Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.3/51-1.1.3.3-b

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**134**]. This is test number [51].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (134)	0.00 (0)
Mathematica	100.00 (134)	0.00 (0)
Maple	64.93 (87)	35.07 (47)
Fricas	40.30 (54)	59.70 (80)
Sympy	40.30 (54)	59.70 (80)
Maxima	29.85 (40)	70.15 (94)
Mupad	25.37 (34)	74.63 (100)
Giac	20.90 (28)	79.10 (106)
Reduce	20.90 (28)	79.10 (106)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

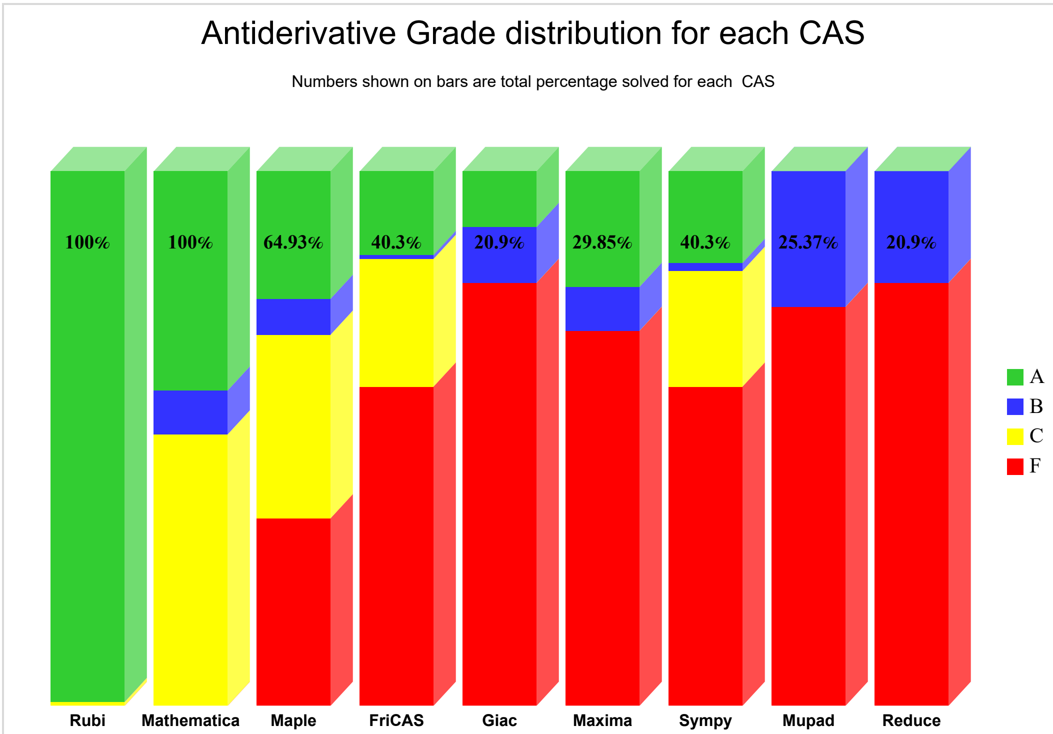
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

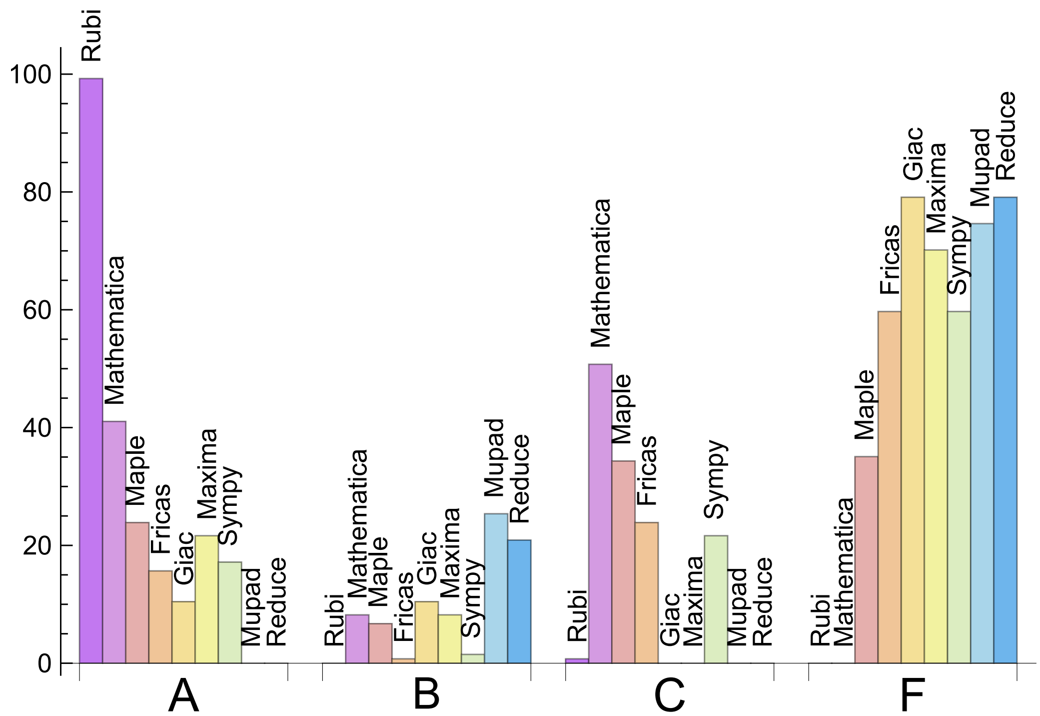
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.254	0.000	0.746	0.000
Mathematica	41.045	8.209	50.746	0.000
Maple	23.881	6.716	34.328	35.075
Maxima	21.642	8.209	0.000	70.149
Sympy	17.164	1.493	21.642	59.701
Fricas	15.672	0.746	23.881	59.701
Giac	10.448	10.448	0.000	79.104
Mupad	0.000	25.373	0.000	74.627
Reduce	0.000	20.896	0.000	79.104

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	47	100.00	0.00	0.00
Fricas	80	35.00	65.00	0.00
Sympy	80	83.75	16.25	0.00
Maxima	94	100.00	0.00	0.00
Mupad	100	0.00	100.00	0.00
Giac	106	100.00	0.00	0.00
Reduce	106	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Giac	0.13
Reduce	0.24
Rubi	0.74
Mupad	0.93
Maple	2.06
Fricas	3.40
Mathematica	6.55
Sympy	11.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	154.43	1.16	112.00	0.94
Maple	207.13	0.93	185.00	0.92
Mathematica	230.75	1.30	178.50	1.00
Maxima	252.47	1.34	236.00	1.27
Rubi	253.97	1.07	175.50	1.00
Giac	344.61	1.53	319.50	1.49
Reduce	694.14	2.84	544.50	2.55
Fricas	889.80	4.01	560.00	3.47
Mupad	3168.79	9.17	730.00	3.98

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

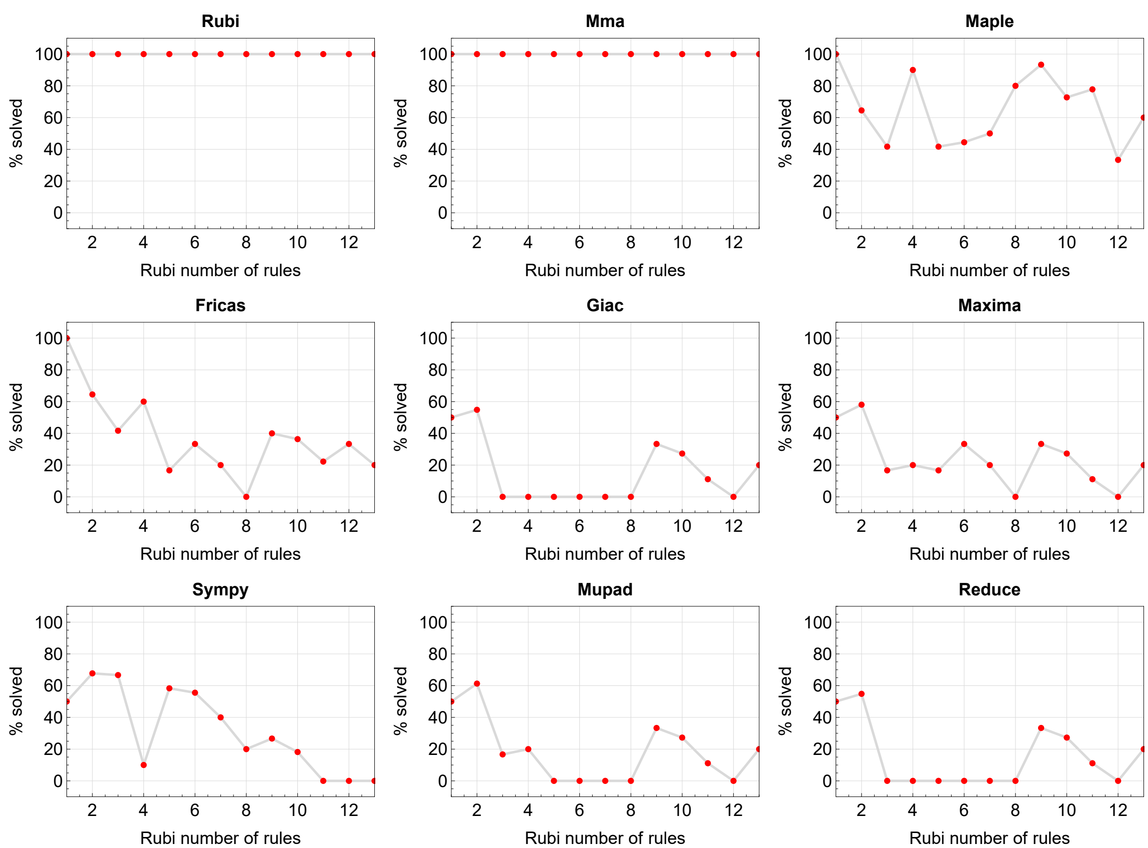


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

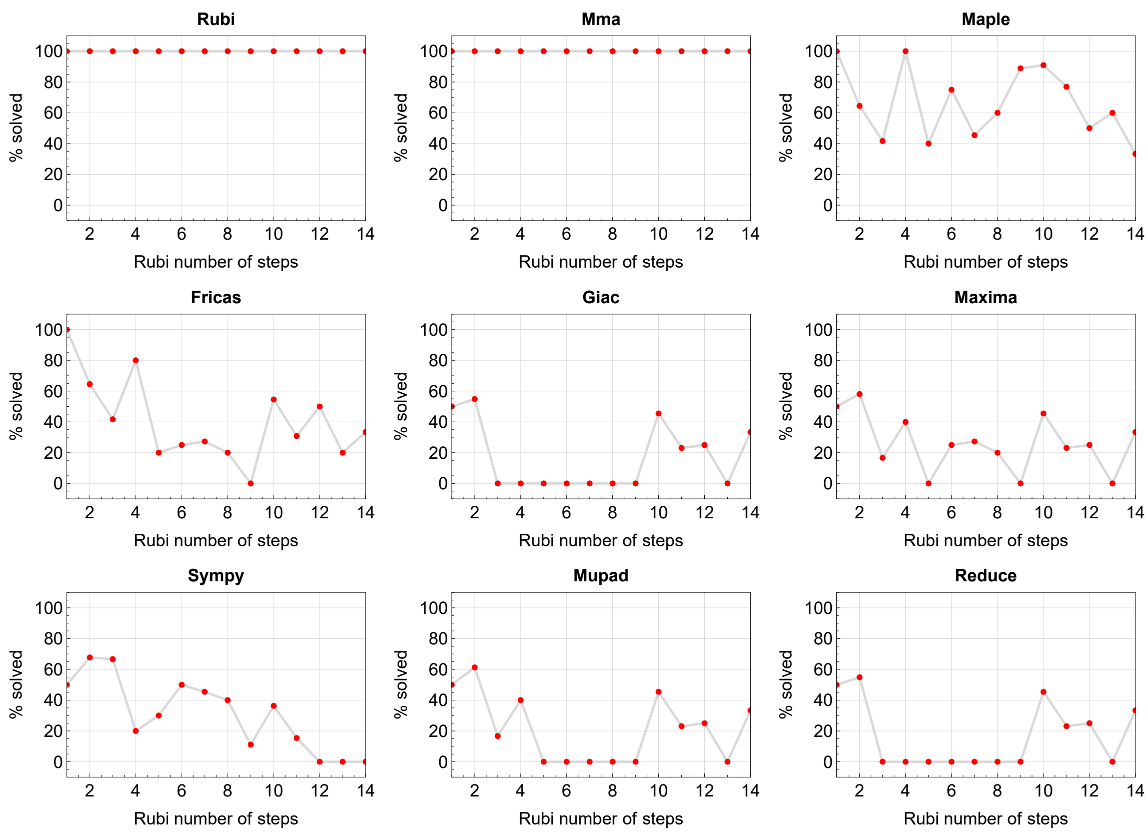


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

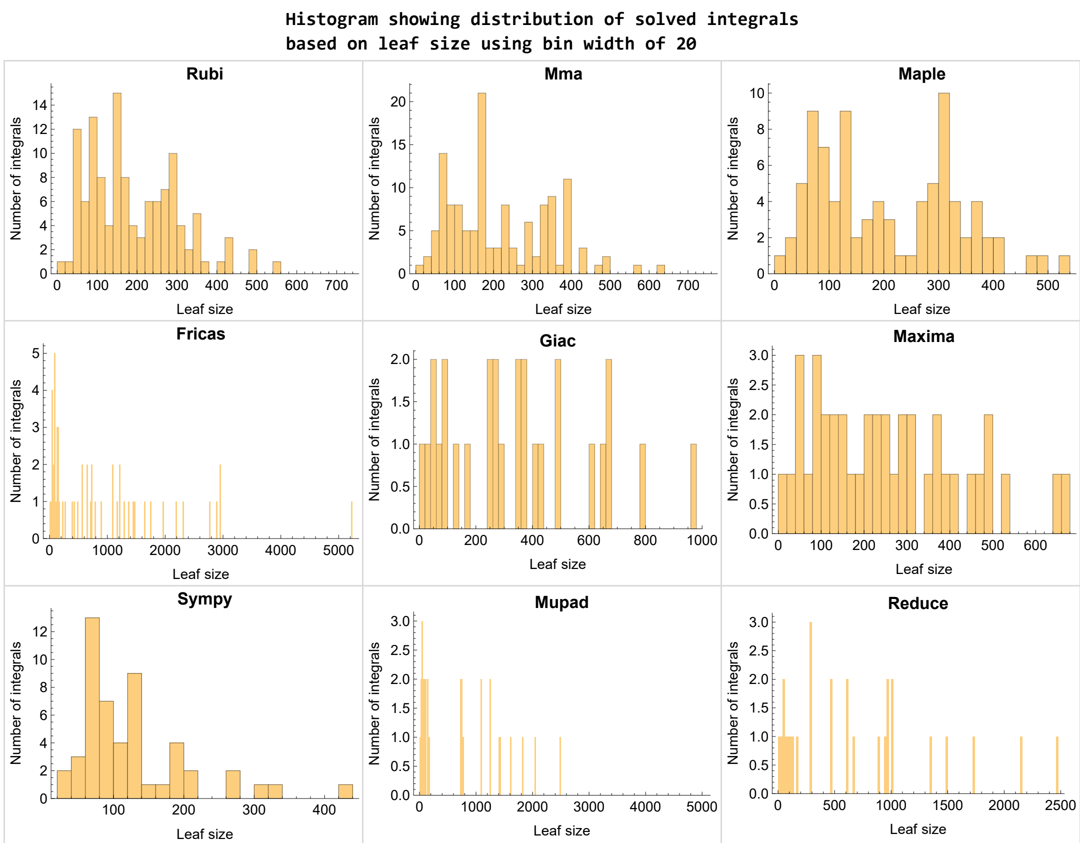


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

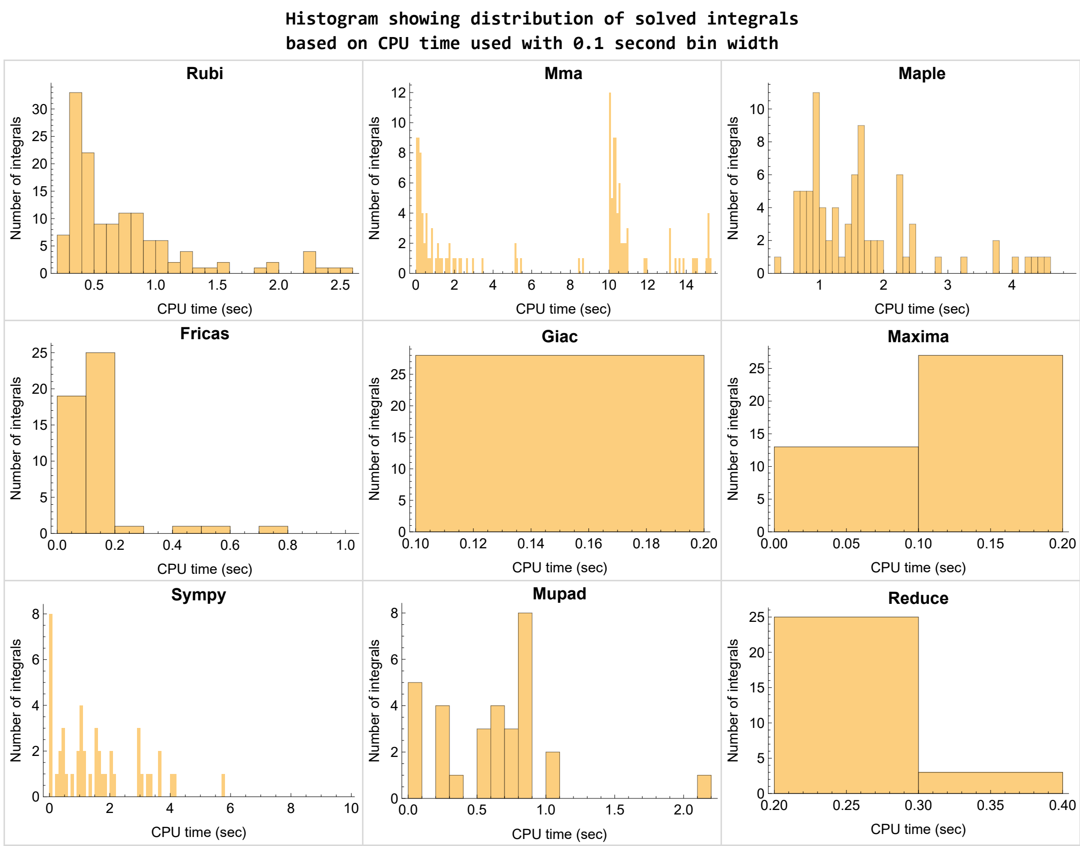


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

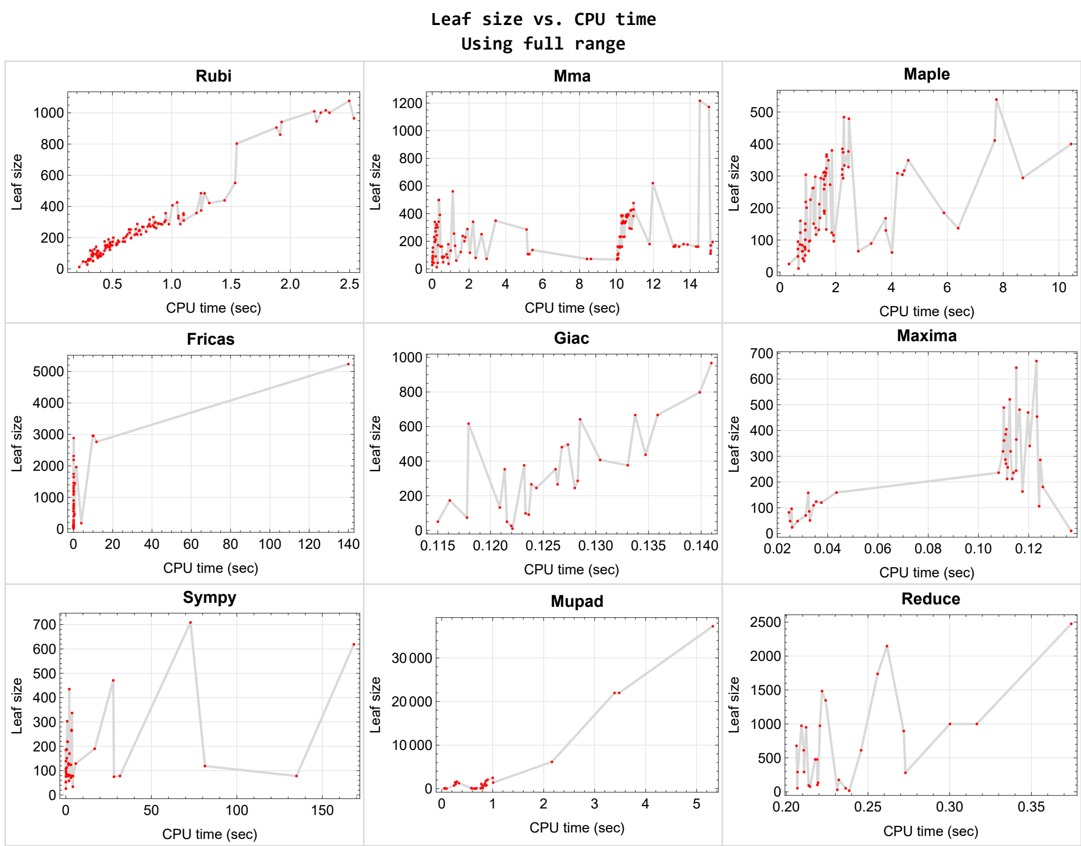


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {62, 85, 86, 89, 90, 97, 99, 102, 109, 114, 121, 124, 125}

Mathematica {28, 30, 31, 32, 34, 35, 37, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 62, 65, 66, 67, 69, 70, 73, 74, 77, 78, 97, 99, 100, 107, 108, 109, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 125, 128, 133, 134}

Maple $\{30, 31, 34, 42, 43, 44, 45, 48, 49, 50, 53, 54, 55, 56\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

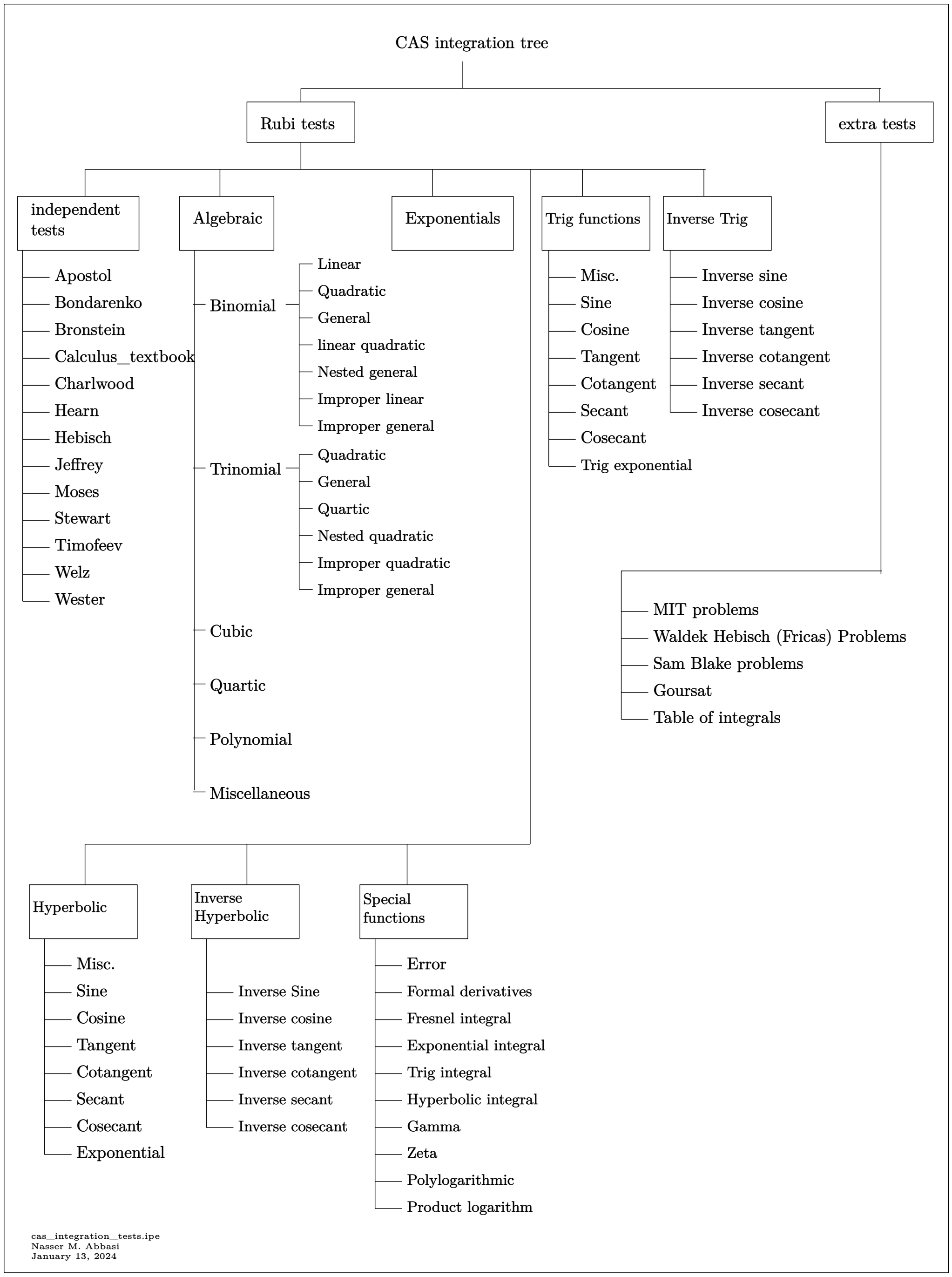
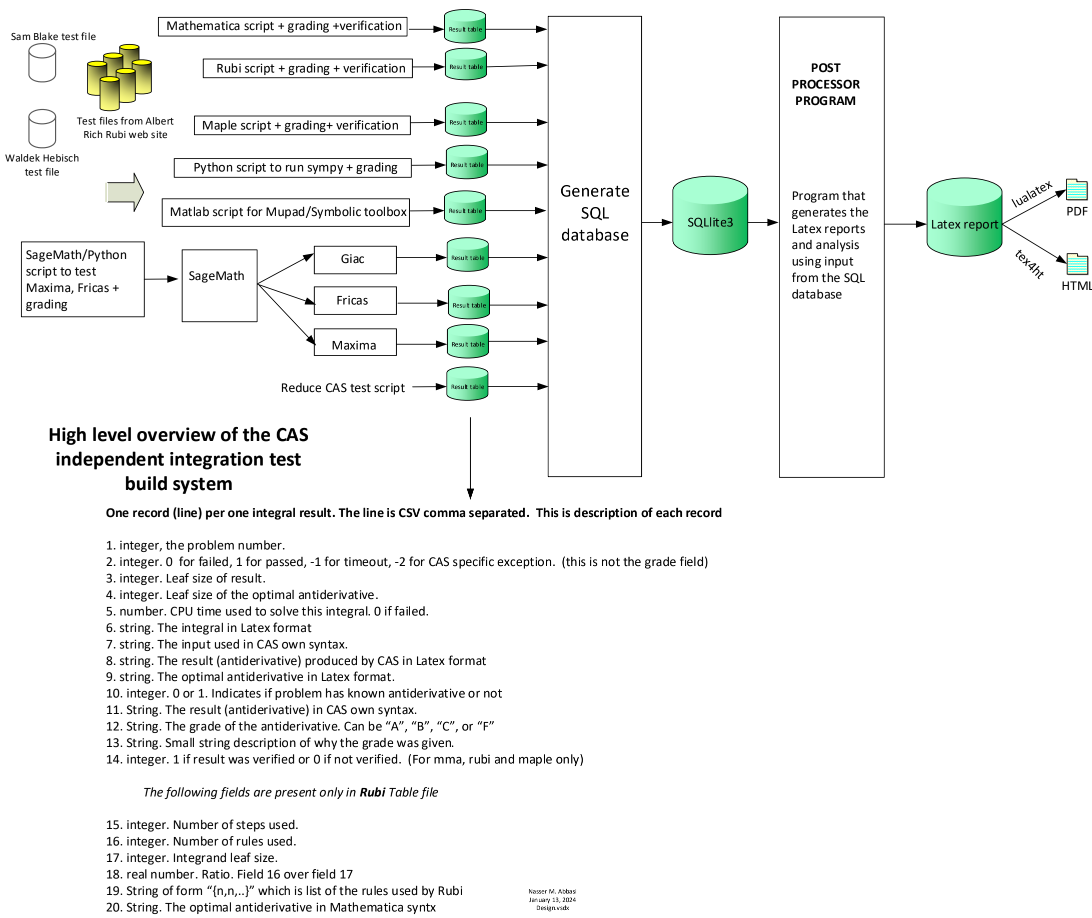


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

B grade { }

C grade { 62 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 41, 60, 63, 64, 67, 68, 71, 72, 75, 76, 79, 80, 81, 82, 83, 84, 91, 92, 93, 94, 95, 96, 126, 127, 129, 130, 131, 132 }

B grade { 65, 66, 69, 70, 73, 74, 77, 78, 128, 133, 134 }

C grade { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 19, 20, 26, 27, 41, 60, 61, 79, 80, 81, 82, 83, 84, 91, 92, 93, 94, 95, 96, 108, 119, 120, 126, 127 }

B grade { 103, 104, 105, 106, 107, 115, 116, 117, 118 }

C grade { 5, 6, 7, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59 }

F normal fail { 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 28, 29, 32, 33, 36, 37, 38, 41, 82, 83, 84, 95, 96 }

B grade { 126 }

C grade { 5, 6, 7, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 60, 61, 79, 80, 81, 91, 92, 93, 94, 103, 104, 115, 116 }

F normal fail { 62, 63, 64, 67, 68, 71, 72, 75, 76, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { 30, 31, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66, 69, 70, 73, 74, 77, 78, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 24, 25, 26, 27, 41, 81, 82, 83, 84, 94, 95, 96 }

B grade { 15, 16, 17, 21, 22, 23, 79, 80, 91, 92, 93 }

C grade { }

F normal fail { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 85, 86, 87, 88, 89, 90, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 14, 19, 25, 41 }

B grade { 5, 12, 13, 15, 16, 17, 18, 20, 21, 22, 23, 24, 26, 27 }

C grade { }

F normal fail { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 41, 82, 83, 84, 95, 96, 132 }

C grade { }

F normal fail { }

F(-1) timedout fail { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25 }

B grade { 82, 83 }

C grade { 28, 29, 32, 33, 38, 41, 63, 64, 67, 68, 72, 76, 79, 80, 81, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 130, 131, 132 }

F normal fail { 30, 31, 34, 35, 36, 37, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 65, 66, 69, 70, 71, 73, 74, 75, 77, 78, 93, 94, 95, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127 }

F(-1) timedout fail { 19, 20, 26, 27, 58, 84, 96, 102, 115, 128, 129, 133, 134 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 41 }

C grade { }

F normal fail { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	101	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	1.07	0.94
time (sec)	N/A	0.421	0.029	0.645	0.026	0.063	0.028	0.123	0.219	0.061

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	72	70	70	76	74	77	66
N.S.	1	1.00	1.00	1.03	1.00	1.00	1.09	1.06	1.10	0.94
time (sec)	N/A	0.376	0.022	0.645	0.031	0.058	0.032	0.118	0.215	0.595

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	53	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	1.06	0.96
time (sec)	N/A	0.336	0.017	0.638	0.025	0.063	0.025	0.122	0.207	0.054

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	29	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	1.04	0.89
time (sec)	N/A	0.288	0.008	0.322	0.026	0.061	0.019	0.122	0.231	0.638

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	220	196	42	212	560	87	245	291	720
N.S.	1	1.34	1.20	0.26	1.29	3.41	0.53	1.49	1.77	4.39
time (sec)	N/A	0.709	0.154	0.816	0.113	0.091	0.279	0.128	0.211	0.801

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	247	212	65	236	648	112	266	612	740
N.S.	1	1.33	1.14	0.35	1.27	3.48	0.60	1.43	3.29	3.98
time (sec)	N/A	0.729	0.192	0.811	0.108	0.082	0.389	0.126	0.246	0.842

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	272	243	84	271	739	151	286	951	762
N.S.	1	1.27	1.14	0.39	1.27	3.45	0.71	1.34	4.44	3.56
time (sec)	N/A	0.798	0.251	0.799	0.111	0.119	0.510	0.128	0.212	0.877

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	160	158	158	185	173	176	146
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.20	1.12	1.14	0.95
time (sec)	N/A	0.533	0.041	0.730	0.032	0.082	0.034	0.116	0.232	0.586

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	123	124	124	139	132	135	116
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.08	1.11	0.95
time (sec)	N/A	0.479	0.030	0.727	0.035	0.061	0.029	0.121	0.219	0.579

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	94	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	1.15	0.91
time (sec)	N/A	0.403	0.022	0.717	0.025	0.079	0.033	0.124	0.214	0.052

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	53	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	1.06	0.96
time (sec)	N/A	0.336	0.012	0.641	0.028	0.073	0.025	0.115	0.236	0.056

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	253	231	78	286	1092	187	353	477	1081
N.S.	1	1.32	1.20	0.41	1.49	5.69	0.97	1.84	2.48	5.63
time (sec)	N/A	0.640	0.118	0.900	0.125	0.106	0.492	0.121	0.218	0.275

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	291	298	101	319	1210	219	376	973	1254
N.S.	1	1.30	1.34	0.45	1.43	5.43	0.98	1.69	4.36	5.62
time (sec)	N/A	0.890	0.200	0.906	0.113	0.103	1.079	0.123	0.209	0.847

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	311	319	131	361	1294	264	407	1484	1401
N.S.	1	1.15	1.18	0.49	1.34	4.79	0.98	1.51	5.50	5.19
time (sec)	N/A	0.947	0.242	0.901	0.110	0.098	3.324	0.130	0.222	1.010

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	332	322	201	489	2190	435	617	896	1822
N.S.	1	1.23	1.19	0.74	1.80	8.08	1.61	2.28	3.31	6.72
time (sec)	N/A	0.845	0.239	0.954	0.110	0.099	2.008	0.118	0.272	0.867

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	288	271	131	385	1642	303	481	678	1433
N.S.	1	1.27	1.19	0.58	1.70	7.23	1.33	2.12	2.99	6.31
time (sec)	N/A	0.710	0.184	0.911	0.111	0.134	0.773	0.127	0.206	0.263

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	253	231	78	287	1093	187	353	477	1081
N.S.	1	1.32	1.20	0.41	1.49	5.69	0.97	1.84	2.48	5.63
time (sec)	N/A	0.654	0.116	0.879	0.111	0.087	0.481	0.126	0.219	0.778

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	219	196	42	212	560	87	245	291	720
N.S.	1	1.34	1.20	0.26	1.29	3.41	0.53	1.49	1.77	4.39
time (sec)	N/A	0.688	0.133	0.829	0.111	0.083	0.320	0.124	0.207	0.242

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	426	340	226	365	1171	0	437	281	6153
N.S.	1	1.30	1.04	0.69	1.12	3.58	0.00	1.34	0.86	18.82
time (sec)	N/A	1.046	0.145	1.094	0.115	0.172	0.000	0.135	0.273	2.162

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	485	498	262	481	2955	0	667	1000	21975
N.S.	1	1.27	1.30	0.69	1.26	7.74	0.00	1.75	2.62	57.53
time (sec)	N/A	1.246	0.354	1.171	0.116	9.883	0.000	0.134	0.300	3.391

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	407	391	304	644	2884	619	798	2146	2490
N.S.	1	1.21	1.16	0.90	1.91	8.56	1.84	2.37	6.37	7.39
time (sec)	N/A	1.005	0.419	0.925	0.115	0.118	168.416	0.140	0.262	1.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	357	341	219	521	2315	471	642	1738	2043
N.S.	1	1.24	1.19	0.76	1.82	8.07	1.64	2.24	6.06	7.12
time (sec)	N/A	0.949	0.352	0.923	0.112	0.117	27.718	0.129	0.256	0.899

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	317	301	151	405	1741	337	496	1347	1616
N.S.	1	1.28	1.21	0.61	1.63	7.02	1.36	2.00	5.43	6.52
time (sec)	N/A	0.813	0.293	0.908	0.111	0.101	3.601	0.127	0.224	0.297

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	291	297	101	319	1210	219	376	973	1254
N.S.	1	1.30	1.33	0.45	1.43	5.43	0.98	1.69	4.36	5.62
time (sec)	N/A	0.885	0.206	0.909	0.110	0.091	0.937	0.133	0.221	0.338

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	247	212	65	236	648	112	266	612	740
N.S.	1	1.33	1.14	0.35	1.27	3.48	0.60	1.43	3.29	3.98
time (sec)	N/A	0.728	0.187	0.795	0.114	0.084	0.425	0.124	0.211	0.875

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	484	499	263	470	2955	0	667	1000	21975
N.S.	1	1.27	1.31	0.69	1.23	7.74	0.00	1.75	2.62	57.53
time (sec)	N/A	1.276	0.367	1.195	0.120	10.161	0.000	0.136	0.317	3.482

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	551	561	298	670	5234	0	967	2475	37266
N.S.	1	1.21	1.23	0.65	1.47	11.48	0.00	2.12	5.43	81.72
time (sec)	N/A	1.535	1.121	1.266	0.123	139.964	0.000	0.141	0.374	5.319

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	200	161	168	0	123	129	0	195	0
N.S.	1	0.99	0.79	0.83	0.00	0.61	0.64	0.00	0.96	0.00
time (sec)	N/A	0.507	13.176	3.786	0.000	0.081	1.512	0.000	0.234	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	142	72	115	0	72	82	0	98	0
N.S.	1	0.97	0.49	0.79	0.00	0.49	0.56	0.00	0.67	0.00
time (sec)	N/A	0.363	8.620	1.916	0.000	0.084	1.120	0.000	0.270	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	749	965	161	273	0	0	0	0	20	0
N.S.	1	1.29	0.21	0.36	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.537	10.298	1.797	0.000	0.000	0.000	0.000	0.227	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	694	1001	232	304	0	0	0	0	31	0
N.S.	1	1.44	0.33	0.44	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.256	10.394	2.230	0.000	0.000	0.000	0.000	1.282	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	176	161	130	0	97	124	0	134	0
N.S.	1	1.07	0.98	0.79	0.00	0.59	0.75	0.00	0.81	0.00
time (sec)	N/A	0.434	13.402	3.793	0.000	0.132	1.618	0.000	0.268	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	73	96	0	58	78	0	65	0
N.S.	1	1.00	0.62	0.81	0.00	0.49	0.66	0.00	0.55	0.00
time (sec)	N/A	0.313	10.046	1.931	0.000	0.104	0.997	0.000	0.231	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	625	860	161	191	0	0	0	0	35	0
N.S.	1	1.38	0.26	0.31	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.915	10.083	1.596	0.000	0.000	0.000	0.000	0.249	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	1016	335	333	0	0	0	0	59	0
N.S.	1	1.41	0.46	0.46	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.299	10.353	2.305	0.000	0.000	0.000	0.000	0.307	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	255	195	294	0	228	0	0	461	0
N.S.	1	1.20	0.92	1.38	0.00	1.07	0.00	0.00	2.16	0.00
time (sec)	N/A	0.696	15.221	8.704	0.000	0.109	0.000	0.000	0.337	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	189	161	185	0	157	0	0	305	0
N.S.	1	1.14	0.97	1.11	0.00	0.95	0.00	0.00	1.84	0.00
time (sec)	N/A	0.467	14.330	5.876	0.000	0.109	0.000	0.000	0.298	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	71	123	0	93	78	0	174	0
N.S.	1	1.00	0.56	0.98	0.00	0.74	0.62	0.00	1.38	0.00
time (sec)	N/A	0.348	10.051	1.854	0.000	0.125	2.952	0.000	0.311	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	1001	331	313	0	0	0	0	59	0
N.S.	1	1.48	0.49	0.46	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.329	10.303	1.606	0.000	0.000	0.000	0.000	0.285	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	784	1077	380	385	0	0	0	0	100	0
N.S.	1	1.37	0.48	0.49	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.496	10.521	2.237	0.000	0.000	0.000	0.000	4.799	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	58	10	16	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	4.83	0.83	1.33	0.83
time (sec)	N/A	0.220	0.250	0.663	0.137	0.089	1.852	0.122	0.239	0.095

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	321	324	290	349	0	0	0	0	342	0
N.S.	1	1.01	0.90	1.09	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	1.056	10.820	4.597	0.000	0.000	0.000	0.000	0.681	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	277	272	341	304	0	0	0	0	198	0
N.S.	1	0.98	1.23	1.10	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.820	10.360	4.388	0.000	0.000	0.000	0.000	0.486	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	232	155	259	0	0	0	0	22	0
N.S.	1	0.97	0.65	1.08	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.645	10.206	1.578	0.000	0.000	0.000	0.000	0.237	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	38	0
N.S.	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.479	10.181	1.589	0.000	0.000	0.000	0.000	0.271	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	269	381	301	0	0	0	0	62	0
N.S.	1	0.96	1.36	1.07	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.761	10.291	1.629	0.000	0.000	0.000	0.000	0.280	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	340	422	361	0	0	0	0	86	0
N.S.	1	1.02	1.26	1.08	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.057	10.765	1.665	0.000	0.000	0.000	0.000	2.320	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	653	946	343	317	0	0	0	0	193	0
N.S.	1	1.45	0.53	0.49	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	2.221	10.508	4.454	0.000	0.000	0.000	0.000	0.658	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	692	906	158	273	0	0	0	0	21	0
N.S.	1	1.31	0.23	0.39	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	1.883	10.197	1.602	0.000	0.000	0.000	0.000	0.252	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	571	803	158	191	0	0	0	0	37	0
N.S.	1	1.41	0.28	0.33	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.548	10.203	1.578	0.000	0.000	0.000	0.000	0.255	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	942	387	310	0	0	0	0	61	0
N.S.	1	1.52	0.62	0.50	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.926	10.269	1.646	0.000	0.000	0.000	0.000	0.328	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	706	1010	427	367	0	0	0	0	85	0
N.S.	1	1.43	0.60	0.52	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	2.202	10.817	1.672	0.000	0.000	0.000	0.000	3.414	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	426	422	477	539	0	0	0	0	0	0
N.S.	1	0.99	1.12	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.316	10.928	7.757	0.000	0.000	0.000	0.000	3.662	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	365	356	396	411	0	0	0	0	0	0
N.S.	1	0.98	1.08	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.099	10.644	7.697	0.000	0.000	0.000	0.000	2.521	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	309	299	342	328	0	0	0	0	1581	0
N.S.	1	0.97	1.11	1.06	0.00	0.00	0.00	0.00	5.12	0.00
time (sec)	N/A	0.862	10.368	2.454	0.000	0.000	0.000	0.000	1.860	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	276	270	233	293	0	0	0	0	32	0
N.S.	1	0.98	0.84	1.06	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.752	10.178	2.262	0.000	0.000	0.000	0.000	0.843	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	286	386	321	0	0	0	0	62	0
N.S.	1	0.92	1.25	1.04	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.810	10.339	2.232	0.000	0.000	0.000	0.000	0.258	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	347	374	374	0	0	0	0	101	0
N.S.	1	0.96	1.03	1.03	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.100	10.567	2.263	0.000	0.000	0.000	0.000	2.343	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	144	0
N.S.	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.446	10.923	2.292	0.000	0.000	0.000	0.000	3.369	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	101	81	89	0	437	0	0	25	0
N.S.	1	0.98	0.79	0.86	0.00	4.24	0.00	0.00	0.24	0.00
time (sec)	N/A	0.334	0.559	3.268	0.000	0.513	0.000	0.000	0.233	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	88	137	0	482	0	0	25	0
N.S.	1	1.00	0.76	1.18	0.00	4.16	0.00	0.00	0.22	0.00
time (sec)	N/A	0.327	0.542	6.386	0.000	0.490	0.000	0.000	0.253	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	84	200	0	0	0	0	0	30	0
N.S.	1	0.64	1.52	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.371	1.753	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	140	179	0	0	0	129	0	174	0
N.S.	1	1.12	1.43	0.00	0.00	0.00	1.03	0.00	1.39	0.00
time (sec)	N/A	0.468	13.646	0.000	0.000	0.000	1.581	0.000	0.280	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	75	0	0	0	82	0	83	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.00	0.00	1.01	0.00
time (sec)	N/A	0.328	10.099	0.000	0.000	0.000	1.056	0.000	0.237	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	21	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.310	10.190	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	233	0	0	0	0	0	32	0
N.S.	1	1.00	3.95	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.304	10.434	0.000	0.000	0.000	0.000	0.000	1.051	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	140	161	0	0	0	124	0	54	0
N.S.	1	1.12	1.29	0.00	0.00	0.00	0.99	0.00	0.43	0.00
time (sec)	N/A	0.470	14.437	0.000	0.000	0.000	1.581	0.000	0.232	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	74	0	0	0	78	0	31	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.95	0.00	0.38	0.00
time (sec)	N/A	0.331	10.047	0.000	0.000	0.000	1.021	0.000	0.250	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	30	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.306	10.217	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	335	0	0	0	0	0	50	0
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.296	10.427	0.000	0.000	0.000	0.000	0.000	0.357	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	153	111	0	0	0	0	0	111	0
N.S.	1	1.15	0.83	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.483	15.104	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	94	67	0	0	0	78	0	69	0
N.S.	1	1.19	0.85	0.00	0.00	0.00	0.99	0.00	0.87	0.00
time (sec)	N/A	0.360	10.045	0.000	0.000	0.000	2.939	0.000	0.211	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	329	0	0	0	0	0	62	0
N.S.	1	1.00	5.31	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.315	10.322	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	380	0	0	0	0	0	104	0
N.S.	1	1.00	6.13	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.306	10.579	0.000	0.000	0.000	0.000	0.000	0.309	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	153	111	0	0	0	0	0	111	0
N.S.	1	1.15	0.83	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.485	15.109	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	71	0	0	0	78	0	69	0
N.S.	1	1.12	0.84	0.00	0.00	0.00	0.92	0.00	0.81	0.00
time (sec)	N/A	0.340	10.046	0.000	0.000	0.000	3.671	0.000	0.273	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	62	0
N.S.	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.312	10.458	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	104	0
N.S.	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.297	10.598	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	116	105	117	257	795	82	0	83	0
N.S.	1	0.93	0.84	0.94	2.06	6.36	0.66	0.00	0.66	0.00
time (sec)	N/A	0.374	0.781	1.293	0.112	0.204	2.111	0.000	0.231	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	94	88	151	181	716	78	0	31	0
N.S.	1	0.99	0.93	1.59	1.91	7.54	0.82	0.00	0.33	0.00
time (sec)	N/A	0.331	0.645	1.204	0.126	0.119	1.302	0.000	0.247	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	82	98	106	276	71	0	69	0
N.S.	1	1.05	0.95	1.14	1.23	3.21	0.83	0.00	0.80	0.00
time (sec)	N/A	0.327	0.829	1.075	0.124	0.117	2.987	0.000	0.209	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	47	37	34	51	54	190	0	109	33
N.S.	1	0.77	0.61	0.56	0.84	0.89	3.11	0.00	1.79	0.54
time (sec)	N/A	0.252	0.891	0.865	0.033	0.104	16.862	0.000	0.220	0.649

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	86	87	709	0	149	87
N.S.	1	0.97	0.66	0.57	0.95	0.96	7.79	0.00	1.64	0.96
time (sec)	N/A	0.334	1.311	0.905	0.033	0.106	72.895	0.000	0.256	0.766

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	80	71	120	121	0	0	189	105
N.S.	1	0.95	0.66	0.59	0.99	1.00	0.00	0.00	1.56	0.87
time (sec)	N/A	0.381	2.361	0.904	0.037	0.122	0.000	0.000	0.255	0.682

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	139	75	0	0	0	170	0	125	0
N.S.	1	0.93	0.50	0.00	0.00	0.00	1.13	0.00	0.83	0.00
time (sec)	N/A	0.471	10.084	0.000	0.000	0.000	2.082	0.000	0.236	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	117	72	0	0	0	82	0	82	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.68	0.00	0.68	0.00
time (sec)	N/A	0.433	8.403	0.000	0.000	0.000	1.178	0.000	0.253	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	0	0	0	78	0	31	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.84	0.00	0.33	0.00
time (sec)	N/A	0.393	10.051	0.000	0.000	0.000	1.010	0.000	0.238	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	72	0	0	0	78	0	69	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.76	0.00	0.68	0.00
time (sec)	N/A	0.399	10.048	0.000	0.000	0.000	4.150	0.000	0.214	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	102	0	0	0	78	0	109	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.59	0.00	0.83	0.00
time (sec)	N/A	0.442	10.065	0.000	0.000	0.000	31.641	0.000	0.220	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	159	110	0	0	0	78	0	149	0
N.S.	1	0.98	0.68	0.00	0.00	0.00	0.48	0.00	0.92	0.00
time (sec)	N/A	0.541	10.102	0.000	0.000	0.000	134.883	0.000	0.226	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	174	167	169	454	1462	129	0	174	0
N.S.	1	0.89	0.85	0.86	2.32	7.46	0.66	0.00	0.89	0.00
time (sec)	N/A	0.502	1.243	1.434	0.123	0.134	5.790	0.000	0.246	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	152	132	132	340	1368	124	0	54	0
N.S.	1	0.97	0.85	0.85	2.18	8.77	0.79	0.00	0.35	0.00
time (sec)	N/A	0.456	1.005	1.391	0.120	0.122	3.004	0.000	0.226	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	146	122	133	244	896	0	0	111	0
N.S.	1	1.14	0.95	1.04	1.91	7.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.455	1.551	1.661	0.115	0.117	0.000	0.000	0.226	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	151	117	138	163	396	0	0	171	0
N.S.	1	1.13	0.87	1.03	1.22	2.96	0.00	0.00	1.28	0.00
time (sec)	N/A	0.455	2.047	1.277	0.118	0.114	0.000	0.000	0.253	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	84	73	66	109	103	0	0	231	149
N.S.	1	0.72	0.62	0.56	0.93	0.88	0.00	0.00	1.97	1.27
time (sec)	N/A	0.332	2.955	1.028	0.034	0.112	0.000	0.000	0.247	0.806

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	150	107	96	159	151	0	0	291	176
N.S.	1	0.93	0.66	0.59	0.98	0.93	0.00	0.00	1.80	1.09
time (sec)	N/A	0.440	5.193	1.050	0.044	0.141	0.000	0.000	0.275	0.778

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	197	176	0	0	0	267	0	242	0
N.S.	1	0.89	0.80	0.00	0.00	0.00	1.21	0.00	1.10	0.00
time (sec)	N/A	0.593	13.861	0.000	0.000	0.000	3.284	0.000	0.304	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	174	179	0	0	0	129	0	174	0
N.S.	1	0.97	0.99	0.00	0.00	0.00	0.72	0.00	0.97	0.00
time (sec)	N/A	0.555	11.801	0.000	0.000	0.000	1.640	0.000	0.285	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	151	161	0	0	0	124	0	54	0
N.S.	1	1.08	1.15	0.00	0.00	0.00	0.89	0.00	0.39	0.00
time (sec)	N/A	0.498	13.103	0.000	0.000	0.000	1.774	0.000	0.217	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	164	171	0	0	0	0	0	111	0
N.S.	1	1.16	1.21	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.537	13.192	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	174	131	0	0	0	0	0	171	0
N.S.	1	1.10	0.83	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.604	15.121	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	197	170	0	0	0	0	0	231	0
N.S.	1	0.97	0.84	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.657	15.119	0.000	0.000	0.000	0.000	0.000	0.295	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	220	288	400	0	1962	0	0	190	0
N.S.	1	1.04	1.36	1.90	0.00	9.30	0.00	0.00	0.90	0.00
time (sec)	N/A	0.740	1.910	10.434	0.000	1.466	0.000	0.000	0.588	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	181	255	309	0	733	0	0	21	0
N.S.	1	1.05	1.47	1.79	0.00	4.24	0.00	0.00	0.12	0.00
time (sec)	N/A	0.548	1.187	4.208	0.000	0.135	0.000	0.000	0.246	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	178	212	0	0	0	0	30	0
N.S.	1	1.00	1.70	2.02	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.372	0.874	1.407	0.000	0.000	0.000	0.000	0.205	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	229	293	0	0	0	0	62	0
N.S.	1	1.09	1.71	2.19	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.457	1.799	1.462	0.000	0.000	0.000	0.000	0.206	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	206	621	300	0	0	0	0	104	0
N.S.	1	1.14	3.45	1.67	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.614	11.975	1.616	0.000	0.000	0.000	0.000	0.278	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	279	1172	349	0	0	0	0	146	0
N.S.	1	1.20	5.03	1.50	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.842	15.021	1.737	0.000	0.000	0.000	0.000	0.219	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	287	294	0	0	0	0	0	330	0
N.S.	1	0.91	0.93	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	1.073	10.711	0.000	0.000	0.000	0.000	0.000	0.760	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	239	346	0	0	0	0	0	190	0
N.S.	1	0.87	1.26	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.784	10.368	0.000	0.000	0.000	0.000	0.000	0.601	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	141	160	0	0	0	0	0	21	0
N.S.	1	0.85	0.96	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.478	10.208	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	225	161	0	0	0	0	0	30	0
N.S.	1	0.87	0.62	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.704	10.073	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	285	332	0	0	0	0	0	62	0
N.S.	1	0.94	1.09	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.919	10.288	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	358	430	0	0	0	0	0	104	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.209	10.939	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	286	349	479	0	2764	0	0	0	0
N.S.	1	1.02	1.25	1.71	0.00	9.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.976	3.446	2.471	0.000	11.756	0.000	0.000	2.503	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	244	341	377	0	1440	0	0	1016	0
N.S.	1	1.06	1.48	1.64	0.00	6.26	0.00	0.00	4.42	0.00
time (sec)	N/A	0.724	2.218	2.453	0.000	0.755	0.000	0.000	0.864	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	142	238	290	0	0	0	0	837	0
N.S.	1	1.05	1.76	2.15	0.00	0.00	0.00	0.00	6.20	0.00
time (sec)	N/A	0.448	1.660	1.566	0.000	0.000	0.000	0.000	0.867	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	251	311	0	0	0	0	50	0
N.S.	1	1.06	1.55	1.92	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.481	2.680	1.562	0.000	0.000	0.000	0.000	0.230	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	220	285	324	0	0	0	0	104	0
N.S.	1	1.07	1.39	1.58	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.626	5.125	1.664	0.000	0.000	0.000	0.000	0.273	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	295	1216	380	0	0	0	0	172	0
N.S.	1	1.11	4.57	1.43	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.856	14.528	1.861	0.000	0.000	0.000	0.000	0.247	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	310	392	0	0	0	0	0	0	0
N.S.	1	0.88	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.101	10.542	0.000	0.000	0.000	0.000	0.000	3.101	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	261	341	0	0	0	0	0	1387	0
N.S.	1	0.88	1.14	0.00	0.00	0.00	0.00	0.00	4.65	0.00
time (sec)	N/A	0.868	10.371	0.000	0.000	0.000	0.000	0.000	2.046	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	268	233	0	0	0	0	0	32	0
N.S.	1	0.87	0.76	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.852	10.252	0.000	0.000	0.000	0.000	0.000	1.005	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	330	300	337	0	0	0	0	0	50	0
N.S.	1	0.91	1.02	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.939	10.413	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	390	374	387	0	0	0	0	0	104	0
N.S.	1	0.96	0.99	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.248	10.643	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	61	0	177	0	0	25	0
N.S.	1	1.00	0.83	1.15	0.00	3.34	0.00	0.00	0.47	0.00
time (sec)	N/A	0.294	0.300	4.014	0.000	4.022	0.000	0.000	0.262	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	65	0	0	0	0	48	0
N.S.	1	1.00	0.84	1.14	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.303	0.598	2.808	0.000	0.000	0.000	0.000	0.231	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	1249	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	15.81	0.00
time (sec)	N/A	0.388	0.324	0.000	0.000	0.000	0.000	0.000	0.598	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	288	137	0	0	0	0	0	0	0
N.S.	1	1.02	0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.907	5.445	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	175	106	0	0	0	119	0	1442	0
N.S.	1	1.01	0.61	0.00	0.00	0.00	0.69	0.00	8.34	0.00
time (sec)	N/A	0.589	5.267	0.000	0.000	0.000	81.328	0.000	0.268	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	0	0	0	75	0	498	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.88	0.00	5.86	0.00
time (sec)	N/A	0.374	0.185	0.000	0.000	0.000	28.132	0.000	0.206	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.275	0.083	0.000	0.000	0.000	4.094	0.000	0.226	0.670

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	21	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.307	0.455	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	32	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.309	0.524	0.000	0.000	0.000	0.000	0.000	0.519	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [27] had the largest ratio of [.684211000000000014]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	17	0.118
2	A	2	2	1.00	17	0.118
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	15	0.133
5	A	10	9	1.34	17	0.529
6	A	10	9	1.33	17	0.529
7	A	11	10	1.27	17	0.588
8	A	2	2	1.00	19	0.105
9	A	2	2	1.00	19	0.105
10	A	2	2	1.00	19	0.105
11	A	2	2	1.00	17	0.118
12	A	2	2	1.32	19	0.105
13	A	2	2	1.30	19	0.105
14	A	11	10	1.15	19	0.526
15	A	2	2	1.23	19	0.105
16	A	2	2	1.27	19	0.105
17	A	2	2	1.32	19	0.105
18	A	10	9	1.34	17	0.529
19	A	10	9	1.30	19	0.474
20	A	11	10	1.27	19	0.526
21	A	2	2	1.21	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.24	19	0.105
23	A	2	2	1.28	19	0.105
24	A	2	2	1.30	19	0.105
25	A	10	9	1.33	17	0.529
26	A	12	11	1.27	19	0.579
27	A	14	13	1.21	19	0.684
28	A	4	4	0.99	21	0.190
29	A	3	3	0.97	19	0.158
30	A	8	8	1.29	21	0.381
31	A	10	10	1.44	21	0.476
32	A	3	3	1.07	21	0.143
33	A	2	2	1.00	19	0.105
34	A	6	6	1.38	21	0.286
35	A	9	9	1.41	21	0.429
36	A	4	4	1.20	21	0.190
37	A	3	3	1.14	21	0.143
38	A	2	2	1.00	19	0.105
39	A	10	10	1.48	21	0.476
40	A	11	11	1.37	21	0.524
41	A	1	1	1.00	15	0.067
42	A	9	9	1.01	23	0.391
43	A	8	8	0.98	23	0.348
44	A	7	7	0.97	23	0.304
45	A	4	4	1.00	23	0.174
46	A	8	8	0.96	23	0.348
47	A	9	9	1.02	23	0.391
48	A	11	11	1.45	22	0.500
49	A	9	9	1.31	22	0.409
50	A	7	7	1.41	22	0.318
51	A	11	11	1.52	22	0.500
52	A	13	13	1.43	22	0.591
53	A	11	11	0.99	23	0.478

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	10	10	0.98	23	0.435
55	A	9	9	0.97	23	0.391
56	A	9	9	0.98	23	0.391
57	A	9	9	0.92	23	0.391
58	A	11	11	0.96	23	0.478
59	A	13	13	1.00	23	0.565
60	A	5	4	0.98	25	0.160
61	A	1	1	1.00	25	0.040
62	C	3	3	0.64	24	0.125
63	A	5	5	1.12	21	0.238
64	A	3	3	1.00	19	0.158
65	A	2	2	1.00	21	0.095
66	A	2	2	1.00	21	0.095
67	A	5	5	1.12	21	0.238
68	A	3	3	1.00	19	0.158
69	A	2	2	1.00	21	0.095
70	A	2	2	1.00	21	0.095
71	A	5	5	1.15	21	0.238
72	A	3	3	1.19	19	0.158
73	A	2	2	1.00	21	0.095
74	A	2	2	1.00	21	0.095
75	A	5	5	1.15	21	0.238
76	A	3	3	1.12	19	0.158
77	A	2	2	1.00	21	0.095
78	A	2	2	1.00	21	0.095
79	A	7	6	0.93	19	0.316
80	A	6	5	0.99	19	0.263
81	A	6	5	1.05	19	0.263
82	A	2	2	0.77	19	0.105
83	A	3	3	0.97	19	0.158
84	A	4	4	0.95	19	0.211
85	A	8	7	0.93	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	7	6	0.97	19	0.316
87	A	6	5	1.00	19	0.263
88	A	6	5	1.00	19	0.263
89	A	7	6	1.00	19	0.316
90	A	8	7	0.98	19	0.368
91	A	8	7	0.89	21	0.333
92	A	7	6	0.97	21	0.286
93	A	8	7	1.14	21	0.333
94	A	7	6	1.13	21	0.286
95	A	3	3	0.72	21	0.143
96	A	4	4	0.93	21	0.190
97	A	9	8	0.89	21	0.381
98	A	8	7	0.97	21	0.333
99	A	7	6	1.08	21	0.286
100	A	7	6	1.16	21	0.286
101	A	7	6	1.10	21	0.286
102	A	8	7	0.97	21	0.333
103	A	12	11	1.04	21	0.524
104	A	10	9	1.05	21	0.429
105	A	5	4	1.00	21	0.190
106	A	6	5	1.09	21	0.238
107	A	9	8	1.14	21	0.381
108	A	11	10	1.20	21	0.476
109	A	14	13	0.91	21	0.619
110	A	11	10	0.87	21	0.476
111	A	5	4	0.85	21	0.190
112	A	10	9	0.87	21	0.429
113	A	12	11	0.94	21	0.524
114	A	14	13	1.00	21	0.619
115	A	13	12	1.02	21	0.571
116	A	11	10	1.06	21	0.476
117	A	6	5	1.05	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.06	21	0.238
119	A	8	7	1.07	21	0.333
120	A	10	9	1.11	21	0.429
121	A	13	12	0.88	21	0.571
122	A	11	10	0.88	21	0.476
123	A	12	11	0.87	21	0.524
124	A	11	10	0.91	21	0.476
125	A	13	12	0.96	21	0.571
126	A	5	4	1.00	17	0.235
127	A	5	4	1.00	26	0.154
128	A	3	3	1.00	19	0.158
129	A	7	7	1.02	19	0.368
130	A	5	5	1.01	19	0.263
131	A	3	3	1.00	17	0.176
132	A	2	2	1.00	9	0.222
133	A	2	2	1.00	19	0.105
134	A	2	2	1.00	19	0.105

CHAPTER 3

Listing of integrals

3.1 $\int (a+bx^4)(c+dx^4)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^4)(c+dx^4)^4 \, dx = ac^4x + \frac{1}{5}c^3(bc+4ad)x^5 + \frac{2}{9}c^2d(2bc+3ad)x^9 + \frac{2}{13}cd^2(3bc+2ad)x^{13} + \frac{1}{17}d^3(4bc+ad)x^{17} + \frac{1}{21}bd^4x^{21}$$

output
```
a*c^4*x+1/5*c^3*(4*a*d+b*c)*x^5+2/9*c^2*d*(3*a*d+2*b*c)*x^9+2/13*c*d^2*(2*
a*d+3*b*c)*x^13+1/17*d^3*(a*d+4*b*c)*x^17+1/21*b*d^4*x^21
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^4 \, dx = ac^4x + \frac{1}{5}c^3(bc+4ad)x^5 + \frac{2}{9}c^2d(2bc+3ad)x^9 + \frac{2}{13}cd^2(3bc+2ad)x^{13} + \frac{1}{17}d^3(4bc+ad)x^{17} + \frac{1}{21}bd^4x^{21}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2
*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^2
1)/21
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^4 dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(c^3x^4(4ad+bc)+2c^2dx^8(3ad+2bc)+d^3x^{16}(ad+4bc)+2cd^2x^{12}(2ad+3bc)+ac^4+bd^4x^{20}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

output

```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2
*c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^2
1)/21
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$a\,c^4x+\left(\frac{4}{5}a\,c^3d+\frac{1}{5}b\,c^4\right)x^5+\left(\frac{2}{3}a\,c^2d^2+\frac{4}{9}b\,c^3d\right)x^9+\left(\frac{4}{13}ac\,d^3+\frac{6}{13}b\,c^2d^2\right)x^{13}+\left(\frac{1}{17}a\,d^4+\right.$
default	$\frac{b\,d^4x^{21}}{21}+\frac{(a\,d^4+4bc\,d^3)x^{17}}{17}+\frac{(4ac\,d^3+6b\,c^2d^2)x^{13}}{13}+\frac{(6a\,c^2d^2+4b\,c^3d)x^9}{9}+\frac{(4a\,c^3d+b\,c^4)x^5}{5}+a\,c^4x$
gosper	$a\,c^4x+\frac{4}{5}x^5a\,c^3d+\frac{1}{5}x^5b\,c^4+\frac{2}{3}x^9a\,c^2d^2+\frac{4}{9}x^9b\,c^3d+\frac{4}{13}x^{13}ac\,d^3+\frac{6}{13}x^{13}b\,c^2d^2+\frac{1}{17}x^{17}a\,d^4$
risch	$a\,c^4x+\frac{4}{5}x^5a\,c^3d+\frac{1}{5}x^5b\,c^4+\frac{2}{3}x^9a\,c^2d^2+\frac{4}{9}x^9b\,c^3d+\frac{4}{13}x^{13}ac\,d^3+\frac{6}{13}x^{13}b\,c^2d^2+\frac{1}{17}x^{17}a\,d^4$
parallelrisch	$a\,c^4x+\frac{4}{5}x^5a\,c^3d+\frac{1}{5}x^5b\,c^4+\frac{2}{3}x^9a\,c^2d^2+\frac{4}{9}x^9b\,c^3d+\frac{4}{13}x^{13}ac\,d^3+\frac{6}{13}x^{13}b\,c^2d^2+\frac{1}{17}x^{17}a\,d^4$
orering	$\frac{x(3315b\,d^4x^{20}+4095a\,d^4x^{16}+16380bc\,d^3x^{16}+21420ac\,d^3x^{12}+32130b\,c^2d^2x^{12}+46410a\,c^2d^2x^8+30940b\,c^3d\,x^8+55692a\,c^3d\,x}{69615}$

input
```
int((b*x^4+a)*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+(4/5*a*c^3*d+1/5*b*c^4)*x^5+(2/3*a*c^2*d^2+4/9*b*c^3*d)*x^9+(4/13*
a*c*d^3+6/13*b*c^2*d^2)*x^13+(1/17*a*d^4+4/17*b*c*d^3)*x^17+1/21*b*d^4*x^2
1
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)(c+dx^4)^4\,dx = \frac{1}{21}\,bd^4x^{21} + \frac{1}{17}\,(4\,bcd^3 + ad^4)x^{17} + \frac{2}{13}\,(3\,bc^2d^2 + 2\,acd^3)x^{13} + \frac{2}{9}\,(2\,bc^3d + 3\,ac^2d^2)x^9 + ac^4x + \frac{1}{5}\,(bc^4 + 4\,ac^3d)x^5$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

output
```
1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*
c*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4
*a*c^3*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int (a+bx^4)(c+dx^4)^4\,dx = ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\cdot\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\cdot\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\cdot\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

output
```
a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*
a*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) +
x**5*(4*a*c**3*d/5 + b*c**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^4)\left(c+dx^4\right)^4\,dx = \frac{1}{21}\,bd^4x^{21} + \frac{1}{17}\left(4\,bcd^3 + ad^4\right)x^{17} + \frac{2}{13}\left(3\,bc^2d^2 + 2\,acd^3\right)x^{13} + \frac{2}{9}\left(2\,bc^3d + 3\,ac^2d^2\right)x^9 + ac^4x + \frac{1}{5}\left(bc^4 + 4\,ac^3d\right)x^5$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

output
```
1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*
c*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4
*a*c^3*d)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int (a+bx^4)\left(c+dx^4\right)^4\,dx = \frac{1}{21}\,bd^4x^{21} + \frac{4}{17}\,bcd^3x^{17} + \frac{1}{17}\,ad^4x^{17} + \frac{6}{13}\,bc^2d^2x^{13} + \frac{4}{13}\,acd^3x^{13} + \frac{4}{9}\,bc^3dx^9 + \frac{2}{3}\,ac^2d^2x^9 + \frac{1}{5}\,bc^4x^5 + \frac{4}{5}\,ac^3dx^5 + ac^4x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

output
```
1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^1
3 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^
5 + 4/5*a*c^3*d*x^5 + a*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int (a+bx^4)(c+dx^4)^4\,dx = x^5\left(\frac{b\,c^4}{5}+\frac{4\,a\,d\,c^3}{5}\right)+x^{17}\left(\frac{a\,d^4}{17}+\frac{4\,b\,c\,d^3}{17}\right)+\frac{b\,d^4\,x^{21}}{21}+a\,c^4\,x+\frac{2\,c^2\,d\,x^9\,(3\,a\,d+2\,b\,c)}{9}+\frac{2\,c\,d^2\,x^{13}\,(2\,a\,d+3\,b\,c)}{13}$$

input
```
int((a + b*x^4)*(c + d*x^4)^4,x)
```

output
```
x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*
d^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(
2*a*d + 3*b*c))/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int (a+bx^4)(c+dx^4)^4\,dx = \frac{x(3315b\,d^4x^{20}+4095a\,d^4x^{16}+16380bc\,d^3x^{16}+21420ac\,d^3x^{12}+32130b\,c^2d^2x^{12}+46410a\,c^2d^2x^8+3094}{69615}$$

input
```
int((b*x^4+a)*(d*x^4+c)^4,x)
```

output
```
(x*(69615*a*c**4 + 55692*a*c**3*d*x**4 + 46410*a*c**2*d**2*x**8 + 21420*a*
c*d**3*x**12 + 4095*a*d**4*x**16 + 13923*b*c**4*x**4 + 30940*b*c**3*d*x**8
 + 32130*b*c**2*d**2*x**12 + 16380*b*c*d**3*x**16 + 3315*b*d**4*x**20))/69
615
```

