2-24abcd+20a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^3}\end{aligned}$$

output
```
-1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^(2/3)/d^2+1/6*b*x*(b*x^3+a)^(5/3)/d+1/
27*b^(2/3)*(20*a^2*d^2-24*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*
x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3-1/3*(-a*d+b*c)^(8/3)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/d^3-1/6
*(-a*d+b*c)^(8/3)*ln(d*x^3+c)/c^(2/3)/d^3+1/2*(-a*d+b*c)^(8/3)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^3-1/18*b^(2/3)*(20*a^2*d^2-2
4*a*b*c*d+9*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.86 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{3b\sqrt[3]{bc-ad}(9b^2c^2-24abcd+20a^2d^2)\,x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

output
```
(3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^
(1/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a
*d)^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b
*c - a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*
(3*b^2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c
- a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7
*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c -
 a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*
x^3)^(1/3)] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + 9*a^3*d^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c
*d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {933, 25, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -\frac{\left(bx^3+a\right)^{2/3}\left(b(6bc-11ad)x^3+a(bc-6ad)\right)}{dx^3+c}dx}{6d}+\frac{bx\left(a+bx^3\right)^{5/3}}{6d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(b(6bc-11ad)x^3+a(bc-6ad)\right)}{dx^3+c}dx}{6d}$$

$$\Big\downarrow \text{1025}$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{\int -\frac{2\left(b\left(9b^2c^2-24abdc+20a^2d^2\right)x^3+a\left(3b^2c^2-7abdc+9a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}+\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}-\frac{2\int \frac{b\left(9b^2c^2-24abdc+20a^2d^2\right)x^3+a\left(3b^2c^2-7abdc+9a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}}{6d}$$

$$\Big\downarrow \text{1026}$$

$$\frac{bx\left(a+bx^3\right)^{5/3}}{6d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(6bc-11ad)}{3d}-\frac{2\left(\frac{b\left(20a^2d^2-24abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{9(bc-ad)^3\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}\right)}{3d}}{6d}$$

$$\Big\downarrow \text{769}$$

$$\frac{bx(a+bx^3)^{5/3}}{6d} - \frac{\frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{3d} - \frac{2\left(\frac{b(20a^2d^2-24abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d} - \frac{9(bc-ad)^3\int\frac{1}{\sqrt[3]{bx^3+a}\cdots}}{d}\cdots\right)}{3d}}{6d}$$

$\big\downarrow$ 901

$$\frac{bx(a+bx^3)^{5/3}}{6d} - \frac{\frac{bx(a+bx^3)^{2/3}(6bc-11ad)}{3d} - \frac{2\left(\frac{b(20a^2d^2-24abcd+9b^2c^2)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d} - \frac{9(bc-ad)^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{\cdots}}{\sqrt[3]{c}\sqrt[3]{\cdots}}\cdots}{\cdots}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{\cdots}}\cdots\right)}{\cdots}\right)}{3d}}{6d}$$

input
```
Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

output
```
(b*x*(a + b*x^3)^(5/3))/(6*d) - ((b*(6*b*c - 11*a*d)*x*(a + b*x^3)^(2/3))/
(3*d) - (2*((-9*(b*c - a*d)^3*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
 + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3
) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d + (b*(9*b^2*c^2 -
 24*a*b*c*d + 20*a^2*d^2)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sq
rt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3
))))/d))/(3*d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{2c\left(-\frac{7\left(\frac{3}{14}bd\,x^{3}+ad-\frac{3}{7}bc\right)dbx\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3}+\left(\frac{20b^{\frac{2}{3}}a^{2}d^{2}}{9}+c\,b^{\frac{5}{3}}\left(bc-\frac{8ad}{3}\right)\right)\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\right.\right.\right.}{}$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(1/3)*(2*c*(-7/3*(3/14*b*d*x^3+a*d-3/7*b*c)*d*b*x*(b*x^
3+a)^(2/3)+(20/9*b^(2/3)*a^2*d^2+c*b^(5/3)*(b*c-8/3*a*d))*(3^(1/2)*arctan(
1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+(b*x^3
+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2))*((a*d-b*c)/c)^(1/3)-(a*d-b*c)^3*(2*arctan(1/3*3^(1/2)*(-2/((a*d-
b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b
*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/d^3/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(273) = 546$.

Time = 5.59 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x))
 + 2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/
3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*l
og((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b
*c - a*d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-
((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a
^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(
2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^
2*d^2*x^4 - 2*(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

Sympy [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{8}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(8/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{14(b\,x^3+a)^{\frac{2}{3}}\,abdx - 6(b\,x^3+a)^{\frac{2}{3}}\,b^2cx + 3(b\,x^3+a)^{\frac{2}{3}}\,b^2d\,x^4 + 18\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+a}\right.}{}$$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

output
```
(14*(a + b*x**3)**(2/3)*a*b*d*x - 6*(a + b*x**3)**(2/3)*b**2*c*x + 3*(a +
b*x**3)**(2/3)*b**2*d*x**4 + 18*int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 +
b*c*x**3 + b*d*x**6),x)*a**3*d**2 - 14*int((a + b*x**3)**(2/3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a**2*b*c*d + 6*int((a + b*x**3)**(2/3)/(a*c
 + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b**2*c**2 + 40*int(((a + b*x**3)**
(2/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a**2*b*d**2 - 48*int
(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*a*b*
*2*c*d + 18*int(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*x**3 + b*c*x**3 + b*
d*x**6),x)*b**3*c**2)/(18*d**2)
```

3.139 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx &= \frac{bx(a+bx^3)^{2/3}}{3d} - \frac{b^{2/3}(3bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} \\ &+ \frac{(bc-ad)^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d^2} + \frac{(bc-ad)^{5/3}\log(c+dx^3)}{6c^{2/3}d^2} \\ &- \frac{(bc-ad)^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d^2} \\ &+ \frac{b^{2/3}(3bc-5ad)\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{6d^2}\end{aligned}$$

output
```
1/3*b*x*(b*x^3+a)^(2/3)/d-1/9*b^(2/3)*(-5*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2+1/3*(-a*d+b*c)^(5/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)
/d^2+1/6*(-a*d+b*c)^(5/3)*ln(d*x^3+c)/c^(2/3)/d^2-1/2*(-a*d+b*c)^(5/3)*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d^2+1/6*b^(2/3)*(-5*a*
d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.91 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{12bdx(a+bx^3)^{2/3} - 4\sqrt{3}b^{2/3}(3bc-5ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - \frac{6\sqrt{-6+6i\sqrt{3}}(bc-a}{}}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

output
```
(12*b*d*x*(a + b*x^3)^(2/3) - 4*Sqrt[3]*b^(2/3)*(3*b*c - 5*a*d)*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - (6*Sqrt[-6 + (6*I)*S
qrt[3]]*(b*c - a*d)^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a
*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/c^(2/3) + 4*b^(
2/3)*(3*b*c - 5*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 + I*Sqr
t[3])*(b*c - a*d)^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3
)*(a + b*x^3)^(1/3)])/c^(2/3) - 2*b^(2/3)*(3*b*c - 5*a*d)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - ((3*I)*(-I + Sqrt[3])
*(b*c - a*d)^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*
(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3
)^(2/3)])/c^(2/3))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {933, 25, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int -\frac{b(3bc-5ad)x^3+a(bc-3ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3d}+\frac{bx(a+bx^3)^{2/3}}{3d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\int \frac{b(3bc-5ad)x^3+a(bc-3ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3d}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3d}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{bx(a+bx^3)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}}{3d}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{bx\left(a+bx^3\right)^{2/3}}{3d}-\frac{\frac{b(3bc-5ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{3(bc-ad)^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\cdots}{\cdots}\right)}{d}}{3d}$$

input
```
Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

output
```
(b*x*(a + b*x^3)^(2/3))/(3*d) - ((-3*(b*c - a*d)^2*(ArcTan[(1 + (2*(b*c -
a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
- a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
 a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3)))
)/d + (b*(3*b*c - 5*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)))/d)/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{5c\left(-\frac{3\left(bx^3+a\right)^{\frac{2}{3}}xbd}{5}+\left(adb^{\frac{2}{3}}-\frac{3b^{\frac{5}{3}}c}{5}\right)\left(\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3 \cdots\right.}{\cdots}\right)}{\cdots}\right)\cdots\right)}{3 \cdots}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-5/3*c*(-3/5*(b*x^3+a)^(2/3)*x*b*d+(a*d*b^(2/3)-3/5*b^(5/3)*c)*(3^(1/
2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3
)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x
^3+a)^(2/3))/x^2)))*((a*d-b*c)/c)^(1/3)+1/2*(a*d-b*c)^2*(2*3^(1/2)*arctan(
1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/
x)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)
*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/((a*d-b
*c)/c)^(1/3)/c/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(220) = 440$.

Time = 0.52 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx + 6\sqrt{3}(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c}{3\,(bc-ad)}\right.}{}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*
d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x -
 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c
 - 5*a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3
*b*c - 5*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x
- (b*x^3 + a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3)
+ (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d^2
```

Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,bx+3\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a^2d-\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abc+5\left(\right.}{3d}$$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(2/3)*b*x + 3*int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 + b*c
*x**3 + b*d*x**6),x)*a**2*d - int((a + b*x**3)**(2/3)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a*b*c + 5*int(((a + b*x**3)**(2/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 3*int(((a + b*x**3)**(2/3)*x**3)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/(3*d)
```

3.140 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

output
```
1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d-
1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^
(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)
/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(
2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}\,x}{\sqrt{3}\sqrt[3]{bc-ad}\,x-(3i+\sqrt{3})\sqrt[3]{}}\right.}{c^{2/3}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
(4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*
d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3))]])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] -
((2*I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I
*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c
 - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c
- a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/
3)]])/c^(2/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {916, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$$

$$\Big\downarrow \text{ 916}$$

$$\frac{b\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \; 769$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{d}$$

$$\Big\downarrow \; 901$$

$$\frac{b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

output
```
-(((b*c - a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^
(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)
^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (b*(ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b
*x^3)^(1/3)]/(2*b^(1/3))))/d
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.40

method	result
pseudoelliptic	$\frac{\frac{(-ad+bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}+\frac{b^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+}{}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1/2*(-a*d+b*c)*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+1/2*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1/3)+ln((((a*d-b*c)/c
)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(a*d-b*c)-b^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)*c*((a*d-b*c)/c)^(1/3)-b^(2/3)*ln((-b^(1/
3)*x+(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(a*d-b*c)*arctan(1/3*3^(1/2
)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2))/((a*d-b*c)/c)^(1/
3)/d/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(186) = 372$.

Time = 0.13 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b}{}\right.}{}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(s
qrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-
1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*
log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2
) + ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2
- 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}d x$$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(2/3)/(c + d*x**3),x)
```

3.141 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

output
```
1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/c^(2/3)/(-a*d+b*c)^(1/3)+1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1
/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}x+(1+\cdots\right.\right.}{12\cdots}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
(-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
- a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt
[3])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(
12*c^(2/3)*(b*c - a*d)^(1/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$\Big\downarrow \text{901}$$

$$\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

output
```
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/
(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d
)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/
3)*(b*c - a*d)^(1/3))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3)
)/((a*d-b*c)/c)^(1/3)/x)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-l
n((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))/((a*d-b*c)/c)^(1/3)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x**3),x)
```

3.142 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/(-a*d+b*c)^(4/3)-1/6*d*l
n(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b
*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})\,d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]
]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*
(-I + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c
 - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d
)^(4/3)))/12
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 907}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*(ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)
^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^
(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c
 - a*d)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right)ca}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d
-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(
1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2
*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/
c/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,ac+(b\,x^3+a)^{\frac{1}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^6}\,d$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c + (a + b*x**3)**(1/3)*a*d*x**3 + (a + b*x**
3)**(1/3)*b*c*x**3 + (a + b*x**3)**(1/3)*b*d*x**6),x)
```

3.143 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 226

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^3)^{4/3}} + \frac{b(3bc-7ad)x}{4a^2(bc-ad)^2\sqrt[3]{a+bx^3}} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}} + \frac{d^2\log(c+dx^3)}{6c^{2/3}(bc-ad)^{7/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}$$

output
```
1/4*b*x/a/(-a*d+b*c)/(b*x^3+a)^(4/3)+1/4*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)
^2/(b*x^3+a)^(1/3)+1/3*d^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/(-a*d+b*c)^(7/3)+1/6*d^2*ln(d*x^3+c)
/c^(2/3)/(-a*d+b*c)^(7/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^
(1/3))/c^(2/3)/(-a*d+b*c)^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{3bx(-8a^2d+3b^2cx^3+ab(4c-7dx^3))}{a^2(bc-ad)^2(a+bx^3)^{4/3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{7/3}} + \frac{2(1+i\sqrt{3})d^2\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{7/3}} - \frac{i(-i+\sqrt{3})d^2\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{7/3}}\right)$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

output
```
((3*b*x*(-8*a^2*d + 3*b^2*c*x^3 + a*b*(4*c - 7*d*x^3)))/(a^2*(b*c - a*d)^2
*(a + b*x^3)^(4/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x
^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(7/3)) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b
*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(c^(2/3)*(
b*c - a*d)^(7/3)) - (I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-
1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt
[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(2/3)*(b*c - a*d)^(7/3)))/12
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {931, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}-\frac{\int -\frac{3bdx^3+3bc-4ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3bdx^3+3bc-4ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int -\frac{4a^2d^2}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{4ad^2\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}+\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^3)^{4/3}(bc-ad)}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{\frac{4ad^2\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}+\frac{bx(3bc-7ad)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(3*b*c - 7*a*d)*x)/(a*(b*c
 - a*d)*(a + b*x^3)^(1/3)) + (4*a*d^2*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)
/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3))
 + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(b*c - a*d)
)/(4*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{3xbc\left(7abd\,x^3-3b^2c\,x^3+8d\,a^2-4abc\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+\left(b\,x^3+a\right)^{\frac{4}{3}}a^2d^2\left(2\arctan\left(\frac{\sqrt{3}\left(-\frac{2\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}+x\right)}{3x}\right)\sqrt{3}+2\ln\left(\left(\frac{ad-bc}{c}\right)\right)\right)}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{4}{3}}(ad-bc)^2c\,a^2}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/6/((a*d-b*c)/c)^(1/3)*(-3/2*x*b*c*(7*a*b*d*x^3-3*b^2*c*x^3+8*a^2*d-4*a*b
*c)*((a*d-b*c)/c)^(1/3)+(b*x^3+a)^(4/3)*a^2*d^2*(2*arctan(1/3*3^(1/2)*(-2/
((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3
)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/(b*x^3+a)^(4/3)/(a*d-b*c)^2/c/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2c+(b\,x^3+a)^{\frac{1}{3}}\,a^2d\,x^3+2\,(b\,x^3+a)^{\frac{1}{3}}\,abc\,x^3+2\,(b\,x^3+a)^{\frac{1}{3}}$$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a**2*c + (a + b*x**3)**(1/3)*a**2*d*x**3 + 2*(a
 + b*x**3)**(1/3)*a*b*c*x**3 + 2*(a + b*x**3)**(1/3)*a*b*d*x**6 + (a + b*x
**3)**(1/3)*b**2*c*x**6 + (a + b*x**3)**(1/3)*b**2*d*x**9),x)
```

3.144 $\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^3)^{7/3}} + \frac{b(6bc-13ad)x}{28a^2(bc-ad)^2(a+bx^3)^{4/3}} + \frac{b(18b^2c^2-57abcd+67a^2d^2)x}{28a^3(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{10/3}} - \frac{d^3\log(c+dx^3)}{6c^{2/3}(bc-ad)^{10/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}$$

output
```
1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^(7/3)+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^3+a)^(4/3)+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d
+b*c)^3/(b*x^3+a)^(1/3)-1/3*d^3*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)
/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(2/3)/(-a*d+b*c)^(10/3)-1/6*d^3*ln(d*
x^3+c)/c^(2/3)/(-a*d+b*c)^(10/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*
x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(10/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.74 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \frac{1}{84}\left(-\frac{3bx(84a^4d^2+18b^4c^2x^6+3ab^3cx^3(14c-19dx^3)+21a^3bd(-4c+7dx^3)\ldots}{a^3(-bc+ad)^3(a+bx^3)^{7/3}}\right.$$

$$+\frac{14\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{10/3}}$$

$$-\frac{14i(-i+\sqrt{3})d^3\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{10/3}}$$

$$+\frac{7(1+i\sqrt{3})d^3\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{10/3}}$$

input
```
Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

output
```
((-3*b*x*(84*a^4*d^2 + 18*b^4*c^2*x^6 + 3*a*b^3*c*x^3*(14*c - 19*d*x^3) +
21*a^3*b*d*(-4*c + 7*d*x^3) + a^2*b^2*(28*c^2 - 133*c*d*x^3 + 67*d^2*x^6))
)/(a^3*(-(b*c) + a*d)^3*(a + b*x^3)^(7/3)) + (14*Sqrt[-6 + (6*I)*Sqrt[3]]*
d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(10/3)) - ((14*I
)*(-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(10/3)) + (7*(1 + I*Sqrt[3])*d^3*L
og[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*
(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(
b*c - a*d)^(10/3)))/84
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {931, 25, 1024, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}-\frac{\int -\frac{6bdx^3+6bc-7ad}{\left(bx^3+a\right)^{7/3}\left(dx^3+c\right)}dx}{7a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6bdx^3+6bc-7ad}{\left(bx^3+a\right)^{7/3}\left(dx^3+c\right)}dx}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(6bc-13ad)}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}-\frac{\int -\frac{3bd(6bc-13ad)x^3+18b^2c^2+28a^2d^2-39abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3bd(6bc-13ad)x^3+18b^2c^2+28a^2d^2-39abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{\int \frac{28a^3d^3}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a\left(a+bx^3\right)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{28a^2d^3\int\frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{bc-ad}}{4a(bc-ad)}+\frac{bx(6bc-13ad)}{4a(a+bx^3)^{4/3}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

$$\Big\downarrow \ 901$$

$$\frac{\frac{\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}-\frac{28a^2d^3\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{bx(6bc-\cdots}{4a(a+bx^3)^{4\cdots}}}{7a(bc-ad)}$$
$$\frac{bx}{7a\left(a+bx^3\right)^{7/3}(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

output
```
(b*x)/(7*a*(b*c - a*d)*(a + b*x^3)^(7/3)) + ((b*(6*b*c - 13*a*d)*x)/(4*a*(
b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2)
*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (28*a^2*d^3*(ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
 - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))
))/(b*c - a*d))/(4*a*(b*c - a*d)))/(7*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{18\left(a^{4}d^{2}-db\left(-\frac{7d\,x^{3}}{4}+c\right)a^{3}+\frac{\left(\frac{67}{28}d^{2}x^{6}-\frac{19}{4}cd\,x^{3}+c^{2}\right)b^{2}a^{2}}{3}+\frac{\left(-\frac{19d\,x^{3}}{14}+c\right)c\,b^{3}x^{3}a}{2}+\frac{3b^{4}c^{2}x^{6}}{14}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}bx+\left(b\,x^{3}+a\right)^{\frac{7}{3}}}{6(}$

input
```
int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/((a*d-b*c)/c)^(1/3)*(18*(a^4*d^2-d*b*(-7/4*d*x^3+c)*a^3+1/3*(67/28*d^
2*x^6-19/4*c*d*x^3+c^2)*b^2*a^2+1/2*(-19/14*d*x^3+c)*c*b^3*x^3*a+3/14*b^4*
c^2*x^6)*c*((a*d-b*c)/c)^(1/3)*b*x+(b*x^3+a)^(7/3)*a^3*d^3*(-2*3^(1/2)*arc
tan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1
/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(
b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/(b*
x^3+a)^(7/3)/c/(a*d-b*c)^3/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx=\int \frac{1}{(a+bx^3)^{\frac{10}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{10}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{10/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)}\,dx = \int \frac{}{(b\,x^3+a)^{\frac{1}{3}}\,a^3c+(b\,x^3+a)^{\frac{1}{3}}\,a^3d\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2bc\,x^3+3\,(b\,x^3+a)}$$

input
```
int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a**3*c + (a + b*x**3)**(1/3)*a**3*d*x**3 + 3*(a
 + b*x**3)**(1/3)*a**2*b*c*x**3 + 3*(a + b*x**3)**(1/3)*a**2*b*d*x**6 + 3*
(a + b*x**3)**(1/3)*a*b**2*c*x**6 + 3*(a + b*x**3)**(1/3)*a*b**2*d*x**9 +
(a + b*x**3)**(1/3)*b**3*c*x**9 + (a + b*x**3)**(1/3)*b**3*d*x**12),x)
```

3.145 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. 2(60) = 120.

Time = 10.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac\left(2a^2d+abdx^3+b^2x^3\left(c+dx^3\right)\right)\text{Appell}\cdots}{\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\cdots\right.}\cdots\right.}{8a\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*
x^3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
b*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/
a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/
(8*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}dx$$

$$\Big\downarrow \ 937$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}\,bx+2\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)a^2d-\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{bd\,x^6+ad\,x^3+bc\,x^3+ac}dx\right)abc+3\Big(}{2d}$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

output
```
((a + b*x**3)**(1/3)*b*x + 2*int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*c
*x**3 + b*d*x**6),x)*a**2*d - int((a + b*x**3)**(1/3)/(a*c + a*d*x**3 + b*
c*x**3 + b*d*x**6),x)*a*b*c + 3*int(((a + b*x**3)**(1/3)*x**3)/(a*c + a*d*
x**3 + b*c*x**3 + b*d*x**6),x)*a*b*d - 2*int(((a + b*x**3)**(1/3)*x**3)/(a
*c + a*d*x**3 + b*c*x**3 + b*d*x**6),x)*b**2*c)/(2*d)
```

3.146 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{x\sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)
^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(59) = 118.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$
$$= \frac{4acx\sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc \text{ Ap}\right.\right.}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{dx^3+c}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**(1/3)/(c + d*x**3),x)
```