3.2 $\int (a+bx^4)(c+dx^4)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^4)(c+dx^4)^3 \, dx = ac^3x + \frac{1}{5}c^2(bc+3ad)x^5 + \frac{1}{3}cd(bc+ad)x^9 + \frac{1}{13}d^2(3bc+ad)x^{13} + \frac{1}{17}bd^3x^{17}$$

output
```
a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)
*x^13+1/17*b*d^3*x^17
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3 \, dx = ac^3x + \frac{1}{5}c^2(bc+3ad)x^5 + \frac{1}{3}cd(bc+ad)x^9 + \frac{1}{13}d^2(3bc+ad)x^{13} + \frac{1}{17}bd^3x^{17}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*
c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)\left(c+dx^4\right)^3 dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(c^2x^4(3ad+bc)+d^2x^{12}(ad+3bc)+3cdx^8(ad+bc)+ac^3+bd^3x^{16}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*
c + a*d)*x^13)/13 + (b*d^3*x^17)/17
```


Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	siz
norman	$a\,c^3x+\left(\frac{3}{5}a\,c^2d+\frac{1}{5}c^3b\right)x^5+\left(\frac{1}{3}ac\,d^2+\frac{1}{3}b\,c^2d\right)x^9+\left(\frac{1}{13}a\,d^3+\frac{3}{13}bc\,d^2\right)x^{13}+\frac{b\,d^3x^{17}}{17}$	72
default	$\frac{b\,d^3x^{17}}{17}+\frac{(a\,d^3+3bc\,d^2)x^{13}}{13}+\frac{(3ac\,d^2+3b\,c^2d)x^9}{9}+\frac{(3a\,c^2d+c^3b)x^5}{5}+a\,c^3x$	73
gosper	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75
risch	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75
parallelrisch	$a\,c^3x+\frac{3}{5}x^5a\,c^2d+\frac{1}{5}x^5c^3b+\frac{1}{3}x^9ac\,d^2+\frac{1}{3}x^9b\,c^2d+\frac{1}{13}x^{13}a\,d^3+\frac{3}{13}x^{13}bc\,d^2+\frac{1}{17}b\,d^3x^{17}$	75
orering	$\frac{x(195b\,d^3x^{16}+255a\,d^3x^{12}+765bc\,d^2x^{12}+1105ac\,d^2x^8+1105b\,c^2d\,x^8+1989a\,c^2d\,x^4+663b\,c^3x^4+3315c^3a)}{3315}$	78

input
```
int((b*x^4+a)*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*c^3*x+(3/5*a*c^2*d+1/5*c^3*b)*x^5+(1/3*a*c*d^2+1/3*b*c^2*d)*x^9+(1/13*a*
d^3+3/13*b*c*d^2)*x^13+1/17*b*d^3*x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)\left(c+dx^4\right)^3\,dx=\frac{1}{17}bd^3x^{17}+\frac{1}{13}\left(3\,bcd^2+ad^3\right)x^{13}+\frac{1}{3}\left(bc^2d+acd^2\right)x^9+\frac{1}{5}\left(bc^3+3\,ac^2d\right)x^5+ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*
x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int (a+bx^4)(c+dx^4)^3\,dx = ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13} + \frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3} + \frac{bc^2d}{3}\right) + x^5\cdot\left(\frac{3ac^2d}{5} + \frac{bc^3}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

output
```
a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c
*d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^3\,dx = \frac{1}{17}\,bd^3x^{17} + \frac{1}{13}\,(3\,bcd^2 + ad^3)x^{13} + \frac{1}{3}\,(bc^2d + acd^2)x^9 + \frac{1}{5}\,(bc^3 + 3\,ac^2d)x^5 + ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*
x^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int (a+bx^4)\,(c+dx^4)^3\,dx = \frac{1}{17}bd^3x^{17} + \frac{3}{13}bcd^2x^{13} + \frac{1}{13}ad^3x^{13} + \frac{1}{3}bc^2dx^9 + \frac{1}{3}acd^2x^9 + \frac{1}{5}bc^3x^5 + \frac{3}{5}ac^2dx^5 + ac^3x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 +
1/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (a+bx^4)\,(c+dx^4)^3\,dx = x^5\left(\frac{b\,c^3}{5}+\frac{3\,a\,d\,c^2}{5}\right)+x^{13}\left(\frac{a\,d^3}{13}+\frac{3\,b\,c\,d^2}{13}\right) + \frac{b\,d^3\,x^{17}}{17}+a\,c^3\,x+\frac{c\,d\,x^9\,(a\,d+b\,c)}{3}$$

input
```
int((a + b*x^4)*(c + d*x^4)^3,x)
```

output
```
x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*
d^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx$$
$$=\frac{x(195b\,d^3x^{16}+255a\,d^3x^{12}+765bc\,d^2x^{12}+1105ac\,d^2x^8+1105b\,c^2d\,x^8+1989a\,c^2d\,x^4+663b\,c^3x^4+331}{3315}$$

input
```
int((b*x^4+a)*(d*x^4+c)^3,x)
```

output
```
(x*(3315*a*c**3 + 1989*a*c**2*d*x**4 + 1105*a*c*d**2*x**8 + 255*a*d**3*x**
12 + 663*b*c**3*x**4 + 1105*b*c**2*d*x**8 + 765*b*c*d**2*x**12 + 195*b*d**
3*x**16))/3315
```

3.3 $\int (a+bx^4)(c+dx^4)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)(c+dx^4)^2 \, dx = ac^2x + \frac{1}{5}c(bc+2ad)x^5 + \frac{1}{9}d(2bc+ad)x^9 + \frac{1}{13}bd^2x^{13}$$

output
```
a*c^2*x+1/5*c*(2*a*d+b*c)*x^5+1/9*d*(a*d+2*b*c)*x^9+1/13*b*d^2*x^13
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)^2 \, dx = ac^2x + \frac{1}{5}c(bc+2ad)x^5 + \frac{1}{9}d(2bc+ad)x^9 + \frac{1}{13}bd^2x^{13}$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)
/13
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4) (c+dx^4)^2 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^8(ad+2bc)+cx^4(2ad+bc)+ac^2+bd^2x^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

input
```
Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)
/13
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^{2} x^{13}}{13}+\frac{\left(a\,d^{2}+2 b c d \right) x^{9}}{9}+\frac{\left(2 a c d +b\,c^{2}\right) x^{5}}{5}+a\,c^{2} x$	49
norman	$\frac{b\,d^{2} x^{13}}{13}+\left(\frac{1}{9} a\,d^{2}+\frac{2}{9} b c d \right) x^{9}+\left(\frac{2}{5} a c d +\frac{1}{5} b\,c^{2}\right) x^{5}+a\,c^{2} x$	49
gosper	$\frac{1}{13} b\,d^{2} x^{13}+\frac{1}{9} x^{9} a\,d^{2}+\frac{2}{9} x^{9} b c d +\frac{2}{5} x^{5} a c d +\frac{1}{5} x^{5} b\,c^{2}+a\,c^{2} x$	51
risch	$\frac{1}{13} b\,d^{2} x^{13}+\frac{1}{9} x^{9} a\,d^{2}+\frac{2}{9} x^{9} b c d +\frac{2}{5} x^{5} a c d +\frac{1}{5} x^{5} b\,c^{2}+a\,c^{2} x$	51
parallelrisch	$\frac{1}{13} b\,d^{2} x^{13}+\frac{1}{9} x^{9} a\,d^{2}+\frac{2}{9} x^{9} b c d +\frac{2}{5} x^{5} a c d +\frac{1}{5} x^{5} b\,c^{2}+a\,c^{2} x$	51
orering	$\frac{x \left(45 b\,d^{2} x^{12}+65 a\,d^{2} x^{8}+130 b c d\,x^{8}+234 x^{4} a c d +117 x^{4} b\,c^{2}+585 a\,c^{2}\right)}{585}$	54

input
```
int((b*x^4+a)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right) \left(c+dx^4\right)^2 \, dx = \frac{1}{13} \, b d^{2} x^{13} + \frac{1}{9} \, {\left(2 \, b c d + a d^{2}\right)} x^{9} + \frac{1}{5} \, {\left(b c^{2} + 2 \, a c d\right)} x^{5} + a c^{2} x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 +
a*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx = ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\cdot\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

output
```
a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
 + b*c**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx = \frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\,\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\,\left(bc^2+2\,acd\right)x^5+ac^2x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 +
a*c^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2\,dx = \frac{1}{13}\,bd^2x^{13}+\frac{2}{9}\,bcdx^9+\frac{1}{9}\,ad^2x^9+\frac{1}{5}\,bc^2x^5+\frac{2}{5}\,acdx^5+ac^2x$$

input
```
integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*
d*x^5 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)(c+dx^4)^2\,dx = x^5\left(\frac{b\,c^2}{5}+\frac{2\,a\,d\,c}{5}\right)+x^9\left(\frac{a\,d^2}{9}+\frac{2\,b\,c\,d}{9}\right)+\frac{b\,d^2\,x^{13}}{13}+a\,c^2\,x$$

input
```
int((a + b*x^4)*(c + d*x^4)^2,x)
```

output
```
x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^1
3)/13 + a*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (a+bx^4)(c+dx^4)^2\,dx = \frac{x(45b\,d^2x^{12}+65a\,d^2x^8+130bcd\,x^8+234acd\,x^4+117b\,c^2x^4+585a\,c^2)}{585}$$

input
```
int((b*x^4+a)*(d*x^4+c)^2,x)
```

output
```
(x*(585*a*c**2 + 234*a*c*d*x**4 + 65*a*d**2*x**8 + 117*b*c**2*x**4 + 130*b
*c*d*x**8 + 45*b*d**2*x**12))/585
```

3.4 $\int (a+bx^4)(c+dx^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^4)(c+dx^4)\,dx = acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

output
```
a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^4)(c+dx^4)\,dx = acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9$$

input
```
Integrate[(a + b*x^4)*(c + d*x^4),x]
```

output
```
a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx^4\right) \, dx$$

$$\Big\downarrow \ 897$$

$$\int \left(x^4(ad+bc)+ac+bdx^8\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

input
```
Int[(a + b*x^4)*(c + d*x^4),x]
```

output
```
a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^5}{5}+\frac{bd\,x^9}{9}$	25
norman	$\frac{bd\,x^9}{9}+\left(\frac{ad}{5}+\frac{bc}{5}\right)x^5+acx$	26
gosper	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
risch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
parallelrisch	$\frac{1}{9}bd\,x^9+\frac{1}{5}x^5ad+\frac{1}{5}x^5bc+acx$	27
orering	$\frac{x(5bd\,x^8+9ad\,x^4+9bc\,x^4+45ac)}{45}$	30

input
```
int((b*x^4+a)*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx=\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^4)(c+dx^4)\,dx = acx + \frac{bdx^9}{9} + x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

input
```
integrate((b*x**4+a)*(d*x**4+c),x)
```

output
```
a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{1}{9}\,bdx^9 + \frac{1}{5}\,(bc+ad)x^5 + acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
```

output
```
1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^4)(c+dx^4)\,dx = \frac{1}{9}\,bdx^9 + \frac{1}{5}\,bcx^5 + \frac{1}{5}\,adx^5 + acx$$

input
```
integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
```

output
```
1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a + bx^4) (c + dx^4) \, dx = \frac{b\,d\,x^9}{9} + \left(\frac{a\,d}{5} + \frac{b\,c}{5}\right) x^5 + a\,c\,x$$

input
```
int((a + b*x^4)*(c + d*x^4),x)
```

output
```
x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int (a + bx^4) (c + dx^4) \, dx = \frac{x(5bd\,x^8 + 9ad\,x^4 + 9bc\,x^4 + 45ac)}{45}$$

input
```
int((b*x^4+a)*(d*x^4+c),x)
```

output
```
(x*(45*a*c + 9*a*d*x**4 + 9*b*c*x**4 + 5*b*d*x**8))/45
```

3.5 $\int \frac{a+bx^4}{c+dx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d}+\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}$$

output
```
b*x/d-1/4*(-a*d+b*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/
d^(5/4)-1/4*(-a*d+b*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)
/d^(5/4)-1/4*(-a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)
*x^2))*2^(1/2)/c^(3/4)/d^(5/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{8bc^{3/4}\sqrt[4]{d}x+2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+\sqrt{2}(bc-ad)}{8c^{3/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^4)/(c + d*x^4),x]
```

output
```
(8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2])/(8*c^(3/4)*d^(5/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{c+dx^4}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{755}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$\downarrow$ 27

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

$\downarrow$ 1103

$$\frac{bx}{d}-\frac{(bc-ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}$$

input
```
Int[(a + b*x^4)/(c + d*x^4),x]
```

output
```
(b*x)/d - ((b*c - a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2
]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(
1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{bx}{d}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\_Z^{4}+c\right)}\frac{(ad-bc)\ln\left(x-\_R\right)}{\_R^{3}}}{4d^{2}}$	42
default	$\frac{bx}{d}+\frac{(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8dc}$	120

input
```
int((b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b*x/d+1/4/d^2*sum((a*d-b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.41

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}\log\left(cd\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)+}{}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + I*d*(-
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c
^3*d^5))^(1/4)*log(I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - I*d*(-(b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5)
)^(1/4)*log(-I*c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) - d*(-(b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*
log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} + \operatorname{RootSum}\left(256t^4c^3d^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{4tcd}{ad-bc}+x\right)\right)\right)$$

input
```
integrate((b*x**4+a)/(d*x**4+c),x)
```

output
```
b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2
*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/
(a*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,d}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(
2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)
*(b*c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/
4)*d^(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4
)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.49

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}bc - (cd^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^2}$$

input
```
integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^
3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^
(1/4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/
4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)
*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) +
 sqrt(c/d))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.39

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)/(c + d*x^4),x)
```

output
```
(b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*
d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*
(-c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i
)/(4*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - (
(16*b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*
c))/(4*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) +
 ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d -
b*c))/(4*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (a
tan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c
*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d
^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 1
6*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3
/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^
2 - 16*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*
b*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c
)*1i)/(4*(-c)^(3/4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\int \frac{a+bx^4}{c+dx^4}\,dx$$
$$= \frac{-2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a+2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b+2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a -}{}$$

input
```
int((b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d
)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((
d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c
+ 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*
x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d*
*(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c -
d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + s
qrt(d)*x**2)*a*d + d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt
(2)*x + sqrt(c) + sqrt(d)*x**2)*b*c + d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/
4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a*d - d**(3/4)*c**(1/4)*sq
rt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b*c + 8*b*
c*d*x)/(8*c*d**2)
```

3.6 $\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = -\frac{(bc-ad)x}{4cd\,(c+dx^4)} - \frac{(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}$$

output
```
-1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*arctan(-1+2^(1/2)*d^(1/4)
*x/c^(1/4))*2^(1/2)/c^(7/4)/d^(5/4)+1/16*(3*a*d+b*c)*arctan(1+2^(1/2)*d^(1
/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/d^(5/4)+1/16*(3*a*d+b*c)*arctanh(2^(1/2)*c^
(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/d^(5/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{-\frac{8c^{3/4}\sqrt[4]{d}(bc-ad)x}{c+dx^4} - 2\sqrt{2}(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt{2}(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) - \sqrt{2}(}{32c^{7/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

output
```
((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(3ad+bc)\int \frac{1}{dx^4+c}dx}{4cd} - \frac{x(bc-ad)}{4cd\,(c+dx^4)}$$

$$\Big\downarrow \text{755}$$

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(3ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(3ad+bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(3ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(3ad+bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4cd}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

input

```
Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

output

```
-1/4*((b*c - a*d)*x)/(c*d*(c + d*x^4)) + ((b*c + 3*a*d)*((-(ArcTan[1 - (Sq
rt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]
*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4))
 + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/
4)*d^(1/4)))/(2*Sqrt[c])))/(4*c*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(3ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{16c\,d^2}$	65
default	$\frac{(ad-bc)x}{4dc(d\,x^4+c)}+\frac{(3ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2d}$	140

input
```
int((b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*d-b*c)/d/c*x/(d*x^4+c)+1/16/c/d^2*sum((3*a*d+b*c)/_R^3*ln(x-_R),_R=
RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.48

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{(cd^2x^4+c^2d)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,\right.\right.}{c^7d^5}}{}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

output

```
1/16*((c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 1
2*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^
5))^(1/4) + (b*c + 3*a*d)*x) - (-I*c*d^2*x^4 - I*c^2*d)*(-(b^4*c^4 + 12*a*
b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^
(1/4)*log(I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a
^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - (I*c*d^2*x^
4 + I*c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*
c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-I*c^2*d*(-(b^4*c^4 + 12*a*b^3*c^
3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)
+ (b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*
a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*
d*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*
a^4*d^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4
 + c^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{x(ad-bc)}{4c^2d+4cd^2x^4} + \text{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4, \left(t \mapsto t\log\left(\frac{16tc^2d}{3ad+bc}\right.\right.\right.$$

input

```
integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

output

```
x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
 81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**
3*d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x}{4\left(cd^2x^4+c^2d\right)} + \frac{\frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc+3\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x\cdots\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\,cd}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arc
tan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(
d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqr
t(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d
)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4))
)/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.43

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,bc+3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^2} - \frac{bcx-adx}{4\,(dx^4+c)cd}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)
^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^
(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3
)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32
*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.98

$$\int \frac{a+bx^4}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)/(c + d*x^4)^2,x)
```

output

```
(atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c
)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(
-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) +
 ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d +
b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/
4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d
+ 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^
(7/4)*d^(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)
/(8*(-c)^(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2
))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5
/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(
-c)^(7/4)*d^(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d
^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)
) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(
12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(
-c)^(7/4)*d^(5/4))))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c
))/(4*c*d*(c + d*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.29

$$\int \frac{a + bx^4}{(c + dx^4)^2} \, dx = \text{Too large to display}$$

input

```
int((b*x^4+a)/(d*x^4+c)^2,x)
```

output
```
( - 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d
)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*c*d - 6*d**(3/4)*c**(1/4)*sqrt(2)*atan
((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*
d**2*x**4 - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) -
2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c**2 - 2*d**(3/4)*c**(1/4)*sqr
t(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqr
t(2)))*b*c*d*x**4 + 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sq
rt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*c*d + 6*d**(3/4)*c**(1
/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1
/4)*sqrt(2)))*a*d**2*x**4 + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**
(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c**2 + 2*d**(3
/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1
/4)*c**(1/4)*sqrt(2)))*b*c*d*x**4 - 3*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**
(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a*c*d - 3*d**(3/4)*c**(
1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*
a*d**2*x**4 - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x
 + sqrt(c) + sqrt(d)*x**2)*b*c**2 - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1
/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b*c*d*x**4 + 3*d**(3/4)*c
**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*
a*c*d + 3*d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + s...
```

3.7 $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 214

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x}{8cd(c+dx^4)^2}+\frac{(bc+7ad)x}{32c^2d(c+dx^4)} - \frac{3(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}$$

output
```
-1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128
*(7*a*d+b*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(11/4)/d^(5/4)
+3/128*(7*a*d+b*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(11/4)/d^
(5/4)+3/128*(7*a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)
*x^2))*2^(1/2)/c^(11/4)/d^(5/4)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{-\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)x}{(c+dx^4)^2}+\frac{8c^{3/4}\sqrt[4]{d}(bc+7ad)x}{c+dx^4}-6\sqrt{2}(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+6\sqrt{2}(bc+7ad)\arctan\left(1\right.}{25}$$

input

```
Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

output

```
((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b
*c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)
*x + Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {910, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(7ad+bc)\int \frac{1}{(dx^4+c)^2}dx}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(7ad+bc)\left(\frac{3\int\frac{1}{dx^4+c}dx}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\,(c+dx^4)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{}{4c(}\right)}{8cd}$$

$$\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\big\downarrow$ 25

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+d}\right)}{8cd}$$

$$\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\big\downarrow$ 27

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx^4)}\right)}{8cd}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{(7ad+bc)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{4c}+\frac{x}{4c(c+dx\cdots}\right.}{8cd}$$

$$\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

input
```
Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

output
```
-1/8*((b*c - a*d)*x)/(c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*(x/(4*c*(c + d*x
^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(
1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))
)/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(4*c)))/(8*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(dx^4+c)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(d\_Z^4+c\right)}\frac{(7ad+bc)\ln\left(x-\_R\right)}{\_R^3}\right)}{128c^2d^2}$
default	$\frac{\frac{(7ad+bc)x^5}{32c^2}+\frac{(11ad-3bc)x}{32cd}}{(dx^4+c)^2}+\frac{3(7ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{256c^3d}$

input
```
int((b*x^4+a)/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+3/128/c^2
/d^2*sum((7*a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.45

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*
a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(
b*c + 7*a*d)*x) - 3*(-I*c^2*d^3*x^8 - 2*I*c^3*d^2*x^4 - I*c^4*d)*(-(b^4*c^
4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4
)/(c^11*d^5))^(1/4)*log(3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^
2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c +
7*a*d)*x) - 3*(I*c^2*d^3*x^8 + 2*I*c^3*d^2*x^4 + I*c^4*d)*(-(b^4*c^4 + 28*
a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11
*d^5))^(1/4)*log(-3*I*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*
d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)
*x) - 3*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d
+ 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)
*log(-3*c^3*d*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3
*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c
^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx = \frac{x^5\cdot\left(7ad^2+bcd\right)+x(11acd-3bc^2)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}$$
$$+\mathrm{RootSum}\Big(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2268ab^3c^3d+81b^4c^4,$$

input
```
integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

output
```
(x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d
**2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 19448
1*a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b*
*3*c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c)
 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{(bcd+7\,ad^2)x^5-(3\,bc^2-11\,acd)x}{32\,(c^2d^3x^8+2\,c^3d^2x^4+c^4d)} + \frac{3\left(\frac{2\,\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\,\sqrt{2}(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc+7\,ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right.}{256\,c^2d}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3
*d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*s
qrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
 - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*s
qrt(d))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)
*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.34

$$\int \frac{a+bx^4}{(c+dx^4)^3}\,dx = \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} - \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+7\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} + \frac{bcdx^5+7\,ad^2x^5-3\,bc^2x+11\,acdx}{32\left(dx^4+c\right)^2c^2d}$$

input
```
integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

output
```
3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^
3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d
)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*
d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3
/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*
c^2*x + 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.56

$$\int \frac{a+bx^4}{(c+dx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((a + b*x^4)/(c + d*x^4)^3,x)
```

output
```
((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*
x^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/
(256*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)
))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*
c^2*d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(
256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3
*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*
c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c
)^(11/4)*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256
*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(
7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4
)*d^(5/4)) - (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4))
)*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^
2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(2
56*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*
d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^
(11/4)*d^(5/4)) - (((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4
) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.44

$$\int \frac{a+bx^4}{(c+dx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((b*x^4+a)/(d*x^4+c)^3,x)
```

output
```
( - 42*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*c**2*d - 84*d**(3/4)*c**(1/4)*sqrt(2)
*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)
))*a*c*d**2*x**4 - 42*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sq
rt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d**3*x**8 - 6*d**(3/4)
*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)
*c**(1/4)*sqrt(2)))*b*c**3 - 12*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c
**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c**2*d*x**4
- 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*
x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c*d**2*x**8 + 42*d**(3/4)*c**(1/4)*sqrt(
2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(
2)))*a*c**2*d + 84*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(
2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*c*d**2*x**4 + 42*d**(3/4)
*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)
*c**(1/4)*sqrt(2)))*a*d**3*x**8 + 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/
4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c**3 + 1
2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)
/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c**2*d*x**4 + 6*d**(3/4)*c**(1/4)*sqrt(2)*
atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))
)*b*c*d**2*x**8 - 21*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)...
```

3.8 $\int (a+bx^4)^2 (c+dx^4)^4 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = a^2c^4x + \frac{2}{5}ac^3(bc+2ad)x^5 + \frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 + \frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} + \frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} + \frac{2}{21}bd^3(2bc+ad)x^{21} + \frac{1}{25}b^2d^4x^{25}$$

output
```
a^2*c^4*x+2/5*a*c^3*(2*a*d+b*c)*x^5+1/9*c^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*
x^9+4/13*c*d*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^13+1/17*d^2*(a^2*d^2+8*a*b*c*d+
6*b^2*c^2)*x^17+2/21*b*d^3*(a*d+2*b*c)*x^21+1/25*b^2*d^4*x^25
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9 +\frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x^{13} +\frac{1}{17}d^2\left(6b^2c^2+8abcd+a^2d^2\right)x^{17} +\frac{2}{21}bd^3(2bc+ad)x^{21}+\frac{1}{25}b^2d^4x^{25}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

output
```
a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*
a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(
6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/2
1 + (b^2*d^4*x^25)/25
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(d^2x^{16}\left(a^2d^2+8abcd+6b^2c^2\right)+4cdx^{12}\left(a^2d^2+3abcd+b^2c^2\right)+c^2x^8\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4+2ac^3x^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

output
```
a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*
a^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(
6*b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/2
1 + (b^2*d^4*x^25)/25
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
norman	$\left(\frac{2}{3}a^2c^2d^2+\frac{8}{9}ab\,c^3d+\frac{1}{9}b^2c^4\right)x^9+a^2c^4x+\left(\frac{4}{5}a^2c^3d+\frac{2}{5}ab\,c^4\right)x^5+\left(\frac{1}{17}a^2d^4+\frac{8}{17}abc\,d^3+\frac{6}{17}b\right.$
default	$\frac{b^2d^4x^{25}}{25}+\frac{(2ab\,d^4+4b^2c\,d^3)x^{21}}{21}+\frac{(a^2d^4+8abc\,d^3+6b^2c^2d^2)x^{17}}{17}+\frac{(4a^2c\,d^3+12ab\,c^2d^2+4b^2c^3d)x^{13}}{13}+\frac{(6a^2c^2d^2+8ab}{9}$
gosper	$\frac{2}{3}x^9a^2c^2d^2+\frac{8}{9}x^9ab\,c^3d+\frac{1}{9}x^9b^2c^4+a^2c^4x+\frac{4}{5}x^5a^2c^3d+\frac{2}{5}x^5ab\,c^4+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d$
risch	$\frac{2}{3}x^9a^2c^2d^2+\frac{8}{9}x^9ab\,c^3d+\frac{1}{9}x^9b^2c^4+a^2c^4x+\frac{4}{5}x^5a^2c^3d+\frac{2}{5}x^5ab\,c^4+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d$
parallelrisch	$\frac{2}{3}x^9a^2c^2d^2+\frac{8}{9}x^9ab\,c^3d+\frac{1}{9}x^9b^2c^4+a^2c^4x+\frac{4}{5}x^5a^2c^3d+\frac{2}{5}x^5ab\,c^4+\frac{1}{17}x^{17}a^2d^4+\frac{8}{17}x^{17}abc\,d$
orering	$\frac{x(13923b^2d^4x^{24}+33150ab\,d^4x^{20}+66300b^2c\,d^3x^{20}+20475a^2d^4x^{16}+163800abc\,d^3x^{16}+122850b^2c^2d^2x^{16}+107100a^2c\,d^3x^{12}+}{}$

input
```
int((b*x^4+a)^2*(d*x^4+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(2/3*a^2*c^2*d^2+8/9*a*b*c^3*d+1/9*b^2*c^4)*x^9+a^2*c^4*x+(4/5*a^2*c^3*d+2
/5*a*b*c^4)*x^5+(1/17*a^2*d^4+8/17*a*b*c*d^3+6/17*b^2*c^2*d^2)*x^17+1/25*b
^2*d^4*x^25+(2/21*a*b*d^4+4/21*b^2*c*d^3)*x^21+(4/13*a^2*c*d^3+12/13*a*b*c
^2*d^2+4/13*b^2*c^3*d)*x^13
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \frac{1}{25}\, b^2 d^4 x^{25} + \frac{2}{21}\left(2\, b^2 c d^3 + abd^4\right) x^{21} + \frac{1}{17}\left(6\, b^2 c^2 d^2 + 8\, abcd^3 + a^2 d^4\right) x^{17} + \frac{4}{13}\left(b^2 c^3 d + 3\, abc^2 d^2 + a^2 c d^3\right) x^{13} + \frac{1}{9}\left(b^2 c^4 + 8\, abc^3 d + 6\, a^2 c^2 d^2\right) x^9 + a^2 c^4 x + \frac{2}{5}\left(abc^4 + 2\, a^2 c^3 d\right) x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

output
```
1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^
2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*
d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x +
2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = a^2 c^4 x + \frac{b^2 d^4 x^{25}}{25} + x^{21} \cdot \left(\frac{2abd^4}{21} + \frac{4b^2 cd^3}{21}\right) + x^{17}\left(\frac{a^2 d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2 c^2 d^2}{17}\right) + x^{13} \cdot \left(\frac{4a^2 cd^3}{13} + \frac{12abc^2 d^2}{13} + \frac{4b^2 c^3 d}{13}\right) + x^9 \cdot \left(\frac{2a^2 c^2 d^2}{3} + \frac{8abc^3 d}{9} + \frac{b^2 c^4}{9}\right) + x^5 \cdot \left(\frac{4a^2 c^3 d}{5} + \frac{2abc^4}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

output
```
a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21
) + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(
4*a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*
c**2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*
b*c**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \frac{1}{25}\, b^2 d^4 x^{25} + \frac{2}{21}\left(2\, b^2 c d^3 + a b d^4\right) x^{21} + \frac{1}{17}\left(6\, b^2 c^2 d^2 + 8\, a b c d^3 + a^2 d^4\right) x^{17} + \frac{4}{13}\left(b^2 c^3 d + 3\, a b c^2 d^2 + a^2 c d^3\right) x^{13} + \frac{1}{9}\left(b^2 c^4 + 8\, a b c^3 d + 6\, a^2 c^2 d^2\right) x^9 + a^2 c^4 x + \frac{2}{5}\left(a b c^4 + 2\, a^2 c^3 d\right) x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

output
```
1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^
2 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*
d^3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x +
2/5*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \frac{1}{25}b^2d^4x^{25} + \frac{4}{21}b^2cd^3x^{21} + \frac{2}{21}abd^4x^{21} + \frac{6}{17}b^2c^2d^2x^{17} + \frac{8}{17}abcd^3x^{17} + \frac{1}{17}a^2d^4x^{17} + \frac{4}{13}b^2c^3dx^{13} + \frac{12}{13}abc^2d^2x^{13} + \frac{4}{13}a^2cd^3x^{13} + \frac{1}{9}b^2c^4x^9 + \frac{8}{9}abc^3dx^9 + \frac{2}{3}a^2c^2d^2x^9 + \frac{2}{5}abc^4x^5 + \frac{4}{5}a^2c^3dx^5 + a^2c^4x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

output
```
1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2
*d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13
+ 12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b
*c^3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a
^2*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = x^9 \left(\frac{2\,a^2\,c^2\,d^2}{3} + \frac{8\,a\,b\,c^3\,d}{9} + \frac{b^2\,c^4}{9}\right) + x^{17} \left(\frac{a^2\,d^4}{17} + \frac{8\,a\,b\,c\,d^3}{17} + \frac{6\,b^2\,c^2\,d^2}{17}\right) + a^2\,c^4\,x + \frac{b^2\,d^4\,x^{25}}{25} + \frac{2\,a\,c^3\,x^5\,(2\,a\,d+b\,c)}{5} + \frac{2\,b\,d^3\,x^{21}\,(a\,d+2\,b\,c)}{21} + \frac{4\,c\,d\,x^{13}\,(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2)}{13}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

output
```
x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/
17 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/2
5 + (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c
*d*x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14

$$\int (a+bx^4)^2 (c+dx^4)^4 \, dx = \frac{x(13923b^2d^4x^{24} + 33150ab\,d^4x^{20} + 66300b^2c\,d^3x^{20} + 20475a^2d^4x^{16} + 163800abc\,d^3x^{16} + 122850b^2c^2d^2x^{16}}{}$$

input
```
int((b*x^4+a)^2*(d*x^4+c)^4,x)
```

output
```
(x*(348075*a**2*c**4 + 278460*a**2*c**3*d*x**4 + 232050*a**2*c**2*d**2*x**
8 + 107100*a**2*c*d**3*x**12 + 20475*a**2*d**4*x**16 + 139230*a*b*c**4*x**
4 + 309400*a*b*c**3*d*x**8 + 321300*a*b*c**2*d**2*x**12 + 163800*a*b*c*d**
3*x**16 + 33150*a*b*d**4*x**20 + 38675*b**2*c**4*x**8 + 107100*b**2*c**3*d
*x**12 + 122850*b**2*c**2*d**2*x**16 + 66300*b**2*c*d**3*x**20 + 13923*b**
2*d**4*x**24))/348075
```

3.9 $\int (a+bx^4)^2 (c+dx^4)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9 + \frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}$$

output
```
a^2*c^3*x+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^9+1/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^13+1/17*b*d^2*(2*a*d+3*b*c)*x^1
7+1/21*b^2*d^3*x^21
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)^3 \, dx = a^2c^3x + \frac{1}{5}ac^2(2bc+3ad)x^5 + \frac{1}{9}c(b^2c^2+6abcd+3a^2d^2)\,x^9 + \frac{1}{13}d(3b^2c^2+6abcd+a^2d^2)\,x^{13} + \frac{1}{17}bd^2(3bc+2ad)x^{17} + \frac{1}{21}b^2d^3x^{21}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*
b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(dx^{12}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^8\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^4(3ad+2bc)+bd^2x^{16}(2ad+3bc)+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*
b*c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
norman	$a^2c^3x+\left(\frac{3}{5}a^2c^2d+\frac{2}{5}ab\,c^3\right)x^5+\left(\frac{1}{3}c\,a^2d^2+\frac{2}{3}ab\,c^2d+\frac{1}{9}b^2c^3\right)x^9+\left(\frac{1}{13}a^2d^3+\frac{6}{13}ac\,d^2b+\frac{3}{13}b$
default	$\frac{b^2d^3x^{21}}{21}+\frac{(2a\,d^3b+3b^2c\,d^2)x^{17}}{17}+\frac{(a^2d^3+6ac\,d^2b+3b^2c^2d)x^{13}}{13}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^9}{9}+\frac{(3a^2c^2d+2ab\,c^3)x^5}{5}$
gosper	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}ac\,d^2$
risch	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}ac\,d^2$
parallelrisch	$a^2c^3x+\frac{3}{5}x^5a^2c^2d+\frac{2}{5}x^5ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{13}x^{13}a^2d^3+\frac{6}{13}x^{13}ac\,d^2$
orering	$\frac{x\left(3315b^2d^3x^{20}+8190ab\,d^3x^{16}+12285b^2c\,d^2x^{16}+5355a^2d^3x^{12}+32130abc\,d^2x^{12}+16065b^2c^2d\,x^{12}+23205a^2c\,d^2x^8+46410ab\right.}{69615}$

input
```
int((b*x^4+a)^2*(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^3*x+(3/5*a^2*c^2*d+2/5*a*b*c^3)*x^5+(1/3*c*a^2*d^2+2/3*a*b*c^2*d+1/9
*b^2*c^3)*x^9+(1/13*a^2*d^3+6/13*a*c*d^2*b+3/13*b^2*c^2*d)*x^13+(2/17*a*d^
3*b+3/17*b^2*c*d^2)*x^17+1/21*b^2*d^3*x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{21} b^2 d^3 x^{21} + \frac{1}{17} \left(3 b^2 c d^2 + 2 a b d^3\right) x^{17} + \frac{1}{13} \left(3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3\right) x^{13} + \frac{1}{9} \left(b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2\right) x^9 + a^2 c^3 x + \frac{1}{5} \left(2 a b c^3 + 3 a^2 c^2 d\right) x^5$$

input

```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

output

```
1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{21}}{21} + x^{17} \cdot \left(\frac{2abd^3}{17} + \frac{3b^2cd^2}{17}\right) + x^{13} \left(\frac{a^2d^3}{13} + \frac{6abcd^2}{13} + \frac{3b^2c^2d}{13}\right) + x^9 \left(\frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9}\right) + x^5 \cdot \left(\frac{3a^2c^2d}{5} + \frac{2abc^3}{5}\right)$$

input

```
integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

output

```
a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17
) + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2
*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*
c**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{21}\,b^2d^3x^{21} + \frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17} + \frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13} + \frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9 + a^2c^3x + \frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2
)*x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx = \frac{1}{21}\,b^2d^3x^{21} + \frac{3}{17}\,b^2cd^2x^{17} + \frac{2}{17}\,abd^3x^{17} + \frac{3}{13}\,b^2c^2dx^{13} + \frac{6}{13}\,abcd^2x^{13} + \frac{1}{13}\,a^2d^3x^{13} + \frac{1}{9}\,b^2c^3x^9 + \frac{2}{3}\,abc^2dx^9 + \frac{1}{3}\,a^2cd^2x^9 + \frac{2}{5}\,abc^3x^5 + \frac{3}{5}\,a^2c^2dx^5 + a^2c^3x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2
*d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*
a*b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 +
a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \,dx = x^9 \left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right) + x^{13}\left(\frac{a^2\,d^3}{13}+\frac{6\,a\,b\,c\,d^2}{13}+\frac{3\,b^2\,c^2\,d}{13}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{21}}{21} + \frac{a\,c^2\,x^5\,(3\,a\,d+2\,b\,c)}{5}+\frac{b\,d^2\,x^{17}\,(2\,a\,d+3\,b\,c)}{17}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

output
```
x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
 (3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a
*c^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \,dx = \frac{x(3315b^2d^3x^{20}+8190ab\,d^3x^{16}+12285b^2c\,d^2x^{16}+5355a^2d^3x^{12}+32130abc\,d^2x^{12}+16065b^2c^2d\,x^{12}+232}{69615}$$

input
```
int((b*x^4+a)^2*(d*x^4+c)^3,x)
```

output
```
(x*(69615*a**2*c**3 + 41769*a**2*c**2*d*x**4 + 23205*a**2*c*d**2*x**8 + 53
55*a**2*d**3*x**12 + 27846*a*b*c**3*x**4 + 46410*a*b*c**2*d*x**8 + 32130*a
*b*c*d**2*x**12 + 8190*a*b*d**3*x**16 + 7735*b**2*c**3*x**8 + 16065*b**2*c
**2*d*x**12 + 12285*b**2*c*d**2*x**16 + 3315*b**2*d**3*x**20))/69615
```

3.10 $\int (a+bx^4)^2 (c+dx^4)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = a^2c^2x + \frac{2}{5}ac(bc+ad)x^5 + \frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9 + \frac{2}{13}bd(bc+ad)x^{13} + \frac{1}{17}b^2d^2x^{17}$$

output
```
a^2*c^2*x+2/5*a*c*(a*d+b*c)*x^5+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+2/13*b
*d*(a*d+b*c)*x^13+1/17*b^2*d^2*x^17
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx = a^2c^2x + \frac{2}{5}ac(bc+ad)x^5 + \frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9 + \frac{2}{13}bd(bc+ad)x^{13} + \frac{1}{17}b^2d^2x^{17}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^8 \left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^{12}(ad+bc)+2acx^4(ad+bc)+b^2d^2x^{16}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$a^2c^2x+\left(\frac{2}{5}a^2cd+\frac{2}{5}b\,c^2a\right)x^5+\left(\frac{1}{9}a^2d^2+\frac{4}{9}abcd+\frac{1}{9}b^2c^2\right)x^9+\left(\frac{2}{13}ab\,d^2+\frac{2}{13}b^2cd\right)x^{13}+\frac{b^2d^2}{17}$
default	$\frac{b^2d^2x^{17}}{17}+\frac{(2ab\,d^2+2b^2cd)x^{13}}{13}+\frac{(a^2d^2+4abcd+b^2c^2)x^9}{9}+\frac{(2a^2cd+2b\,c^2a)x^5}{5}+a^2c^2x$
gosper	$a^2c^2x+\frac{2}{5}x^5a^2cd+\frac{2}{5}x^5b\,c^2a+\frac{1}{9}x^9a^2d^2+\frac{4}{9}x^9abcd+\frac{1}{9}x^9b^2c^2+\frac{2}{13}x^{13}ab\,d^2+\frac{2}{13}x^{13}b^2cd+\frac{1}{17}$
risch	$a^2c^2x+\frac{2}{5}x^5a^2cd+\frac{2}{5}x^5b\,c^2a+\frac{1}{9}x^9a^2d^2+\frac{4}{9}x^9abcd+\frac{1}{9}x^9b^2c^2+\frac{2}{13}x^{13}ab\,d^2+\frac{2}{13}x^{13}b^2cd+\frac{1}{17}$
parallelrisch	$a^2c^2x+\frac{2}{5}x^5a^2cd+\frac{2}{5}x^5b\,c^2a+\frac{1}{9}x^9a^2d^2+\frac{4}{9}x^9abcd+\frac{1}{9}x^9b^2c^2+\frac{2}{13}x^{13}ab\,d^2+\frac{2}{13}x^{13}b^2cd+\frac{1}{17}$
orering	$\frac{x(585b^2d^2x^{16}+1530ab\,d^2x^{12}+1530b^2cd\,x^{12}+1105a^2d^2x^8+4420abcd\,x^8+1105b^2c^2x^8+3978a^2cd\,x^4+3978ab\,c^2x^4+9945a^2c^2}{9945}$

input
```
int((b*x^4+a)^2*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c^2*x+(2/5*a^2*c*d+2/5*b*c^2*a)*x^5+(1/9*a^2*d^2+4/9*a*b*c*d+1/9*b^2*c
^2)*x^9+(2/13*a*b*d^2+2/13*b^2*c*d)*x^13+1/17*b^2*d^2*x^17
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^2\,dx=\frac{1}{17}b^2d^2x^{17}+\frac{2}{13}\left(b^2cd+abd^2\right)x^{13}+\frac{1}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^9+\frac{2}{5}\left(abc^2+a^2cd\right)x^5+a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13} \cdot \left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5 \cdot \left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
 x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a
*b*c**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx = \frac{1}{17}\,b^2d^2x^{17} + \frac{2}{13}\,\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\,\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\,\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \,dx = \frac{1}{17}b^2d^2x^{17} + \frac{2}{13}b^2cdx^{13} + \frac{2}{13}abd^2x^{13} + \frac{1}{9}b^2c^2x^9 + \frac{4}{9}abcdx^9 + \frac{1}{9}a^2d^2x^9 + \frac{2}{5}abc^2x^5 + \frac{2}{5}a^2cdx^5 + a^2c^2x$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^
9 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5
+ a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \,dx = x^9\left(\frac{a^2\,d^2}{9}+\frac{4\,a\,b\,c\,d}{9}+\frac{b^2\,c^2}{9}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^{17}}{17} + \frac{2\,a\,c\,x^5\,\left(a\,d+b\,c\right)}{5}+\frac{2\,b\,d\,x^{13}\,\left(a\,d+b\,c\right)}{13}$$

input
```
int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

output
```
x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^1
7)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int (a+bx^4)^2 (c+dx^4)^2 \, dx$$
$$= \frac{x(585b^2d^2x^{16}+1530ab\,d^2x^{12}+1530b^2cd\,x^{12}+1105a^2d^2x^8+4420abcd\,x^8+1105b^2c^2x^8+3978a^2cd\,x^4+}{9945}$$

input
```
int((b*x^4+a)^2*(d*x^4+c)^2,x)
```

output
```
(x*(9945*a**2*c**2 + 3978*a**2*c*d*x**4 + 1105*a**2*d**2*x**8 + 3978*a*b*c
**2*x**4 + 4420*a*b*c*d*x**8 + 1530*a*b*d**2*x**12 + 1105*b**2*c**2*x**8 +
 1530*b**2*c*d*x**12 + 585*b**2*d**2*x**16))/9945
```

3.11 $\int (a+bx^4)^2 (c+dx^4)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)^2 (c+dx^4)\, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

output
```
a^2*c*x+1/5*a*(a*d+2*b*c)*x^5+1/9*b*(2*a*d+b*c)*x^9+1/13*b^2*d*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4)\, dx = a^2cx + \frac{1}{5}a(2bc+ad)x^5 + \frac{1}{9}b(bc+2ad)x^9 + \frac{1}{13}b^2dx^{13}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)
/13
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2c+bx^8(2ad+bc)+ax^4(ad+2bc)+b^2dx^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

input
```
Int[(a + b*x^4)^2*(c + d*x^4),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)
/13
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^{13}}{13}+\frac{(2abd+b^2c)x^9}{9}+\frac{(d\,a^2+2abc)x^5}{5}+a^2cx$	49
norman	$\frac{b^2 d\,x^{13}}{13}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{5}d\,a^2+\frac{2}{5}abc\right)x^5+a^2cx$	49
gosper	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5d\,a^2+\frac{2}{5}x^5abc+a^2cx$	51
risch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5d\,a^2+\frac{2}{5}x^5abc+a^2cx$	51
parallelrisch	$\frac{1}{13}b^2d\,x^{13}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{5}x^5d\,a^2+\frac{2}{5}x^5abc+a^2cx$	51
orering	$\frac{x(45d\,b^2x^{12}+130abd\,x^8+65b^2c\,x^8+117a^2d\,x^4+234abc\,x^4+585a^2c)}{585}$	54

input
```
int((b*x^4+a)^2*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)\,dx=\frac{1}{13}\,b^2dx^{13}+\frac{1}{9}\,(b^2c+2\,abd)x^9+\frac{1}{5}\,(2\,abc+a^2d)x^5+a^2cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 +
a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (a+bx^4)^2 (c+dx^4) \, dx = a^2cx + \frac{b^2dx^{13}}{13} + x^9 \cdot \left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^5 \left(\frac{a^2d}{5} + \frac{2abc}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(d*x**4+c),x)
```

output
```
a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5
+ 2*a*b*c/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (c+dx^4) \, dx = \frac{1}{13} b^2dx^{13} + \frac{1}{9} (b^2c + 2\,abd)x^9 + \frac{1}{5} (2\,abc + a^2d)x^5 + a^2cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

output
```
1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 +
a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (c+dx^4) \, dx = \frac{1}{13} b^2dx^{13} + \frac{1}{9} b^2cx^9 + \frac{2}{9} abdx^9 + \frac{2}{5} abcx^5 + \frac{1}{5} a^2dx^5 + a^2cx$$

input
```
integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

output
```
1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*
d*x^5 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (c+dx^4) \, dx = x^5 \left(\frac{d\,a^2}{5} + \frac{2\,b\,c\,a}{5}\right) + x^9 \left(\frac{c\,b^2}{9} + \frac{2\,a\,d\,b}{9}\right) + \frac{b^2\,d\,x^{13}}{13} + a^2\,c\,x$$

input
```
int((a + b*x^4)^2*(c + d*x^4),x)
```

output
```
x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^1
3)/13 + a^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (a+bx^4)^2 (c+dx^4) \, dx = \frac{x(45b^2d\,x^{12} + 130abd\,x^8 + 65b^2c\,x^8 + 117a^2d\,x^4 + 234abc\,x^4 + 585a^2c)}{585}$$

input
```
int((b*x^4+a)^2*(d*x^4+c),x)
```

output
```
(x*(585*a**2*c + 117*a**2*d*x**4 + 234*a*b*c*x**4 + 130*a*b*d*x**8 + 65*b*
*2*c*x**8 + 45*b**2*d*x**12))/585
```

3.12 $\int \frac{(a+bx^4)^2}{c+dx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{(a+bx^4)^2}{c+dx^4} dx = -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^5}{5d} - \frac{(bc-ad)^2 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc-ad)^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*arctan(-1+2^(1/2)*d^(
1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/d^(9/4)+1/4*(-a*d+b*c)^2*arctan(1+2^(1/2)*
d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/d^(9/4)+1/4*(-a*d+b*c)^2*arctanh(2^(1/2
)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(3/4)/d^(9/4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$

$$= \frac{-40bc^{3/4}\sqrt[4]{d}(bc-2ad)x+8b^2c^{3/4}d^{5/4}x^5-10\sqrt{2}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+10\sqrt{2}(bc-ad)^2\,\mathrm{arct}}{}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

output
```
(-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(
b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b
*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^
(3/4)*d^(9/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^4)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (
Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b
*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqr
t[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result
risch	$\frac{b^2x^5}{5d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}+\frac{\sum_{\_R=\operatorname{RootOf}(d\_Z^4+c)}\frac{(a^2d^2-2abcd+b^2c^2)\ln(x-\_R)}{\_R^3}}{4d^3}$
default	$\frac{b(\frac{1}{5}bd\,x^5+2adx-bcx)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)(\frac{c}{d})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-(\frac{c}{d})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{c}{d})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{c}{d})^{\frac{1}{4}}}-1\right.}{8d^2c}$

input
```
int((b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*c*x+1/4/d^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1092, normalized size of antiderivative = 5.69

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/20*(4*b^2*d*x^5 + 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*d^2*(-(b^8*c^
8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^3*d^9))^(1/4)*log(I*c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*x) - 5*I*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-I*c*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^...
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2x^5}{5d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right)$$
$$+\text{RootSum}\Big(256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c),x)
```

output
```
b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9
+ a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3
*d**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**
6*d**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*
d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx = \frac{b^2dx^5 - 5\,(b^2c - 2\,abd)x}{5\,d^2} + \frac{\frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2-2\,a}{}}{8\,d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/
4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(
1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqr
t(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4
)))/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(143) = 286$.

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$$
$$= \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^3}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{8\,cd^3}$$
$$+\frac{b^2d^4x^5-5\,b^2cd^3x+10\,abd^4x}{5\,d^5}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c
*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c
*d + (c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(
c*d^3) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c
*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
 + 1/5*(b^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1081, normalized size of antiderivative = 5.63

$$\int \frac{(a + bx^4)^2}{c + dx^4} dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^2/(c + d*x^4),x)
```

output
```
(b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d
 - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/
4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)
^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i
)/((-c)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9
/4)) - ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)
^2*1i)/(2*(-c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^
2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))
/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3
+ 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b
^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.48

$$\int \frac{(a + bx^4)^2}{c + dx^4} dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^2/(d*x^4+c),x)
```

output
```
( - 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d**2 + 20*d**(3/4)*c**(1/4)*sqrt(2
)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2
)))*a*b*c*d - 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2)
 - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c**2 + 10*d**(3/4)*c**(1
/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1
/4)*sqrt(2)))*a**2*d**2 - 20*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(
1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c*d + 10*d**(
3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(
1/4)*c**(1/4)*sqrt(2)))*b**2*c**2 - 5*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**
(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**2*d**2 + 10*d**(3/4)
*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x
**2)*a*b*c*d - 5*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2
)*x + sqrt(c) + sqrt(d)*x**2)*b**2*c**2 + 5*d**(3/4)*c**(1/4)*sqrt(2)*log(
d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**2*d**2 - 10*d**(3
/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x
**2)*a*b*c*d + 5*d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x
 + sqrt(c) + sqrt(d)*x**2)*b**2*c**2 + 80*a*b*c*d**2*x - 40*b**2*c**2*d*x
+ 8*b**2*c*d**2*x**5)/(40*c*d**3)
```

3.13 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 223

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2} dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{4cd^2(c+dx^4)} + \frac{(bc-ad)(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}} - \frac{(bc-ad)(5bc+3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}$$

output
```
b^2*x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)
*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/d^(9/4)-1/16*(-a*d+b
*c)*(3*a*d+5*b*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/d^(9
/4)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/
2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/d^(9/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$= \frac{32b^2\sqrt[4]{d}x + \frac{8\sqrt[4]{d}(bc-ad)^2x}{c(c+dx^4)} + \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}}{32d^{9/4}}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

output
```
(32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[
2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1
/4)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d -
3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4)
 - (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$\Big\downarrow \; 915$$
$$\int \left(\frac{b^2}{d^2} - \frac{-a^2d^2+2bdx^4(bc-ad)+b^2c^2}{d^2(c+dx^4)^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{(bc-ad)(3ad+5bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{x(bc-ad)^2}{4cd^2\left(c+dx^4\right)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(
9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
- ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

method	result
risch	$\frac{b^2x}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{4c\,d^2(d\,x^4+c)}+\frac{\sum_{\_R=\mathrm{RootOf}(d\_Z^4+c)}\frac{(3a^2d^2+2abcd-5b^2c^2)\ln(x-\_R)}{\_R^3}}{16d^3c}$
default	$\frac{b^2x}{d^2}+\frac{\frac{(a^2d^2-2abcd+b^2c^2)x}{4c(d\,x^4+c)}+\frac{(3a^2d^2+2abcd-5b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}}{d^2}$

input
```
int((b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/d^2/(d*x^4+c)+1/16/d^3/c*sum
((3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 1210, normalized size of antiderivative = 5.43

$$\int\frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*(16*b^2*c*d*x^5 + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c
^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/
(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b
^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)
 - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (-I*c*d^3*x^4 - I*c^2*d^2)*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^
3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(I*c^2*d^2*(-(625*b^8*c^8 -
1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 +
81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (I
*c*d^3*x^4 + I*c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^
6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 -
 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(
-I*c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*
c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*
a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a...
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{4c^2d^2+4cd^3x^4}$$
$$+\text{RootSum}\left(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+646a^4b^4c^4d^4+1640a^3b\right.$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

output
```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**
3*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7
 - 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*
d**4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c*
*7*d + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a
*b*c*d - 5*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(cd^3x^4+c^2d^2)} + \frac{b^2x}{d^2} - \frac{\frac{2\,\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x+\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{d}x-\sqrt{2}c^{\frac14}d^{\frac14}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(5\,b^{\cdots}}{}}{32\,cd^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 -
1/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*
sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1
/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d
^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4
)) - sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(174) = 348$.

Time = 0.12 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$$
$$= \frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3}$$
$$+ \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(dx^4+c)cd^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*
d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^
3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^
2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^
2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a
^2*d^2*x)/((d*x^4 + c)*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 1254, normalized size of antiderivative = 5.62

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4))
+ (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*
d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3
- 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*
a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*
c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((
x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b
*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2
*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(
16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2
 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b
*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a
*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d +
5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 -
26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d -
 b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) +
(((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 973, normalized size of antiderivative = 4.36

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^2/(d*x^4+c)^2,x)
```

output
```
( - 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d
)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*c*d**2 - 6*d**(3/4)*c**(1/4)*sqrt(2
)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2
)))*a**2*d**3*x**4 - 4*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*s
qrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c**2*d - 4*d**(3/4)
*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)
*c**(1/4)*sqrt(2)))*a*b*c*d**2*x**4 + 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d
**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*
c**3 + 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sq
rt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c**2*d*x**4 + 6*d**(3/4)*c**(1/
4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/
4)*sqrt(2)))*a**2*c*d**2 + 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(
1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d**3*x**4 +
4*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)
/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c**2*d + 4*d**(3/4)*c**(1/4)*sqrt(2)*ata
n((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a
*b*c*d**2*x**4 - 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt
(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c**3 - 10*d**(3/4)*c*
*(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c*
*(1/4)*sqrt(2)))*b**2*c**2*d*x**4 - 3*d**(3/4)*c**(1/4)*sqrt(2)*log( - ...
```

3.14 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\begin{aligned}\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx &= \frac{(bc-ad)^2x}{8cd^2(c+dx^4)^2} - \frac{(bc-ad)(9bc+7ad)x}{32c^2d^2(c+dx^4)} \\ &\quad - \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ &\quad + \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} \\ &\quad + \frac{(5b^2c^2+6abcd+21a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}\end{aligned}$$

output
```
1/8*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+9*b*c)*x/c^2/d
^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(-1+2^(1/2)*d^(1
/4)*x/c^(1/4))*2^(1/2)/c^(11/4)/d^(9/4)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*
c^2)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(11/4)/d^(9/4)+1/128*(2
1*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+
d^(1/2)*x^2))*2^(1/2)/c^(11/4)/d^(9/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$
$$= \frac{\frac{32c^{7/4}\sqrt[4]{d}(bc-ad)^2x}{(c+dx^4)^2} - \frac{8c^{3/4}\sqrt[4]{d}(9b^2c^2-2abcd-7a^2d^2)x}{c+dx^4} - 2\sqrt{2}(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

output
```
((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(
9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 +
 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[
2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/
4)*d^(9/4))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {930, 910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$
$$\Big\downarrow \text{930}$$
$$\frac{\int \frac{b(5bc+3ad)x^4+a(bc+7ad)}{(dx^4+c)^2}dx}{8cd} - \frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$$\Big\downarrow \text{ 910}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\int \frac{1}{dx^4+c}dx-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-x\left(-7\cdots\right.}{8cd}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2d}{c}+2ab+\frac{5b^2c}{d}\right)}{4(c+dx^4)}}{8cd}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\downarrow$ 1479

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{8cd}$$

$$\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{8cd}$$

$$\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-\frac{x\left(-\frac{7a^2 \cdots}{c}\right.}{4(\cdots}}{8cd}$$

$$\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

$\Big\downarrow$ 1103

$$\frac{\frac{1}{4}\left(\frac{21a^2d}{c}+6ab+\frac{5b^2c}{d}\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\cdots}{8cd}$$

$$\frac{x(a+bx^4)(bc-ad)}{8cd(c+dx^4)^2}$$

input
```
Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

output
```
-1/8*((b*c - a*d)*x*(a + b*x^4))/(c*d*(c + d*x^4)^2) + (-1/4*((2*a*b + (5*
b^2*c)/d - (7*a^2*d)/c)*x)/(c + d*x^4) + ((6*a*b + (5*b^2*c)/d + (21*a^2*d
)/c)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))
) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*
Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(S
qrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/4)/(8*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\,\_Z^4+c\right)}\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128c^2d^3}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^5}{32c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)x}{32c\,d^2}}{\left(d\,x^4+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\ldots\right.\right.}{256c^3d^2}$

input
```
int((b*x^4+a)^2/(d*x^4+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d
-5*b^2*c^2)/c/d^2*x)/(d*x^4+c)^2+1/128/c^2/d^3*sum((21*a^2*d^2+6*a*b*c*d+5
*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 1294, normalized size of antiderivative = 4.79

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
-1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - (c^2*d^4*x^8 + 2*c
^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^
3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^1
1*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^
6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^
3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/
(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (-I*c^2*d^4*
x^8 - 2*I*c^3*d^3*x^4 - I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 159
00*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 1769
04*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*
a^8*d^8)/(c^11*d^9))^(1/4)*log(I*c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d
 + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
+ 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*x)
 + (I*c^2*d^4*x^8 + 2*I*c^3*d^3*x^4 + I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b
^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*
c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c
*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-I*c^3*d^2*(-(625*b^8*c^8 + 3
000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 11280...
```

Sympy [A] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \frac{x^5\cdot(7a^2d^3+2abcd^2-9b^2c^2d)+x(11a^2cd^2-6abc^2d-5b^2c^3)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8} + \mathrm{RootSum}\Big(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7bcd^7+280476a^6b^2c^2d^6+176904a^5b^3c^3d^5+11$$

input
```
integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

output
```
(x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6
*a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d*
*4*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*
a**7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 +
112806*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c
**6*d**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c*
*3*d**2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(9\,b^2c^2d-2\,abcd^2-7\,a^2d^3)x^5+(5\,b^2c^3+6\,abc^2d-11\,a^2cd^2)x}{32\,(c^2d^4x^8+2\,c^3d^3x^4+c^4d^2)} + \frac{\frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(5}{}}{256\,c^2d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
-1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^
2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*
sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)
*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)
*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(
c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*l
og(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) -
sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(
1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx$$

$$= \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}$$
$$+\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}$$
$$+\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}$$
$$-\frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256\,c^3d^3}$$
$$-\frac{9\,b^2c^2dx^5-2\,abcd^2x^5-7\,a^2d^3x^5+5\,b^2c^3x+6\,abc^2dx-11\,a^2cd^2x}{32\,(dx^4+c)^2c^2d^2}$$

input
```
integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*
a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d
)^(1/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 +
 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*
x^5 - 7*a^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^
4 + c)^2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 1401, normalized size of antiderivative = 5.19

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

output
```
- ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
 9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((
((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*
c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2
*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2
 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c
)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 6
0*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6
*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a
*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)
) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*
(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3
 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(
256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)
)))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (a
tan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6
*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1484, normalized size of antiderivative = 5.50

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^2/(d*x^4+c)^3,x)
```

output

```
( - 42*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*c**2*d**2 - 84*d**(3/4)*c**(1/4)*s
qrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*s
qrt(2)))*a**2*c*d**3*x**4 - 42*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c*
*(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d**4*x**8
- 12*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)
*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c**3*d - 24*d**(3/4)*c**(1/4)*sqrt(2)
*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)
))*a*b*c**2*d**2*x**4 - 12*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/
4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c*d**3*x**8 - 1
0*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)
/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c**4 - 20*d**(3/4)*c**(1/4)*sqrt(2)*ata
n((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b
**2*c**3*d*x**4 - 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqr
t(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c**2*d**2*x**8 + 42*
d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(
d**(1/4)*c**(1/4)*sqrt(2)))*a**2*c**2*d**2 + 84*d**(3/4)*c**(1/4)*sqrt(2)*
atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))
)*a**2*c*d**3*x**4 + 42*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*
sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d**4*x**8 + 12...
```

3.15 $\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3} + \frac{d^3(4bc-ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} - \frac{(bc-ad)^4\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{17/4}} + \frac{(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{17/4}}$$

output
```
d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^13/b+1/4*(-
a*d+b*c)^4*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(17/4)+1
/4*(-a*d+b*c)^4*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(17/
4)+1/4*(-a*d+b*c)^4*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2
))*2^(1/2)/a^(3/4)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$= \frac{4680\sqrt[4]{b}d(4b^3c^3-6ab^2c^2d+4a^2bcd^2-a^3d^3)\,x+936b^{5/4}d^2(6b^2c^2-4abcd+a^2d^2)\,x^5+520b^{9/4}d^3(4bc-}{}$$

input
```
Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

output
```
(4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x +
936*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4
*b*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcT
an[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*
d)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (58
5*Sqrt[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/a^(3/4))/(4680*b^(17/4))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d(2bc-ad)\,(a^2d^2-2abcd+2b^2c^2)}{b^4}+\frac{d^2x^4\,(a^2d^2-4abcd+6b^2c^2)}{b^3}+\frac{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3}{b^4\,(a+bx^4)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4}{2\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

input
```
Int[(c + d*x^4)^4/(a + b*x^4),x]
```

output
```
(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^
2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) +
(d^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) +
((b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(3/4)*b^(17/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.74

method	result
risch	$\frac{d^4x^{13}}{13b} - \frac{d^4x^9a}{9b^2} + \frac{4d^3x^9c}{9b} - \frac{4d^3ac\,x^5}{5b^2} + \frac{6d^2c^2x^5}{5b} + \frac{d^4a^2x^5}{5b^3} - \frac{d^4a^3x}{b^4} + \frac{4d^3a^2cx}{b^3} - \frac{6d^2a\,c^2x}{b^2} + \frac{4d\,c^3x}{b} + \frac{-\sum_{R=\mathrm{RootO}}}{}$
default	$-\frac{d\left(-\frac{b^3d^3x^{13}}{13} + \frac{\left((ad-2bc)b^2d^2-2b^3c\,d^2\right)x^9}{9} + \frac{\left(2(ad-2bc)b^2cd-db\left(a^2d^2-2abcd+2b^2c^2\right)\right)x^5}{5} + (ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)x\right)}{b^4} + -$

input
```
int((d*x^4+c)^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/13*d^4*x^13/b-1/9*d^4/b^2*x^9*a+4/9*d^3/b*x^9*c-4/5*d^3/b^2*a*c*x^5+6/5*
d^2/b*c^2*x^5+1/5*d^4/b^3*a^2*x^5-d^4/b^4*a^3*x+4*d^3/b^3*a^2*c*x-6*d^2/b^
2*a*c^2*x+4*d/b*c^3*x+1/4/b^5*sum((a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2
-4*a*b^3*c^3*d+b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 2190, normalized size of antiderivative = 8.08

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*
c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*b^4*(-(b^16*c^16 - 16*a*b^1
5*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c
^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*
c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^
6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^
3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))
^(1/4)*log(a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 -
 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 +
 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 -
11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 +
1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 1
6*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x) + 585*I*b^4*(-(b^16*c^16
 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820
*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 114
40*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*
a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*
a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/
(a^3*b^17))^(1/4)*log(I*a*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2...
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right) + x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right) + \text{RootSum}\Bigg(256t^4a^3b^{17}+a^{16}d^{16}-16a^{15}bcd^{15}+120a^{14}b^2c^2d^{14}-560a^{13}b^3c^3d^{13}+1820a^{12}b^4c^4d^{12}-436 + \frac{d^4x^{13}}{13b}$$

input
```
integrate((d*x**4+c)**4/(b*x**4+a),x)
```

output
```
x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*
c*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/
b**3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a
**16*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b*
*3*c**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 +
8008*a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c
**8*d**8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a
**5*b**11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d
**3 + 120*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda
(_t, _t*log(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d*
*2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(218) = 436$.

Time = 0.11 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.80

$$\int \frac{(c + dx^4)^4}{a + bx^4}\,dx$$
$$= \frac{45\,b^3d^4x^{13} + 65\,(4\,b^3cd^3 - ab^2d^4)x^9 + 117\,(6\,b^3c^2d^2 - 4\,ab^2cd^3 + a^2bd^4)x^5 + 585\,(4\,b^3c^3d - 6\,ab^2c^2d^2 + 4}{585\,b^4}$$
$$+ \frac{\frac{2\sqrt{2}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)\,a}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2
*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2
 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3
*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
 sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
1/4)))/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(218) = 436$.