3.147 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^
(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx =$$

$$-\frac{4acx \operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(a+bx^3)^{2/3}(c+dx^3)\left(-4ac \operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad \operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*
x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{2}{3}}\,(d\,x^{3}+c)}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)} \,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}} c+\left(b\,x^3+a\right)^{\frac{2}{3}} d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x**3),x)
```

3.148 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a
)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(62) = 124.

Time = 10.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\frac{x\left(-\frac{bdx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(4ac(2ad-b(2c+dx^3))\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3}\right.\right.}{(c+dx^3)\left(4ac\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},\right.\right.}\right.}{8a(-bc}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

output
```
(x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3,
2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*Appell
F1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3,
 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3,
2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}(dx^3+c)}dx}{a(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)}d x$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{3}}(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)}\,dx=\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,ac+(b\,x^3+a)^{\frac{2}{3}}\,ad\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,bc\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,bd\,x^6}\,d$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a*c + (a + b*x**3)**(2/3)*a*d*x**3 + (a + b*x**
3)**(2/3)*b*c*x**3 + (a + b*x**3)**(2/3)*b*d*x**6),x)
```

3.149 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3
+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 429 vs. 2(62) = 124.

Time = 10.83 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.92

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx =$$

$$-\frac{x\left(\frac{bd(-4bc+9ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c} + \frac{4\left(4ac(10a^3d^2+4b^3cx^3(2c+dx^3)-a^2bd(20c+dx^3)+ab^2(10c^2-12cdx^3\right.}{(a+bx^3)(c+dx^3)\left(-\right.}\right.}{40}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

output
```
-1/40*(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/
3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*
c*x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3
- 9*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*Appe
llF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*Appe
llF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^3
)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{8}{3}}\,(d\,x^{3}+c)}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{8}{3}}\,(c+dx^3)}\,dx$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

output
```
Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2c+(b\,x^3+a)^{\frac{2}{3}}\,a^2d\,x^3+2\,(b\,x^3+a)^{\frac{2}{3}}\,abc\,x^3+2\,(b\,x^3+a)^{\frac{2}{3}}$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**2*c + (a + b*x**3)**(2/3)*a**2*d*x**3 + 2*(a
 + b*x**3)**(2/3)*a*b*c*x**3 + 2*(a + b*x**3)**(2/3)*a*b*d*x**6 + (a + b*x
**3)**(2/3)*b**2*c*x**6 + (a + b*x**3)**(2/3)*b**2*d*x**9),x)
```

3.150 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 351

$$\begin{aligned}\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx &= \frac{b(2bc-ad)x(a+bx^3)^{2/3}}{3cd^2} \\ &- \frac{(bc-ad)x(a+bx^3)^{5/3}}{3cd\,(c+dx^3)} - \frac{2b^{5/3}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^3} \\ &+ \frac{2(bc-ad)^{5/3}(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^3} \\ &+ \frac{(bc-ad)^{5/3}(3bc+ad)\log(c+dx^3)}{9c^{5/3}d^3} \\ &- \frac{(bc-ad)^{5/3}(3bc+ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}d^3} \\ &+ \frac{b^{5/3}(3bc-4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3d^3}\end{aligned}$$

output
```
1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3
)/c/d/(d*x^3+c)-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/
3)/d^3+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1/3*(-a*d+
b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5
/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.02 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{1}{18}\left(\frac{6x(a+bx^3)^{2/3}\left(b^2+\frac{(bc-ad)^2}{c(c+dx^3)}\right)}{d^2}\right.$$

$$-\frac{9b^3x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}$$

$$+\frac{12ab^2x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd\sqrt[3]{a+bx^3}}$$

$$+\frac{2a^3\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+a}}\right.\right.}{c^{5/3}\sqrt[3]{bc-ad}}$$

$$-\frac{2ab^2\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{}}{\sqrt[3]{}}\right.\right.}{d^2\sqrt[3]{bc-ad}}$$

$$+\frac{2a^2b\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(b+ax^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{b+}}\right.\right.}{c^{2/3}d\sqrt[3]{bc-ad}}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

output
```
((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^
3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/
3)) + (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3))
- (2*a*b^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) +
 (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)
) + (2*a^2*b*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)
))/18
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{(bx^3+a)^{2/3}(3b(2bc-ad)x^3+a(bc+2ad))}{dx^3+c}dx}{3cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{3cd(c+dx^3)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{\int -\frac{6\left(b^2c(3bc-4ad)x^3+a\left(b^2c^2-abdc-a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3d}+\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}}{3cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{3cd(c+dx^3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\int \frac{b^2c(3bc-4ad)x^3+a\left(b^2c^2-abdc-a^2d^2\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \; 1026$$

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)^2(ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}\right)}{d}}{3cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \; 769$$

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2(ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c}}{d}\right.}{d}}{3cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{bx(a+bx^3)^{2/3}(2bc-ad)}{d}-\frac{2\left(\frac{b^2c(3bc-4ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2(ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-\cdots}}{\sqrt[3]{c}\sqrt[3]{a+\cdots}}\cdots}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-\cdots}}\cdots\right)}{\cdots}\right)}{d}}{3cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{3cd(c+dx^3)}$$

input
```
Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

output
```
-1/3*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)) + ((b*(2*b*c - a*
d)*x*(a + b*x^3)^(2/3))/d - (2*(-(((b*c - a*d)^2*(3*b*c + a*d)*(ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c
^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) -
 Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c -
a*d)^(1/3))))/d) + (b^2*c*(3*b*c - 4*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)]/(2*b^(1/3))))/d))/d)/(3*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c^{2}\left(d\,x^{3}+c\right)\left(a\,b^{\frac{5}{3}} d-\frac{3 b^{\frac{8}{3}} c}{4}\right)\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{9}+\frac{2\left(d\,x^{3}+c\right)(ad+3bc)(ad-bc)^{2}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x \ldots}{\ldots}\right)}{9}\ldots$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)*(2*((a*d-b*c)/c)^(1/3)*c^2*(d*x^3+c)*(a*b^(5/3)*d-
3/4*b^(8/3)*c)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/
x^2)+(d*x^3+c)*(a*d+3*b*c)*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)
^(1/3))/x)-4*((a*d-b*c)/c)^(1/3)*c^2*(d*x^3+c)*(a*b^(5/3)*d-3/4*b^(8/3)*c)
*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)-4*((a*d-b*c)/
c)^(1/3)*c^2*(d*x^3+c)*(a*b^(5/3)*d-3/4*b^(8/3)*c)*ln((-b^(1/3)*x+(b*x^3+a
)^(1/3))/x)+3/2*c*d*(b*x^3+a)^(2/3)*(2*b^2*c^2-2*d*b*(-1/2*b*x^3+a)*c+a^2*
d^2)*x*((a*d-b*c)/c)^(1/3)+(a*d-b*c)^2*(a*d+3*b*c)*(d*x^3+c)*(arctan(1/3*3
^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)-1/2*ln((((a*d
-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/
x^2)))/d^3/c^2/(d*x^3+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(291) = 582.

Time = 3.52 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.33

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b
*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(
-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4
*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sq
rt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^
3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 -
 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*
d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^
(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*
d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 +
(2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c
^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

output
```
( - 9*(a + b*x**3)**(2/3)*a**2*b*d*x + 4*(a + b*x**3)**(2/3)*a*b**2*c*x +
2*(a + b*x**3)**(2/3)*a*b**2*d*x**4 - 3*(a + b*x**3)**(2/3)*b**3*c*x**4 +
12*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d*
*3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x*
*9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**5*c
*d**3 + 12*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2
*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b
*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x
)*a**5*d**4*x**3 - 18*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d*
*2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x
**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d
**2*x**9),x)*a**4*b*c**2*d**2 - 18*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d
+ 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 +
 a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6
 - 3*b**2*c*d**2*x**9),x)*a**4*b*c*d**3*x**3 - 8*int((a + b*x**3)**(2/3)/(
2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b
*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b*
*2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**3*b**2*c**3*d - 8*int((a + b*x*
*3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*
c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c...
```

3.151 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 301

$$\begin{aligned}\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2} dx = &-\frac{(bc-ad)x(a+bx^3)^{2/3}}{3cd(c+dx^3)} + \frac{b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} \\ &- \frac{(bc-ad)^{2/3}(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}d^2} \\ &- \frac{(bc-ad)^{2/3}(3bc+2ad)\log(c+dx^3)}{18c^{5/3}d^2} \\ &+ \frac{(bc-ad)^{2/3}(3bc+2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}d^2} \\ &- \frac{b^{5/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^2}\end{aligned}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^3+a)^(2/3)/c/d/(d*x^3+c)+1/3*b^(5/3)*arctan(1/3*(1+
2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2-1/9*(-a*d+b*c)^(2/3)*(2*
a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))*3^(1/2)/c^(5/3)/d^2-1/18*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln(d*x^3+c)/
c^(5/3)/d^2+1/6*(-a*d+b*c)^(2/3)*(2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/
3)-(b*x^3+a)^(1/3))/c^(5/3)/d^2-1/2*b^(5/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))
/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.56 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \frac{-\frac{12d(bc-ad)x(a+bx^3)^{2/3}}{c(c+dx^3)} + 12\sqrt{3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \frac{2i\left(3i+\sqrt{3}\right)\left(3b^2c^2-abcd-2a^2\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

output
```
((-12*d*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*(c + d*x^3)) + 12*Sqrt[3]*b^(5
/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + ((2*I)
*(3*I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*ArcTanh[(I + ((-I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(c^(5/3)
*(b*c - a*d)^(1/3)) - 12*b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - (
(2*I)*(-I + Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*c - a*d)^(
1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(5/3)*(b*c - a*d)^
(1/3)) + 6*b^(5/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)] + ((1 + I*Sqrt[3])*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)*Log[2*(b*
c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x
^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(c^(5/3)*(b*c - a*
d)^(1/3)))/(36*d^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {930, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{3b^2cx^3+a(bc+2ad)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)}{3cd(c+dx^3)}$$

$$\Big\downarrow \text{1026}$$

$$\frac{\frac{3b^2c\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)(2ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)}{3cd(c+dx^3)}$$

$$\Big\downarrow \text{769}$$

$$\frac{\frac{3b^2c\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)(2ad+3bc)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)}{3cd(c+dx^3)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{3b^2c\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)(2ad+3bc)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\cdots\right)}{d}}{3cd}$$

$$\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)}{3cd\left(c+dx^3\right)}$$

input
```
Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

output
```
-1/3*((b*c - a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (-(((b*c - a*d)
*(3*b*c + 2*a*d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)
^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*
c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^
3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (3*b^2*c*(ArcTan[(1 + (2*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x)
+ (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d)/(3*c*d)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$-\frac{-\frac{3 b^{\frac{5}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c^{2} \left(d\,x^{3}+c\right) \left(\frac{a d -b c}{c}\right)^{\frac{1}{3}}}{2}-2 \left(a d -b c \right) \left(a d +\frac{3 b c}{2}\right) \left(d\,x^{3}+c \right) \ln\left(\frac{\left(\frac{a d -b c}{c}\right)^{\frac{1}{3}} x +\left(b\,x \right.}{x}\right.}{}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-3/2*b^(5/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/
3))/x^2)*c^2*(d*x^3+c)*((a*d-b*c)/c)^(1/3)-2*(a*d-b*c)*(a*d+3/2*b*c)*(d*x^
3+c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3*b^(5/3)*3^(1/2)*((a*d
-b*c)/c)^(1/3)*c^2*(d*x^3+c)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)
+x)/x)+3*b^(5/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*c^2*(d*x^3+c)*((a*d-b*
c)/c)^(1/3)+(a*d-b*c)*(-3*d*(b*x^3+a)^(2/3)*x*c*((a*d-b*c)/c)^(1/3)+(a*d+3
/2*b*c)*(d*x^3+c)*(-2*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)
^(1/3)+x)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^
3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))))/((a*d-b*c)/c)^(1/3)/d^2/c^2/(d*x^3+c
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(248) = 496$.

Time = 0.38 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.10

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
-1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sq
rt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b
*c - a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sq
rt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*
d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x
^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) +
3*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^
(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^
3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((
b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(
1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(
b*c - a*d))/x^2))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{5}{3}}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(5/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

output
```
( - (a + b*x**3)**(2/3)*a*b*x + 4*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d +
 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 +
a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6
- 3*b**2*c*d**2*x**9),x)*a**4*c*d**2 + 4*int((a + b*x**3)**(2/3)/(2*a**2*c
**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*
x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*
d*x**6 - 3*b**2*c*d**2*x**9),x)*a**4*d**3*x**3 - 10*int((a + b*x**3)**(2/3
)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*
a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6
*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**3*b*c**2*d - 10*int((a + b*x
**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b
*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**
3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**3*b*c*d**2*x**3 +
6*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**
3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**
9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**2*b*
*2*c**3 + 6*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 +
2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*
b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),
x)*a**2*b**2*c**2*d*x**3 + 2*int(((a + b*x**3)**(2/3)*x**3)/(c**2 + 2*c...
```

3.152 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{2a\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log(c+dx^3)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)+2/9*a*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x
/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b*c)^(1/3)+1/9*a*
ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)-1/3*a*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(
b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.78 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}}{c+dx^3} - \frac{2\sqrt{-6+6i\sqrt{3}}a\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2\left(a+i\sqrt{3}a\right)\log\left(\right.}{}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

output
```
((6*c^(2/3)*x*(a + b*x^3)^(2/3))/(c + d*x^3) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*a*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sq
rt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) + (2*(a + I*Sqrt[3]*
a)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(1/3) - (I*(-I + Sqrt[3])*a*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1
 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[
3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a*d)^(1/3))/(18*c^(5/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a(dx^3+c)}}dx}{3c} + \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

$$\Big\downarrow \ 901$$

$$\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*
d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d
)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3
*c)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{-\frac{a \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) \left(d\,x^{3}+c\right)}{9}+\frac{2a \ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) \left(d\,x^{3}+c\right)}{9}+\frac{x\left(b\,x^{3}+a\right)}{}}{c^{2}(d\,x^{3}+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)*(-1/2*a*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^
(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*(d*x^3+c)+a*ln((((a*d-b*c)/c
)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(d*x^3+c)+3/2*x*(b*x^3+a)^(2/3)*c*((a*d-b*c)
/c)^(1/3)+a*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/
x)*(d*x^3+c)*3^(1/2))/c^2/(d*x^3+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{{\left(d\,x^3+c\right)}^2}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx=\frac{-(b\,x^3+a)^{\frac{2}{3}}\,bx+4\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{2ab\,d^3x^9-3b^2c\,d^2x^9+2a^2d^3x^6+abc\,d^2x^6-6b^2c^2d\,x^6+4a^2c\,d^2x^3-4ab\,c^2d\,x^3-3b^2c^3x}\right.}{}$$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

output
```
( - (a + b*x**3)**(2/3)*b*x + 4*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4
*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*
b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 -
3*b**2*c*d**2*x**9),x)*a**3*c*d**2 + 4*int((a + b*x**3)**(2/3)/(2*a**2*c**
2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x*
*3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*
x**6 - 3*b**2*c*d**2*x**9),x)*a**3*d**3*x**3 - 10*int((a + b*x**3)**(2/3)/
(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c**3 - 4*a*
b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*x**3 - 6*b
**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**2*b*c**2*d - 10*int((a + b*x**
3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*x**6 - 3*a*b*c
**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9 - 3*b**2*c**3*
x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a**2*b*c*d**2*x**3 + 6*
int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**2*d**3*
x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**3*x**9
 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a*b**2*c*
*3 + 6*int((a + b*x**3)**(2/3)/(2*a**2*c**2*d + 4*a**2*c*d**2*x**3 + 2*a**
2*d**3*x**6 - 3*a*b*c**3 - 4*a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + 2*a*b*d**
3*x**9 - 3*b**2*c**3*x**3 - 6*b**2*c**2*d*x**6 - 3*b**2*c*d**2*x**9),x)*a*
b**2*c**2*d*x**3)/(2*a*c*d + 2*a*d**2*x**3 - 3*b*c**2 - 3*b*c*d*x**3)
```

3.153 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = -\frac{dx(a+bx^3)^{2/3}}{3c(bc-ad)\,(c+dx^3)} + \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}$$

output
```
-1/3*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)+1/9*(-2*a*d+3*b*c)*arctan(
1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5
/3)/(-a*d+b*c)^(4/3)+1/18*(-2*a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4
/3)-1/6*(-2*a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5
/3)/(-a*d+b*c)^(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$= \frac{-12c^{2/3}d\sqrt[3]{bc-ad}x(a+bx^3)^{2/3}+2(3-i\sqrt{3})(3bc-2ad)(c+dx^3)\operatorname{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
(-12*c^(2/3)*d*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(2/3) + 2*(3 - I*Sqrt[3])*(
3*b*c - 2*a*d)*(c + d*x^3)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]] + 2*(1 + I*Sqrt[3])*(3*b*c - 2*a*
d)*(c + d*x^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*
x^3)^(1/3)] - I*(-I + Sqrt[3])*(3*b*c - 2*a*d)*(c + d*x^3)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1
/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^
(4/3)*(c + d*x^3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {907, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{ 901}$$

$$\frac{(3bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d
)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c -
a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^
(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{-\frac{(d x^3+c)\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b x^3+a\right)^{\frac{1}{3}}x+\left(b x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9}+\frac{2(d x^3+c)\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b x^3+a\right.}{x}\right.}{9}}{(ad-bc)c^2(d x^3+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/((a*d-b*c)/c)^(1/3)*(-1/2*(d*x^3+c)*(a*d-3/2*b*c)*ln((((a*d-b*c)/c)^(2
/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(d*x^3
+c)*(a*d-3/2*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+3/2*d*(b*x
^3+a)^(2/3)*x*c*((a*d-b*c)/c)^(1/3)+(d*x^3+c)*arctan(1/3*3^(1/2)*(-2/((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)*(a*d-3/2*b*c))/(a*d-b*c)/c^2/
(d*x^3+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,cd\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,d^2x^6}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c**2 + 2*(a + b*x**3)**(1/3)*c*d*x**3 + (a + b*
x**3)**(1/3)*d**2*x**6),x)
```

3.154 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{b(3bc+ad)x}{3ac(bc-ad)^2\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{2d(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} - \frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}$$

output

```
1/3*b*(a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*d+b*c)/
(b*x^3+a)^(1/3)/(d*x^3+c)-2/9*d*(-a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b*c)^(7/3)-1
/9*d*(-a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(7/3)+1/3*d*(-a*d+3*b*c)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \frac{\frac{6c^{2/3}x(a^2d^2+abd^2x^3+3b^2c(c+dx^3))}{a(bc-ad)^2\sqrt[3]{a+bx^3}(c+dx^3)} + \frac{2i(3i+\sqrt{3})d(3bc-ad)\operatorname{arctanh}\left(\frac{i+\frac{(-i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}}}{}$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output
```
((6*c^(2/3)*x*(a^2*d^2 + a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3)))/(a*(b*c - a*d
)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((2*I)*(3*I + Sqrt[3])*d*(3*b*c - a*d
)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/
3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3])*d*(-3*b*c + a*d)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(7/3) + ((1 + I*Sqrt[3])*d*(3*b*c - a*d)*Log[2*(b*c - a*d)^(2/3)*
x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(
I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(18*c^(5/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {931, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^3+3bc-2ad}{(bx^3+a)^{4/3}(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\int \frac{2ad(3bc-ad)}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{a(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{2d(3bc-ad)\int \frac{1}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{bc-ad}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{bx(ad+3bc)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{2d(3bc-ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{3c(bc-ad)} - \frac{dx}{3c\sqrt[3]{a+bx^3}\,(c+dx^3)\,(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((b*(3*b*c + a*
d)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (2*d*(3*b*c - a*d)*(ArcTan[(1 +
(2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(
2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - L
og[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*
d)^(1/3))))/(b*c - a*d))/(3*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{-2\left(b\,x^{3}+a\right)^{\frac{1}{3}} a d \left(d\,x^{3}+c\right)\left(a d-3 b c\right)\ln\left(\frac{\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-3 c\left(a\left(b\,x^{3}+a\right) d^{2}+3 x^{3} b^{2} c d+3 b^{2} c^{2}\right) x\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}}+a \left(}{9\left(\frac{a d-b c}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}\right.}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-2*(b*x^3+a)^(1/3)*a*d*(d*x^3+c)*(a*d-3*b*c)*ln((((a*d-b*c)/c)^(1/3)
*x+(b*x^3+a)^(1/3))/x)-3*c*(a*(b*x^3+a)*d^2+3*x^3*b^2*c*d+3*b^2*c^2)*x*((a
*d-b*c)/c)^(1/3)+a*(-2*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a
)^(1/3)+x)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x
^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d*(d*x^3+c)*(a*d-3*b*c)*(b*x^3+a)^(1/
3))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/c^2/(d*x^3+c)/(a*d-b*c)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a\,c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,acd\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,a\,d^2x^6+(b\,x^3+a)^{\frac{1}{3}}}$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c**2 + 2*(a + b*x**3)**(1/3)*a*c*d*x**3 + (a
+ b*x**3)**(1/3)*a*d**2*x**6 + (a + b*x**3)**(1/3)*b*c**2*x**3 + 2*(a + b*
x**3)**(1/3)*b*c*d*x**6 + (a + b*x**3)**(1/3)*b*d**2*x**9),x)
```

3.155 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 324

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \frac{b(3bc+4ad)x}{12ac(bc-ad)^2(a+bx^3)^{4/3}} + \frac{b(9b^2c^2-33abcd-4a^2d^2)x}{12a^2c(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{dx}{3c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)} + \frac{d^2(9bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{10/3}} - \frac{d^2(9bc-2ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}$$

output
```
1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^(4/3)+1/12*b*(-4*a^2*d^2
-33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^(1/3)-1/3*d*x/c/(-a*
d+b*c)/(b*x^3+a)^(4/3)/(d*x^3+c)+1/9*d^2*(-2*a*d+9*b*c)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(5/3)/(-a*d+b
*c)^(10/3)+1/18*d^2*(-2*a*d+9*b*c)*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(10/3)-1
/6*d^2*(-2*a*d+9*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/
3)/(-a*d+b*c)^(10/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.86 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \frac{\frac{3c^{2/3}x(4a^4d^3+8a^3bd^3x^3-9b^4c^2x^3(c+dx^3)+4a^2b^2d(9c^2+9cdx^3+d^2x^6)+3ab^3c(-4c^2+7cdx^3+11d^2x^6}{a^2(-bc+ad)^3(a+bx^3)^{4/3}(c+dx^3)}}{}$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

output
```
((3*c^(2/3)*x*(4*a^4*d^3 + 8*a^3*b*d^3*x^3 - 9*b^4*c^2*x^3*(c + d*x^3) + 4
*a^2*b^2*d*(9*c^2 + 9*c*d*x^3 + d^2*x^6) + 3*a*b^3*c*(-4*c^2 + 7*c*d*x^3 +
 11*d^2*x^6)))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((2*
I)*(3*I + Sqrt[3])*d^2*(-9*b*c + 2*a*d)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/
3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(10/3)
+ (2*(1 + I*Sqrt[3])*d^2*(9*b*c - 2*a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(10/3) + ((1 + I*Sqrt[3
])*d^2*(-9*b*c + 2*a*d)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(
1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a +
b*x^3)^(2/3)])/(b*c - a*d)^(10/3))/(36*c^(5/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {931, 1024, 25, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-6bdx^3+3bc-2ad}{(bx^3+a)^{7/3}(dx^3+c)}dx}{3c(bc-ad)} - \frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)} - \frac{\int -\frac{3bd(3bc+4ad)x^3+9b^2c^2+8a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3bd(3bc+4ad)x^3+9b^2c^2+8a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)}dx}{4a(bc-ad)} + \frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\int -\frac{4a^2d^2(9bc-2ad)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{a(bc-ad)}}{4a(bc-ad)} + \frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{4ad^2(9bc-2ad)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{bc-ad} + \frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}}{4a(bc-ad)} + \frac{bx(4ad+3bc)}{4a(a+bx^3)^{4/3}(bc-ad)}}{3c(bc-ad)} - \frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}(bc-ad)}+\frac{4ad^2(9bc-2ad)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{b}{4a(a+}}{3c(bc-ad)}$$

$$\frac{dx}{3c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((b*(3*b*c + 4*
a*d)*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((b*(9*b^2*c^2 - 33*a*b*c*d
- 4*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + (4*a*d^2*(9*b*c - 2*a*
d)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c -
 a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c
^(2/3)*(b*c - a*d)^(1/3))))/(b*c - a*d))/(4*a*(b*c - a*d)))/(3*c*(b*c - a*
d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{-3cx\left(a^4d^3+2a^3b\,d^3x^3+9d\left(\frac{1}{9}d^2x^6+cd\,x^3+c^2\right)b^2a^2-3c\left(d\,x^3+c\right)\left(-\frac{11d\,x^3}{4}+c\right)b^3a-\frac{9b^4c^2x^3\left(d\,x^3+c\right)}{4}\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}-\cdots}{9\left(\cdots\right.}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9/((a*d-b*c)/c)^(1/3)*(-3*c*x*(a^4*d^3+2*a^3*b*d^3*x^3+9*d*(1/9*d^2*x^6
+c*d*x^3+c^2)*b^2*a^2-3*c*(d*x^3+c)*(-11/4*d*x^3+c)*b^3*a-9/4*b^4*c^2*x^3*
(d*x^3+c))*((a*d-b*c)/c)^(1/3)-1/2*(b*x^3+a)^(4/3)*a^2*d^2*(d*x^3+c)*(2*a*
d-9*b*c)*(2*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/
x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/
c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))
/(b*x^3+a)^(4/3)/c^2/(d*x^3+c)/(a*d-b*c)^3/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx = \int \frac{}{(b\,x^3+a)^{\frac{1}{3}}\,a^2c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,a^2cd\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,a^2d^2x^6+2\,(b\,x^3+}$$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a**2*c**2 + 2*(a + b*x**3)**(1/3)*a**2*c*d*x**3
 + (a + b*x**3)**(1/3)*a**2*d**2*x**6 + 2*(a + b*x**3)**(1/3)*a*b*c**2*x**
3 + 4*(a + b*x**3)**(1/3)*a*b*c*d*x**6 + 2*(a + b*x**3)**(1/3)*a*b*d**2*x*
*9 + (a + b*x**3)**(1/3)*b**2*c**2*x**6 + 2*(a + b*x**3)**(1/3)*b**2*c*d*x
**9 + (a + b*x**3)**(1/3)*b**2*d**2*x**12),x)
```

3.156 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^
3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(60) = 120.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.68

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \frac{x\left(b(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac(3a^2d-b^2cx^3+ab\right.}{(c+dx^3)}\right.}{}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

output
```
(x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3
)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*
x^3*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c
+ d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] +
x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a +
 b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx$$

$$\Big\downarrow \ 937$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{(dx^3+c)^2}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{4}{3}}}{(d\,x^{3}+c)^{2}}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} \, dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{(c+dx^3)^2} \, dx$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(4/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} \, dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^2} \, dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} \, dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^2} \, dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

output
```
( - (a + b*x**3)**(1/3)*a*b*x + int((a + b*x**3)**(1/3)/(a**2*c**2*d + 2*a
**2*c*d**2*x**3 + a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2
*x**6 + a*b*d**3*x**9 - b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*
x**9),x)*a**4*c*d**2 + int((a + b*x**3)**(1/3)/(a**2*c**2*d + 2*a**2*c*d**
2*x**3 + a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a
*b*d**3*x**9 - b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*x**9),x)*
a**4*d**3*x**3 - int((a + b*x**3)**(1/3)/(a**2*c**2*d + 2*a**2*c*d**2*x**3
 + a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a*b*d**
3*x**9 - b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*x**9),x)*a**3*b
*c**2*d - int((a + b*x**3)**(1/3)/(a**2*c**2*d + 2*a**2*c*d**2*x**3 + a**2
*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a*b*d**3*x**9
- b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*x**9),x)*a**3*b*c*d**2
*x**3 - int(((a + b*x**3)**(1/3)*x**6)/(a**2*c**2*d + 2*a**2*c*d**2*x**3 +
 a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a*b*d**3*
x**9 - b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*x**9),x)*a*b**3*c
**2*d - int(((a + b*x**3)**(1/3)*x**6)/(a**2*c**2*d + 2*a**2*c*d**2*x**3 +
 a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a*b*d**3*
x**9 - b**2*c**3*x**3 - 2*b**2*c**2*d*x**6 - b**2*c*d**2*x**9),x)*a*b**3*c
*d**2*x**3 + int(((a + b*x**3)**(1/3)*x**6)/(a**2*c**2*d + 2*a**2*c*d**2*x
**3 + a**2*d**3*x**6 - a*b*c**3 - a*b*c**2*d*x**3 + a*b*c*d**2*x**6 + a...
```

3.157 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$
$$= \frac{x\left(\frac{bx^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}+\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d}{}\right.}{-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\right.\right.}\right.}{c+dx^3}\right.}{12\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
-((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -
((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{(dx^3+c)^2}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{\left(d\,x^3+c\right)^2}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx = \int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{d^2x^6+2cd\,x^3+c^2}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

output
```
int((a + b*x**3)**(1/3)/(c**2 + 2*c*d*x**3 + d**2*x**6),x)
```

3.158 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a
)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 393 vs. 2(59) = 118.

Time = 10.33 (sec) , antiderivative size = 393, normalized size of antiderivative = 6.66

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \frac{4acx\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(4c(-3bc+3ad+bdx^3)+bdx^3\left(1\right.\right.}{12c^2(bc}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

output
```
(4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b
*c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1
[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) +
 b*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x
^3)/a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])
)/(12*c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3,
2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2
, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b
*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)^2}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,c^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,cd\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,d^2x^6}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c**2 + 2*(a + b*x**3)**(2/3)*c*d*x**3 + (a + b*
x**3)**(2/3)*d**2*x**6),x)
```

3.159 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,2,4/3,-b*x^3/a,-d*x^3/c)/a/c^2/(b*x^3
+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 386 vs. 2(62) = 124.

Time = 10.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.23

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \frac{x\left(bd(3bc+2ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c(16ac(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

output
```
(x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d
*x^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a
*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1
[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)
^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}\left(dx^3+c\right)^2}dx}{a\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(d\,x^{3}+c\right)^{2}}d x$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{5/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a\,c^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,acd\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,a\,d^2x^6+(b\,x^3+a)^{\frac{2}{3}}$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a*c**2 + 2*(a + b*x**3)**(2/3)*a*c*d*x**3 + (a
+ b*x**3)**(2/3)*a*d**2*x**6 + (a + b*x**3)**(2/3)*b*c**2*x**3 + 2*(a + b*
x**3)**(2/3)*b*c*d*x**6 + (a + b*x**3)**(2/3)*b*d**2*x**9),x)
```

3.160 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x
^3+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 550 vs. 2(62) = 124.

Time = 11.06 (sec) , antiderivative size = 550, normalized size of antiderivative = 8.87

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2} dx = \frac{\frac{bd(-6b^2c^2+21abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3} + \frac{4c\left(-4acx(15a^4d^3-\right.}{}}{}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

output
```
((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appe
llF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*
c*(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c
+ 4*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c
^2 + 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((
d*x^3)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3)
 + a^2*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d
*x^3 + 7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^
3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(
(b*c - a*d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3
)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)]))))/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)^2}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{8}{3}} \left(d\,x^{3}+c\right)^{2}}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \int \frac{1}{(a+bx^3)^{\frac{8}{3}}(c+dx^3)^2}\,dx$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

output
```
Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^2}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^2}\,dx = \int \frac{}{(b\,x^3+a)^{\frac{2}{3}}\,a^2c^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,a^2cd\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,a^2d^2x^6+2\,(b\,x^3+}$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**2*c**2 + 2*(a + b*x**3)**(2/3)*a**2*c*d*x**3
 + (a + b*x**3)**(2/3)*a**2*d**2*x**6 + 2*(a + b*x**3)**(2/3)*a*b*c**2*x**
3 + 4*(a + b*x**3)**(2/3)*a*b*c*d*x**6 + 2*(a + b*x**3)**(2/3)*a*b*d**2*x*
*9 + (a + b*x**3)**(2/3)*b**2*c**2*x**6 + 2*(a + b*x**3)**(2/3)*b**2*c*d*x
**9 + (a + b*x**3)**(2/3)*b**2*d**2*x**12),x)
```

3.161 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 541

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = -\frac{b(2bc-ad)\left(18b^2c^2-18abcd-5a^2d^2\right)x(a+bx^3)^{2/3}}{18c^2d^4} + \frac{b(18b^2c^2-10abcd-5a^2d^2)\,x(a+bx^3)^{5/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{11/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(12bc+5ad)x(a+bx^3)^{8/3}}{18c^2d^2\,(c+dx^3)} + \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}d^5} - \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^5} - \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^5} + \frac{(bc-ad)^{8/3}\left(54b^2c^2+18abcd+5a^2d^2\right)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^5} - \frac{b^{8/3}(54b^2c^2-126abcd+77a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18d^5}$$

output
```
-1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/
c^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^
3-1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d
+12*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)+1/27*b^(8/3)*(77*a^2*d^2-126*
a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^
(1/2)/d^5-1/27*(-a*d+b*c)^(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/
3)/d^5-1/54*(-a*d+b*c)^(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)
/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2
*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^5
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 12.47 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

output
```
((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
 a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*
x^3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*
(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)])/(d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3
)) + (10*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3
)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1
/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3))
+ (36*a*b^4*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
+ a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3
)) - (72*a^2*b^3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c
^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)
^(1/3)) + (30*a^3*b^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c...
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 505, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {930, 1023, 27, 1025, 27, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{\left(bx^3+a\right)^{8/3}\left(6b(2bc-ad)x^3+a(bc+5ad)\right)}{\left(dx^3+c\right)^2}dx}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1023$$

$$\frac{\frac{x\left(a+bx^3\right)^{8/3}\left(\frac{5a^2d}{c}+7ab-\frac{12b^2c}{d}\right)}{3\left(c+dx^3\right)}-\frac{\int -\frac{2\left(bx^3+a\right)^{5/3}\left(3b\left(18b^2c^2-10abdc-5a^2d^2\right)x^3+a\left(6b^2c^2-2abdc+5a^2d^2\right)\right)}{dx^3+c}dx}{3cd}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{\left(bx^3+a\right)^{5/3}\left(3b\left(18b^2c^2-10abdc-5a^2d^2\right)x^3+a\left(6b^2c^2-2abdc+5a^2d^2\right)\right)}{dx^3+c}dx}{3cd}+\frac{x\left(a+bx^3\right)^{8/3}\left(\frac{5a^2d}{c}+7ab-\frac{12b^2c}{d}\right)}{3\left(c+dx^3\right)}}{6cd}-\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{2\left(\frac{\int -\frac{3\left(bx^3+a\right)^{2/3}\left(3b(2bc-ad)\left(18b^2c^2-18abdc-5a^2d^2\right)x^3+a\left(18b^3c^3-22ab^2dc^2-a^2bd^2c-10a^3d^3\right)\right)}{dx^3+c}dx}{6d}+\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}\right)}{3cd}+}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(3b(2bc-ad)\left(18b^2c^2-18abdc-5a^2d^2\right)x^3+a\left(18b^3c^3-22ab^2dc^2-a^2bd^2c-10a^3d^3\right)\right)}{dx^3+c}dx}{2d}\right)}{3cd}+\frac{x}{\cdots}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\big\downarrow$ 1025

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\frac{\int -\frac{6\left(b^3c^2\left(54b^2c^2-126abdc+77a^2d^2\right)x^3+a\left(18b^4c^4-36ab^3dc^3+15a^2b^2d^2c^2+3a^3bd^3c+5a^4d^4\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3d}+\frac{bx\left(a+b\cdots\right.}{\cdots}}{2d}\cdots\right.}{3cd}\cdots}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\int \frac{b^3c^2\left(54b^2c^2-126abdc+77a^2d^2\right)x^3+a\left(18b^4c^4-36ab\cdots\right.}{\sqrt[3]{bx^3+a}\left(dx^3\cdots\right.}\cdots}{d}}{2d}\cdots\right.}{3cd}\cdots}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\big\downarrow$ 1026

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}}dx\cdots}{d}\cdots\right.}{2d}\cdots\right.}{3cd}\cdots}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 769

$$\frac{\frac{2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{a+\ldots}}}{\sqrt{\ldots}}\right)}{\sqrt{3}\sqrt[3]{b}\ldots}\ldots\right)}{d}\ldots\right)}{\ldots}\ldots\right)}{3cd}\ldots}{6c\ldots}$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 901

$$2\left(\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{2d}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{b^3c^2\left(77a^2d^2-126abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{a+\ldots}}}{\sqrt{\ldots}}\right)}{\sqrt{3}\sqrt[3]{b}\ldots}\ldots\right)}{d}\ldots\right)}{\ldots}\ldots\right)$$

$$\frac{x\left(a+bx^3\right)^{11/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(11/3))/(c*d*(c + d*x^3)^2) + (((7*a*b - (
12*b^2*c)/d + (5*a^2*d)/c)*x*(a + b*x^3)^(8/3))/(3*(c + d*x^3)) + (2*((b*(
18*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(5/3))/(2*d) - ((b*(2*b
*c - a*d)*(18*b^2*c^2 - 18*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/d - (
2*(-(((b*c - a*d)^3*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*
(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[
((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^
(1/3))))/d) + (b^3*c^2*(54*b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*(ArcTan[(1
+ (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d))/d)/(2*d)))/(3*c*d))/(6*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$-\frac{2\left(\frac{5(ad-bc)^3\left(a^2d^2+\frac{18}{5}abcd+\frac{54}{5}b^2c^2\right)\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{108}-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^3\left(d\right.}{}\right.}{}$

input
```
int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/((a*d-b*c)/c)^(1/3)*(5/108*(a*d-b*c)^3*(a^2*d^2+18/5*a*b*c*d+54/5*b^2*c
^2)*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^
(1/3)*x+(b*x^3+a)^(2/3))/x^2)-1/2*((a*d-b*c)/c)^(1/3)*c^3*(d*x^3+c)^2*(77/
54*a^2*b^(8/3)*d^2+(b*c-7/3*a*d)*c*b^(11/3))*ln((b^(2/3)*x^2+b^(1/3)*(b*x^
3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-5/54*(a*d-b*c)^3*(a^2*d^2+18/5*a*b*c*d+
54/5*b^2*c^2)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x
)/x)*(d*x^3+c)^2*3^(1/2)-5/54*(a*d-b*c)^3*(a^2*d^2+18/5*a*b*c*d+54/5*b^2*c
^2)*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+((a*d-b*c)/c
)^(1/3)*c*(c^2*(d*x^3+c)^2*3^(1/2)*(77/54*a^2*b^(8/3)*d^2+(b*c-7/3*a*d)*c*
b^(11/3))*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+c^2*
(d*x^3+c)^2*(77/54*a^2*b^(8/3)*d^2+(b*c-7/3*a*d)*c*b^(11/3))*ln((-b^(1/3)*
x+(b*x^3+a)^(1/3))/x)-2/9*(b*x^3+a)^(2/3)*d*(-9/2*b^4*c^5+9*(-3/4*b*x^3+a)
*d*b^3*c^4-15/4*d^2*(2/5*b^2*x^6-11/3*a*b*x^3+a^2)*b^2*c^3-3/4*d^3*(-1/2*b
^3*x^9-13/3*a*b^2*x^6+8*a^2*b*x^3+a^3)*b*c^2+a^3*d^4*(3/4*b*x^3+a)*c+5/8*a
^4*d^5*x^3)*x))/c^3/d^5/(d*x^3+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1555 vs. $2(473) = 946$.

Time = 88.77 (sec) , antiderivative size = 1555, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-1/54*(2*sqrt(3)*(54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3
*b*c^3*d^3 + 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b
^2*c^2*d^4 + 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c
^4*d^2 + 23*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/
((b*c - a*d)*x)) + 2*sqrt(3)*(54*b^4*c^6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^
4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^2*d^4)*x^6 + 2*
(54*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3)*(-b^2)^(1/3)*
arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)
) - 2*(54*b^4*c^6 - 90*a*b^3*c^5*d + 23*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d^3
+ 5*a^4*c^2*d^4 + (54*b^4*c^4*d^2 - 90*a*b^3*c^3*d^3 + 23*a^2*b^2*c^2*d^4
+ 8*a^3*b*c*d^5 + 5*a^4*d^6)*x^6 + 2*(54*b^4*c^5*d - 90*a*b^3*c^4*d^2 + 23
*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 + 5*a^4*c*d^5)*x^3)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(54*b^4*c^6 - 126*a*b^3*c^5*d
+ 77*a^2*b^2*c^4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*c^3*d^3 + 77*a^2*b^2*c^
2*d^4)*x^6 + 2*(54*b^4*c^5*d - 126*a*b^3*c^4*d^2 + 77*a^2*b^2*c^3*d^3)*x^3
)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (54*b^4*c^
6 - 126*a*b^3*c^5*d + 77*a^2*b^2*c^4*d^2 + (54*b^4*c^4*d^2 - 126*a*b^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{14}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{14/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \text{too large to display}$$

input