Time = 0.12 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.28

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^
3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5)
+ 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*
b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a
^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5
) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(
a*b^3)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)
*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(
2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*
a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log
(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^1
3 + 260*b^12*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*
b^11*c*d^3*x^5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^
2*d^2*x + 2340*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 1822, normalized size of antiderivative = 6.72

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^4/(a + b*x^4),x)
```

output
```
x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b
) - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*
d^3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*
c*d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)
/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^
4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a
)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8
*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^
3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(
-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a
*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.31

$$\int \frac{(c + dx^4)^4}{a + bx^4} \, dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^4/(b*x^4+a),x)
```

output
```
( - 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**4 + 4680*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sq
rt(2)))*a**3*b*c*d**3 - 7020*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d*
*2 + 4680*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**3*d - 1170*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*b**4*c**4 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**4 - 4680
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c*d**3 + 7020*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*c**2*d**2 - 4680*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**3*d +
 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*c**4 - 585*b**(3/4)*a**(1/4)*sqrt(2
)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**4*d**4 +
 2340*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(
a) + sqrt(b)*x**2)*a**3*b*c*d**3 - 3510*b**(3/4)*a**(1/4)*sqrt(2)*log( ...
```

3.16 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 227

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^9}{9b}-\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}$$

output

```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/5*d^2*(-a*d+3*b*c)*x^5/b^2+1/9*d^3
*x^9/b+1/4*(-a*d+b*c)^3*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/
4)/b^(13/4)+1/4*(-a*d+b*c)^3*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a
^(3/4)/b^(13/4)+1/4*(-a*d+b*c)^3*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2
)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^4)^3}{a+bx^4} \, dx$$
$$= \frac{360a^{3/4}\sqrt[4]{b}d(3b^2c^2-3abcd+a^2d^2)x-72a^{3/4}b^{5/4}d^2(-3bc+ad)x^5+40a^{3/4}b^{9/4}d^3x^9-90\sqrt{2}(bc-ad)^3 a}{}$$

input
```
Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

output
```
(360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^
(5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*
c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d
)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*
d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*
b^(13/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{a+bx^4} \, dx$$
$$\Big\downarrow \text{915}$$
$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3\,(a+bx^4)}+\frac{d^2x^4(3bc-ad)}{b^2}+\frac{d^3x^8}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

input
```
Int[(c + d*x^4)^3/(a + b*x^4),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b
^2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)
) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(4*Sqrt[2]*a^(3/4)*b^(13/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.58

method	result
risch	$\frac{d^3x^9}{9b} - \frac{d^3a\,x^5}{5b^2} + \frac{3d^2c\,x^5}{5b} + \frac{d^3a^2x}{b^3} - \frac{3d^2acx}{b^2} + \frac{3d\,c^2x}{b} + \frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)} \frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(x-\_\right.}{\_R^3}}{4b^4}$
default	$\frac{d\left(\frac{1}{9}b^2d^2x^9-\frac{1}{5}ab\,d^2x^5+\frac{3}{5}b^2cd\,x^5+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3} + \frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+}\right.\right.}{8b^3a}$

input
```
int((d*x^4+c)^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^3*x^9/b-1/5*d^3/b^2*a*x^5+3/5*d^2/b*c*x^5+d^3/b^3*a^2*x-3*d^2/b^2*a*
c*x+3*d/b*c^2*x+1/4/b^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 1642, normalized size of antiderivative = 7.23

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 45*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d
^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d +
 3*a^2*b*c*d^2 - a^3*d^3)*x) - 45*I*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a^3*b^13))^(1/4)*log(I*a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x) + 45*I*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*...
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)$$
$$+\mathrm{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-220a^9b^3c^3d^9+495a^8b^4c^4d^8-792a^7b^5c^5\right.$$
$$+\frac{d^3x^9}{9b}$$

input
```
integrate((d*x**4+c)**3/(b*x**4+a),x)
```

output
```
x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/
b**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11
*b*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8
*b**4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*
a**5*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 +
66*a**2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t
*log(-4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c*
*3) + x))) + d**3*x**9/(9*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(176) = 352$.

Time = 0.11 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \frac{5\,b^2d^3x^9 + 9\,(3\,b^2cd^2 - abd^3)x^5 + 45\,(3\,b^2c^2d - 3\,abcd^2 + a^2d^3)x}{45\,b^3} + \frac{\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt
(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(
1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sq
rt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.12

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}$$
$$+ \frac{5b^8d^3x^9 + 27b^8cd^2x^5 - 9ab^7d^3x^5 + 135b^8c^2dx - 135ab^7cd^2x + 45a^2b^6d^3x}{45b^9}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c
^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a*b^4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 -
3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*
a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^
8*d^3*x^9 + 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b
^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1433, normalized size of antiderivative = 6.31

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^3/(a + b*x^4),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
 - (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2
*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(
(-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a
^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3
+ ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*
d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((
((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*
d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(1
3/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*
b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)
^(3/4)*b^(13/4))))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x
*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3
 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.99

$$\int \frac{(c+dx^4)^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^3/(b*x^4+a),x)
```

output
```
(90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*d**3 - 270*b**(3/4)*a**(1/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b*c*d**2 + 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c**2*d - 90*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**3*c**3 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*d**
3 + 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c*d**2 - 270*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*a*b**2*c**2*d + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c**3 + 45*b*
*(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(b)*x**2)*a**3*d**3 - 135*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1
/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*b*c*d**2 + 135*b**(3/4)*a**(1
/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a
*b**2*c**2*d - 45*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a) + sqrt(b)*x**2)*b**3*c**3 - 45*b**(3/4)*a**(1/4)*sqrt(2)*lo
g(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**3*d**3 + 135...
```

3.17 $\int \frac{(c+dx^4)^2}{a+bx^4} dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{(c+dx^4)^2}{a+bx^4} dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc-ad)^2 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc-ad)^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/5*d^2*x^5/b+1/4*(-a*d+b*c)^2*arctan(-1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+1/4*(-a*d+b*c)^2*arctan(1+2^(1/2)*b
^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+1/4*(-a*d+b*c)^2*arctanh(2^(1/2)
*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$$
$$= \frac{-40a^{3/4}\sqrt[4]{b}d(-2bc+ad)x+8a^{3/4}b^{5/4}d^2x^5-10\sqrt{2}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}(bc-ad)^2\,\text{ar}}{}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

output
```
(-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*S
qrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(
b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a
^(3/4)*b^(9/4))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 915}$$
$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^4)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^4}{b}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2}{2\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2
]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

method	result
risch	$\frac{d^{2} x^{5}}{5 b}-\frac{d^{2} a x}{b^{2}}+\frac{2 d c x}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)} \frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln \left(x-\_R\right)}{\_R^{3}}}{4 b^{3}}$
default	$-\frac{d\left(-\frac{1}{5} b d x^{5}+a d x-2 b c x\right)}{b^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln \left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8 b^{2} a}$

input
```
int((d*x^4+c)^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*c*x+1/4/b^3*sum((a^2*d^2-2*a*b*c*d+b^2*c^2
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 1093, normalized size of antiderivative = 5.69

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/20*(4*b*d^2*x^5 + 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 5*I*b^2*(-(b^8*c^
8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(a^3*b^9))^(1/4)*log(I*a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*x) - 5*I*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-I*a*b^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^...
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)$$
$$+\text{RootSum}\left(256t^4a^3b^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b\right.$$
$$+\frac{d^2x^5}{5b}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a
**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b
**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7
*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d
+ b**2*c**2) + x))) + d**2*x**5/(5*b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(143) = 286$.

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \frac{bd^2x^5 + 5\,(2\,bcd - ad^2)x}{5\,b^2} + \frac{\frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(b^2c^2-2\,ab}{}}{8\,b^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(
1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4
)))/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(143) = 286$.

Time = 0.13 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c^2 - 2(ab^3)^{\frac{1}{4}}abcd + (ab^3)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$
$$+ \frac{b^4d^2x^5 + 10b^4cdx - 5ab^3d^2x}{5b^5}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*
b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c
*d + (a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a
*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
+ 1/5*(b^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1081, normalized size of antiderivative = 5.63

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^2/(a + b*x^4),x)
```

output
```
(d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b
 - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/
4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*
c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)
^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i
)/((-a)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2
*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9
/4)) - ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)
^2*1i)/(2*(-a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^
2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))
/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2
+ 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b
^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx^4)^2}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^2/(b*x^4+a),x)
```

output
```
( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d**2 + 20*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b*c*d - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c**2 + 10*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*d**2 - 20*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*d + 10*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**2*c**2 - 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*d**2 + 10*b**(3/4)
*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x
**2)*a*b*c*d - 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(b)*x**2)*b**2*c**2 + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(
b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*d**2 - 10*b**(3
/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x
**2)*a*b*c*d + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x
 + sqrt(a) + sqrt(b)*x**2)*b**2*c**2 - 40*a**2*b*d**2*x + 80*a*b**2*c*d*x
+ 8*a*b**2*d**2*x**5)/(40*a*b**3)
```

3.18 $\int \frac{c+dx^4}{a+bx^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int \frac{c+dx^4}{a+bx^4} \, dx = \frac{dx}{b} - \frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
d*x/b+1/4*(-a*d+b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/
b^(5/4)+1/4*(-a*d+b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)
/b^(5/4)+1/4*(-a*d+b*c)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)
*x^2))*2^(1/2)/a^(3/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{8a^{3/4}\sqrt[4]{b}dx - 2\sqrt{2}(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}(bc-ad)}{8a^{3/4}b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4),x]
```

output
```
(8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(8*a^(3/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{a+bx^4}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(bc-ad)\int \frac{1}{bx^4+a}dx}{b} + \frac{dx}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{b} + \frac{dx}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(bc-ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{(bc-ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{dx}{b}$$

input
```
Int[(c + d*x^4)/(a + b*x^4),x]
```

output
```
(d*x)/b + ((b*c - a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2
]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$	42
default	$\frac{dx}{b}+\frac{(-ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ba}$	120

input
```
int((d*x^4+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+1/4/b^2*sum((-a*d+b*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.41

$$\int \frac{c+dx^4}{a+bx^4}\,dx = -\frac{b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)}{}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) + I*b*(
-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
a^3*b^5))^(1/4)*log(I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - I*b*(-(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5
))^(1/4)*log(-I*a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - b*(-(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)
*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3b^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(-\frac{4tab}{ad-bc}+x\right)\right)\right) + \frac{dx}{b}$$

input
```
integrate((d*x**4+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c*
*2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d -
b*c) + x))) + d*x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} + \frac{\frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*(b*c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(117) = 234$.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}bc - (ab^3)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^2}$$

input
```
integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^
3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/
4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.39

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)/(a + b*x^4),x)
```

output
```
(d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^
2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*
(-a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i
)/(4*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - (
(16*a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*
c))/(4*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) +
 ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d -
b*c))/(4*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (a
tan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b
^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b
^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 1
6*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3
/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*
d - 16*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*
a^2*b^2*d - 16*a*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c
)*1i)/(4*(-a)^(3/4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.77

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d-2b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2}{}$$

input
```
int((d*x^4+c)/(b*x^4+a),x)
```

output

```
(2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c - 2
*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/
(b**(1/4)*a**(1/4)*sqrt(2)))*a*d + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c + b**
(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt
(b)*x**2)*a*d - b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)
*x + sqrt(a) + sqrt(b)*x**2)*b*c - b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*d + b**(3/4)*a**(1/4)*sqrt(
2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c + 8*a*b*d
*x)/(8*a*b**2)
```

3.19 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 327

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}+\frac{b^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{d^{3/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

output

```
1/4*b^(3/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*c
)+1/4*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*
c)-1/4*d^(3/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d+
b*c)-1/4*d^(3/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d
+b*c)+1/4*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))
*2^(1/2)/a^(3/4)/(-a*d+b*c)-1/4*d^(3/4)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/
(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(3/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2}{}$$

input
```
Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^
(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {917, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \text{917}$$

$$\frac{b\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{755}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$\Big\downarrow$ 1476

$$\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$\Big\downarrow$ 1082

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$\Big\downarrow$ 217

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1479}$$

$$\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \ 27$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \ 1103$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

method	result
default	$\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)c}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)\right.}{}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{7}-4c\,d^{3}a^{6}b+6c^{2}d^{2}a^{5}b^{2}-4c^{3}d\,a^{4}b^{3}+a^{3}b^{4}c^{4}\right)\_Z^{4}+b^{3}\right)}\_R\ln\left(\left(\left(-a^{7}d^{7}+4a^{6}bc\,d^{6}-6a^{5}b^{2}c^{2}d^{5}+3a^{4}b^{3}c^{3}d^{4}+3a^{3}b^{4}\right.\right.\right.\right.}{}$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1
/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*
2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(
1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b
)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1171, normalized size of antiderivative = 3.58

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*
b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^
7*d^4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4
))^(1/4)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*
c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^
(1/4)*log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2
*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*l
og(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^
2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*
x - (I*b*c^2 - I*a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^4)(c + dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8(bc-ad)}$$
$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8(bc-ad)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*s
qrt(2)*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sq
rt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d)
- 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d
) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2
)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^
2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1
/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
^2 - sqrt(2)*a*c*d)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 6153, normalized size of antiderivative = 18.82

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*
a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c
^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3
/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*
b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4
*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2
*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 -
4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a
^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6
+ 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 10
24*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1
024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3
/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^
2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 -
20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 2
0480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x
*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{2}\left(2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2d^{\frac{3}{4}}c^{\frac{1}{4}}atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a+2d^{\frac{3}{4}}c^{\frac{1}{4}}\cdots}{}$$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(2)*(2*b**(3/4)*a**(1/4)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*b**(3/4)*a**(1/4)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*d**(3/4)*c
**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*
sqrt(2)))*a + 2*d**(3/4)*c**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt
(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a + b**(3/4)*a**(1/4)*log( - b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c - b**(3/4)*a**(1/4)*log(b**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c - d**(3/4)*c**(1/4)*l
og( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a + d**(3/4)*c
**(1/4)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a))/(8*a
*c*(a*d - b*c))
```

3.20 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 382

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx =& -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{d^{3/4}(7bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} \\ &- \frac{d^{3/4}(7bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} \\ &+ \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ &- \frac{d^{3/4}(7bc-3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^(7/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^
(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*c)^2+1/4*b^(7/4)*arctan(1+2^(1/2)*b^(1/4)*x
/a^(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*c)^2-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctan(
-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/(-a*d+b*c)^2-1/16*d^(3/4)*(-
3*a*d+7*b*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/(-a*d+b*c
)^2+1/4*b^(7/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2
^(1/2)/a^(3/4)/(-a*d+b*c)^2-1/16*d^(3/4)*(-3*a*d+7*b*c)*arctanh(2^(1/2)*c^
(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$$

$$= \frac{8a^{3/4}c^{3/4}d(-bc+ad)x - 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\arctan\left(1+\right.}{}$$

input
```
Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

output
```
(8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x
^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c
 + d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3
/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] +
 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c
+ d*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*
a^(3/4)*d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d
*x^4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*
c^(7/4)*(b*c - a*d)^2*(c + d*x^4))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {931, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)\,(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-3bdx^4+4bc-3ad}{(bx^4+a)(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{4b^2c\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d(7bc-3ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{4b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{4b^2c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\cdots\right)}{bc-ad}}{4c(bc-ad)}$$

$$\frac{dx}{4c\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{4b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 217

$$\frac{\frac{4b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1479

$$\frac{\frac{4b^2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 25

$$\frac{4b^2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\int\frac{\cdots}{\sqrt[4]{d}\cdots}\cdots\right)}{\cdots}$$

$$\frac{\cdots}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{4b^2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\cdots}{\cdots}\right)}{\cdots}$$

$$\frac{\cdots}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1103

$$\frac{4b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\frac{\arctan\left(\cdots\right)}{\sqrt{2}\cdots}\cdots}{\cdots}\right)}{\cdots}$$

$$\frac{\cdots}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b^2*c*((-(ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(7*b*c - 3*a*d)*((-(ArcTan[1 - (Sqr
t[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*
d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4))
+ Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4
)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.69

method	result
default	$\frac{d\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\frac{(3ad-7bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}\right)}{(ad-bc)^2}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d/(a*d-b*c)^2*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^(1
/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/
(c/d)^(1/4)*x-1)))+1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.88 (sec) , antiderivative size = 2955, normalized size of antiderivative = 7.74

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6
*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^
6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a
*c^2*d)*log(b^2*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*
d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^
9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d
 + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2
 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b
^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 +
b*c^3 - a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5
*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^
5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*
a^2*b*c*d + a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^
6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 +
 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(-I*(b*c^2*d - a*c
*d^2)*x^4 - I*b*c^3 + I*a*c^2*d)*log(b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^
7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 5
6*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)
*(I*a*b^2*c^2 - 2*I*a^2*b*c*d + I*a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4
*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = -\frac{dx}{4\left((bc^2d-acd^2)x^4+bc^3-ac^2d\right)} + \frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{\right.}}{}}{8\left(b^2c^2-2\,abcd+a^2d^2\right)} - \frac{\frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(7\,bcd-3\,ad^2)\log\left(\sqrt{d}x^2\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/3
2*(2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)
*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c
^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(286) = 572$.

Time = 0.13 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a
*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^
(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 -
2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqr
t(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d +
 sqrt(2)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2
*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(
c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/
4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(
7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4)
 + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2
) + 1/16*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a
^2*c^2*d^2) - 1/4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 21975, normalized size of antiderivative = 57.53

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

output
```
2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288
*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458
7520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6
 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65
536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 367001
6*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 +
 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10
)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16
 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2
 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
+ 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*
d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^
12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6
*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*
d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*
b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13
*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1000, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^4+a)/(d*x^4+c)^2,x)
```

output

```
( - 8*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c**3 - 8*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b
*c**2*d*x**4 + 8*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c**3 + 8*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*b*c**2*d*x**4 - 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1
/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*c*d - 6*d**(3
/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1
/4)*c**(1/4)*sqrt(2)))*a**2*d**2*x**4 + 14*d**(3/4)*c**(1/4)*sqrt(2)*atan(
(d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b
*c**2 + 14*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*s
qrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c*d*x**4 + 6*d**(3/4)*c**(1/4)*
sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*
sqrt(2)))*a**2*c*d + 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*s
qrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d**2*x**4 - 14*d**
(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**
(1/4)*c**(1/4)*sqrt(2)))*a*b*c**2 - 14*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**
(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b*c*d
*x**4 - 4*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x ...
```

3.21 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 337

$$\begin{aligned}\int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx &= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} \\ &+ \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5}{5b^4} + \frac{d^4(5bc-2ad)x^9}{9b^3} + \frac{d^5x^{13}}{13b^2} \\ &+ \frac{(bc-ad)^5x}{4ab^5\,(a+bx^4)} - \frac{(bc-ad)^4(3bc+17ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ &+ \frac{(bc-ad)^4(3bc+17ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{21/4}} \\ &+ \frac{(bc-ad)^4(3bc+17ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{21/4}}\end{aligned}$$

output
```
d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/5*d^3*(3
*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^5/b^4+1/9*d^4*(-2*a*d+5*b*c)*x^9/b^3+1/1
3*d^5*x^13/b^2+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^4+a)+1/16*(-a*d+b*c)^4*(17*a*
d+3*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(21/4)+1/1
6*(-a*d+b*c)^4*(17*a*d+3*b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/
a^(7/4)/b^(21/4)+1/16*(-a*d+b*c)^4*(17*a*d+3*b*c)*arctanh(2^(1/2)*a^(1/4)*
b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(21/4)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$$

$$= \frac{18720\sqrt[4]{b}d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x+3744b^{5/4}d^3(10b^2c^2-10abcd+3a^2d^2)\,x^5+2080b}{}$$

input
```
Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

output
```
(18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d
^3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*
b^(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(
b*c - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*
a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c
 - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4
) - (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c
+ 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))
/(18720*b^(21/4))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{b^4}+\frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{5bdx^4(bc-ad)^4+(4ad\cdots}{b^5(a+bx^4\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^4(17ad+3bc)}{8\sqrt{2}a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}+\frac{d^3x^5(3a^2d^2-10abcd+10b^2c^2)}{5b^4}+\frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5}+\frac{x(bc-ad)^5}{4ab^5(a+bx^4)}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{d^5x^{13}}{13b^2}$$

input
```
Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

output
```
(d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (
d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a
*d)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b
*x^4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c
 - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(21/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.90

method	result
risch	$\frac{d^5 x^{13}}{13 b^2}-\frac{2 d^5 a\, x^9}{9 b^3}+\frac{5 d^4 c\, x^9}{9 b^2}+\frac{3 d^5 a^2 x^5}{5 b^4}-\frac{2 d^4 a c\, x^5}{b^3}+\frac{2 d^3 c^2 x^5}{b^2}-\frac{4 d^5 a^3 x}{b^5}+\frac{15 d^4 a^2 c x}{b^4}-\frac{20 d^3 a\, c^2 x}{b^3}+\frac{10 d^2 c^3 x}{b^2}-$
default	$-\frac{d^2\left(-\frac{1}{13} b^3 d^3 x^{13}+\frac{2}{9} a\, b^2 d^3 x^9-\frac{5}{9} b^3 c\, d^2 x^9-\frac{3}{5} a^2 b\, d^3 x^5+2 a\, b^2 c\, d^2 x^5-2 b^3 c^2 d\, x^5+4 a^3 d^3 x-15 a^2 b c\, d^2 x+20 a\, b^2 c^2 d x-10 b^3 c^3 x\right)}{b^5}+$

input
```
int((d*x^4+c)^5/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*d^5*x^13/b^2-2/9*d^5/b^3*a*x^9+5/9*d^4/b^2*c*x^9+3/5*d^5/b^4*a^2*x^5-
2*d^4/b^3*a*c*x^5+2*d^3/b^2*c^2*x^5-4*d^5/b^5*a^3*x+15*d^4/b^4*a^2*c*x-20*
d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/4*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2
*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/b^5/(b*x^4+a)+1/16/b^6/
a*sum((17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d^3-50*a^2*b^3*c^3*d^2+5*a
*b^4*c^4*d+3*b^5*c^5)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 2884, normalized size of antiderivative = 8.56

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 2
08*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a
*b^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 +
 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*
b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264
*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7
 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*
b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12
 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a
^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^1
7 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(
a^7*b^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^
2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 132
64*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d
^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^1
0*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^
12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144
*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d
^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)
/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + ...
```

Sympy [A] (verification not implemented)

Time = 168.42 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = x^9\left(-\frac{2ad^5}{9b^3}+\frac{5cd^4}{9b^2}\right)+x^5\cdot\left(\frac{3a^2d^5}{5b^4}-\frac{2acd^4}{b^3}+\frac{2c^2d^3}{b^2}\right)$$
$$+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)$$
$$+\frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{4a^2b^5+4ab^6x^4}$$
$$+\text{RootSum}\left(65536t^4a^7b^{21}+83521a^{20}d^{20}-1277380a^{19}bcd^{19}+9094830a^{18}b^2c^2d^{18}-39944900a^{17}b^3c^3d^1\right.$$
$$+\frac{d^5x^{13}}{13b^2}$$

input
```
integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

output
```
x**9*(-2*a*d**5/(9*b**3) + 5*c*d**4/(9*b**2)) + x**5*(3*a**2*d**5/(5*b**4)
 - 2*a*c*d**4/b**3 + 2*c**2*d**3/b**2) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*
d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a
**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*
c**4*d + b**5*c**5)/(4*a**2*b**5 + 4*a*b**6*x**4) + RootSum(65536*_t**4*a*
*7*b**21 + 83521*a**20*d**20 - 1277380*a**19*b*c*d**19 + 9094830*a**18*b**
2*c**2*d**18 - 39944900*a**17*b**3*c**3*d**17 + 120836285*a**16*b**4*c**4*
d**16 - 266040144*a**15*b**5*c**5*d**15 + 438700840*a**14*b**6*c**6*d**14
- 548231440*a**13*b**7*c**7*d**13 + 517559250*a**12*b**8*c**8*d**12 - 3609
00280*a**11*b**9*c**9*d**11 + 174873556*a**10*b**10*c**10*d**10 - 48530040
*a**9*b**11*c**11*d**9 - 500110*a**8*b**12*c**12*d**8 + 6137200*a**7*b**13
*c**13*d**7 - 1960920*a**6*b**14*c**14*d**6 - 13264*a**5*b**15*c**15*d**5
+ 132205*a**4*b**16*c**16*d**4 - 15780*a**3*b**17*c**17*d**3 - 4050*a**2*b
**18*c**18*d**2 + 540*a*b**19*c**19*d + 81*b**20*c**20, Lambda(_t, _t*log(
16*_t*a**2*b**5/(17*a**5*d**5 - 65*a**4*b*c*d**4 + 90*a**3*b**2*c**2*d**3
- 50*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 3*b**5*c**5) + x))) + d**5*x*
*13/(13*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(282) = 564$.

Time = 0.12 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{4\,(ab^6x^4 + a^2b^5)}$$
$$+ \frac{45\,b^3d^5x^{13} + 65\,(5\,b^3cd^4 - 2\,ab^2d^5)x^9 + 117\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^5 + 585\,(10\,b^3c^3d^2 - 20\,a\cdots}{585\,b^5}$$
$$+ \frac{\frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3c^3d^2+90\,a^3b^2c^2d^3-65\,a^4bcd^4+17\,a^5d^5)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,b^5c^5+5\,ab^4c^4d-50\,a^2b^3\cdots}{\cdots}}{\cdots}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5
*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 +
 65*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4
 + 3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*
d^4 - 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d -
 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d
 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
 sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*
d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 798 vs. $2(282) = 564$.

Time = 0.14 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.37

$$\int \frac{(c + dx^4)^5}{(a + bx^4)^2} dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(
a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^
3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/
4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^
2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(
a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4
)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^
2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3
)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c
^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 +
 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^2*b^6) + 1/4*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3
*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*
(45*b^24*d^5*x^13 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^
2*d^3*x^5 - 1170*a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d
^2*x - 11700*a*b^23*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*
x)/b^26
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 2490, normalized size of antiderivative = 7.39

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

output
```
x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2
*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b
^2) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b
^3 - (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x
^13)/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^
2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((
((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^
3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 -
13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b
*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5
*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*
c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-
a)^(7/4)*b^(21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d
^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14
770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*
b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*
c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a
*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a
*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^
10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - ...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2146, normalized size of antiderivative = 6.37

$$\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^5/(b*x^4+a)^2,x)
```

output
```
( - 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**6*d**5 + 76050*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**5*b*c*d**4 - 19890*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*b*d**5*x*
*4 - 105300*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b**2*c**2*d**3 + 76050*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*a**4*b**2*c*d**4*x**4 + 58500*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**3*b**3*c**3*d**2 - 105300*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**3*c
**2*d**3*x**4 - 5850*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*c**4*d + 58500*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**4*c**3*d**2*x**4 - 3510*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*a*b**5*c**5 - 5850*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**5*c**4*d
*x**4 - 3510*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) ...
```

3.22 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 287

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} - \frac{(bc-ad)^3(3bc+13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(3bc+13ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(3bc+13ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}$$

output
```
d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9
*d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+
3*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(17/4)+1/16*
(-a*d+b*c)^3*(13*a*d+3*b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^
(7/4)/b^(17/4)+1/16*(-a*d+b*c)^3*(13*a*d+3*b*c)*arctanh(2^(1/2)*a^(1/4)*b^
(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(17/4)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$= \frac{1440\sqrt[4]{b}d^2(6b^2c^2-8abcd+3a^2d^2)\,x+576b^{5/4}d^3(2bc-ad)x^5+160b^{9/4}d^4x^9+\frac{360\sqrt[4]{b}(bc-ad)^4x}{a(a+bx^4)}+\frac{90\sqrt{2}(-bc+}{}}{}$$

input
```
Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

output
```
(1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*
(2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a
*(a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c +
13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(
b*c) + a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \; 915$$

$$\int \left(\frac{d^2(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{4bdx^4(bc-ad)^3+(bc-ad)^3(3ad+bc)}{b^4(a+bx^4)^2}+\frac{2d^3x^4(2bc-ad)}{b^3}+\frac{d^4x^8}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^3(13ad+3bc)}{8\sqrt{2}a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4}+\frac{x(bc-ad)^4}{4ab^4(a+bx^4)}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{d^4x^9}{9b^2}$$

input

```
Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

output

```
(d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5
)/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) -
((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
8*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3
*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
(16*Sqrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4)
)
```

Defintions of rubi rules used

rule 915

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.92 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.76

method	result
risch	$\frac{d^4x^9}{9b^2}-\frac{2d^4a\,x^5}{5b^3}+\frac{4d^3c\,x^5}{5b^2}+\frac{3d^4a^2x}{b^4}-\frac{8d^3acx}{b^3}+\frac{6d^2c^2x}{b^2}+\frac{(d^4a^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4)x}{4a\,b^4(b\,x^4+a)}-\frac{\_R\text{=Roo}}{}$
default	$\frac{d^2\left(\frac{1}{9}b^2d^2x^9-\frac{2}{5}ab\,d^2x^5+\frac{4}{5}b^2cd\,x^5+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4}-\frac{-\frac{\left(d^4a^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+c^4b^4\right)x}{4a\left(b\,x^4+a\right)}+\frac{\left(13d^4a^4-36a^3bc\right.}{}}{}$

input

```
int((d*x^4+c)^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/9*d^4*x^9/b^2-2/5*d^4/b^3*a*x^5+4/5*d^3/b^2*c*x^5+3*d^4/b^4*a^2*x-8*d^3/
b^3*a*c*x+6*d^2/b^2*c^2*x+1/4*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/a*x/b^4/(b*x^4+a)-1/16/b^5/a*sum((13*a^4*d^4-36*a^3*b*
c*d^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/_R^3*ln(x-_R),_R=RootOf(
_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 2315, normalized size of antiderivative = 8.07

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*
(30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 45*(a*b^5*x^4 +
 a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8
304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5
- 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d
^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b
^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577
784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17)
)^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^
14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11
*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486
160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d
^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/
(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a
^3*b*c*d^3 - 13*a^4*d^4)*x) - 45*(I*a*b^5*x^4 + I*a^2*b^4)*(-(81*b^16*c^16
 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 4
5724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^
6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*
d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*...
```

Sympy [A] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = x^5\left(-\frac{2ad^4}{5b^3}+\frac{4cd^3}{5b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)$$
$$+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{4a^2b^4+4ab^5x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^7b^{17}+28561a^{16}d^{16}-316368a^{15}bcd^{15}+1577784a^{14}b^2c^2d^{14}-4651504a^{13}b^3c^3d^{13}+\right.$$
$$+\frac{d^4x^9}{9b^2}$$

input
```
integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

output
```
x**5*(-2*a*d**4/(5*b**3) + 4*c*d**3/(5*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*
c*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(4*a**2*b**4 + 4*a*b**5*x**4
) + RootSum(65536*_t**4*a**7*b**17 + 28561*a**16*d**16 - 316368*a**15*b*c*
d**15 + 1577784*a**14*b**2*c**2*d**14 - 4651504*a**13*b**3*c**3*d**13 + 89
23164*a**12*b**4*c**4*d**12 - 11486160*a**11*b**5*c**5*d**11 + 9723912*a**
10*b**6*c**6*d**10 - 4810608*a**9*b**7*c**7*d**9 + 617958*a**8*b**8*c**8*d
**8 + 772112*a**7*b**9*c**9*d**7 - 434808*a**6*b**10*c**10*d**6 + 20400*a*
*5*b**11*c**11*d**5 + 45724*a**4*b**12*c**12*d**4 - 8304*a**3*b**13*c**13*
d**3 - 2376*a**2*b**14*c**14*d**2 + 432*a*b**15*c**15*d + 81*b**16*c**16,
Lambda(_t, _t*log(-16*_t*a**2*b**4/(13*a**4*d**4 - 36*a**3*b*c*d**3 + 30*a
**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - 3*b**4*c**4) + x))) + d**4*x**9/(9*
b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(234) = 468$.

Time = 0.11 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{4\,(ab^5x^4 + a^2b^4)} + \frac{5\,b^2d^4x^9 + 18\,(2\,b^2cd^3 - abd^4)x^5 + 45\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{45\,b^4} + \frac{\frac{2\,\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bcd^3-13\,a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(3\,b^4c^4+4\,ab^3c^3d-30\,a^2b^2c^2d^2+36\,a^3bc\cdots}{\sqrt{a}\sqrt{\cdots}}}{\cdots}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^
4)*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqr
t(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13
*a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 +
 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30
*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*
b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(234) = 468$.

Time $= 0.13$ (sec) , antiderivative size $= 642$, normalized size of antiderivative $= 2.24$

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(
a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(
1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*
c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 -
13*(a*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^
(1/4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^
3*b*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/
4)*a*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b
*c*d^3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a^2*b^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x -
 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 +
 36*b^16*c*d^3*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c
*d^3*x + 135*a^2*b^14*d^4*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 2043, normalized size of antiderivative = 7.12

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

output
```
x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b
^2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x
*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/
(4*a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b
^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*
d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5
) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c
^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d -
b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 +
 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d
^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^
7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*
a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7
/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4))
)/((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^
3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 2
4*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3
*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^
3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(
-a)^(7/4)*b^(17/4)) - (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1738, normalized size of antiderivative = 6.06

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^4/(b*x^4+a)^2,x)
```

output

```
(1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*d**4 - 3240*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(
2)))*a**4*b*c*d**3 + 1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*b*d**4*x**4 + 2
700*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b**2*c**2*d**2 - 3240*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a**3*b**2*c*d**3*x**4 - 360*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*
b**3*c**3*d + 2700*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**3*c**2*d**2*x**4 -
270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**4*c**4 - 360*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b**4*c**3*d*x**4 - 270*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**5*c**4*x**4 -
1170*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)
*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**5*d**4 + 3240*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqr...
```

3.23 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 248

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc-ad)^3x}{4ab^3(a+bx^4)} - \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a
)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/
2)/a^(7/4)/b^(13/4)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1+2^(1/2)*b^(1/4)
*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(13/4)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*arctanh
(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$

$$= \frac{160\sqrt[4]{b}d^2(3bc-2ad)x+32b^{5/4}d^3x^5+\frac{40\sqrt[4]{b}(bc-ad)^3x}{a(a+bx^4)}-\frac{30\sqrt{2}(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{30\sqrt{2}(bc-ad)^2(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

output
```
(160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
 - a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*
(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[
2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^4(bc-ad)^2+(bc-ad)^2(2ad+bc)}{b^3(a+bx^4)^2}+\frac{d^3x^4}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3(a+bx^4)}+\frac{d^3x^5}{5b^2}$$

input
```
Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b
^3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
 - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a
*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^
(7/4)*b^(13/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.61

method	result
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4a\,b^3(b\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\ln}{\_R^3}\right.}{16b^4a}$
default	$-\frac{d^2\left(-\frac{1}{5}bd\,x^5+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{4a\left(b\,x^4+a\right)}+\frac{3\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{32a^2}}{b^3}$

input
```
int((d*x^4+c)^3/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*d^3*x^5/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/a*x/b^3/(b*x^4+a)+3/16/b^4/a*sum((3*a^3*d^3-5*a^2*b*c*
d^2+a*b^2*c^2*d+b^3*c^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 1741, normalized size of antiderivative = 7.02

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 15*(a*b^4*
x^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*
a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*
c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^
9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))
^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2
 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^
6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*
c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*
b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) -
15*(-I*a*b^4*x^4 - I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932
*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^
12)/(a^7*b^13))^(1/4)*log(3*I*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*
a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7
*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^
8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81
*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 +
 3*a^3*d^3)*x) - 15*(I*a*b^4*x^4 + I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c...
```

Sympy [A] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{4a^2b^3+4ab^4x^4}$$
$$+\text{RootSum}\Big(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}-156492a^9b^3c^3d^9+8415$$
$$+\frac{d^3x^5}{5b^2}$$

input
```
integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t
**4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b
**2*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 +
26568*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c*
*7*d**5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2
*b**10*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*lo
g(16*_t*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b*
*3*c**3) + x))) + d**3*x**5/(5*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(197) = 394$.

Time = 0.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x}{4\,(ab^4x^4 + a^2b^3)} + \frac{bd^3x^5 + 5\,(3\,bcd^2 - 2\,ad^3)x}{5\,b^3} + \frac{3\left(\frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\cdots\right)}{2\sqrt{\sqrt{\cdots}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right)}{\cdots}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2
*b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*
(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a
^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2
*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^
2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(3/4)*b^(1/4)))/(a*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(197) = 394$.

Time = 0.13 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$$
$$= \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5\,(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^4}$$
$$+ \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5\,(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^4}$$
$$+ \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5\,(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$
$$- \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 + (ab^3)^{\frac{1}{4}}ab^2c^2d - 5\,(ab^3)^{\frac{1}{4}}a^2bcd^2 + 3\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$
$$+ \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{4\,(bx^4+a)ab^3} + \frac{b^8d^3x^5 + 15\,b^8cd^2x - 10\,ab^7d^3x}{5\,b^{10}}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3
)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*
c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b
^3)^(1/4)*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
+ 1/4*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4
+ a)*a*b^3) + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1616, normalized size of antiderivative = 6.52

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^3/(a + b*x^4)^2,x)
```

output
```
(d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a
*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 -
 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))
/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 1
2*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7
/4)*b^(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 -
9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d
 - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d
^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4
)))*3i)/(16*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9
*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^
2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3
*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(
16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a
*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d
^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (
3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d -
60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*
d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1347, normalized size of antiderivative = 5.43

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^3/(b*x^4+a)^2,x)
```

output
```
( - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3 + 150*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(
2)))*a**3*b*c*d**2 - 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x**4 - 30*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d + 150*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(
2)))*a**2*b**2*c*d**2*x**4 - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**3 -
30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**2*d*x**4 - 30*b**(3/4)*a**(1/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*s
qrt(2)))*b**4*c**3*x**4 + 90*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3 - 150*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b
**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c*d**2 + 90*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*
a**3*b*d**3*x**4 + 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d - 15...
```

3.24 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 223

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2(a+bx^4)}-\frac{(bc-ad)(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{9/4}}$$

output

```
d^2*x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)
*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(9/4)+1/16*(-a*d+b
*c)*(5*a*d+3*b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(9
/4)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/
2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$= \frac{32\sqrt[4]{b}d^2x + \frac{8\sqrt[4]{b}(bc-ad)^2x}{a(a+bx^4)} + \frac{2\sqrt{2}(-3b^2c^2-2abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\sqrt{2}(3b^2c^2+2abcd-5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{a^{7/4}}}{32b^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

output
```
(32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[
2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/
4) + (Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \ 915$$
$$\int \left(\frac{-a^2d^2+2bdx^4(bc-ad)+b^2c^2}{b^2(a+bx^4)^2}+\frac{d^2}{b^2}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)(5ad+3bc)}{8\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2(a+bx^4)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*
c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(
9/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
+ ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

method	result
risch	$\frac{d^2 x}{b^2}+\frac{(a^2 d^2-2abcd+b^2 c^2)x}{4a\,b^2(b\,x^4+a)}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\frac{(5a^2d^2-2abcd-3b^2c^2)\ln(x-\_R)}{\_R^3}}{16b^3a}$
default	$\frac{d^2 x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{4a(b\,x^4+a)}+\frac{(5a^2d^2-2abcd-3b^2c^2)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{32a^2}}{b^2}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/b^2/(b*x^4+a)-1/16/b^3/a*sum
((5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 1210, normalized size of antiderivative = 5.43

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*a*b*d^2*x^5 - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7
*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/
(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6
*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)
 - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (I*a*b^3*x^4 + I*a^2*b^2)*(-(8
1*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(I*a^2*b^2*(-(81*b^8*c^8 + 21
6*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^
4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) - (-I
*a*b^3*x^4 - I*a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*
d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 9
00*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(
-I*a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3
*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^
2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*
a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b...
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{4a^2b^2+4ab^3x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b^4c^4d^4-984a\right.$$
$$+\frac{d^2x}{b^2}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + Root
Sum(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*
b**2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*
a**3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**
8*c**8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b*
*2*c**2) + x))) + d**2*x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{4\,(ab^3x^4+a^2b^2)} + \frac{d^2x}{b^2} + \frac{\frac{2\,\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(3\,b^2c^2+2\,abcd-5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3\,b^2}{}}{32\,ab^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 +
1/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*
sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d
^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4
)) - sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(174) = 348$.

Time = 0.13 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$$
$$= \frac{d^2x}{b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c^2 + 2\,(ab^3)^{\frac{1}{4}}\,abcd - 5\,(ab^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$+ \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\,(bx^4+a)ab^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*
d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^
3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^
2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a
^2*d^2*x)/((b*x^4 + a)*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1254, normalized size of antiderivative = 5.62

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4))
+ (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2
*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*
(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(
25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*
d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2
+ 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a
*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2
 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b
*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(
16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*
b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b)
 + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))
/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d +
3*b*c)*1i)/(8*(-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((
x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b
*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d
^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) +
((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 973, normalized size of antiderivative = 4.36

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^2,x)
```

output
```
(10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*d**2 - 4*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*
a**2*b*c*d + 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*d**2*x**4 - 6*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*b**2*c**2 - 4*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c*d*
x**4 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c**2*x**4 - 10*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)
*sqrt(2)))*a**3*d**2 + 4*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c*d - 10*b**(3
/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*a**2*b*d**2*x**4 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
b**2*c**2 + 4*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c*d*x**4 + 6*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**
(1/4)*sqrt(2)))*b**3*c**2*x**4 + 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**...
```

3.25 $\int \frac{c+dx^4}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{c+dx^4}{(a+bx^4)^2} dx = \frac{(bc-ad)x}{4ab(a+bx^4)} - \frac{(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
1/4*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*arctan(-1+2^(1/2)*b^(1/4)*
x/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/16*(a*d+3*b*c)*arctan(1+2^(1/2)*b^(1/
4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/16*(a*d+3*b*c)*arctanh(2^(1/2)*a^(
1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{-\frac{8a^{3/4}\sqrt[4]{b}(-bc+ad)x}{a+bx^4} - 2\sqrt{2}(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \sqrt{2}}{32a^{7/4}b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*
d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+3bc)\int \frac{1}{bx^4+a}dx}{4ab} + \frac{x(bc-ad)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \text{755}$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{(ad+3bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(ad+3bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(bc-ad)}{4ab\,(a+bx^4)}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

output
```
((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) + ((3*b*c + a*d)*((-(ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a])))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result	size
risch	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^4+a\right)}\frac{(ad+3bc)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$	65
default	$-\frac{(ad-bc)x}{4ba(b\,x^4+a)}+\frac{(ad+3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2b}$	140

input
```
int((d*x^4+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a*d-b*c)/b/a*x/(b*x^4+a)+1/16/a/b^2*sum((a*d+3*b*c)/_R^3*ln(x-_R),_R
=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.48

$$\int\frac{c+dx^4}{(a+bx^4)^2}\,dx$$
$$=\frac{(ab^2x^4+a^2b)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+}{a^7b^5}\right.\right.}{}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*((a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4
+ 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^
5))^(1/4) + (3*b*c + a*d)*x) - (-I*a*b^2*x^4 - I*a^2*b)*(-(81*b^4*c^4 + 10
8*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^
(1/4)*log(I*a^2*b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) - (I*a*b^2*x^
4 + I*a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-I*a^2*b*(-(81*b^4*c^4 + 108*a*b^
3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)
+ (3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*
b*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
a^4*d^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4
 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{x(-ad+bc)}{4a^2b+4ab^2x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}\right.\right.\right.$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

output
```
x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5
+ a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*
d + 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx = \frac{(bc-ad)x}{4\left(ab^2x^4+a^2b\right)} + \frac{\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3bc+ad)\log\bigl(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)
*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))
/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.43

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,bc+(ab^3)^{\frac{1}{4}}\,ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^2} + \frac{bcx-adx}{4\,(bx^4+a)ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^
3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3
)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32
*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.98

$$\int \frac{c+dx^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)/(a + b*x^4)^2,x)
```

output
```
(atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c
)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(
-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) +
 ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*
b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/
4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2
+ 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^
(7/4)*b^(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)
/(8*(-a)^(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d
))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5
/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(
-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c
^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)
) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(
12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(
-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c
))/(4*a*b*(a + b*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.29

$$\int \frac{c + dx^4}{(a + bx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d*x^4+c)/(b*x^4+a)^2,x)
```

output

```
( - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 6*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b*c - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 - 6*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*b**2*c*x**4 + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 6*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*b*c + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 + 6*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*b**2*c*x**4 - b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1
/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*d - 3*b**(3/4)*a**(1
/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a
*b*c - b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(b)*x**2)*a*b*d*x**4 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b**2*c*x**4 + b**(3/4)*a**
(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*
*2*d + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sq...
```

3.26 $\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 382

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{d^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}$$

output
```
1/4*b*x/a/(-a*d+b*c)/(b*x^4+a)+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctan(-1+2^(1/
2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(-a*d+b*c)^2+1/16*b^(3/4)*(-7*a*d+3*
b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(-a*d+b*c)^2+1/4*
d^(7/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d+b*c)^2+
1/4*d^(7/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d+b*c)
^2+1/16*b^(3/4)*(-7*a*d+3*b*c)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+
b^(1/2)*x^2))*2^(1/2)/a^(7/4)/(-a*d+b*c)^2+1/4*d^(7/4)*arctanh(2^(1/2)*c^(
1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(3/4)/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$$

$$= \frac{8a^{3/4}bc^{3/4}(bc-ad)x-2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad)(a+bx^4)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}b^{3/4}c^{3/4}(3bc-7ad}{}$$

input
```
Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

output
```
(8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*
a*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/
4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3
/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*
Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4
)*(b*c - a*d)^2*(a + b*x^4))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {931, 25, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{4a(a+bx^4)(bc-ad)}-\frac{\int -\frac{3bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)}dx}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{4ad^2\int \frac{1}{dx^4+c}dx}{bc-ad}+\frac{b(3bc-7ad)\int \frac{1}{bx^4+a}dx}{bc-ad}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{4ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\int \cdots\right)}{bc-ad}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\downarrow \ 1082$$

$$\frac{\frac{4ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\cdots\right)}{\cdots}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\downarrow \ 217$$

$$\frac{\frac{4ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\cdots\right)}{bc-ad}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$$\downarrow \ 1479$$

$$\frac{\frac{4ad^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\cdots\right)}{\cdots}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 25

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\int \cdots}{\sqrt[4]{b}\cdots}\cdots\right)}{\cdots}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{4ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}\cdots}{\cdots}\cdots\right)}{\cdots}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 1103

$$\frac{\frac{4ad^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b(3bc-7ad)\left(\frac{\arctan\left(\cdots\right.}{\sqrt{\cdots}}\right.}{\cdots}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

output

```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*(3*b*c - 7*a*d)*((-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[
2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (4*a*d^2*((-(ArcTan[1 - (Sqrt[2
]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c
] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d
^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.69

method	result
default	$\frac{d^2\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac14}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}-1\right)\right)}{8(ad-bc)^2c}-\frac{b\left(\frac{(ad-bc)x}{4a\left(b\,x^4+a\right)}+\frac{(7ad-3bc)\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\left(\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)^2/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(
c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))-b/(a*d-b*c)^2*(1/4*(a*d-b*
c)/a*x/(b*x^4+a)+1/32*(7*a*d-3*b*c)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)
^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 2955, normalized size of antiderivative = 7.74

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/16*(4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5
*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c -
 a^3*d)*log(d^2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2
- 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^
2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d +
 a^2*c*d^2)) - 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 5
6*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c
^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 +
a^2*b*c - a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 +
28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^
9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*
a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(-I*(a*b^2*c - a^2
*b*d)*x^4 - I*a^2*b*c + I*a^3*d)*log(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^1
0*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^
5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)
*(I*b^2*c^3 - 2*I*a*b*c^2*d + I*a^2*c*d^2)) + 4*(-d^7/(b^8*c^11 - 8*a*b^7*
c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx$$
$$= \frac{\left(\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{32\,(ab^2c^2-2\,a^2bcd+a^3d^2)}$$
$$+\frac{bx}{4\,((ab^2c-a^2bd)x^4+a^2bc-a^3d)}$$
$$+\frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\right.}{}}{8\,(b^2c^2-2\,abcd+a^2d^2)}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(
a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^
3*d^2) + 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt
(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sq
rt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - s
qrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(286) = 572$.

Time = 0.14 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c
*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/
(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^
(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*
sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqr
t(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + s
qrt(2)*a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2
 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(
a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(
3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2
) - 1/16*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(
2)*a^4*d^2) + 1/4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 21975, normalized size of antiderivative = 57.53

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)} dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

output
```
2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*
b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458
7520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^
6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16
- (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^
9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(
81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2
- 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^
7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^
4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a
^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^
3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^
8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*
c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^
13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 409
6*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 25
3952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 +
 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^
12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1000, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+bx^4)^2\,(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^4+a)^2/(d*x^4+c),x)
```

output
```
(14*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d - 6*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a
*b*c**2 + 14*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*d*x**4 - 6*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*b**2*c**2*x**4 - 14*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d + 6*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b
**(1/4)*a**(1/4)*sqrt(2)))*a*b*c**2 - 14*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c
*d*x**4 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c**2*x**4 - 8*d**(3/4)*c**(1/
4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/
4)*sqrt(2)))*a**3*d - 8*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*
sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*d*x**4 + 8*d**(
3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(
1/4)*c**(1/4)*sqrt(2)))*a**3*d + 8*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4
)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*d*x*
*4 + 7*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + s...
```

3.27 $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 456

$$\begin{aligned}\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx &= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} \\
&\quad - \frac{b^{7/4}(3bc-11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad + \frac{b^{7/4}(3bc-11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad - \frac{d^{7/4}(11bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} \\
&\quad + \frac{d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} \\
&\quad + \frac{b^{7/4}(3bc-11ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} \\
&\quad + \frac{d^{7/4}(11bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}\end{aligned}$$

output
```
1/4*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^4+c)+1/4*b*x/a/(-a*d+b*c)/(b*x^4+a
)/(d*x^4+c)+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/
4))*2^(1/2)/a^(7/4)/(-a*d+b*c)^3+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+2^(
1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(-a*d+b*c)^3+1/16*d^(7/4)*(-3*a*d+
11*b*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/(-a*d+b*c)^3+
1/16*d^(7/4)*(-3*a*d+11*b*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c
^(7/4)/(-a*d+b*c)^3+1/16*b^(7/4)*(-11*a*d+3*b*c)*arctanh(2^(1/2)*a^(1/4)*b
^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/(-a*d+b*c)^3+1/16*d^(7/4)*
(-3*a*d+11*b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2
^(1/2)/c^(7/4)/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \frac{1}{32}\left(\frac{8b^2x}{a(bc-ad)^2\left(a+bx^4\right)}+\frac{8d^2x}{c(bc-ad)^2\left(c+dx^4\right)} + \frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(bc-ad)^3} + \frac{2\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}(-bc+ad)^3} + \frac{2\sqrt{2}d^{7/4}(-11bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3} + \frac{2\sqrt{2}d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(-3bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{7/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{c^{7/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

output
```
((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*d)^2*(c +
 d*x^4)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) +
(2*Sqrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4
)*(-3*b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(a^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/4)*(-(b*c) + a*d)^3) +
(Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(c^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(b*c
 - a*d)^3))/32
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {931, 25, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)}-\frac{\int -\frac{7bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)^2}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7bdx^4+3bc-4ad}{(bx^4+a)(dx^4+c)^2}dx}{4a(bc-ad)}+\frac{bx}{4a(a+bx^4)(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\int \frac{4\left(3bd(bc+ad)x^4+3b^2c^2+3a^2d^2-8abcd\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{4c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3bd(bc+ad)x^4+3b^2c^2+3a^2d^2-8abcd}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1020$$

$$\frac{\frac{\frac{b^2c(3bc-11ad)\int \frac{1}{bx^4+a}dx}{bc-ad}+\frac{ad^2(11bc-3ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 755$$

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^4)(bc-ad)}}{4a(bc-ad)}+$$
$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}\cdots}{2\sqrt{d}}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{c(bc-ad)}\cdots}{4a(bc-ad)}$$
$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\cdots\right.}{\cdots}}{c(bc-ad)}\cdots}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 217

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{c(bc-ad)}\cdots}{4a(bc-ad)}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 1479

$$\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-\cdots}{\cdots}}{c(bc-ad)}\cdots}{4a(bc\cdots}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\downarrow$ 25

$$\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\cdots\right.}{c(bc-ad)}}{4a(bc-a\cdots}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{\cdots}}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x\cdots}{\sqrt[4]{d}}}\cdots}{2\sqrt{2}\sqrt[4]{c}\cdots}\right.}{c(bc-ad)}}{4a(bc-ad)}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\cdots\right.}{c(bc-ad)}}{4a(bc-a\cdots}$$

$$\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

output
```
(b*x)/(4*a*(b*c - a*d)*(a + b*x^4)*(c + d*x^4)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^4)) + ((b^2*c*(3*b*c - 11*a*d)*((-(ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(
1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (a*d^2*(11*b*c - 3*a*d)*((-(ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt
[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/
4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^
(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(c*(b*c - a*d)))/(4*a*(b*c - a*
d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.65

method	result
default	$\frac{d^2\left(\frac{(ad-bc)x}{4c\left(d\,x^4+c\right)}+\frac{(3ad-11bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2}\right)}{(ad-bc)^3}+\frac{b^2\left(\frac{(ad-bc)x}{4a\left(b\,x^4+a\right)}\right.}{}$
risch	Expression too large to display

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/c*x/(d*x^4+c)+1/32*(3*a*d-11*b*c)/c^2*(c/d)
^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4
)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x-1)))+b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x/(b*x^4+a)+1/32*(1
1*a*d-3*b*c)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 139.96 (sec) , antiderivative size = 5234, normalized size of antiderivative = 11.48

$$\int\frac{1}{(a+bx^4)^2\,(c+dx^4)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} dx = \text{Timed out}$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)
*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1
/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt
(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a)
)/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*
c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2
 + a^2*d^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b
*c^2*d^2 + a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arcta
n(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sq
rt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(
2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sq
rt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3
*c*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(358) = 716$.

Time = 0.14 (sec) , antiderivative size = 967, normalized size of antiderivative = 2.12

$$\int \frac{1}{(a+bx^4)^2\,(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^
3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1
/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*s
qrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c
*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 -
sqrt(2)*a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3
*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d
^3) + 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^
2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2
*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - s
qrt(2)*a^5*d^3) + 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*
b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d
^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4...
```

Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 37266, normalized size of antiderivative = 81.72

$$\int \frac{1}{(a+bx^4)^2 (c+dx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
5*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d +
b*c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 6553
6*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 144
17920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14
*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^
8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 7
86432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*
b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7
*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 324403
20*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^1
0 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d
^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*
b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11
)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^
8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 ...
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 2475, normalized size of antiderivative = 5.43

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^2,x)
```

output

```
( - 22*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c**3*d - 22*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sq
rt(2)))*a**2*b*c**2*d**2*x**4 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c**4
- 16*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)
*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c**3*d*x**4 - 22*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b**2*c**2*d**2*x**8 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c**4
*x**4 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c**3*d*x**8 + 22*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*a**2*b*c**3*d + 22*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c**2*d
**2*x**4 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c**4 + 16*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*b**2*c**3*d*x**4 + 22*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*...
```

3.28 $\int \sqrt{a+bx^4}\left(c+dx^4\right)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2 \, dx = \frac{1}{231}\left(77c^2-\frac{ad(22bc-5ad)}{b^2}\right)x\sqrt{a+bx^4} + \frac{d(22bc-5ad)x\left(a+bx^4\right)^{3/2}}{77b^2} + \frac{d^2x^5\left(a+bx^4\right)^{3/2}}{11b} + \frac{a^{3/4}\left(77b^2c^2-22abcd+5a^2d^2\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^4}}$$

output
```
1/231*(77*c^2-a*d*(-5*a*d+22*b*c)/b^2)*x*(b*x^4+a)^(1/2)+1/77*d*(-5*a*d+22
*b*c)*x*(b*x^4+a)^(3/2)/b^2+1/11*d^2*x^5*(b*x^4+a)^(3/2)/b+1/231*a^(3/4)*(
5*a^2*d^2-22*a*b*c*d+77*b^2*c^2)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)
+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(
1/2))/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$= \frac{x\sqrt{a+bx^4}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{13}{4},-\frac{bx^4}{a}\right)+4bx^4(7c^2+10cdx^4+3\right.}{585a\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^4]*(c + d*x^4)^2,x]
```

output
```
(x*Sqrt[a + b*x^4]*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Hypergeometric2
F1[-1/2, 1/4, 13/4, -((b*x^4)/a)] + 4*b*x^4*(7*c^2 + 10*c*d*x^4 + 3*d^2*x^
8)*Hypergeometric2F1[1/2, 5/4, 17/4, -((b*x^4)/a)] + 8*b*x^4*(c + d*x^4)^2
*HypergeometricPFQ[{1/2, 5/4, 2}, {1, 17/4}, -((b*x^4)/a)]))/(585*a*Sqrt[1
 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {933, 913, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int \sqrt{bx^4+a}\left(5d(3bc-ad)x^4+c(11bc-ad)\right)dx}{11b}+\frac{dx\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}{11b}$$
$$\Big\downarrow \text{913}$$

$$\frac{\frac{(5a^2d^2-22abcd+77b^2c^2)\int\sqrt{bx^4+a}dx}{7b}+\frac{5dx\left(a+bx^4\right)^{3/2}(3bc-ad)}{7b}}{11b}+\frac{dx\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}{11b}$$

$$\Big\downarrow \; 748$$

$$\frac{\frac{(5a^2d^2-22abcd+77b^2c^2)\left(\frac{2}{3}a\int\frac{1}{\sqrt{bx^4+a}}dx+\frac{1}{3}x\sqrt{a+bx^4}\right)}{7b}+\frac{5dx\left(a+bx^4\right)^{3/2}(3bc-ad)}{7b}}{11b}+\frac{dx\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}{11b}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{(5a^2d^2-22abcd+77b^2c^2)\left(\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{3}x\sqrt{a+bx^4}\right)}{7b}+\frac{5dx\left(a+bx^4\right)^{3/2}(3bc-ad)}{7b}}{11b}+\frac{dx\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}{11b}$$

input
```
Int[Sqrt[a + b*x^4]*(c + d*x^4)^2,x]
```

output
```
(d*x*(a + b*x^4)^(3/2)*(c + d*x^4))/(11*b) + ((5*d*(3*b*c - a*d)*x*(a + b*
x^4)^(3/2))/(7*b) + ((77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*((x*Sqrt[a + b*
x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqr
t[a + b*x^4])))/(7*b))/(11*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{x(-21b^2d^2x^8-6ab\,d^2x^4-66b^2cd\,x^4+10a^2d^2-44abcd-77b^2c^2)\sqrt{b\,x^4+a}}{231b^2}+\frac{2a(5a^2d^2-22abcd+77b^2c^2)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{231b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
elliptic	$\frac{d^2x^9\sqrt{b\,x^4+a}}{11}+\frac{\left(\frac{2}{11}a\,d^2+2bcd\right)x^5\sqrt{b\,x^4+a}}{7b}+\frac{\left(2acd+b\,c^2-\frac{5a\left(\frac{2}{11}a\,d^2+2bcd\right)}{7b}\right)x\sqrt{b\,x^4+a}}{3b}+\frac{\left(a\,c^2-\frac{a\left(2acd+b\,c^2-\frac{5a\left(\frac{2}{11}a\,d^2+\right.}{7b}\right.}{3b}\right.}{}$
default	$c^2\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d^2\left(\frac{x^9\sqrt{b\,x^4+a}}{11}+\frac{2a\,x^5\sqrt{b\,x^4+a}}{77b}-\frac{10a^2x\sqrt{b\,x^4}}{231b^2}\right.$

input
```
int((b*x^4+a)^(1/2)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/231*x*(-21*b^2*d^2*x^8-6*a*b*d^2*x^4-66*b^2*c*d*x^4+10*a^2*d^2-44*a*b*c
*d-77*b^2*c^2)/b^2*(b*x^4+a)^(1/2)+2/231*a*(5*a^2*d^2-22*a*b*c*d+77*b^2*c^
2)/b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1
/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.61

$$\int\sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$=\frac{2\,(77\,b^2c^2-22\,abcd+5\,a^2d^2)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(21\,b^2d^2x^9+6\,(11\,b^2cd+abd^2)x^5+(}{231\,b^2}$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/231*(2*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*sqrt(b)*(-a/b)^(3/4)*ellipt
ic_f(arcsin((-a/b)^(1/4)/x), -1) + (21*b^2*d^2*x^9 + 6*(11*b^2*c*d + a*b*d
^2)*x^5 + (77*b^2*c^2 + 44*a*b*c*d - 10*a^2*d^2)*x)*sqrt(b*x^4 + a))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx = \frac{\sqrt{a}c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt{a}d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/2)*(d*x**4+c)**2,x)
```

output
```
sqrt(a)*c**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(5/4)) + sqrt(a)*c*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,
), b*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + sqrt(a)*d**2*x**9*gamma(9/4)
*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx = \int \sqrt{bx^4+a}\left(dx^4+c\right)^2\,dx$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \sqrt{bx^4+a}\left(dx^4+c\right)^2\,dx$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)^2\,dx$$

input
```
int((a + b*x^4)^(1/2)*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/2)*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$=\frac{-10\sqrt{b\,x^4+a}\,a^2d^2x+44\sqrt{b\,x^4+a}\,abcdx+6\sqrt{b\,x^4+a}\,ab\,d^2x^5+77\sqrt{b\,x^4+a}\,b^2c^2x+66\sqrt{b\,x^4+a}\,b^2cd}{231}$$

input
```
int((b*x^4+a)^(1/2)*(d*x^4+c)^2,x)
```

output

```
( - 10*sqrt(a + b*x**4)*a**2*d**2*x + 44*sqrt(a + b*x**4)*a*b*c*d*x + 6*sq
rt(a + b*x**4)*a*b*d**2*x**5 + 77*sqrt(a + b*x**4)*b**2*c**2*x + 66*sqrt(a
 + b*x**4)*b**2*c*d*x**5 + 21*sqrt(a + b*x**4)*b**2*d**2*x**9 + 10*int(sqr
t(a + b*x**4)/(a + b*x**4),x)*a**3*d**2 - 44*int(sqrt(a + b*x**4)/(a + b*x
**4),x)*a**2*b*c*d + 154*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b**2*c**2)
/(231*b**2)
```

3.29 $\int \sqrt{a+bx^4}(c+dx^4)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx = \frac{(7bc-ad)x\sqrt{a+bx^4}}{21b}+\frac{dx\left(a+bx^4\right)^{3/2}}{7b} + \frac{a^{3/4}(7bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/21*(-a*d+7*b*c)*x*(b*x^4+a)^(1/2)/b+1/7*d*x*(b*x^4+a)^(3/2)/b+1/21*a^(3/
4)*(-a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(5/4)/(b*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx = \frac{x\sqrt{a+bx^4}\left(d(a+bx^4)+\frac{(7bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{7b}$$

input
```
Integrate[Sqrt[a + b*x^4]*(c + d*x^4),x]
```

output
```
(x*Sqrt[a + b*x^4]*(d*(a + b*x^4) + ((7*b*c - a*d)*Hypergeometric2F1[-1/2,
 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(7*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(7bc-ad)\int \sqrt{bx^4+a}dx}{7b}+\frac{dx\left(a+bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(7bc-ad)\left(\frac{2}{3}a\int \frac{1}{\sqrt{bx^4+a}}dx+\frac{1}{3}x\sqrt{a+bx^4}\right)}{7b}+\frac{dx\left(a+bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bc-ad)\left(\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{1}{3}x\sqrt{a+bx^4}\right)}{7b}+\frac{dx\left(a+bx^4\right)^{3/2}}{7b}$$

input
```
Int[Sqrt[a + b*x^4]*(c + d*x^4),x]
```

output
```
(d*x*(a + b*x^4)^(3/2))/(7*b) + ((7*b*c - a*d)*((x*Sqrt[a + b*x^4])/3 + (a
^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4]
)))/(7*b)
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x(3bd\,x^4+2ad+7bc)\sqrt{b\,x^4+a}}{21b}-\frac{2a(ad-7bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$\frac{d\,x^5\sqrt{b\,x^4+a}}{7}+\frac{\left(\frac{2ad}{7}+bc\right)x\sqrt{b\,x^4+a}}{3b}+\frac{\left(ac-\frac{a\left(\frac{2ad}{7}+bc\right)}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{x^5\sqrt{b\,x^4+a}}{7}+\frac{2ax\sqrt{b\,x^4+a}}{21b}-\frac{2a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.$

input
```
int((b*x^4+a)^(1/2)*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(3*b*d*x^4+2*a*d+7*b*c)/b*(b*x^4+a)^(1/2)-2/21*a*(a*d-7*b*c)/b/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(
1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx = \frac{2\,(7\,bc-ad)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1\right)+\left(3\,bdx^5+(7\,bc+2\,ad)x\right)\sqrt{bx^4+a}}{21\,b}$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/21*(2*(7*b*c - a*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/
x), -1) + (3*b*d*x^5 + (7*b*c + 2*a*d)*x)*sqrt(b*x^4 + a))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/2)*(d*x**4+c),x)
```

output
```
sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx = \int \sqrt{bx^4+a}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(d*x^4 + c), x)
```

Giac [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx=\int \sqrt{bx^4+a}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/2)*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx=\int \sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)\,dx$$

input
```
int((a + b*x^4)^(1/2)*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)*(c + d*x^4), x)
```

Reduce [F]

$$\int \sqrt{a+bx^4}\left(c+dx^4\right)\,dx$$
$$=\frac{2\sqrt{b\,x^4+a}\,adx+7\sqrt{b\,x^4+a}\,bcx+3\sqrt{b\,x^4+a}\,bd\,x^5-2\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a^2d+14\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)abc}{21b}$$

input
```
int((b*x^4+a)^(1/2)*(d*x^4+c),x)
```

output
```
(2*sqrt(a + b*x**4)*a*d*x + 7*sqrt(a + b*x**4)*b*c*x + 3*sqrt(a + b*x**4)*
b*d*x**5 - 2*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*d + 14*int(sqrt(a +
 b*x**4)/(a + b*x**4),x)*a*b*c)/(21*b)
```

3.30 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 749

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx \\
&= \frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} + \frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} \\
&+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} \\
&- \frac{b^{3/4}(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/4*(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(3/4)/d^(3/4)+1/4*(-a*d+b*c)^(1/2)*arctanh((-a*d+b*c)^(1/2)*
x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/d^(3/4)+1/2*b^(3/4)*(a^(1
/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d/(b*x^4+a)^(1/2)-1/2*b^
(3/4)*(-a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d/
(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*(-a*d+b
*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ellipt
icPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(
1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/
(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))/d/(b*x^4+a)^(1/2)+1/8*(b^(1/2)*(-c)^(
1/2)-a^(1/2)*d^(1/2))*(-a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)
+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^
(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2
*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))/d/(b*x^4+
a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{5acx\sqrt{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,A\right.\right.}$$

input
```
Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

output
```
(5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^
4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x
^4)/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {922, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}dx$$

$$\Big\downarrow \text{922}$$

$$\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \text{761}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \text{925}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}$$

$$\Big\downarrow \text{1541}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\sqrt{\cdots}\right.}{\cdots}}{2c \cdots}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\cdots\right.}{\cdots}}{2c}\cdots\right)}{d}$$

$$\Big\downarrow \; 761$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\cdots\right)}{d}$$

$$\Big\downarrow \; 2221$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}+\frac{\sqrt[4]{\cdots}}{\cdots}\cdots\right)}{\cdots}$$

$$\Big\downarrow \; 2223$$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}} -$$

$$(bc-ad)\left(\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{2c}\right.$$

input
```
Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

output
```
(b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*
x^4]) - ((b*c - a*d)*(((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*
x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(S
qrt[b] - (Sqrt[a]*Sqrt[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/
4)*d^(1/4)*Sqrt[a + b*x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqr
t[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(
Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4
*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b
]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt
[d])*Sqrt[d]*(((-c)^(1/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqr
t[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[a] - (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c]
+ Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.36

method	result
default	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d+c\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\cdots}}{\cdots}\right)}{\cdots}}{8d^2}$
elliptic	$\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d+c\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}+\frac{2\_\alpha^3 d\sqrt{1-\cdots}}{\cdots}\right)}{\cdots}}{8d^2}$

input
```
int((b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)
*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*
_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a
^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(
1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

output
```
Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{bx^4+a}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{b\,x^4+a}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt{b\,x^4+a}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(1/2)/(d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(c + d*x**4),x)
```

3.31 $\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 694

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \frac{x\sqrt{a+bx^4}}{4c(c+dx^4)} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}d^{3/4}\sqrt{bc-ad}}$$
$$- \frac{(bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}d^{3/4}\sqrt{bc-ad}}$$
$$+ \frac{a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c(bc+ad)\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d\sqrt{a+bx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d\sqrt{a+bx^4}}$$

output
```
1/4*x*(b*x^4+a)^(1/2)/c/(d*x^4+c)-1/16*(-3*a*d+b*c)*arctan((-a*d+b*c)^(1/2
)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(7/4)/d^(3/4)/(-a*d+b*c)^(1/2
)-1/16*(-3*a*d+b*c)*arctanh((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(7/4)/d^(3/4)/(-a*d+b*c)^(1/2)+1/2*a^(3/4)*b^(3/4)*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c/(a*d+b*c)/(b*x^4+a)^(1/2)+1/32*(b^
(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*(-3*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x
^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(
1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1
/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^2/(b^(1/2)*(-c)^(1/2)-a^(1/2)*d
^(1/2))/d/(b*x^4+a)^(1/2)+1/32*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*(-3*a*
d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ell
ipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d
^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/
c^2/(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))/d/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx$$
$$= \frac{x\left(\frac{bx^4\sqrt{1+\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{5\left(\frac{a+bx^4}{c}+\frac{15a^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2ad\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.}\right.}{c+dx^4}\right.}{20\sqrt{a+bx^4}}$$

input
```
Integrate[Sqrt[a + b*x^4]/(c + d*x^4)^2,x]
```

output
```
(x*((b*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c + (15*a^2*AppellF1[1/4, 1/2, 1, 5/4,
-((b*x^4)/a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a
), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -
((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
)))/(c + d*x^4)))/(20*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {929, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{929}$$

$$\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}-\frac{\int -\frac{bx^4+3a}{\sqrt{bx^4+a}(dx^4+c)}dx}{4c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bx^4+3a}{\sqrt{bx^4+a}(dx^4+c)}dx}{4c}+\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{d}-\frac{(bc-3ad)\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{d}}{4c}+\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-3ad)\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{d}}{4c}+\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}}{4c}+\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \cdots}{ad+b\cdots}}{2c}\cdots\right.}{\cdots}}{4c}$$

$$\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\cdots}{\left(1-\cdots\right.}}{ad+bc}}{2c}\cdots\right.}{\cdots}}{4c}$$

$$\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\,\text{ar}\cdots\right.}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \; 2221$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{(bc-3ad)\left(\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)}{2c}\right.}{}$$

$$\frac{x\sqrt{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\downarrow$ 2223

$$\frac{\sqrt{bx^4+a}x}{4c\left(dx^4+c\right)}+$$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}} - \frac{(bc-3ad)\left(\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\operatorname{arcta}\right.}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}\right.}{}$$

input
```
Int[Sqrt[a + b*x^4]/(c + d*x^4)^2,x]
```

output
```
(x*Sqrt[a + b*x^4])/(4*c*(c + d*x^4)) + ((b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4]) - ((b*c - 3*a*d)*(((b^(1/4)
*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sq
rt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt[d])/Sqrt
[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(
d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1
/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d
]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4
])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a +
b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1/4)*(Sqrt
[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(
1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] - (Sqrt[b]
*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.44

method	result
default	$\frac{x\sqrt{b\,x^4+a}}{4c(d\,x^4+c)}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4d+c\right)}\frac{(-3ad+bc)\left(-\frac{\text{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+c}}\right.}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{}$
elliptic	$\frac{x\sqrt{b\,x^4+a}}{4c(d\,x^4+c)}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4d+c\right)}\frac{(-3ad+bc)\left(-\frac{\text{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{b\,x^4+c}}\right.}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{}$

input
```
int((b*x^4+a)^(1/2)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(b*x^4+a)^(1/2)/c/(d*x^4+c)+1/4/c/d*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((-3*a*d+b*c)/_al
pha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-b*c
)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-I*
b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)
*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*d,(-I
/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+c
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(1/2)/(d*x**4+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt{bx^4+a}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt{bx^4+a}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt{b\,x^4+a}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/2)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^4}}{(c+dx^4)^2}\,dx=\int \frac{\sqrt{b\,x^4+a}}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int((b*x^4+a)^(1/2)/(d*x^4+c)^2,x)
```

output
```
int(sqrt(a + b*x**4)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.32 $\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \frac{d(14bc-5ad)x\sqrt{a+bx^4}}{21b^2} + \frac{d^2x^5\sqrt{a+bx^4}}{7b} + \frac{(21b^2c^2-14abcd+5a^2d^2)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42\sqrt[4]{a}b^{9/4}\sqrt{a+bx^4}}$$

output
```
1/21*d*(-5*a*d+14*b*c)*x*(b*x^4+a)^(1/2)/b^2+1/7*d^2*x^5*(b*x^4+a)^(1/2)/b
+1/42*(5*a^2*d^2-14*a*b*c*d+21*b^2*c^2)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(
a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),
1/2*2^(1/2))/a^(1/4)/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \frac{x\sqrt{1+\frac{bx^4}{a}}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{13}{4},-\frac{bx^4}{a}\right)-4bx^4(7c^2+10cdx^4+3d\right.}{585a\sqrt{a+bx^4}}$$

input

```
Integrate[(c + d*x^4)^2/Sqrt[a + b*x^4],x]
```

output

```
(x*Sqrt[1 + (b*x^4)/a]*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Hypergeomet
ric2F1[1/4, 1/2, 13/4, -((b*x^4)/a)] - 4*b*x^4*(7*c^2 + 10*c*d*x^4 + 3*d^2
*x^8)*Hypergeometric2F1[5/4, 3/2, 17/4, -((b*x^4)/a)] - 8*b*x^4*(c + d*x^4
)^2*HypergeometricPFQ[{5/4, 3/2, 2}, {1, 17/4}, -((b*x^4)/a)]))/(585*a*Sqr
t[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {933, 913, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \frac{d(11bc-5ad)x^4+c(7bc-ad)}{\sqrt{bx^4+a}}dx}{7b}+\frac{dx\sqrt{a+bx^4}(c+dx^4)}{7b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{(5a^2d^2-14abcd+21b^2c^2)\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}+\frac{dx\sqrt{a+bx^4}(11bc-5ad)}{3b}}{7b}+\frac{dx\sqrt{a+bx^4}(c+dx^4)}{7b}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5a^2d^2-14abcd+21b^2c^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}+\frac{dx\sqrt{a+bx^4}(11bc-5ad)}{3b}}{7b}+\frac{dx\sqrt{a+bx^4}\left(c+dx^4\right)}{7b}$$

input
```
Int[(c + d*x^4)^2/Sqrt[a + b*x^4],x]
```

output
```
(d*x*Sqrt[a + b*x^4]*(c + d*x^4))/(7*b) + ((d*(11*b*c - 5*a*d)*x*Sqrt[a +
b*x^4])/(3*b) + ((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4]))/(7*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{xd\left(-3bd\,x^{4}+5ad-14bc\right)\sqrt{b\,x^{4}+a}}{21b^{2}}+\frac{\left(5a^{2}d^{2}-14abcd+21b^{2}c^{2}\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
elliptic	$\frac{d^{2}x^{5}\sqrt{b\,x^{4}+a}}{7b}+\frac{\left(2cd-\frac{5d^{2}a}{7b}\right)x\sqrt{b\,x^{4}+a}}{3b}+\frac{\left(c^{2}-\frac{a\left(2cd-\frac{5d^{2}a}{7b}\right)}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
default	$\frac{c^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+d^{2}\left(\frac{x^{5}\sqrt{b\,x^{4}+a}}{7b}-\frac{5ax\sqrt{b\,x^{4}+a}}{21b^{2}}+\frac{5a^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{Elliptic}}{21b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*d*(-3*b*d*x^4+5*a*d-14*b*c)/b^2*(b*x^4+a)^(1/2)+1/21*(5*a^2*d^2-14
*a*b*c*d+21*b^2*c^2)/b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2
))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \frac{\left(c+dx^4\right)^2}{\sqrt{a+bx^4}}\,dx = \frac{(21\,b^2c^2-14\,abcd+5\,a^2d^2)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+(3\,abd^2x^5+(14\,abcd-5\,a^2d^2)x)\sqrt{bx^4}}{21\,ab^2}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/21*((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*sqrt(b)*(-a/b)^(3/4)*elliptic_
f(arcsin((-a/b)^(1/4)/x), -1) + (3*a*b*d^2*x^5 + (14*a*b*c*d - 5*a^2*d^2)*
x)*sqrt(b*x^4 + a))/(a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(1/2),x)
```

output
```
c**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(5/4)) + c*d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d**2*x**9*gamma(9/4)*hyper((1/
2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \int \frac{(d\,x^4+c)^2}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{\sqrt{a+bx^4}}\,dx = \frac{-5\sqrt{b\,x^4+a}\,a\,d^2x+14\sqrt{b\,x^4+a}\,bcdx+3\sqrt{b\,x^4+a}\,b\,d^2x^5+5\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a^2d^2-14\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a}{21b^2}$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a + b*x**4)*a*d**2*x + 14*sqrt(a + b*x**4)*b*c*d*x + 3*sqrt(a +
 b*x**4)*b*d**2*x**5 + 5*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*d**2 -
14*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b*c*d + 21*int(sqrt(a + b*x**4)/
(a + b*x**4),x)*b**2*c**2)/(21*b**2)
```

3.33 $\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{dx\sqrt{a+bx^4}}{3b} + \frac{(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/3*d*x*(b*x^4+a)^(1/2)/b+1/6*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4
)),1/2*2^(1/2))/a^(1/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{dx(a+bx^4)+(3bc-ad)x\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{3b\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/Sqrt[a + b*x^4],x]
```

output
```
(d*x*(a + b*x^4) + (3*b*c - a*d)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^4)/a)])/(3*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(3bc-ad)\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}+\frac{dx\sqrt{a+bx^4}}{3b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}+\frac{dx\sqrt{a+bx^4}}{3b}$$

input
```
Int[(c + d*x^4)/Sqrt[a + b*x^4],x]
```

output
```
(d*x*Sqrt[a + b*x^4])/(3*b) + ((3*b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(6*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

method	result	siz
elliptic	$\frac{d x \sqrt{b\,x^4+a}}{3 b}+\frac{\left(c-\frac{a d}{3 b}\right) \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	96
risch	$\frac{d x \sqrt{b\,x^4+a}}{3 b}-\frac{(a d-3 b c) \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	99
default	$\frac{c \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+d \left(\frac{x \sqrt{b\,x^4+a}}{3 b}-\frac{a \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)$	16

input
```
int((d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x*(b*x^4+a)^(1/2)/b+(c-1/3*a*d/b)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(
1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}adx + (3\,bc-ad)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)}{3\,ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(b*x^4 + a)*a*d*x + (3*b*c - a*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx=\frac{\sqrt{b\,x^4+a}\,dx-\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)ad+3\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)bc}{3b}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*d*x - int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*d + 3*int(s
qrt(a + b*x**4)/(a + b*x**4),x)*b*c)/(3*b)
```

3.34 $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx$

Optimal result

Integrand size = 21, antiderivative size = 625

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx \\
&= -\frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} \\
&\quad + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{a+bx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{a+bx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-1/4*d^(1/4)*arctan((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))
/(-c)^(3/4)/(-a*d+b*c)^(1/2)-1/4*d^(1/4)*arctanh((-a*d+b*c)^(1/2)*x/(-c)^(
1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(1/2)+1/2*b^(3/4)*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(a*d+b*c)/(b*x^4+a)^(1/
2)-1/8*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+
a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4
))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)
/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2
))/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(
1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/2)*(-c)^(1/2)
+a^(1/2)*d^(1/2))/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = -\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\,(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx}{c}\right.\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a
+ b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

$$\Big\downarrow \ 925$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}$$

$$\Big\downarrow \ 1541$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}$$

$\Big\downarrow$ 2221

$$\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}}{2c}+$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\frac{\sqrt[4]{-c}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\arctan\cdots}{2\sqrt[4]{d}\sqrt{bc}\cdots}\cdots\right.}{\cdots}}{2c}$$

$\Big\downarrow$ 2223

$$\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx\cdots}{\left(\sqrt{b}x^2\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}}{2c}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\sqrt[4]{-c}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\arctan\cdots}{2\sqrt[4]{d}\sqrt{bc}\cdots}\cdots\right.}{\cdots}}{2c}$$

input
```
Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

output
```
((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt
[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt
[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d]
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-
c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[
a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]
*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*
Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1
/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] -
 (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(
4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.31

method	result	siz
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} d+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 b \,x^{2} \_\alpha^{2}+2 a}{2 \sqrt{\frac{a d -b c}{d}}\, \sqrt{b \,x^{4}+a}}\right)}{\sqrt{\frac{a d -b c}{d}}}+\frac{2 \_\alpha^{3} d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticPi}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, \frac{i \sqrt{a}\, \_\alpha^{2} d}{\sqrt{b}\, c}, \frac{\sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, c \sqrt{b \,x^{4}+a}}}{\_\alpha^{3}}}{8 d}$	19
elliptic	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4} d+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 b \,x^{2} \_\alpha^{2}+2 a}{2 \sqrt{\frac{a d -b c}{d}}\, \sqrt{b \,x^{4}+a}}\right)}{\sqrt{\frac{a d -b c}{d}}}+\frac{2 \_\alpha^{3} d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticPi}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, \frac{i \sqrt{a}\, \_\alpha^{2} d}{\sqrt{b}\, c}, \frac{\sqrt{-\frac{i \sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, c \sqrt{b \,x^{4}+a}}}{\_\alpha^{3}}}{8 d}$	19

input
```
int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2
+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_al
pha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/
(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_
alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

output
```
Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx=\int \frac{1}{\sqrt{b\,x^4+a}\,\left(d\,x^4+c\right)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)}\,dx=\int \frac{\sqrt{b\,x^4+a}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.35 $\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 723

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx$$

$$= -\frac{dx\sqrt{a+bx^4}}{4c(bc-ad)\,(c+dx^4)} + \frac{\sqrt[4]{d}(5bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}(bc-ad)^{3/2}}$$

$$+ \frac{\sqrt[4]{d}(5bc-3ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}(bc-ad)^{3/2}}$$

$$+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c(bc+ad)\sqrt{a+bx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{a+bx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{a+bx^4}}$$

output
```
-1/4*d*x*(b*x^4+a)^(1/2)/c/(-a*d+b*c)/(d*x^4+c)+1/16*d^(1/4)*(-3*a*d+5*b*c
)*arctan((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(7/4)
/(-a*d+b*c)^(3/2)+1/16*d^(1/4)*(-3*a*d+5*b*c)*arctanh((-a*d+b*c)^(1/2)*x/(
-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(7/4)/(-a*d+b*c)^(3/2)+1/2*b^(3/4)
*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJa
cobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c/(a*d+b*c)/(b*x^4
+a)^(1/2)-1/32*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(a^(1/2
)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*
arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(
1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^2/(b^(1/2)*
(-c)^(1/2)-a^(1/2)*d^(1/2))/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/32*(b^(1/2)*(-c)^
(1/2)-a^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2
),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^2/(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))/(-
a*d+b*c)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \frac{x\left(\frac{bdx^4\sqrt{1+\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad}+\frac{c\left(25ac(-4bc+4ad+bdx^4)\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-10dx^4(a+bx^4)\left(2ad\operatorname{App}\right.\right.}{(bc-ad)(c+dx^4)\left(-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2x^4\left(2ad\operatorname{AppellF1}\left(\right.\right.\right.}\right.}{20c^2\sqrt{a+bx^4}}$$

input
```
Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)^2),x]
```

output
```
(x*((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a),
-((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*Appe
llF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 10*d*x^4*(a + b*x^4)*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*Appell
F1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d*x^
4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*Appell
F1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*Sqrt[a + b*x
^4])
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 1016, normalized size of antiderivative = 1.41, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {931, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-bdx^4+4bc-3ad}{\sqrt{bx^4+a}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx-b\int \frac{1}{\sqrt{bx^4+a}}dx}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{(5bc-3ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 1541

$$\frac{(5bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{a}\sqrt{d}\left(\ldots\right.}{\ldots}}{\ldots}\right.}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{(5bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}+\frac{\frac{\sqrt{b}c\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\ldots\right.}{\ldots}}{2c}\right.}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 761

$$\frac{(5bc-3ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{ad+bc}}{2c}\ldots\right.}{\ldots}$$

$$\frac{dx\sqrt{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 2221

$$(5bc-3ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\sqrt[4]{-c}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)a}{2\sqrt[4]{}}\right.}{}}{2c}\right.$$

$$\frac{dx\sqrt{bx^4+a}}{4c(bc-ad)\left(dx^4+c\right)}$$

$\downarrow$ 2223

$$(5bc-3ad)\left(\frac{\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}+\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\sqrt{d}\left(\frac{\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{-c}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)}{}\right.}{}}{2c}\right.$$

$$\frac{dx\sqrt{bx^4+a}}{4c(bc-ad)\left(dx^4+c\right)}$$

input
```
Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x*Sqrt[a + b*x^4])/(c*(b*c - a*d)*(c + d*x^4)) + (-1/2*(b^(3/4)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*Sqrt[a + b*x^4]) + (5*b*c
- 3*a*d)*(((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sq
rt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sq
rt[a]*Sqrt[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*S
qrt[a + b*x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c
])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt
[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(
1/4)*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a
]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b
*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]
*(((-c)^(1/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]
*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + (
(Sqrt[a] - (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.46

method	result
default	$\frac{dx\sqrt{b\,x^4+a}}{4c(ad-bc)(d\,x^4+c)}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d+c\right)}\frac{(-3ad+5bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2b\,x^2 ...}{2\sqrt{\frac{ad-b ...}{d}}...}\right)}{\sqrt{\frac{ad-b ...}{d}}} ...\right)}{...}}{...}$
elliptic	$\frac{dx\sqrt{b\,x^4+a}}{4c(ad-bc)(d\,x^4+c)}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d+c\right)}\frac{(-3ad+5bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2b\,x^2 ...}{2\sqrt{\frac{ad-b ...}{d}}...}\right)}{\sqrt{\frac{ad-b ...}{d}}} ...\right)}{...}}{...}$

input

```
int(1/(b*x^4+a)^(1/2)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/4/c*d/(a*d-b*c)*x*(b*x^4+a)^(1/2)/(d*x^4+c)+1/4*b/c/(a*d-b*c)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^
(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*su
m((-3*a*d+5*b*c)/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/
a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^
(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt{bx^4+a}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,d^2x^{12}+a\,d^2x^8+2bcd\,x^8+2acd\,x^4+b\,c^2x^4+a\,c^2}dx$$

input
```
int(1/(b*x^4+a)^(1/2)/(d*x^4+c)^2,x)
```

output
```
int(sqrt(a + b*x**4)/(a*c**2 + 2*a*c*d*x**4 + a*d**2*x**8 + b*c**2*x**4 +
2*b*c*d*x**8 + b*d**2*x**12),x)
```

3.36 $\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \frac{(bc-ad)^3x}{2ab^3\sqrt{a+bx^4}} + \frac{d^2(7bc-4ad)x\sqrt{a+bx^4}}{7b^3} + \frac{d^3x^5\sqrt{a+bx^4}}{7b^2} + \frac{(7b^3c^3+21ab^2c^2d-35a^2bcd^2+15a^3d^3)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{28a^{5/4}b^{13/4}\sqrt{a+bx^4}}$$

output

```
1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a)^(1/2)+1/7*d^2*(-4*a*d+7*b*c)*x*(b*x^4+a
)^(1/2)/b^3+1/7*d^3*x^5*(b*x^4+a)^(1/2)/b^2+1/28*(15*a^3*d^3-35*a^2*b*c*d^
2+21*a*b^2*c^2*d+7*b^3*c^3)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))
/a^(5/4)/b^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(7(bc-ad)^3x+2ad^2(7bc-4ad)x(a+bx^4)+2abd^3x^5(a+bx^4)\right)-i(7b^3c^3+21\ldots}{14a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}b^3\sqrt{a-\ldots}}$$

input
```
Integrate[(c + d*x^4)^3/(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[(I*Sqrt[b])/Sqrt[a]]*(7*(b*c - a*d)^3*x + 2*a*d^2*(7*b*c - 4*a*d)*x*
(a + b*x^4) + 2*a*b*d^3*x^5*(a + b*x^4)) - I*(7*b^3*c^3 + 21*a*b^2*c^2*d -
 35*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[b])/Sqrt[a]]*x], -1])/(14*a*Sqrt[(I*Sqrt[b])/Sqrt[a]]*b^3*Sqrt[a
+ b*x^4])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {930, 1025, 913, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{(dx^4+c)(c(bc+ad)-d(7bc-9ad)x^4)}{\sqrt{bx^4+a}}dx}{2ab}+\frac{x(c+dx^4)^2(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 1025}$$

$$\frac{\frac{\int \frac{c(7b^2c^2+14abdc-9a^2d^2)-3d(7bc-5ad)(bc-3ad)x^4}{\sqrt{bx^4+a}}dx}{7b}-\frac{dx\sqrt{a+bx^4}(c+dx^4)(7bc-9ad)}{7b}}{2ab}+\frac{x(c+dx^4)^2(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$\Big\downarrow$ 913

$$\frac{\frac{\frac{(15a^3d^3-35a^2bcd^2+21ab^2c^2d+7b^3c^3)\int \frac{1}{\sqrt{bx^4+a}}dx}{b}-\frac{dx\sqrt{a+bx^4}(7bc-5ad)(bc-3ad)}{b}}{7b}-\frac{dx\sqrt{a+bx^4}(c+dx^4)(7bc-9ad)}{7b}}{2ab}+\frac{x(c+dx^4)^2(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$\Big\downarrow$ 761

$$\frac{\frac{\frac{(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}(15a^3d^3-35a^2bcd^2+21ab^2c^2d+7b^3c^3)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}(7bc-5ad)(bc-3ad)}{b}}{7b}-\frac{dx\sqrt{a+bx^4}(c+d}{7b}}{2ab}$$
$$\frac{x(c+dx^4)^2(bc-ad)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)^3/(a + b*x^4)^(3/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4)^2)/(2*a*b*Sqrt[a + b*x^4]) + (-1/7*(d*(7*b*c -
9*a*d)*x*Sqrt[a + b*x^4]*(c + d*x^4))/b + (-((d*(7*b*c - 5*a*d)*(b*c - 3*a
*d)*x*Sqrt[a + b*x^4])/b) + ((7*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2
+ 15*a^3*d^3)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(5/4)*
Sqrt[a + b*x^4]))/(7*b))/(2*a*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.70 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.38

method	result
elliptic	$-\frac{\left(a^{3} d^{3}-3 a^{2} b c\, d^{2}+3 a\, b^{2} c^{2} d-b^{3} c^{3}\right) x}{2 b^{3} a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{d^{3} x^{5} \sqrt{b\, x^{4}+a}}{7 b^{2}}+\frac{\left(-\frac{d^{2} \left(a d-3 b c\right)}{b^{2}}-\frac{5 a\, d^{3}}{7 b^{2}}\right) x \sqrt{b\, x^{4}+a}}{3 b}+\frac{\left(\frac{d \left(a^{2} d^{2}-3 a b c d+3 b^{2} c^{2}\right)}{b^{3}}-\frac{a^{3} d^{3}}{\cdots}\right.}{\cdots}$
default	$c^{3} \left(\frac{x}{2 a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{2 a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\, x^{4}+a}}\right)+d^{3} \left(-\frac{a^{2} x}{2 b^{3} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{5} \sqrt{b\, x^{4}+a}}{7 b^{2}}-\frac{4 a x \sqrt{b \cdots}}{7 b \cdots}\right.$
risch	$-\frac{d^{2} x \left(-b d\, x^{4}+4 a d-7 b c\right) \sqrt{b\, x^{4}+a}}{7 b^{3}}+\frac{b d \left(11 a^{2} d^{2}-28 a b c d+21 b^{2} c^{2}\right) \left(-\frac{x}{2 b \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{2 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\, x^{4}+a}} \cdots\right.}{\cdots}$

input
```
int((d*x^4+c)^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x/((x^4+a/b)*b)^(
1/2)+1/7*d^3*x^5*(b*x^4+a)^(1/2)/b^2+1/3*(-1/b^2*d^2*(a*d-3*b*c)-5/7*a/b^2
*d^3)/b*x*(b*x^4+a)^(1/2)+(d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)/b^3-1/2*(a^3*d^
3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a/b^3-1/3*(-1/b^2*d^2*(a*d-3*b*c)-5
/7*a/b^2*d^3)/b*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/
2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \frac{(7\,ab^3c^3+21\,a^2b^2c^2d-35\,a^3bcd^2+15\,a^4d^3+(7\,b^4c^3+21\,ab^3c^2d-35\,a^2b^2cd^2+15\,a^3}{}$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/14*((7*a*b^3*c^3 + 21*a^2*b^2*c^2*d - 35*a^3*b*c*d^2 + 15*a^4*d^3 + (7*b
^4*c^3 + 21*a*b^3*c^2*d - 35*a^2*b^2*c*d^2 + 15*a^3*b*d^3)*x^4)*sqrt(b)*(-
a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (2*a^2*b^2*d^3*x^9 + 2
*(7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^5 + (7*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 3
5*a^3*b*c*d^2 - 15*a^4*d^3)*x)*sqrt(b*x^4 + a))/(a^2*b^4*x^4 + a^3*b^3)
```

Sympy [F]

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{(c+dx^4)^3}{(a+bx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**4+c)**3/(b*x**4+a)**(3/2),x)
```

output
```
Integral((c + d*x**4)**3/(a + b*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^3/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^3/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{(d\,x^4+c)^3}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x^4)^3/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x^4)^3/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^3}{(a+bx^4)^{3/2}}\,dx = \frac{-15\sqrt{b\,x^4+a}\,a^2d^3x+35\sqrt{b\,x^4+a}\,abc\,d^2x-3\sqrt{b\,x^4+a}\,ab\,d^3x^5-21\sqrt{b\,x^4+a}\,b^2c^2d}{}$$

input
```
int((d*x^4+c)^3/(b*x^4+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**4)*a**2*d**3*x + 35*sqrt(a + b*x**4)*a*b*c*d**2*x - 3
*sqrt(a + b*x**4)*a*b*d**3*x**5 - 21*sqrt(a + b*x**4)*b**2*c**2*d*x + 7*sq
rt(a + b*x**4)*b**2*c*d**2*x**5 + sqrt(a + b*x**4)*b**2*d**3*x**9 + 15*int
(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**4*d**3 - 35*int(sq
rt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*c*d**2 + 15*int(s
qrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*d**3*x**4 + 21*i
nt(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*c**2*d -
35*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*c*d**
2*x**4 + 7*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*
c**3 + 21*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*c
**2*d*x**4 + 7*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b**
4*c**3*x**4)/(7*b**3*(a + b*x**4))
```

3.37 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx = \frac{(bc-ad)^2x}{2ab^2\sqrt{a+bx^4}} + \frac{d^2x\sqrt{a+bx^4}}{3b^2} + \frac{(3b^2c^2+6abcd-5a^2d^2)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(1/2)+1/3*d^2*x*(b*x^4+a)^(1/2)/b^2+1/1
2*(-5*a^2*d^2+6*a*b*c*d+3*b^2*c^2)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/
2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2
^(1/2))/a^(5/4)/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^4}{a}}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{2},\frac{13}{4},-\frac{bx^4}{a}\right)-12bx\right.}{}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^4)/a]*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Hypergeomet
ric2F1[1/4, 3/2, 13/4, -((b*x^4)/a)] - 12*b*x^4*(7*c^2 + 10*c*d*x^4 + 3*d^
2*x^8)*Hypergeometric2F1[5/4, 5/2, 17/4, -((b*x^4)/a)] - 24*b*x^4*(c + d*x
^4)^2*HypergeometricPFQ[{5/4, 2, 5/2}, {1, 17/4}, -((b*x^4)/a)]))/(585*a^2
*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {930, 913, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{c(bc+ad)-d(3bc-5ad)x^4}{\sqrt{bx^4+a}}dx}{2ab}+\frac{x(c+dx^4)(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(-5a^2d^2+6abcd+3b^2c^2)\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}-\frac{dx\sqrt{a+bx^4}(3bc-5ad)}{3b}}{2ab}+\frac{x(c+dx^4)(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(-5a^2d^2+6abcd+3b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{dx\sqrt{a+bx^4}(3bc-5ad)}{3b}}{2ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(3/2),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(2*a*b*Sqrt[a + b*x^4]) + (-1/3*(d*(3*b*c - 5*
a*d)*x*Sqrt[a + b*x^4])/b + ((3*b^2*c^2 + 6*a*b*c*d - 5*a^2*d^2)*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4]))/(2*a*
b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{(a^2d^2-2abcd+b^2c^2)x}{2b^2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{d^2x\sqrt{b\,x^4+a}}{3b^2}+\frac{\left(-\frac{d(ad-2bc)}{b^2}+\frac{a^2d^2-2abcd+b^2c^2}{2a\,b^2}-\frac{a\,d^2}{3b^2}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c^2\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d^2\left(\frac{xa}{2b^2\sqrt{(x^4+\frac{a}{b})b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{}}{}}}{}\right.$
risch	$\frac{d^2x\sqrt{b\,x^4+a}}{3b^2}-\frac{a^2d^2\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+2bd(2ad-3bc)\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.}{3b^2}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/((x^4+a/b)*b)^(1/2)+1/3*d^2*x*(b*x
^4+a)^(1/2)/b^2+(-d*(a*d-2*b*c)/b^2+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b^2-
1/3*a/b^2*d^2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(
1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx=\frac{(3\,ab^2c^2+6\,a^2bcd-5\,a^3d^2+(3\,b^3c^2+6\,ab^2cd-5\,a^2bd^2)x^4)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right.}{x}\right.\right.}{6\,(a^2b^3x^4+a^3b^2)}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*((3*a*b^2*c^2 + 6*a^2*b*c*d - 5*a^3*d^2 + (3*b^3*c^2 + 6*a*b^2*c*d - 5
*a^2*b*d^2)*x^4)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -
1) + (2*a^2*b*d^2*x^5 + (3*a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2)*x)*sqrt(b*
x^4 + a))/(a^2*b^3*x^4 + a^3*b^2)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(3/2),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx=\int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx=\int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/2}}\,dx=\frac{5\sqrt{b\,x^4+a}\,a\,d^2x-6\sqrt{b\,x^4+a}\,bcdx+\sqrt{b\,x^4+a}\,b\,d^2x^5-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^3d^2 -}{}$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(3/2),x)
```

output

```
(5*sqrt(a + b*x**4)*a*d**2*x - 6*sqrt(a + b*x**4)*b*c*d*x + sqrt(a + b*x**
4)*b*d**2*x**5 - 5*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)
*a**3*d**2 + 6*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**
2*b*c*d - 5*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b
*d**2*x**4 + 3*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b
**2*c**2 + 6*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**
2*c*d*x**4 + 3*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b**
3*c**2*x**4)/(3*b**2*(a + b*x**4))
```

3.38 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/2}} \, dx = \frac{(bc-ad)x}{2ab\sqrt{a+bx^4}} + \frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-a*d+b*c)*x/a/b/(b*x^4+a)^(1/2)+1/4*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*(
(b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*
x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/2}} \, dx = \frac{x\left(bc-ad+(bc+ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{2ab\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(x*(b*c - a*d + (b*c + a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2
, 5/4, -((b*x^4)/a)]))/(2*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{bx^4+a}}dx}{2ab}+\frac{x(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{a+bx^4}}+\frac{x(bc-ad)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
((b*c - a*d)*x)/(2*a*b*Sqrt[a + b*x^4]) + ((b*c + a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

method	result
elliptic	$-\frac{x(ad-bc)}{2ba\sqrt{(x^4+\frac{a}{b})b}}+\frac{\left(\frac{d}{b}-\frac{ad-bc}{2ba}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{E}}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*x/a*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+(d/b-1/2*(a*d-b*c)/b/a)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{((b^2c+abd)x^4+abc+a^2d)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-\sqrt{bx^4+a}(b^2c-abd)x}{2\,(ab^3x^4+a^2b^2)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((b^2*c + a*b*d)*x^4 + a*b*c + a^2*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_
f(arcsin(x*(-b/a)^(1/4)), -1) - sqrt(b*x^4 + a)*(b^2*c - a*b*d)*x)/(a*b^3*
x^4 + a^2*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,dx+\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2d+\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)abc+\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4\cdots}\right.}{b\,(b\,x^4+a)}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d*x + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*
x**8),x)*a**2*d + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*
a*b*c + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b*d*x**4
 + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b**2*c*x**4)/(b
*(a + b*x**4))
```

3.39 $\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 677

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx &= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} \\
&+ \frac{d^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} + \frac{d^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} \\
&+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{a+bx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/2*b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/2)+1/4*d^(5/4)*arctan((-a*d+b*c)^(1/2)*x
/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/4)/(-a*d+b*c)^(3/2)+1/4*d^(5/
4)*arctanh((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(3/
4)/(-a*d+b*c)^(3/2)+1/4*b^(3/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+
b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1
/2))/a^(5/4)/(a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(
1/2))*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*El
lipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)
*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4
)/c/(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))/(-a*d+b*c)/(b*x^4+a)^(1/2)+1/8*(b
^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(
1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4
*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2)
,1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))/(-a*d
+b*c)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \frac{x\left(-\frac{bdx^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(2ad-b\left(2c+dx^4\right)\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4}\right.}{\left(c+dx^4\right)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\right.\right.}\right.}{10a(-bc}$$

input
```
Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

output
```
(x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/
2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1
/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2,
2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*
x^4)/a), -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}-\frac{\int -\frac{bdx^4+bc-2ad}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^4+bc-2ad}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx-2ad\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\int \frac{1}{\sqrt{bx^4+a}(dx^4+c)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\downarrow \text{1541}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\cdots}{\sqrt{a}\left(1-\cdots\right.}}{ad+bc}}{2c}\right.}{2a(bc-ad)}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\downarrow \text{27}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int\frac{\sqrt{b}x^2\cdots}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\cdots\right.}}{ad+bc}}{2c}\right.}{2a(bc-ad)}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\downarrow \text{761}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\cdots\right.}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\cdots}{2c}\right.}{\cdots}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$$\downarrow \text{2221}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-2ad\left(\frac{\frac{\sqrt{d}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{-c}\right)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{ad+bc}+\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(}}}{}}{2c}\right.$$

$$\frac{bx}{2a\sqrt{a+bx^4}(bc-ad)}$$

$\downarrow$ 2223

$$\frac{bx}{2a(bc-ad)\sqrt{bx^4+a}}+$$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{bx^4+a}}-2ad\left(\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt[4]{a}(bc+ad)\sqrt{bx^4+a}}\right.$$

input
```
Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) - 2*a*d*(((b^(1/4)*c*(Sqrt[
b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqr
t[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a]*Sqrt[d])/Sqrt[-c])*Arc
Tanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(d^(1/4)*S
qrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[
b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(b*c
 + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) -
 ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((-c)^(1/4)*(Sqrt[b]*Sqrt[
-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt
[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] - (Sqrt[b]*Sqrt[-c]
)/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.46

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{}\right.}{}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d+c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+}{2\sqrt{\frac{ad-bc}{d}}\sqrt{b}}\right)}{\sqrt{\frac{ad-bc}{d}}}}{}\right.}{}$

input
```
int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x/a/(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*sum(1/_alpha
^3/(a*d-b*c)*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((
a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/
c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a
)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/
c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{2}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/2}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 +
 b**2*c*x**8 + b**2*d*x**12),x)
```

3.40 $\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 784

$$
\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx &= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^4}} \\
&- \frac{dx}{4c(bc-ad)\sqrt{a+bx^4}\,(c+dx^4)} - \frac{3d^{5/4}(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}(bc-ad)^{5/2}} \\
&- \frac{3d^{5/4}(3bc-ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{16(-c)^{7/4}(bc-ad)^{5/2}} \\
&+ \frac{b^{3/4}(bc-2ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c(bc-ad)(bc+ad)\sqrt{a+bx^4}} \\
&+ \frac{3\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)d(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{a+bx^4}} \\
&+ \frac{3\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)d(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}},2\arctan\left(\cdots\right.\right.}{32\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)(bc-ad)^2\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)/
(b*x^4+a)^(1/2)/(d*x^4+c)-3/16*d^(5/4)*(-a*d+3*b*c)*arctan((-a*d+b*c)^(1/2
)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/(-c)^(7/4)/(-a*d+b*c)^(5/2)-3/16*d
^(5/4)*(-a*d+3*b*c)*arctanh((-a*d+b*c)^(1/2)*x/(-c)^(1/4)/d^(1/4)/(b*x^4+a
)^(1/2))/(-c)^(7/4)/(-a*d+b*c)^(5/2)+1/4*b^(3/4)*(-2*a*d+b*c)*(a^(1/2)+b^(
1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arct
an(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c/(-a*d+b*c)/(a*d+b*c)/(b*x^4+a
)^(1/2)+3/32*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))*d*(-a*d+3*b*c)*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*(-c)^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/
2)/b^(1/2)/(-c)^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^2/(b^(1/2)*(-
c)^(1/2)-a^(1/2)*d^(1/2))/(-a*d+b*c)^2/(b*x^4+a)^(1/2)+3/32*(b^(1/2)*(-c)^
(1/2)-a^(1/2)*d^(1/2))*d*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
4*(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/(-c)^(1/2)/d^(1/2
),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^2/(b^(1/2)*(-c)^(1/2)+a^(1/2)*d^(1/2))/(-
a*d+b*c)^2/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \frac{x\left(bd(2bc+ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c\left(25ac(4a^2d^2\right.}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)^2),x]
```

output
```
(x*(b*d*(2*b*c + a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(4*a^2*d^2 + a*b*d*(-8*c + d*x^4)
+ 2*b^2*c*(2*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] - 10*x^4*(a^2*d^2 + a*b*d^2*x^4 + 2*b^2*c*(c + d*x^4))*(2*a*d*Appell
F1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1
/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2,
2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*
x^4)/a), -((d*x^4)/c)])))))/(20*a*c^2*(b*c - a*d)^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {931, 1024, 27, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-5bdx^4+4bc-3ad}{\left(bx^4+a\right)^{3/2}\left(dx^4+c\right)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(ad+2bc)}{a\sqrt{a+bx^4}(bc-ad)}-\frac{\int -\frac{2\left(bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd\right)}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{2a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a+bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{b(ad+2bc)\int \frac{1}{\sqrt{bx^4+a}}dx-3ad(3bc-ad)\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a+bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 761

$$\frac{\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+2bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-3ad(3bc-ad)\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a+bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 925

$$\frac{\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+2bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-3ad(3bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{1}{\left(\frac{\sqrt{d}x^2}{\sqrt{-c}}+1\right)\sqrt{bx^4+a}}dx}{2c}\right)}{a(bc-ad)}+\cdots}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1541

$$\frac{\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+2bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-3ad(3bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{a}\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\cdots}{\cdots}}{2c}\cdots\right.}{a(bc-ad)}}{4c(bc-a\cdots}$$

$$\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+2bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-3ad(3bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\int\frac{1}{\sqrt{bx^4+a}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\int}{ad+}}{2c}\right.}{a(bc-ad)}$$

$$4c(bc-ad)$$

$$\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 761

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+2bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}-3ad(3bc-ad)\left(\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\sqrt{b}c\right)\text{EllipticF}}{2\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}\right.}{}$$

$$\frac{dx}{4c\sqrt{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

$\big\downarrow$ 2221

$$\frac{b(2bc+ad)x}{a(bc-ad)\sqrt{bx^4+a}}+\frac{\frac{b^{3/4}(2bc+ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{bx^4+a}}-3ad(3bc-ad)\left(\frac{\sqrt[4]{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)}{2\sqrt[4]{a}}\right.}{}$$

$$\frac{dx}{4c(bc-ad)\sqrt{bx^4+a}\left(dx^4+c\right)}$$

$\big\downarrow$ 2223

$$\frac{\frac{b(2bc+ad)x}{a(bc-ad)\sqrt{bx^4+a}}+\frac{\frac{b^{3/4}(2bc+ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{bx^4+a}}-3ad(3bc-ad)\left(\frac{\sqrt[4]{b}c\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\cdots}}{2\sqrt[4]{a}(\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{dx}{4c(bc-ad)\sqrt{bx^4+a}\,(dx^4+c)}$$

input
```
Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a + b*x^4]*(c + d*x^4)) + ((b*(2*b*c + a*d)
*x)/(a*(b*c - a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(2*b*c + a*d)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) - 3*a*d*(3*b*c -
a*d)*(((b^(1/4)*c*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b
]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*Sqrt[d]*(-1/2*((-c)^(3/4)*(Sqrt[b] - (Sqrt[a
]*Sqrt[d])/Sqrt[-c])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[
a + b*x^4])])/(d^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[a] + (Sqrt[b]*Sqrt[-c])/S
qrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(Sqrt[a]*Sqrt[b]*
Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)
*Sqrt[a + b*x^4])))/(b*c + a*d))/(2*c) + ((b^(1/4)*(Sqrt[b]*c + Sqrt[a]*Sq
rt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(b*c +
 a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*Sqrt[d]*(((
-c)^(1/4)*(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/
((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*d^(1/4)*Sqrt[b*c - a*d]) + ((Sqr
t[a] - (Sqrt[b]*Sqrt[-c])/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.49

method	result
default	$\frac{d^{2} x \sqrt{b\,x^{4}+a}}{4 c \left(a d -b c \right)^{2} \left(d\,x^{4}+c \right)}+\frac{b^{2} x}{2 a \left(a d -b c \right)^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\left(\frac{b d}{4 \left(a d -b c \right)^{2} c}+\frac{b^{2}}{2 \left(a d -b c \right)^{2} a}\right) \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}-$
elliptic	$\frac{d^{2} x \sqrt{b\,x^{4}+a}}{4 c \left(a d -b c \right)^{2} \left(d\,x^{4}+c \right)}+\frac{b^{2} x}{2 a \left(a d -b c \right)^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\left(\frac{b d}{4 \left(a d -b c \right)^{2} c}+\frac{b^{2}}{2 \left(a d -b c \right)^{2} a}\right) \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}-$

input
```
int(1/(b*x^4+a)^(3/2)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c*d^2/(a*d-b*c)^2*x*(b*x^4+a)^(1/2)/(d*x^4+c)+1/2*b^2*x/a/(a*d-b*c)^2/
((x^4+a/b)*b)^(1/2)+(1/4*b*d/(a*d-b*c)^2/c+1/2*b^2/(a*d-b*c)^2/a)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2)
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*su
m((-a*d+3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/
a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^
(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d+c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{2}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{3/2}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(3/2)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)^2}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2d^2x^{16}+2ab\,d^2x^{12}+2b^2cd\,x^{12}+a^2d^2x^8+4abcd\,x^8+b^2c^2x^8+2a^2cd\,x}$$

input
```
int(1/(b*x^4+a)^(3/2)/(d*x^4+c)^2,x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*c**2 + 2*a**2*c*d*x**4 + a**2*d**2*x**8 + 2*a*b
*c**2*x**4 + 4*a*b*c*d*x**8 + 2*a*b*d**2*x**12 + b**2*c**2*x**8 + 2*b**2*c
*d*x**12 + b**2*d**2*x**16),x)
```

3.41 $\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx = -\frac{x}{\sqrt{1+x^4}}$$

output
```
-x/(x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}} \, dx = -\frac{x}{\sqrt{1+x^4}}$$

input
```
Integrate[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

output
```
-(x/Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4-1}{\left(x^4+1\right)^{3/2}}dx$$
$$\Big\downarrow \text{908}$$
$$-\frac{x}{\sqrt{x^4+1}}$$

input
```
Int[(-1 + x^4)/(1 + x^4)^(3/2),x]
```

output
```
-(x/Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 908
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{x}{\sqrt{x^4+1}}$	11
default	$-\frac{x}{\sqrt{x^4+1}}$	11
trager	$-\frac{x}{\sqrt{x^4+1}}$	11
risch	$-\frac{x}{\sqrt{x^4+1}}$	11
elliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
pseudoelliptic	$-\frac{x}{\sqrt{x^4+1}}$	11
meijerg	$-x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-x^4\right)+\frac{x^5\,\text{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^4\right)}{5}$	32
orering	$-\frac{x(x^4-1)}{\sqrt{x^4+1}\,(-1+x)(1+x)(x^2+1)}$	33

input
```
int((x^4-1)/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-x/sqrt(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)} - \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((x**4-1)/(x**4+1)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gam
ma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-x/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
integrate((x^4-1)/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-x/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{\sqrt{x^4+1}}$$

input
```
int((x^4 - 1)/(x^4 + 1)^(3/2),x)
```

output
```
-x/(x^4 + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{-1+x^4}{(1+x^4)^{3/2}}\,dx = -\frac{\sqrt{x^4+1}\,x}{x^4+1}$$

input
```
int((x^4-1)/(x^4+1)^(3/2),x)
```

output
```
( - sqrt(x**4 + 1)*x)/(x**4 + 1)
```

3.42 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 321

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2}+\frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21d^3\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$

output
```
-1/21*b*(-13*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/d^2+1/7*b*x*(-b*x^4+a)^(3/2)/d+
1/21*a^(1/4)*b^(3/4)*(47*a^2*d^2-56*a*b*c*d+21*b^2*c^2)*(1-b*x^4/a)^(1/2)*
EllipticF(b^(1/4)*x/a^(1/4),I)/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)
^3*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)
/c^(1/2),I)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*(1-b*x
^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)/b^(1/4)/c/d^3/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.90

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{x\left(5b(-a+bx^4)(7bc-16ad+3bdx^4)-\frac{b(21b^2c^2-56abcd+47a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\right.}{c}\right.}{}$$

input
```
Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

output
```
(x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*
b*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*
x^4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Ap
pellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*Appell
F1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/
2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/
a, (d*x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {933, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{bx(a-bx^4)^{3/2}}{7d}-\frac{\int \frac{\sqrt{a-bx^4}(a(bc-7ad)-b(7bc-13ad)x^4)}{c-dx^4}dx}{7d}$$

$$\Big\downarrow \text{1025}$$

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\int\frac{a\left(7b^2c^2-16abdc+21a^2d^2\right)-b\left(21b^2c^2-56abdc+47a^2d^2\right)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{3d}}{7d}$$

$\downarrow$ 1021

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{b\left(47a^2d^2-56abcd+21b^2c^2\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{7d}$$

$\downarrow$ 765

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{7d}$$

$\downarrow$ 762

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{7d}$$

$\downarrow$ 925

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\int\frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\cdots\right.}{d}}{3d}}{7d}$$

$\downarrow$ 27

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\cdots\right.}{d}}{3d}}{7d}$$

$$\Big\downarrow \ 1543$$

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}}{2\sqrt{c}\sqrt{a-bx^4}}\right.}{d}}{3d}}{7d}$$

$$\Big\downarrow \ 1542$$

$$\frac{bx\left(a-bx^4\right)^{3/2}}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-21(bc-ad)^3\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}\right.}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right.}{3d}}{7d}$$

input

```
Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

output

```
(b*x*(a - b*x^4)^(3/2))/(7*d) - ((b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(3
*d) - ((a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) -
 (21*(b*c - a*d)^3*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqr
t[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a
 - b*x^4])))/d)/(3*d))/(7*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.60 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.09

method	result
risch	$\frac{bx\left(-3bd\,x^{4}+16ad-7bc\right)\sqrt{-b\,x^{4}+a}}{21d^{2}}+\frac{\frac{b\left(47a^{2}d^{2}-56abcd+21b^{2}c^{2}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{21\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b\right.}{}}{}$
default	$-\frac{b^{2}x^{5}\sqrt{-b\,x^{4}+a}}{7d}-\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)x\sqrt{-b\,x^{4}+a}}{3b}+\frac{\left(\frac{b\left(3a^{2}d^{2}-3abcd+b^{2}c^{2}\right)}{d^{3}}+\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$-\frac{b^{2}x^{5}\sqrt{-b\,x^{4}+a}}{7d}-\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)x\sqrt{-b\,x^{4}+a}}{3b}+\frac{\left(\frac{b\left(3a^{2}d^{2}-3abcd+b^{2}c^{2}\right)}{d^{3}}+\frac{\left(-\frac{b^{2}(3ad-bc)}{d^{2}}+\frac{5b^{2}a}{7d}\right)a}{3b}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$

input
```
int((-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/21*b*x*(-3*b*d*x^4+16*a*d-7*b*c)*(-b*x^4+a)^(1/2)/d^2+1/21/d^2*(b*(47*a^
2*d^2-56*a*b*c*d+21*b^2*c^2)/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^
(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1
/a^(1/2)*b^(1/2))^(1/2),I)-21/8*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/d^2*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*
x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)
*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/
(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_a
lpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = -\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

input
```
integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

output
```
-Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqr
t(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-
c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx=\int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx=\int -\frac{(-bx^4+a)^{\frac{5}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx=\int \frac{(a-b\,x^4)^{5/2}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(5/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(5/2)/(c - d*x^4), x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx = \frac{16\sqrt{-b\,x^4+a}\,abdx-7\sqrt{-b\,x^4+a}\,b^2cx-3\sqrt{-b\,x^4+a}\,b^2d\,x^5+21\left(\int \frac{\sqrt{-b\,x^4+a}}{bd\,x^8-ad\,x^4-bc\,x^4\ldots}\right.}{}$$

input
```
int((-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

output
```
(16*sqrt(a - b*x**4)*a*b*d*x - 7*sqrt(a - b*x**4)*b**2*c*x - 3*sqrt(a - b*
x**4)*b**2*d*x**5 + 21*int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*x**4 + b
*d*x**8),x)*a**3*d**2 - 16*int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*x**4
 + b*d*x**8),x)*a**2*b*c*d + 7*int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*
x**4 + b*d*x**8),x)*a*b**2*c**2 - 47*int((sqrt(a - b*x**4)*x**4)/(a*c - a*
d*x**4 - b*c*x**4 + b*d*x**8),x)*a**2*b*d**2 + 56*int((sqrt(a - b*x**4)*x*
*4)/(a*c - a*d*x**4 - b*c*x**4 + b*d*x**8),x)*a*b**2*c*d - 21*int((sqrt(a
- b*x**4)*x**4)/(a*c - a*d*x**4 - b*c*x**4 + b*d*x**8),x)*b**3*c**2)/(21*d
**2)
```

3.43 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 277

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}$$

output
```
1/3*b*x*(-b*x^4+a)^(1/2)/d-1/3*a^(1/4)*b^(3/4)*(-5*a*d+3*b*c)*(1-b*x^4/a)^
(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*
d+b*c)^2*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b
^(1/2)/c^(1/2),I)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(-a*d+b*c)^2*
(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(
1/2),I)/b^(1/4)/c/d^2/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.23

$$\int \frac{\left(a-bx^4\right)^{3/2}}{c-dx^4}\,dx =$$
$$-\frac{x\left(\frac{b(-3bc+5ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(3a^2d-abdx^4+b^2x^4\left(-c+dx^4\right)\right)\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4(a-\cdots\right.}{\left(-c+dx^4\right)\left(5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\mathrm{A}\cdots\right.\right.}\right.}{15d\sqrt{a-bx^4}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
-1/15*(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4
*(-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*
(a - b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^
4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d
*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(d*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {933, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^4\right)^{3/2}}{c-dx^4}dx$$
$$\Big\downarrow \; 933$$

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{3d}$$

$\Big\downarrow$ 1021

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{b(3bc-5ad)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$\Big\downarrow$ 765

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$\Big\downarrow$ 762

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}$$

$\Big\downarrow$ 925

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{3d}$$

$\Big\downarrow$ 27

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{3d}$$

$\Big\downarrow$ 1543

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}}{3d}$$

$\Big\downarrow$ 1542

$$\frac{bx\sqrt{a-bx^4}}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\sqrt[4]{a}\sqrt{1-\frac{bx}{a}}\right)}{d}}{3d}$$

input
```
Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
(b*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (
b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4])
- (3*(b*c - a*d)^2*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqr
t[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a
 - b*x^4])))/d)/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.39 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.10

method	result
risch	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\frac{b(5ad-3bc)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{-\cdots}{2\sqrt{\cdots}}\right)}{\sqrt{\cdots}}}{\cdots}\cdots\right.}{\cdots}}{3d}$
default	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\cdots\right.}{\cdots}$
elliptic	$\frac{bx\sqrt{-b\,x^4+a}}{3d}+\frac{\left(\frac{b(2ad-bc)}{d^2}-\frac{ba}{3d}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\cdots\right.}{\cdots}$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x*(-b*x^4+a)^(1/2)/d+1/3/d*(b*(5*a*d-3*b*c)/d/(1/a^(1/2)*b^(1/2))^(1
/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)
^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/8*(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^2*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*
b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/
2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*
_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = -\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int \left(-\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\right)dx$$

input
```
integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
 b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(-bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(3/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(3/2)/(c - d*x^4), x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx = \frac{\sqrt{-b\,x^4+a}\,bx+3\left(\int \frac{\sqrt{-b\,x^4+a}}{bd\,x^8-ad\,x^4-bc\,x^4+ac}dx\right)a^2d-\left(\int \frac{\sqrt{-b\,x^4+a}}{bd\,x^8-ad\,x^4-bc\,x^4+ac}dx\right)abc-5\left(\int\right.}{3d}$$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

output
```
(sqrt(a - b*x**4)*b*x + 3*int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*x**4
+ b*d*x**8),x)*a**2*d - int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*x**4 +
b*d*x**8),x)*a*b*c - 5*int((sqrt(a - b*x**4)*x**4)/(a*c - a*d*x**4 - b*c*x
**4 + b*d*x**8),x)*a*b*d + 3*int((sqrt(a - b*x**4)*x**4)/(a*c - a*d*x**4 -
 b*c*x**4 + b*d*x**8),x)*b**2*c)/(3*d)
```

3.44 $\int \frac{\sqrt{a-bx^4}}{c-dx^4} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4} \, dx = \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}$$

output
```
a^(1/4)*b^(3/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/d/(-b*x^4
+a)^(1/2)-1/2*a^(1/4)*(-a*d+b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^
(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)-1/2
*a^(1/4)*(-a*d+b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)
*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx =$$

$$-\frac{5acx\sqrt{a-bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{App}\right.\right.}$$

input
```
Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

output
```
(-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c
])/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
+ 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Ap
pellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {922, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}dx$$

$$\Big\downarrow \text{ 922}$$

$$\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d}$$

$$\Big\downarrow \text{925}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)}{d}$$

input
```
Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

output
```
(a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)]
, -1])/(d*Sqrt[a - b*x^4]) - ((b*c - a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*El
lipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.58 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.08

method	result
default	$\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\,\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\,\_\alpha^3d\sqrt{1-\frac{\sqrt{b}}{\sqrt{}}}}{}\right.}{\_\alpha^3}}{8d^2}$
elliptic	$\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2\,\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-b\,x^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\,\_\alpha^3d\sqrt{1-\frac{\sqrt{b}}{\sqrt{}}}}{}\right.}{\_\alpha^3}}{8d^2}$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
b/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-
1/8/d^2*sum((a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_al
pha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2
))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2)
)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = -\int \frac{\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

input
```
integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int \frac{\sqrt{a-b\,x^4}}{c-d\,x^4}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(c - d*x^4),x)
```

output
```
int((a - b*x^4)^(1/2)/(c - d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-d\,x^4+c}dx$$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a - b*x**4)/(c - d*x**4),x)
```

3.45 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx$

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

output
```
1/2*a^(1/4)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2
)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*(1-b*x^4/a)^(1
/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4
)/c/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = -\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\,(-c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\right.\right.}$$

input
```
Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^
4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appell
F1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx$$

$$\Big\downarrow \text{925}$$

$$\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

input
```
Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

output
```
(a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[
c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a
^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]),
 ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.13

method	result	si
default	$-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}d-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}d\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\,\_\alpha^{2}d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^{4}+a}}}{\_\alpha^{3}}}{8d}$	1
elliptic	$-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}d-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^{4}+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}-\frac{2\_\alpha^{3}d\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\frac{\sqrt{a}\,\_\alpha^{2}d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,c\sqrt{-b\,x^{4}+a}}}{\_\alpha^{3}}}{8d}$	1

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x
^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*
_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_al
pha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = -\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{-bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int \frac{1}{\sqrt{a-b\,x^4}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{bd\,x^8-ad\,x^4-bc\,x^4+ac}dx$$

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a - b*x**4)/(a*c - a*d*x**4 - b*c*x**4 + b*d*x**8),x)
```

3.46 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-bx^4}}$$

output
```
1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(1/2)+1/2*b^(3/4)*(1-b*x^4/a)^(1/2)*Ellipt
icF(b^(1/4)*x/a^(1/4),I)/a^(3/4)/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d
*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c
^(1/2),I)/b^(1/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*d*(1-b*x^4/a)^
(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1
/4)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \frac{5acx\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(-2bc+2ad+bdx^4)+bdx^4\sqrt{1-\cdots}\right.}{10ac(-bc+ad)\sqrt{\cdots}}$$

input
```
Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c +
2*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4,
1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqr
t[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c
 + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*
(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {931, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-bdx^4+bc-2ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{2a(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-2ad\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\operatorname{Elliptic}\cdots}{2\cdots}\right.}{2a(bc-ad)}$$
$$\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^
4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] - 2*a*d*
((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt
[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (
a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/(2*a*(
b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.07

method	result
default	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-\cdots}}\right)}{\sqrt{\frac{ad-bc}{d}}}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$-\frac{bx}{2a(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2(ad-bc)a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2\cdots}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-\cdots}}\right)}{\sqrt{\frac{ad-bc}{d}}}\cdots}{\cdots}\right)}{\cdots}$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x/a/(a*d-b*c)/(-(x^4-a/b)*b)^(1/2)-1/2*b/(a*d-b*c)/a/(1/a^(1/2)*b^(
1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/_alpha^3
/(a*d-b*c)*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a
*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/
c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(
1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(
-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d
-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = -\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*
sqrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8-a^2d\,x^4-2abc\,x^4+a^2c}dx$$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*c - a**2*d*x**4 - 2*a*b*c*x**4 + 2*a*b*d*x**8 +
 b**2*c*x**8 - b**2*d*x**12),x)
```

3.47 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 334

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx &= \frac{bx}{6a(bc-ad)(a-bx^4)^{3/2}} \\ &+ \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} \\ &+ \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}(bc-ad)^2\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^(3/2)+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b
*c)^2/(-b*x^4+a)^(1/2)+1/12*b^(3/4)*(-11*a*d+5*b*c)*(1-b*x^4/a)^(1/2)*Elli
pticF(b^(1/4)*x/a^(1/4),I)/a^(7/4)/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/
4)*d^2*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(
1/2)/c^(1/2),I)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)+1/2*a^(1/4)*d^2*(1
-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/
2),I)/b^(1/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.76 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \frac{x\left(\frac{bd(-5bc+11ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac\left(12a^3d^2+a^2bd(-24c+dx^4)+5\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9
/4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*
x^4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^
8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4
)*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/
a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/
(60*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {931, 1024, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-5bdx^4+5bc-6ad}{(a-bx^4)^{3/2}(c-dx^4)}dx}{6a(bc-ad)}+\frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\int \frac{-bd(5bc-11ad)x^4+5b^2c^2+12a^2d^2-11abcd}{\sqrt{a-bx^4}(c-dx^4)}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx+b(5bc-11ad)\int \frac{1}{\sqrt{a-bx^4}}dx}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx+\frac{b\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a(a-bx^4)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\frac{bx(5bc-11ad)}{2a\sqrt{a-bx^4}(bc-ad)}}{6a(bc-ad)}+\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{ 1543}$$

$$\frac{\frac{12a^2d^2\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{2a(bc-ad)}+\cdots}{6a(bc-ad)}$$

$$\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{ 1542}$$

$$\frac{\frac{12a^2d^2\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad}{\sqrt{}}}{2a(bc-ad)}}{6a(bc-ad)}$$

$$\frac{bx}{6a\left(a-bx^4\right)^{3/2}(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + ((b*(5*b*c - 11*a*d)*x)/(2*a*(
b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + 1
2*a^2*d^2*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b
*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b
]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]
)))/(2*a*(b*c - a*d)))/(6*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.66 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.08

method	result
default	$-\frac{x\sqrt{-b\,x^4+a}}{6a(ad-bc)b\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{d\left(\ldots\right)}{\ldots}$
elliptic	$-\frac{x\sqrt{-b\,x^4+a}}{6a(ad-bc)b\left(x^4-\frac{a}{b}\right)^2}-\frac{bx(11ad-5bc)}{12a^2(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b(11ad-5bc)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad-bc)^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{d\left(\ldots\right)}{\ldots}$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*x/a/(a*d-b*c)/b*(-b*x^4+a)^(1/2)/(x^4-a/b)^2-1/12*b*x/a^2*(11*a*d-5*b
*c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2
/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^
(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8
*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2)
)^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(
1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx =$$

$$-\int \frac{1}{-a^2c\sqrt{a-bx^4}+a^2dx^4\sqrt{a-bx^4}+2abcx^4\sqrt{a-bx^4}-2abdx^8\sqrt{a-bx^4}-b^2cx^8\sqrt{a-bx^4}+b^2dx^{12}\sqrt{a}}$$

input
```
integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a**2*c*sqrt(a - b*x**4) + a**2*d*x**4*sqrt(a - b*x**4) + 2*a
*b*c*x**4*sqrt(a - b*x**4) - 2*a*b*d*x**8*sqrt(a - b*x**4) - b**2*c*x**8*s
qrt(a - b*x**4) + b**2*d*x**12*sqrt(a - b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{5/2}\,(c-dx^4)}\,dx = \int -\frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}\,(c-dx^4)}\,dx = \int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)),x)
```

output
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^4)^{5/2}\,(c-dx^4)}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^3d\,x^{16}-3a\,b^2d\,x^{12}-b^3c\,x^{12}+3a^2bd\,x^8+3a\,b^2c\,x^8-a^3d\,x^4-3a^2bc\,x^4}$$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a - b*x**4)/(a**3*c - a**3*d*x**4 - 3*a**2*b*c*x**4 + 3*a**2*b*d*
x**8 + 3*a*b**2*c*x**8 - 3*a*b**2*d*x**12 - b**3*c*x**12 + b**3*d*x**16),x
)
```

3.48 $\int \frac{(a+bx^4)^{3/2}}{c-dx^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 653

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = -\frac{bx\sqrt{a+bx^4}}{3d} + \frac{(bc+ad)^{3/2}\arctan\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}d^{7/4}}$$
$$+ \frac{(bc+ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}d^{7/4}}$$
$$+ \frac{2a^{3/4}b^{3/4}(bc+2ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3d(bc-ad)\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)(bc+ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)d^2\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)(bc+ad)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right.}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)d^2\sqrt{a+bx^4}}$$

output
```
-1/3*b*x*(b*x^4+a)^(1/2)/d+1/4*(a*d+b*c)^(3/2)*arctan((a*d+b*c)^(1/2)*x/c^
(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/d^(7/4)+1/4*(a*d+b*c)^(3/2)*arctanh
((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/d^(7/4)+2/3*a^
(3/4)*b^(3/4)*(2*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2
)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/d
/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))*(a*d+b*c
)^2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ellipt
icPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2
))^2/a^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/
2)*c^(1/2)-a^(1/2)*d^(1/2))/d^2/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)-a^(1/
2)*d^(1/2))*(a*d+b*c)^2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*
x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*c^(
1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/
4)/b^(1/4)/c/(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))/d^2/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.51 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \frac{x\left(\frac{b(3bc+5ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\mathrm{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{-25ac(3a^2d+abdx^4+b^2x^4(-c+dx^4))\,\mathrm{AppellF1}\left(\frac{1}{4},\right.}{(c-dx^4)\left(-5ac\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},\right.\right.}\right.}{15d}$$

input
```
Integrate[(a + b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
(x*((b*(3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
-((b*x^4)/a), (d*x^4)/c])/c + (-25*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^4*(-c
+ d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c] + 10*b*x^4*(
a + b*x^4)*(-c + d*x^4)*(-2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (
d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c]))/((c
- d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c] + 2*x
^4*(-2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] + b*c*Appel
lF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c])))))/(15*d*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {933, 25, 1021, 761, 925, 27, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx$$

$$\Big\downarrow \text{ 933}$$

$$-\frac{\int -\frac{b(3bc+5ad)x^4+a(bc+3ad)}{\sqrt{bx^4+a}(c-dx^4)}dx}{3d}-\frac{bx\sqrt{a+bx^4}}{3d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b(3bc+5ad)x^4+a(bc+3ad)}{\sqrt{bx^4+a}(c-dx^4)}dx}{3d}-\frac{bx\sqrt{a+bx^4}}{3d}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{\frac{3(ad+bc)^2\int \frac{1}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx}{d}-\frac{b(5ad+3bc)\int \frac{1}{\sqrt{bx^4+a}}dx}{d}}{3d}-\frac{bx\sqrt{a+bx^4}}{3d}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{3(ad+bc)^2\int \frac{1}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5ad+3bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}}{3d}-\frac{bx\sqrt{a+bx^4}}{3d}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{\frac{3(ad+bc)^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5ad+3bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\right.}{2\sqrt[4]{a}d\sqrt{a+bx^4}}}{3d}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{3(ad+bc)^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5ad+3bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\cdots\right.}{2\sqrt[4]{a}d\sqrt{a+bx^4}}}{3d}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\Big\downarrow$ 1541

$$\frac{\frac{3(ad+bc)^2\left(\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\cdots}}{\cdots}}{3d}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{3(ad+bc)^2\left(\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\cdots}{2\cdots}}{3d}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\Big\downarrow$ 761

$$\frac{\frac{3(ad+bc)^2\left(\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{Ellipti}\cdots}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\cdots\right.}\cdots\right)}{d}\cdots}{3d}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\Big\downarrow$ 2221

$$\frac{\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{Ellipti}}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\right.}}{3(ad+bc)^2}$$

$$\frac{bx\sqrt{a+bx^4}}{3d}$$

$\downarrow$ 2223

$$\frac{\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\frac{\sqrt{d}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}}-\right.}{4\sqrt{b}\sqrt{c}\sqrt{}}\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx^4+a}}\right.}{\sqrt{b}\sqrt{}}}{2\sqrt{c}}}{3(bc+ad)^2}$$

$$\frac{bx\sqrt{bx^4+a}}{3d}$$

input
```
Int[(a + b*x^4)^(3/2)/(c - d*x^4),x]
```

output
```
-1/3*(b*x*Sqrt[a + b*x^4])/d + (-1/2*(b^(3/4)*(3*b*c + 5*a*d)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*d*Sqrt[a + b*x^4]) + (3*(b*c + a*d)^
2*(((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*
Sqrt[c] - Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) - (Sqrt[d]*(-1/2*((Sqrt[b]*Sqr
t[c] - Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a
 + b*x^4])])/(c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] + Sqrt[
b]/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*Sqrt[c])/Sqrt[a] - Sqrt[d])^2)/(S
qrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b
^(1/4)*Sqrt[a + b*x^4])))/(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d]))/(2*Sqrt[c])
 + ((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*
Sqrt[c] + Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) + (Sqrt[d]*(((Sqrt[b]*Sqrt[c]
+ Sqrt[a]*Sqrt[d])*ArcTanh[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b
*x^4])])/(2*c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] - Sqrt[b]
/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1021

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{bx\sqrt{b\,x^4+a}}{3d}-\frac{\frac{b(5ad+3bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{3\left(a^2d^2+2abcd+b^2c^2\right)\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^4d-c\right)}-\frac{\text{arctanh}\left(-\right.}{}\right.}{}}{3d}$
default	$-\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(-\frac{b(2ad+bc)}{d^2}+\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2+2abcd+b^2c^2\right)}{}}{}$
elliptic	$-\frac{bx\sqrt{b\,x^4+a}}{3d}+\frac{\left(-\frac{b(2ad+bc)}{d^2}+\frac{ba}{3d}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2+2abcd+b^2c^2\right)}{}}{}$

input
```
int((b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*x*(b*x^4+a)^(1/2)/d-1/3/d*(b*(5*a*d+3*b*c)/d/(I/a^(1/2)*b^(1/2))^(1
/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4
+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/8*(a^2*d^2+2*a*b*c*d+
b^2*c^2)/d^2*sum(1/_alpha^3*(-1/((a*d+b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^
2*b*x^2+2*a)/((a*d+b*c)/d)^(1/2)/(b*x^4+a)^(1/2))-2/(I/a^(1/2)*b^(1/2))^(1
/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))
^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b
^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = -\int \frac{a\sqrt{a+bx^4}}{-c+dx^4}\,dx - \int \frac{bx^4\sqrt{a+bx^4}}{-c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(a*sqrt(a + b*x**4)/(-c + d*x**4), x) - Integral(b*x**4*sqrt(a +
b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate((b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \int -\frac{(bx^4+a)^{\frac{3}{2}}}{dx^4-c}\,dx$$

input
```
integrate((b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-(b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \int \frac{(b\,x^4+a)^{3/2}}{c-d\,x^4}\,dx$$

input
```
int((a + b*x^4)^(3/2)/(c - d*x^4),x)
```

output
```
int((a + b*x^4)^(3/2)/(c - d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/2}}{c-dx^4}\,dx = \frac{-\sqrt{b\,x^4+a}\,bx+3\left(\int \frac{\sqrt{b\,x^4+a}}{-bd\,x^8-ad\,x^4+bc\,x^4+ac}dx\right)a^2d+\left(\int \frac{\sqrt{b\,x^4+a}}{-bd\,x^8-ad\,x^4+bc\,x^4+ac}dx\right)abc+5}{3d}$$

input
```
int((b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

output
```
( - sqrt(a + b*x**4)*b*x + 3*int(sqrt(a + b*x**4)/(a*c - a*d*x**4 + b*c*x*
*4 - b*d*x**8),x)*a**2*d + int(sqrt(a + b*x**4)/(a*c - a*d*x**4 + b*c*x**4
 - b*d*x**8),x)*a*b*c + 5*int((sqrt(a + b*x**4)*x**4)/(a*c - a*d*x**4 + b*
c*x**4 - b*d*x**8),x)*a*b*d + 3*int((sqrt(a + b*x**4)*x**4)/(a*c - a*d*x**
4 + b*c*x**4 - b*d*x**8),x)*b**2*c)/(3*d)
```

3.49 $\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 692

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \frac{\sqrt{bc+ad}\arctan\left(\frac{\sqrt{bc+ad}x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}d^{3/4}} + \frac{\sqrt{bc+ad}\,\text{arctanh}\left(\frac{\sqrt{bc+ad}x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}d^{3/4}} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} + \frac{b^{3/4}(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d(bc-ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc-ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc-ad)\sqrt{a+bx^4}}$$

output
```
1/4*(a*d+b*c)^(1/2)*arctan((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/(b*x^4+a)^(1/
2))/c^(3/4)/d^(3/4)+1/4*(a*d+b*c)^(1/2)*arctanh((a*d+b*c)^(1/2)*x/c^(1/4)/
d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/d^(3/4)-1/2*b^(3/4)*(a^(1/2)+b^(1/2)*x^2)
*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4
)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d/(b*x^4+a)^(1/2)+1/2*b^(3/4)*(a*d+b*c)*
(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJac
obiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/d/(-a*d+b*c)/(b*x^4
+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))^2*(a*d+b*c)*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(
b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/d/(-a*d+b*c)/(b*x^4+a)^(
1/2)-1/8*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2))^2*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^
2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/4*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/c^(
1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/d/(-a*d+b*c)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx$$

$$= \frac{5acx\sqrt{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\,\text{Appell}\right.\right.}$$

input
```
Integrate[Sqrt[a + b*x^4]/(c - d*x^4),x]
```

output
```
(5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)
/c])/((c - d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)
/c] + 2*x^4*(2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] +
b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {922, 761, 925, 27, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx$$

$$\Big\downarrow \ 922$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{bx^4+a}(c-dx^4)}dx}{d}-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{d}$$

$$\Big\downarrow \ 761$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{bx^4+a}(c-dx^4)}dx}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 925$$

$$\frac{(ad+bc)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2c}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 1541$$

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{a}\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{a}\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)}{d}-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{E}}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{\right.}}\right.}{d}$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2221}$$

$$(ad+bc)\left(\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{E}\cdots}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{\cdots}\right.}\cdots\right.$$

$$\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 2223$$

$$(bc+ad)\left(\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\frac{\sqrt{d}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}}\cdots\right.}{4\sqrt{b}\sqrt{\cdots}}\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx^4+a}}\cdots\right.}{\sqrt{\cdots}}}{2\sqrt{c}}\cdots\right.$$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{bx^4+a}}$$

input
```
Int[Sqrt[a + b*x^4]/(c - d*x^4),x]
```

output
```
-1/2*(b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*d*Sqrt[a +
 b*x^4]) + ((b*c + a*d)*(((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) - (Sqrt[
d]*(-1/2*((Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c + a*d]*x)/(
c^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((S
qrt[a]/Sqrt[c] + Sqrt[b]/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*Sqrt[c])/Sq
rt[a] - Sqrt[d])^2)/(Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(Sqrt[b]*Sqrt[c] - Sqrt[a]
*Sqrt[d]))/(2*Sqrt[c]) + ((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4
)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) + (Sqrt[
d]*(((Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])*ArcTanh[(Sqrt[b*c + a*d]*x)/(c^(1
/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqr
t[a]/Sqrt[c] - Sqrt[b]/Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2
/(4*Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.39

method	result
default	$-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}d-c\right)}\frac{(ad+bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad+bc}{d}}\,\sqrt{b\,x^{4}+a}}\right)}{\sqrt{\frac{ad+bc}{d}}}-\frac{2\_\alpha^{3}d\sqrt{1-}}{}\right.}{}}{8d^{2}}$
elliptic	$-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}d-c\right)}\frac{(ad+bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2b\,x^{2}\_\alpha^{2}+2a}{2\sqrt{\frac{ad+bc}{d}}\,\sqrt{b\,x^{4}+a}}\right)}{\sqrt{\frac{ad+bc}{d}}}-\frac{2\_\alpha^{3}d\sqrt{1-}}{}\right.}{}}{8d^{2}}$

input
```
int((b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2
)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)-1/8/d^2*sum((a*d+b*c)/_alpha^3*(-1/((a*d+b*c)/d)^(1/2)*arctanh(1/2*(2*
_alpha^2*b*x^2+2*a)/((a*d+b*c)/d)^(1/2)/(b*x^4+a)^(1/2))-2/(I/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a
^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^
(1/2)/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = -\int \frac{\sqrt{a+bx^4}}{-c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(sqrt(a + b*x**4)/(-c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(b*x^4 + a)/(d*x^4 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{dx^4-c}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-sqrt(b*x^4 + a)/(d*x^4 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \int \frac{\sqrt{b\,x^4+a}}{c-d\,x^4}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(c - d*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)/(c - d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^4}}{c-dx^4}\,dx = \int \frac{\sqrt{b\,x^4+a}}{-d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(c - d*x**4),x)
```