```
int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

output

```
( - 90*(a + b*x**3)**(2/3)*a**5*b*d**3*x - 90*(a + b*x**3)**(2/3)*a**4*b**
2*c*d**2*x - 315*(a + b*x**3)**(2/3)*a**4*b**2*d**3*x**4 + 364*(a + b*x**3
)**(2/3)*a**3*b**3*c**2*d*x + 725*(a + b*x**3)**(2/3)*a**3*b**3*c*d**2*x**
4 + 130*(a + b*x**3)**(2/3)*a**3*b**3*d**3*x**7 - 168*(a + b*x**3)**(2/3)*
a**2*b**4*c**3*x - 483*(a + b*x**3)**(2/3)*a**2*b**4*c**2*d*x**4 - 372*(a
+ b*x**3)**(2/3)*a**2*b**4*c*d**2*x**7 + 15*(a + b*x**3)**(2/3)*a**2*b**4*
d**3*x**10 + 126*(a + b*x**3)**(2/3)*a*b**5*c**3*x**4 + 378*(a + b*x**3)**
(2/3)*a*b**5*c**2*d*x**7 - 36*(a + b*x**3)**(2/3)*a*b**5*c*d**2*x**10 - 10
8*(a + b*x**3)**(2/3)*b**6*c**3*x**7 + 27*(a + b*x**3)**(2/3)*b**6*c**2*d*
x**10 + 450*int((a + b*x**3)**(2/3)/(5*a**3*c**3*d**2 + 15*a**3*c**2*d**3*
x**3 + 15*a**3*c*d**4*x**6 + 5*a**3*d**5*x**9 - 12*a**2*b*c**4*d - 31*a**2
*b*c**3*d**2*x**3 - 21*a**2*b*c**2*d**3*x**6 + 3*a**2*b*c*d**4*x**9 + 5*a*
*2*b*d**5*x**12 + 9*a*b**2*c**5 + 15*a*b**2*c**4*d*x**3 - 9*a*b**2*c**3*d*
*2*x**6 - 27*a*b**2*c**2*d**3*x**9 - 12*a*b**2*c*d**4*x**12 + 9*b**3*c**5*
x**3 + 27*b**3*c**4*d*x**6 + 27*b**3*c**3*d**2*x**9 + 9*b**3*c**2*d**3*x**
12),x)*a**9*c**2*d**6 + 900*int((a + b*x**3)**(2/3)/(5*a**3*c**3*d**2 + 15
*a**3*c**2*d**3*x**3 + 15*a**3*c*d**4*x**6 + 5*a**3*d**5*x**9 - 12*a**2*b*
c**4*d - 31*a**2*b*c**3*d**2*x**3 - 21*a**2*b*c**2*d**3*x**6 + 3*a**2*b*c*
d**4*x**9 + 5*a**2*b*d**5*x**12 + 9*a*b**2*c**5 + 15*a*b**2*c**4*d*x**3 -
9*a*b**2*c**3*d**2*x**6 - 27*a*b**2*c**2*d**3*x**9 - 12*a*b**2*c*d**4*x...
```

3.162 $\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 458

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \frac{b(18b^2c^2-7abcd-5a^2d^2)\,x(a+bx^3)^{2/3}}{18c^2d^3} - \frac{(bc-ad)x(a+bx^3)^{8/3}}{6cd\,(c+dx^3)^2} - \frac{(bc-ad)(9bc+5ad)x(a+bx^3)^{5/3}}{18c^2d^2\,(c+dx^3)} - \frac{b^{8/3}(9bc-11ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^4} + \frac{(bc-ad)^{5/3}\left(27b^2c^2+12abcd+5a^2d^2\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^4} + \frac{(bc-ad)^{5/3}\left(27b^2c^2+12abcd+5a^2d^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^4} - \frac{(bc-ad)^{5/3}\left(27b^2c^2+12abcd+5a^2d^2\right)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^4} + \frac{b^{8/3}(9bc-11ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6d^4}$$

output
```
1/18*b*(-5*a^2*d^2-7*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c^2/d^3-1/6*(-a
*d+b*c)*x*(b*x^3+a)^(8/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+9*b*c)*x*
(b*x^3+a)^(5/3)/c^2/d^2/(d*x^3+c)-1/9*b^(8/3)*(-11*a*d+9*b*c)*arctan(1/3*(
1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^4+1/27*(-a*d+b*c)^(5/3)*
(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/
3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)/d^4+1/54*(-a*d+b*c)^(5/3)*(5*
a^2*d^2+12*a*b*c*d+27*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^4-1/18*(-a*d+b*c)^(5/
3)*(5*a^2*d^2+12*a*b*c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+
a)^(1/3))/c^(8/3)/d^4+1/6*b^(8/3)*(-11*a*d+9*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^
(1/3))/d^4
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 12.18 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

output
```
((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*
(b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b
*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^
3*(a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1
/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a
^4*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/
3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] +
 Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(
1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)
*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a
^2*b^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)
^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) +
(4*a^3*b*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/...
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {930, 1023, 25, 1025, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{(bx^3+a)^{5/3}(3b(3bc-ad)x^3+a(bc+5ad))}{(dx^3+c)^2}dx}{6cd}-\frac{x(a+bx^3)^{8/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \text{1023}$$

$$\frac{\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}-\frac{\int -\frac{\left(bx^3+a\right)^{2/3}\left(3b\left(18b^2c^2-7abdc-5a^2d^2\right)x^3+a\left(9b^2c^2-abdc+10a^2d^2\right)\right)}{dx^3+c}dx}{3cd}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(bx^3+a\right)^{2/3}\left(3b\left(18b^2c^2-7abdc-5a^2d^2\right)x^3+a\left(9b^2c^2-abdc+10a^2d^2\right)\right)}{dx^3+c}dx}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 1025

$$\frac{\frac{\frac{\int -\frac{6\left(3b^3c^2(9bc-11ad)x^3+a\left(9b^3c^3-8ab^2dc^2-2a^2bd^2c-5a^3d^3\right)\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3d}+\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\int \frac{3b^3c^2(9bc-11ad)x^3+a\left(9b^3c^3-8ab^2dc^2-2a^2bd^2c-5a^3d^3\right)}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}}{3cd}+\frac{x(a+bx^3)^{5/3}\left(\frac{5a^2d}{c}+4ab-\frac{9b^2c}{d}\right)}{3(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 1026

$$\frac{\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+27b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{d}\right)}{d}}{3cd}+\ldots}{6cd}-\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\Big\downarrow$ 769

$$\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+\cdots\right.}{\cdots}\right.}{d}}{3cd}$$

$$6cd$$

$$\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{d}-\frac{2\left(\frac{3b^3c^2(9bc-11ad)\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)^2\left(5a^2d^2+12abcd+\cdots\right.}{\cdots}\right.}{d}}{3cd}$$

$$6cd$$

$$\frac{x\left(a+bx^3\right)^{8/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(8/3))/(c*d*(c + d*x^3)^2) + (((4*a*b - (9
*b^2*c)/d + (5*a^2*d)/c)*x*(a + b*x^3)^(5/3))/(3*(c + d*x^3)) + ((b*(18*b^
2*c^2 - 7*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/d - (2*(-(((b*c - a*d)
^2*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)
*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/
3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3
)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) + (3*
b^3*c^2*(9*b*c - 11*a*d)*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3)
)))/d))/d)/(3*c*d))/(6*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$-\frac{\frac{5(ad-bc)^2\left(a^2d^2+\frac{12}{5}abcd+\frac{27}{5}b^2c^2\right)\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54}+\frac{11\left(a b^{\frac{8}{3}} d-\frac{9b^{\frac{11}{3}}c}{11}\right)\left(\cdots\right.}{\cdots}}{\cdots}$

input
```
int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-b*c)^2*(a^2*d^2+12/5*a*b*c*d+27/5*b^2*
c^2)*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)
^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+33/10*(a*b^(8/3)*d-9/11*b^(11/3)*c)*((a*d-b
*c)/c)^(1/3)*c^3*(d*x^3+c)^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*
x^3+a)^(2/3))/x^2)+(a*d-b*c)^2*(a^2*d^2+12/5*a*b*c*d+27/5*b^2*c^2)*arctan(
1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*(d*x^3+c)^2*3^(1
/2)+(a*d-b*c)^2*(a^2*d^2+12/5*a*b*c*d+27/5*b^2*c^2)*(d*x^3+c)^2*ln((((a*d-
b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5*((a*d-b*c)/c)^(1/3)*c*(-11/4*(a*b
^(8/3)*d-9/11*b^(11/3)*c)*c^2*(d*x^3+c)^2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b
*x^3+a)^(1/3)/b^(1/3)+x)/x)-11/4*(a*b^(8/3)*d-9/11*b^(11/3)*c)*c^2*(d*x^3+
c)^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+d*(b*x^3+a)^(2/3)*(9/4*c^4*b^3-2*d
*(-27/16*b*x^3+a)*b^2*c^3-1/2*(-3/2*b^2*x^6+25/4*a*b*x^3+a^2)*d^2*b*c^2+a^
2*d^3*(5/8*b*x^3+a)*c+5/8*a^3*d^4*x^3)*x))/d^4/(d*x^3+c)^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. $2(394) = 788$.

Time = 19.59 (sec) , antiderivative size = 1246, normalized size of antiderivative = 2.72

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2
*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6
 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3
)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c -
a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^
3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3
)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^
2)^(1/3))/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^
3*c^2*d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5
)*x^6 + 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4
)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b
^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^
3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^
3 + a)^(1/3)*b)/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a
*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*l
og(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(
2/3)*b)/x^2) + (27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{11}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx=\int \frac{(b\,x^3+a)^{11/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3}\,dx=\text{too large to display}$$

input
```
int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

output
```
( - 24*(a + b*x**3)**(2/3)*a**4*b*d**2*x - 54*(a + b*x**3)**(2/3)*a**3*b**
2*d**2*x**4 + 28*(a + b*x**3)**(2/3)*a**2*b**3*c**2*x + 89*(a + b*x**3)**(
2/3)*a**2*b**3*c*d*x**4 + 10*(a + b*x**3)**(2/3)*a**2*b**3*d**2*x**7 - 21*
(a + b*x**3)**(2/3)*a*b**4*c**2*x**4 - 24*(a + b*x**3)**(2/3)*a*b**4*c*d*x
**7 + 18*(a + b*x**3)**(2/3)*b**5*c**2*x**7 + 150*int((a + b*x**3)**(2/3)/
(5*a**3*c**3*d**2 + 15*a**3*c**2*d**3*x**3 + 15*a**3*c*d**4*x**6 + 5*a**3*
d**5*x**9 - 12*a**2*b*c**4*d - 31*a**2*b*c**3*d**2*x**3 - 21*a**2*b*c**2*d
**3*x**6 + 3*a**2*b*c*d**4*x**9 + 5*a**2*b*d**5*x**12 + 9*a*b**2*c**5 + 15
*a*b**2*c**4*d*x**3 - 9*a*b**2*c**3*d**2*x**6 - 27*a*b**2*c**2*d**3*x**9 -
 12*a*b**2*c*d**4*x**12 + 9*b**3*c**5*x**3 + 27*b**3*c**4*d*x**6 + 27*b**3
*c**3*d**2*x**9 + 9*b**3*c**2*d**3*x**12),x)*a**8*c**2*d**5 + 300*int((a +
 b*x**3)**(2/3)/(5*a**3*c**3*d**2 + 15*a**3*c**2*d**3*x**3 + 15*a**3*c*d**
4*x**6 + 5*a**3*d**5*x**9 - 12*a**2*b*c**4*d - 31*a**2*b*c**3*d**2*x**3 -
21*a**2*b*c**2*d**3*x**6 + 3*a**2*b*c*d**4*x**9 + 5*a**2*b*d**5*x**12 + 9*
a*b**2*c**5 + 15*a*b**2*c**4*d*x**3 - 9*a*b**2*c**3*d**2*x**6 - 27*a*b**2*
c**2*d**3*x**9 - 12*a*b**2*c*d**4*x**12 + 9*b**3*c**5*x**3 + 27*b**3*c**4*
d*x**6 + 27*b**3*c**3*d**2*x**9 + 9*b**3*c**2*d**3*x**12),x)*a**8*c*d**6*x
**3 + 150*int((a + b*x**3)**(2/3)/(5*a**3*c**3*d**2 + 15*a**3*c**2*d**3*x*
*3 + 15*a**3*c*d**4*x**6 + 5*a**3*d**5*x**9 - 12*a**2*b*c**4*d - 31*a**2*b
*c**3*d**2*x**3 - 21*a**2*b*c**2*d**3*x**6 + 3*a**2*b*c*d**4*x**9 + 5*a...
```

3.163 $\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 391

$$\begin{aligned}
\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = &-\frac{(bc-ad)x(a+bx^3)^{5/3}}{6cd\,(c+dx^3)^2} \\
&-\frac{(bc-ad)(6bc+5ad)x(a+bx^3)^{2/3}}{18c^2d^2\,(c+dx^3)}+\frac{b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} \\
&-\frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}d^3} \\
&-\frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\log\left(c+dx^3\right)}{54c^{8/3}d^3} \\
&+\frac{(bc-ad)^{2/3}\,(9b^2c^2+6abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3} \\
&-\frac{b^{8/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d^3}
\end{aligned}$$

output
```
-1/6*(-a*d+b*c)*x*(b*x^3+a)^(5/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6
*b*c)*x*(b*x^3+a)^(2/3)/c^2/d^2/(d*x^3+c)+1/3*b^(8/3)*arctan(1/3*(1+2*b^(1
/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^3-1/27*(-a*d+b*c)^(2/3)*(5*a^2*d
^2+6*a*b*c*d+9*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+
a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)/d^3-1/54*(-a*d+b*c)^(2/3)*(5*a^2*d^2+6*
a*b*c*d+9*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^3+1/18*(-a*d+b*c)^(2/3)*(5*a^2*d^
2+6*a*b*c*d+9*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8
/3)/d^3-1/2*b^(8/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/d^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.12 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(-bc+ad)x(a+bx^3)^{2/3}(3bc(2c+3dx^3)+ad(8c+5dx^3))}{d^2(c+dx^3)^2}+\frac{27b^3c^{5/3}x^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a}\right.}{d^2\sqrt[3]{a+bx^3}}}{}$$

input
```
Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*
d*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)
/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a
+ b*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^
(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/
3) + (6*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b +
a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) +
(c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3))
 + (2*a^2*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b +
 a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^
3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(
1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108
*c^(8/3))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 1023, 27, 1026, 769, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3} \, dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{(bx^3+a)^{2/3}(6b^2cx^3+a(bc+5ad))}{(dx^3+c)^2}dx}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$$\Big\downarrow \text{1023}$$

$$\frac{-\frac{\int -\frac{2(9b^3c^2x^3+a(3b^2c^2+ad(bc+5ad)))}{\sqrt[3]{bx^3+a}(dx^3+c)}dx}{3cd}-\frac{x(a+bx^3)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x(a+bx^3)^{5/3}(bc-ad)}{6cd(c+dx^3)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{9b^3c^2x^3+a\left(3b^2c^2+ad(bc+5ad)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 1026

$$\frac{\frac{2\left(\frac{9b^3c^2\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}\right)}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}(bc-ad)(5ad+6bc)}{3cd(c+dx^3)}}{6cd}-\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 769

$$\frac{\frac{2\left(\frac{9b^3c^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{d}\right)}{3cd}-\frac{x\left(a+bx^3\right)^{2/3}}{3c}}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

$\downarrow$ 901

$$\frac{\frac{2\left(\frac{9b^3c^2\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{d}-\frac{(bc-ad)\left(5a^2d^2+6abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx\cdots\right.}{6c^{2/3}\sqrt[3]{bc\cdots}}\cdots\right)}{d}\cdots\right)}{3cd}\cdots}{6cd}$$

$$\frac{x\left(a+bx^3\right)^{5/3}(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

output
```
-1/6*((b*c - a*d)*x*(a + b*x^3)^(5/3))/(c*d*(c + d*x^3)^2) + (-1/3*((b*c -
 a*d)*(6*b*c + 5*a*d)*x*(a + b*x^3)^(2/3))/(c*d*(c + d*x^3)) + (2*(-(((b*c
 - a*d)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^
(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(
1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/d) +
(9*b^3*c^2*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]
*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/d))/(3*c*d
))/(6*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1023
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(
p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$-\frac{\frac{5(ad-bc)\left(d\,x^{3}+c\right)^{2}\left(a^{2}d^{2}+\frac{6}{5}abcd+\frac{9}{5}b^{2}c^{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{54}+\frac{b^{\frac{8}{3}}c^{3}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right.}{}}{}$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*(-1/2*(a*d-b*c)*(d*x^3+c)^2*(a^2*d^2+6/5*a*b*c*d+
9/5*b^2*c^2)*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2)+9/10*b^(8/3)*c^3*((a*d-b*c)/c)^(1/3)*(d*x^3+c)^
2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-b*c
)*(d*x^3+c)^2*(a^2*d^2+6/5*a*b*c*d+9/5*b^2*c^2)*3^(1/2)*arctan(1/3*3^(1/2)
*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)+(a*d-b*c)*(d*x^3+c)^2*(a^2*
d^2+6/5*a*b*c*d+9/5*b^2*c^2)*ln(((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)
+12/5*((a*d-b*c)/c)^(1/3)*(-3/4*b^(8/3)*3^(1/2)*c^2*(d*x^3+c)^2*arctan(1/3
*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)-3/4*b^(8/3)*c^2*(d*x^3+c)^2*ln((
-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+(a*d-b*c)*(b*x^3+a)^(2/3)*(3/4*b*c^2+d*(9/8
*b*x^3+a)*c+5/8*a*d^2*x^3)*d*x)*c)/d^3/(d*x^3+c)^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(334) = 668.

Time = 2.34 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.44

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^
4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c
*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3
)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*
c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*
a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d +
6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/
3)*(b*c - a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^
2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x
^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b
*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d
^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^
2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a
*b*c*d^3 - 5*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx=\int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx=\int \frac{(bx^3+a)^{\frac{8}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx=\int \frac{(b\,x^3+a)^{8/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

output
```
(2*(a + b*x**3)**(2/3)*a**3*d*x - 6*(a + b*x**3)**(2/3)*a**2*b*c*x - 3*(a
+ b*x**3)**(2/3)*a**2*b*d*x**4 - 2*(a + b*x**3)**(2/3)*a*b**2*c*x**4 + 2*i
nt(((a + b*x**3)**(2/3)*x**6)/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x**6 + d**3
*x**9),x)*a*b**2*c**3*d + 4*int(((a + b*x**3)**(2/3)*x**6)/(c**3 + 3*c**2*
d*x**3 + 3*c*d**2*x**6 + d**3*x**9),x)*a*b**2*c**2*d**2*x**3 + 2*int(((a +
 b*x**3)**(2/3)*x**6)/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x**6 + d**3*x**9),x
)*a*b**2*c*d**3*x**6 - 6*int(((a + b*x**3)**(2/3)*x**6)/(c**3 + 3*c**2*d*x
**3 + 3*c*d**2*x**6 + d**3*x**9),x)*b**3*c**4 - 12*int(((a + b*x**3)**(2/3
)*x**6)/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x**6 + d**3*x**9),x)*b**3*c**3*d*
x**3 - 6*int(((a + b*x**3)**(2/3)*x**6)/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x
**6 + d**3*x**9),x)*b**3*c**2*d**2*x**6 + 10*int(((a + b*x**3)**(2/3)*x**3
)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x*
*9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**1
2 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2
*c*d**3*x**12),x)*a**5*c**2*d**3 + 20*int(((a + b*x**3)**(2/3)*x**3)/(a**2
*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*
a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**12 - 3*b
**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2*c*d**3
*x**12),x)*a**5*c*d**4*x**3 + 10*int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3
*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*...
```

3.164 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} + \frac{5a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2\log(c+dx^3)}{54c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}$$

output
```
1/6*x*(b*x^3+a)^(5/3)/c/(d*x^3+c)^2+5/18*a*x*(b*x^3+a)^(2/3)/c^2/(d*x^3+c)
+5/27*a^2*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1
/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(1/3)+5/54*a^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b
*c)^(1/3)-5/18*a^2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/
(-a*d+b*c)^(1/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(8acx+3bcx^4+5adx^4)}{(c+dx^3)^2} - \frac{10\sqrt{-6+6i\sqrt{3}}a^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}}{}$$

input
```
Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*(a + b*x^3)^(2/3)*(8*a*c*x + 3*b*c*x^4 + 5*a*d*x^4))/(c + d*x^
3)^2 - (10*Sqrt[-6 + (6*I)*Sqrt[3]]*a^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sq
rt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]]/(
b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*a^2*Log[2*(b*c - a*d)^(1/3)*x + (1
+ I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]]/(b*c - a*d)^(1/3) - ((5*I)*(-I +
Sqrt[3])*a^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3
)]]/(b*c - a*d)^(1/3))/(108*c^(8/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {903, 903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{5a\int \frac{(bx^3+a)^{2/3}}{(dx^3+c)^2}dx}{6c} + \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

$$\Big\downarrow \text{903}$$

$$\frac{5a\left(\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a}\,(dx^3+c)}dx}{3c}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}\right)}{6c}+\frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

$$\Big\downarrow \text{901}$$

$$\frac{5a\left(\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}\right)}{6c}+\frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

input