3.50 $\int \frac{1}{\sqrt{a+bx^4}(c-dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 571

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = \frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}\sqrt{bc+ad}} + \frac{\sqrt[4]{d}\,\text{arctanh}\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}\sqrt{bc+ad}}$$
$$+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(bc-ad)\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)\sqrt{a+bx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)\sqrt{a+bx^4}}$$

output
```
1/4*d^(1/4)*arctan((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3
/4)/(a*d+b*c)^(1/2)+1/4*d^(1/4)*arctanh((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/
(b*x^4+a)^(1/2))/c^(3/4)/(a*d+b*c)^(1/2)+1/2*b^(3/4)*(a^(1/2)+b^(1/2)*x^2)
*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4
)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*
c^(1/2)+a^(1/2)*d^(1/2))^2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)
*c^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a
^(1/4)/b^(1/4)/c/(-a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)-a^(1/2)*d
^(1/2))^2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*c^(1/2)+a^(1/2)*d
^(1/2))^2/a^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(
-a*d+b*c)/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\,(c-dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(-2ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^4]*(c - d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c])/(Sqrt[a + b
*x^4]*(c - d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)
/c] + 2*x^4*(-2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.41, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {925, 27, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}dx$$

$$\Big\downarrow \text{925}$$

$$\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)}-\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{2221}$$

$$\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)}-\frac{\sqrt{d}\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}}-\sqrt{d}\right)^2}{4\sqrt{b}\sqrt{c}\sqrt{d}}\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}\right.}{\sqrt{b}\sqrt{c}-\sqrt{}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{2223}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\frac{\sqrt{d}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}}-\sqrt{d}\right)^2}{4\sqrt{b}\sqrt{c}\sqrt{d}}\right.}{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{bx^4+a}}\right.}{\sqrt{b}\sqrt{c}-\sqrt{}}}{2\sqrt{c}}$$

$$\frac{\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}+\frac{\sqrt{d}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)\text{arctanh}\left(\frac{\sqrt{bc+ad}x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{2\sqrt[4]{c}\sqrt[4]{d}\sqrt{bc+ad}}+\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\right.}{}\right.}{\sqrt{b}\sqrt{c}+\sqrt{}}}{2\sqrt{c}}$$

input
```
Int[1/(Sqrt[a + b*x^4]*(c - d*x^4)),x]
```

output
```
((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqr
t[c] - Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) - (Sqrt[d]*(-1/2*((Sqrt[b]*Sqrt[c
] - Sqrt[a]*Sqrt[d])*ArcTan[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a +
b*x^4])])/(c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] + Sqrt[b]/
Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^
2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*Sqrt[c])/Sqrt[a] - Sqrt[d])^2)/(Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1
/4)*Sqrt[a + b*x^4])))/(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d]))/(2*Sqrt[c]) +
((b^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqr
t[c] + Sqrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) + (Sqrt[d]*(((Sqrt[b]*Sqrt[c] + S
qrt[a]*Sqrt[d])*ArcTanh[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(2*c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] - Sqrt[b]/Sq
rt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[
a + b*x^4])))/(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d]))/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.33

method	result
default	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 d-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 b\,x^2\_\alpha^2+2 a}{2\sqrt{\frac{a d+b c}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{a d+b c}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},-\frac{i\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$
elliptic	$-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 d-c\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 b\,x^2\_\alpha^2+2 a}{2\sqrt{\frac{a d+b c}{d}}\,\sqrt{b\,x^4+a}}\right)}{\sqrt{\frac{a d+b c}{d}}}-\frac{2\_\alpha^3 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},-\frac{i\sqrt{a}\,\_\alpha^2 d}{\sqrt{b}\,c},\frac{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{b\,x^4+a}}}{\_\alpha^3}}{8d}$

input
```
int(1/(b*x^4+a)^(1/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*sum(1/_alpha^3*(-1/((a*d+b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^
2+2*a)/((a*d+b*c)/d)^(1/2)/(b*x^4+a)^(1/2))-2/(I/a^(1/2)*b^(1/2))^(1/2)*_a
lpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)
/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2)
*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = -\int \frac{1}{-c\sqrt{a+bx^4}+dx^4\sqrt{a+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-c*sqrt(a + b*x**4) + d*x**4*sqrt(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(b*x^4 + a)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = \int -\frac{1}{\sqrt{bx^4+a}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/(sqrt(b*x^4 + a)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,\left(c-d\,x^4\right)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c - d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^4}\,(c-dx^4)}\,dx = \int \frac{\sqrt{b\,x^4+a}}{-bd\,x^8-ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(1/(b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(a*c - a*d*x**4 + b*c*x**4 - b*d*x**8),x)
```

3.51 $\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 620

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx &= \frac{bx}{2a(bc+ad)\sqrt{a+bx^4}} \\ &+ \frac{d^{5/4}\arctan\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}(bc+ad)^{3/2}} + \frac{d^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}(bc+ad)^{3/2}} \\ &+ \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} \\ &- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}} \\ &- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2 d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*b*x/a/(a*d+b*c)/(b*x^4+a)^(1/2)+1/4*d^(5/4)*arctan((a*d+b*c)^(1/2)*x/c
^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/(a*d+b*c)^(3/2)+1/4*d^(5/4)*arctan
h((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/(a*d+b*c)^(3/
2)+1/4*b^(3/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(-a*
d+b*c)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))^2*d*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2))^2/a^(1/2)/
b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(-a*d+b*c)/(a*d+b*c
)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2))^2*d*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))^2/a^(1/2)/b^(1/2
)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(-a*d+b*c)/(a*d+b*c)/(b*x
^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.27 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = \frac{-5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(5c(2bc+2ad-bdx^4)+bdx^4\sqrt{1+}\right.}{10ac(bc+ad)\sqrt{}}$$

input
```
Integrate[1/((a + b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c]*(5*c*(2*b*c
+ 2*a*d - b*d*x^4) + b*d*x^4*Sqrt[1 + (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4
, 1/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^
4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c])
*(-2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] + b*c*AppellF
1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c]))/(10*a*c*(b*c + a*d)*Sqrt[a
+ b*x^4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x
^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c]
- b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {931, 25, 1021, 761, 925, 27, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}-\frac{\int -\frac{-bdx^4+bc+2ad}{\sqrt{bx^4+a}(c-dx^4)}dx}{2a(ad+bc)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-bdx^4+bc+2ad}{\sqrt{bx^4+a}(c-dx^4)}dx}{2a(ad+bc)}+\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{2ad\int \frac{1}{\sqrt{bx^4+a}(c-dx^4)}dx+b\int \frac{1}{\sqrt{bx^4+a}}dx}{2a(ad+bc)}+\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$$\Big\downarrow \text{761}$$

$$\frac{2ad\int \frac{1}{\sqrt{bx^4+a}(c-dx^4)}dx+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$$\Big\downarrow \text{925}$$

$$\frac{2ad\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2c}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 27

$$\frac{2ad\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 1541

$$\frac{2ad\left(\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{a}\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\cdots}}{\cdots}}{2a(ad+bc)}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 27

$$\frac{2ad\left(\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{E}\cdots}{2\sqrt[4]{a}\sqrt{\cdots}}}{2a(ad+bc)}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 761

$$\frac{2ad\left(\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(\cdots\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{\cdots}\right)}\cdots}{\cdots}\right)}{2a(ad+bc)}$$

$$\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 2221

$$2ad\left(\frac{\frac{\sqrt{d}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(\ldots\right.}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\ldots\right.}\right.$$

$$\frac{bx}{2a\sqrt{a+bx^4}(ad+bc)}$$

$\big\downarrow$ 2223

$$\frac{bx}{2a(bc+ad)\sqrt{bx^4+a}}+$$

$$\frac{b^{3/4}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{bx^4+a}}+2ad\left(\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\ldots\right.$$

input
```
Int[1/((a + b*x^4)^(3/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(2*a*(b*c + a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) + 2*a*d*(((b^(1/4)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqr
t[d])*Sqrt[a + b*x^4]) - (Sqrt[d]*(-1/2*((Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d
])*ArcTan[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(c^(1/4)
*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] + Sqrt[b]/Sqrt[d])*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-1/4
*(Sqrt[a]*((Sqrt[b]*Sqrt[c])/Sqrt[a] - Sqrt[d])^2)/(Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4
])))/(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d]))/(2*Sqrt[c]) + ((b^(1/4)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqr
t[d])*Sqrt[a + b*x^4]) + (Sqrt[d]*(((Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])*Ar
cTanh[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2*c^(1/4)*d
^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] - Sqrt[b]/Sqrt[d])*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])))/(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
 + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.50

method	result
default	$\frac{bx}{2a(ad+bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad+bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d-c\right)}-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad+bc}{d}}\,\sqrt{b\,x^4}}\right)}{\sqrt{\frac{ad+bc}{d}}}\right)}{}$
elliptic	$\frac{bx}{2a(ad+bc)\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a(ad+bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d-c\right)}-\frac{\operatorname{arctanh}\left(\frac{2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad+bc}{d}}\,\sqrt{b\,x^4}}\right)}{\sqrt{\frac{ad+bc}{d}}}\right)}{}$

input
```
int(1/(b*x^4+a)^(3/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x/a/(a*d+b*c)/((x^4+a/b)*b)^(1/2)+1/2*b/a/(a*d+b*c)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/
(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/(a*d+b*
c)/_alpha^3*(-1/((a*d+b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a
*d+b*c)/d)^(1/2)/(b*x^4+a)^(1/2))-2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c
*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)
^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2)*_alpha^2/
c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = -\int \frac{1}{-ac\sqrt{a+bx^4}+adx^4\sqrt{a+bx^4}-bcx^4\sqrt{a+bx^4}+bdx^8\sqrt{a+bx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a*c*sqrt(a + b*x**4) + a*d*x**4*sqrt(a + b*x**4) - b*c*x**4*
sqrt(a + b*x**4) + b*d*x**8*sqrt(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = \int -\frac{1}{(bx^4+a)^{\frac{3}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/2}(c-dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a + b*x^4)^(3/2)*(c - d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(3/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/2}\,(c-dx^4)}\,dx = \int \frac{\sqrt{b\,x^4+a}}{-b^2d\,x^{12}-2abd\,x^8+b^2c\,x^8-a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(1/(b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*c - a**2*d*x**4 + 2*a*b*c*x**4 - 2*a*b*d*x**8 +
 b**2*c*x**8 - b**2*d*x**12),x)
```

3.52 $\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 706

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \frac{bx}{6a(bc+ad)(a+bx^4)^{3/2}} + \frac{b(5bc+11ad)x}{12a^2(bc+ad)^2\sqrt{a+bx^4}} + \frac{d^{9/4}\arctan\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}(bc+ad)^{5/2}} + \frac{d^{9/4}\text{arctanh}\left(\frac{\sqrt{bc+ad}\,x}{\sqrt[4]{c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4c^{3/4}(bc+ad)^{5/2}} + \frac{b^{3/4}(5bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}(bc-ad)(bc+ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)(bc+ad)^2\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)d^2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{b}\sqrt{c}+\sqrt{a}\sqrt{d}\right)(bc+ad)^2\sqrt{a+bx^4}}$$

output
```
1/6*b*x/a/(a*d+b*c)/(b*x^4+a)^(3/2)+1/12*b*(11*a*d+5*b*c)*x/a^2/(a*d+b*c)^
2/(b*x^4+a)^(1/2)+1/4*d^(9/4)*arctan((a*d+b*c)^(1/2)*x/c^(1/4)/d^(1/4)/(b*
x^4+a)^(1/2))/c^(3/4)/(a*d+b*c)^(5/2)+1/4*d^(9/4)*arctanh((a*d+b*c)^(1/2)*
x/c^(1/4)/d^(1/4)/(b*x^4+a)^(1/2))/c^(3/4)/(a*d+b*c)^(5/2)+1/24*b^(3/4)*(a
*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4)/(-a*d+b*c
)/(a*d+b*c)/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))*d^2*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin
(2*arctan(b^(1/4)*x/a^(1/4))),-1/4*(b^(1/2)*c^(1/2)-a^(1/2)*d^(1/2))^2/a^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/4)/c/(b^(1/2)*c^(1/
2)-a^(1/2)*d^(1/2))/(a*d+b*c)^2/(b*x^4+a)^(1/2)-1/8*(b^(1/2)*c^(1/2)-a^(1/
2)*d^(1/2))*d^2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^
(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(b^(1/2)*c^(1/2)+a^(
1/2)*d^(1/2))^2/a^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^(1/4)/b^(1/
4)/c/(b^(1/2)*c^(1/2)+a^(1/2)*d^(1/2))/(a*d+b*c)^2/(b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.82 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.60

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx =$$

$$-\frac{x\left(\frac{bd(5bc+11ad)x^4\sqrt{1+\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac\left(12a^3d^2+5b^3cx^4\left(2c-dx^4\right)+a^2bd\left(24c-dx^4\right)+ab^2\left(12c^2+15cdx^4-11d^2\right.\right.\right.}{(a+bx^4)(-c+dx^4)\left(5ac\right.}\right.}{60a}$$

input
```
Integrate[1/((a + b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
-1/60*(x*((b*d*(5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2,
 1, 9/4, -((b*x^4)/a), (d*x^4)/c])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c*x^4
*(2*c - d*x^4) + a^2*b*d*(24*c - d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*
d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c] + 2*b*x^4*(-
c + d*x^4)*(13*a^2*d + 5*b^2*c*x^4 + a*b*(7*c + 11*d*x^4))*(-2*a*d*AppellF
1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), (d*x^4)/c])))/((a + b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[
1/4, 1/2, 1, 5/4, -((b*x^4)/a), (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/
2, 2, 9/4, -((b*x^4)/a), (d*x^4)/c] - b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*
x^4)/a), (d*x^4)/c])))))/(a^2*(b*c + a*d)^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1010, normalized size of antiderivative = 1.43, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {931, 25, 1024, 25, 1021, 761, 925, 27, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c-dx^4\right)}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}-\frac{\int -\frac{-5bdx^4+5bc+6ad}{\left(bx^4+a\right)^{3/2}\left(c-dx^4\right)}dx}{6a(ad+bc)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-5bdx^4+5bc+6ad}{\left(bx^4+a\right)^{3/2}\left(c-dx^4\right)}dx}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(11ad+5bc)}{2a\sqrt{a+bx^4}(ad+bc)}-\frac{\int -\frac{-bd(5bc+11ad)x^4+5b^2c^2+12a^2d^2+11abcd}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx}{2a(ad+bc)}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{-bd(5bc+11ad)x^4+5b^2c^2+12a^2d^2+11abcd}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx}{2a(ad+bc)}+\frac{bx(11ad+5bc)}{2a\sqrt{a+bx^4}(ad+bc)}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \ 1021$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx+b(11ad+5bc)\int \frac{1}{\sqrt{bx^4+a}}dx}{2a(ad+bc)}+\frac{bx(11ad+5bc)}{2a\sqrt{a+bx^4}(ad+bc)}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{12a^2d^2\int \frac{1}{\sqrt{bx^4+a}\left(c-dx^4\right)}dx+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(11ad+5bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{bx(11ad+5bc)}{2a\sqrt{a+bx^4}(ad+bc)}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \ 925$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2c}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(11ad+5bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{b\cdots}{2a\sqrt{\cdots}}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{12a^2d^2\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(11ad+5bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}}}{2a(ad+bc)}+\frac{b\cdots}{2a\sqrt{\cdots}}}{6a(ad+bc)}+\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$$\Big\downarrow \ 1541$$

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{a}\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{a}\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx}{\left(\sqrt{a}+\sqrt{b}\right.}}}{}}{2a(ad+bc)}}{6a(ad+bc)}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$\downarrow$ 27

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}}{2\sqrt{c}}+\frac{\frac{\sqrt{b}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}-\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{bx^4+a}}dx}{\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}}}{2\sqrt{c}}\right)+\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(11ad+}{2\sqrt[4]{a}\sqrt{a}}}{2a(ad+bc)}}{6a(ad+bc)}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$\downarrow$ 761

$$\frac{\frac{12a^2d^2\left(\frac{\frac{\sqrt{d}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{bx^4+a}}dx}{\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}}{2\sqrt{c}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(\right.}{2\sqrt[4]{a}\sqrt{a+bx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\right.}\right.}{2a(ad+bc)}}{6a(}$$

$$\frac{bx}{6a\left(a+bx^4\right)^{3/2}(ad+bc)}$$

$\downarrow$ 2221

$$\frac{b(5bc+11ad)x}{2a(bc+ad)\sqrt{bx^4+a}}+\frac{\frac{bx}{6a(bc+ad)\left(bx^4+a\right)^{3/2}}+\frac{12a^2\left(\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\frac{\sqrt{d}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{Elliptic}\ldots}{4\sqrt[4]{a}\sqrt[4]{\ldots}}\ldots\right)}{2\sqrt{c}\ldots}\right)}{\ldots}}{\ldots}$$

$\Big\downarrow$ 2223

$$\frac{b(5bc+11ad)x}{2a(bc+ad)\sqrt{bx^4+a}}+\frac{\frac{bx}{6a(bc+ad)\left(bx^4+a\right)^{3/2}}+\frac{12a^2\left(\frac{\sqrt[4]{b}\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{b}\sqrt{c}-\sqrt{a}\sqrt{d}\right)\sqrt{bx^4+a}}-\frac{\sqrt{d}\left(\frac{\left(\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{b}}{\sqrt{d}}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{Elliptic}\ldots}{4\sqrt[4]{a}\sqrt[4]{\ldots}}\ldots\right)}{2\sqrt{c}\ldots}\right)}{\ldots}}{\ldots}$$

input
```
Int[1/((a + b*x^4)^(5/2)*(c - d*x^4)),x]
```

output
```
(b*x)/(6*a*(b*c + a*d)*(a + b*x^4)^(3/2)) + ((b*(5*b*c + 11*a*d)*x)/(2*a*(
b*c + a*d)*Sqrt[a + b*x^4]) + ((b^(3/4)*(5*b*c + 11*a*d)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[a + b*x^4]) + 12*a^2*d^2*(((b^(1/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] - S
qrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) - (Sqrt[d]*(-1/2*((Sqrt[b]*Sqrt[c] - Sqrt
[a]*Sqrt[d])*ArcTan[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b*x^4])]
)/(c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] + Sqrt[b]/Sqrt[d])
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[a]*((Sqrt[b]*Sqrt[c])/Sqrt[a] - Sqrt[d])^2)/(Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(1/4)*Sqrt
[a + b*x^4])))/(Sqrt[b]*Sqrt[c] - Sqrt[a]*Sqrt[d]))/(2*Sqrt[c]) + ((b^(1/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[b]*Sqrt[c] + S
qrt[a]*Sqrt[d])*Sqrt[a + b*x^4]) + (Sqrt[d]*(((Sqrt[b]*Sqrt[c] + Sqrt[a]*S
qrt[d])*ArcTanh[(Sqrt[b*c + a*d]*x)/(c^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(2
*c^(1/4)*d^(1/4)*Sqrt[b*c + a*d]) + ((Sqrt[a]/Sqrt[c] - Sqrt[b]/Sqrt[d])*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[c]*Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.52

method	result
default	$\frac{x\sqrt{b\,x^4+a}}{6ab(ad+bc)\left(x^4+\frac{a}{b}\right)^2}+\frac{bx(11ad+5bc)}{12a^2(ad+bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{b(11ad+5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad+bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{d\Big(\ \_\alpha=\mathrm{Ro}}{}$
elliptic	$\frac{x\sqrt{b\,x^4+a}}{6ab(ad+bc)\left(x^4+\frac{a}{b}\right)^2}+\frac{bx(11ad+5bc)}{12a^2(ad+bc)^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{b(11ad+5bc)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2(ad+bc)^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{d\Big(\ \_\alpha=\mathrm{Ro}}{}$

input

```
int(1/(b*x^4+a)^(5/2)/(-d*x^4+c),x,method=_RETURNVERBOSE)
```

output

```
1/6*x/a/b/(a*d+b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/12*b*x/a^2*(11*a*d+5*b*c
)/(a*d+b*c)^2/((x^4+a/b)*b)^(1/2)+1/12*b/a^2*(11*a*d+5*b*c)/(a*d+b*c)^2/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a
^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8
*d*sum(1/(a*d+b*c)^2/_alpha^3*(-1/((a*d+b*c)/d)^(1/2)*arctanh(1/2*(2*_alph
a^2*b*x^2+2*a)/((a*d+b*c)/d)^(1/2)/(b*x^4+a)^(1/2))-2/(I/a^(1/2)*b^(1/2))^
(1/2)*_alpha^3*d/c*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2
))^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)
/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx =$$
$$-\int \frac{1}{-a^2c\sqrt{a+bx^4}+a^2dx^4\sqrt{a+bx^4}-2abcx^4\sqrt{a+bx^4}+2abdx^8\sqrt{a+bx^4}-b^2cx^8\sqrt{a+bx^4}+b^2dx^{12}\sqrt{a}}$$

input
```
integrate(1/(b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

output
```
-Integral(1/(-a**2*c*sqrt(a + b*x**4) + a**2*d*x**4*sqrt(a + b*x**4) - 2*a
*b*c*x**4*sqrt(a + b*x**4) + 2*a*b*d*x**8*sqrt(a + b*x**4) - b**2*c*x**8*s
qrt(a + b*x**4) + b**2*d*x**12*sqrt(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \int -\frac{1}{(bx^4+a)^{\frac{5}{2}}(dx^4-c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/2}\,(c-d\,x^4)}\,dx$$

input
```
int(1/((a + b*x^4)^(5/2)*(c - d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(5/2)*(c - d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/2}(c-dx^4)}\,dx = \int \frac{\sqrt{b\,x^4+a}}{-b^3d\,x^{16}-3a\,b^2d\,x^{12}+b^3c\,x^{12}-3a^2bd\,x^8+3a\,b^2c\,x^8-a^3d\,x^4+3a^2bc\,x$$

input
```
int(1/(b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

output
```
int(sqrt(a + b*x**4)/(a**3*c - a**3*d*x**4 + 3*a**2*b*c*x**4 - 3*a**2*b*d*
x**8 + 3*a*b**2*c*x**8 - 3*a*b**2*d*x**12 + b**3*c*x**12 - b**3*d*x**16),x
)
```

3.53 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 426

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = -\frac{b(77b^2c^2-122abcd+21a^2d^2)\,x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x(a-bx^4)^{3/2}}{28cd^2} - \frac{(bc-ad)x(a-bx^4)^{5/2}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{84cd^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)^3(11bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^4\sqrt{a-bx^4}}$$

output
```
-1/84*b*(21*a^2*d^2-122*a*b*c*d+77*b^2*c^2)*x*(-b*x^4+a)^(1/2)/c/d^3+1/28*
b*(-7*a*d+11*b*c)*x*(-b*x^4+a)^(3/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a)^(5/
2)/c/d/(-d*x^4+c)+1/84*a^(1/4)*b^(3/4)*(21*a^3*d^3+349*a^2*b*c*d^2-553*a*b
^2*c^2*d+231*b^3*c^3)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/c/d
^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*(1-b*x^4/a)^(1
/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/
4)/c^2/d^4/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-a*d+b*c)^3*(3*a*d+11*b*c)*(1-b*x
^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)/b^(1/4)/c^2/d^4/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.93 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.12

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(231b^3c^3-553ab^2c^2d+349a^2bcd^2+21a^3d^3)\,x^5\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx(-84a^4d^3\right.}{}}{}$$

input
```
Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

output
```
-1/420*(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x
^5*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] +
(5*c*(5*a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b
^3*c*d*x^4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*
x^8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4
)*(-63*a^2*b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-7
7*c^2 + 44*c*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4
)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/
((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x
^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {930, 25, 1025, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}dx$$

$$\Big\downarrow \ 930$$

$$-\frac{\int -\frac{\left(a-bx^4\right)^{3/2}\left(a(bc+3ad)-b(11bc-7ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(a-bx^4\right)^{3/2}\left(a(bc+3ad)-b(11bc-7ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \ 1025$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\int \frac{\sqrt{a-bx^4}\left(a\left(11b^2c^2-14abdc-21a^2d^2\right)-b\left(77b^2c^2-122abdc+21a^2d^2\right)x^4\right)}{c-dx^4}dx}{7d}}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \ 1025$$

$$\frac{\frac{bx\left(a-bx^4\right)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\int \frac{a\left(77b^3c^3-155ab^2dc^2+63a^2bd^2c+63a^3d^3\right)-b\left(231b^3c^3-553ab^2dc^2+349a^2bd^2c+2\cdots\right.}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{3d}}{7d}}{4cd}-\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \ 1021$$

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{b\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{21(bc-ad)^3(3ad+11bc\cdots}{\cdots}}{3d}}{7d}}{4cd}\cdots$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\big\downarrow$ 765

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{21(bc-ad)^3(\cdots}{\cdots}}{3d}}{7d}}{4cd}\cdots$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\big\downarrow$ 762

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{d\sqrt{a-bx^4}}\cdots}{3d}}{7d}}{4cd}\cdots$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\big\downarrow$ 925

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{3d}-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{d\sqrt{a-bx^4}}\cdots}{7d}}{4cd}\cdots$$

$$\frac{x\left(a-bx^4\right)^{5/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{d\sqrt{a-bx^4}}\ldots}{7d}}{4cd}$$

$$\frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{d\sqrt{a-bx^4}}\ldots}{7d}}{4cd}$$

$$\frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{bx(a-bx^4)^{3/2}(11bc-7ad)}{7d}-\frac{\frac{bx\sqrt{a-bx^4}(21a^2d^2-122abcd+77b^2c^2)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(21a^3d^3+349a^2bcd^2-553ab^2c^2d+231b^3c^3)\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{d\sqrt{a-bx^4}}\ldots}{\ldots}}{4c\ldots}$$

$$\frac{x(a-bx^4)^{5/2}(bc-ad)}{4cd(c-dx^4)}$$

input
```
Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a - b*x^4)^(5/2))/(c*d*(c - d*x^4)) + ((b*(11*b*c - 7
*a*d)*x*(a - b*x^4)^(3/2))/(7*d) - ((b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*
d^2)*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a*b^2
*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSi
n[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - (21*(b*c - a*d)^3*(11*b
*c + 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(
Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a -
 b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^
4]))/d)/(3*d))/(7*d))/(4*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.76 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.27

method	result
default	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+\right.}{d^4}\right.}{}$
elliptic	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x\sqrt{-b\,x^4+a}}{4c\,d^3(-d\,x^4+c)}-\frac{b^3x^5\sqrt{-b\,x^4+a}}{7d^2}-\frac{\left(-\frac{2b^3(2ad-bc)}{d^3}+\frac{5b^3a}{7d^2}\right)x\sqrt{-b\,x^4+a}}{3b}+\frac{\left(\frac{b^2\left(6a^2d^2-8abcd+\right.}{d^4}\right.}{}$
risch	$21\left(d^4a^4-4a^3bc\,d^3\right.$ $\frac{b^2x(-3bd\,x^4+23ad-14bc)\sqrt{-b\,x^4+a}}{21d^3}+\frac{\frac{b^2\left(103a^2d^2-154abcd+63b^2c^2\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{d\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}+}{}$

input

```
int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)
/(-d*x^4+c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5
/7*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^
4+1/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^
3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2
/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^5*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a
^2*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)
*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))
-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1
+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/
2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/
2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{7}{2}}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

output
```
Integral((a - b*x**4)**(7/2)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{7}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-b\,x^4)^{7/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx = \text{too large to display}$$

input
```
int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x)
```

output
```
( - 84*sqrt(a - b*x**4)*a**3*b*d**2*x + 115*sqrt(a - b*x**4)*a**2*b**2*c*d
*x - 69*sqrt(a - b*x**4)*a**2*b**2*d**2*x**5 - 55*sqrt(a - b*x**4)*a*b**3*
c**2*x + 102*sqrt(a - b*x**4)*a*b**3*c*d*x**5 + 9*sqrt(a - b*x**4)*a*b**3*
d**2*x**9 - 33*sqrt(a - b*x**4)*b**4*c**2*x**5 - 9*sqrt(a - b*x**4)*b**4*c
*d*x**9 + 63*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2
*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12
 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**6*c*d**4
 - 63*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x
**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2
*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**6*d**5*x**4 - 4
2*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8
- a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**
3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**5*b*c**2*d**3 + 42*
int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 -
a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*
x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**5*b*c*d**4*x**4 - 136
*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 -
 a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3
*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**4*b**2*c**3*d**2 + 1
36*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x...
```

3.54 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 365

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd\,(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2(7bc+3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

output
```
1/12*b*(-3*a*d+7*b*c)*x*(-b*x^4+a)^(1/2)/c/d^2-1/4*(-a*d+b*c)*x*(-b*x^4+a)
^(3/2)/c/d/(-d*x^4+c)-1/12*a^(1/4)*b^(3/4)*(-3*a^2*d^2-26*a*b*c*d+21*b^2*c
^2)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/c/d^3/(-b*x^4+a)^(1/2
)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1
/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/d^3/(-b*x^4+
a)^(1/2)+1/8*a^(1/4)*(-a*d+b*c)^2*(3*a*d+7*b*c)*(1-b*x^4/a)^(1/2)*Elliptic
Pi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/d^3/(-
b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.64 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx =$$

$$-\frac{b(-21b^2c^2+26abcd+3a^2d^2)\,x^5\sqrt{1-\frac{bx^4}{a}}\,\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(5acx\left(12a^3d^2+2ab^2cdx^4-3a^2bd^2x^4+\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

output
```
-1/60*(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*Ap
pellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2
 + 2*a*b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF
1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d
+ 3*a^2*d^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*
x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
))/((-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(c^2*d^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {930, 25, 1025, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} \, dx$$

$$\Big\downarrow \; 930$$

$$-\frac{\int -\frac{\sqrt{a-bx^4}\left(a(bc+3ad)-b(7bc-3ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\sqrt{a-bx^4}\left(a(bc+3ad)-b(7bc-3ad)x^4\right)}{c-dx^4}dx}{4cd}-\frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \; 1025$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\int \frac{a\left(7b^2c^2-6abdc-9a^2d^2\right)-b\left(21b^2c^2-26abdc-3a^2d^2\right)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{3d}}{4cd}-\frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \; 1021$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{b\left(-3a^2d^2-26abcd+21b^2c^2\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)^2(3ad+7bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x(a-bx^4)^{3/2}(bc-ad)}{4cd(c-dx^4)}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\int\frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}\right.}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right.}{d}}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1543}$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{\cdots}}}{2\sqrt{c}\sqrt{a-bx^4}}\right.}{3d}}{4cd}-\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{bx\sqrt{a-bx^4}(7bc-3ad)}{3d}-\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)^2(3ad+7bc)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(\cdots\right.}{2\sqrt[4]{b}\cdots}\right.}{3d}}{4cd}$$

$$\frac{x\left(a-bx^4\right)^{3/2}(bc-ad)}{4cd\left(c-dx^4\right)}$$

input

```
Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

output

```
-1/4*((b*c - a*d)*x*(a - b*x^4)^(3/2))/(c*d*(c - d*x^4)) + ((b*(7*b*c - 3*
a*d)*x*Sqrt[a - b*x^4])/(3*d) - ((a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d
 - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(d*Sqrt[a - b*x^4]) - (3*(b*c - a*d)^2*(7*b*c + 3*a*d)*((a^(1/4)*Sqrt[
1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(
b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1
- (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d)/(3*d))/(4*c*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
 - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.70 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.13

method	result
default	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-b\,x^4+a}}{4c\,d^2(-d\,x^4+c)}+\frac{b^2x\sqrt{-b\,x^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{b\left(a^2d^2-2abcd+b^2c^2\right)}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{Ellipt}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
elliptic	$\frac{(a^2d^2-2abcd+b^2c^2)x\sqrt{-b\,x^4+a}}{4c\,d^2(-d\,x^4+c)}+\frac{b^2x\sqrt{-b\,x^4+a}}{3d^2}+\frac{\left(\frac{b^2(3ad-2bc)}{d^3}+\frac{b\left(a^2d^2-2abcd+b^2c^2\right)}{4d^3c}-\frac{b^2a}{3d^2}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{Ellipt}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
risch	$\frac{b^2x\sqrt{-b\,x^4+a}}{3d^2}+\frac{3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(-\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)\left(d\,x^4-c\right)}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum}{\_\alpha=\mathrm{RootOf}}\right.}{}$

input
```
int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/3*b^
2/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^3*(a^2*d^2-2*a*b*c
*d+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1
/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a
^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^
2*d+7*b^3*c^3)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b
*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)
/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_
alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{5}{2}}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

output
```
Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{5}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-b\,x^4)^{5/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(5/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(5/2)/(c - d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = \text{too large to display}$$

input

```
int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

output

```
( - 9*sqrt(a - b*x**4)*a**2*b*d*x + 5*sqrt(a - b*x**4)*a*b**2*c*x - 3*sqrt
(a - b*x**4)*a*b**2*d*x**5 + 3*sqrt(a - b*x**4)*b**3*c*x**5 + 9*int(sqrt(a
 - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 +
 a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b
**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**5*c*d**3 - 9*int(sqrt(a - b*x**
4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**
2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2
*d*x**8 + b**2*c*d**2*x**12),x)*a**5*d**4*x**4 - 9*int(sqrt(a - b*x**4)/(a
**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x
**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x*
*8 + b**2*c*d**2*x**12),x)*a**4*b*c**2*d**2 + 9*int(sqrt(a - b*x**4)/(a**2
*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4
 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8
+ b**2*c*d**2*x**12),x)*a**4*b*c*d**3*x**4 - 5*int(sqrt(a - b*x**4)/(a**2*
c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4
+ a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 +
 b**2*c*d**2*x**12),x)*a**3*b**2*c**3*d + 5*int(sqrt(a - b*x**4)/(a**2*c**
2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a
*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b*
*2*c*d**2*x**12),x)*a**3*b**2*c**2*d**2*x**4 + 5*int(sqrt(a - b*x**4)/(...
```

3.55 $\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\,(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}} - \frac{3\sqrt[4]{a}(bc-ad)(bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}}$$

output

```
-1/4*(-a*d+b*c)*x*(-b*x^4+a)^(1/2)/c/d/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(a*d
+3*b*c)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/c/d^2/(-b*x^4+a)^
(1/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4
)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/d^2/(-b*x^4+a)
^(1/2)-3/8*a^(1/4)*(-a*d+b*c)*(a*d+b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/
4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/d^2/(-b*x^4+a)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.11

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \frac{x\left(-b(3bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d+b\right.}{}\right.}{20c^2}$$

input
```
Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

output
```
(x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a
*b*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a
*d)*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a
, (d*x^4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {930, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$-\frac{\int -\frac{a(bc+3ad)-b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\,(c-dx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{a(bc+3ad)-b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 1021

$$\frac{\frac{b(ad+3bc)\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 765

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 762

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 925

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{4cd}-\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$\Big\downarrow$ 1543

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\cdots}}}{2\sqrt{c}\sqrt{a-bx^4}}\cdots\right)}{d}}{4cd}$$

$$\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1542$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{3(bc-ad)(ad+bc)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\cdots}{\cdots}\right)}{d}}{4cd}$$

$$\frac{x\sqrt{a-bx^4}(bc-ad)}{4cd\left(c-dx^4\right)}$$

input
```
Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*Sqrt[a - b*x^4])/(c*d*(c - d*x^4)) + ((a^(1/4)*b^(3/4)
*(3*b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
-1])/(d*Sqrt[a - b*x^4]) - (3*(b*c - a*d)*(b*c + a*d)*((a^(1/4)*Sqrt[1 - (
b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*
x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/d)/(4*c*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.06

method	result
default	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(\ldots\right.}{\ldots}\right.}{\ldots}$
elliptic	$\frac{(ad-bc)x\sqrt{-b\,x^4+a}}{4dc(-d\,x^4+c)}+\frac{\left(\frac{b^2}{d^2}+\frac{b(ad-bc)}{4d^2c}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{3\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{\left(a^2d^2-b^2c^2\right)\left(\ldots\right.}{\ldots}\right.}{\ldots}$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*d-b*c)/d/c*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+(b^2/d^2+1/4*b/d^2*(a*d-b*
c)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x
^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I
)-3/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctan
h(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a
^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2
)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/
2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1
/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-bx^4)^{\frac{3}{2}}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

output
```
Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(3/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(3/2)/(c - d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

output

```
( - 2*sqrt(a - b*x**4)*a*b*x + 3*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**
2*c*d**2*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x
**8 - a*b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x
**12),x)*a**4*c*d**2 - 3*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2
*x**4 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*
b*d**3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)
*a**4*d**3*x**4 - 4*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4
 + a**2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**
3*x**12 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**3
*b*c**2*d + 4*int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**
2*d**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**1
2 + b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**3*b*c*d
**2*x**4 + int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d
**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 +
 b**2*c**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**2*b**2*c**
3 - int(sqrt(a - b*x**4)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d**3*x**
8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**12 + b**2*c
**3*x**4 - 2*b**2*c**2*d*x**8 + b**2*c*d**2*x**12),x)*a**2*b**2*c**2*d*x**
4 + int((sqrt(a - b*x**4)*x**8)/(a**2*c**2*d - 2*a**2*c*d**2*x**4 + a**2*d
**3*x**8 - a*b*c**3 + a*b*c**2*d*x**4 + a*b*c*d**2*x**8 - a*b*d**3*x**1...
```

3.56 $\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

output
```
1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4*a^(1/4)*b^(3/4)*(1-b*x^4/a)^(1/2)*
EllipticF(b^(1/4)*x/a^(1/4),I)/c/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*
c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)
/c^(1/2),I)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)-1/8*a^(1/4)*(-3*a*d+b*c)*(1-b*x
^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I
)/b^(1/4)/c^2/d/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx = \frac{x\left(-\frac{5(a-bx^4)}{c}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}(c-dx^4)\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{75a^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx}{a}\right.}{5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+2x^4\left(2ad\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\right.\right.}\right.}{20\sqrt{a-bx^4}\left(-c+dx^4\right)}$$

input
```
Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

output
```
(x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5
/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1
, 5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
 (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a -
b*x^4]*(-c + d*x^4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {929, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx$$

$$\Big\downarrow \; 929$$

$$\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}-\frac{\int -\frac{3a-bx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{4c}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3a-bx^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1021$$

$$\frac{\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{d}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$$\Big\downarrow \; 1543$$

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{d}}{4c}+\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\ldots\right.}{2\sqrt[4]{b}\ldots}\right.}{d}}{4c}$$

$$\frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}$$

input
```
Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

output
```
(x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + ((a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(d*Sqrt[a - b*x^4]) - ((b*
c - 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(S
qrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a -
b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4
])))/d)/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.06

method	result
default	$\frac{x\sqrt{-b\,x^4+a}}{4c(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right.}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{}$
elliptic	$\frac{x\sqrt{-b\,x^4+a}}{4c(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4cd\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4d-c\right)}\frac{(3ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{-2b\,x^2\_\alpha^2+2a}{2\sqrt{\frac{ad-bc}{d}}\,\sqrt{-b\,x^4+a}}\right.}{\sqrt{\frac{ad-bc}{d}}}\right.}{}}{}$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x*(-b*x^4+a)^(1/2)/c/(-d*x^4+c)+1/4/c/d*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1
-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)
*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((3*a*d-b*c)/_alph
a^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)
/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^
(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*El
lipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1
/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{a-bx^4}}{(-c+dx^4)^2}\,dx$$

input
```
integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{(dx^4-c)^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{a-b\,x^4}}{(c-d\,x^4)^2}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(c - d*x^4)^2,x)
```

output
```
int((a - b*x^4)^(1/2)/(c - d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{d^2x^8-2cd\,x^4+c^2}dx$$

input
```
int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

output
```
int(sqrt(a - b*x**4)/(c**2 - 2*c*d*x**4 + d**2*x**8),x)
```

3.57 $\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\begin{aligned}\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\,(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)\sqrt{a-bx^4}}\\
&+\frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}}\\
&+\frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)\sqrt{a-bx^4}}\end{aligned}$$

output
```
-1/4*d*x*(-b*x^4+a)^(1/2)/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^(1/4)*b^(3/4)*(1-b
*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/c/(-a*d+b*c)/(-b*x^4+a)^(1/2)
+1/8*a^(1/4)*(-3*a*d+5*b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4)
,-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1/
2)+1/8*a^(1/4)*(-3*a*d+5*b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/
4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)/(-b*x^4+a)^(1
/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx$$
$$= \frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)\left(-5c(4bc-4ad+bdx^4)+bdx^4\sqrt{1-\frac{bx^4}{a}}(-c+dx^4)\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2}\right.\right.}{20c^2(bc-ad)\sqrt{a-bx^4}\,(-c+dx^4)\left(5ac\,A\right.}$$

input
```
Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

output
```
(5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1
/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt
[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)
/c])*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF
1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b
*x^4]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
 + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*App
ellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {931, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx$$
$$\Big\downarrow \text{931}$$

$$-\frac{\int -\frac{bdx^4+4bc-3ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{bdx^4+4bc-3ad}{\sqrt{a-bx^4}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1021

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-b\int \frac{1}{\sqrt{a-bx^4}}dx}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 765

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 762

$$\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 925

$$\frac{(5bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{(5bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{4c(bc-ad)}-\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1543

$$\frac{(5bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{\sqrt{a-bx^4}}}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1542

$$\frac{(5bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}\right)-\frac{\sqrt[4]{a}}{}}{4c(bc-ad)}$$

$$\frac{dx\sqrt{a-bx^4}}{4c\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

output
```
-1/4*(d*x*Sqrt[a - b*x^4])/(c*(b*c - a*d)*(c - d*x^4)) + (-((a^(1/4)*b^(3/
4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a
- b*x^4]) + (5*b*c - 3*a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sq
rt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^
(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)
*c*Sqrt[a - b*x^4])))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.04

method	result
default	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2}{2\sqrt{\frac{ad-bc}{d}}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right)}{}}{}$
elliptic	$\frac{dx\sqrt{-b\,x^4+a}}{4c(ad-bc)(-d\,x^4+c)}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{4c(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4d-c\right)}\frac{(3ad-5bc)\left(-\frac{\operatorname{arctanh}\left(\frac{-2b\,x^2}{2\sqrt{\frac{ad-bc}{d}}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right)}{}}{}$

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c*d/(a*d-b*c)*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/4*b/c/(a*d-b*c)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*sum
((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*
_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(
1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1
/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)
/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{a-bx^4}\,(-c+dx^4)^2}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(1/(sqrt(a - b*x**4)*(-c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}\,(c-dx^4)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = \int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = \int \frac{1}{\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(1/2)*(c - d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b\,d^2x^{12}+a\,d^2x^8+2bcd\,x^8-2acd\,x^4-b\,c^2x^4+a\,c^2}dx$$

input
```
int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

output
```
int(sqrt(a - b*x**4)/(a*c**2 - 2*a*c*d*x**4 + a*d**2*x**8 - b*c**2*x**4 +
2*b*c*d*x**8 - b*d**2*x**12),x)
```

3.58 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx &= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\,(c-dx^4)} \\ &+ \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4a^{3/4}c(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}} \\ &- \frac{3\sqrt[4]{a}d(3bc-ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/4*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-a*d+b*c)
/(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/4*b^(3/4)*(a*d+2*b*c)*(1-b*x^4/a)^(1/2)*Ell
ipticF(b^(1/4)*x/a^(1/4),I)/a^(3/4)/c/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/8*a^
(1/4)*d*(-a*d+3*b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/
2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)-3/
8*a^(1/4)*d*(-a*d+3*b*c)*(1-b*x^4/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^
(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^2/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.57 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \frac{x\left(-bd(2bc+ad)x^4\sqrt{1-\frac{bx^4}{a}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{c\left(25ac(4a^2d^2+\right.}{}\right.}{}$$

input
```
Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

output
```
(x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
 (b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) -
a*b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10
*x^4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4,
 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^
4)/a, (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)
/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4
)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2
*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {931, 25, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$-\frac{\int -\frac{5bdx^4+4bc-3ad}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5bdx^4+4bc-3ad}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{\int \frac{2\left(-bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{2a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-bd(2bc+ad)x^4+2b^2c^2+3a^2d^2-8abcd}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{\frac{b(ad+2bc)\int \frac{1}{\sqrt{a-bx^4}}dx-3ad(3bc-ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-3ad(3bc-ad)\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\int\frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\int\frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int\frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)}{a(bc-ad)}+\frac{bx(ad+2bc)}{a\sqrt{a-bx^4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1543$$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1542$$

$$\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}-3ad(3bc-ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{Ellip}\cdots}{\cdots}\right.}{a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\sqrt{a-bx^4}\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + ((b*(2*b*c + a*d)
*x)/(a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(2*b*c + a*d)*Sqrt
[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4
] - 3*a*d*(3*b*c - a*d)*((a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a
]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4
)*c*Sqrt[a - b*x^4]) + (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*S
qrt[a - b*x^4])))/(a*(b*c - a*d)))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.26 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.03

method	result
default	$\frac{d^2 x\sqrt{-b x^4+a}}{4c(ad-bc)^2(-d x^4+c)}+\frac{b^2 x}{2a(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{bd}{4(ad-bc)^2c}+\frac{b^2}{2(ad-bc)^2a}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b x^4+a}}+$
elliptic	$\frac{d^2 x\sqrt{-b x^4+a}}{4c(ad-bc)^2(-d x^4+c)}+\frac{b^2 x}{2a(ad-bc)^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(\frac{bd}{4(ad-bc)^2c}+\frac{b^2}{2(ad-bc)^2a}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b x^4+a}}+$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/c*d^2/(a*d-b*c)^2*x*(-b*x^4+a)^(1/2)/(-d*x^4+c)+1/2*b^2*x/a/(a*d-b*c)^
2/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d/(a*d-b*c)^2/c+1/2*b^2/(a*d-b*c)^2/a)/(1/a^
(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*su
m((a*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2
*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^
(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(
1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2
)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2
))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(3/2)*(c - d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2d^2x^{16}-2ab\,d^2x^{12}-2b^2cd\,x^{12}+a^2d^2x^8+4abcd\,x^8+b^2c^2x^8-2a^2cd\,x}$$

input
```
int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*c**2 - 2*a**2*c*d*x**4 + a**2*d**2*x**8 - 2*a*b
*c**2*x**4 + 4*a*b*c*d*x**8 - 2*a*b*d**2*x**12 + b**2*c**2*x**8 - 2*b**2*c
*d*x**12 + b**2*d**2*x**16),x)
```

3.59 $\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 439

$$\begin{aligned}\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx &= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2(a-bx^4)^{3/2}} \\ &+ \frac{b(5b^2c^2-17abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)(a-bx^4)^{3/2}(c-dx^4)} \\ &+ \frac{b^{3/4}(5b^2c^2-17abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}} \\ &+ \frac{\sqrt[4]{a}d^2(13bc-3ad)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^(3/2)+1/12*b*(-3*a^2*d^
2-17*a*b*c*d+5*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)-1/4*d*x/c/(-
a*d+b*c)/(-b*x^4+a)^(3/2)/(-d*x^4+c)+1/12*b^(3/4)*(-3*a^2*d^2-17*a*b*c*d+5
*b^2*c^2)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(7/4)/c/(-a*d
+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*(1-b*x^4/a)^(1/2)
*EllipticPi(b^(1/4)*x/a^(1/4),-a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/
c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)+1/8*a^(1/4)*d^2*(-3*a*d+13*b*c)*(1-b*x^4
/a)^(1/2)*EllipticPi(b^(1/4)*x/a^(1/4),a^(1/2)*d^(1/2)/b^(1/2)/c^(1/2),I)/
b^(1/4)/c^2/(-a*d+b*c)^3/(-b*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.92 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \frac{x\left(\frac{bd(-5b^2c^2+17abcd+3a^2d^2)x^4\sqrt{1-\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2}+5\left(\frac{5b^3c}{a^2}-\frac{17b^2d}{a}-\right.\right.}{}$$

input
```
Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

output
```
(x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*App
ellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^
2 - (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*
a*d^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 1
7*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*
x^4)/a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x
^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))))/(60*(b
*c - a*d)^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {931, 25, 1024, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$-\frac{\int -\frac{9bdx^4+4bc-3ad}{(a-bx^4)^{5/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9bdx^4+4bc-3ad}{(a-bx^4)^{5/2}(c-dx^4)}dx}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\int \frac{2\left(-5bd(2bc+3ad)x^4+10b^2c^2+9a^2d^2-24abcd\right)}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{6a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-5bd(2bc+3ad)x^4+10b^2c^2+9a^2d^2-24abcd}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}dx}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{\int \frac{2\left(-bd\left(5b^2c^2-17abdc-3a^2d^2\right)x^4+5b^3c^3-9a^3d^3+36a^2bcd^2-17ab^2c^2d\right)}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{2a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a-bx^4)^{3/2}(c-dx^4)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{-bd\left(5b^2c^2-17abdc-3a^2d^2\right)x^4+5b^3c^3-9a^3d^3+36a^2bcd^2-17ab^2c^2d}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 1021$$

$$\frac{\frac{\frac{b\left(-3a^2d^2-17abcd+5b^2c^2\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\frac{\frac{b\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(3ad+2bc)}{3a(a-bx^4)^{3/2}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{\frac{3a^2d^2(13bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{a\sqrt{a-bx^4}(bc-ad)}}{3a(bc-ad)}+\frac{}{3}}{4c(bc-ad)}-\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 925$$

$$\frac{\frac{\frac{3a^2d^2(13bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2c}+\frac{\int \frac{\sqrt{c}}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2c}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)}+}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3a^2d^2(13bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{a-bx^4}}dx}{2\sqrt{c}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}}{a(bc-ad)}+}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1543

$$\frac{\frac{\frac{3a^2d^2(13bc-3ad)\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{c}-\sqrt{d}x^2\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}+\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2\sqrt{c}\sqrt{a-bx^4}}\right)+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\text{arc}\right.}{\sqrt{a-bx^4}}}{a(bc-ad)}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

$\downarrow$ 1542

$$\frac{\frac{\frac{\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}+3a^2d^2(13bc-3ad)\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\right.}{a(bc-ad)}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)(bc-ad)}$$

input
```
Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + ((b*(2*b*c + 3*
a*d)*x)/(3*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + ((b*(5*b^2*c^2 - 17*a*b*c*d
- 3*a^2*d^2)*x)/(a*(b*c - a*d)*Sqrt[a - b*x^4]) + ((a^(1/4)*b^(3/4)*(5*b^2
*c^2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + 3*a^2*d^2*(13*b*c - 3*a*d)*((a^(1/4
)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), A
rcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(1/4)*
Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin
[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])))/(a*(b*c - a*d)
))/(3*a*(b*c - a*d)))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.29 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.10

method	result
default	$-\frac{b\,d^{3} x \sqrt{-b\,x^{4}+a}}{4(ad-bc)c(a^{2}d^{2}-2abcd+b^{2}c^{2})(bd\,x^{4}-bc)}+\frac{x\sqrt{-b\,x^{4}+a}}{6(ad-bc)^{2} a\left(x^{4}-\frac{a}{b}\right)^{2}}+\frac{b^{2} x(17ad-5bc)}{12a^{2}(ad-bc)^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^{2}}{4(ad-bc)c\left(a^{2}d^{2}-2abcd\right.}\right.}{}$
elliptic	$-\frac{b\,d^{3} x \sqrt{-b\,x^{4}+a}}{4(ad-bc)c(a^{2}d^{2}-2abcd+b^{2}c^{2})(bd\,x^{4}-bc)}+\frac{x\sqrt{-b\,x^{4}+a}}{6(ad-bc)^{2} a\left(x^{4}-\frac{a}{b}\right)^{2}}+\frac{b^{2} x(17ad-5bc)}{12a^{2}(ad-bc)^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\left(\frac{b\,d^{2}}{4(ad-bc)c\left(a^{2}d^{2}-2abcd\right.}\right.}{}$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*d^3/(a*d-b*c)/c/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-b*x^4+a)^(1/2)/(b*d
*x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-a/b)^2+1/12*b^2*x/a^2*
(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^(1/2)+(1/4*b*d^2/(a*d-b*c)/c/(a^
2*d^2-2*a*b*c*d+b^2*c^2)+1/12*b^2/a^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c*d*sum
((3*a*d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(
-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*
b^(1/2))^(1/2)*_alpha^3*d/c*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1
/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1
/2))),_alpha=RootOf(_Z^4*d-c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-bx^4)^{\frac{5}{2}}(-c+dx^4)^2}\,dx$$

input
```
integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

output
```
Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{5}{2}}(dx^4-c)^2}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2}\,dx = \int \frac{1}{(a-b\,x^4)^{5/2}\,(c-d\,x^4)^2}\,dx$$

input
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x)
```

output
```
int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx = \int \frac{\sqrt{-b\,x^4+}}{-b^3d^2x^{20}+3a\,b^2d^2x^{16}+2b^3cd\,x^{16}-3a^2b\,d^2x^{12}-6a\,b^2cd\,x^{12}-b^3c^2x^{12}}$$

input
```
int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

output
```
int(sqrt(a - b*x**4)/(a**3*c**2 - 2*a**3*c*d*x**4 + a**3*d**2*x**8 - 3*a**
2*b*c**2*x**4 + 6*a**2*b*c*d*x**8 - 3*a**2*b*d**2*x**12 + 3*a*b**2*c**2*x*
*8 - 6*a*b**2*c*d*x**12 + 3*a*b**2*d**2*x**16 - b**3*c**2*x**12 + 2*b**3*c
*d*x**16 - b**3*d**2*x**20),x)
```

3.60 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

output
```
1/4*arctan(2^(1/2)*a^(1/4)*b^(1/4)*x/(b*x^4+a)^(1/2))*2^(1/2)/a^(1/4)/b^(1
/4)/c+1/4*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(b*x^4+a)^(1/2))*2^(1/2)/a^(1/
4)/b^(1/4)/c
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

output
```
(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]] + ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*c)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {920, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$$

$$\Big\downarrow \ 920$$

$$\frac{\int \frac{1}{1-\frac{4abx^4}{\left(bx^4+a\right)^2}}d\frac{x}{\sqrt{bx^4+a}}}{c}$$

$$\Big\downarrow \ 756$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}}d\frac{x}{\sqrt{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}+1}d\frac{x}{\sqrt{bx^4+a}}}{c}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{2\sqrt{a}\sqrt{b}x^2}{bx^4+a}}d\frac{x}{\sqrt{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{c}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{c}$$

input
```
Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

output
```
(ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a
^(1/4)*b^(1/4)))/c
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 920
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[a/c
  Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\sqrt{2}\left(-2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)+\ln\left(\frac{-\sqrt{2}\,(ab)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}{\sqrt{2}\,(ab)^{\frac{1}{4}}x-\sqrt{b\,x^{4}+a}}\right)\right)}{8c(ab)^{\frac{1}{4}}}$	89
default	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c(ab)^{\frac{1}{4}}}$	94
elliptic	$-\frac{\left(2\arctan\left(\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x(ab)^{\frac{1}{4}}}\right)-\ln\left(\frac{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^{4}+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)\right)\sqrt{2}}{8c(ab)^{\frac{1}{4}}}$	94

input
```
int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)*(-2*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4))+ln((-2^(
1/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2))/(2^(1/2)*(a*b)^(1/4)*x-(b*x^4+a)^(1/2)
)))/c/(a*b)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 437, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}-x^2\right)}{bx^4-a}\right)$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

output
```
1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b
*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(
a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4
))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/
4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) +
x^2))/(b*x^4 - a)) - 1/4*I*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^
(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) - 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4)
)^(1/4) - sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a)) +
1/4*I*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(1
/(a*b*c^4))^(3/4) + 2*I*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(b*x^4
 + a)*(a*c^2*sqrt(1/(a*b*c^4)) - x^2))/(b*x^4 - a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = -\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

input
```
integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

output
```
-Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
```

output
```
-integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

input
```
integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
```

output
```
integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

input
```
int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
```

output
```
int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx=\frac{\int \frac{\sqrt{b\,x^4+a}}{-b\,x^4+a}dx}{c}$$

input
```
int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x)
```

output
```
int(sqrt(a + b*x**4)/(a - b*x**4),x)/c
```

3.61 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

output
```
1/2*arctan(b^(1/4)*x*(a^(1/2)+b^(1/2)*x^2)/a^(1/4)/(-b*x^4+a)^(1/2))/a^(1/
4)/b^(1/4)/c+1/2*arctanh(b^(1/4)*x*(a^(1/2)-b^(1/2)*x^2)/a^(1/4)/(-b*x^4+a
)^(1/2))/a^(1/4)/b^(1/4)/c
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\left(\arctan\left(\frac{(1+i)\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a-bx^4}}\right)-i\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-bx^4}}{\sqrt[4]{a}\sqrt[4]{b}x}\right)\right)}{\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

output
```
((1/4 - I/4)*(ArcTan[((1 + I)*a^(1/4)*b^(1/4)*x)/Sqrt[a - b*x^4]] - I*ArcT
an[((1/2 + I/2)*Sqrt[a - b*x^4])/(a^(1/4)*b^(1/4)*x)]))/(a^(1/4)*b^(1/4)*c
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{921\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$$

$$\Big\downarrow \; 921$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

input
```
Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

output
```
ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a
^(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*S
qrt[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Defintions of rubi rules used

rule 921
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Maple [A] (verified)

Time = 6.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

method	result	size
pseudoelliptic	$-\frac{\ln\left(\frac{-b\,x^4+2x^2\sqrt{ab}-2(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}\,x+a}{-b\,x^4+2(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}\,x+2x^2\sqrt{ab}+a}\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)}{8(ab)^{\frac{1}{4}}c}$	137
default	$-\frac{\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141
elliptic	$-\frac{\ln\left(\frac{\frac{-b\,x^4+a}{2x^2}-\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}{\frac{-b\,x^4+a}{2x^2}+\frac{(ab)^{\frac{1}{4}}\sqrt{-b\,x^4+a}}{x}+\sqrt{ab}}\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{-b\,x^4+a}}{x(ab)^{\frac{1}{4}}}-1\right)}{8c(ab)^{\frac{1}{4}}}$	141

input
```
int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a*b)^(1/4)*(ln((-b*x^4+2*x^2*(a*b)^(1/2)-2*(a*b)^(1/4)*(-b*x^4+a)^(1
/2)*x+a)/(-b*x^4+2*(a*b)^(1/4)*(-b*x^4+a)^(1/2)*x+2*x^2*(a*b)^(1/2)+a))+2*
arctan((-b*x^4+a)^(1/2)/x/(a*b)^(1/4)+1)-2*arctan(-(-b*x^4+a)^(1/2)/x/(a*b
)^(1/4)+1))/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = -\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}+\cdots}{bx^4+a}\right.$$
$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}-\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\cdots}{bx^4+a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}+2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}-\cdots}{bx^4+a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{-4i\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\sqrt{-bx^4+a}ac^2\sqrt{-\frac{1}{abc^4}}-2i\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\cdots}{bx^4+a}\right.$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

output
```
-1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/
(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/
4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*(
1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^
4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*
x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) - 1/4*I*(1/4)^
(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))
^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) + 2*I*(1/4)^(1/4)*a*c*x
*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a)) + 1/4*I*(1/4)^(
1/4)*(-1/(a*b*c^4))^(1/4)*log((-4*I*(1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))
^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4)) - 2*I*(1/4)^(1/4)*a*c*x
*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^4 + a))
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

input
```
integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

output
```
Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

input
```
int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
```

output
```
int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx = \frac{\int \frac{\sqrt{-b\,x^4+a}}{b\,x^4+a}dx}{c}$$

input
```
int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x)
```

output
```
int(sqrt(a - b*x**4)/(a + b*x**4),x)/c
```

3.62 $\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx = \frac{cx\sqrt{1-\frac{d^2x^8}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},\frac{d^2x^8}{c^2}\right)}{\sqrt{c-dx^4}\sqrt{c+dx^4}} + \frac{dx^5\sqrt{1-\frac{d^2x^8}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},\frac{d^2x^8}{c^2}\right)}{5\sqrt{c-dx^4}\sqrt{c+dx^4}}$$

output
```
c*x*(1-d^2*x^8/c^2)^(1/2)*hypergeom([1/8, 1/2],[9/8],d^2*x^8/c^2)/(-d*x^4+
c)^(1/2)/(d*x^4+c)^(1/2)+1/5*d*x^5*(1-d^2*x^8/c^2)^(1/2)*hypergeom([1/2, 5
/8],[13/8],d^2*x^8/c^2)/(-d*x^4+c)^(1/2)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.75 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx$$

$$= \frac{5c^2x\sqrt{c+dx^4}\sqrt{1-\frac{dx^4}{c}}\sqrt{1-\frac{d^2x^8}{c^2}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},-\frac{1}{2},\frac{5}{4},\frac{dx^4}{c},-\frac{dx^4}{c}\right)}{(c-dx^4)^{3/2}\sqrt{1+\frac{dx^4}{c}}\left(5c\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},-\frac{1}{2},\frac{5}{4},\frac{dx^4}{c},-\frac{dx^4}{c}\right)+2dx^4\left(\text{AppellF1}\left(\frac{5}{4},\frac{3}{2},-\frac{1}{2},\frac{9}{4},\frac{dx^4}{c},-\frac{dx^4}{c}\right)+\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^4]/Sqrt[c - d*x^4],x]
```

output
```
(5*c^2*x*Sqrt[c + d*x^4]*Sqrt[1 - (d*x^4)/c]*Sqrt[1 - (d^2*x^8)/c^2]*Appel
lF1[1/4, 1/2, -1/2, 5/4, (d*x^4)/c, -((d*x^4)/c)])/((c - d*x^4)^(3/2)*Sqrt
[1 + (d*x^4)/c]*(5*c*AppellF1[1/4, 1/2, -1/2, 5/4, (d*x^4)/c, -((d*x^4)/c)
] + 2*d*x^4*(AppellF1[5/4, 3/2, -1/2, 9/4, (d*x^4)/c, -((d*x^4)/c)] + Hype
rgeometricPFQ[{1/2, 5/8}, {13/8}, (d^2*x^8)/c^2])))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{1-\frac{dx^4}{c}}\int \frac{\sqrt{dx^4+c}}{\sqrt{1-\frac{dx^4}{c}}}dx}{\sqrt{c-dx^4}}$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{c+dx^4}\sqrt{1-\frac{dx^4}{c}}\int \frac{\sqrt{\frac{dx^4}{c}+1}}{\sqrt{1-\frac{dx^4}{c}}}dx}{\sqrt{c-dx^4}\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{c+dx^4}\sqrt{1-\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},-\frac{1}{2},\frac{5}{4},\frac{dx^4}{c},-\frac{dx^4}{c}\right)}{\sqrt{c-dx^4}\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^4]/Sqrt[c - d*x^4],x]
```

output
```
(x*Sqrt[c + d*x^4]*Sqrt[1 - (d*x^4)/c]*AppellF1[1/4, 1/2, -1/2, 5/4, (d*x^
4)/c, -((d*x^4)/c)])/(Sqrt[c - d*x^4]*Sqrt[1 + (d*x^4)/c])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\sqrt{-d\,x^{4}+c}}dx$$

input
```
int((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x)
```

output
```
int((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx=\int \frac{\sqrt{dx^4+c}}{\sqrt{-dx^4+c}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(d*x^4 + c)*sqrt(-d*x^4 + c)/(d*x^4 - c), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx=\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(-d*x**4+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**4)/sqrt(c - d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx=\int \frac{\sqrt{dx^4+c}}{\sqrt{-dx^4+c}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/sqrt(-d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx = \int \frac{\sqrt{dx^4+c}}{\sqrt{-dx^4+c}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/sqrt(-d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}}{\sqrt{c-d\,x^4}}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(c - d*x^4)^(1/2),x)
```

output
```
int((c + d*x^4)^(1/2)/(c - d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{c-dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,\sqrt{-d\,x^4+c}}{-d\,x^4+c}dx$$

input
```
int((d*x^4+c)^(1/2)/(-d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*sqrt(c - d*x**4))/(c - d*x**4),x)
```

3.63 $\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\begin{aligned}\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx &= \frac{3d(62bc-15ad)x\left(a+bx^4\right)^{4/3}}{589b^2}+\frac{3d^2x^5\left(a+bx^4\right)^{4/3}}{31b}\\ &\quad+\frac{\left(589c^2-\frac{3ad(62bc-15ad)}{b^2}\right)x\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{589\sqrt[3]{1+\frac{bx^4}{a}}}\end{aligned}$$

output