```
Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

output

```
(x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*((x*(a + b*x^3)^(2/3))/(3
*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^
3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a
+ b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c)))/(6*c)
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)a^{2}\left(d\,x^{3}+c\right)^{2}}{54}+\frac{5\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a^{2}\left(d\,x^{3}+c\right)^{2}}{27}+\frac{4\left(\ldots\right.}{}}{c^{3}\left(d\,x^{3}+c\right)^{2}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*(-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(
1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2*(d*x^3+c)^2+ln((((a*d-b*c
)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2*(d*x^3+c)^2+12/5*(1/8*(5*a*d+3*b*c)*x
^3+a*c)*c*(b*x^3+a)^(2/3)*x*((a*d-b*c)/c)^(1/3)+a^2*arctan(1/3*3^(1/2)*(-2
/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*(d*x^3+c)^2*3^(1/2))/c^3/(d*x^3
+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{5/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx = \text{too large to display}$$

input
```
int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

output
```
(2*(a + b*x**3)**(2/3)*a**2*d*x - 6*(a + b*x**3)**(2/3)*a*b*c*x - 3*(a + b
*x**3)**(2/3)*a*b*d*x**4 - (a + b*x**3)**(2/3)*b**2*c*x**4 + 10*int(((a +
b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*
x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x
**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2
*d**2*x**9 - 3*b**2*c*d**3*x**12),x)*a**4*c**2*d**3 + 20*int(((a + b*x**3)
**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 +
a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a
*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x
**9 - 3*b**2*c*d**3*x**12),x)*a**4*c*d**4*x**3 + 10*int(((a + b*x**3)**(2/
3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*
d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d*
*4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 -
 3*b**2*c*d**3*x**12),x)*a**4*d**5*x**6 - 50*int(((a + b*x**3)**(2/3)*x**3
)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x*
*9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**1
2 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2
*c*d**3*x**12),x)*a**3*b*c**3*d**2 - 100*int(((a + b*x**3)**(2/3)*x**3)/(a
**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 -
 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**12...
```

3.165 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 256

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3} \, dx = \frac{x(a+bx^3)^{2/3}}{6c\,(c+dx^3)^2} + \frac{(3bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)\,(c+dx^3)} + \frac{a(6bc-5ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc-5ad)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc-5ad)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}$$

output
```
1/6*x*(b*x^3+a)^(2/3)/c/(d*x^3+c)^2+1/18*(-5*a*d+3*b*c)*x*(b*x^3+a)^(2/3)/
c^2/(-a*d+b*c)/(d*x^3+c)+1/27*a*(-5*a*d+6*b*c)*arctan(1/3*(1+2*(-a*d+b*c)^
(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(4/3)
+1/54*a*(-5*a*d+6*b*c)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(4/3)-1/18*a*(-5*a*d
+6*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^
(4/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.23 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \frac{\frac{6c^{2/3}x(a+bx^3)^{2/3}(3bc(2c+dx^3)-ad(8c+5dx^3))}{(bc-ad)(c+dx^3)^2} + \frac{2i\left(3i+\sqrt{3}\right)a(-6bc+5ad)\text{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right.}{(bc-ad)^{4/3}}}{}$$

input
```
Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

output
```
((6*c^(2/3)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + d*x^3) - a*d*(8*c + 5*d*x^3)
))/((b*c - a*d)*(c + d*x^3)^2) + ((2*I)*(3*I + Sqrt[3])*a*(-6*b*c + 5*a*d)
*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/((b*c - a*d)^(1/3
)*x))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*a*(6*b*c - 5*a*d)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(4/3) + ((1 + I*Sqrt[3])*a*(-6*b*c + 5*a*d)*Log[2*(b*c - a*d)^(2/
3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(108*c^(8/3
))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {907, 903, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{(6bc-5ad)\int \frac{\left(bx^3+a\right)^{2/3}}{\left(dx^3+c\right)^2}dx}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 903$$

$$\frac{(6bc-5ad)\left(\frac{2a\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}\right)}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{(6bc-5ad)\left(\frac{2a\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}\right)}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

input
```
Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

output
```
-1/6*(d*x*(a + b*x^3)^(5/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a
*d)*((x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*(ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c
 - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))
))/(3*c)))/(6*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-\frac{5a\left(ad-\frac{6bc}{5}\right)\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54}+\frac{5a\left(ad-\frac{6bc}{5}\right)\left(dx^3+c\right)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\right.}{x}\right.}{27}}{(ad-bc)c^3(d}$

input
```
int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27*(-1/2*a*(a*d-6/5*b*c)*(d*x^3+c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b
*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+a*(a*d-6/5*b*c)*(d*x^
3+c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5*c*(1/8*(5*a*d^2-
3*b*c*d)*x^3+c*(a*d-3/4*b*c))*(b*x^3+a)^(2/3)*x*((a*d-b*c)/c)^(1/3)+a*(a*d
-6/5*b*c)*arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)
*(d*x^3+c)^2*3^(1/2))/((a*d-b*c)/c)^(1/3)/(a*d-b*c)/c^3/(d*x^3+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{(c+dx^3)^3}\,dx$$

input
```
integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

output
```
Integral((a + b*x**3)**(2/3)/(c + d*x**3)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx = \text{too large to display}$$

input

```
int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

output

```
(2*(a + b*x**3)**(2/3)*a*d*x - 6*(a + b*x**3)**(2/3)*b*c*x - 3*(a + b*x**3
)**(2/3)*b*d*x**4 + 10*int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**
2*c**2*d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*
b*c**3*d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 -
 9*b**2*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2*c*d**3*x**12),x)*a**3
*c**2*d**3 + 20*int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*
d**2*x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*
d*x**3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2
*c**3*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2*c*d**3*x**12),x)*a**3*c*d**4
*x**3 + 10*int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*
x**3 + 3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**
3 - 6*a*b*c**2*d**2*x**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3
*d*x**6 - 9*b**2*c**2*d**2*x**9 - 3*b**2*c*d**3*x**12),x)*a**3*d**5*x**6 -
 52*int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 +
3*a**2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a
*b*c**2*d**2*x**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6
 - 9*b**2*c**2*d**2*x**9 - 3*b**2*c*d**3*x**12),x)*a**2*b*c**3*d**2 - 104*
int(((a + b*x**3)**(2/3)*x**3)/(a**2*c**3*d + 3*a**2*c**2*d**2*x**3 + 3*a*
*2*c*d**3*x**6 + a**2*d**4*x**9 - 3*a*b*c**4 - 8*a*b*c**3*d*x**3 - 6*a*b*c
**2*d**2*x**6 + a*b*d**4*x**12 - 3*b**2*c**4*x**3 - 9*b**2*c**3*d*x**6 ...
```

3.166 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 307

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = -\frac{dx(a+bx^3)^{2/3}}{6c(bc-ad)\,(c+dx^3)^2} - \frac{d(9bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)^2\,(c+dx^3)} + \frac{(9b^2c^2-12abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{7/3}} + \frac{(9b^2c^2-12abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{7/3}} - \frac{(9b^2c^2-12abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}$$

output
```
-1/6*d*x*(b*x^3+a)^(2/3)/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*
(b*x^3+a)^(2/3)/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^
2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2)
)*3^(1/2)/c^(8/3)/(-a*d+b*c)^(7/3)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*l
n(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(7/3)-1/18*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(7/3)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.67 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$

$$= \frac{\frac{6c^{2/3}dx(a+bx^3)^{2/3}(-3bc(4c+3dx^3)+ad(8c+5dx^3))}{(bc-ad)^2(c+dx^3)^2} + \frac{2\left(3-i\sqrt{3}\right)(9b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{i+\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}x}}{\sqrt{3}}\right)}{(bc-ad)^{7/3}} +}{}$$

input
```
Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

output
```
((6*c^(2/3)*d*x*(a + b*x^3)^(2/3)*(-3*b*c*(4*c + 3*d*x^3) + a*d*(8*c + 5*d
*x^3)))/((b*c - a*d)^2*(c + d*x^3)^2) + (2*(3 - I*Sqrt[3])*(9*b^2*c^2 - 12
*a*b*c*d + 5*a^2*d^2)*ArcTanh[(I + ((-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3))/((b*c - a*d)^(1/3)*x))/Sqrt[3]])/(b*c - a*d)^(7/3) + (2*(1 + I*Sqrt[3]
)*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(7/3) - (I*(-I + Sqrt[3])
*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 -
I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])
*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(7/3))/(108*c^(8/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {931, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-3bdx^3+6bc-5ad}{\sqrt[3]{bx^3+a}(dx^3+c)^2}dx}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{2\left(9b^2c^2-12abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(5a^2d^2-12abcd+9b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \; 901$$

$$\frac{\frac{2\left(5a^2d^2-12abcd+9b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}\right)}{3c(bc-ad)}-\frac{dx\left(a+bx^3\right)^{2/3}(9bc-5ad)}{3c(c+dx^3)(bc-ad)}}{6c(bc-ad)} \frac{dx\left(a+bx^3\right)^{2/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)^2) + (-1/3*(d*(9*b
*c - 5*a*d)*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + (2*(9*b^2*c
^2 - 12*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c
 + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d)))/(
6*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{\frac{5\left(d\,x^{3}+c\right)^{2}\left(a^{2}d^{2}-\frac{12}{5}abcd+\frac{9}{5}b^{2}c^{2}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{27}+\frac{4cd\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(-\frac{3b\,c^{2}}{2}+d\left(-\frac{9b\,x^{3}}{8}+a\right)c+\frac{5a\,d^{2}x^{3}}{8}\right)x\left(\frac{ad-bc}{c}\cdots\right.}{9}\cdots}{\left(\frac{ad-bc}{c}\right)\cdots}$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
5/27/((a*d-b*c)/c)^(1/3)*((d*x^3+c)^2*(a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*l
n((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+12/5*c*d*(b*x^3+a)^(2/3)*(-3/
2*b*c^2+d*(-9/8*b*x^3+a)*c+5/8*a*d^2*x^3)*x*((a*d-b*c)/c)^(1/3)+(d*x^3+c)^
2*(a^2*d^2-12/5*a*b*c*d+9/5*b^2*c^2)*(arctan(1/3*3^(1/2)*(-2/((a*d-b*c)/c)
^(1/3)*(b*x^3+a)^(1/3)+x)/x)*3^(1/2)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/(a*d-b*c)^2/c^3/(
d*x^3+c)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)^3}\,dx$$
$$= \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,c^2d\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,c\,d^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,d^3x^9}dx$$

input
```
int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c**3 + 3*(a + b*x**3)**(1/3)*c**2*d*x**3 + 3*(a
 + b*x**3)**(1/3)*c*d**2*x**6 + (a + b*x**3)**(1/3)*d**3*x**9),x)
```

3.167 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 366

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \frac{b(18b^2c^2+15abcd-5a^2d^2)\,x}{18ac^2(bc-ad)^3\sqrt[3]{a+bx^3}} - \frac{dx}{6c(bc-ad)\sqrt[3]{a+bx^3}\,(c+dx^3)^2} - \frac{d(12bc-5ad)x}{18c^2(bc-ad)^2\sqrt[3]{a+bx^3}\,(c+dx^3)} - \frac{d(27b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{10/3}} - \frac{d(27b^2c^2-18abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}} + \frac{d(27b^2c^2-18abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{10/3}}$$

output
```
1/18*b*(-5*a^2*d^2+15*a*b*c*d+18*b^2*c^2)*x/a/c^2/(-a*d+b*c)^3/(b*x^3+a)^(
1/3)-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(1/3)/(d*x^3+c)^2-1/18*d*(-5*a*d+12*b*
c)*x/c^2/(-a*d+b*c)^2/(b*x^3+a)^(1/3)/(d*x^3+c)-1/27*d*(5*a^2*d^2-18*a*b*c
*d+27*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))
*3^(1/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(10/3)-1/54*d*(5*a^2*d^2-18*a*b*c*d+2
7*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(10/3)+1/18*d*(5*a^2*d^2-18*a*b*
c*d+27*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a
*d+b*c)^(10/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.17 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx =$$
$$-\frac{65c^2(a+bx^3)^2\left(-14000a^2c^5-21896abc^5x^3-48104a^2c^4dx^3-8391b^2c^5x^6-70802abc^4dx^6-60807a^2c^3\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

output
```
-1/16380*(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104
*a^2*c^4*d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^
2*x^6 - 24417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^
9 - 23409*b^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12
- 7425*b^2*c^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c +
 d*x^3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*
x^3 + 33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*
x^9))*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) +
 486*(b*c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1
, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)^3*x^8*
(a + b*x^3)^(7/3)*(c + d*x^3)^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {931, 1024, 27, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-6bdx^3+6bc-5ad}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)^2}dx}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{\int \frac{d\left(a(12bc-5ad)-3b(6bc+ad)x^3\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\int \frac{a(12bc-5ad)-3b(6bc+ad)x^3}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\left(\frac{\int \frac{2a\left(27b^2c^2-18abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{x\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\left(\frac{2a\left(5a^2d^2-18abcd+27b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}-\frac{x\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}}{6c(bc-ad)}-$$

$$\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

$\downarrow$ 901

$$\frac{\frac{bx(ad+6bc)}{a\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)}-\frac{d\left(\frac{2a\left(5a^2d^2-18abcd+27b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a\ldots}\right)}{2c^{2/3}\sqrt[3]{bc-a\ldots}}\ldots\right)}{3c(bc-ad)}\ldots\right)}{a(bc-ad)}}{6c(bc-ad)}$$

$$\frac{dx}{6c\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

input

```
Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + ((b*(6*b*c +
a*d)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) - (d*(-1/3*((18*b^2*
c^2 + 15*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x
^3)) + (2*a*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c -
 a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))
/(3*c*(b*c - a*d))))/(a*(b*c - a*d)))/(6*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{\frac{5ad\left(a^2d^2-\frac{18}{5}abcd+\frac{27}{5}b^2c^2\right)\left(dx^3+c\right)^2\left(bx^3+a\right)^{\frac{1}{3}}\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{54}+\frac{5\arctan\left(\frac{\sqrt{3}\left(-\right.}{}\right)}{}}{}$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
4/9/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(-5/24*a*d*(a^2*d^2-18/5*a*b*c*d+2
7/5*b^2*c^2)*(d*x^3+c)^2*(b*x^3+a)^(1/3)*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d
-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+5/12*arctan(1/3*3^(
1/2)*(-2/((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)+x)/x)*a*d*(a^2*d^2-18/5*a*b*c
*d+27/5*b^2*c^2)*(d*x^3+c)^2*(b*x^3+a)^(1/3)*3^(1/2)+5/12*a*d*(a^2*d^2-18/
5*a*b*c*d+27/5*b^2*c^2)*(d*x^3+c)^2*(b*x^3+a)^(1/3)*ln(((((a*d-b*c)/c)^(1/3
)*x+(b*x^3+a)^(1/3))/x)+c*(5/8*a^2*x^3*(b*x^3+a)*d^4+a*(-15/8*b*x^3+a)*c*(
b*x^3+a)*d^3-9/4*b*c^2*(b^2*x^6+a*b*x^3+a^2)*d^2-9/2*b^3*c^3*d*x^3-9/4*c^4
*b^3)*x*((a*d-b*c)/c)^(1/3))/(d*x^3+c)^2/c^3/(a*d-b*c)^3/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx = \int \frac{}{(b\,x^3+a)^{\frac{1}{3}}\,a\,c^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,c^2 d\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,ac\,d^2x^6+(b\,x^3+}$$

input
```
int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*c**3 + 3*(a + b*x**3)**(1/3)*a*c**2*d*x**3 +
3*(a + b*x**3)**(1/3)*a*c*d**2*x**6 + (a + b*x**3)**(1/3)*a*d**3*x**9 + (a
 + b*x**3)**(1/3)*b*c**3*x**3 + 3*(a + b*x**3)**(1/3)*b*c**2*d*x**6 + 3*(a
 + b*x**3)**(1/3)*b*c*d**2*x**9 + (a + b*x**3)**(1/3)*b*d**3*x**12),x)
```

3.168 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 442

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{b(9b^2c^2+36abcd-10a^2d^2)\,x}{36ac^2(bc-ad)^3(a+bx^3)^{4/3}} + \frac{b(27b^3c^3-135ab^2c^2d-42a^2bcd^2+10a^3d^3)\,x}{36a^2c^2(bc-ad)^4\sqrt[3]{a+bx^3}} - \frac{dx}{6c(bc-ad)(a+bx^3)^{4/3}(c+dx^3)^2} - \frac{5d(3bc-ad)x}{18c^2(bc-ad)^2(a+bx^3)^{4/3}(c+dx^3)} + \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{13/3}} + \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{13/3}} - \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\log\left(\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{13/3}}$$

output
```
1/36*b*(-10*a^2*d^2+36*a*b*c*d+9*b^2*c^2)*x/a/c^2/(-a*d+b*c)^3/(b*x^3+a)^(
4/3)+1/36*b*(10*a^3*d^3-42*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x/a^2/c
^2/(-a*d+b*c)^4/(b*x^3+a)^(1/3)-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^(4/3)/(d*x^
3+c)^2-5/18*d*(-a*d+3*b*c)*x/c^2/(-a*d+b*c)^2/(b*x^3+a)^(4/3)/(d*x^3+c)+1/
27*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c^(8/3)/(-a*d+b*c)^(13/3)+1/54
*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(13/
3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3
)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(13/3)
```

Mathematica [A] (warning: unable to verify)

Time = 15.98 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \frac{1}{36}x(a+bx^3)^{2/3}\left(-\frac{9b^3}{a(-bc+ad)^3(a+bx^3)^2}+\frac{27b^3(bc-5ad)}{a^2(bc-ad)^4(a+bx^3)}-\frac{6d^3}{c(bc-ad)^3(c+dx^3)^2}+\frac{2d^3(-21bc+5a\ldots}{c^2(bc-ad)^4(c+\ldots}\right. + \frac{d^2(54b^2c^2-24abcd+5a^2d^2)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{b+ax^3}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{b+ax^3}}\right)+\log\left(c^{2/3}\ldots\right.\right.}{54c^{8/3}(bc-ad)^{13/3}}$$

input
```
Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