```
3/589*d*(-15*a*d+62*b*c)*x*(b*x^4+a)^(4/3)/b^2+3/31*d^2*x^5*(b*x^4+a)^(4/3
)/b+1/589*(589*c^2-3*a*d*(-15*a*d+62*b*c)/b^2)*x*(b*x^4+a)^(1/3)*hypergeom
([-1/3, 1/4],[5/4],-b*x^4/a)/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 13.65 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx = \frac{x\sqrt[3]{a+bx^4}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Gamma}\left(-\frac{1}{3}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{13}{4},-\frac{bx^4}{a}\right)-8bx^4(7c\right.}{585}$$

input
```
Integrate[(a + b*x^4)^(1/3)*(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/3)*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Gamma[-1/3]*H
ypergeometric2F1[-1/3, 1/4, 13/4, -((b*x^4)/a)] - 8*b*x^4*(7*c^2 + 10*c*d*
x^4 + 3*d^2*x^8)*Gamma[2/3]*Hypergeometric2F1[2/3, 5/4, 17/4, -((b*x^4)/a)
] - 16*b*x^4*(c + d*x^4)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 5/4, 2}, {1,
 17/4}, -((b*x^4)/a)]))/(585*a*(1 + (b*x^4)/a)^(1/3)*Gamma[-1/3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{3\int \frac{1}{3}\sqrt[3]{bx^4+a}\left(d(43bc-15ad)x^4+c(31bc-3ad)\right)dx}{31b}+\frac{3dx\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}{31b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt[3]{bx^4+a}\left(d(43bc-15ad)x^4+c(31bc-3ad)\right)dx}{31b}+\frac{3dx\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}{31b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{(45a^2d^2-186abcd+589b^2c^2)\int \sqrt[3]{bx^4+a}dx}{19b}+\frac{3dx(a+bx^4)^{4/3}(43bc-15ad)}{19b}}{31b}+\frac{3dx\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}{31b}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{\sqrt[3]{a+bx^4}(45a^2d^2-186abcd+589b^2c^2)\int \sqrt[3]{\frac{bx^4}{a}+1}dx}{19b\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{3dx(a+bx^4)^{4/3}(43bc-15ad)}{19b}}{31b}+\frac{3dx\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}{31b}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{x\sqrt[3]{a+bx^4}(45a^2d^2-186abcd+589b^2c^2)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{19b\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{3dx(a+bx^4)^{4/3}(43bc-15ad)}{19b}}{31b}+\frac{3dx\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}{31b}$$

input
```
Int[(a + b*x^4)^(1/3)*(c + d*x^4)^2,x]
```

output
```
(3*d*x*(a + b*x^4)^(4/3)*(c + d*x^4))/(31*b) + ((3*d*(43*b*c - 15*a*d)*x*(
a + b*x^4)^(4/3))/(19*b) + ((589*b^2*c^2 - 186*a*b*c*d + 45*a^2*d^2)*x*(a
+ b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)])/(19*b*(1 +
 (b*x^4)/a)^(1/3)))/(31*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 :> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{3}} \left(d\,x^{4}+c\right)^{2}dx$$

input
```
int((b*x^4+a)^(1/3)*(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(1/3)*(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right)^2\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt[3]{a}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt[3]{a}d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(d*x**4+c)**2,x)
```

output
```
a**(1/3)*c**2*x*gamma(1/4)*hyper((-1/3, 1/4), (5/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**(1/3)*c*d*x**5*gamma(5/4)*hyper((-1/3, 5/4), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**(1/3)*d**2*x**9*gamma(9
/4)*hyper((-1/3, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2 \, dx = \int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right)^2 \, dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/3)*(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2 \, dx = \int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right)^2 \, dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/3)*(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx=\int {\left(b\,x^4+a\right)}^{1/3}\,{\left(d\,x^4+c\right)}^2\,dx$$

input
```
int((a + b*x^4)^(1/3)*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/3)*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$=\frac{-180(b\,x^4+a)^{\frac{1}{3}}\,a^2d^2x+744(b\,x^4+a)^{\frac{1}{3}}\,abcdx+84(b\,x^4+a)^{\frac{1}{3}}\,ab\,d^2x^5+1767(b\,x^4+a)^{\frac{1}{3}}\,b^2c^2x+1302(b}{}$$

input
```
int((b*x^4+a)^(1/3)*(d*x^4+c)^2,x)
```

output
```
( - 180*(a + b*x**4)**(1/3)*a**2*d**2*x + 744*(a + b*x**4)**(1/3)*a*b*c*d*
x + 84*(a + b*x**4)**(1/3)*a*b*d**2*x**5 + 1767*(a + b*x**4)**(1/3)*b**2*c
**2*x + 1302*(a + b*x**4)**(1/3)*b**2*c*d*x**5 + 399*(a + b*x**4)**(1/3)*b
**2*d**2*x**9 + 180*int((a + b*x**4)**(1/3)/(a + b*x**4),x)*a**3*d**2 - 74
4*int((a + b*x**4)**(1/3)/(a + b*x**4),x)*a**2*b*c*d + 2356*int((a + b*x**
4)**(1/3)/(a + b*x**4),x)*a*b**2*c**2)/(4123*b**2)
```

3.64 $\int \sqrt[3]{a+bx^4}(c+dx^4)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\, dx = \frac{3dx(a+bx^4)^{4/3}}{19b}+\frac{(19bc-3ad)x\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{19b\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
3/19*d*x*(b*x^4+a)^(4/3)/b+1/19*(-3*a*d+19*b*c)*x*(b*x^4+a)^(1/3)*hypergeo
m([-1/3, 1/4],[5/4],-b*x^4/a)/b/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\, dx = \frac{3x\sqrt[3]{a+bx^4}\left(d(a+bx^4)+\frac{(19bc-3ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{1+\frac{bx^4}{a}}}\right)}{19b}$$

input
```
Integrate[(a + b*x^4)^(1/3)*(c + d*x^4),x]
```

output
```
(3*x*(a + b*x^4)^(1/3)*(d*(a + b*x^4) + ((19*b*c - 3*a*d)*Hypergeometric2F
1[-1/3, 1/4, 5/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^(1/3))))/(19*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(19bc-3ad)\int \sqrt[3]{bx^4+a}dx}{19b}+\frac{3dx\left(a+bx^4\right)^{4/3}}{19b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^4}(19bc-3ad)\int \sqrt[3]{\frac{bx^4}{a}+1}dx}{19b\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{3dx\left(a+bx^4\right)^{4/3}}{19b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^4}(19bc-3ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{19b\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{3dx\left(a+bx^4\right)^{4/3}}{19b}$$

input
```
Int[(a + b*x^4)^(1/3)*(c + d*x^4),x]
```

output
```
(3*d*x*(a + b*x^4)^(4/3))/(19*b) + ((19*b*c - 3*a*d)*x*(a + b*x^4)^(1/3)*H
ypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)])/(19*b*(1 + (b*x^4)/a)^(1/3
))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{3}} \left(d\,x^{4}+c\right)dx$$

input
```
int((b*x^4+a)^(1/3)*(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(1/3)*(d*x^4+c),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right) dx = \int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right) dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/3)*(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right) dx = \frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt[3]{a}dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(d*x**4+c),x)
```

output
```
a**(1/3)*c*x*gamma(1/4)*hyper((-1/3, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(1/3)*d*x**5*gamma(5/4)*hyper((-1/3, 5/4), (9/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/3)*(d*x^4 + c), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{1}{3}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/3)*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/3)*(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\,dx=\int {\left(b\,x^4+a\right)}^{1/3}\,\left(d\,x^4+c\right)\,dx$$

input
```
int((a + b*x^4)^(1/3)*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/3)*(c + d*x^4), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^4}\left(c+dx^4\right)\,dx$$
$$= \frac{12(b\,x^4+a)^{\frac{1}{3}}\,adx+57(b\,x^4+a)^{\frac{1}{3}}\,bcx+21(b\,x^4+a)^{\frac{1}{3}}\,bd\,x^5-12\left(\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}}dx\right)a^2d+76\left(\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}}dx\right)abc}{133b}$$

input
```
int((b*x^4+a)^(1/3)*(d*x^4+c),x)
```

output
```
(12*(a + b*x**4)**(1/3)*a*d*x + 57*(a + b*x**4)**(1/3)*b*c*x + 21*(a + b*x
**4)**(1/3)*b*d*x**5 - 12*int((a + b*x**4)**(1/3)/(a + b*x**4),x)*a**2*d +
 76*int((a + b*x**4)**(1/3)/(a + b*x**4),x)*a*b*c)/(133*b)
```

3.65 $\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4} \, dx = \frac{x\sqrt[3]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
x*(b*x^4+a)^(1/3)*AppellF1(1/4,-1/3,1,5/4,-b*x^4/a,-d*x^4/c)/c/(1+b*x^4/a)
^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4} \, dx$$
$$= \frac{15acx\sqrt[3]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(15ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(-3ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{3},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\right.\right.}$$

input
```
Integrate[(a + b*x^4)^(1/3)/(c + d*x^4),x]
```

output
```
(15*a*c*x*(a + b*x^4)^(1/3)*AppellF1[1/4, -1/3, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)])/((c + d*x^4)*(15*a*c*AppellF1[1/4, -1/3, 1, 5/4, -((b*x^4)/a),
-((d*x^4)/c)] + 4*x^4*(-3*a*d*AppellF1[5/4, -1/3, 2, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)] + b*c*AppellF1[5/4, 2/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}dx$$

$$\Big\downarrow \textbf{937}$$

$$\frac{\sqrt[3]{a+bx^4}\int \frac{\sqrt[3]{\frac{bx^4}{a}+1}}{dx^4+c}dx}{\sqrt[3]{\frac{bx^4}{a}+1}}$$

$$\Big\downarrow \textbf{936}$$

$$\frac{x\sqrt[3]{a+bx^4}\operatorname{AppellF1}\left(\frac{1}{4},-\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c\sqrt[3]{\frac{bx^4}{a}+1}}$$

input
```
Int[(a + b*x^4)^(1/3)/(c + d*x^4),x]
```

output
```
(x*(a + b*x^4)^(1/3)*AppellF1[1/4, -1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c
)])/(c*(1 + (b*x^4)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{1}{3}}}{d\,x^{4}+c}dx$$

input
```
int((b*x^4+a)^(1/3)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(1/3)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/3)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(1/3)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{3}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/3)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{3}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/3)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\left(b\,x^4+a\right)^{1/3}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(1/3)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/3)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\left(b\,x^4+a\right)^{\frac{1}{3}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(1/3)/(d*x^4+c),x)
```

output
```
int((a + b*x**4)**(1/3)/(c + d*x**4),x)
```

3.66 $\int \frac{\sqrt[3]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx=\frac{x\sqrt[3]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
x*(b*x^4+a)^(1/3)*AppellF1(1/4,-1/3,2,5/4,-b*x^4/a,-d*x^4/c)/c^2/(1+b*x^4/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(59) = 118.

Time = 10.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt[3]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx$$

$$=\frac{x\left(\frac{bx^4\left(1+\frac{bx^4}{a}\right)^{2/3}\text{AppellF1}\left(\frac{5}{4},\frac{2}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{3\left(\frac{a+bx^4}{c}+\frac{45a^2\,\text{AppellF1}\left(\frac{1}{4},\frac{2}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\cdots\right.}{15ac\,\text{AppellF1}\left(\frac{1}{4},\frac{2}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4x^4\left(3ad\,\text{AppellF1}\left(\frac{5}{4},\frac{2}{3},2,\frac{9}{4},-\frac{bx^4}{a},-\cdots\right.\right.}\right.}{c+dx^4}\right.}{12\left(a+bx^4\right)^{2/3}}$$

input
```
Integrate[(a + b*x^4)^(1/3)/(c + d*x^4)^2,x]
```

output
```
(x*((b*x^4*(1 + (b*x^4)/a)^(2/3)*AppellF1[5/4, 2/3, 1, 9/4, -((b*x^4)/a),
-((d*x^4)/c)])/c^2 + (3*((a + b*x^4)/c + (45*a^2*AppellF1[1/4, 2/3, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)])/(15*a*c*AppellF1[1/4, 2/3, 1, 5/4, -((b*x^4
)/a), -((d*x^4)/c)] - 4*x^4*(3*a*d*AppellF1[5/4, 2/3, 2, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)] + 2*b*c*AppellF1[5/4, 5/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)
/c)]))))/(c + d*x^4)))/(12*(a + b*x^4)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2} dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^4}\int \frac{\sqrt[3]{\frac{bx^4}{a}+1}}{(dx^4+c)^2}dx}{\sqrt[3]{\frac{bx^4}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2\sqrt[3]{\frac{bx^4}{a}+1}}$$

input
```
Int[(a + b*x^4)^(1/3)/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/3)*AppellF1[1/4, -1/3, 2, 5/4, -((b*x^4)/a), -((d*x^4)/c
)])/(c^2*(1 + (b*x^4)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{3}}}{\left(d\,x^{4}+c\right)^{2}}dx$$

input
```
int((b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(1/3)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(1/3)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{3}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/3)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{3}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/3)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{1/3}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(1/3)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/3)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{\frac{1}{3}}}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int((b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**4)**(1/3)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.67 $\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\begin{aligned}\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx &= \frac{3d(46bc-15ad)x(a+bx^4)^{2/3}}{253b^2}+\frac{3d^2x^5(a+bx^4)^{2/3}}{23b}\\&\quad+\frac{\left(253c^2-\frac{3ad(46bc-15ad)}{b^2}\right)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{253\sqrt[3]{a+bx^4}}\end{aligned}$$

output
```
3/253*d*(-15*a*d+46*b*c)*x*(b*x^4+a)^(2/3)/b^2+3/23*d^2*x^5*(b*x^4+a)^(2/3
)/b+1/253*(253*c^2-3*a*d*(-15*a*d+46*b*c)/b^2)*x*(1+b*x^4/a)^(1/3)*hyperge
om([1/4, 1/3],[5/4],-b*x^4/a)/(b*x^4+a)^(1/3)
```

Mathematica [A] (warning: unable to verify)

Time = 14.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx$$
$$= \frac{x\sqrt[3]{1+\frac{bx^4}{a}}\left(39a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{13}{4},-\frac{bx^4}{a}\right)-8bx^4(7c^2+10cdx^4+3\right.}{1755a\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(1/3),x]
```

output
```
(x*(1 + (b*x^4)/a)^(1/3)*(39*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Hypergeom
etric2F1[1/4, 1/3, 13/4, -((b*x^4)/a)] - 8*b*x^4*(7*c^2 + 10*c*d*x^4 + 3*d
^2*x^8)*Hypergeometric2F1[5/4, 4/3, 17/4, -((b*x^4)/a)] - 16*b*x^4*(c + d*
x^4)^2*HypergeometricPFQ[{5/4, 4/3, 2}, {1, 17/4}, -((b*x^4)/a)]))/(1755*a
*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx$$
$$\Big\downarrow \ 933$$
$$\frac{3\int \frac{5d(7bc-3ad)x^4+c(23bc-3ad)}{3\sqrt[3]{bx^4+a}}dx}{23b}+\frac{3dx\left(a+bx^4\right)^{2/3}\left(c+dx^4\right)}{23b}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{5d(7bc-3ad)x^4+c(23bc-3ad)}{\sqrt[3]{bx^4+a}}dx}{23b}+\frac{3dx\left(a+bx^4\right)^{2/3}\left(c+dx^4\right)}{23b}$$

$$\Big\downarrow \; 913$$

$$\frac{\frac{\left(45a^2d^2-138abcd+253b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^4+a}}dx}{11b}+\frac{15dx\left(a+bx^4\right)^{2/3}(7bc-3ad)}{11b}}{23b}+\frac{3dx\left(a+bx^4\right)^{2/3}\left(c+dx^4\right)}{23b}$$

$$\Big\downarrow \; 779$$

$$\frac{\frac{\sqrt[3]{\frac{bx^4}{a}+1}\left(45a^2d^2-138abcd+253b^2c^2\right)\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}}dx}{11b\sqrt[3]{a+bx^4}}+\frac{15dx\left(a+bx^4\right)^{2/3}(7bc-3ad)}{11b}}{23b}+\frac{3dx\left(a+bx^4\right)^{2/3}\left(c+dx^4\right)}{23b}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\left(45a^2d^2-138abcd+253b^2c^2\right)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}}+\frac{15dx\left(a+bx^4\right)^{2/3}(7bc-3ad)}{11b}}{23b}+\frac{3dx\left(a+bx^4\right)^{2/3}\left(c+dx^4\right)}{23b}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^4)^(2/3)*(c + d*x^4))/(23*b) + ((15*d*(7*b*c - 3*a*d)*x*(a
 + b*x^4)^(2/3))/(11*b) + ((253*b^2*c^2 - 138*a*b*c*d + 45*a^2*d^2)*x*(1 +
 (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(11*b*(a
 + b*x^4)^(1/3)))/(23*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/3),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)/(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[3]{a}\Gamma\left(\frac{9}{4}\right)} + \frac{d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(1/3),x)
```

output
```
c**2*x*gamma(1/4)*hyper((1/4, 1/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/3)*gamma(5/4)) + c*d*x**5*gamma(5/4)*hyper((1/3, 5/4), (9/4,), b*x**4
*exp_polar(I*pi)/a)/(2*a**(1/3)*gamma(9/4)) + d**2*x**9*gamma(9/4)*hyper((
1/3, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{1/3}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(1/3),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[3]{a+bx^4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/3),x)
```

output
```
int(x**8/(a + b*x**4)**(1/3),x)*d**2 + 2*int(x**4/(a + b*x**4)**(1/3),x)*c
*d + int(1/(a + b*x**4)**(1/3),x)*c**2
```

3.68 $\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx=\frac{3dx\left(a+bx^4\right)^{2/3}}{11b}+\frac{(11bc-3ad)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}}$$

output
```
3/11*d*x*(b*x^4+a)^(2/3)/b+1/11*(-3*a*d+11*b*c)*x*(1+b*x^4/a)^(1/3)*hyperg
eom([1/4, 1/3],[5/4],-b*x^4/a)/b/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx=\frac{3dx\left(a+bx^4\right)+(11bc-3ad)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^4) + (11*b*c - 3*a*d)*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometr
ic2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(11*b*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(11bc-3ad)\int \frac{1}{\sqrt[3]{bx^4+a}}dx}{11b}+\frac{3dx\left(a+bx^4\right)^{2/3}}{11b}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}(11bc-3ad)\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}}dx}{11b\sqrt[3]{a+bx^4}}+\frac{3dx\left(a+bx^4\right)^{2/3}}{11b}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}(11bc-3ad)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}}+\frac{3dx\left(a+bx^4\right)^{2/3}}{11b}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^4)^(2/3))/(11*b) + ((11*b*c - 3*a*d)*x*(1 + (b*x^4)/a)^(1/
3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(11*b*(a + b*x^4)^(1/3)
)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/3),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)/(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{3}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(1/3),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 1/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/3)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((1/3, 5/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(1/3)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{d\,x^4+c}{(b\,x^4+a)^{1/3}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(1/3),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(1/3), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{\sqrt[3]{a+bx^4}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/3),x)
```

output
```
int(x**4/(a + b*x**4)**(1/3),x)*d + int(1/(a + b*x**4)**(1/3),x)*c
```

3.69 $\int \frac{1}{\sqrt[3]{a+bx^4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx=\frac{x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c\sqrt[3]{a+bx^4}}$$

output
```
x*(1+b*x^4/a)^(1/3)*AppellF1(1/4,1/3,1,5/4,-b*x^4/a,-d*x^4/c)/c/(b*x^4+a)^
(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx=$$

$$-\frac{15acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt[3]{a+bx^4}\,(c+dx^4)\left(-15ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(3ad\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{3},2,\frac{9}{4},-\frac{bx^4}{a},-\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^4)^(1/3)*(c + d*x^4)),x]
```

output
```
(-15*a*c*x*AppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b
*x^4)^(1/3)*(c + d*x^4)*(-15*a*c*AppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a),
-((d*x^4)/c)] + 4*x^4*(3*a*d*AppellF1[5/4, 1/3, 2, 9/4, -((b*x^4)/a), -((d
*x^4)/c)] + b*c*AppellF1[5/4, 4/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}(dx^4+c)}dx}{\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c\sqrt[3]{a+bx^4}}$$

input
```
Int[1/((a + b*x^4)^(1/3)*(c + d*x^4)),x]
```

output
```
(x*(1 + (b*x^4)/a)^(1/3)*AppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(c*(a + b*x^4)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{4}+a)^{\frac{1}{3}}\,(d\,x^{4}+c)}dx$$

input
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/3)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(1/3)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/3)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/3)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{1/3}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/3)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/3)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,c+(b\,x^4+a)^{\frac{1}{3}}\,d\,x^4}dx$$

input
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/3)*c + (a + b*x**4)**(1/3)*d*x**4),x)
```

3.70 $\int \frac{1}{\sqrt[3]{a+bx^4}(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \frac{x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2\sqrt[3]{a+bx^4}}$$

output
```
x*(1+b*x^4/a)^(1/3)*AppellF1(1/4,1/3,2,5/4,-b*x^4/a,-d*x^4/c)/c^2/(b*x^4+a
)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(59) = 118$.

Time = 10.43 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.68

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx$$
$$= \frac{x\left(\frac{bdx^4\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad} + \frac{c\left(225ac(-4bc+4ad+bdx^4)\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-60dx^4(a+bx^4)\left(3\right.\right.}{(bc-ad)(c+dx^4)\left(-15ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(3ad\,\text{App}\right.\right.}\right.}{60c^2\sqrt[3]{a+bx^4}}$$

input
```
Integrate[1/((a + b*x^4)^(1/3)*(c + d*x^4)^2),x]
```

output
```
(x*((b*d*x^4*(1 + (b*x^4)/a)^(1/3)*AppellF1[5/4, 1/3, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(225*a*c*(-4*b*c + 4*a*d + b*d*x^4)*A
ppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 60*d*x^4*(a + b*x^
4)*(3*a*d*AppellF1[5/4, 1/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*App
ellF1[5/4, 4/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d
*x^4)*(-15*a*c*AppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 4*
x^4*(3*a*d*AppellF1[5/4, 1/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*Ap
pellF1[5/4, 4/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*c^2*(a + b*x
^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}(dx^4+c)^2}dx}{\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2\sqrt[3]{a+bx^4}}$$

input
```
Int[1/((a + b*x^4)^(1/3)*(c + d*x^4)^2),x]
```

output
```
(x*(1 + (b*x^4)/a)^(1/3)*AppellF1[1/4, 1/3, 2, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(c^2*(a + b*x^4)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{3}} \left(d\,x^{4}+c\right)^{2}}dx$$

input
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/3)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(1/3)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/3)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/3)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{1/3}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(1/3)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(1/3)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,c^2+2\,(b\,x^4+a)^{\frac{1}{3}}\,cd\,x^4+(b\,x^4+a)^{\frac{1}{3}}\,d^2x^8}dx$$

input
```
int(1/(b*x^4+a)^(1/3)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(1/3)*c**2 + 2*(a + b*x**4)**(1/3)*c*d*x**4 + (a + b*
x**4)**(1/3)*d**2*x**8),x)
```

3.71 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \frac{3(bc-ad)^2x}{4ab^2\sqrt[3]{a+bx^4}} + \frac{3d^2x(a+bx^4)^{2/3}}{11b^2} + \frac{(11b^2c^2+66abcd-45a^2d^2)\,x\sqrt[3]{1+\frac{bx^4}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{44ab^2\sqrt[3]{a+bx^4}}$$

output
```
3/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(1/3)+3/11*d^2*x*(b*x^4+a)^(2/3)/b^2+1/
44*(-45*a^2*d^2+66*a*b*c*d+11*b^2*c^2)*x*(1+b*x^4/a)^(1/3)*hypergeom([1/4,
 1/3],[5/4],-b*x^4/a)/a/b^2/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 15.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \frac{x\left(33b^2c^2+45a^2d^2+6abd(-11c+2dx^4)+(11b^2c^2+66abcd-45a^2d^2)\sqrt[3]{1+\frac{bx^4}{a}}\ \text{H}\right.}{44ab^2\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(4/3),x]
```

output
```
(x*(33*b^2*c^2 + 45*a^2*d^2 + 6*a*b*d*(-11*c + 2*d*x^4) + (11*b^2*c^2 + 66
*a*b*c*d - 45*a^2*d^2)*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5
/4, -((b*x^4)/a)]))/(44*a*b^2*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {930, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^{4/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{3\int \frac{c(bc+3ad)-d(11bc-15ad)x^4}{3\sqrt[3]{bx^4+a}}dx}{4ab}+\frac{3x\left(c+dx^4\right)(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{c(bc+3ad)-d(11bc-15ad)x^4}{\sqrt[3]{bx^4+a}}dx}{4ab}+\frac{3x\left(c+dx^4\right)(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{\left(-45a^2d^2+66abcd+11b^2c^2\right)\int \frac{1}{\sqrt[3]{bx^4+a}}dx}{11b}-\frac{3dx\left(a+bx^4\right)^{2/3}(11bc-15ad)}{11b}}{4ab}+\frac{3x\left(c+dx^4\right)(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\sqrt[3]{\frac{bx^4}{a}+1}\left(-45a^2d^2+66abcd+11b^2c^2\right)\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}}dx}{11b\sqrt[3]{a+bx^4}}-\frac{3dx\left(a+bx^4\right)^{2/3}\left(11bc-15ad\right)}{11b}}{4ab}+\frac{3x\left(c+dx^4\right)\left(bc-ad\right)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \textbf{778}$$

$$\frac{\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\left(-45a^2d^2+66abcd+11b^2c^2\right)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}}-\frac{3dx\left(a+bx^4\right)^{2/3}\left(11bc-15ad\right)}{11b}}{4ab}+\frac{3x\left(c+dx^4\right)\left(bc-ad\right)}{4ab\sqrt[3]{a+bx^4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(4/3),x]
```

output
```
(3*(b*c - a*d)*x*(c + d*x^4))/(4*a*b*(a + b*x^4)^(1/3)) + ((-3*d*(11*b*c -
 15*a*d)*x*(a + b*x^4)^(2/3))/(11*b) + ((11*b^2*c^2 + 66*a*b*c*d - 45*a^2*
d^2)*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)
])/(11*b*(a + b*x^4)^(1/3)))/(4*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^{4}+c)^{2}}{(b\,x^{4}+a)^{\frac{4}{3}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(4/3),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx=\int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^
4 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(4/3),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(4/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(4/3),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{4/3}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(4/3),x)
```

output
```
int(x**8/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*d**2 + 2*
int(x**4/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c*d + int
(1/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c**2
```

3.72 $\int \frac{c+dx^4}{(a+bx^4)^{4/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}} \, dx = -\frac{3dx}{b\sqrt[3]{a+bx^4}} + \frac{(bc+3ad)x\sqrt[3]{1+\frac{bx^4}{a}} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{ab\sqrt[3]{a+bx^4}}$$

output
```
-3*d*x/b/(b*x^4+a)^(1/3)+(3*a*d+b*c)*x*(1+b*x^4/a)^(1/3)*hypergeom([1/4, 4
/3],[5/4],-b*x^4/a)/a/b/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}} \, dx = \frac{-3adx + (bc+3ad)x\sqrt[3]{1+\frac{bx^4}{a}} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{ab\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(4/3),x]
```

output
```
(-3*a*d*x + (b*c + 3*a*d)*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 4
/3, 5/4, -((b*x^4)/a)])/(a*b*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(3ad+bc)\int \frac{1}{\sqrt[3]{bx^4+a}}dx}{4ab}+\frac{3x(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}(3ad+bc)\int \frac{1}{\sqrt[3]{\frac{bx^4}{a}+1}}dx}{4ab\sqrt[3]{a+bx^4}}+\frac{3x(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}(3ad+bc)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{4ab\sqrt[3]{a+bx^4}}+\frac{3x(bc-ad)}{4ab\sqrt[3]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(4/3),x]
```

output
```
(3*(b*c - a*d)*x)/(4*a*b*(a + b*x^4)^(1/3)) + ((b*c + 3*a*d)*x*(1 + (b*x^4
)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(4*a*b*(a + b*x
^4)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((d*x^4+c)/(b*x^4+a)^(4/3),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(2/3)*(d*x^4 + c)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{4}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{4}{3}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(4/3),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 4/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
4/3)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 4/3), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(4/3)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx=\int \frac{d\,x^4+c}{(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(4/3),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(4/3), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{4/3}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(4/3),x)
```

output
```
int(x**4/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*d + int(1
/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c
```

3.73 $\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \frac{x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{1}{4},\frac{4}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac\sqrt[3]{a+bx^4}}$$

output
```
x*(1+b*x^4/a)^(1/3)*AppellF1(1/4,4/3,1,5/4,-b*x^4/a,-d*x^4/c)/a/c/(b*x^4+a
)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(62) = 124.

Time = 10.32 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.31

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \frac{x\left(-\frac{bdx^4\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{75ac(-4bc+4ad-3bdx^4)\,\text{AppellF1}\left(\frac{1}{4},\right.}{(c+dx^4)\left(15ac\,\text{AppellF1}\left(\frac{1}{4},\right.\right.}\right.}{20a(-b}$$

input
```
Integrate[1/((a + b*x^4)^(4/3)*(c + d*x^4)),x]
```

output
```
(x*(-((b*d*x^4*(1 + (b*x^4)/a)^(1/3)*AppellF1[5/4, 1/3, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c) + (75*a*c*(-4*b*c + 4*a*d - 3*b*d*x^4)*AppellF1[1/4,
 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 60*b*x^4*(c + d*x^4)*(3*a*d*Ap
pellF1[5/4, 1/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 4
/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))/((c + d*x^4)*(15*a*c*AppellF1[1/
4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 4*x^4*(3*a*d*AppellF1[5/4, 1
/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 4/3, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)])))))/(20*a*(-(b*c) + a*d)*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{4/3}\left(dx^4+c\right)}dx}{a\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\text{AppellF1}\left(\frac{1}{4},\frac{4}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac\sqrt[3]{a+bx^4}}$$

input
```
Int[1/((a + b*x^4)^(4/3)*(c + d*x^4)),x]
```

output
```
(x*(1 + (b*x^4)/a)^(1/3)*AppellF1[1/4, 4/3, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*c*(a + b*x^4)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{4}{3}}\left(d\,x^{4}+c\right)}dx$$

input
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{4/3}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{4}{3}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(4/3)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(4/3)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{4}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(4/3)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{4}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(4/3)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{4/3}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(4/3)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(4/3)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,ac+(b\,x^4+a)^{\frac{1}{3}}\,ad\,x^4+(b\,x^4+a)^{\frac{1}{3}}\,bc\,x^4+(b\,x^4+a)^{\frac{1}{3}}\,bd\,x^8}\,d$$

input
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/3)*a*c + (a + b*x**4)**(1/3)*a*d*x**4 + (a + b*x**
4)**(1/3)*b*c*x**4 + (a + b*x**4)**(1/3)*b*d*x**8),x)
```

3.74 $\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx=\frac{x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{1}{4},\frac{4}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac^2\sqrt[3]{a+bx^4}}$$

output
```
x*(1+b*x^4/a)^(1/3)*AppellF1(1/4,4/3,2,5/4,-b*x^4/a,-d*x^4/c)/a/c^2/(b*x^4
+a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(62) = 124.

Time = 10.58 (sec) , antiderivative size = 380, normalized size of antiderivative = 6.13

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx=\frac{x\left(bd(3bc+ad)x^4\sqrt[3]{1+\frac{bx^4}{a}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c(225ac(4a^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x^4)^(4/3)*(c + d*x^4)^2),x]
```

output
```
(x*(b*d*(3*b*c + a*d)*x^4*(1 + (b*x^4)/a)^(1/3)*AppellF1[5/4, 1/3, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)] + (c*(225*a*c*(4*a^2*d^2 + a*b*d*(-8*c + d*x^
4) + b^2*c*(4*c + 3*d*x^4))*AppellF1[1/4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*
x^4)/c)] - 60*x^4*(a^2*d^2 + a*b*d^2*x^4 + 3*b^2*c*(c + d*x^4))*(3*a*d*App
ellF1[5/4, 1/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 4/
3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(15*a*c*AppellF1[1/
4, 1/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 4*x^4*(3*a*d*AppellF1[5/4, 1
/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 4/3, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)]))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{4/3}(dx^4+c)^2}dx}{a\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{\frac{bx^4}{a}+1}\text{AppellF1}\left(\frac{1}{4},\frac{4}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac^2\sqrt[3]{a+bx^4}}$$

input
```
Int[1/((a + b*x^4)^(4/3)*(c + d*x^4)^2),x]
```

output
```
(x*(1 + (b*x^4)/a)^(1/3)*AppellF1[1/4, 4/3, 2, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*c^2*(a + b*x^4)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{4}{3}} \left(d\,x^{4}+c\right)^{2}}dx$$

input
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{4/3}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{4}{3}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(4/3)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(4/3)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{4}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(4/3)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{4/3}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{4}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(4/3)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{4/3}\,\left(d\,x^4+c\right)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(4/3)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(4/3)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^4\right)^{4/3}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{3}}\,a\,c^2+2\left(b\,x^4+a\right)^{\frac{1}{3}}\,acd\,x^4+\left(b\,x^4+a\right)^{\frac{1}{3}}\,a\,d^2x^8+\left(b\,x^4+a\right)^{\frac{1}{3}}$$

input
```
int(1/(b*x^4+a)^(4/3)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(1/3)*a*c**2 + 2*(a + b*x**4)**(1/3)*a*c*d*x**4 + (a
+ b*x**4)**(1/3)*a*d**2*x**8 + (a + b*x**4)**(1/3)*b*c**2*x**4 + 2*(a + b*
x**4)**(1/3)*b*c*d*x**8 + (a + b*x**4)**(1/3)*b*d**2*x**12),x)
```

3.75 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \frac{3(bc-ad)^2x}{8ab^2\left(a+bx^4\right)^{2/3}} + \frac{3d^2x\sqrt[3]{a+bx^4}}{7b^2} + \frac{\left(35b^2c^2+42abcd-45a^2d^2\right)x\left(1+\frac{bx^4}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{56ab^2\left(a+bx^4\right)^{2/3}}$$

output
```
3/8*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(2/3)+3/7*d^2*x*(b*x^4+a)^(1/3)/b^2+1/5
6*(-45*a^2*d^2+42*a*b*c*d+35*b^2*c^2)*x*(1+b*x^4/a)^(2/3)*hypergeom([1/4,
2/3],[5/4],-b*x^4/a)/a/b^2/(b*x^4+a)^(2/3)
```

Mathematica [A] (verified)

Time = 15.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \frac{x\left(21b^2c^2+45a^2d^2+6abd(-7c+4dx^4)+\left(35b^2c^2+42abcd-45a^2d^2\right)\left(1+\frac{bx^4}{a}\right)^{2/3}\text{H}\right.}{56ab^2\left(a+bx^4\right)^{2/3}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(5/3),x]
```

output
```
(x*(21*b^2*c^2 + 45*a^2*d^2 + 6*a*b*d*(-7*c + 4*d*x^4) + (35*b^2*c^2 + 42*
a*b*c*d - 45*a^2*d^2)*(1 + (b*x^4)/a)^(2/3)*Hypergeometric2F1[1/4, 2/3, 5/
4, -((b*x^4)/a)]))/(56*a*b^2*(a + b*x^4)^(2/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {930, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx$$

$$\Big\downarrow \text{930}$$

$$\frac{3\int \frac{c(5bc+3ad)-d(7bc-15ad)x^4}{3(bx^4+a)^{2/3}}dx}{8ab}+\frac{3x(c+dx^4)(bc-ad)}{8ab(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{c(5bc+3ad)-d(7bc-15ad)x^4}{(bx^4+a)^{2/3}}dx}{8ab}+\frac{3x(c+dx^4)(bc-ad)}{8ab(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(-45a^2d^2+42abcd+35b^2c^2)\int \frac{1}{(bx^4+a)^{2/3}}dx}{7b}-\frac{3dx\sqrt[3]{a+bx^4}(7bc-15ad)}{7b}}{8ab}+\frac{3x(c+dx^4)(bc-ad)}{8ab(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\left(\frac{bx^4}{a}+1\right)^{2/3}(-45a^2d^2+42abcd+35b^2c^2)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{2/3}}dx}{7b(a+bx^4)^{2/3}}-\frac{3dx\sqrt[3]{a+bx^4}(7bc-15ad)}{7b}}{8ab}+\frac{3x(c+dx^4)(bc-ad)}{8ab(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x\left(\frac{bx^4}{a}+1\right)^{2/3}(-45a^2d^2+42abcd+35b^2c^2)\text{ Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{7b(a+bx^4)^{2/3}}-\frac{3dx\sqrt[3]{a+bx^4}(7bc-15ad)}{7b}}{8ab}+\frac{3x\left(c+dx^4\right)(bc-ad)}{8ab\left(a+bx^4\right)^{2/3}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(5/3),x]
```

output
```
(3*(b*c - a*d)*x*(c + d*x^4))/(8*a*b*(a + b*x^4)^(2/3)) + ((-3*d*(7*b*c -
15*a*d)*x*(a + b*x^4)^(1/3))/(7*b) + ((35*b^2*c^2 + 42*a*b*c*d - 45*a^2*d^
2)*x*(1 + (b*x^4)/a)^(2/3)*Hypergeometric2F1[1/4, 2/3, 5/4, -((b*x^4)/a)])
/(7*b*(a + b*x^4)^(2/3)))/(8*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{\frac{5}{3}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(5/3),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/3)/(b^2*x^8 + 2*a*b*x^
4 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(5/3),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(5/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{5/3}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(5/3),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(5/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/3}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{2}{3}}\,a+(b\,x^4+a)^{\frac{2}{3}}\,b\,x^4}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{2}{3}}\,a+(b\,x^4+a)^{\frac{2}{3}}\,b\,x^4}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}\,a+(b\,x^4+a)^{\frac{2}{3}}\,b\,x^4}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(5/3),x)
```

output
```
int(x**8/((a + b*x**4)**(2/3)*a + (a + b*x**4)**(2/3)*b*x**4),x)*d**2 + 2*
int(x**4/((a + b*x**4)**(2/3)*a + (a + b*x**4)**(2/3)*b*x**4),x)*c*d + int
(1/((a + b*x**4)**(2/3)*a + (a + b*x**4)**(2/3)*b*x**4),x)*c**2
```

3.76 $\int \frac{c+dx^4}{(a+bx^4)^{5/3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}} dx = -\frac{3dx}{5b(a+bx^4)^{2/3}} + \frac{(5bc+3ad)x\left(1+\frac{bx^4}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{5ab(a+bx^4)^{2/3}}$$

output
```
-3/5*d*x/b/(b*x^4+a)^(2/3)+1/5*(3*a*d+5*b*c)*x*(1+b*x^4/a)^(2/3)*hypergeom
([1/4, 5/3],[5/4],-b*x^4/a)/a/b/(b*x^4+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}} dx = \frac{-3adx+(5bc+3ad)x\left(1+\frac{bx^4}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{5ab(a+bx^4)^{2/3}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(5/3),x]
```

output
```
(-3*a*d*x + (5*b*c + 3*a*d)*x*(1 + (b*x^4)/a)^(2/3)*Hypergeometric2F1[1/4,
 5/3, 5/4, -((b*x^4)/a)])/(5*a*b*(a + b*x^4)^(2/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(3ad+5bc)\int \frac{1}{(bx^4+a)^{2/3}}dx}{8ab}+\frac{3x(bc-ad)}{8ab\,(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{2/3}(3ad+5bc)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{2/3}}dx}{8ab\,(a+bx^4)^{2/3}}+\frac{3x(bc-ad)}{8ab\,(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\left(\frac{bx^4}{a}+1\right)^{2/3}(3ad+5bc)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{8ab\,(a+bx^4)^{2/3}}+\frac{3x(bc-ad)}{8ab\,(a+bx^4)^{2/3}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(5/3),x]
```

output
```
(3*(b*c - a*d)*x)/(8*a*b*(a + b*x^4)^(2/3)) + ((5*b*c + 3*a*d)*x*(1 + (b*x
^4)/a)^(2/3)*Hypergeometric2F1[1/4, 2/3, 5/4, -((b*x^4)/a)])/(8*a*b*(a + b
*x^4)^(2/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{5}{3}}}d x$$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/3),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/3)*(d*x^4 + c)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{3}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{3}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{3}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(5/3),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 5/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/3)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 5/3), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(5/3)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\int \frac{d\,x^4+c}{(b\,x^4+a)^{5/3}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(5/3),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(5/3), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/3}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{2}{3}}\,a+(b\,x^4+a)^{\frac{2}{3}}\,b\,x^4}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}\,a+(b\,x^4+a)^{\frac{2}{3}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/3),x)
```

output

```
int(x**4/((a + b*x**4)**(2/3)*a + (a + b*x**4)**(2/3)*b*x**4),x)*d + int(1
/((a + b*x**4)**(2/3)*a + (a + b*x**4)**(2/3)*b*x**4),x)*c
```

3.77 $\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \frac{x\left(1+\frac{bx^4}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{4},\frac{5}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac\,(a+bx^4)^{2/3}}$$

output
```
x*(1+b*x^4/a)^(2/3)*AppellF1(1/4,5/3,1,5/4,-b*x^4/a,-d*x^4/c)/a/c/(b*x^4+a
)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 332 vs. 2(62) = 124.

Time = 10.46 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \frac{x\left(-\frac{bdx^4\left(1+\frac{bx^4}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{5}{4},\frac{2}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{3\left(5ac(-8bc+8ad-3bdx^4)\operatorname{AppellF1}\left(\frac{1}{4},\right.\right.}{(c+dx^4)\left(15ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{2}{3}\right.\right.}\right.}{8a(-bc}$$

input
```
Integrate[1/((a + b*x^4)^(5/3)*(c + d*x^4)),x]
```

output
```
(x*(-((b*d*x^4*(1 + (b*x^4)/a)^(2/3)*AppellF1[5/4, 2/3, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c) + (3*(5*a*c*(-8*b*c + 8*a*d - 3*b*d*x^4)*AppellF1[1/
4, 2/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 4*b*x^4*(c + d*x^4)*(3*a*d*A
ppellF1[5/4, 2/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*c*AppellF1[5/4
, 5/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(15*a*c*AppellF
1[1/4, 2/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 4*x^4*(3*a*d*AppellF1[5/
4, 2/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*c*AppellF1[5/4, 5/3, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^4)^(2/3
))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/3}(dx^4+c)}dx}{a(a+bx^4)^{2/3}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\left(\frac{bx^4}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{4},\frac{5}{3},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac(a+bx^4)^{2/3}}$$

input
```
Int[1/((a + b*x^4)^(5/3)*(c + d*x^4)),x]
```

output
```
(x*(1 + (b*x^4)/a)^(2/3)*AppellF1[1/4, 5/3, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*c*(a + b*x^4)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{4}+a)^{\frac{5}{3}}\,(d\,x^{4}+c)}dx$$

input
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/3}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{3}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/3)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(5/3)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/3)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{3}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/3)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/3}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(5/3)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(5/3)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}\,ac+(b\,x^4+a)^{\frac{2}{3}}\,ad\,x^4+(b\,x^4+a)^{\frac{2}{3}}\,bc\,x^4+(b\,x^4+a)^{\frac{2}{3}}\,bd\,x^8}\,d$$

input
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(2/3)*a*c + (a + b*x**4)**(2/3)*a*d*x**4 + (a + b*x**
4)**(2/3)*b*c*x**4 + (a + b*x**4)**(2/3)*b*d*x**8),x)
```

3.78 $\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx=\frac{x\left(1+\frac{bx^4}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{4},\frac{5}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac^2(a+bx^4)^{2/3}}$$

output
```
x*(1+b*x^4/a)^(2/3)*AppellF1(1/4,5/3,2,5/4,-b*x^4/a,-d*x^4/c)/a/c^2/(b*x^4
+a)^(2/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 386 vs. $2(62) = 124$.

Time = 10.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.23

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx=\frac{x\left(bd(3bc+2ad)x^4\left(1+\frac{bx^4}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{5}{4},\frac{2}{3},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c(45ac(}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(5/3)*(c + d*x^4)^2),x]
```

output
```
(x*(b*d*(3*b*c + 2*a*d)*x^4*(1 + (b*x^4)/a)^(2/3)*AppellF1[5/4, 2/3, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(45*a*c*(8*a^2*d^2 + 2*a*b*d*(-8*c + d
*x^4) + b^2*c*(8*c + 3*d*x^4))*AppellF1[1/4, 2/3, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] - 12*x^4*(2*a^2*d^2 + 2*a*b*d^2*x^4 + 3*b^2*c*(c + d*x^4))*(3*
a*d*AppellF1[5/4, 2/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*c*AppellF
1[5/4, 5/3, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(15*a*c*Ap
pellF1[1/4, 2/3, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 4*x^4*(3*a*d*Appell
F1[5/4, 2/3, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*c*AppellF1[5/4, 5/3
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*
x^4)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^4\right)^{5/3}\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{2/3}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/3}\left(dx^4+c\right)^2}dx}{a\left(a+bx^4\right)^{2/3}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\left(\frac{bx^4}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{4},\frac{5}{3},2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ac^2\left(a+bx^4\right)^{2/3}}$$

input
```
Int[1/((a + b*x^4)^(5/3)*(c + d*x^4)^2),x]
```

output
```
(x*(1 + (b*x^4)/a)^(2/3)*AppellF1[1/4, 5/3, 2, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/(a*c^2*(a + b*x^4)^(2/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a \right)^{\frac{5}{3}} \left(d\,x^{4}+c \right)^{2}}d x$$

input
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{3}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/3)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(5/3)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/3)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{3}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/3)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx=\int \frac{1}{(b\,x^4+a)^{5/3}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(5/3)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(5/3)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/3}(c+dx^4)^2}\,dx=\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}\,a\,c^2+2\,(b\,x^4+a)^{\frac{2}{3}}\,acd\,x^4+(b\,x^4+a)^{\frac{2}{3}}\,a\,d^2x^8+(b\,x^4+a)^{\frac{2}{3}}$$

input
```
int(1/(b*x^4+a)^(5/3)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(2/3)*a*c**2 + 2*(a + b*x**4)**(2/3)*a*c*d*x**4 + (a
+ b*x**4)**(2/3)*a*d**2*x**8 + (a + b*x**4)**(2/3)*b*c**2*x**4 + 2*(a + b*
x**4)**(2/3)*b*c*d*x**8 + (a + b*x**4)**(2/3)*b*d**2*x**12),x)
```

3.79 $\int (a+bx^4)^{3/4}(c+dx^4)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int (a+bx^4)^{3/4}(c+dx^4)\,dx=\frac{(8bc-ad)x(a+bx^4)^{3/4}}{32b}+\frac{dx(a+bx^4)^{7/4}}{8b}+\frac{3a(8bc-ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}+\frac{3a(8bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}$$

output
```
1/32*(-a*d+8*b*c)*x*(b*x^4+a)^(3/4)/b+1/8*d*x*(b*x^4+a)^(7/4)/b+3/64*a*(-a
*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)+3/64*a*(-a*d+8*b*c)*ar
ctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int (a+bx^4)^{3/4}(c+dx^4)\,dx=\frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}(8bc+3ad+4bdx^4)-3a(-8bc+ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-3a(-8bc+ad}{64b^{5/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)*(c + d*x^4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(8*b*c + 3*a*d + 4*b*d*x^4) - 3*a*(-8*b*c +
 a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - 3*a*(-8*b*c + a*d)*ArcTanh[(
b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(5/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {913, 748, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right) \, dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(8bc-ad)\int \left(bx^4+a\right)^{3/4}dx}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(8bc-ad)\left(\frac{3}{4}a\int \frac{1}{\sqrt[4]{bx^4+a}}dx+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

$$\Big\downarrow \text{770}$$

$$\frac{(8bc-ad)\left(\frac{3}{4}a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

$$\Big\downarrow \text{756}$$

$$\frac{(8bc-ad)\left(\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+$$
$$\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

$$\Big\downarrow \text{216}$$

$$\frac{(8bc-ad)\left(\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8bc-ad)\left(\frac{3}{4}a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}}{8b}$$

input
```
Int[(a + b*x^4)^(3/4)*(c + d*x^4),x]
```

output
```
(d*x*(a + b*x^4)^(7/4))/(8*b) + ((8*b*c - a*d)*((x*(a + b*x^4)^(3/4))/4 +
(3*a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)
*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/4))/(8*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(\frac{d\,x^{4}}{2}+c\right)x\,b^{\frac{5}{4}}}{4}+\frac{3a\left(2\left(b\,x^{4}+a\right)^{\frac{3}{4}}x d\,b^{\frac{1}{4}}+\left(a d-8 b c\right)\left(\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\frac{\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{2}\right)\right)}{64}}{b^{\frac{5}{4}}}$	117

input
```
int((b*x^4+a)^(3/4)*(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
3/64*(16/3*(b*x^4+a)^(3/4)*(1/2*d*x^4+c)*x*b^(5/4)+a*(2*(b*x^4+a)^(3/4)*x*
d*b^(1/4)+(a*d-8*b*c)*(arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-1/2*ln((-b^(1/4
)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))))))/b^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 795, normalized size of antiderivative = 6.36

$$\int (a+bx^4)^{3/4} (c+dx^4) \, dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(3/4)*(d*x^4+c),x, algorithm="fricas")
```

output
```
1/128*(3*b*((4096*a^4*b^4*c^4 - 2048*a^5*b^3*c^3*d + 384*a^6*b^2*c^2*d^2 -
 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^(1/4)*log(-27*(b^4*x*((4096*a^4*b^4*c^4 -
2048*a^5*b^3*c^3*d + 384*a^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^
(3/4) + (512*a^3*b^3*c^3 - 192*a^4*b^2*c^2*d + 24*a^5*b*c*d^2 - a^6*d^3)*(
b*x^4 + a)^(1/4))/x) - 3*b*((4096*a^4*b^4*c^4 - 2048*a^5*b^3*c^3*d + 384*a
^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^(1/4)*log(27*(b^4*x*((4096
*a^4*b^4*c^4 - 2048*a^5*b^3*c^3*d + 384*a^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 +
 a^8*d^4)/b^5)^(3/4) - (512*a^3*b^3*c^3 - 192*a^4*b^2*c^2*d + 24*a^5*b*c*d
^2 - a^6*d^3)*(b*x^4 + a)^(1/4))/x) - 3*I*b*((4096*a^4*b^4*c^4 - 2048*a^5*
b^3*c^3*d + 384*a^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^(1/4)*log
(-27*(I*b^4*x*((4096*a^4*b^4*c^4 - 2048*a^5*b^3*c^3*d + 384*a^6*b^2*c^2*d^
2 - 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^(3/4) + (512*a^3*b^3*c^3 - 192*a^4*b^2*
c^2*d + 24*a^5*b*c*d^2 - a^6*d^3)*(b*x^4 + a)^(1/4))/x) + 3*I*b*((4096*a^4
*b^4*c^4 - 2048*a^5*b^3*c^3*d + 384*a^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 + a^8
*d^4)/b^5)^(1/4)*log(-27*(-I*b^4*x*((4096*a^4*b^4*c^4 - 2048*a^5*b^3*c^3*d
 + 384*a^6*b^2*c^2*d^2 - 32*a^7*b*c*d^3 + a^8*d^4)/b^5)^(3/4) + (512*a^3*b
^3*c^3 - 192*a^4*b^2*c^2*d + 24*a^5*b*c*d^2 - a^6*d^3)*(b*x^4 + a)^(1/4))/
x) + 4*(4*b*d*x^5 + (8*b*c + 3*a*d)*x)*(b*x^4 + a)^(3/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right) dx = \frac{a^{\frac{3}{4}} c x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{4}} d x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)*(d*x**4+c),x)
```

output
```
a**(3/4)*c*x*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(3/4)*d*x**5*gamma(5/4)*hyper((-3/4, 5/4), (9/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(101) = 202$.

Time = 0.11 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.06

$$\int \left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\,dx =$$

$$-\frac{1}{16}\left(3\,a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)+\frac{4\left(bx^4+a\right)^{\frac{3}{4}}a}{\left(b-\frac{bx^4+a}{x^4}\right)x^3}\right)c$$

$$+\frac{1}{128}\left(\frac{3\,a^2\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b}+\frac{4\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3}+\frac{3\left(bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}\right)}{b^3-\frac{2\left(bx^4+a\right)b^2}{x^4}+\frac{\left(bx^4+a\right)^2b}{x^8}}\right)d$$

input

```
integrate((b*x^4+a)^(3/4)*(d*x^4+c),x, algorithm="maxima")
```

output

```
-1/16*(3*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) + 4*(b*
x^4 + a)^(3/4)*a/((b - (b*x^4 + a)/x^4)*x^3))*c + 1/128*(3*a^2*(2*arctan((
b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/
x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 4*((b*x^4 + a)^(3/4)*a^2*
b/x^3 + 3*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^3 - 2*(b*x^4 + a)*b^2/x^4 + (b*x^4
 + a)^2*b/x^8))*d
```

Giac [F]

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)\,dx=\int (bx^4+a)^{\frac{3}{4}}(dx^4+c)\,dx$$

input
```
integrate((b*x^4+a)^(3/4)*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)\,dx=\int \left(b\,x^4+a\right)^{3/4}\,\left(d\,x^4+c\right)\,dx$$

input
```
int((a + b*x^4)^(3/4)*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(3/4)*(c + d*x^4), x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)\,dx=\frac{3(b\,x^4+a)^{\frac{3}{4}}\,adx+8(b\,x^4+a)^{\frac{3}{4}}\,bcx+4(b\,x^4+a)^{\frac{3}{4}}\,bd\,x^5-3\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a^2d+24\left(\int \frac{1}{(b\,x^4+}\right.}{32b}$$

input
```
int((b*x^4+a)^(3/4)*(d*x^4+c),x)
```

output
```
(3*(a + b*x**4)**(3/4)*a*d*x + 8*(a + b*x**4)**(3/4)*b*c*x + 4*(a + b*x**4
)**(3/4)*b*d*x**5 - 3*int((a + b*x**4)**(3/4)/(a + b*x**4),x)*a**2*d + 24*
int((a + b*x**4)**(3/4)/(a + b*x**4),x)*a*b*c)/(32*b)
```

3.80 $\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}} dx = \frac{dx(a+bx^4)^{3/4}}{4b} + \frac{(4bc-ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}} + \frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

output
```
1/4*d*x*(b*x^4+a)^(3/4)/b+1/8*(-a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4
))/b^(5/4)+1/8*(-a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}} dx = \frac{2\sqrt[4]{b}dx(a+bx^4)^{3/4} + (4bc-ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + (4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*d*x*(a + b*x^4)^(3/4) + (4*b*c - a*d)*ArcTan[(b^(1/4)*x)/(a + b
*x^4)^(1/4)] + (4*b*c - a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^
(5/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {913, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \textbf{913}$$

$$\frac{(4bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \textbf{770}$$

$$\frac{(4bc-ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \textbf{756}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\downarrow \text{219}$$

$$\frac{(4bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx(a+bx^4)^{3/4}}{4b}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(1/4),x]
```

output
```
(d*x*(a + b*x^4)^(3/4))/(4*b) + ((4*b*c - a*d)*(ArcTan[(b^(1/4)*x)/(a + b*
x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4
))))/(4*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.59

method	result
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x d\,b^{\frac{1}{4}}+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a d-8\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) b c-\ln\left(\frac{-b^{\frac{1}{4}} x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a d+4\ln\left(\frac{-b^{\frac{1}{4}} x-\left(b\,x^{4}\cdots\right.}{b^{\frac{1}{4}} x-\left(b\,x^{4}+\cdots\right.}\right.}{16 b^{\frac{5}{4}}}$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(b*x^4+a)^(3/4)*x*d*b^(1/4)+2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*
a*d-8*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*b*c-ln((-b^(1/4)*x-(b*x^4+a)^(1/
4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))*a*d+4*ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^
(1/4)*x-(b*x^4+a)^(1/4)))*b*c)/b^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 716, normalized size of antiderivative = 7.54

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(3/4)*d*x + b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2
*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(1/4)*log(-(b^4*x*((256*b^4*
c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5
)^(3/4) + (64*b^3*c^3 - 48*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4
+ a)^(1/4))/x) - b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 -
16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(1/4)*log((b^4*x*((256*b^4*c^4 - 256*a*b^3*
c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^
3*c^3 - 48*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x) +
 I*b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3
 + a^4*d^4)/b^5)^(1/4)*log((I*b^4*x*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a
^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^3*c^3 - 48*a
*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x) - I*b*((256*b
^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/
b^5)^(1/4)*log((-I*b^4*x*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*
d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^3*c^3 - 48*a*b^2*c^2*d
+ 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(1/4)*gamma(9/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.91

$$\int \frac{c + dx^4}{\sqrt[4]{a + bx^4}}\, dx = \frac{1}{16}\, d\left(\frac{a\left(\frac{2 \arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b} - \frac{4\left(bx^4 + a\right)^{\frac{3}{4}} a}{\left(b^2 - \frac{(bx^4+a)b}{x^4}\right) x^3}\right) - \frac{1}{4}\, c\left(\frac{2 \arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/16*d*(a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 4*(b
*x^4 + a)^(3/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^3)) - 1/4*c*(2*arctan((b*x^
4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(
b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))
```

Giac [F]

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{d\,x^4+c}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(1/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**4/(a + b*x**4)**(1/4),x)*d + int(1/(a + b*x**4)**(1/4),x)*c
```

3.81 $\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx=\frac{(bc-ad)x}{ab\sqrt[4]{a+bx^4}}+\frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}+\frac{d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

output
```
(-a*d+b*c)*x/a/b/(b*x^4+a)^(1/4)+1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b
^(5/4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx=\frac{\frac{2\sqrt[4]{b}(bc-ad)x}{a\sqrt[4]{a+bx^4}}+d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*(b*c - a*d)*x)/(a*(a + b*x^4)^(1/4)) + d*ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)] + d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {910, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{5/4}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{d\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{d\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(5/4),x]
```

output
```
((b*c - a*d)*x)/(a*b*(a + b*x^4)^(1/4)) + (d*(ArcTan[(b^(1/4)*x)/(a + b*x^
4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))
))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

method	result	size
pseudoelliptic	$\frac{\frac{(-ad+bc)x}{b\left(b\,x^{4}+a\right)^{\frac{1}{4}}}-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)ad}{2b^{\frac{5}{4}}}+\frac{\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)ad}{4b^{\frac{5}{4}}}}{a}$	98

input
```
int((d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
((-a*d+b*c)/b*x/(b*x^4+a)^(1/4)-1/2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a*
d/b^(5/4)+1/4*ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))
*a*d/b^(5/4))/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.21

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \frac{4\,(bx^4+a)^{\frac{3}{4}}(bc-ad)x+(ab^2x^4+a^2b)\left(\frac{d^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(\frac{d^4}{b^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}d^3}{x}\right)-(ab^2x^4+\cdots}{\cdots}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/4*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + (a*b^2*x^4 + a^2*b)*(d^4/b^5)^(1/
4)*log((b^4*x*(d^4/b^5)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - (a*b^2*x^4 + a
^2*b)*(d^4/b^5)^(1/4)*log(-(b^4*x*(d^4/b^5)^(3/4) - (b*x^4 + a)^(1/4)*d^3)
/x) + (-I*a*b^2*x^4 - I*a^2*b)*(d^4/b^5)^(1/4)*log((I*b^4*x*(d^4/b^5)^(3/4
) + (b*x^4 + a)^(1/4)*d^3)/x) + (I*a*b^2*x^4 + I*a^2*b)*(d^4/b^5)^(1/4)*lo
g((-I*b^4*x*(d^4/b^5)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x))/(a*b^2*x^4 + a^2*
b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right)}{4a^{\frac{5}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x*gamma(1/4)/(4*a**(5/4)*(1 + b*x**4/a)**(1/4)*gamma(5/4)) + d*x**5*gamm
a(5/4)*hyper((5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/4)*gam
ma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{4}\,d\left(\frac{\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{b}+\frac{4\,x}{(bx^4+a)^{\frac{1}{4}}b}\right)+\frac{cx}{(bx^4+a)^{\frac{1}{4}}a}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/4*d*((2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
 (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 4*x/((
b*x^4 + a)^(1/4)*b)) + c*x/((b*x^4 + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(5/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.82 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}}\,dx=\frac{(bc-ad)x}{5ab\left(a+bx^4\right)^{5/4}}+\frac{(4bc+ad)x}{5a^2b\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-a*d+b*c)*x/a/b/(b*x^4+a)^(5/4)+1/5*(a*d+4*b*c)*x/a^2/b/(b*x^4+a)^(1/
4)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}}\,dx=\frac{x(5ac+4bcx^4+adx^4)}{5a^2\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(9/4),x]
```

output
```
(x*(5*a*c + 4*b*c*x^4 + a*d*x^4))/(5*a^2*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{4c\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{4cx}{5a^2\sqrt[4]{a+bx^4}}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(9/4),x]
```

output
```
(4*c*x)/(5*a^2*(a + b*x^4)^(1/4)) + (x*(c + d*x^4))/(5*a*(a + b*x^4)^(5/4)
)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
trager	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
pseudoelliptic	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
orering	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34

input
```
int((d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/5*x*(a*d*x^4+4*b*c*x^4+5*a*c)/(b*x^4+a)^(5/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = \frac{((4\,bc+ad)x^5+5\,acx)(bx^4+a)^{\frac{3}{4}}}{5\,(a^2b^2x^8+2\,a^3bx^4+a^4)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/5*((4*b*c + a*d)*x^5 + 5*a*c*x)*(b*x^4 + a)^(3/4)/(a^2*b^2*x^8 + 2*a^3*b
*x^4 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(53) = 106$.

Time = 16.86 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.11

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = c\left(\frac{5ax\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)} + \frac{4bx^5\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right)}{4a^{\frac{9}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+4a^{\frac{5}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*(5*a*x*gamma(1/4)/(16*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a*
*(9/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4)) + 4*b*x**5*gamma(1/4)/(16*
a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/4)*b*x**4*(1 + b*x**
4/a)**(1/4)*gamma(9/4))) + d*x**5*gamma(5/4)/(4*a**(9/4)*(1 + b*x**4/a)**(
1/4)*gamma(9/4) + 4*a**(5/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = -\frac{\left(b-\frac{5\,(bx^4+a)}{x^4}\right)cx^5}{5\,(bx^4+a)^{\frac{5}{4}}a^2} + \frac{dx^5}{5\,(bx^4+a)^{\frac{5}{4}}a}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/5*(b - 5*(b*x^4 + a)/x^4)*c*x^5/((b*x^4 + a)^(5/4)*a^2) + 1/5*d*x^5/((b
*x^4 + a)^(5/4)*a)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(9/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\frac{5\,a\,c\,x+a\,d\,x^5+4\,b\,c\,x^5}{5\,a^2\,(b\,x^4+a)^{5/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(9/4),x)
```

output
```
(5*a*c*x + a*d*x^5 + 4*b*c*x^5)/(5*a^2*(a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(1/4)*a**2 + 2*(a +
 b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.83 $\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx=\frac{(bc-ad)x}{9ab\left(a+bx^4\right)^{9/4}}+\frac{(8bc+ad)x}{45a^2b\left(a+bx^4\right)^{5/4}}+\frac{4(8bc+ad)x}{45a^3b\sqrt[4]{a+bx^4}}$$

output
```
1/9*(-a*d+b*c)*x/a/b/(b*x^4+a)^(9/4)+1/45*(a*d+8*b*c)*x/a^2/b/(b*x^4+a)^(5
/4)+4/45*(a*d+8*b*c)*x/a^3/b/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx=\frac{45a^2cx+72abcx^5+9a^2dx^5+32b^2cx^9+4abdx^9}{45a^3\left(a+bx^4\right)^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(13/4),x]
```

output
```
(45*a^2*c*x + 72*a*b*c*x^5 + 9*a^2*d*x^5 + 32*b^2*c*x^9 + 4*a*b*d*x^9)/(45
*a^3*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+8bc)\int \frac{1}{(bx^4+a)^{9/4}}dx}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+8bc)\left(\frac{4\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a(a+bx^4)^{5/4}}\right)(ad+8bc)}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x)/(9*a*b*(a + b*x^4)^(9/4)) + ((8*b*c + a*d)*(x/(5*a*(a + b*
x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(9*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{d\,x^{4}}{5}+c\right)a^{2}+\frac{8\left(\frac{d\,x^{4}}{18}+c\right)b\,x^{4}a}{5}+\frac{32b^{2}c\,x^{8}}{45}\right)x}{\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	52
gosper	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57
trager	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57
orering	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57

input
```
int((d*x^4+c)/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
((1/5*d*x^4+c)*a^2+8/5*(1/18*d*x^4+c)*b*x^4*a+32/45*b^2*c*x^8)/(b*x^4+a)^(
9/4)*x/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \frac{(4\,(8\,b^2c+abd)x^9+9\,(8\,abc+a^2d)x^5+45\,a^2cx)(bx^4+a)^{\frac{3}{4}}}{45\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/45*(4*(8*b^2*c + a*b*d)*x^9 + 9*(8*a*b*c + a^2*d)*x^5 + 45*a^2*c*x)*(b*x
^4 + a)^(3/4)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(83) = 166.