output
```
(x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^
3*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^
3*(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3
))))/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^
(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a
*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^
3)^(1/3)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {931, 1024, 27, 1024, 25, 27, 1024, 27, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-9bdx^3+6bc-5ad}{\left(bx^3+a\right)^{7/3}\left(dx^3+c\right)^2}dx}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(2ad+3bc)}{2a\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}-\frac{\int -\frac{2\left(6bd(3bc+2ad)x^3+9b^2c^2+10a^2d^2-24abcd\right)}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)^2}dx}{4a(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{6bd(3bc+2ad)x^3+9b^2c^2+10a^2d^2-24abcd}{\left(bx^3+a\right)^{4/3}\left(dx^3+c\right)^2}dx}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}-\frac{\int -\frac{d\left(3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{d\left(3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{d\int \frac{3b\left(9b^2c^2-42abdc-2a^2d^2\right)x^3+a\left(9b^2c^2+36abdc-10a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)^2}dx}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+3bc)}{2a(a+bx^3)^{4/3}(c+dx^3)(bc-ad)}}{6c(bc-ad)}-\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\Big\downarrow$ 1024

$$\frac{\frac{d\left(\frac{\int \frac{4a^2d\left(54b^2c^2-24abdc+5a^2d^2\right)}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}+\frac{x\left(a+bx^3\right)^{2/3}\left(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}+\frac{bx(2ad+}{2a(a+bx^3)^{4/3}(c+}}{6c(bc-ad)}$$

$$\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\left(\frac{4a^2d\left(5a^2d^2-24abcd+54b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^3+a}\left(dx^3+c\right)}dx}{3c(bc-ad)}+\frac{x\left(a+bx^3\right)^{2/3}\left(10a^3d^3-42a^2bcd^2-135ab^2c^2d+27b^3c^3\right)}{3c\left(c+dx^3\right)(bc-ad)}\right)}{a(bc-ad)}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}}{2a(bc-ad)}}{6c(bc-ad)}$$

$$\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

$$\Big\downarrow \text{901}$$

$$\frac{\frac{\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{a\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}+\frac{d\left(\frac{4a^2d\left(5a^2d^2-24abcd+54b^2c^2\right)\left(\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a\cdots}\right)}{2c^{2/3}\sqrt[3]{bc-ad\cdots}}\cdots\right)}{3c(bc-ad)}\cdots\right)}{a(bc-ad)}\cdots}{2a(bc-ad)}\cdots}{6c(bc-ad)}\cdots$$

$$\frac{dx}{6c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

output
```
-1/6*(d*x)/(c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)^2) + ((b*(3*b*c +
2*a*d)*x)/(2*a*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + ((b*(9*b^2*c^2
 - 42*a*b*c*d - 2*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)
) + (d*(((27*b^3*c^3 - 135*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 10*a^3*d^3)*x*(a
 + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + (4*a^2*d*(54*b^2*c^2 - 24
*a*b*c*d + 5*a^2*d^2)*(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3
]/(6*c^(2/3)*(b*c - a*d)^(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a +
 b*x^3)^(1/3)]/(2*c^(2/3)*(b*c - a*d)^(1/3))))/(3*c*(b*c - a*d))))/(a*(b*c
 - a*d)))/(2*a*(b*c - a*d)))/(6*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{-24\left(d^4\left(c+\frac{5d\,x^3}{8}\right)a^5-3d^3\left(-\frac{5}{12}d^2x^6+\frac{5}{24}cd\,x^3+c^2\right)b\,a^4-6\left(\frac{5d\,x^3}{6}+c\right)\left(-\frac{d\,x^3}{8}+c\right)d^3b^2x^3a^3-9\left(\frac{7}{12}d^2x^6+\frac{3}{2}cd\,x^3+c^2\right)\left(\right.\right.}{}$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/54/((a*d-b*c)/c)^(1/3)*(-24*(d^4*(c+5/8*d*x^3)*a^5-3*d^3*(-5/12*d^2*x^6
+5/24*c*d*x^3+c^2)*b*a^4-6*(5/6*d*x^3+c)*(-1/8*d*x^3+c)*d^3*b^2*x^3*a^3-9*
(7/12*d^2*x^6+3/2*c*d*x^3+c^2)*(1/2*d*x^3+c)*c*d*b^3*a^2+9/4*c^2*(d*x^3+c)
^2*(-15/4*d*x^3+c)*b^4*a+27/16*b^5*c^3*x^3*(d*x^3+c)^2)*c*x*((a*d-b*c)/c)^
(1/3)+a^2*d^2*(b*x^3+a)^(4/3)*(d*x^3+c)^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2
)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))
/((a*d-b*c)/c)^(1/3)/x)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a
)^(1/3))/x)))/(b*x^3+a)^(4/3)/(d*x^3+c)^2/c^3/(a*d-b*c)^4/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}\,(c+dx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(a+bx^3)^{\frac{7}{3}}(c+dx^3)^3}\,dx$$

input
```
integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
```

output
```
Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{7}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx=\int \frac{1}{(b\,x^3+a)^{7/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3}\,dx=\int \frac{}{(b\,x^3+a)^{\frac{1}{3}}\,a^2c^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2c^2d\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2c\,d^2x^6+(b\,x^3}$$

input
```
int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a**2*c**3 + 3*(a + b*x**3)**(1/3)*a**2*c**2*d*x
**3 + 3*(a + b*x**3)**(1/3)*a**2*c*d**2*x**6 + (a + b*x**3)**(1/3)*a**2*d*
*3*x**9 + 2*(a + b*x**3)**(1/3)*a*b*c**3*x**3 + 6*(a + b*x**3)**(1/3)*a*b*
c**2*d*x**6 + 6*(a + b*x**3)**(1/3)*a*b*c*d**2*x**9 + 2*(a + b*x**3)**(1/3
)*a*b*d**3*x**12 + (a + b*x**3)**(1/3)*b**2*c**3*x**6 + 3*(a + b*x**3)**(1
/3)*b**2*c**2*d*x**9 + 3*(a + b*x**3)**(1/3)*b**2*c*d**2*x**12 + (a + b*x*
*3)**(1/3)*b**2*d**3*x**15),x)
```

3.169 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^
3/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(60) = 120.

Time = 10.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} dx = \frac{x\left(b(2bc+5ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left((a+bx^3)(-bc(c-2a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

output
```
(x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + (4*c*((a + b*x^3)*(-(b*c*(c - 2*d*x^3)) + a
*d*(8*c + 5*d*x^3)) + (4*a^2*c*(b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
-((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)]))))/(c + d*x^3)^2))/(72*c^3*d*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{(dx^3+c)^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)^{3}}dx$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(4/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx=\text{too large to display}$$

input
```
int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

output
```
( - 2*(a + b*x**3)**(1/3)*a*b*x + 25*int((a + b*x**3)**(1/3)/(5*a**2*c**3*
d + 15*a**2*c**2*d**2*x**3 + 15*a**2*c*d**3*x**6 + 5*a**2*d**4*x**9 - 2*a*
b*c**4 - a*b*c**3*d*x**3 + 9*a*b*c**2*d**2*x**6 + 13*a*b*c*d**3*x**9 + 5*a
*b*d**4*x**12 - 2*b**2*c**4*x**3 - 6*b**2*c**3*d*x**6 - 6*b**2*c**2*d**2*x
**9 - 2*b**2*c*d**3*x**12),x)*a**4*c**2*d**2 + 50*int((a + b*x**3)**(1/3)/
(5*a**2*c**3*d + 15*a**2*c**2*d**2*x**3 + 15*a**2*c*d**3*x**6 + 5*a**2*d**
4*x**9 - 2*a*b*c**4 - a*b*c**3*d*x**3 + 9*a*b*c**2*d**2*x**6 + 13*a*b*c*d*
*3*x**9 + 5*a*b*d**4*x**12 - 2*b**2*c**4*x**3 - 6*b**2*c**3*d*x**6 - 6*b**
2*c**2*d**2*x**9 - 2*b**2*c*d**3*x**12),x)*a**4*c*d**3*x**3 + 25*int((a +
b*x**3)**(1/3)/(5*a**2*c**3*d + 15*a**2*c**2*d**2*x**3 + 15*a**2*c*d**3*x*
*6 + 5*a**2*d**4*x**9 - 2*a*b*c**4 - a*b*c**3*d*x**3 + 9*a*b*c**2*d**2*x**
6 + 13*a*b*c*d**3*x**9 + 5*a*b*d**4*x**12 - 2*b**2*c**4*x**3 - 6*b**2*c**3
*d*x**6 - 6*b**2*c**2*d**2*x**9 - 2*b**2*c*d**3*x**12),x)*a**4*d**4*x**6 -
 10*int((a + b*x**3)**(1/3)/(5*a**2*c**3*d + 15*a**2*c**2*d**2*x**3 + 15*a
**2*c*d**3*x**6 + 5*a**2*d**4*x**9 - 2*a*b*c**4 - a*b*c**3*d*x**3 + 9*a*b*
c**2*d**2*x**6 + 13*a*b*c*d**3*x**9 + 5*a*b*d**4*x**12 - 2*b**2*c**4*x**3
- 6*b**2*c**3*d*x**6 - 6*b**2*c**2*d**2*x**9 - 2*b**2*c*d**3*x**12),x)*a**
3*b*c**3*d - 20*int((a + b*x**3)**(1/3)/(5*a**2*c**3*d + 15*a**2*c**2*d**2
*x**3 + 15*a**2*c*d**3*x**6 + 5*a**2*d**4*x**9 - 2*a*b*c**4 - a*b*c**3*d*x
**3 + 9*a*b*c**2*d**2*x**6 + 13*a*b*c*d**3*x**9 + 5*a*b*d**4*x**12 - 2*...
```

3.170 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx=\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 407 vs. $2(59) = 118$.

Time = 10.65 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$$
$$=\frac{-b(-4bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16acx\left(-b^2cx^3(7c+4dx^3)+3a^2d(6c+5dx^3)+a\ldots\right.\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

output
```
(-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3))
 + 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(a + b*x^3)*(-(b*
c*(7*c + 4*d*x^3)) + a*d*(8*c + 5*d*x^3))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -(
(b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3
)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*
x^3)/c)]))))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^3} \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{\left(dx^3+c\right)^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^3} \text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)^{3}}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x**3)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d^3x^9+3c\,d^2x^6+3c^2d\,x^3+c^3}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

output
```
int((a + b*x**3)**(1/3)/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x**6 + d**3*x**9)
,x)
```

3.171 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a
)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(59) = 118.

Time = 10.79 (sec) , antiderivative size = 418, normalized size of antiderivative = 7.08

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3} dx = \frac{x\left(5bd(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c(-}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

output
```
(x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3)
+ b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 +
5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3
*(a + b*x^3)*(a*d*(8*c + 5*d*x^3) - b*c*(13*c + 10*d*x^3))*(3*a*d*AppellF1
[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3,
 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(72*c^3*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}(dx^3+c)^3}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{2}{3}}\,(d\,x^{3}+c)^{3}}dx$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\left(d\,x^3+c\right)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}c^3+3\left(b\,x^3+a\right)^{\frac{2}{3}}c^2d\,x^3+3\left(b\,x^3+a\right)^{\frac{2}{3}}c\,d^2x^6+\left(b\,x^3+a\right)^{\frac{2}{3}}}$$

input
```
int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c**3 + 3*(a + b*x**3)**(2/3)*c**2*d*x**3 + 3*(a
 + b*x**3)**(2/3)*c*d**2*x**6 + (a + b*x**3)**(2/3)*d**3*x**9),x)
```

3.172 $\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\,(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3
+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 531 vs. $2(62) = 124$.

Time = 11.00 (sec) , antiderivative size = 531, normalized size of antiderivative = 8.56

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \frac{\frac{bd(-9b^2c^2-16abcd+5a^2d^2)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(-bc+ad)^3} - \frac{4c\left(4acx\left(3a^3d^3(6c+\right.\right.}{}}{}$$

input
```
Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

output
```
((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appe
llF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*
c*(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 1
6*d^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2
- 43*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c
*d^2*x^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(
3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Appel
lF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d
*x^3)^2*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^
3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*
x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{5/3}(dx^3+c)^3}dx}{a(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{5}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(d\,x^{3}+c\right)^{3}}dx$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx=\int \frac{}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,a\,c^3+3\left(b\,x^3+a\right)^{\frac{2}{3}}\,a\,c^2 d\,x^3+3\left(b\,x^3+a\right)^{\frac{2}{3}}\,ac\,d^2x^6+\left(b\,x^3+\right.}$$

input
```
int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a*c**3 + 3*(a + b*x**3)**(2/3)*a*c**2*d*x**3 +
3*(a + b*x**3)**(2/3)*a*c*d**2*x**6 + (a + b*x**3)**(2/3)*a*d**3*x**9 + (a
 + b*x**3)**(2/3)*b*c**3*x**3 + 3*(a + b*x**3)**(2/3)*b*c**2*d*x**6 + 3*(a
 + b*x**3)**(2/3)*b*c*d**2*x**9 + (a + b*x**3)**(2/3)*b*d**3*x**12),x)
```

3.173 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x
^3+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. 2(62) = 124.

Time = 11.69 (sec) , antiderivative size = 515, normalized size of antiderivative = 8.31

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \frac{x\left(bd(36b^3c^3-171ab^2c^2d-110a^2bcd^2+25a^3d^3)\,x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\right.}{}$$

input
```
Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

output
```
(x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*
(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c
 + d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*
c*d*x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^
2*b^3*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*
x^3) + (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^
3*b*c*d^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*
x^3)^2))/(360*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{8/3}(dx^3+c)^3}dx}{a^2(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{8}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3(a+bx^3)^{2/3}}$$

input
```
Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{8}{3}}\,(d\,x^{3}+c)^{3}}dx$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

output
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}\,(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(a+bx^3)^{\frac{8}{3}}(c+dx^3)^3}\,dx$$

input
```
integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

output
```
Integral(1/((a + b*x**3)**(8/3)*(c + d*x**3)**3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx = \int \frac{1}{(bx^3+a)^{\frac{8}{3}}(dx^3+c)^3}\,dx$$

input
```
integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx=\int \frac{1}{(b\,x^3+a)^{8/3}\,(d\,x^3+c)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x)
```

output
```
int(1/((a + b*x^3)^(8/3)*(c + d*x^3)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx=\int \frac{}{(b\,x^3+a)^{\frac{2}{3}}\,a^2c^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a^2c^2d\,x^3+3\,(b\,x^3+a)^{\frac{2}{3}}\,a^2c\,d^2x^6+(b\,x^3}$$

input
```
int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

output
```
int(1/((a + b*x**3)**(2/3)*a**2*c**3 + 3*(a + b*x**3)**(2/3)*a**2*c**2*d*x
**3 + 3*(a + b*x**3)**(2/3)*a**2*c*d**2*x**6 + (a + b*x**3)**(2/3)*a**2*d*
*3*x**9 + 2*(a + b*x**3)**(2/3)*a*b*c**3*x**3 + 6*(a + b*x**3)**(2/3)*a*b*
c**2*d*x**6 + 6*(a + b*x**3)**(2/3)*a*b*c*d**2*x**9 + 2*(a + b*x**3)**(2/3
)*a*b*d**3*x**12 + (a + b*x**3)**(2/3)*b**2*c**3*x**6 + 3*(a + b*x**3)**(2
/3)*b**2*c**2*d*x**9 + 3*(a + b*x**3)**(2/3)*b**2*c*d**2*x**12 + (a + b*x*
*3)**(2/3)*b**2*d**3*x**15),x)
```

3.174 $\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx=\frac{x(a+bx^3)^{5/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}(c+dx^3)^{19/12}}$$

output
```
x*(b*x^3+a)^(5/4)*hypergeom([-5/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))
/c/(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)/(d*x^3+c)^(19/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx=\frac{ax\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\sqrt[4]{1+\frac{bx^3}{a}}\,(c+dx^3)^{7/12}}$$

input
```
Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

output
```
(a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c
+ d*x^3)^(7/12))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.85, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx$$

$$\Big\downarrow \text{903}$$

$$\frac{15a\int \frac{\sqrt[4]{bx^3+a}}{(dx^3+c)^{19/12}}dx}{19c}+\frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

$$\Big\downarrow \text{903}$$

$$\frac{15a\left(\frac{3a\int \frac{1}{(bx^3+a)^{3/4}(dx^3+c)^{7/12}}dx}{7c}+\frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}}\right)}{19c}+\frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

$$\Big\downarrow \text{905}$$

$$\frac{15a\left(\frac{3ax(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{7c^2(a+bx^3)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}}\right)}{19c}+\frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

input
```
Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

output
```
(4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (15*a*((4*x*(a + b*x^
3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/(a*(c + d*x^3
)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))))/(19*c)
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{5}{4}}}{(d\,x^3+c)^{\frac{31}{12}}}dx$$

input
```
int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

output
```
int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(bx^3+a)^{\frac{5}{4}}}{(dx^3+c)^{\frac{31}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \int \frac{(b\,x^3+a)^{5/4}}{(d\,x^3+c)^{31/12}}\,dx$$

input
```
int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)
```

output
```
int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}}\,dx = \text{too large to display}$$

input
```
int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

output

```
( - 68*(c + d*x**3)**(1/4)*(a + b*x**3)**(1/4)*a**3*d**3*x - 164*(c + d*x*
*3)**(1/4)*(a + b*x**3)**(1/4)*a**2*b*c*d**2*x - 136*(c + d*x**3)**(1/4)*(
a + b*x**3)**(1/4)*a**2*b*d**3*x**4 - 20*(c + d*x**3)**(1/4)*(a + b*x**3)*
*(1/4)*a*b**2*c**2*d*x - 4*(c + d*x**3)**(1/4)*(a + b*x**3)**(1/4)*a*b**2*
c*d**2*x**4 - 28*(c + d*x**3)**(1/4)*(a + b*x**3)**(1/4)*b**3*c**3*x - 28*
(c + d*x**3)**(1/4)*(a + b*x**3)**(1/4)*b**3*c**2*d*x**4 + 63869*(c + d*x*
*3)**(5/6)*int((a + b*x**3)**(1/4)/(221*(c + d*x**3)**(7/12)*a**3*c**2*d**
2 + 442*(c + d*x**3)**(7/12)*a**3*c*d**3*x**3 + 221*(c + d*x**3)**(7/12)*a
**3*d**4*x**6 - 28*(c + d*x**3)**(7/12)*a**2*b*c**3*d + 165*(c + d*x**3)**
(7/12)*a**2*b*c**2*d**2*x**3 + 414*(c + d*x**3)**(7/12)*a**2*b*c*d**3*x**6
 + 221*(c + d*x**3)**(7/12)*a**2*b*d**4*x**9 - 49*(c + d*x**3)**(7/12)*a*b
**2*c**4 - 126*(c + d*x**3)**(7/12)*a*b**2*c**3*d*x**3 - 105*(c + d*x**3)*
*(7/12)*a*b**2*c**2*d**2*x**6 - 28*(c + d*x**3)**(7/12)*a*b**2*c*d**3*x**9
 - 49*(c + d*x**3)**(7/12)*b**3*c**4*x**3 - 98*(c + d*x**3)**(7/12)*b**3*c
**3*d*x**6 - 49*(c + d*x**3)**(7/12)*b**3*c**2*d**2*x**9),x)*a**6*c**2*d**
5 + 63869*(c + d*x**3)**(5/6)*int((a + b*x**3)**(1/4)/(221*(c + d*x**3)**(
7/12)*a**3*c**2*d**2 + 442*(c + d*x**3)**(7/12)*a**3*c*d**3*x**3 + 221*(c
+ d*x**3)**(7/12)*a**3*d**4*x**6 - 28*(c + d*x**3)**(7/12)*a**2*b*c**3*d +
 165*(c + d*x**3)**(7/12)*a**2*b*c**2*d**2*x**3 + 414*(c + d*x**3)**(7/12)
*a**2*b*c*d**3*x**6 + 221*(c + d*x**3)**(7/12)*a**2*b*d**4*x**9 - 49*(c...
```

3.175 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx = \frac{x(a+bx^3)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}(c+dx^3)^{13/12}}$$

output
```
x*(b*x^3+a)^(3/4)*hypergeom([-3/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))
/c/(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)/(d*x^3+c)^(13/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx = \frac{x(a+bx^3)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

input
```
Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

output
```
(x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3
)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$$

$$\Big\downarrow \; 903$$

$$\frac{9a\int \frac{1}{\sqrt[4]{bx^3+a}(dx^3+c)^{13/12}}dx}{13c}+\frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

$$\Big\downarrow \; 905$$

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}+\frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

input
```
Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

output
```
(4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3
))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*
x^3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{3}{4}}}{(d\,x^{3}+c)^{\frac{25}{12}}}dx$$

input
```
int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

output
```
int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*
c^2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx = \int \frac{(bx^3+a)^{\frac{3}{4}}}{(dx^3+c)^{\frac{25}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx=\int \frac{(b\,x^3+a)^{3/4}}{(d\,x^3+c)^{25/12}}\,dx$$

input
```
int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x)
```

output
```
int((a + b*x^3)^(3/4)/(c + d*x^3)^(25/12), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx=\text{too large to display}$$

input
```
int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

output

```
( - 4*(c + d*x**3)**(3/4)*(a + b*x**3)**(3/4)*b*x - 136*(c + d*x**3)**(5/6
)*int(((c + d*x**3)**(2/3)*(a + b*x**3)**(3/4)*x**6)/(17*(c + d*x**3)**(3/
4)*a**2*c**2*d + 34*(c + d*x**3)**(3/4)*a**2*c*d**2*x**3 + 17*(c + d*x**3)
**(3/4)*a**2*d**3*x**6 - 5*(c + d*x**3)**(3/4)*a*b*c**3 + 7*(c + d*x**3)**
(3/4)*a*b*c**2*d*x**3 + 29*(c + d*x**3)**(3/4)*a*b*c*d**2*x**6 + 17*(c + d
*x**3)**(3/4)*a*b*d**3*x**9 - 5*(c + d*x**3)**(3/4)*b**2*c**3*x**3 - 10*(c
 + d*x**3)**(3/4)*b**2*c**2*d*x**6 - 5*(c + d*x**3)**(3/4)*b**2*c*d**2*x**
9),x)*a*b**2*c*d**2 - 136*(c + d*x**3)**(5/6)*int(((c + d*x**3)**(2/3)*(a
+ b*x**3)**(3/4)*x**6)/(17*(c + d*x**3)**(3/4)*a**2*c**2*d + 34*(c + d*x**
3)**(3/4)*a**2*c*d**2*x**3 + 17*(c + d*x**3)**(3/4)*a**2*d**3*x**6 - 5*(c
+ d*x**3)**(3/4)*a*b*c**3 + 7*(c + d*x**3)**(3/4)*a*b*c**2*d*x**3 + 29*(c
+ d*x**3)**(3/4)*a*b*c*d**2*x**6 + 17*(c + d*x**3)**(3/4)*a*b*d**3*x**9 -
5*(c + d*x**3)**(3/4)*b**2*c**3*x**3 - 10*(c + d*x**3)**(3/4)*b**2*c**2*d*
x**6 - 5*(c + d*x**3)**(3/4)*b**2*c*d**2*x**9),x)*a*b**2*d**3*x**3 + 40*(c
 + d*x**3)**(5/6)*int(((c + d*x**3)**(2/3)*(a + b*x**3)**(3/4)*x**6)/(17*(
c + d*x**3)**(3/4)*a**2*c**2*d + 34*(c + d*x**3)**(3/4)*a**2*c*d**2*x**3 +
 17*(c + d*x**3)**(3/4)*a**2*d**3*x**6 - 5*(c + d*x**3)**(3/4)*a*b*c**3 +
7*(c + d*x**3)**(3/4)*a*b*c**2*d*x**3 + 29*(c + d*x**3)**(3/4)*a*b*c*d**2*
x**6 + 17*(c + d*x**3)**(3/4)*a*b*d**3*x**9 - 5*(c + d*x**3)**(3/4)*b**2*c
**3*x**3 - 10*(c + d*x**3)**(3/4)*b**2*c**2*d*x**6 - 5*(c + d*x**3)**(3...
```