Time = 72.90 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*(45*a**5*x*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) +
192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**
2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*
x**4/a)**(1/4)*gamma(13/4)) + 117*a**4*b*x**5*gamma(1/4)/(64*a**(33/4)*(1
+ b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4
)*gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4)
+ 64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4)) + 104*a**3*b*
*2*x**9*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a
**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**2*x**
8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*x**4/
a)**(1/4)*gamma(13/4)) + 32*a**2*b**3*x**13*gamma(1/4)/(64*a**(33/4)*(1 +
b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*
gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) +
64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4))) + d*(9*a*x**5*
gamma(5/4)/(16*a**(17/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 32*a**(13/4)*
b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 16*a**(9/4)*b**2*x**8*(1 + b*x*
*4/a)**(1/4)*gamma(13/4)) + 4*b*x**9*gamma(5/4)/(16*a**(17/4)*(1 + b*x**4/
a)**(1/4)*gamma(13/4) + 32*a**(13/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13
/4) + 16*a**(9/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = -\frac{\left(5\,b - \frac{9\,(bx^4+a)}{x^4}\right)dx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^2} + \frac{\left(5\,b^2 - \frac{18\,(bx^4+a)b}{x^4} + \frac{45\,(bx^4+a)^2}{x^8}\right)cx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^3}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
-1/45*(5*b - 9*(b*x^4 + a)/x^4)*d*x^9/((b*x^4 + a)^(9/4)*a^2) + 1/45*(5*b^
2 - 18*(b*x^4 + a)*b/x^4 + 45*(b*x^4 + a)^2/x^8)*c*x^9/((b*x^4 + a)^(9/4)*
a^3)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(13/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \frac{4\,a\,d\,x\,(b\,x^4+a)^2-5\,a^3\,d\,x+32\,b\,c\,x\,(b\,x^4+a)^2+a^2\,d\,x\,(b\,x^4+a)+5\,a^2\,b\,c\,x+8\,a}{45\,a^3\,b\,(b\,x^4+a)^{9/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(13/4),x)
```

output
```
(4*a*d*x*(a + b*x^4)^2 - 5*a^3*d*x + 32*b*c*x*(a + b*x^4)^2 + a^2*d*x*(a +
 b*x^4) + 5*a^2*b*c*x + 8*a*b*c*x*(a + b*x^4))/(45*a^3*b*(a + b*x^4)^(9/4)
)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output

```
int(x**4/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
int(1/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3*(a
 + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.84 $\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx=\frac{(bc-ad)x}{13ab\,(a+bx^4)^{13/4}}+\frac{(12bc+ad)x}{117a^2b\,(a+bx^4)^{9/4}}+\frac{8(12bc+ad)x}{585a^3b\,(a+bx^4)^{5/4}}+\frac{32(12bc+ad)x}{585a^4b\sqrt[4]{a+bx^4}}$$

output
```
1/13*(-a*d+b*c)*x/a/b/(b*x^4+a)^(13/4)+1/117*(a*d+12*b*c)*x/a^2/b/(b*x^4+a
)^(9/4)+8/585*(a*d+12*b*c)*x/a^3/b/(b*x^4+a)^(5/4)+32/585*(a*d+12*b*c)*x/a
^4/b/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx=\frac{384b^3cx^{13}+32ab^2x^9(39c+dx^4)+52a^2bx^5(27c+2dx^4)+117a^3(5cx+dx^5)}{585a^4\,(a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(17/4),x]
```

output
```
(384*b^3*c*x^13 + 32*a*b^2*x^9*(39*c + d*x^4) + 52*a^2*b*x^5*(27*c + 2*d*x
^4) + 117*a^3*(5*c*x + d*x^5))/(585*a^4*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+12bc)\int \frac{1}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{8\int \frac{1}{\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{8\left(\frac{4\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)(ad+12bc)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x)/(13*a*b*(a + b*x^4)^(13/4)) + ((12*b*c + a*d)*(x/(9*a*(a +
 b*x^4)^(9/4)) + (8*(x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^
(1/4))))/(9*a)))/(13*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{d\,x^4}{5}+c\right)a^3+\frac{12b\left(\frac{2d\,x^4}{27}+c\right)x^4a^2}{5}+\frac{32\left(\frac{d\,x^4}{39}+c\right)b^2x^8a}{15}+\frac{128b^3c\,x^{12}}{195}\right)x}{(b\,x^4+a)^{\frac{13}{4}}a^4}$	71
gosper	$\frac{x(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585a^3c)}{585(b\,x^4+a)^{\frac{13}{4}}a^4}$	81
trager	$\frac{x(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585a^3c)}{585(b\,x^4+a)^{\frac{13}{4}}a^4}$	81
orering	$\frac{x(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585a^3c)}{585(b\,x^4+a)^{\frac{13}{4}}a^4}$	81

input
```
int((d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
((1/5*d*x^4+c)*a^3+12/5*b*(2/27*d*x^4+c)*x^4*a^2+32/15*(1/39*d*x^4+c)*b^2*
x^8*a+128/195*b^3*c*x^12)/(b*x^4+a)^(13/4)*x/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \frac{(32\,(12\,b^3c+ab^2d)x^{13}+104\,(12\,ab^2c+a^2bd)x^9+117\,(12\,a^2bc+a^3d)x^5+585\,a^3cx)(}{585\,(a^4b^4x^{16}+4\,a^5b^3x^{12}+6\,a^6b^2x^8+4\,a^7bx^4+a^8)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/585*(32*(12*b^3*c + a*b^2*d)*x^13 + 104*(12*a*b^2*c + a^2*b*d)*x^9 + 117
*(12*a^2*b*c + a^3*d)*x^5 + 585*a^3*c*x)*(b*x^4 + a)^(3/4)/(a^4*b^4*x^16 +
 4*a^5*b^3*x^12 + 6*a^6*b^2*x^8 + 4*a^7*b*x^4 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^4}{(a + bx^4)^{17/4}}\, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{c + dx^4}{(a + bx^4)^{17/4}}\, dx = \frac{\left(45\, b^2 - \frac{130\, (bx^4 + a) b}{x^4} + \frac{117\, (bx^4 + a)^2}{x^8}\right) dx^{13}}{585\, (bx^4 + a)^{\frac{13}{4}}\, a^3} - \frac{\left(15\, b^3 - \frac{65\, (bx^4 + a) b^2}{x^4} + \frac{117\, (bx^4 + a)^2 b}{x^8} - \frac{195\, (bx^4 + a)^3}{x^{12}}\right) cx^{13}}{195\, (bx^4 + a)^{\frac{13}{4}}\, a^4}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
1/585*(45*b^2 - 130*(b*x^4 + a)*b/x^4 + 117*(b*x^4 + a)^2/x^8)*d*x^13/((b*
x^4 + a)^(13/4)*a^3) - 1/195*(15*b^3 - 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4
 + a)^2*b/x^8 - 195*(b*x^4 + a)^3/x^12)*c*x^13/((b*x^4 + a)^(13/4)*a^4)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{(b\,x^4+a)^{13/4}}+\frac{x\,(a\,d+12\,b\,c)}{117\,a^2\,b\,(b\,x^4+a)^{9/4}} + \frac{x\,(8\,a\,d+96\,b\,c)}{585\,a^3\,b\,(b\,x^4+a)^{5/4}}+\frac{x\,(32\,a\,d+384\,b\,c)}{585\,a^4\,b\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(17/4),x)
```

output
```
(x*(c/(13*a) - d/(13*b)))/(a + b*x^4)^(13/4) + (x*(a*d + 12*b*c))/(117*a^2
*b*(a + b*x^4)^(9/4)) + (x*(8*a*d + 96*b*c))/(585*a^3*b*(a + b*x^4)^(5/4))
 + (x*(32*a*d + 384*b*c))/(585*a^4*b*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \left(\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{1}\right.\right.$$
$$+\left(\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.\right.$$

input
```
int((d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(1/((a + b*x**4)**(1/4)*a**4 +
4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 +
 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x)*c
```

3.85 $\int (a+bx^4)^{5/4} (c+dx^4) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int (a+bx^4)^{5/4} (c+dx^4) \, dx = \frac{a(10bc-ad)x\sqrt[4]{a+bx^4}}{24b} + \frac{(10bc-ad)x(a+bx^4)^{5/4}}{60b} + \frac{dx(a+bx^4)^{9/4}}{10b} - \frac{a^{3/2}(10bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24\sqrt{b}\,(a+bx^4)^{3/4}}$$

output

```
1/24*a*(-a*d+10*b*c)*x*(b*x^4+a)^(1/4)/b+1/60*(-a*d+10*b*c)*x*(b*x^4+a)^(5
/4)/b+1/10*d*x*(b*x^4+a)^(9/4)/b-1/24*a^(3/2)*(-a*d+10*b*c)*(1+a/b/x^4)^(3
/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(
b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\frac{x\sqrt[4]{a+bx^4}\left(d\left(a+bx^4\right)^2-\frac{a(-10bc+ad)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{10b}$$

input
```
Integrate[(a + b*x^4)^(5/4)*(c + d*x^4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*(d*(a + b*x^4)^2 - (a*(-10*b*c + a*d)*Hypergeometric2
F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(10*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {913, 748, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(10bc-ad)\int \left(bx^4+a\right)^{5/4}dx}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\int \sqrt[4]{bx^4+a}dx+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{748}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\left(\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{768}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2(a+bx^4)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{858}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2(a+bx^4)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{807}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4(a+bx^4)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

$$\Big\downarrow \text{229}$$

$$\frac{(10bc-ad)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}}{10b}$$

input

```
Int[(a + b*x^4)^(5/4)*(c + d*x^4),x]
```

output
```
(d*x*(a + b*x^4)^(9/4))/(10*b) + ((10*b*c - a*d)*((x*(a + b*x^4)^(5/4))/6
+ (5*a*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x
^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))))/
6))/(10*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(d\,x^{4}+c\right)dx$$

input
```
int((b*x^4+a)^(5/4)*(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(5/4)*(d*x^4+c),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*d*x^8 + (b*c + a*d)*x^4 + a*c)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\frac{a^{\frac{5}{4}}cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{\frac{5}{4}}dx^5\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt[4]{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt[4]{a}bdx^9\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)*(d*x**4+c),x)
```

output
```
a**(5/4)*c*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(5/4)*d*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**(1/4)*b*c*x**5*gamma(5/4)*hy
per((-1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**(1/
4)*b*d*x**9*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*(d*x^4 + c), x)
```

Giac [F]

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\int \left(b\,x^4+a\right)^{5/4}\left(d\,x^4+c\right)\,dx$$

input
```
int((a + b*x^4)^(5/4)*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(5/4)*(c + d*x^4), x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^{5/4}\left(c+dx^4\right)\,dx=\frac{5(b\,x^4+a)^{\frac{1}{4}}\,a^2dx+70(b\,x^4+a)^{\frac{1}{4}}\,abcx+22(b\,x^4+a)^{\frac{1}{4}}\,abd\,x^5+20(b\,x^4+a)^{\frac{1}{4}}\,b^2c\,x^5+12(b\,x^4}{120b}$$

input
```
int((b*x^4+a)^(5/4)*(d*x^4+c),x)
```

output
```
(5*(a + b*x**4)**(1/4)*a**2*d*x + 70*(a + b*x**4)**(1/4)*a*b*c*x + 22*(a +
 b*x**4)**(1/4)*a*b*d*x**5 + 20*(a + b*x**4)**(1/4)*b**2*c*x**5 + 12*(a +
b*x**4)**(1/4)*b**2*d*x**9 - 5*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**
3*d + 50*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**2*b*c)/(120*b)
```

3.86 $\int \sqrt[4]{a+bx^4}(c+dx^4)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \sqrt[4]{a+bx^4}(c+dx^4)\,dx = \frac{(6bc-ad)x\sqrt[4]{a+bx^4}}{12b}+\frac{dx(a+bx^4)^{5/4}}{6b} - \frac{\sqrt{a}(6bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
1/12*(-a*d+6*b*c)*x*(b*x^4+a)^(1/4)/b+1/6*d*x*(b*x^4+a)^(5/4)/b-1/12*a^(1/
2)*(-a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x
^2/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx$$
$$=\frac{x\sqrt[4]{a+bx^4}\left(d(a+bx^4)+\frac{(6bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{6b}$$

input
```
Integrate[(a + b*x^4)^(1/4)*(c + d*x^4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*(d*(a + b*x^4) + ((6*b*c - a*d)*Hypergeometric2F1[-1/
4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {913, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx$$

$$\Big\downarrow \; 913$$

$$\frac{(6bc-ad)\int \sqrt[4]{bx^4+a}dx}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

$$\Big\downarrow \; 748$$

$$\frac{(6bc-ad)\left(\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

$$\Big\downarrow \; 768$$

$$\frac{(6bc-ad)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2(a+bx^4)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{(6bc-ad)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2(a+bx^4)^{3/4}}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{(6bc-ad)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4(a+bx^4)^{3/4}}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

$$\Big\downarrow \text{229}$$

$$\frac{(6bc-ad)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}}{6b}$$

input
```
Int[(a + b*x^4)^(1/4)*(c + d*x^4),x]
```

output
```
(d*x*(a + b*x^4)^(5/4))/(6*b) + ((6*b*c - a*d)*((x*(a + b*x^4)^(1/4))/2 -
(Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[
b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))))/(6*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(d\,x^{4}+c\right)dx$$

input
```
int((b*x^4+a)^(1/4)*(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(1/4)*(d*x^4+c),x)
```

Fricas [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx=\frac{\sqrt[4]{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt[4]{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)*(d*x**4+c),x)
```

output
```
a**(1/4)*c*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(1/4)*d*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*(d*x^4 + c), x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)\,dx=\int \left(b\,x^4+a\right)^{1/4}\,\left(d\,x^4+c\right)\,dx$$

input
```
int((a + b*x^4)^(1/4)*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/4)*(c + d*x^4), x)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^4}\,(c+dx^4)\ dx$$
$$=\frac{(b\,x^4+a)^{\frac{1}{4}}\,adx+6(b\,x^4+a)^{\frac{1}{4}}\,bcx+2(b\,x^4+a)^{\frac{1}{4}}\,bd\,x^5-\left(\int\frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2d+6\left(\int\frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)abc}{12b}$$

input
```
int((b*x^4+a)^(1/4)*(d*x^4+c),x)
```

output
```
((a + b*x**4)**(1/4)*a*d*x + 6*(a + b*x**4)**(1/4)*b*c*x + 2*(a + b*x**4)*
*(1/4)*b*d*x**5 - int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**2*d + 6*int((
a + b*x**4)**(1/4)/(a + b*x**4),x)*a*b*c)/(12*b)
```

3.87 $\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx=\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{(2bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
1/2*d*x*(b*x^4+a)^(1/4)/b-1/2*(-a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJa
cobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^4+a)^
(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx=\frac{dx(a+bx^4)+(2bc-ad)x\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{2b\,(a+bx^4)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(3/4),x]
```

output
```
(d*x*(a + b*x^4) + (2*b*c - a*d)*x*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1
[1/4, 3/4, 5/4, -((b*x^4)/a)])/(2*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {913, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(2bc-ad)\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}+\frac{dx\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\,(a+bx^4)^{3/4}}+\frac{dx\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{858}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\,(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\,(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{a}\sqrt{b}\,(a+bx^4)^{3/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(3/4),x]
```

output
```
(d*x*(a + b*x^4)^(1/4))/(2*b) - ((2*b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*E
llipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[a]*Sqrt[b]*(a + b*x^
4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{dx^{4}+c}{\left(bx^{4}+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(3/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**4/(a + b*x**4)**(3/4),x)*d + int(1/(a + b*x**4)**(3/4),x)*c
```

3.88 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{7/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{7/4}} \, dx=\frac{(bc-ad)x}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(2bc+ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
1/3*(-a*d+b*c)*x/a/b/(b*x^4+a)^(3/4)-1/3*(a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3
*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/b^(1/2)/
(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{7/4}} \, dx=\frac{x\left(bc-ad+(2bc+ad)\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{3ab\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(7/4),x]
```

output
```
(x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, -((b*x^4)/a)]))/(3*a*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {910, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+2bc)\int \frac{1}{(bx^4+a)^{3/4}}dx}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3ab\left(a+bx^4\right)^{3/4}}+\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x)/(3*a*b*(a + b*x^4)^(3/4)) - ((2*b*c + a*d)*(1 + a/(b*x^4))
^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*Sqrt[
b]*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^4+c}{(b\,x^4+a)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(7/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(7/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**4/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.89 $\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \frac{(bc-ad)x}{7ab\,(a+bx^4)^{7/4}} + \frac{(6bc+ad)x}{21a^2b\,(a+bx^4)^{3/4}} - \frac{2(6bc+ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
1/7*(-a*d+b*c)*x/a/b/(b*x^4+a)^(7/4)+1/21*(a*d+6*b*c)*x/a^2/b/(b*x^4+a)^(3
/4)-2/21*(a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1
/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \frac{-2a^2dx+6b^2cx^5+abx(9c+dx^4)+2(6bc+ad)x(a+bx^4)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeom}}{21a^2b\,(a+bx^4)^{7/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(11/4),x]
```

output
```
(-2*a^2*d*x + 6*b^2*c*x^5 + a*b*x*(9*c + d*x^4) + 2*(6*b*c + a*d)*x*(a + b
*x^4)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)]
)/(21*a^2*b*(a + b*x^4)^(7/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {910, 749, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{11/4}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+6bc)\int \frac{1}{\left(bx^4+a\right)^{7/4}}dx}{7ab}+\frac{x(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+6bc)\left(\frac{2\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}+\frac{x}{3a\left(a+bx^4\right)^{3/4}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{(ad+6bc)\left(\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}+\frac{x}{3a\left(a+bx^4\right)^{3/4}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(ad+6bc)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a(a+bx^4)^{3/4}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^4)^{7/4}}$$

$\Big\downarrow$ 807

$$\frac{(ad+6bc)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^4)^{7/4}}$$

$\Big\downarrow$ 229

$$\frac{(ad+6bc)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}\right)}{7ab}+\frac{x(bc-ad)}{7ab\,(a+bx^4)^{7/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(11/4),x]
```

output
```
((b*c - a*d)*x)/(7*a*b*(a + b*x^4)^(7/4)) + ((6*b*c + a*d)*(x/(3*a*(a + b*
x^4)^(3/4)) - (2*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a
]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/(7*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{11}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(11/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(11/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(11/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2*b*x
^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{11}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{11}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(11/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 11/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(11/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 11/4), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*a**(11/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(11/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(11/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(11/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{11/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(11/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(11/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{11/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b^2x^8}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(11/4),x)
```

output
```
int(x**4/((a + b*x**4)**(3/4)*a**2 + 2*(a + b*x**4)**(3/4)*a*b*x**4 + (a +
 b*x**4)**(3/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(3/4)*a**2 + 2*(a +
 b*x**4)**(3/4)*a*b*x**4 + (a + b*x**4)**(3/4)*b**2*x**8),x)*c
```

3.90 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{15/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{15/4}} \, dx = \frac{(bc-ad)x}{11ab\left(a+bx^4\right)^{11/4}} + \frac{(10bc+ad)x}{77a^2b\left(a+bx^4\right)^{7/4}} + \frac{2(10bc+ad)x}{77a^3b\left(a+bx^4\right)^{3/4}} - \frac{4(10bc+ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right), 2\right)}{77a^{7/2}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
1/11*(-a*d+b*c)*x/a/b/(b*x^4+a)^(11/4)+1/77*(a*d+10*b*c)*x/a^2/b/(b*x^4+a)
^(7/4)+2/77*(a*d+10*b*c)*x/a^3/b/(b*x^4+a)^(3/4)-4/77*(a*d+10*b*c)*(1+a/b/
x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^
(7/2)/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{15/4}} \, dx = \frac{-77a^3dx+(10bc+ad)x\left(7a^2+10a\left(a+bx^4\right)+20\left(a+bx^4\right)^2+40\left(a+bx^4\right)^2\left(1+\frac{bx^4}{a}\right.\right.}{770a^3b\left(a+bx^4\right)^{11/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(15/4),x]
```

output
```
(-77*a^3*d*x + (10*b*c + a*d)*x*(7*a^2 + 10*a*(a + b*x^4) + 20*(a + b*x^4)
^2 + 40*(a + b*x^4)^2*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/
4, -((b*x^4)/a)]))/(770*a^3*b*(a + b*x^4)^(11/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {910, 749, 749, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+10bc)\int \frac{1}{(bx^4+a)^{11/4}}dx}{11ab}+\frac{x(bc-ad)}{11ab(a+bx^4)^{11/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+10bc)\left(\frac{6\int \frac{1}{\left(bx^4+a\right)^{7/4}}dx}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab(a+bx^4)^{11/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+10bc)\left(\frac{6\left(\frac{2\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}+\frac{x}{3a\left(a+bx^4\right)^{3/4}}\right)}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab(a+bx^4)^{11/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{(ad+10bc)\left(\frac{6\left(\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}+\frac{x}{3a\left(a+bx^4\right)^{3/4}}\right)}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{(ad+10bc)\left(\frac{6\left(\frac{x}{3a\left(a+bx^4\right)^{3/4}}-\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}\right)}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{(ad+10bc)\left(\frac{6\left(\frac{x}{3a\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}\right)}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{(ad+10bc)\left(\frac{6\left(\frac{x}{3a\left(a+bx^4\right)^{3/4}}-\frac{2\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}\right)}{7a}+\frac{x}{7a(a+bx^4)^{7/4}}\right)}{11ab}+\frac{x(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(15/4),x]
```

output
```
((b*c - a*d)*x)/(11*a*b*(a + b*x^4)^(11/4)) + ((10*b*c + a*d)*(x/(7*a*(a +
 b*x^4)^(7/4)) + (6*(x/(3*a*(a + b*x^4)^(3/4)) - (2*Sqrt[b]*(1 + a/(b*x^4)
)^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a +
 b*x^4)^(3/4))))/(7*a)))/(11*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{15}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(15/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \int \frac{dx^{4}+c}{(bx^{4}+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^4*x^16 + 4*a*b^3*x^12 + 6*a^2*b^
2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 134.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{15}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{15}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{15}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{15}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(15/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 15/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(15/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 15/4), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*a**(15/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{15/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(15/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(15/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{15/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2 b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}dx\right) d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2 b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}dx\right) c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(15/4),x)
```

output
```
int(x**4/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3*x**12),x)*d +
int(1/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*a**2*b*x**4 + 3*(a
 + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3*x**12),x)*c
```

3.91 $\int (a+bx^4)^{3/4} (c+dx^4)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 196

$$\int (a+bx^4)^{3/4} (c+dx^4)^2 \, dx = \frac{1}{384}\left(96c^2-\frac{ad(24bc-5ad)}{b^2}\right)x(a+bx^4)^{3/4} + \frac{d(24bc-5ad)x(a+bx^4)^{7/4}}{96b^2} + \frac{d^2x^5(a+bx^4)^{7/4}}{12b} + \frac{a(96b^2c^2-24abcd+5a^2d^2)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}} + \frac{a(96b^2c^2-24abcd+5a^2d^2)\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}$$

output
```
1/384*(96*c^2-a*d*(-5*a*d+24*b*c)/b^2)*x*(b*x^4+a)^(3/4)+1/96*d*(-5*a*d+24
*b*c)*x*(b*x^4+a)^(7/4)/b^2+1/12*d^2*x^5*(b*x^4+a)^(7/4)/b+1/256*a*(5*a^2*
d^2-24*a*b*c*d+96*b^2*c^2)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+1/256
*a*(5*a^2*d^2-24*a*b*c*d+96*b^2*c^2)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^
(9/4)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2 \, dx = \frac{2\sqrt[4]{b}x\left(a+bx^4\right)^{3/4}\left(-15a^2d^2+12abd(6c+dx^4)+32b^2(3c^2+3cdx^4+d^2x^8)\right)+3a(96b^2c^2-2\cdots}{768b^{9\cdots}}$$

input
```
Integrate[(a + b*x^4)^(3/4)*(c + d*x^4)^2,x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(-15*a^2*d^2 + 12*a*b*d*(6*c + d*x^4) + 32*
b^2*(3*c^2 + 3*c*d*x^4 + d^2*x^8)) + 3*a*(96*b^2*c^2 - 24*a*b*c*d + 5*a^2*
d^2)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 3*a*(96*b^2*c^2 - 24*a*b*c*d
+ 5*a^2*d^2)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(768*b^(9/4))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {933, 913, 748, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2 \, dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \left(bx^4+a\right)^{3/4} \left(d(16bc-5ad)x^4+c(12bc-ad)\right)dx}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\int \left(bx^4+a\right)^{3/4}dx}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \ 748$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\left(\frac{3}{4}a\int \frac{1}{\sqrt[4]{bx^4+a}}dx+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \; 770$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\left(\frac{3}{4}a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\left(\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(5a^2d^2-24abcd+96b^2c^2\right)\left(\frac{3}{4}a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}\right)}{8b}+\frac{dx\left(a+bx^4\right)^{7/4}(16bc-5ad)}{8b}}{12b}+\frac{dx\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}{12b}$$

input
```
Int[(a + b*x^4)^(3/4)*(c + d*x^4)^2,x]
```

output
```
(d*x*(a + b*x^4)^(7/4)*(c + d*x^4))/(12*b) + ((d*(16*b*c - 5*a*d)*x*(a + b
*x^4)^(7/4))/(8*b) + ((96*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*((x*(a + b*x^4
)^(3/4))/4 + (3*a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + Arc
Tanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/4))/(8*b))/(12*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\frac{3 a d \left(\frac{d x^{4}}{6}+c\right) x \left(b x^{4}+a\right)^{\frac{3}{4}} b^{\frac{5}{4}}}{16}+\frac{\left(\frac{1}{3} d^{2} x^{8}+c d x^{4}+c^{2}\right) x \left(b x^{4}+a\right)^{\frac{3}{4}} b^{\frac{9}{4}}}{4}+\frac{5 a \left(-4 \left(b x^{4}+a\right)^{\frac{3}{4}} x a d^{2} b^{\frac{1}{4}}+\left(\ln \left(\frac{-b^{\frac{1}{4}} x-\left(b x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x-\left(b x^{4}+a\right)^{\frac{1}{4}}}\right)-2 \operatorname{arc}\right.\right.}{512}}{b^{\frac{9}{4}}}$

input
```
int((b*x^4+a)^(3/4)*(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
5/512*(96/5*a*d*(1/6*d*x^4+c)*x*(b*x^4+a)^(3/4)*b^(5/4)+128/5*(1/3*d^2*x^8
+c*d*x^4+c^2)*x*(b*x^4+a)^(3/4)*b^(9/4)+a*(-4*(b*x^4+a)^(3/4)*x*a*d^2*b^(1
/4)+(ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan
(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*(a^2*d^2-24/5*a*b*c*d+96/5*b^2*c^2)))/b^(9/
4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 1462, normalized size of antiderivative = 7.46

$$\int (a+bx^4)^{3/4} (c+dx^4)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(3/4)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/1536*(3*b^2*((84934656*a^4*b^8*c^8 - 84934656*a^5*b^7*c^7*d + 49545216*a
^6*b^6*c^6*d^2 - 18579456*a^7*b^5*c^5*d^3 + 5031936*a^8*b^4*c^4*d^4 - 9676
80*a^9*b^3*c^3*d^5 + 134400*a^10*b^2*c^2*d^6 - 12000*a^11*b*c*d^7 + 625*a^
12*d^8)/b^9)^(1/4)*log((b^7*x*((84934656*a^4*b^8*c^8 - 84934656*a^5*b^7*c^
7*d + 49545216*a^6*b^6*c^6*d^2 - 18579456*a^7*b^5*c^5*d^3 + 5031936*a^8*b^
4*c^4*d^4 - 967680*a^9*b^3*c^3*d^5 + 134400*a^10*b^2*c^2*d^6 - 12000*a^11*
b*c*d^7 + 625*a^12*d^8)/b^9)^(3/4) + (884736*a^3*b^6*c^6 - 663552*a^4*b^5*
c^5*d + 304128*a^5*b^4*c^4*d^2 - 82944*a^6*b^3*c^3*d^3 + 15840*a^7*b^2*c^2
*d^4 - 1800*a^8*b*c*d^5 + 125*a^9*d^6)*(b*x^4 + a)^(1/4))/x) - 3*b^2*((849
34656*a^4*b^8*c^8 - 84934656*a^5*b^7*c^7*d + 49545216*a^6*b^6*c^6*d^2 - 18
579456*a^7*b^5*c^5*d^3 + 5031936*a^8*b^4*c^4*d^4 - 967680*a^9*b^3*c^3*d^5
+ 134400*a^10*b^2*c^2*d^6 - 12000*a^11*b*c*d^7 + 625*a^12*d^8)/b^9)^(1/4)*
log(-(b^7*x*((84934656*a^4*b^8*c^8 - 84934656*a^5*b^7*c^7*d + 49545216*a^6
*b^6*c^6*d^2 - 18579456*a^7*b^5*c^5*d^3 + 5031936*a^8*b^4*c^4*d^4 - 967680
*a^9*b^3*c^3*d^5 + 134400*a^10*b^2*c^2*d^6 - 12000*a^11*b*c*d^7 + 625*a^12
*d^8)/b^9)^(3/4) - (884736*a^3*b^6*c^6 - 663552*a^4*b^5*c^5*d + 304128*a^5
*b^4*c^4*d^2 - 82944*a^6*b^3*c^3*d^3 + 15840*a^7*b^2*c^2*d^4 - 1800*a^8*b*
c*d^5 + 125*a^9*d^6)*(b*x^4 + a)^(1/4))/x) - 3*I*b^2*((84934656*a^4*b^8*c^
8 - 84934656*a^5*b^7*c^7*d + 49545216*a^6*b^6*c^6*d^2 - 18579456*a^7*b^5*c
^5*d^3 + 5031936*a^8*b^4*c^4*d^4 - 967680*a^9*b^3*c^3*d^5 + 134400*a^10...
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2 \, dx = \frac{a^{\frac{3}{4}} c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{4} \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{4}} c d x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{a^{\frac{3}{4}} d^2 x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)*(d*x**4+c)**2,x)
```

output
```
a**(3/4)*c**2*x*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**(3/4)*c*d*x**5*gamma(5/4)*hyper((-3/4, 5/4), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**(3/4)*d**2*x**9*gamma(9
/4)*hyper((-3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(168) = 336$.

Time = 0.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.32

$$\int \left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2\,dx =$$

$$-\frac{1}{16}\left(3\,a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)+\frac{4\left(bx^4+a\right)^{\frac{3}{4}}a}{\left(b-\frac{bx^4+a}{x^4}\right)x^3}\right)c^2$$

$$+\frac{1}{64}\left(\frac{3\,a^2\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b}+\frac{4\left(\frac{\left(bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3}+\frac{3\left(bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}\right)}{b^3-\frac{2\left(bx^4+a\right)b^2}{x^4}+\frac{\left(bx^4+a\right)^2b}{x^8}}\right)cd$$

$$-\frac{1}{1536}\left(\frac{15\,a^3\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}+\frac{4\left(\frac{5\left(bx^4+a\right)^{\frac{3}{4}}a^3b^2}{x^3}+\frac{42\left(bx^4+a\right)^{\frac{7}{4}}a^3b}{x^7}-\frac{15\left(bx^4+a\right)^{\frac{11}{4}}}{x^{11}}\right.}{b^5-\frac{3\left(bx^4+a\right)b^4}{x^4}+\frac{3\left(bx^4+a\right)^2b^3}{x^8}-\frac{\left(bx^4+a\right)^3b^2}{x^{12}}}\right.$$

input
```
integrate((b*x^4+a)^(3/4)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(3*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) + 4*(b*
x^4 + a)^(3/4)*a/((b - (b*x^4 + a)/x^4)*x^3))*c^2 + 1/64*(3*a^2*(2*arctan(
(b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)
/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 4*((b*x^4 + a)^(3/4)*a^2
*b/x^3 + 3*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^3 - 2*(b*x^4 + a)*b^2/x^4 + (b*x^
4 + a)^2*b/x^8))*c*d - 1/1536*(15*a^3*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)
*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)
^(1/4)/x))/b^(1/4))/b^2 + 4*(5*(b*x^4 + a)^(3/4)*a^3*b^2/x^3 + 42*(b*x^4 +
 a)^(7/4)*a^3*b/x^7 - 15*(b*x^4 + a)^(11/4)*a^3/x^11)/(b^5 - 3*(b*x^4 + a)
*b^4/x^4 + 3*(b*x^4 + a)^2*b^3/x^8 - (b*x^4 + a)^3*b^2/x^12))*d^2
```

Giac [F]

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2 \, dx = \int \left(bx^4+a\right)^{\frac{3}{4}} \left(dx^4+c\right)^2 dx$$

input
```
integrate((b*x^4+a)^(3/4)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2 \, dx = \int \left(b\,x^4+a\right)^{3/4}\,\left(d\,x^4+c\right)^2 \,dx$$

input
```
int((a + b*x^4)^(3/4)*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(3/4)*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int (a+bx^4)^{3/4} (c+dx^4)^2\, dx = \frac{-15(b\,x^4+a)^{\frac{3}{4}}\, a^2 d^2 x + 72(b\,x^4+a)^{\frac{3}{4}}\, abcdx + 12(b\,x^4+a)^{\frac{3}{4}}\, ab\,d^2 x^5 + 96(b\,x^4+a)^{\frac{3}{4}}\, b^2 c^2 x + 9}{}$$

input
```
int((b*x^4+a)^(3/4)*(d*x^4+c)^2,x)
```

output
```
( - 15*(a + b*x**4)**(3/4)*a**2*d**2*x + 72*(a + b*x**4)**(3/4)*a*b*c*d*x
+ 12*(a + b*x**4)**(3/4)*a*b*d**2*x**5 + 96*(a + b*x**4)**(3/4)*b**2*c**2*
x + 96*(a + b*x**4)**(3/4)*b**2*c*d*x**5 + 32*(a + b*x**4)**(3/4)*b**2*d**
2*x**9 + 15*int((a + b*x**4)**(3/4)/(a + b*x**4),x)*a**3*d**2 - 72*int((a
+ b*x**4)**(3/4)/(a + b*x**4),x)*a**2*b*c*d + 288*int((a + b*x**4)**(3/4)/
(a + b*x**4),x)*a*b**2*c**2)/(384*b**2)
```

3.92 $\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \frac{d(16bc-5ad)x(a+bx^4)^{3/4}}{32b^2} + \frac{d^2x^5(a+bx^4)^{3/4}}{8b} + \frac{(32b^2c^2-16abcd+5a^2d^2)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}} + \frac{(32b^2c^2-16abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}$$

output

```
1/32*d*(-5*a*d+16*b*c)*x*(b*x^4+a)^(3/4)/b^2+1/8*d^2*x^5*(b*x^4+a)^(3/4)/b
+1/64*(5*a^2*d^2-16*a*b*c*d+32*b^2*c^2)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/
b^(9/4)+1/64*(5*a^2*d^2-16*a*b*c*d+32*b^2*c^2)*arctanh(b^(1/4)*x/(b*x^4+a)
^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \frac{2\sqrt[4]{b}dx(a+bx^4)^{3/4}(16bc-5ad+4bdx^4)+(32b^2c^2-16abcd+5a^2d^2)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+(32b^2c^2-}{64b^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*d*x*(a + b*x^4)^(3/4)*(16*b*c - 5*a*d + 4*b*d*x^4) + (32*b^2*c^
2 - 16*a*b*c*d + 5*a^2*d^2)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (32*b^
2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(6
4*b^(9/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {933, 913, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int \frac{d(12bc-5ad)x^4+c(8bc-ad)}{\sqrt[4]{bx^4+a}}dx}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{(5a^2d^2-16abcd+32b^2c^2)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}+\frac{dx(a+bx^4)^{3/4}(12bc-5ad)}{4b}}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

$$\Big\downarrow \text{770}$$

$$\frac{\frac{(5a^2d^2-16abcd+32b^2c^2)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}+\frac{dx(a+bx^4)^{3/4}(12bc-5ad)}{4b}}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{(5a^2d^2-16abcd+32b^2c^2)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}+\frac{dx(a+bx^4)^{3/4}(12bc-5ad)}{4b}}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(5a^2d^2-16abcd+32b^2c^2)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx(a+bx^4)^{3/4}(12bc-5ad)}{4b}}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(5a^2d^2-16abcd+32b^2c^2)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx(a+bx^4)^{3/4}(12bc-5ad)}{4b}}{8b}+\frac{dx(a+bx^4)^{3/4}(c+dx^4)}{8b}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(1/4),x]
```

output
```
(d*x*(a + b*x^4)^(3/4)*(c + d*x^4))/(8*b) + ((d*(12*b*c - 5*a*d)*x*(a + b*
x^4)^(3/4))/(4*b) + ((32*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*(ArcTan[(b^(1/4
)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4
)]/(2*b^(1/4))))/(4*b))/(8*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{5\left(-\frac{32\left(\frac{d\,x^4}{4}+c\right)\left(b\,x^4+a\right)^{\frac{3}{4}}d x\,b^{\frac{5}{4}}}{5}+2\left(b\,x^4+a\right)^{\frac{3}{4}}x a\,d^2 b^{\frac{1}{4}}+\left(\arctan\left(\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\frac{\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^4+a\right)^{\frac{1}{4}}}\right)}{2}\right)\left(a^2 d^2-\frac{16}{5}\right.}{64 b^{\frac{9}{4}}}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-5/64*(-32/5*(1/4*d*x^4+c)*(b*x^4+a)^(3/4)*d*x*b^(5/4)+2*(b*x^4+a)^(3/4)*x
*a*d^2*b^(1/4)+(arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-1/2*ln((-b^(1/4)*x-(b*
x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4))))*(a^2*d^2-16/5*a*b*c*d+32/5*b^2
*c^2))/b^(9/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1368, normalized size of antiderivative = 8.77

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/128*(b^2*((1048576*b^8*c^8 - 2097152*a*b^7*c^7*d + 2228224*a^2*b^6*c^6*d
^2 - 1507328*a^3*b^5*c^5*d^3 + 710656*a^4*b^4*c^4*d^4 - 235520*a^5*b^3*c^3
*d^5 + 54400*a^6*b^2*c^2*d^6 - 8000*a^7*b*c*d^7 + 625*a^8*d^8)/b^9)^(1/4)*
log((b^7*x*((1048576*b^8*c^8 - 2097152*a*b^7*c^7*d + 2228224*a^2*b^6*c^6*d
^2 - 1507328*a^3*b^5*c^5*d^3 + 710656*a^4*b^4*c^4*d^4 - 235520*a^5*b^3*c^3
*d^5 + 54400*a^6*b^2*c^2*d^6 - 8000*a^7*b*c*d^7 + 625*a^8*d^8)/b^9)^(3/4)
+ (32768*b^6*c^6 - 49152*a*b^5*c^5*d + 39936*a^2*b^4*c^4*d^2 - 19456*a^3*b
^3*c^3*d^3 + 6240*a^4*b^2*c^2*d^4 - 1200*a^5*b*c*d^5 + 125*a^6*d^6)*(b*x^4
 + a)^(1/4))/x) - b^2*((1048576*b^8*c^8 - 2097152*a*b^7*c^7*d + 2228224*a^
2*b^6*c^6*d^2 - 1507328*a^3*b^5*c^5*d^3 + 710656*a^4*b^4*c^4*d^4 - 235520*
a^5*b^3*c^3*d^5 + 54400*a^6*b^2*c^2*d^6 - 8000*a^7*b*c*d^7 + 625*a^8*d^8)/
b^9)^(1/4)*log(-(b^7*x*((1048576*b^8*c^8 - 2097152*a*b^7*c^7*d + 2228224*a
^2*b^6*c^6*d^2 - 1507328*a^3*b^5*c^5*d^3 + 710656*a^4*b^4*c^4*d^4 - 235520
*a^5*b^3*c^3*d^5 + 54400*a^6*b^2*c^2*d^6 - 8000*a^7*b*c*d^7 + 625*a^8*d^8)
/b^9)^(3/4) - (32768*b^6*c^6 - 49152*a*b^5*c^5*d + 39936*a^2*b^4*c^4*d^2 -
 19456*a^3*b^3*c^3*d^3 + 6240*a^4*b^2*c^2*d^4 - 1200*a^5*b*c*d^5 + 125*a^6
*d^6)*(b*x^4 + a)^(1/4))/x) - I*b^2*((1048576*b^8*c^8 - 2097152*a*b^7*c^7*
d + 2228224*a^2*b^6*c^6*d^2 - 1507328*a^3*b^5*c^5*d^3 + 710656*a^4*b^4*c^4
*d^4 - 235520*a^5*b^3*c^3*d^5 + 54400*a^6*b^2*c^2*d^6 - 8000*a^7*b*c*d^7 +
 625*a^8*d^8)/b^9)^(1/4)*log((I*b^7*x*((1048576*b^8*c^8 - 2097152*a*b^7...
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)} + \frac{d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(1/4),x)
```

output
```
c**2*x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/4)*gamma(5/4)) + c*d*x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4
*exp_polar(I*pi)/a)/(2*a**(1/4)*gamma(9/4)) + d**2*x**9*gamma(9/4)*hyper((
1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/4)*gamma(13/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(132) = 264$.

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = -\frac{1}{128}\,d^2\left(\frac{5\,a^2\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}-\frac{4\left(\frac{9\,(bx^4+a)^{\frac{3}{4}}a^2b}{x^3}-\frac{5\,(bx^4+a)^{\frac{7}{4}}a^2}{x^7}\right)}{b^4-\frac{2\,(bx^4+a)b^3}{x^4}+\frac{(bx^4+a)^2b^2}{x^8}}\right) + \frac{1}{8}\,cd\left(\frac{a\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b}-\frac{4\,(bx^4+a)^{\frac{3}{4}}a}{\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x^3}\right) - \frac{1}{4}\,c^2\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/128*d^2*(5*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-
(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/
b^2 - 4*(9*(b*x^4 + a)^(3/4)*a^2*b/x^3 - 5*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^4
 - 2*(b*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8)) + 1/8*c*d*(a*(2*arctan(
(b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)
/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 4*(b*x^4 + a)^(3/4)*a/((
b^2 - (b*x^4 + a)*b/x^4)*x^3)) - 1/4*c^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1
/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 +
 a)^(1/4)/x))/b^(1/4))
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(1/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(1/4),x)
```

output
```
int(x**8/(a + b*x**4)**(1/4),x)*d**2 + 2*int(x**4/(a + b*x**4)**(1/4),x)*c
*d + int(1/(a + b*x**4)**(1/4),x)*c**2
```

3.93 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \frac{(bc-ad)^2x}{ab^2\sqrt[4]{a+bx^4}} + \frac{d^2x(a+bx^4)^{3/4}}{4b^2} + \frac{d(8bc-5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}} + \frac{d(8bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

output
```
(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(1/4)+1/4*d^2*x*(b*x^4+a)^(3/4)/b^2+1/8*d*(
-5*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+1/8*d*(-5*a*d+8*b*
c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(4b^2c^2+5a^2d^2+abd(-8c+dx^4))}{a\sqrt[4]{a+bx^4}} + d(8bc-5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + d(8bc-5ad)\text{arc}}{8b^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*x*(4*b^2*c^2 + 5*a^2*d^2 + a*b*d*(-8*c + d*x^4)))/(a*(a + b*x^
4)^(1/4)) + d*(8*b*c - 5*a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + d*(8
*b*c - 5*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(9/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {930, 27, 913, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(ac-(4bc-5ad)x^4)}{\sqrt[4]{bx^4+a}}dx}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{ac-(4bc-5ad)x^4}{\sqrt[4]{bx^4+a}}dx}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{913}$$

$$\frac{d\left(\frac{a(8bc-5ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}-\frac{x(a+bx^4)^{3/4}(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{d\left(\frac{a(8bc-5ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}-\frac{x(a+bx^4)^{3/4}(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{d\left(\frac{a(8bc-5ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}-\frac{x(a+bx^4)^{3/4}(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 216$$

$$\frac{d\left(\frac{a(8bc-5ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}-\frac{x(a+bx^4)^{3/4}(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{a(8bc-5ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}-\frac{x(a+bx^4)^{3/4}(4bc-5ad)}{4b}\right)}{ab}+\frac{x(c+dx^4)(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(5/4),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(a*b*(a + b*x^4)^(1/4)) + (d*(-1/4*((4*b*c - 5
*a*d)*x*(a + b*x^4)^(3/4))/b + (a*(8*b*c - 5*a*d)*(ArcTan[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(
1/4))))/(4*b)))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{5\left(\frac{32ad\left(-\frac{dx^4}{8}+c\right)x\,b^{\frac{5}{4}}}{5}-\frac{16b^{\frac{9}{4}}c^2x}{5}+ad\left(-4adx\,b^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}\left(ad-\frac{8bc}{5}\right)\left(\ln\left(\frac{-b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(bx^4+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{(bx^4+}{b^{\frac{1}{4}}}\right.\right.\right.\right.}{16\left(bx^4+a\right)^{\frac{1}{4}}b^{\frac{9}{4}}a}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-5/16*(32/5*a*d*(-1/8*d*x^4+c)*x*b^(5/4)-16/5*b^(9/4)*c^2*x+a*d*(-4*a*d*x*
b^(1/4)+(b*x^4+a)^(1/4)*(a*d-8/5*b*c)*(ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^
(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))))/(b*x^4+
a)^(1/4)/b^(9/4)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 896, normalized size of antiderivative = 7.00

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/16*((a*b^3*x^4 + a^2*b^2)*((4096*b^4*c^4*d^4 - 10240*a*b^3*c^3*d^5 + 960
0*a^2*b^2*c^2*d^6 - 4000*a^3*b*c*d^7 + 625*a^4*d^8)/b^9)^(1/4)*log(-(b^7*x
*((4096*b^4*c^4*d^4 - 10240*a*b^3*c^3*d^5 + 9600*a^2*b^2*c^2*d^6 - 4000*a^
3*b*c*d^7 + 625*a^4*d^8)/b^9)^(3/4) + (512*b^3*c^3*d^3 - 960*a*b^2*c^2*d^4
 + 600*a^2*b*c*d^5 - 125*a^3*d^6)*(b*x^4 + a)^(1/4))/x) - (a*b^3*x^4 + a^2
*b^2)*((4096*b^4*c^4*d^4 - 10240*a*b^3*c^3*d^5 + 9600*a^2*b^2*c^2*d^6 - 40
00*a^3*b*c*d^7 + 625*a^4*d^8)/b^9)^(1/4)*log((b^7*x*((4096*b^4*c^4*d^4 - 1
0240*a*b^3*c^3*d^5 + 9600*a^2*b^2*c^2*d^6 - 4000*a^3*b*c*d^7 + 625*a^4*d^8
)/b^9)^(3/4) - (512*b^3*c^3*d^3 - 960*a*b^2*c^2*d^4 + 600*a^2*b*c*d^5 - 12
5*a^3*d^6)*(b*x^4 + a)^(1/4))/x) - (-I*a*b^3*x^4 - I*a^2*b^2)*((4096*b^4*c
^4*d^4 - 10240*a*b^3*c^3*d^5 + 9600*a^2*b^2*c^2*d^6 - 4000*a^3*b*c*d^7 + 6
25*a^4*d^8)/b^9)^(1/4)*log((I*b^7*x*((4096*b^4*c^4*d^4 - 10240*a*b^3*c^3*d
^5 + 9600*a^2*b^2*c^2*d^6 - 4000*a^3*b*c*d^7 + 625*a^4*d^8)/b^9)^(3/4) - (
512*b^3*c^3*d^3 - 960*a*b^2*c^2*d^4 + 600*a^2*b*c*d^5 - 125*a^3*d^6)*(b*x^
4 + a)^(1/4))/x) - (I*a*b^3*x^4 + I*a^2*b^2)*((4096*b^4*c^4*d^4 - 10240*a*
b^3*c^3*d^5 + 9600*a^2*b^2*c^2*d^6 - 4000*a^3*b*c*d^7 + 625*a^4*d^8)/b^9)^
(1/4)*log((-I*b^7*x*((4096*b^4*c^4*d^4 - 10240*a*b^3*c^3*d^5 + 9600*a^2*b^
2*c^2*d^6 - 4000*a^3*b*c*d^7 + 625*a^4*d^8)/b^9)^(3/4) - (512*b^3*c^3*d^3
- 960*a*b^2*c^2*d^4 + 600*a^2*b*c*d^5 - 125*a^3*d^6)*(b*x^4 + a)^(1/4))/x)
 + 4*(a*b*d^2*x^5 + (4*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)*x)*(b*x^4 + a)^...
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(5/4),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(5/4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(106) = 212$.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \frac{1}{16}d^2\left(\frac{4\left(4\,ab-\frac{5\,(bx^4+a)a}{x^4}\right)}{\frac{(bx^4+a)^{\frac{1}{4}}b^3}{x}-\frac{(bx^4+a)^{\frac{5}{4}}b^2}{x^5}}+\frac{5\,a\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right) - \frac{1}{2}cd\left(\frac{\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{b}+\frac{4\,x}{(bx^4+a)^{\frac{1}{4}}b}\right)+\frac{c^2x}{(bx^4+a)^{\frac{1}{4}}a}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/16*d^2*(4*(4*a*b - 5*(b*x^4 + a)*a/x^4)/((b*x^4 + a)^(1/4)*b^3/x - (b*x^
4 + a)^(5/4)*b^2/x^5) + 5*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/
4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))
/b^(1/4))/b^2) - 1/2*c*d*((2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4)
 + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b
^(1/4))/b + 4*x/((b*x^4 + a)^(1/4)*b)) + c^2*x/((b*x^4 + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(5/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(5/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d**2 + 2*
int(x**4/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c*d + int
(1/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c**2
```

3.94 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \frac{(bc-ad)^2x}{5ab^2(a+bx^4)^{5/4}} + \frac{2(bc-ad)(2bc+3ad)x}{5a^2b^2\sqrt[4]{a+bx^4}} + \frac{d^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{9/4}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{9/4}}$$

output
```
1/5*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(5/4)+2/5*(-a*d+b*c)*(3*a*d+2*b*c)*x/a^
2/b^2/(b*x^4+a)^(1/4)+1/2*d^2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+1/
2*d^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \frac{\frac{2\sqrt[4]{b}(bc-ad)x(5abc+5a^2d+4b^2cx^4+6abdx^4)}{a^2(a+bx^4)^{5/4}} + 5d^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + 5d^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+b}}\right.}{10b^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(9/4),x]
```

output
```
((2*b^(1/4)*(b*c - a*d)*x*(5*a*b*c + 5*a^2*d + 4*b^2*c*x^4 + 6*a*b*d*x^4))
/(a^2*(a + b*x^4)^(5/4)) + 5*d^2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 5
*d^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(10*b^(9/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 910, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{5ad^2x^4+c(4bc+ad)}{(bx^4+a)^{5/4}}dx}{5ab}+\frac{x(c+dx^4)(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \ 910$$

$$\frac{\frac{5ad^2\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}+\frac{x(bc-ad)(5ad+4bc)}{ab\sqrt[4]{a+bx^4}}}{5ab}+\frac{x(c+dx^4)(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \ 770$$

$$\frac{\frac{5ad^2\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}+\frac{x(bc-ad)(5ad+4bc)}{ab\sqrt[4]{a+bx^4}}}{5ab}+\frac{x(c+dx^4)(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \ 756$$

$$\frac{\frac{5ad^2\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{b}+\frac{x(bc-ad)(5ad+4bc)}{ab\sqrt[4]{a+bx^4}}}{5ab}+$$
$$\frac{x(c+dx^4)(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\downarrow 216$$

$$\frac{\frac{5ad^2\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)(5ad+4bc)}{ab\sqrt[4]{a+bx^4}}}{5ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}$$

$$\downarrow 219$$

$$\frac{\frac{5ad^2\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)(5ad+4bc)}{ab\sqrt[4]{a+bx^4}}}{5ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(5*a*b*(a + b*x^4)^(5/4)) + (((b*c - a*d)*(4*b
*c + 5*a*d)*x)/(a*b*(a + b*x^4)^(1/4)) + (5*a*d^2*(ArcTan[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(
1/4))))/b)/(5*a*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{4ac\left(\frac{2d\,x^{4}}{5}+c\right)x\,b^{\frac{9}{4}}+\frac{16b^{\frac{13}{4}}c^{2}x^{5}}{5}+a^{2}d^{2}\left(-\frac{24b^{\frac{5}{4}}x^{5}}{5}-4xa\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{5}{4}}\left(\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right.\right.\right.}{4\left(b\,x^{4}+a\right)^{\frac{5}{4}}b^{\frac{9}{4}}a^{2}}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/4/(b*x^4+a)^(5/4)*(4*a*c*(2/5*d*x^4+c)*x*b^(9/4)+16/5*b^(13/4)*c^2*x^5+a
^2*d^2*(-24/5*b^(5/4)*x^5-4*x*a*b^(1/4)+(b*x^4+a)^(5/4)*(ln((-b^(1/4)*x-(b
*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)
^(1/4)))))/b^(9/4)/a^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.96

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \frac{5\,(a^2b^4x^8+2\,a^3b^3x^4+a^4b^2)\left(\frac{d^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{b^7x\left(\frac{d^8}{b^9}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}d^6}{x}\right)-5\,(a^2b^4x^8+2\,a^3b^3x^4 \cdots}{}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/20*(5*(a^2*b^4*x^8 + 2*a^3*b^3*x^4 + a^4*b^2)*(d^8/b^9)^(1/4)*log((b^7*x
*(d^8/b^9)^(3/4) + (b*x^4 + a)^(1/4)*d^6)/x) - 5*(a^2*b^4*x^8 + 2*a^3*b^3*
x^4 + a^4*b^2)*(d^8/b^9)^(1/4)*log(-(b^7*x*(d^8/b^9)^(3/4) - (b*x^4 + a)^(
1/4)*d^6)/x) - 5*(I*a^2*b^4*x^8 + 2*I*a^3*b^3*x^4 + I*a^4*b^2)*(d^8/b^9)^(
1/4)*log((I*b^7*x*(d^8/b^9)^(3/4) + (b*x^4 + a)^(1/4)*d^6)/x) - 5*(-I*a^2*
b^4*x^8 - 2*I*a^3*b^3*x^4 - I*a^4*b^2)*(d^8/b^9)^(1/4)*log((-I*b^7*x*(d^8/
b^9)^(3/4) + (b*x^4 + a)^(1/4)*d^6)/x) + 4*(2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2)*x^5 + 5*(a*b^2*c^2 - a^3*d^2)*x)*(b*x^4 + a)^(3/4))/(a^2*b^4*x^8
 + 2*a^3*b^3*x^4 + a^4*b^2)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x***4+c)**2/(b*x**4+a)**(9/4),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(9/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = -\frac{\left(b-\frac{5\,(bx^4+a)}{x^4}\right)c^2x^5}{5\,(bx^4+a)^{\frac{5}{4}}a^2}+\frac{2\,cdx^5}{5\,(bx^4+a)^{\frac{5}{4}}a} - \frac{1}{20}\left(\frac{4\left(b+\frac{5\,(bx^4+a)}{x^4}\right)x^5}{(bx^4+a)^{\frac{5}{4}}b^2}+\frac{5\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)d^2$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/5*(b - 5*(b*x^4 + a)/x^4)*c^2*x^5/((b*x^4 + a)^(5/4)*a^2) + 2/5*c*d*x^5
/((b*x^4 + a)^(5/4)*a) - 1/20*(4*(b + 5*(b*x^4 + a)/x^4)*x^5/((b*x^4 + a)^
(5/4)*b^2) + 5*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^
(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2
)*d^2
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(9/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(9/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d**2 + 2*int(x**4/((a + b*x**4)**(1/4)*a**2
+ 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c*d +
 int(1/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a + b
*x**4)**(1/4)*b**2*x**8),x)*c**2
```

3.95 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}} \, dx = \frac{(bc-ad)^2x}{9ab^2(a+bx^4)^{9/4}} + \frac{2(bc-ad)(4bc+5ad)x}{45a^2b^2(a+bx^4)^{5/4}} + \frac{(32b^2c^2+8abcd+5a^2d^2)\,x}{45a^3b^2\sqrt[4]{a+bx^4}}$$

output
```
1/9*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(9/4)+2/45*(-a*d+b*c)*(5*a*d+4*b*c)*x/a
^2/b^2/(b*x^4+a)^(5/4)+1/45*(5*a^2*d^2+8*a*b*c*d+32*b^2*c^2)*x/a^3/b^2/(b*
x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}} \, dx = \frac{32b^2c^2x^9+8abcx^5(9c+dx^4)+a^2(45c^2x+18cdx^5+5d^2x^9)}{45a^3(a+bx^4)^{9/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(13/4),x]
```

output
```
(32*b^2*c^2*x^9 + 8*a*b*c*x^5*(9*c + d*x^4) + a^2*(45*c^2*x + 18*c*d*x^5 +
 5*d^2*x^9))/(45*a^3*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}dx$$

$$\Big\downarrow \; 903$$

$$\frac{8c\int \frac{dx^4+c}{(bx^4+a)^{9/4}}dx}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}$$

$$\Big\downarrow \; 903$$

$$\frac{8c\left(\frac{4c\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}$$

$$\Big\downarrow \; 746$$

$$\frac{8c\left(\frac{4cx}{5a^2\sqrt[4]{a+bx^4}}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(13/4),x]
```

output
```
(x*(c + d*x^4)^2)/(9*a*(a + b*x^4)^(9/4)) + (8*c*((4*c*x)/(5*a^2*(a + b*x^
4)^(1/4)) + (x*(c + d*x^4))/(5*a*(a + b*x^4)^(5/4))))/(9*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{1}{9}d^{2}x^{8}+\frac{2}{5}c d\,x^{4}+c^{2}\right)a^{2}+\frac{8\left(\frac{d\,x^{4}}{9}+c\right)c b\,x^{4}a}{5}+\frac{32b^{2}c^{2}x^{8}}{45}\right)x}{\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	66
gosper	$\frac{x\left(5a^{2}d^{2}x^{8}+8abcd\,x^{8}+32b^{2}c^{2}x^{8}+18a^{2}cd\,x^{4}+72ab\,c^{2}x^{4}+45a^{2}c^{2}\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	76
trager	$\frac{x\left(5a^{2}d^{2}x^{8}+8abcd\,x^{8}+32b^{2}c^{2}x^{8}+18a^{2}cd\,x^{4}+72ab\,c^{2}x^{4}+45a^{2}c^{2}\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	76
orering	$\frac{x\left(5a^{2}d^{2}x^{8}+8abcd\,x^{8}+32b^{2}c^{2}x^{8}+18a^{2}cd\,x^{4}+72ab\,c^{2}x^{4}+45a^{2}c^{2}\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	76

input
```
int((d*x^4+c)^2/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
((1/9*d^2*x^8+2/5*c*d*x^4+c^2)*a^2+8/5*(1/9*d*x^4+c)*c*b*x^4*a+32/45*b^2*c
^2*x^8)/(b*x^4+a)^(9/4)*x/a^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = \frac{((32\,b^2c^2+8\,abcd+5\,a^2d^2)x^9+18\,(4\,abc^2+a^2cd)x^5+45\,a^2c^2x)(bx^4+a)^{\frac{3}{4}}}{45\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/45*((32*b^2*c^2 + 8*a*b*c*d + 5*a^2*d^2)*x^9 + 18*(4*a*b*c^2 + a^2*c*d)*
x^5 + 45*a^2*c^2*x)*(b*x^4 + a)^(3/4)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^
5*b*x^4 + a^6)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(13/4),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(13/4), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = -\frac{2\left(5\,b-\frac{9\,(bx^4+a)}{x^4}\right)cdx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^2} + \frac{d^2x^9}{9\,(bx^4+a)^{\frac{9}{4}}a} + \frac{\left(5\,b^2-\frac{18\,(bx^4+a)b}{x^4}+\frac{45\,(bx^4+a)^2}{x^8}\right)c^2x^9}{45\,(bx^4+a)^{\frac{9}{4}}a^3}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
-2/45*(5*b - 9*(b*x^4 + a)/x^4)*c*d*x^9/((b*x^4 + a)^(9/4)*a^2) + 1/9*d^2*
x^9/((b*x^4 + a)^(9/4)*a) + 1/45*(5*b^2 - 18*(b*x^4 + a)*b/x^4 + 45*(b*x^4
 + a)^2/x^8)*c^2*x^9/((b*x^4 + a)^(9/4)*a^3)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(13/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = \frac{5\,a^4\,d^2\,x+5\,a^2\,d^2\,x\,{\left(b\,x^4+a\right)}^2+32\,b^2\,c^2\,x\,{\left(b\,x^4+a\right)}^2+5\,a^2\,b^2\,c^2\,x-10\,a^3\,d^2\,x\,\left(b\,x^4\right.}{45\,a^3\,b^2\,(}$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(13/4),x)
```

output
```
(5*a^4*d^2*x + 5*a^2*d^2*x*(a + b*x^4)^2 + 32*b^2*c^2*x*(a + b*x^4)^2 + 5*
a^2*b^2*c^2*x - 10*a^3*d^2*x*(a + b*x^4) + 8*a*b^2*c^2*x*(a + b*x^4) - 10*
a^3*b*c*d*x + 8*a*b*c*d*x*(a + b*x^4)^2 + 2*a^2*b*c*d*x*(a + b*x^4))/(45*a
^3*b^2*(a + b*x^4)^(9/4))
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)cd$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(13/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d**2
 + 2*int(x**4/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**
4 + 3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)
*c*d + int(1/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4
 + 3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*
c**2
```

3.96 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \frac{(bc-ad)^2x}{13ab^2(a+bx^4)^{13/4}} + \frac{2(bc-ad)(6bc+7ad)x}{117a^2b^2(a+bx^4)^{9/4}} + \frac{(96b^2c^2+16abcd+5a^2d^2)\,x}{585a^3b^2(a+bx^4)^{5/4}} + \frac{4(96b^2c^2+16abcd+5a^2d^2)\,x}{585a^4b^2\sqrt[4]{a+bx^4}}$$

output
```
1/13*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(13/4)+2/117*(-a*d+b*c)*(7*a*d+6*b*c)*
x/a^2/b^2/(b*x^4+a)^(9/4)+1/585*(5*a^2*d^2+16*a*b*c*d+96*b^2*c^2)*x/a^3/b^
2/(b*x^4+a)^(5/4)+4/585*(5*a^2*d^2+16*a*b*c*d+96*b^2*c^2)*x/a^4/b^2/(b*x^4
+a)^(1/4)
```

Mathematica [A] (verified)

Time = 5.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \frac{384b^3c^2x^{13}+32ab^2cx^9(39c+2dx^4)+4a^2bx^5(351c^2+52cdx^4+5d^2x^8)+13a^3(45c^2x}{585a^4(a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(17/4),x]
```

output
```
(384*b^3*c^2*x^13 + 32*a*b^2*c*x^9*(39*c + 2*d*x^4) + 4*a^2*b*x^5*(351*c^2
 + 52*c*d*x^4 + 5*d^2*x^8) + 13*a^3*(45*c^2*x + 18*c*d*x^5 + 5*d^2*x^9))/(
585*a^4*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {907, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{(12bc-13ad)\int \frac{(dx^4+c)^2}{(bx^4+a)^{13/4}}dx}{13a(bc-ad)}+\frac{bx(c+dx^4)^3}{13a(a+bx^4)^{13/4}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(12bc-13ad)\left(\frac{8c\int \frac{dx^4+c}{(bx^4+a)^{9/4}}dx}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}\right)}{13a(bc-ad)}+\frac{bx(c+dx^4)^3}{13a(a+bx^4)^{13/4}(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{(12bc-13ad)\left(\frac{8c\left(\frac{4c\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}\right)}{13a(bc-ad)}+\frac{bx(c+dx^4)^3}{13a(a+bx^4)^{13/4}(bc-ad)}$$

$$\Big\downarrow \text{746}$$

$$\frac{(12bc-13ad)\left(\frac{8c\left(\frac{4cx}{5a^2\sqrt[4]{a+bx^4}}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x(c+dx^4)^2}{9a(a+bx^4)^{9/4}}\right)}{13a(bc-ad)}+\frac{bx(c+dx^4)^3}{13a\,(a+bx^4)^{13/4}\,(bc-ad)}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(17/4),x]
```

output
```
(b*x*(c + d*x^4)^3)/(13*a*(b*c - a*d)*(a + b*x^4)^(13/4)) + ((12*b*c - 13*
a*d)*((x*(c + d*x^4)^2)/(9*a*(a + b*x^4)^(9/4)) + (8*c*((4*c*x)/(5*a^2*(a
+ b*x^4)^(1/4)) + (x*(c + d*x^4))/(5*a*(a + b*x^4)^(5/4))))/(9*a)))/(13*a*
(b*c - a*d))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{9}d^{2}x^{8}+\frac{2}{5}cd\,x^{4}+c^{2}\right)a^{3}+\frac{12\left(\frac{5}{351}d^{2}x^{8}+\frac{4}{27}cd\,x^{4}+c^{2}\right)b\,x^{4}a^{2}}{5}+\frac{32\left(\frac{2d\,x^{4}}{39}+c\right)c\,b^{2}x^{8}a}{15}+\frac{128b^{3}c^{2}x^{12}}{195}\right)}{\left(b\,x^{4}+a\right)^{\frac{13}{4}}a^{4}}$
gosper	$\frac{x\left(20a^{2}b\,d^{2}x^{12}+64a\,b^{2}cd\,x^{12}+384b^{3}c^{2}x^{12}+65a^{3}d^{2}x^{8}+208a^{2}bcd\,x^{8}+1248a\,b^{2}c^{2}x^{8}+234a^{3}cd\,x^{4}+1404a^{2}b\,c^{2}x^{4}+585a^{3}c^{2}\right)}{585\left(b\,x^{4}+a\right)^{\frac{13}{4}}a^{4}}$
trager	$\frac{x\left(20a^{2}b\,d^{2}x^{12}+64a\,b^{2}cd\,x^{12}+384b^{3}c^{2}x^{12}+65a^{3}d^{2}x^{8}+208a^{2}bcd\,x^{8}+1248a\,b^{2}c^{2}x^{8}+234a^{3}cd\,x^{4}+1404a^{2}b\,c^{2}x^{4}+585a^{3}c^{2}\right)}{585\left(b\,x^{4}+a\right)^{\frac{13}{4}}a^{4}}$
orering	$\frac{x\left(20a^{2}b\,d^{2}x^{12}+64a\,b^{2}cd\,x^{12}+384b^{3}c^{2}x^{12}+65a^{3}d^{2}x^{8}+208a^{2}bcd\,x^{8}+1248a\,b^{2}c^{2}x^{8}+234a^{3}cd\,x^{4}+1404a^{2}b\,c^{2}x^{4}+585a^{3}c^{2}\right)}{585\left(b\,x^{4}+a\right)^{\frac{13}{4}}a^{4}}$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^4+a)^(13/4)*x*((1/9*d^2*x^8+2/5*c*d*x^4+c^2)*a^3+12/5*(5/351*d^2*x^
8+4/27*c*d*x^4+c^2)*b*x^4*a^2+32/15*(2/39*d*x^4+c)*c*b^2*x^8*a+128/195*b^3
*c^2*x^12)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^{17/4}}\,dx = \frac{\left(4\left(96\,b^3c^2+16\,ab^2cd+5\,a^2bd^2\right)x^{13}+13\left(96\,ab^2c^2+16\,a^2bcd+5\,a^3d^2\right)x^9+585\,a^3c^2 \cdots}{585\left(a^4b^4x^{16}+4\,a^5b^3x^{12}+6\,a^6b^2x^8+4\,a^7bx^4+a \cdots\right.}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/585*(4*(96*b^3*c^2 + 16*a*b^2*c*d + 5*a^2*b*d^2)*x^13 + 13*(96*a*b^2*c^2
 + 16*a^2*b*c*d + 5*a^3*d^2)*x^9 + 585*a^3*c^2*x + 234*(6*a^2*b*c^2 + a^3*
c*d)*x^5)*(b*x^4 + a)^(3/4)/(a^4*b^4*x^16 + 4*a^5*b^3*x^12 + 6*a^6*b^2*x^8
 + 4*a^7*b*x^4 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = -\frac{\left(9\,b-\frac{13\,(bx^4+a)}{x^4}\right)d^2x^{13}}{117\,(bx^4+a)^{\frac{13}{4}}a^2} + \frac{2\left(45\,b^2-\frac{130\,(bx^4+a)b}{x^4}+\frac{117\,(bx^4+a)^2}{x^8}\right)cdx^{13}}{585\,(bx^4+a)^{\frac{13}{4}}a^3} - \frac{\left(15\,b^3-\frac{65\,(bx^4+a)b^2}{x^4}+\frac{117\,(bx^4+a)^2b}{x^8}-\frac{195\,(bx^4+a)^3}{x^{12}}\right)c^2x^{13}}{195\,(bx^4+a)^{\frac{13}{4}}a^4}$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/117*(9*b - 13*(b*x^4 + a)/x^4)*d^2*x^13/((b*x^4 + a)^(13/4)*a^2) + 2/58
5*(45*b^2 - 130*(b*x^4 + a)*b/x^4 + 117*(b*x^4 + a)^2/x^8)*c*d*x^13/((b*x^
4 + a)^(13/4)*a^3) - 1/195*(15*b^3 - 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4 +
 a)^2*b/x^8 - 195*(b*x^4 + a)^3/x^12)*c^2*x^13/((b*x^4 + a)^(13/4)*a^4)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{(b\,x^4+a)^{13/4}} - \frac{x\left(\frac{d^2}{9\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{117\,a^2\,b^2}\right)}{(b\,x^4+a)^{9/4}} + \frac{x\,(5\,a^2\,d^2+16\,a\,b\,c\,d+96\,b^2\,c^2)}{585\,a^3\,b^2\,(b\,x^4+a)^{5/4}} + \frac{x\,(20\,a^2\,d^2+64\,a\,b\,c\,d+384\,b^2\,c^2)}{585\,a^4\,b^2\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(17/4),x)
```

output
```
(x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^4)^(13/4)
- (x*(d^2/(9*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(117*a^2*b^2)))/(a
+ b*x^4)^(9/4) + (x*(5*a^2*d^2 + 96*b^2*c^2 + 16*a*b*c*d))/(585*a^3*b^2*(a
 + b*x^4)^(5/4)) + (x*(20*a^2*d^2 + 384*b^2*c^2 + 64*a*b*c*d))/(585*a^4*b^
2*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1}\right.$$
$$+2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^1}\right.$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(17/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d**2 + 2*int(x**4/((a + b*x**4)**(1/4)
*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**
2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**
16),x)*c*d + int(1/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*
b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b*
*3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x)*c**2
```

3.97 $\int (a+bx^4)^{5/4} (c+dx^4)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 221

$$\int (a+bx^4)^{5/4} (c+dx^4)^2 \, dx = \frac{a(140b^2c^2-28abcd+5a^2d^2)\, x\sqrt[4]{a+bx^4}}{336b^2} + \frac{1}{840}\left(140c^2-\frac{ad(28bc-5ad)}{b^2}\right) x\left(a+bx^4\right)^{5/4} + \frac{d(28bc-5ad)x(a+bx^4)^{9/4}}{140b^2} + \frac{d^2x^5(a+bx^4)^{9/4}}{14b} - \frac{a^{3/2}(140b^2c^2-28abcd+5a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{336b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output