3.176 $\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx = \frac{x\sqrt[4]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\left(c+dx^3\right)^{7/12}}$$

output
```
x*(b*x^3+a)^(1/4)*hypergeom([-1/4, 1/3],[4/3],-(-a*d+b*c)*x^3/a/(d*x^3+c))
/c/(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)/(d*x^3+c)^(7/12)
```

Mathematica [A] (warning: unable to verify)

Time = 3.86 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx = \frac{x\sqrt[4]{a+bx^3}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{1+\frac{bx^3}{a}}\left(c+dx^3\right)^{7/12}}$$

input
```
Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

output
```
(x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*
x^3)^(7/12))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx$$

$$\Big\downarrow \text{ 903}$$

$$\frac{3a\int \frac{1}{(bx^3+a)^{3/4}(dx^3+c)^{7/12}}dx}{7c}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

$$\Big\downarrow \text{ 905}$$

$$\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

input
```
Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

output
```
(4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{4}}}{\left(d\,x^{3}+c\right)^{\frac{19}{12}}}d x$$

input
```
int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

output
```
int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
 x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
```

output
```
Integral((a + b*x**3)**(1/4)/(c + d*x**3)**(19/12), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{4}}}{(dx^3+c)^{\frac{19}{12}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/4}}{\left(d\,x^3+c\right)^{19/12}}\,dx$$

input
```
int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x)
```

output
```
int((a + b*x^3)^(1/4)/(c + d*x^3)^(19/12), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}}\,dx=\int \frac{\left(b\,x^3+a\right)^{\frac{1}{4}}}{\left(d\,x^3+c\right)^{\frac{7}{12}}c+\left(d\,x^3+c\right)^{\frac{7}{12}}d\,x^3}dx$$

input
```
int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

output
```
int((a + b*x**3)**(1/4)/((c + d*x**3)**(7/12)*c + (c + d*x**3)**(7/12)*d*x
**3),x)
```

3.177 $\int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

output
```
x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3],[4/3],-(-a*d+b*c)*x
^3/a/(d*x^3+c))/c/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

Mathematica [A] (warning: unable to verify)

Time = 3.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\left(1+\frac{dx^3}{c}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}$$

input
```
Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3
)^(13/12))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx$$

$$\Big\downarrow \text{905}$$

$$\frac{x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

input
```
Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^
(1/12))
```

Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{1}{4}}\,(d\,x^3+c)^{\frac{13}{12}}}dx$$

input
```
int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

output
```
int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d
^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
```

output
```
Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{4}}(dx^3+c)^{\frac{13}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/4}\,(d\,x^3+c)^{13/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x)
```

output
```
int(1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^3}\,(c+dx^3)^{13/12}}\,dx = \int \frac{1}{(d\,x^3+c)^{\frac{1}{12}}\,(b\,x^3+a)^{\frac{1}{4}}\,c+(d\,x^3+c)^{\frac{1}{12}}\,(b\,x^3+a)^{\frac{1}{4}}\,d\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

output
```
int(1/((c + d*x**3)**(1/12)*(a + b*x**3)**(1/4)*c + (c + d*x**3)**(1/12)*(
a + b*x**3)**(1/4)*d*x**3),x)
```

3.178 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx=\frac{x\sqrt[4]{a+bx^3}\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{7/12}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{a(c+dx^3)^{7/12}}$$

output
```
x*(b*x^3+a)^(1/4)*(a*(d*x^3+c)/c/(b*x^3+a))^(7/12)*hypergeom([1/3, 7/12],[
4/3],(-a*d+b*c)*x^3/c/(b*x^3+a))/a/(d*x^3+c)^(7/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx=\frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}\sqrt[4]{1+\frac{dx^3}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}$$

input
```
Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3
)^(7/12))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}dx$$

$$\Big\downarrow \text{905}$$

$$\frac{x(c+dx^3)^{5/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{3/4}}$$

input
```
Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometr
ic2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)
^(3/4))
```

Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a)^{\frac{3}{4}}\,(d\,x^{3}+c)^{\frac{7}{12}}}dx$$

input
```
int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

output
```
int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{3/4}\,(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
 a*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{3/4}\,(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{3}{4}}\,(c+dx^3)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
```

output
```
Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{4}}(dx^3+c)^{\frac{7}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(b\,x^3+a)^{3/4}\,(d\,x^3+c)^{7/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x)
```

output
```
int(1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx = \int \frac{1}{(d\,x^3+c)^{\frac{7}{12}}(b\,x^3+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

output
```
int(1/((c + d*x**3)**(7/12)*(a + b*x**3)**(3/4)),x)
```

3.179 $\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x\sqrt[12]{\frac{a(c+dx^3)}{c(a+bx^3)}}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{3},\frac{4}{3},\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{a\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

output
```
x*(a*(d*x^3+c)/c/(b*x^3+a))^(1/12)*hypergeom([1/12, 1/3],[4/3],(-a*d+b*c)*
x^3/c/(b*x^3+a))/a/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)
```

Mathematica [A] (warning: unable to verify)

Time = 3.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}\sqrt[4]{1+\frac{dx^3}{c}}}$$

input
```
Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*
x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3
)/c)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx$$

$$\Big\downarrow \; 905$$

$$\frac{x\left(c+dx^3\right)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c\left(a+bx^3\right)^{5/4}}$$

input
```
Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

output
```
(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeomet
ric2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3
)^(5/4))
```

Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{5}{4}} \left(d\,x^{3}+c\right)^{\frac{1}{12}}}dx$$

input
```
int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

output
```
int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\ \sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b
*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\ \sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{5}{4}}\ \sqrt[12]{c+dx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

output
```
Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{5}{4}}(dx^3+c)^{\frac{1}{12}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{5/4}\,(d\,x^3+c)^{1/12}}\,dx$$

input
```
int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x)
```

output
```
int(1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{5/4}\sqrt[12]{c+dx^3}}\,dx = \int \frac{1}{(d\,x^3+c)^{\frac{1}{12}}\,(b\,x^3+a)^{\frac{1}{4}}\,a+(d\,x^3+c)^{\frac{1}{12}}\,(b\,x^3+a)^{\frac{1}{4}}\,b\,x^3}dx$$

input
```
int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

output
```
int(1/((c + d*x**3)**(1/12)*(a + b*x**3)**(1/4)*a + (c + d*x**3)**(1/12)*(
a + b*x**3)**(1/4)*b*x**3),x)
```

3.180 $\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \frac{x(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(-\frac{5}{12},\frac{1}{3},\frac{4}{3},\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{a\,(a+bx^3)^{3/4}\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{5/12}}$$

output
```
x*(d*x^3+c)^(5/12)*hypergeom([-5/12, 1/3],[4/3],(-a*d+b*c)*x^3/c/(b*x^3+a)
)/a/(b*x^3+a)^(3/4)/(a*(d*x^3+c)/c/(b*x^3+a))^(5/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a\,(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

input
```
Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

output
```
(x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3
)/c)^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx$$

$$\Big\downarrow \text{903}$$

$$\frac{5c\int \frac{1}{\left(bx^3+a\right)^{3/4}\left(dx^3+c\right)^{7/12}}dx}{9a}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

input
```
Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

output
```
(4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(
a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{5}{12}}}{(b\,x^{3}+a)^{\frac{7}{4}}}dx$$

input
```
int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

output
```
int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
 x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(c+dx^3)^{\frac{5}{12}}}{(a+bx^3)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
```

output
```
Integral((c + d*x**3)**(5/12)/(a + b*x**3)**(7/4), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(dx^3+c)^{\frac{5}{12}}}{(bx^3+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(d\,x^3+c)^{5/12}}{(b\,x^3+a)^{7/4}}\,dx$$

input
```
int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x)
```

output
```
int((c + d*x^3)^(5/12)/(a + b*x^3)^(7/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}}\,dx = \int \frac{(d\,x^3+c)^{\frac{5}{12}}}{(b\,x^3+a)^{\frac{3}{4}}\,a+(b\,x^3+a)^{\frac{3}{4}}\,b\,x^3}dx$$

input
```
int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

output
```
int((c + d*x**3)**(5/12)/((a + b*x**3)**(3/4)*a + (a + b*x**3)**(3/4)*b*x*
*3),x)
```

3.181 $\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{x(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(-\frac{11}{12},\frac{1}{3},\frac{4}{3},\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{a\,(a+bx^3)^{5/4}\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{11/12}}$$

output
```
x*(d*x^3+c)^(11/12)*hypergeom([-11/12, 1/3],[4/3],(-a*d+b*c)*x^3/c/(b*x^3+
a))/a/(b*x^3+a)^(5/4)/(a*(d*x^3+c)/c/(b*x^3+a))^(11/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \frac{x\sqrt[4]{1+\frac{bx^3}{a}}(c+dx^3)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{9}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2\sqrt[4]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{5/4}}$$

input
```
Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

output
```
(x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*
x^3)/c)^(5/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{11c\int \frac{1}{(bx^3+a)^{5/4}\sqrt[12]{dx^3+c}}dx}{15a}+\frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{11x(c+dx^3)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{15a(a+bx^3)^{5/4}}+\frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}$$

input
```
Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

output
```
(4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4
/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(d\,x^{3}+c)^{\frac{11}{12}}}{(b\,x^{3}+a)^{\frac{9}{4}}}dx$$

input
```
int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

output
```
int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*
a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(dx^3+c)^{\frac{11}{12}}}{(bx^3+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(d\,x^3+c)^{11/12}}{(b\,x^3+a)^{9/4}}\,dx$$

input
```
int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x)
```

output
```
int((c + d*x^3)^(11/12)/(a + b*x^3)^(9/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}}\,dx = \int \frac{(d\,x^3+c)^{\frac{11}{12}}}{(b\,x^3+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{4}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{4}}\,b^2x^6}dx$$

input
```
int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

output
```
int((c + d*x**3)**(11/12)/((a + b*x**3)**(1/4)*a**2 + 2*(a + b*x**3)**(1/4
)*a*b*x**3 + (a + b*x**3)**(1/4)*b**2*x**6),x)
```

3.182 $\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{x(c+dx^3)^{17/12} \text{Hypergeometric2F1}\left(-\frac{17}{12},\frac{1}{3},\frac{4}{3},\frac{(bc-ad)x^3}{c(a+bx^3)}\right)}{a(a+bx^3)^{7/4}\left(\frac{a(c+dx^3)}{c(a+bx^3)}\right)^{17/12}}$$

output
```
x*(d*x^3+c)^(17/12)*hypergeom([-17/12, 1/3],[4/3],(-a*d+b*c)*x^3/c/(b*x^3+
a))/a/(b*x^3+a)^(7/4)/(a*(d*x^3+c)/c/(b*x^3+a))^(17/12)
```

Mathematica [A] (warning: unable to verify)

Time = 5.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} \, dx = \frac{cx\left(1+\frac{bx^3}{a}\right)^{3/4}(c+dx^3)^{5/12} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{4},\frac{4}{3},\frac{(-bc+ad)x^3}{a(c+dx^3)}\right)}{a^2(a+bx^3)^{3/4}\left(1+\frac{dx^3}{c}\right)^{3/4}}$$

input
```
Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

output
```
(c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (
d*x^3)/c)^(3/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {903, 903, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{17c\int \frac{\left(dx^3+c\right)^{5/12}}{\left(bx^3+a\right)^{7/4}}dx}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{903}$$

$$\frac{17c\left(\frac{5c\int \frac{1}{\left(bx^3+a\right)^{3/4}\left(dx^3+c\right)^{7/12}}dx}{9a}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}\right)}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{905}$$

$$\frac{17c\left(\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{4}{3},-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}\right)}{21a}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

input
```
Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

output
```
(4*x*(c + d*x^3)^(17/12))/(21*a*(a + b*x^3)^(7/4)) + (17*c*((4*x*(c + d*x^
3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a*(c + d*x^3))
)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))))/(21*a)
```

Defintions of rubi rules used

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(d\,x^3+c)^{\frac{17}{12}}}{(b\,x^3+a)^{\frac{11}{4}}}dx$$

input
```
int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

output
```
int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*
a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(dx^3+c)^{\frac{17}{12}}}{(bx^3+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \int \frac{(d\,x^3+c)^{17/12}}{(b\,x^3+a)^{11/4}}\,dx$$

input
```
int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)
```

output
```
int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}}\,dx = \left(\int \frac{(d\,x^3+c)^{\frac{5}{12}}}{(b\,x^3+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^3+a)^{\frac{3}{4}}\,ab\,x^3+(b\,x^3+a)^{\frac{3}{4}}\,b^2x^6}dx\right)c + \left(\int \frac{(d\,x^3+c)^{\frac{5}{12}}\,x^3}{(b\,x^3+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^3+a)^{\frac{3}{4}}\,ab\,x^3+(b\,x^3+a)^{\frac{3}{4}}\,b^2x^6}dx\right)d$$

input
```
int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

output
```
int((c + d*x**3)**(5/12)/((a + b*x**3)**(3/4)*a**2 + 2*(a + b*x**3)**(3/4)
*a*b*x**3 + (a + b*x**3)**(3/4)*b**2*x**6),x)*c + int(((c + d*x**3)**(5/12
)*x**3)/((a + b*x**3)**(3/4)*a**2 + 2*(a + b*x**3)**(3/4)*a*b*x**3 + (a +
b*x**3)**(3/4)*b**2*x**6),x)*d
```

3.183 $\int (a+bx^3)^m (c+dx^3)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} (c+dx^3)^p \left(1+\frac{dx^3}{c}\right)^{-p} \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

output
```
x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3,-m,-p,4/3,-b*x^3/a,-d*x^3/c)/((1+b*
x^3/a)^m)/((1+d*x^3/c)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^3)^m (c+dx^3)^p \, dx = \frac{4acx(a+bx^3)^m (c+dx^3)^p \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\,\text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(bcm\,\text{AppellF1}\left(\frac{4}{3},1-m,-p,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adp\,\text{App}\right.}$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

output
```
(4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/
a), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^
3)/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx$$

$$\Big\downarrow \text{937}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \int \left(\frac{bx^3}{a}+1\right)^m \left(dx^3+c\right)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \int \left(\frac{bx^3}{a}+1\right)^m \left(\frac{dx^3}{c}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(c+dx^3\right)^p \left(\frac{dx^3}{c}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{3},-m,-p,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

output
```
(x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -(
(d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{p}d x$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx=\int \left(bx^{3}+a\right)^{m}\left(dx^{3}+c\right)^{p}\,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(bx^3+a\right)^m \left(dx^3+c\right)^p \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^p \, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^p,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^p, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

output
```
((c + d*x**3)**p*(a + b*x**3)**m*x + 9*int(((c + d*x**3)**p*(a + b*x**3)**
m*x**3)/(3*a*c*m + 3*a*c*p + a*c + 3*a*d*m*x**3 + 3*a*d*p*x**3 + a*d*x**3
+ 3*b*c*m*x**3 + 3*b*c*p*x**3 + b*c*x**3 + 3*b*d*m*x**6 + 3*b*d*p*x**6 + b
*d*x**6),x)*a*d*m**2 + 9*int(((c + d*x**3)**p*(a + b*x**3)**m*x**3)/(3*a*c
*m + 3*a*c*p + a*c + 3*a*d*m*x**3 + 3*a*d*p*x**3 + a*d*x**3 + 3*b*c*m*x**3
 + 3*b*c*p*x**3 + b*c*x**3 + 3*b*d*m*x**6 + 3*b*d*p*x**6 + b*d*x**6),x)*a*
d*m*p + 3*int(((c + d*x**3)**p*(a + b*x**3)**m*x**3)/(3*a*c*m + 3*a*c*p +
a*c + 3*a*d*m*x**3 + 3*a*d*p*x**3 + a*d*x**3 + 3*b*c*m*x**3 + 3*b*c*p*x**3
 + b*c*x**3 + 3*b*d*m*x**6 + 3*b*d*p*x**6 + b*d*x**6),x)*a*d*m + 9*int(((c
 + d*x**3)**p*(a + b*x**3)**m*x**3)/(3*a*c*m + 3*a*c*p + a*c + 3*a*d*m*x**
3 + 3*a*d*p*x**3 + a*d*x**3 + 3*b*c*m*x**3 + 3*b*c*p*x**3 + b*c*x**3 + 3*b
*d*m*x**6 + 3*b*d*p*x**6 + b*d*x**6),x)*b*c*m*p + 9*int(((c + d*x**3)**p*(
a + b*x**3)**m*x**3)/(3*a*c*m + 3*a*c*p + a*c + 3*a*d*m*x**3 + 3*a*d*p*x**
3 + a*d*x**3 + 3*b*c*m*x**3 + 3*b*c*p*x**3 + b*c*x**3 + 3*b*d*m*x**6 + 3*b
*d*p*x**6 + b*d*x**6),x)*b*c*p**2 + 3*int(((c + d*x**3)**p*(a + b*x**3)**m
*x**3)/(3*a*c*m + 3*a*c*p + a*c + 3*a*d*m*x**3 + 3*a*d*p*x**3 + a*d*x**3 +
 3*b*c*m*x**3 + 3*b*c*p*x**3 + b*c*x**3 + 3*b*d*m*x**6 + 3*b*d*p*x**6 + b*
d*x**6),x)*b*c*p + 9*int(((c + d*x**3)**p*(a + b*x**3)**m)/(3*a*c*m + 3*a*
c*p + a*c + 3*a*d*m*x**3 + 3*a*d*p*x**3 + a*d*x**3 + 3*b*c*m*x**3 + 3*b*c*
p*x**3 + b*c*x**3 + 3*b*d*m*x**6 + 3*b*d*p*x**6 + b*d*x**6),x)*a*c*m**2...
```

3.184 $\int (a+bx^3)^m (c+dx^3)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\begin{aligned}
&\int (a+bx^3)^m (c+dx^3)^3 \, dx \\
&= \frac{d(28a^2d^2-12abcd(10+3m)+3b^2c^2(70+51m+9m^2))\,x(a+bx^3)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} \\
&\quad - \frac{d^2(7ad-3bc(10+3m))x^4(a+bx^3)^{1+m}}{b^2(7+3m)(10+3m)} + \frac{d^3x^7(a+bx^3)^{1+m}}{b(10+3m)} \\
&\quad + \frac{\left(b^3c^3(70+51m+9m^2) - \frac{ad(28a^2d^2-12abcd(10+3m)+3b^2c^2(70+51m+9m^2))}{4+3m}\right) x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{Hyperg}}{b^3(7+3m)(10+3m)}
\end{aligned}$$

output
```
d*(28*a^2*d^2-12*a*b*c*d*(10+3*m)+3*b^2*c^2*(9*m^2+51*m+70))*x*(b*x^3+a)^(
1+m)/b^3/(4+3*m)/(7+3*m)/(10+3*m)-d^2*(7*a*d-3*b*c*(10+3*m))*x^4*(b*x^3+a)
^(1+m)/b^2/(7+3*m)/(10+3*m)+d^3*x^7*(b*x^3+a)^(1+m)/b/(10+3*m)+(b^3*c^3*(9
*m^2+51*m+70)-a*d*(28*a^2*d^2-12*a*b*c*d*(10+3*m)+3*b^2*c^2*(9*m^2+51*m+70
))/(4+3*m))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^3/(7+3*m)/
(10+3*m)/((1+b*x^3/a)^m)
```

Mathematica [A] (verified)

Time = 7.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.48

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 dx$$
$$= \frac{1}{140}x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(140c^3\,\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right) + dx^3\left(105c^2\,\text{Hypergeometric2F1}\left(\frac{4}{3},-m,\frac{7}{3},-\frac{bx^3}{a}\right) +2dx^3\left(30c\,\text{Hypergeometric2F1}\left(\frac{7}{3},-m,\frac{10}{3},-\frac{bx^3}{a}\right)+7dx^3\,\text{Hypergeometric2F1}\left(\frac{10}{3},-m,\frac{13}{3},-\frac{bx^3}{a}\right)\right.\right.\right.$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] +
d*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30
*c*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric
2F1[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^3 \left(a+bx^3\right)^m dx$$
$$\Big\downarrow \; 933$$
$$\frac{\int -\left(bx^3+a\right)^m \left(dx^3+c\right) \left(d(7ad-bc(3m+16))x^3+c(ad-bc(3m+10))\right) dx}{b(3m+10)}+\frac{dx\left(c+dx^3\right)^2 \left(a+bx^3\right)^{m+1}}{b(3m+10)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\int\left(bx^3+a\right)^m\left(dx^3+c\right)\left(d(7ad-bc(3m+16))x^3+c(ad-bc(3m+10))\right)dx}{b(3m+10)}$$

$$\Big\downarrow \; 1025$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{\int -\left(bx^3+a\right)^m\left(d\left(b^2\left(9m^2+60m+118\right)c^2-abd(15m+92)c+28a^2d^2\right)x^3+c\left(b^2\left(9m^2+51m+70\right)c^2-abd(6m+23)c+7a^2d^2\right)\right)dx}{b(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+b\right.}{}}{b(3m+10)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\int\left(bx^3+a\right)^m\left(d\left(b^2\left(9m^2+60m+118\right)c^2-abd(15m+92)c+28a^2d^2\right)x^3+c\left(b^2\left(9m^2+51m+70\right)c^2-ab\right.}{b(3m+7)}}{b(3m+10)}$$

$$\Big\downarrow \; 913$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{\left(28a^3d^3-12a^2bcd^2(3m+10)+3ab^2c^2\right.}{}}{b(3m+7)}}{b(3m+10)}$$

$$\Big\downarrow \; 779$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-1\right.}{}}{b(3m+7)}}{b(3m+10)}$$

$$\Big\downarrow \; 778$$

$$\frac{dx\left(c+dx^3\right)^2\left(a+bx^3\right)^{m+1}}{b(3m+10)}-\frac{\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}(7ad-bc(3m+16))}{b(3m+7)}-\frac{\frac{dx\left(a+bx^3\right)^{m+1}\left(28a^2d^2-abcd(15m+92)+b^2c^2\left(9m^2+60m+118\right)\right)}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(28a^3d^3-\right.}{}}{}}{b(3m+10)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

output
```
(d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((d*(7*a*d - b*c*
(16 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) - ((d*(28*a^2
*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a + b*x^3)^(
1 + m))/(b*(4 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*a*b^2
*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2 + 27*m^3))*x*(
a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b*(4 + 3*m)*(
1 + (b*x^3)/a)^m))/(b*(7 + 3*m)))/(b*(10 + 3*m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{3}dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3\,dx=\int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m\,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \int \left(b\,x^3+a\right)^m \left(d\,x^3+c\right)^3 \, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx = \text{too large to display}$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

output

```
(84*(a + b*x**3)**m*a**3*d**3*m*x - 108*(a + b*x**3)**m*a**2*b*c*d**2*m**2
*x - 360*(a + b*x**3)**m*a**2*b*c*d**2*m*x - 63*(a + b*x**3)**m*a**2*b*d**
3*m**2*x**4 - 21*(a + b*x**3)**m*a**2*b*d**3*m*x**4 + 81*(a + b*x**3)**m*a
*b**2*c**2*d*m**3*x + 459*(a + b*x**3)**m*a*b**2*c**2*d*m**2*x + 630*(a +
b*x**3)**m*a*b**2*c**2*d*m*x + 81*(a + b*x**3)**m*a*b**2*c*d**2*m**3*x**4
+ 297*(a + b*x**3)**m*a*b**2*c*d**2*m**2*x**4 + 90*(a + b*x**3)**m*a*b**2*
c*d**2*m*x**4 + 27*(a + b*x**3)**m*a*b**2*d**3*m**3*x**7 + 45*(a + b*x**3)
**m*a*b**2*d**3*m**2*x**7 + 12*(a + b*x**3)**m*a*b**2*d**3*m*x**7 + 27*(a
+ b*x**3)**m*b**3*c**3*m**3*x + 189*(a + b*x**3)**m*b**3*c**3*m**2*x + 414
*(a + b*x**3)**m*b**3*c**3*m*x + 280*(a + b*x**3)**m*b**3*c**3*x + 81*(a +
 b*x**3)**m*b**3*c**2*d*m**3*x**4 + 486*(a + b*x**3)**m*b**3*c**2*d*m**2*x
**4 + 783*(a + b*x**3)**m*b**3*c**2*d*m*x**4 + 210*(a + b*x**3)**m*b**3*c*
*2*d*x**4 + 81*(a + b*x**3)**m*b**3*c*d**2*m**3*x**7 + 405*(a + b*x**3)**m
*b**3*c*d**2*m**2*x**7 + 486*(a + b*x**3)**m*b**3*c*d**2*m*x**7 + 120*(a +
 b*x**3)**m*b**3*c*d**2*x**7 + 27*(a + b*x**3)**m*b**3*d**3*m**3*x**10 + 1
08*(a + b*x**3)**m*b**3*d**3*m**2*x**10 + 117*(a + b*x**3)**m*b**3*d**3*m*
x**10 + 28*(a + b*x**3)**m*b**3*d**3*x**10 - 6804*int((a + b*x**3)**m/(81*
a*m**4 + 594*a*m**3 + 1431*a*m**2 + 1254*a*m + 280*a + 81*b*m**4*x**3 + 59
4*b*m**3*x**3 + 1431*b*m**2*x**3 + 1254*b*m*x**3 + 280*b*x**3),x)*a**4*d**
3*m**5 - 49896*int((a + b*x**3)**m/(81*a*m**4 + 594*a*m**3 + 1431*a*m**...
```

3.185 $\int (a+bx^3)^m (c+dx^3)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = -\frac{2d(2ad-bc(7+3m))x(a+bx^3)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{d^2x^4(a+bx^3)^{1+m}}{b(7+3m)} + \frac{(4a^2d^2-2abcd(7+3m)+b^2c^2(28+33m+9m^2))\,x(a+bx^3)^m\left(1+\frac{bx^3}{a}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{3},\right.}{b^2(4+3m)(7+3m)}$$

output
```
-2*d*(2*a*d-b*c*(7+3*m))*x*(b*x^3+a)^(1+m)/b^2/(4+3*m)/(7+3*m)+d^2*x^4*(b*
x^3+a)^(1+m)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33*m+28
))*x*(b*x^3+a)^m*hypergeom([1/3, -m],[4/3],-b*x^3/a)/b^2/(4+3*m)/(7+3*m)/(
(1+b*x^3/a)^m)
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \frac{1}{14}x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(14c^2 \text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right) + dx^3\left(7c\,\text{Hypergeometric2F1}\left(\frac{4}{3},-m,\frac{7}{3},-\frac{bx^3}{a}\right) + 2dx^3 \text{Hypergeometric2F1}\left(\frac{7}{3},-m,\frac{10}{3},-\frac{bx^3}{a}\right)\right)\right)$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeo
metric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^3)^2 (a+bx^3)^m \, dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int -(bx^3+a)^m \left(d(4ad-bc(3m+10))x^3+c(ad-bc(3m+7))\right)dx}{b(3m+7)}+\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\int (bx^3+a)^m \left(d(4ad-bc(3m+10))x^3+c(ad-bc(3m+7))\right)dx}{b(3m+7)}$$

$$\Big\downarrow \; 913$$

$$\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx(a+bx^3)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\int (bx^3+a)^m dx}{b(3m+4)}}{b(3m+7)}$$

$$\Big\downarrow \; 779$$

$$\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx(a+bx^3)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\int \left(\frac{bx^3}{a}+1\right)^m dx}{b(3m+4)}}{b(3m+7)}$$

$$\Big\downarrow \; 778$$

$$\frac{dx\left(c+dx^3\right)\left(a+bx^3\right)^{m+1}}{b(3m+7)}-\frac{\frac{dx(a+bx^3)^{m+1}(4ad-bc(3m+10))}{b(3m+4)}-\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\right.}{b(3m+4)}}{b(3m+7)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

output
```
(d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) - ((d*(4*a*d - b*c*(10
 + 3*m))*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) - ((4*a^2*d^2 - 2*a*b*c*d*(7
 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeometric2F1[1
/3, -m, 4/3, -((b*x^3)/a)])/(b*(4 + 3*m)*(1 + (b*x^3)/a)^m))/(b*(7 + 3*m))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)^{2}d x$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \int \left(dx^3+c\right)^2 \left(bx^3+a\right)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 92.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.70

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \frac{a^m c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^m c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^m d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{7}{3}, -m \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c)**2,x)
```

output
```
a**m*c**2*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) + 2*a**m*c*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**m*d**2*x**7*gamma(7/3)*hyper((7/3
, -m), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \int (dx^3+c)^2 (bx^3+a)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \int (dx^3+c)^2 (bx^3+a)^m \, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^m (c+dx^3)^2 \, dx = \int (b\,x^3+a)^m \, (d\,x^3+c)^2 \, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

output
```
( - 12*(a + b*x**3)**m*a**2*d**2*m*x + 18*(a + b*x**3)**m*a*b*c*d*m**2*x +
 42*(a + b*x**3)**m*a*b*c*d*m*x + 9*(a + b*x**3)**m*a*b*d**2*m**2*x**4 + 3
*(a + b*x**3)**m*a*b*d**2*m*x**4 + 9*(a + b*x**3)**m*b**2*c**2*m**2*x + 33
*(a + b*x**3)**m*b**2*c**2*m*x + 28*(a + b*x**3)**m*b**2*c**2*x + 18*(a +
b*x**3)**m*b**2*c*d*m**2*x**4 + 48*(a + b*x**3)**m*b**2*c*d*m*x**4 + 14*(a
 + b*x**3)**m*b**2*c*d*x**4 + 9*(a + b*x**3)**m*b**2*d**2*m**2*x**7 + 15*(
a + b*x**3)**m*b**2*d**2*m*x**7 + 4*(a + b*x**3)**m*b**2*d**2*x**7 + 324*i
nt((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*x*
*3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**3*d**2*m**4 + 1296*
int((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*x
**3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**3*d**2*m**3 + 1404
*int((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*
x**3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**3*d**2*m**2 + 336
*int((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*
x**3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**3*d**2*m - 486*in
t((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*x**
3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**2*b*c*d*m**5 - 3078*
int((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m**3*x
**3 + 108*b*m**2*x**3 + 117*b*m*x**3 + 28*b*x**3),x)*a**2*b*c*d*m**4 - 664
2*int((a + b*x**3)**m/(27*a*m**3 + 108*a*m**2 + 117*a*m + 28*a + 27*b*m...
```

3.186 $\int (a+bx^3)^m (c+dx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int (a+bx^3)^m (c+dx^3)\, dx = \frac{dx(a+bx^3)^{1+m}}{b(4+3m)} + \left(c - \frac{ad}{4b+3bm}\right) x(a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left(\frac{1}{3}, -m, \frac{4}{3}, -\frac{bx^3}{a}\right)$$

output
```
d*x*(b*x^3+a)^(1+m)/b/(4+3*m)+(c-a*d/(3*b*m+4*b))*x*(b*x^3+a)^m*hypergeom(
[1/3, -m],[4/3],-b*x^3/a)/((1+b*x^3/a)^m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int (a+bx^3)^m (c+dx^3)\, dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \left(d(a+bx^3)\left(1+\frac{bx^3}{a}\right)^m + (-ad+bc(4+3m)) \text{Hypergeometric2F1}\left(\frac{1}{3}, -m, \frac{4}{3}, \ldots\right.\right.}{b(4+3m)}$$

input
```
Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m
))*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^
3)/a)^m)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)\left(a+bx^3\right)^m dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{3bm+4b}\right)\int \left(bx^3+a\right)^m dx+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right)\int \left(\frac{bx^3}{a}+1\right)^m dx+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-m,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

input
```
Int[(a + b*x^3)^m*(c + d*x^3),x]
```

output
```
(d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a
+ b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^
m
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{m} \left(d\,x^{3}+c\right)dx$$

input
```
int((b*x^3+a)^m*(d*x^3+c),x)
```

output
```
int((b*x^3+a)^m*(d*x^3+c),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\,dx=\int \left(dx^3+c\right)\left(bx^3+a\right)^m\,dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\, dx = \frac{a^m c x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^m d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{4}{3}, -m \\ \frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**m*(d*x**3+c),x)
```

output
```
a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)\, dx = \int \left(dx^3+c\right)\left(bx^3+a\right)^m\, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Giac [F]

$$\int (a+bx^3)^m (c+dx^3)\, dx = \int (dx^3+c)(bx^3+a)^m\, dx$$

input
```
integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^3)^m (c+dx^3)\, dx = \int (b\,x^3+a)^m\,(d\,x^3+c)\, dx$$

input
```
int((a + b*x^3)^m*(c + d*x^3),x)
```

output
```
int((a + b*x^3)^m*(c + d*x^3), x)
```

Reduce [F]

$$\int (a+bx^3)^m (c+dx^3)\, dx = \frac{3(b\,x^3+a)^m\,admx+3(b\,x^3+a)^m\,bcmx+4(b\,x^3+a)^m\,bcx+3(b\,x^3+a)^m\,bdm\,x^4+(b\,x^3+a)^m\,bd\,x^4-}{}$$

input
```
int((b*x^3+a)^m*(d*x^3+c),x)
```

output
```
(3*(a + b*x**3)**m*a*d*m*x + 3*(a + b*x**3)**m*b*c*m*x + 4*(a + b*x**3)**m
*b*c*x + 3*(a + b*x**3)**m*b*d*m*x**4 + (a + b*x**3)**m*b*d*x**4 - 27*int(
(a + b*x**3)**m/(9*a*m**2 + 15*a*m + 4*a + 9*b*m**2*x**3 + 15*b*m*x**3 + 4
*b*x**3),x)*a**2*d*m**3 - 45*int((a + b*x**3)**m/(9*a*m**2 + 15*a*m + 4*a
 + 9*b*m**2*x**3 + 15*b*m*x**3 + 4*b*x**3),x)*a**2*d*m**2 - 12*int((a + b*x
**3)**m/(9*a*m**2 + 15*a*m + 4*a + 9*b*m**2*x**3 + 15*b*m*x**3 + 4*b*x**3)
,x)*a**2*d*m + 81*int((a + b*x**3)**m/(9*a*m**2 + 15*a*m + 4*a + 9*b*m**2*
x**3 + 15*b*m*x**3 + 4*b*x**3),x)*a*b*c*m**4 + 243*int((a + b*x**3)**m/(9*
a*m**2 + 15*a*m + 4*a + 9*b*m**2*x**3 + 15*b*m*x**3 + 4*b*x**3),x)*a*b*c*m
**3 + 216*int((a + b*x**3)**m/(9*a*m**2 + 15*a*m + 4*a + 9*b*m**2*x**3 + 1
5*b*m*x**3 + 4*b*x**3),x)*a*b*c*m**2 + 48*int((a + b*x**3)**m/(9*a*m**2 +
15*a*m + 4*a + 9*b*m**2*x**3 + 15*b*m*x**3 + 4*b*x**3),x)*a*b*c*m)/(b*(9*m
**2 + 15*m + 4))
```

3.187 $\int \frac{(a+bx^3)^m}{c+dx^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{c+dx^3} dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{c+dx^3} dx =$$

$$-\frac{4acx(a+bx^3)^m \text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(-bcm\,\text{AppellF1}\left(\frac{4}{3},1-m,1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx}{c}\right.\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^m}{c+dx^3}dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\int \frac{\left(\frac{bx^3}{a}+1\right)^m}{dx^3+c}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\text{AppellF1}\left(\frac{1}{3},-m,1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3),x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
*(1 + (b*x^3)/a)^m)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^3+a)^m}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c),x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx=\int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(bx^3+a)^m}{dx^3+c}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^m}{d\,x^3+c}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3),x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3), x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^m}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^m}{d\,x^3+c}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c),x)
```

output
```
int((a + b*x**3)**m/(c + d*x**3),x)
```

3.188 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2} \, dx = -\frac{4acx(a+bx^3)^m \operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^2 \left(-4ac \operatorname{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right) - 3x^3 \left(bcm \operatorname{AppellF1}\left(\frac{4}{3},1-m,2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\int \frac{\left(\frac{bx^3}{a}+1\right)^m}{(dx^3+c)^2}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\text{AppellF1}\left(\frac{1}{3},-m,2,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
^2*(1 + (b*x^3)/a)^m)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx=\int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^2}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3)^2,x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^2}\,dx = \int \frac{(b\,x^3+a)^m}{d^2x^6+2cd\,x^3+c^2}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

output
```
int((a + b*x**3)**m/(c**2 + 2*c*d*x**3 + d**2*x**6),x)
```

3.189 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3} \, dx = \frac{x(a+bx^3)^m \left(1+\frac{bx^3}{a}\right)^{-m} \operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

output
```
x*(b*x^3+a)^m*AppellF1(1/3,-m,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/((1+b*x^3/a)^m)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3} \, dx = -\frac{4acx(a+bx^3)^m \operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)^3 \left(-4ac \operatorname{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3 \left(bcm \operatorname{AppellF1}\left(\frac{4}{3},1-m,3,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.\right.}$$

input
```
Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

output
```
(-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x
^3)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\int \frac{\left(\frac{bx^3}{a}+1\right)^m}{(dx^3+c)^3}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^3)^m\left(\frac{bx^3}{a}+1\right)^{-m}\text{AppellF1}\left(\frac{1}{3},-m,3,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

output
```
(x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c
^3*(1 + (b*x^3)/a)^m)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{m}}{(d\,x^{3}+c)^{3}}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

output
```
int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx=\int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(bx^3+a)^m}{(dx^3+c)^3}\,dx$$

input
```
integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^m}{(d\,x^3+c)^3}\,dx$$

input
```
int((a + b*x^3)^m/(c + d*x^3)^3,x)
```

output
```
int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^m}{(c+dx^3)^3}\,dx = \int \frac{(b\,x^3+a)^m}{d^3x^9+3c\,d^2x^6+3c^2d\,x^3+c^3}dx$$

input
```
int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

output
```
int((a + b*x**3)**m/(c**3 + 3*c**2*d*x**3 + 3*c*d**2*x**6 + d**3*x**9),x)
```

3.190 $\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx$

Optimal result

Integrand size = 50, antiderivative size = 53

$$\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x(a+bx^3)^{-\frac{bc}{3bc-3ad}} (c+dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

output
```
x*(d*x^3+c)^(a*d/(-3*a*d+3*b*c))/a/c/((b*x^3+a)^(b*c/(-3*a*d+3*b*c)))
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^{-1-\frac{bc}{3bc-3ad}} (c+dx^3)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x(a+bx^3)^{\frac{bc}{-3bc+3ad}} (c+dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

input
```
Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)
/(3*b*c - 3*a*d)),x]
```

output
```
(x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)
))/(a*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^{-\frac{bc}{3bc-3ad}-1}(c+dx^3)^{\frac{ad}{3bc-3ad}-1}dx$$

$$\Big\downarrow \; 906$$

$$\frac{x(a+bx^3)^{-\frac{bc}{3bc-3ad}}(c+dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

input
```
Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*
c - 3*a*d)),x]
```

output
```
(x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3
*a*d)))
```

Defintions of rubi rules used

rule 906
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
gosper	$\frac{x\left(b\,x^{3}+a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}}\left(d\,x^{3}+c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$	71
orering	$\frac{\left(b\,x^{3}+a\right)\left(d\,x^{3}+c\right)x\left(b\,x^{3}+a\right)^{-1-\frac{bc}{-3ad+3bc}}\left(d\,x^{3}+c\right)^{-1+\frac{ad}{-3ad+3bc}}}{ac}$	72

input
```
int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x,
method=_RETURNVERBOSE)
```

output
```
x/a/c*(b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*
b*c)/(a*d-b*c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{bdx^7+(bc+ad)x^4+acx}{\left(bx^3+a\right)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}\left(dx^3+c\right)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="fricas")
```

output
```
(b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
 - a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+
3*b*c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx$$
$$=\int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx$$
$$=\int \left(bx^3+a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

input
```
integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*
c)),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
 - a*d) - 1), x)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx = \frac{x\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}+\frac{x^4\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\,\left(a\,d+b\,c\right)}{a\,c}+\frac{b\,d\,x^7\,\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{\left(d\,x^3+c\right)^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

input
```
int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*
b*c) + 1),x)
```

output
```
(x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*
d - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d -
3*b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} \, dx = \int \frac{\left(b\,x^3+a\right)^{\frac{bc}{3ad-3bc}}}{\left(d\,x^3+c\right)^{\frac{ad}{3ad-3bc}} ac+\left(d\,x^3+c\right)^{\frac{ad}{3ad-3bc}} ad\,x^3+\left(d\,x^3+c\right)^{\frac{ad}{3ad-3bc}} bc\,x^3+\left(d\,x^3+c\right)^{\frac{ad}{3ad-3bc}} bd\,x^6} dx$$

input
```
int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x)
```

output
```
int((a + b*x**3)**((b*c)/(3*a*d - 3*b*c))/((c + d*x**3)**((a*d)/(3*a*d - 3
*b*c))*a*c + (c + d*x**3)**((a*d)/(3*a*d - 3*b*c))*a*d*x**3 + (c + d*x**3)
**((a*d)/(3*a*d - 3*b*c))*b*c*x**3 + (c + d*x**3)**((a*d)/(3*a*d - 3*b*c))
*b*d*x**6),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file