```
1/336*a*(5*a^2*d^2-28*a*b*c*d+140*b^2*c^2)*x*(b*x^4+a)^(1/4)/b^2+1/840*(14
0*c^2-a*d*(-5*a*d+28*b*c)/b^2)*x*(b*x^4+a)^(5/4)+1/140*d*(-5*a*d+28*b*c)*x
*(b*x^4+a)^(9/4)/b^2+1/14*d^2*x^5*(b*x^4+a)^(9/4)/b-1/336*a^(3/2)*(5*a^2*d
^2-28*a*b*c*d+140*b^2*c^2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcco
t(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.86 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.80

$$\int (a+bx^4)^{5/4}(c+dx^4)^2\,dx = \frac{x\sqrt[4]{a+bx^4}\left(13a(45c^2+18cdx^4+5d^2x^8)\operatorname{Gamma}\left(-\frac{5}{4}\right)\operatorname{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{13}{4},-\frac{bx^4}{a}\right)\right.}{}$$

input
```
Integrate[(a + b*x^4)^(5/4)*(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/4)*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Gamma[-5/4]*H
ypergeometric2F1[-5/4, 1/4, 13/4, -((b*x^4)/a)] - 8*b*x^4*(7*c^2 + 10*c*d*
x^4 + 3*d^2*x^8)*Gamma[-1/4]*Hypergeometric2F1[-1/4, 5/4, 17/4, -((b*x^4)/
a)] - 16*b*x^4*(c + d*x^4)^2*Gamma[-1/4]*HypergeometricPFQ[{-1/4, 5/4, 2},
 {1, 17/4}, -((b*x^4)/a)]))/(585*(1 + (b*x^4)/a)^(1/4)*Gamma[-5/4])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {933, 913, 748, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^{5/4}(c+dx^4)^2\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int (bx^4+a)^{5/4}(d(18bc-5ad)x^4+c(14bc-ad))\,dx}{14b}+\frac{dx(a+bx^4)^{9/4}(c+dx^4)}{14b}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{\left(5a^2d^2-28abcd+140b^2c^2\right)\int \left(bx^4+a\right)^{5/4}dx}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}{14b}$$

$\downarrow$ 748

$$\frac{\frac{\left(5a^2d^2-28abcd+140b^2c^2\right)\left(\frac{5}{6}a\int \sqrt[4]{bx^4+a}dx+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}{14b}$$

$\downarrow$ 748

$$\frac{\frac{\left(5a^2d^2-28abcd+140b^2c^2\right)\left(\frac{5}{6}a\left(\frac{1}{2}a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}{14b}$$

$\downarrow$ 768

$$\frac{\frac{\left(5a^2d^2-28abcd+140b^2c^2\right)\left(\frac{5}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2\left(a+bx^4\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}{14b}$$

$\downarrow$ 858

$$\frac{\frac{\left(5a^2d^2-28abcd+140b^2c^2\right)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2\left(a+bx^4\right)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}\right)}{10b}+\frac{dx\left(a+bx^4\right)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}{14b}$$

$\downarrow$ 807

$$\frac{\frac{(5a^2d^2-28abcd+140b^2c^2)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4(a+bx^4)^{3/4}}\right)+\frac{1}{6}x(a+bx^4)^{5/4}\right)}{10b}+\frac{dx(a+bx^4)^{9/4}(18bc-5ad)}{10b}}{14b}+\frac{dx(a+bx^4)^{9/4}(c+dx^4)}{14b}$$

$$\Big\downarrow \textbf{229}$$

$$\frac{\frac{(5a^2d^2-28abcd+140b^2c^2)\left(\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)+\frac{1}{6}x(a+bx^4)^{5/4}\right)}{10b}+\frac{dx(a+bx^4)^{9/4}(18}{10b}}{14b}$$
$$\frac{dx(a+bx^4)^{9/4}(c+dx^4)}{14b}$$

input
```
Int[(a + b*x^4)^(5/4)*(c + d*x^4)^2,x]
```

output
```
(d*x*(a + b*x^4)^(9/4)*(c + d*x^4))/(14*b) + ((d*(18*b*c - 5*a*d)*x*(a + b
*x^4)^(9/4))/(10*b) + ((140*b^2*c^2 - 28*a*b*c*d + 5*a^2*d^2)*((x*(a + b*x
^4)^(5/4))/6 + (5*a*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*
x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x
^4)^(3/4))))/6))/(10*b))/(14*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(d\,x^{4}+c\right)^{2}d x$$

input
```
int((b*x^4+a)^(5/4)*(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(5/4)*(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^{5/4} \left(c+dx^4\right)^2 \, dx = \int \left(bx^4+a\right)^{\frac{5}{4}} \left(dx^4+c\right)^2 dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*c^2 + 2*a*c*d)*x^4 + a*c
^2)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.21

$$\int \left(a+bx^4\right)^{5/4} \left(c+dx^4\right)^2 \, dx = \frac{a^{\frac{5}{4}} c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{5}{4}} c d x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{a^{\frac{5}{4}} d^2 x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{\sqrt[4]{a} b c^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt[4]{a} b c d x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{\sqrt[4]{a} b d^2 x^{13} \Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)*(d*x**4+c)**2,x)
```

output
```
a**(5/4)*c**2*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**(5/4)*c*d*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**(5/4)*d**2*x**9*gamma(9
/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
+ a**(1/4)*b*c**2*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(9/4)) + a**(1/4)*b*c*d*x**9*gamma(9/4)*hyper((-1/4,
9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(2*gamma(13/4)) + a**(1/4)*b*d**2
*x**13*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(17/4))
```

Maxima [F]

$$\int (a+bx^4)^{5/4} (c+dx^4)^2 \, dx = \int (bx^4+a)^{\frac{5}{4}} (dx^4+c)^2 \, dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*(d*x^4 + c)^2, x)
```

Giac [F]

$$\int (a+bx^4)^{5/4} (c+dx^4)^2 \, dx = \int (bx^4+a)^{\frac{5}{4}} (dx^4+c)^2 \, dx$$

input
```
integrate((b*x^4+a)^(5/4)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^{5/4} \left(c+dx^4\right)^2 \, dx = \int \left(b\,x^4+a\right)^{5/4}\,\left(d\,x^4+c\right)^2 \,dx$$

input
```
int((a + b*x^4)^(5/4)*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(5/4)*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^{5/4} \left(c+dx^4\right)^2 \, dx = \frac{-25(b\,x^4+a)^{\frac{1}{4}}\,a^3d^2x+140(b\,x^4+a)^{\frac{1}{4}}\,a^2bcdx+10(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,d^2x^5+980(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2c}{}$$

input
```
int((b*x^4+a)^(5/4)*(d*x^4+c)^2,x)
```

output
```
( - 25*(a + b*x**4)**(1/4)*a**3*d**2*x + 140*(a + b*x**4)**(1/4)*a**2*b*c*
d*x + 10*(a + b*x**4)**(1/4)*a**2*b*d**2*x**5 + 980*(a + b*x**4)**(1/4)*a*
b**2*c**2*x + 616*(a + b*x**4)**(1/4)*a*b**2*c*d*x**5 + 180*(a + b*x**4)**
(1/4)*a*b**2*d**2*x**9 + 280*(a + b*x**4)**(1/4)*b**3*c**2*x**5 + 336*(a +
 b*x**4)**(1/4)*b**3*c*d*x**9 + 120*(a + b*x**4)**(1/4)*b**3*d**2*x**13 +
25*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**4*d**2 - 140*int((a + b*x**4
)**(1/4)/(a + b*x**4),x)*a**3*b*c*d + 700*int((a + b*x**4)**(1/4)/(a + b*x
**4),x)*a**2*b**2*c**2)/(1680*b**2)
```

3.98 $\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 180

$$\begin{aligned}\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx &= \frac{\left(12b^2c^2-4abcd+a^2d^2\right)x\sqrt[4]{a+bx^4}}{24b^2}+\frac{d(4bc-ad)x\left(a+bx^4\right)^{5/4}}{12b^2}+\frac{d^2x^5\left(a+bx^4\right)^{5/4}}{10b}\\ &\quad -\frac{\sqrt{a}\left(12b^2c^2-4abcd+a^2d^2\right)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24b^{3/2}\left(a+bx^4\right)^{3/4}}\end{aligned}$$

output

```
1/24*(a^2*d^2-4*a*b*c*d+12*b^2*c^2)*x*(b*x^4+a)^(1/4)/b^2+1/12*d*(-a*d+4*b
*c)*x*(b*x^4+a)^(5/4)/b^2+1/10*d^2*x^5*(b*x^4+a)^(5/4)/b-1/24*a^(1/2)*(a^2
*d^2-4*a*b*c*d+12*b^2*c^2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcco
t(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.80 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$=\frac{x\sqrt[4]{a+bx^4}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Gamma}\left(-\frac{1}{4}\right)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{13}{4},-\frac{bx^4}{a}\right)-8bx^4(7c\ldots}{585\ldots}$$

input
```
Integrate[(a + b*x^4)^(1/4)*(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/4)*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Gamma[-1/4]*H
ypergeometric2F1[-1/4, 1/4, 13/4, -((b*x^4)/a)] - 8*b*x^4*(7*c^2 + 10*c*d*
x^4 + 3*d^2*x^8)*Gamma[3/4]*Hypergeometric2F1[3/4, 5/4, 17/4, -((b*x^4)/a)
] - 16*b*x^4*(c + d*x^4)^2*Gamma[3/4]*HypergeometricPFQ[{3/4, 5/4, 2}, {1,
 17/4}, -((b*x^4)/a)]))/(585*a*(1 + (b*x^4)/a)^(1/4)*Gamma[-1/4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {933, 913, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int \sqrt[4]{bx^4+a}\left(d(14bc-5ad)x^4+c(10bc-ad)\right)dx}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$
$$\Big\downarrow \text{913}$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\int \sqrt[4]{bx^4+a}dx}{6b}+\frac{dx(a+bx^4)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

$$\Big\downarrow \; 748$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\left(\frac{1}{2}a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{6b}+\frac{dx(a+bx^4)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

$$\Big\downarrow \; 768$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2\left(a+bx^4\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{6b}+\frac{dx(a+bx^4)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

$$\Big\downarrow \; 858$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2\left(a+bx^4\right)^{3/4}}\right)}{6b}+\frac{dx(a+bx^4)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

$$\Big\downarrow \; 807$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4\left(a+bx^4\right)^{3/4}}\right)}{6b}+\frac{dx(a+bx^4)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

$$\Big\downarrow \; 229$$

$$\frac{\frac{5(a^2d^2-4abcd+12b^2c^2)\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\left(a+bx^4\right)^{3/4}}\right)}{6b}+\frac{dx\left(a+bx^4\right)^{5/4}(14bc-5ad)}{6b}}{10b}+\frac{dx\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}{10b}$$

input
```
Int[(a + b*x^4)^(1/4)*(c + d*x^4)^2,x]
```

output
```
(d*x*(a + b*x^4)^(5/4)*(c + d*x^4))/(10*b) + ((d*(14*b*c - 5*a*d)*x*(a + b
*x^4)^(5/4))/(6*b) + (5*(12*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*((x*(a + b*x^4)
^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sq
rt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))))/(6*b))/(10*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(d\,x^{4}+c\right)^{2}d x$$

input
```
int((b*x^4+a)^(1/4)*(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(1/4)*(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \left(bx^{4}+a\right)^{\frac{1}{4}}\left(dx^{4}+c\right)^{2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.72

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx = \frac{\sqrt[4]{a}c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt[4]{a}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{\sqrt[4]{a}d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)*(d*x**4+c)**2,x)
```

output
```
a**(1/4)*c**2*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**(1/4)*c*d*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**(1/4)*d**2*x**9*gamma(9
/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2\,dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2\,dx$$

input
```
integrate((b*x^4+a)^(1/4)*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx=\int \left(b\,x^4+a\right)^{1/4}\,\left(d\,x^4+c\right)^2\,dx$$

input
```
int((a + b*x^4)^(1/4)*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/4)*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^4}\left(c+dx^4\right)^2\,dx$$

$$=\frac{-5(b\,x^4+a)^{\frac{1}{4}}\,a^2d^2x+20(b\,x^4+a)^{\frac{1}{4}}\,abcdx+2(b\,x^4+a)^{\frac{1}{4}}\,ab\,d^2x^5+60(b\,x^4+a)^{\frac{1}{4}}\,b^2c^2x+40(b\,x^4+a)^{\frac{1}{4}}}{}$$

input
```
int((b*x^4+a)^(1/4)*(d*x^4+c)^2,x)
```

output

```
( - 5*(a + b*x**4)**(1/4)*a**2*d**2*x + 20*(a + b*x**4)**(1/4)*a*b*c*d*x +
 2*(a + b*x**4)**(1/4)*a*b*d**2*x**5 + 60*(a + b*x**4)**(1/4)*b**2*c**2*x
+ 40*(a + b*x**4)**(1/4)*b**2*c*d*x**5 + 12*(a + b*x**4)**(1/4)*b**2*d**2*
x**9 + 5*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**3*d**2 - 20*int((a + b
*x**4)**(1/4)/(a + b*x**4),x)*a**2*b*c*d + 60*int((a + b*x**4)**(1/4)/(a +
 b*x**4),x)*a*b**2*c**2)/(120*b**2)
```

3.99 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \frac{d(12bc-5ad)x\sqrt[4]{a+bx^4}}{12b^2} + \frac{d^2x^5\sqrt[4]{a+bx^4}}{6b} - \frac{(12b^2c^2-12abcd+5a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{a}b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/12*d*(-5*a*d+12*b*c)*x*(b*x^4+a)^(1/4)/b^2+1/6*d^2*x^5*(b*x^4+a)^(1/4)/b
-1/12*(5*a^2*d^2-12*a*b*c*d+12*b^2*c^2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacob
iAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(3/2)/(b*x^4+a)^(3/
4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \frac{x\left(1+\frac{bx^4}{a}\right)^{3/4}\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{13}{4},-\frac{bx^4}{a}\right)-6\right.}{}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(3/4),x]
```

output
```
(x*(1 + (b*x^4)/a)^(3/4)*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8)*Hypergeom
etric2F1[1/4, 3/4, 13/4, -((b*x^4)/a)] - 6*b*x^4*(7*c^2 + 10*c*d*x^4 + 3*d
^2*x^8)*Hypergeometric2F1[5/4, 7/4, 17/4, -((b*x^4)/a)] - 12*b*x^4*(c + d*
x^4)^2*HypergeometricPFQ[{5/4, 7/4, 2}, {1, 17/4}, -((b*x^4)/a)]))/(585*a*
(a + b*x^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {933, 913, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int \frac{5d(2bc-ad)x^4+c(6bc-ad)}{\left(bx^4+a\right)^{3/4}}dx}{6b}+\frac{dx\sqrt[4]{a+bx^4}\left(c+dx^4\right)}{6b}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{\left(5a^2d^2-12abcd+12b^2c^2\right)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2b}+\frac{5dx\sqrt[4]{a+bx^4}(2bc-ad)}{2b}}{6b}+\frac{dx\sqrt[4]{a+bx^4}\left(c+dx^4\right)}{6b}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(5a^2d^2-12abcd+12b^2c^2\right)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}+\frac{5dx\sqrt[4]{a+bx^4}(2bc-ad)}{2b}}{6b}+\frac{dx\sqrt[4]{a+bx^4}\left(c+dx^4\right)}{6b}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{\frac{5dx\sqrt[4]{a+bx^4}(2bc-ad)}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(5a^2d^2-12abcd+12b^2c^2)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}}{6b}+\frac{dx\sqrt[4]{a+bx^4}(c+dx^4)}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{5dx\sqrt[4]{a+bx^4}(2bc-ad)}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(5a^2d^2-12abcd+12b^2c^2)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}}{6b}+\frac{dx\sqrt[4]{a+bx^4}(c+dx^4)}{6b}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{5dx\sqrt[4]{a+bx^4}(2bc-ad)}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(5a^2d^2-12abcd+12b^2c^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{a}\sqrt{b}(a+bx^4)^{3/4}}}{6b}+\frac{dx\sqrt[4]{a+bx^4}(c+dx^4)}{6b}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(3/4),x]
```

output
```
(d*x*(a + b*x^4)^(1/4)*(c + d*x^4))/(6*b) + ((5*d*(2*b*c - a*d)*x*(a + b*x
^4)^(1/4))/(2*b) - ((12*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[a]*Sqrt[b
]*(a + b*x^4)^(3/4)))/(6*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \frac{c^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)} + \frac{cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{d^2x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(3/4),x)
```

output
```
c**2*x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/4)*gamma(5/4)) + c*d*x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4
*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(9/4)) + d**2*x**9*gamma(9/4)*hyper((
3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(3/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(3/4),x)
```

output
```
int(x**8/(a + b*x**4)**(3/4),x)*d**2 + 2*int(x**4/(a + b*x**4)**(3/4),x)*c
*d + int(1/(a + b*x**4)**(3/4),x)*c**2
```

3.100 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \frac{(bc-ad)^2x}{3ab^2(a+bx^4)^{3/4}} + \frac{d^2x\sqrt[4]{a+bx^4}}{2b^2} - \frac{(4b^2c^2+4abcd-5a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{6a^{3/2}b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(3/4)+1/2*d^2*x*(b*x^4+a)^(1/4)/b^2-1/6
*(-5*a^2*d^2+4*a*b*c*d+4*b^2*c^2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/
2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 13.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \frac{x\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Gamma}\left(\frac{3}{4}\right)\left(13a(45c^2+18cdx^4+5d^2x^8)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{1}{4}\right.\right.}{}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(7/4),x]
```

output
```
(x*(1 + (b*x^4)/a)^(3/4)*Gamma[3/4]*(13*a*(45*c^2 + 18*c*d*x^4 + 5*d^2*x^8
)*Hypergeometric2F1[1/4, 7/4, 13/4, -((b*x^4)/a)] - 14*b*x^4*(7*c^2 + 10*c
*d*x^4 + 3*d^2*x^8)*Hypergeometric2F1[5/4, 11/4, 17/4, -((b*x^4)/a)] - 28*
b*x^4*(c + d*x^4)^2*HypergeometricPFQ[{5/4, 2, 11/4}, {1, 17/4}, -((b*x^4)
/a)]))/(780*a^2*(a + b*x^4)^(3/4)*Gamma[7/4])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 913, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{c(2bc+ad)-d(2bc-5ad)x^4}{(bx^4+a)^{3/4}}dx}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{(-5a^2d^2+4abcd+4b^2c^2)\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}-\frac{dx\sqrt[4]{a+bx^4}(2bc-5ad)}{2b}}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(-5a^2d^2+4abcd+4b^2c^2)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}-\frac{dx\sqrt[4]{a+bx^4}(2bc-5ad)}{2b}}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(-5a^2d^2+4abcd+4b^2c^2)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}-\frac{dx\sqrt[4]{a+bx^4}(2bc-5ad)}{2b}}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$\Big\downarrow$ 807

$$\frac{-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(-5a^2d^2+4abcd+4b^2c^2)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}-\frac{dx\sqrt[4]{a+bx^4}(2bc-5ad)}{2b}}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$\Big\downarrow$ 229

$$\frac{-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(-5a^2d^2+4abcd+4b^2c^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{a}\sqrt{b}(a+bx^4)^{3/4}}-\frac{dx\sqrt[4]{a+bx^4}(2bc-5ad)}{2b}}{3ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(3*a*b*(a + b*x^4)^(3/4)) + (-1/2*(d*(2*b*c -
5*a*d)*x*(a + b*x^4)^(1/4))/b - ((4*b^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*(1 +
a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqr
t[a]*Sqrt[b]*(a + b*x^4)^(3/4)))/(3*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^
4 + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(7/4),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(7/4), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(7/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c^2$$

input

```
int((d*x^4+c)^2/(b*x^4+a)^(7/4),x)
```

output

```
int(x**8/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d**2 + 2*
int(x**4/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c*d + int
(1/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c**2
```

3.101 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \frac{(bc-ad)^2x}{7ab^2(a+bx^4)^{7/4}} + \frac{2(bc-ad)(3bc+4ad)x}{21a^2b^2(a+bx^4)^{3/4}} - \frac{(12b^2c^2+4abcd+5a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/7*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(7/4)+2/21*(-a*d+b*c)*(4*a*d+3*b*c)*x/a
^2/b^2/(b*x^4+a)^(3/4)-1/21*(5*a^2*d^2+4*a*b*c*d+12*b^2*c^2)*(1+a/b/x^4)^(
3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/
b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \frac{x\Big(3a(bc-ad)^2+2(3b^2c^2+abcd-4a^2d^2)(a+bx^4)+(12b^2c^2+4abcd+5a^2d^2)(a-}{21a^2b^2(a+bx^4)^{7/4}}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(11/4),x]
```

output
```
(x*(3*a*(b*c - a*d)^2 + 2*(3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*(a + b*x^4) +
(12*b^2*c^2 + 4*a*b*c*d + 5*a^2*d^2)*(a + b*x^4)*(1 + (b*x^4)/a)^(3/4)*Hyp
ergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)]))/(21*a^2*b^2*(a + b*x^4)^(7/4
))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {930, 910, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(2bc+5ad)x^4+c(6bc+ad)}{(bx^4+a)^{7/4}}dx}{7ab}+\frac{x(c+dx^4)(bc-ad)}{7ab(a+bx^4)^{7/4}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{1}{3}\left(\frac{12bc^2}{a}+\frac{5ad^2}{b}+4cd\right)\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{x\left(\frac{6bc^2}{a}-\frac{5ad^2}{b}-cd\right)}{3(a+bx^4)^{3/4}}}{7ab}+\frac{x(c+dx^4)(bc-ad)}{7ab(a+bx^4)^{7/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(\frac{12bc^2}{a}+\frac{5ad^2}{b}+4cd\right)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3(a+bx^4)^{3/4}}+\frac{x\left(\frac{6bc^2}{a}-\frac{5ad^2}{b}-cd\right)}{3(a+bx^4)^{3/4}}}{7ab}+\frac{x(c+dx^4)(bc-ad)}{7ab(a+bx^4)^{7/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{x\left(\frac{6bc^2}{a}-\frac{5ad^2}{b}-cd\right)}{3(a+bx^4)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(\frac{12bc^2}{a}+\frac{5ad^2}{b}+4cd\right)\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3(a+bx^4)^{3/4}}}{7ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

$\Big\downarrow$ 807

$$\frac{\frac{x\left(\frac{6bc^2}{a}-\frac{5ad^2}{b}-cd\right)}{3(a+bx^4)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(\frac{12bc^2}{a}+\frac{5ad^2}{b}+4cd\right)\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6(a+bx^4)^{3/4}}}{7ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

$\Big\downarrow$ 229

$$\frac{\frac{x\left(\frac{6bc^2}{a}-\frac{5ad^2}{b}-cd\right)}{3(a+bx^4)^{3/4}}-\frac{\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(\frac{12bc^2}{a}+\frac{5ad^2}{b}+4cd\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3\sqrt{a}(a+bx^4)^{3/4}}}{7ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{7ab\left(a+bx^4\right)^{7/4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(11/4),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(7*a*b*(a + b*x^4)^(7/4)) + ((((6*b*c^2)/a - c
*d - (5*a*d^2)/b)*x)/(3*(a + b*x^4)^(3/4)) - (Sqrt[b]*((12*b*c^2)/a + 4*c*
d + (5*a*d^2)/b)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[
b]*x^2)]/2, 2])/(3*Sqrt[a]*(a + b*x^4)^(3/4)))/(7*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(d\,x^{4}+c)^{2}}{(b\,x^{4}+a)^{\frac{11}{4}}}dx$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(11/4),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(11/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(11/4),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/4)/(b^3*x^12 + 3*a*b^2
*x^8 + 3*a^2*b*x^4 + a^3), x)
```

Sympy [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \int \frac{(c+dx^4)^2}{(a+bx^4)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(11/4),x)
```

output
```
Integral((c + d*x**4)**2/(a + b*x**4)**(11/4), x)
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(11/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(11/4), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{11}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(11/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(11/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{11/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(11/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(11/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{11/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b^2x^8}dx\right)d^2 + 2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b^2x^8}dx\right)cd + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b^2x^8}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(11/4),x)
```

output
```
int(x**8/((a + b*x**4)**(3/4)*a**2 + 2*(a + b*x**4)**(3/4)*a*b*x**4 + (a +
 b*x**4)**(3/4)*b**2*x**8),x)*d**2 + 2*int(x**4/((a + b*x**4)**(3/4)*a**2
+ 2*(a + b*x**4)**(3/4)*a*b*x**4 + (a + b*x**4)**(3/4)*b**2*x**8),x)*c*d +
 int(1/((a + b*x**4)**(3/4)*a**2 + 2*(a + b*x**4)**(3/4)*a*b*x**4 + (a + b
*x**4)**(3/4)*b**2*x**8),x)*c**2
```

3.102 $\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\begin{aligned}\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx &= \frac{(bc-ad)^2x}{11ab^2\,(a+bx^4)^{11/4}} \\ &+ \frac{2(bc-ad)(5bc+6ad)x}{77a^2b^2\,(a+bx^4)^{7/4}} + \frac{(60b^2c^2+12abcd+5a^2d^2)\,x}{231a^3b^2\,(a+bx^4)^{3/4}} \\ &- \frac{2(60b^2c^2+12abcd+5a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231a^{7/2}b^{3/2}\,(a+bx^4)^{3/4}}\end{aligned}$$

output
```
1/11*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)^(11/4)+2/77*(-a*d+b*c)*(6*a*d+5*b*c)*x
/a^2/b^2/(b*x^4+a)^(7/4)+1/231*(5*a^2*d^2+12*a*b*c*d+60*b^2*c^2)*x/a^3/b^2
/(b*x^4+a)^(3/4)-2/231*(5*a^2*d^2+12*a*b*c*d+60*b^2*c^2)*(1+a/b/x^4)^(3/4)
*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(7/2)/b^(3
/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \frac{x\Big(21a^2(bc-ad)^2+6a(5b^2c^2+abcd-6a^2d^2)\,(a+bx^4)+(60b^2c^2+12abcd+5a^2d^2}{2}$$

input
```
Integrate[(c + d*x^4)^2/(a + b*x^4)^(15/4),x]
```

output
```
(x*(21*a^2*(b*c - a*d)^2 + 6*a*(5*b^2*c^2 + a*b*c*d - 6*a^2*d^2)*(a + b*x^
4) + (60*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*(a + b*x^4)^2 + 2*(60*b^2*c^2 +
 12*a*b*c*d + 5*a^2*d^2)*(a + b*x^4)^2*(1 + (b*x^4)/a)^(3/4)*Hypergeometri
c2F1[1/4, 3/4, 5/4, -((b*x^4)/a)]))/(231*a^3*b^2*(a + b*x^4)^(11/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {930, 910, 749, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{d(6bc+5ad)x^4+c(10bc+ad)}{(bx^4+a)^{11/4}}dx}{11ab}+\frac{x(c+dx^4)\,(bc-ad)}{11ab\,(a+bx^4)^{11/4}}$$

$$\Big\downarrow \text{910}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\int \frac{1}{(bx^4+a)^{7/4}}dx+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x(c+dx^4)\,(bc-ad)}{11ab\,(a+bx^4)^{11/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\left(\frac{2\int\frac{1}{(bx^4+a)^{3/4}}dx}{3a}+\frac{x}{3a(a+bx^4)^{3/4}}\right)+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\left(\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a(a+bx^4)^{3/4}}+\frac{x}{3a(a+bx^4)^{3/4}}\right)+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a(a+bx^4)^{3/4}}\right)+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}\right)+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{1}{7}\left(\frac{60bc^2}{a}+\frac{5ad^2}{b}+12cd\right)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}\right)+\frac{5x(bc-ad)(ad+2bc)}{7ab(a+bx^4)^{7/4}}}{11ab}+\frac{x\left(c+dx^4\right)(bc-ad)}{11ab\left(a+bx^4\right)^{11/4}}$$

input
```
Int[(c + d*x^4)^2/(a + b*x^4)^(15/4),x]
```

output
```
((b*c - a*d)*x*(c + d*x^4))/(11*a*b*(a + b*x^4)^(11/4)) + ((5*(b*c - a*d)*
(2*b*c + a*d)*x)/(7*a*b*(a + b*x^4)^(7/4)) + (((60*b*c^2)/a + 12*c*d + (5*
a*d^2)/b)*(x/(3*a*(a + b*x^4)^(3/4)) - (2*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^
3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3
/4))))/7)/(11*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{\left(d\,x^{4}+c\right)^{2}}{\left(b\,x^{4}+a\right)^{\frac{15}{4}}}d x$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(15/4),x)
```

output
```
int((d*x^4+c)^2/(b*x^4+a)^(15/4),x)
```

Fricas [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(15/4),x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^(1/4)/(b^4*x^16 + 4*a*b^3
*x^12 + 6*a^2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**2/(b*x**4+a)**(15/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(15/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(15/4), x)
```

Giac [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \int \frac{(dx^4+c)^2}{(bx^4+a)^{\frac{15}{4}}}\,dx$$

input
```
integrate((d*x^4+c)^2/(b*x^4+a)^(15/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2/(b*x^4 + a)^(15/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \int \frac{(d\,x^4+c)^2}{(b\,x^4+a)^{15/4}}\,dx$$

input
```
int((c + d*x^4)^2/(a + b*x^4)^(15/4),x)
```

output
```
int((c + d*x^4)^2/(a + b*x^4)^(15/4), x)
```

Reduce [F]

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^{15/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}dx\right.$$
$$+2\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}dx\right)cd$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{3}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{12}}dx\right)c^2$$

input
```
int((d*x^4+c)^2/(b*x^4+a)^(15/4),x)
```

output
```
int(x**8/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3*x**12),x)*d**2
 + 2*int(x**4/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*a**2*b*x**
4 + 3*(a + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3*x**12),x)
*c*d + int(1/((a + b*x**4)**(3/4)*a**3 + 3*(a + b*x**4)**(3/4)*a**2*b*x**4
 + 3*(a + b*x**4)**(3/4)*a*b**2*x**8 + (a + b*x**4)**(3/4)*b**3*x**12),x)*
c**2
```

3.103 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{bx(a+bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}$$

output
```
1/4*b*x*(b*x^4+a)^(3/4)/d-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x
^4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/d^2-1/8*b^(3/4)*(-7*a*d+4*b*c)*arctanh(b^(1/4)*x/(b
*x^4+a)^(1/4))/d^2+1/2*(-a*d+b*c)^(7/4)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)
/(b*x^4+a)^(1/4))/c^(3/4)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{2bdx(a+bx^4)^{3/4} - b^{3/4}(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{(2+2i)(bc-ad)^{7/4}\arctan\left(\frac{(1-i)\sqrt[4]{\cdots}}{\sqrt[4]{c}\sqrt[4]{\cdots}}\cdots\right)}{\cdots}\cdots}{\cdots}$$

input
```
Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

output
```
(2*b*d*x*(a + b*x^4)^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTan[(((1 - I)*(b*c - a*
d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(
1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4) - b^(3/4)*(4*b*c - 7*a*d)*ArcTanh
[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((2 + 2*I)*(b*c - a*d)^(7/4)*ArcTanh[(((
1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/
4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(3/4))/(8*d^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {933, 25, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{b(4bc-7ad)x^4+a(bc-4ad)}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{4d} + \frac{bx(a+bx^4)^{3/4}}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\int \frac{b(4bc-7ad)x^4+a(bc-4ad)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4d}$$

$$\Big\downarrow \text{1026}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \text{770}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \text{756}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \text{216}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \text{219}$$

$$\frac{bx(a+bx^4)^{3/4}}{4d}-\frac{\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4d}$$

$$\Big\downarrow \text{902}$$

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}}{4d}$$

$\downarrow$ 756

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{d}}{4d}$$

$\downarrow$ 218

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}}{4d}$$

$\downarrow$ 221

$$\frac{\frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{b(4bc-7ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{4(bc-ad)^2\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}}{4d}$$

input
```
Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

output
```
(b*x*(a + b*x^4)^(3/4))/(4*d) - ((b*(4*b*c - 7*a*d)*(ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b
^(1/4))))/d - (4*(b*c - a*d)^2*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/d)/(4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(167) = 334.

Time = 10.43 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.90

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{2}\,(ad-bc)^{2} \ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)}{2}+\sqrt{2}\,(ad-bc)^{2} \arctan\left(\frac{\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}\right)}{}$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/((a*d-b*c)/c)^(1/4)*(1/2*2^(1/2)*(a*d-b*c)^2*ln((-((a*d-b*c)/c)^(1/4)
*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d
-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)
^(1/2)))+2^(1/2)*(a*d-b*c)^2*arctan((2^(1/2)*(b*x^4+a)^(1/4)-((a*d-b*c)/c)
^(1/4)*x)/((a*d-b*c)/c)^(1/4)/x)+2^(1/2)*(a*d-b*c)^2*arctan((2^(1/2)*(b*x^
4+a)^(1/4)+((a*d-b*c)/c)^(1/4)*x)/((a*d-b*c)/c)^(1/4)/x)+7/2*c*((-1/2*b^(3
/4)*a*d+2/7*b^(7/4)*c)*ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a
)^(1/4)))+(b^(3/4)*a*d-4/7*b^(7/4)*c)*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-
2/7*(b*x^4+a)^(3/4)*x*b*d)*((a*d-b*c)/c)^(1/4))/d^2/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 1962, normalized size of antiderivative = 9.30

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(3/4)*b*x + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 +
7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*
b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^
5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7
*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^
4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*
c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*
d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) + 4*I*d*((b^7*c^7 - 7*a*b^6*c^6*d + 2
1*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c
^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*log((I*c^2*d^6*x*((b^7*
c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4)
- (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a
^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(1/4))/x) - 4*I*d*((b^7*c^7 - 7*a*b^6*c^
6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21...
```

Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx=\int \frac{(a+bx^4)^{\frac{7}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{7/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(7/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx = \frac{(b x^4+a)^{\frac{3}{4}} bx + 4\left(\int \frac{(b x^4+a)^{\frac{3}{4}}}{bd x^8+ad x^4+bc x^4+ac}dx\right) a^2d - \left(\int \frac{(b x^4+a)^{\frac{3}{4}}}{bd x^8+ad x^4+bc x^4+ac}dx\right) abc + 7\left(\right.}{4d}$$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

output
```
((a + b*x**4)**(3/4)*b*x + 4*int((a + b*x**4)**(3/4)/(a*c + a*d*x**4 + b*c
*x**4 + b*d*x**8),x)*a**2*d - int((a + b*x**4)**(3/4)/(a*c + a*d*x**4 + b*
c*x**4 + b*d*x**8),x)*a*b*c + 7*int(((a + b*x**4)**(3/4)*x**4)/(a*c + a*d*
x**4 + b*c*x**4 + b*d*x**8),x)*a*b*d - 4*int(((a + b*x**4)**(3/4)*x**4)/(a
*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)*b**2*c)/(4*d)
```

3.104 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \frac{b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

output

```
1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arcta
n((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d+1/2*b^(3/4)*arctan
h(b^(1/4)*x/(b*x^4+a)^(1/4))/d-1/2*(-a*d+b*c)^(3/4)*arctanh((-a*d+b*c)^(1/
4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = -\frac{-2b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\frac{(1+i)\left((-1+i)b^{3/4}c^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+(bc-ad)^{3/4}\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\cdots}{\cdots}}{2x}\right)\cdots\right)}{c^{3/4}\cdots}}{4d}$$

input
```
Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

output
```
-1/4*(-2*b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((1 + I)*((-1 +
I)*b^(3/4)*c^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (b*c - a*d)^(3/
4)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - (
(1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (b*c - a*d)
^(3/4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)
) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/c^(3/4
))/d
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {916, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \; 916$$

$$\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \; 770$$

$$\frac{b\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \; 756$$

$$\frac{b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}$$

$$\Big\downarrow \; 902$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}$$

$$\Big\downarrow \; 756$$

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-adx^2}}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-adx^2}}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{d}$$

$\Big\downarrow$ 218

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-adx^2}}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}$$

$\Big\downarrow$ 221

$$\frac{b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}$$

input
```
Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

output
```
(b*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x
)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/d - ((b*c - a*d)*(ArcTan[((b*c - a*d)^(
1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTa
nh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*
d)^(1/4))))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 916
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[b/d   Int[(a + b*x^n)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*
x^n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c -
a*d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(133) = 266.

Time = 4.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.79

method	result
pseudoelliptic	$\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} c \left(2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)\right) b^{\frac{3}{4}}+\frac{\sqrt{2}\,(ad-bc)\left(2\arctan\left(-\frac{\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}+1\right)-2\arctan}{}}{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}dc}$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)/c)^(1/4)*(-((a*d-b*c)/c)^(1/4)*c*(2*arctan(1/b^(1/4)/x*(b*x
^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(1/4)*x-(b*x^4+a)^(1/4)))
*b^(3/4)+1/2*2^(1/2)*(a*d-b*c)*(2*arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x
^4+a)^(1/4)/x+1)-2*arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)
-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^
2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*
c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))))/d/c
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.24

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)
*log((c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*
d^4))^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(
-(c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4)
)^(3/4) - (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*I*((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((I
*c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) - 1/4*I*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((-I
*c^2*d^3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
^(3/4) + (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/
d^4)^(1/4)*log((d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b
^3/d^4)^(1/4)*log(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x) - 1/
4*I*(b^3/d^4)^(1/4)*log((I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/
x) + 1/4*I*(b^3/d^4)^(1/4)*log((-I*d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/
4)*b^2)/x)
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{3}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(3/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{\frac{3}{4}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

output
```
int((a + b*x**4)**(3/4)/(c + d*x**4),x)
```

3.105 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

output
```
1/2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^
(1/4)+1/2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*
d+b*c)^(1/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)\left(\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)+\operatorname{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}\,x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right. \cdots}{c^{3/4}\sqrt[4]{bc-ad}}$$

input
```
Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

output
```
((1/4 + I/4)*(ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)
^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] +
ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1
 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)]))/(c^(3/4)*(b*c
 - a*d)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{c-\frac{x^4(bc-ad)}{a+bx^4}}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

input
```
Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

output
```
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c
- a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]
/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(81) = 162$.

Time = 1.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.02

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)+2\arctan\left(\frac{\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}+1\right)-2\arctan\left(-\frac{\sqrt{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x} \right.\right.}{8\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c}$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*d-b*c)/c)^(1/4)*2^(1/2)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)
*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(
b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+2*arcta
n(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-2*arctan(-2^(1/2)/((a*d
-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1))/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,c+(b\,x^4+a)^{\frac{1}{4}}\,d\,x^4}dx$$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*c + (a + b*x**4)**(1/4)*d*x**4),x)
```

3.106 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

output
```
b*x/a/(-a*d+b*c)/(b*x^4+a)^(1/4)-1/2*d*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(
b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)-1/2*d*arctanh((-a*d+b*c)^(1/4)*x/
c^(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(5/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \frac{1}{4}\left(\frac{4bx}{(abc-a^2d)\sqrt[4]{a+bx^4}} - \frac{(1+i)d\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}} - \frac{(1+i)d\,\text{arctanh}\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}+\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{c^{3/4}(bc-ad)^{5/4}}\right)$$

input
```
Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

output
```
((4*b*x)/((a*b*c - a^2*d)*(a + b*x^4)^(1/4)) - ((1 + I)*d*ArcTan[(((1 - I)
*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a
+ b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(3/4)*(b*c - a*d)^(5/4)) - (
(1 + I)*d*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1
/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(
3/4)*(b*c - a*d)^(5/4)))/4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {907, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{907}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{902}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}$$

$$\Big\downarrow \text{756}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}$$

$$\Big\downarrow \text{221}$$

$$\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*(ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*
c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4
))))/(b*c - a*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(108) = 216$.

Time = 1.46 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.19

method	result
pseudoelliptic	$\frac{\arctan\left(-\frac{\sqrt{2}\,(b\,x^4+a)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}+1\right)ad\sqrt{2}\,(b\,x^4+a)^{\frac{1}{4}}-\arctan\left(\frac{\sqrt{2}\,(b\,x^4+a)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}+1\right)ad\sqrt{2}\,(b\,x^4+a)^{\frac{1}{4}}-\frac{\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}(b\,x^4+a)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}(b\,x^4+a)^{\frac{1}{4}}}\right.}{}}{4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}(b\,x^4+a)^{\frac{1}{4}}(ad-bc)ca}$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)*a*d*2^(1/2)*
(b*x^4+a)^(1/4)-arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)*a*
d*2^(1/2)*(b*x^4+a)^(1/4)-1/2*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^
4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))*a*d*2^(1/2)
*(b*x^4+a)^(1/4)-4*b*x*c*((a*d-b*c)/c)^(1/4))/((a*d-b*c)/c)^(1/4)/(b*x^4+a
)^(1/4)/(a*d-b*c)/c/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,ac+(b\,x^4+a)^{\frac{1}{4}}\,ad\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,bc\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,bd\,x^8}\,d$$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*c + (a + b*x**4)**(1/4)*a*d*x**4 + (a + b*x**
4)**(1/4)*b*c*x**4 + (a + b*x**4)**(1/4)*b*d*x**8),x)
```

3.107 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 180

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{bx}{5a(bc-ad)(a+bx^4)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}$$

output
```
1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^(5/4)+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)
^2/(b*x^4+a)^(1/4)+1/2*d^2*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/
4))/c^(3/4)/(-a*d+b*c)^(9/4)+1/2*d^2*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b
*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(9/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.98 (sec) , antiderivative size = 621, normalized size of antiderivative = 3.45

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \frac{-585c^4(bc-ad)x^4(a+bx^4)^2 - 936c^3d(bc-ad)x^8(a+bx^4)^2 - 416c^2d^2(bc-\cdots}{\cdots}$$

input
```
Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

output
```
(-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b
*x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4
)^3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925
*c^5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, (
(b*c - a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hyperge
ometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c
- a*d)^3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b
*x^4))] + 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*
c - a*d)*x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric
2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)
^3*x^12*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a
 + b*x^4))] + 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {
1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*H
ypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4)
)])/(325*c^4*(b*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {931, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bdx^4+4bc-5ad}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4bdx^4+4bc-5ad}{(bx^4+a)^{5/4}(dx^4+c)}dx}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{1024}$$

$$\frac{\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int -\frac{5a^2d^2}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{5ad^2\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{902}$$

$$\frac{\frac{5ad^2\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{756}$$

$$\frac{\frac{5ad^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{\frac{5ad^2\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{5ad^2\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx(4bc-9ad)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(4*b*c - 9*a*d)*x)/(a*(b*c
 - a*d)*(a + b*x^4)^(1/4)) + (5*a*d^2*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/
4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d
)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(b
*c - a*d))/(5*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(148) = 296$.

Time = 1.62 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.67

method	result
pseudoelliptic	$\frac{\frac{a^{2} d^{2} \left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(2 \arctan\left(-\frac{\sqrt{2}\,\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}+1\right)-2 \arctan\left(\frac{\sqrt{2}\,\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} x}+1\right)-\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{}}\right.}{2}}{4 \left(\frac{ad-bc}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(ad-bc\right)^{2} c\,a^{2}}$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(5/4)*(1/2*a^2*d^2*(b*x^4+a)^(5/4)*(2*ar
ctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-2*arctan(2^(1/2)/((
a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^
(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/((((a*d-b*c)/c)^(1
/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))))*2
^(1/2)-8*(d*a^2-1/2*b*(-9/5*d*x^4+c)*a-2/5*b^2*c*x^4)*((a*d-b*c)/c)^(1/4)*
c*b*x)/(a*d-b*c)^2/c/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{9/4}\,(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2c+(b\,x^4+a)^{\frac{1}{4}}\,a^2d\,x^4+2\,(b\,x^4+a)^{\frac{1}{4}}\,abc\,x^4+2\,(b\,x^4+a)^{\frac{1}{4}}$$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**2*c + (a + b*x**4)**(1/4)*a**2*d*x**4 + 2*(a
 + b*x**4)**(1/4)*a*b*c*x**4 + 2*(a + b*x**4)**(1/4)*a*b*d*x**8 + (a + b*x
**4)**(1/4)*b**2*c*x**8 + (a + b*x**4)**(1/4)*b**2*d*x**12),x)
```

3.108 $\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} \\ &+ \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+113a^2d^2)\,x}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}} \\ &- \frac{d^3\arctan\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}\end{aligned}$$

output
```
1/9*b*x/a/(-a*d+b*c)/(b*x^4+a)^(9/4)+1/45*b*(-17*a*d+8*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^4+a)^(5/4)+1/45*b*(113*a^2*d^2-100*a*b*c*d+32*b^2*c^2)*x/a^3/(-a
*d+b*c)^3/(b*x^4+a)^(1/4)-1/2*d^3*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4
+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)-1/2*d^3*arctanh((-a*d+b*c)^(1/4)*x/c^
(1/4)/(b*x^4+a)^(1/4))/c^(3/4)/(-a*d+b*c)^(13/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 15.02 (sec) , antiderivative size = 1172, normalized size of antiderivative = 5.03

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

output
```
-1/11475*(-16575*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 - 39780*c^4*d*(b*c -
a*d)^2*x^12*(a + b*x^4)^2 - 35360*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^2
 - 10880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 - 29835*c^6*(b*c - a*d)*
x^4*(a + b*x^4)^3 - 71604*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 - 63648*c^4*
d^2*(b*c - a*d)*x^12*(a + b*x^4)^3 - 19584*c^3*d^3*(b*c - a*d)*x^16*(a + b
*x^4)^3 - 149175*c^7*(a + b*x^4)^4 - 358020*c^6*d*x^4*(a + b*x^4)^4 - 3182
40*c^5*d^2*x^8*(a + b*x^4)^4 - 97920*c^4*d^3*x^12*(a + b*x^4)^4 + 149175*c
^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a +
b*x^4))] + 358020*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, (
(b*c - a*d)*x^4)/(c*(a + b*x^4))] + 318240*c^5*d^2*x^8*(a + b*x^4)^4*Hyper
geometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 97920*c^4*d
^3*x^12*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*
(a + b*x^4))] + 13620*c^3*(b*c - a*d)^4*x^16*Hypergeometric2F1[2, 17/4, 21
/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 36900*c^2*d*(b*c - a*d)^4*x^20*Hy
pergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 33840*
c*d^2*(b*c - a*d)^4*x^24*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4
)/(c*(a + b*x^4))] + 10560*d^3*(b*c - a*d)^4*x^28*Hypergeometric2F1[2, 17/
4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 6480*c^3*(b*c - a*d)^4*x^16*
HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4
))] + 18720*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 17/4}, {1...
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {931, 25, 1024, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}-\frac{\int -\frac{8bdx^4+8bc-9ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{9a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8bdx^4+8bc-9ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bd(8bc-17ad)x^4+32b^2c^2+45a^2d^2-68abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4bd(8bc-17ad)x^4+32b^2c^2+45a^2d^2-68abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int \frac{45a^3d^3}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a(a+bx^4)^{9/4}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\int\frac{1}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$$\Big\downarrow \ 902$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$$\Big\downarrow \ 756$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{bx(113a^2d^2-100abcd+32b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{45a^2d^3\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(8bc-17ad)}{5a(a+bx^4)^{5/4}(bc-ad)}}{9a(bc-ad)}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + ((b*(8*b*c - 17*a*d)*x)/(5*a*(
b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d^
2)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (45*a^2*d^3*(ArcTan[((b*c - a*d)
^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + Arc
Tanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
a*d)^(1/4))))/(b*c - a*d))/(5*a*(b*c - a*d)))/(9*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.50

method	result
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} a^{3} d^{3} \left(\ln\left(\frac{-\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\, x +\sqrt{\frac{a d -b c}{c}}\, x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}} \sqrt{2}\, x +\sqrt{\frac{a d -b c}{c}}\, x^{2}+\sqrt{b\,x^{4}+a}}\right)+2 \arctan\left(\frac{\sqrt{2}\, \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{\left(\frac{a d -b c}{c}\right)^{\frac{1}{4}} x}+1\right)-2 \arctan\left(-\right.}{8 \left(\frac{a d -b c}{c}\right)}$

input
```
int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/((a*d-b*c)/c)^(1/4)/(b*x^4+a)^(9/4)*((b*x^4+a)^(9/4)*a^3*d^3*(ln((-((
a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4
+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1
/2)*x^2+(b*x^4+a)^(1/2)))+2*arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(
1/4)/x+1)-2*arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1))*2^(1
/2)+24*((a*d-b*c)/c)^(1/4)*c*(a^4*d^2-d*b*(-9/5*d*x^4+c)*a^3+1/3*b^2*(113/
45*d^2*x^8-5*c*d*x^4+c^2)*a^2+8/15*c*b^3*x^4*(-25/18*d*x^4+c)*a+32/135*b^4
*c^2*x^8)*b*x)/(a*d-b*c)^3/c/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{13}{4}}(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{13}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{13/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)}\,dx = \int \frac{}{(b\,x^4+a)^{\frac{1}{4}}\,a^3c+(b\,x^4+a)^{\frac{1}{4}}\,a^3d\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2bc\,x^4+3\,(b\,x^4+a)}$$

input
```
int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**3*c + (a + b*x**4)**(1/4)*a**3*d*x**4 + 3*(a
 + b*x**4)**(1/4)*a**2*b*c*x**4 + 3*(a + b*x**4)**(1/4)*a**2*b*d*x**8 + 3*
(a + b*x**4)**(1/4)*a*b**2*c*x**8 + 3*(a + b*x**4)**(1/4)*a*b**2*d*x**12 +
 (a + b*x**4)**(1/4)*b**3*c*x**12 + (a + b*x**4)**(1/4)*b**3*d*x**16),x)
```

3.109 $\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 316

$$\begin{aligned}\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx =& -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2}+\frac{bx(a+bx^4)^{5/4}}{6d}\\&+\frac{\sqrt{a}b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12d^2(a+bx^4)^{3/4}}\\&+\frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2}\\&+\frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd^2}\end{aligned}$$

output
```
-1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/d^2+1/6*b*x*(b*x^4+a)^(5/4)/d+1/
12*a^(1/2)*b^(3/2)*(-11*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1
/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/d^2/(b*x^4+a)^(3/4)+1/2*(-a*d+b*c)
^2*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4
),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d^2+1/2*(-a*d+b*c)^2*(a/(
b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d
+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.71 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \frac{x\left(5b(a+bx^4)(-6bc+13ad+2bdx^4)+\frac{b(12b^2c^2-30abcd+23a^2d^2)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{AppellF1}\left(\frac{5}{4},\frac{3}{4},1\right.}{c}\right.}{}$$

input
```
Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

output
```
(x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*
b*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2
*d^2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)
*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a
*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3
/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {933, 25, 1025, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{\sqrt[4]{bx^4+a}\,(b(6bc-11ad)x^4+a(bc-6ad))}{dx^4+c}dx}{6d}+\frac{bx(a+bx^4)^{5/4}}{6d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\int \frac{\sqrt[4]{bx^4+a}\left(b(6bc-11ad)x^4+a(bc-6ad)\right)}{dx^4+c}dx}{6d}$$

$$\Big\downarrow \; 1025$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{\int -\frac{b\left(12b^2c^2-30abdc+23a^2d^2\right)x^4+a\left(6b^2c^2-13abdc+12a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}+\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}}{6d}$$

$$\Big\downarrow \; 25$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{\int \frac{b\left(12b^2c^2-30abdc+23a^2d^2\right)x^4+a\left(6b^2c^2-13abdc+12a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}}{6d}$$

$$\Big\downarrow \; 404$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-ab(6bc-11ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2d}}{6d}$$

$$\Big\downarrow \; 768$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$$\Big\downarrow \; 858$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$$\Big\downarrow \; 807$$

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\Big\downarrow$ 229

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12(bc-ad)^2\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\Big\downarrow$ 923

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\,\text{Ellip}\cdots}{\left(a+bx^4\right)^{3/4}}}{2d}}{6d}$$

$\Big\downarrow$ 925

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\cdots}{2c}\cdots\right)}{2d}}{6d}$$

$\Big\downarrow$ 27

$$\frac{bx\left(a+bx^4\right)^{5/4}}{6d}-\frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d}-\frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\cdots}{2\sqrt{c}}\cdots\right)}{2d}}{6d}$$

$\Big\downarrow$ 1542

$$\frac{bx(a+bx^4)^{5/4}}{6d} - \frac{\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{2d} - \frac{12\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}}{\sqrt[4]{b}}\right.\right.}{2\sqrt[4]{b}c}\right.}{2d}}{6d}$$

input
```
Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

output
```
(b*x*(a + b*x^4)^(5/4))/(6*d) - ((b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/
(2*d) - ((Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a + b*x^4)^(3/4) + 12*(b*c - a*
d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sq
rt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)
+ EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*
x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(2*d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{9}{4}}}{d\,x^{4}+c}dx$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx=\int \frac{(a+bx^4)^{\frac{9}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(9/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx=\int \frac{(bx^4+a)^{\frac{9}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{9/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(9/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(9/4)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{9/4}}{c+dx^4}\,dx = \frac{13(b\,x^4+a)^{\frac{1}{4}}\,abdx - 6(b\,x^4+a)^{\frac{1}{4}}\,b^2cx + 2(b\,x^4+a)^{\frac{1}{4}}\,b^2d\,x^5 + 12\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{bd\,x^8+ad\,x^4+bc\,x^4+a}\right.}{}$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

output
```
(13*(a + b*x**4)**(1/4)*a*b*d*x - 6*(a + b*x**4)**(1/4)*b**2*c*x + 2*(a +
b*x**4)**(1/4)*b**2*d*x**5 + 12*int((a + b*x**4)**(1/4)/(a*c + a*d*x**4 +
b*c*x**4 + b*d*x**8),x)*a**3*d**2 - 13*int((a + b*x**4)**(1/4)/(a*c + a*d*
x**4 + b*c*x**4 + b*d*x**8),x)*a**2*b*c*d + 6*int((a + b*x**4)**(1/4)/(a*c
 + a*d*x**4 + b*c*x**4 + b*d*x**8),x)*a*b**2*c**2 + 23*int(((a + b*x**4)**
(1/4)*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)*a**2*b*d**2 - 30*int
(((a + b*x**4)**(1/4)*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)*a*b*
*2*c*d + 12*int(((a + b*x**4)**(1/4)*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*
d*x**8),x)*b**3*c**2)/(12*d**2)
```

3.110 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2d\,(a+bx^4)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}cd}$$

output
```
1/2*b*x*(b*x^4+a)^(1/4)/d-1/2*a^(1/2)*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*Invers
eJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/d/(b*x^4+a)^(3/4)-1/2*(
-a*d+b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+
a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d-1/2*(-a*d+b*c)*(
a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-
a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(-5ac(2a^2d+abdx^4+b^2x^4(c+dx^4))\,\text{Appell}\right.}{(c+dx^4)\left(-5ac\,\text{AppellF1}\right.}\right.}{10}$$

input
```
Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

output
```
(x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/
4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*
x^4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] +
b*x^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/
a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^
4)/c)])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/
(10*d*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {924, 748, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$$

$$\Big\downarrow \text{924}$$

$$\frac{b\int \sqrt[4]{bx^4+a}dx}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{748}$$

$$\frac{b\left(\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{768}$$

$$\frac{b\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2(a+bx^4)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{807}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{229}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{(bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{d}$$

$$\Big\downarrow \text{923}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}$$

$$\Big\downarrow \text{925}$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{d}$$

$$\Big\downarrow \; 1542$$

$$\frac{b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2(a+bx^4)^{3/4}}\right)}{d}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right.}{d}$$

input
```
Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

output
```
(b*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*E
llipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))))/d -
((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c -
a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(
1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)
/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 924
```
Int[((a_) + (b_.)*(x_)^4)^(5/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/
d   Int[(a + b*x^4)^(1/4), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^4)^(1
/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}{d\,x^{4}+c}d x$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(a+bx^4)^{\frac{5}{4}}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(5/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(5/4)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,bx+2\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)a^2d-\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)abc+3\Bigg(}{2d}$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

output
```
((a + b*x**4)**(1/4)*b*x + 2*int((a + b*x**4)**(1/4)/(a*c + a*d*x**4 + b*c
*x**4 + b*d*x**8),x)*a**2*d - int((a + b*x**4)**(1/4)/(a*c + a*d*x**4 + b*
c*x**4 + b*d*x**8),x)*a*b*c + 3*int(((a + b*x**4)**(1/4)*x**4)/(a*c + a*d*
x**4 + b*c*x**4 + b*d*x**8),x)*a*b*d - 2*int(((a + b*x**4)**(1/4)*x**4)/(a
*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)*b**2*c)/(2*d)
```

3.111 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c}$$

output
```
1/2*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/
4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c+1/2*(a/(b*x^4+a))^(1/2)*
(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1
/2)/c^(1/2),I)/b^(1/4)/c
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$
$$= \frac{5acx\sqrt[4]{a+bx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(-4ad\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bc\,\text{Ap}\right.\right.}$$

input
```
Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

output
```
(5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*
x^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x
^4)/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$
$$\Big\downarrow \text{ 923}$$
$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}$$
$$\Big\downarrow \text{ 925}$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)$$

$$\Big\downarrow 27$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow 1542$$

$$\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right.\right.}{2\sqrt[4]{b}c}\right.$$

input
```
Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

output
```
Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]
*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + Ell
ipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^
(1/4)], -1]/(2*b^(1/4)*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

input
```
integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

output
```
Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\int \frac{{\left(b\,x^4+a\right)}^{1/4}}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^(1/4)/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^(1/4)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx=\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{d\,x^{4}+c}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

output
```
int((a + b*x**4)**(1/4)/(c + d*x**4),x)
```

3.112 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 259

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)}$$

output
```
-b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1
/2)),2^(1/2))/a^(1/2)/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/2*d*(a/(b*x^4+a))^(1/2)
*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^
(1/2)/c^(1/2),I)/b^(1/4)/c/(-a*d+b*c)-1/2*d*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^
(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2
),I)/b^(1/4)/c/(-a*d+b*c)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(a+bx^4)^{3/4}(c+dx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*
x^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {926, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{926}$$

$$\frac{b\int \frac{1}{(bx^4+a)^{3/4}}dx}{bc-ad}-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{768}$$

$$\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \; 858$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 807$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 229$$

$$-\frac{d\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 923$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \color{blue}{1542}$$

$$-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

output
```
-((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin
[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c -
 a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(
1/4)*c)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 926
```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x], x] - Simp[d/(b*c - a*d)   In
t[(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*
c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(d\,x^{4}+c\right)}dx$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}\,(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{4}}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{3/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,c+(b\,x^4+a)^{\frac{3}{4}}\,d\,x^4}dx$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*c + (a + b*x**4)**(3/4)*d*x**4),x)
```

3.113 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 304

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} \\ &- \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(bc-ad)^2(a+bx^4)^{3/4}} \\ &+ \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2} \\ &+ \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}\end{aligned}$$

output

```
1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^(3/4)-1/3*b^(3/2)*(-5*a*d+2*b*c)*(1+a/b/x^4
)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/
2)/(-a*d+b*c)^2/(b*x^4+a)^(3/4)+1/2*d^2*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2
)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I
)/b^(1/4)/c/(-a*d+b*c)^2+1/2*d^2*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*Ellip
ticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4
)/c/(-a*d+b*c)^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.09

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \frac{x\left(-\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}+\frac{5\left(5ac(3ad-b(3c+dx^4))\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4}\right.\right.}{(c+dx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4}\right.\right.}\right.}{15a(-bc}$$

input
```
Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

output
```
(x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)
/a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4,
3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4
, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4,
 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4,
2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {931, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{3a(a+bx^4)^{3/4}(bc-ad)} - \frac{\int -\frac{2bdx^4+2bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bdx^4+2bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{3ad^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{b(2bc-5ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{3ad^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{3ad^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{3ad^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{3ad^2\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$$\Big\downarrow \text{923}$$

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$\big\downarrow$ 925

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\cdots}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\cdots}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

$\big\downarrow$ 1542

$$\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)\cdots}{\cdots}}{3a(bc-ad)}$$

$$\frac{bx}{3a\left(a+bx^4\right)^{3/4}(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(2*b*c - 5*a*d)*(1
 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sq
rt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) + (3*a*d^2*Sqrt[a/(a + b*x^4)]*Sqrt[
a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/
4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(
Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)
))/(b*c - a*d))/(3*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^4+a)^{\frac{7}{4}}\,(d\,x^4+c)}dx$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{7}{4}}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{7/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,ac+(b\,x^4+a)^{\frac{3}{4}}\,ad\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,bc\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,bd\,x^8}\,d$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*c + (a + b*x**4)**(3/4)*a*d*x**4 + (a + b*x**
4)**(3/4)*b*c*x**4 + (a + b*x**4)**(3/4)*b*d*x**8),x)
```

3.114 $\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 357

$$\begin{aligned}\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx &= \frac{bx}{7a(bc-ad)\,(a+bx^4)^{7/4}} \\ &+ \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2\,(a+bx^4)^{3/4}} \\ &- \frac{b^{3/2}(12b^2c^2-38abcd+47a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}(bc-ad)^3\,(a+bx^4)^{3/4}} \\ &- \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3} \\ &- \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{2\sqrt[4]{b}c(bc-ad)^3}\end{aligned}$$

output
```
1/7*b*x/a/(-a*d+b*c)/(b*x^4+a)^(7/4)+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*
c)^2/(b*x^4+a)^(3/4)-1/21*b^(3/2)*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/
b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
a^(5/2)/(-a*d+b*c)^3/(b*x^4+a)^(3/4)-1/2*d^3*(a/(b*x^4+a))^(1/2)*(b*x^4+a)
^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1
/2),I)/b^(1/4)/c/(-a*d+b*c)^3-1/2*d^3*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*
EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b
^(1/4)/c/(-a*d+b*c)^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.94 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{x\left(-\frac{2bd(-6bc+13ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}-\frac{5\left(5ac\left(21a^3d^2+6b^3cx^4(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

output
```
(x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*
x^4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4
 - 13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b
*x^4*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*A
ppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4
, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*A
ppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4
, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(a
+ b*x^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {931, 25, 1024, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}-\frac{\int -\frac{6bdx^4+6bc-7ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{7a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{6bdx^4+6bc-7ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{7a(bc-ad)}+\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}-\frac{\int -\frac{2bd(6bc-13ad)x^4+12b^2c^2+21a^2d^2-26abcd}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{3a(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{2bd(6bc-13ad)x^4+12b^2c^2+21a^2d^2-26abcd}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}$$

$$\Big\downarrow \text{ 404}$$

$$\frac{\frac{\frac{b\left(47a^2d^2-38abcd+12b^2c^2\right)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}-\frac{21a^2d^3\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a(a+bx^4)^{7/4}(bc-ad)}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\frac{\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 858

$$\frac{\frac{-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 807

$$\frac{\frac{-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{21a^2d^3\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 229

$$\frac{\frac{-\frac{21a^2d^3\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}+\frac{bx(6bc-13ad)}{3a(a+bx^4)^{3/4}(bc-ad)}}{7a(bc-ad)}+\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 923

$$\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 925

$$-\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}}{}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}}{}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

$\Big\downarrow$ 1542

$$-\frac{\frac{-\frac{21a^2d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3}}{}}{3a(bc-ad)}}{7a(bc-ad)}$$

$$\frac{bx}{7a\left(a+bx^4\right)^{7/4}(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

output
```
(b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + ((b*(6*b*c - 13*a*d)*x)/(3*a*(
b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*
a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]
/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (21*a^2*d^3*Sqrt[a/(a +
 b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])),
 ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]
/(2*b^(1/4)*c)))/(b*c - a*d))/(3*a*(b*c - a*d)))/(7*a*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{11}{4}} \left(d\,x^{4}+c\right)}dx$$

input
```
int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

output
```
int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{11/4}\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{11/4}\,(c+dx^4)}\,dx = \int \frac{1}{(a+bx^4)^{\frac{11}{4}}\,(c+dx^4)}\,dx$$

input
```
integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

output
```
Integral(1/((a + b*x**4)**(11/4)*(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{11}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(bx^4+a)^{\frac{11}{4}}(dx^4+c)}\,dx$$

input
```
integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{11/4}\,(d\,x^4+c)}\,dx$$

input
```
int(1/((a + b*x^4)^(11/4)*(c + d*x^4)),x)
```

output
```
int(1/((a + b*x^4)^(11/4)*(c + d*x^4)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a^2c+(b\,x^4+a)^{\frac{3}{4}}\,a^2d\,x^4+2\,(b\,x^4+a)^{\frac{3}{4}}\,abc\,x^4+2\,(b\,x^4+a)^{\frac{3}{4}}}$$

input
```
int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a**2*c + (a + b*x**4)**(3/4)*a**2*d*x**4 + 2*(a
 + b*x**4)**(3/4)*a*b*c*x**4 + 2*(a + b*x**4)**(3/4)*a*b*d*x**8 + (a + b*x
**4)**(3/4)*b**2*c*x**8 + (a + b*x**4)**(3/4)*b**2*d*x**12),x)
```

3.115 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 280

$$\begin{aligned}\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx &= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} \\ &- \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd\,(c+dx^4)} - \frac{b^{7/4}(8bc-11ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\ &+ \frac{(bc-ad)^{7/4}(8bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3} \\ &- \frac{b^{7/4}(8bc-11ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} \\ &+ \frac{(bc-ad)^{7/4}(8bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}\end{aligned}$$

output
```
1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^(3/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(7/4
)/c/d/(d*x^4+c)-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/
4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c^(1/
4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3-1/8*b^(7/4)*(-11*a*d+8*b*c)*arctanh(b^(1/4
)*x/(b*x^4+a)^(1/4))/d^3+1/8*(-a*d+b*c)^(7/4)*(3*a*d+8*b*c)*arctanh((-a*d+
b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)dx(a+bx^4)^{3/4}(-2abcd+a^2d^2+b^2c(2c+dx^4))}{c(c+dx^4)} - (1-i)b^{7/4}(8bc-11ad)\arctan\right.}{}$$

input
```
Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

output
```
((1/16 + I/16)*(((2 - 2*I)*d*x*(a + b*x^4)^(3/4)*(-2*a*b*c*d + a^2*d^2 + b
^2*c*(2*c + d*x^4)))/(c*(c + d*x^4)) - (1 - I)*b^(7/4)*(8*b*c - 11*a*d)*Ar
cTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*A
rcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1 +
 I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/c^(7/4) - (1 - I
)*b^(7/4)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(
1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^
(1/4))/(2*x)])/c^(7/4)))/d^3
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {930, 1025, 27, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{(bx^4+a)^{3/4}(4b(2bc-ad)x^4+a(bc+3ad))}{dx^4+c}dx}{4cd}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{\frac{\int -\frac{4\left(b^2c(8bc-11ad)x^4+a\left(2b^2c^2-2abdc-3a^2d^2\right)\right)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4d}+\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}}{4cd}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\int \frac{b^2c(8bc-11ad)x^4+a\left(2b^2c^2-2abdc-3a^2d^2\right)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{4cd}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \text{1026}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{d}}{4cd}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \text{770}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)^2(3ad+8bc)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{d}}{d}}{4cd}-\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int\frac{1}{\sqrt[4]{bx^4+a}(\ldots}}{d}}{d}}{4cd}$$
$$\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int\frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}\ldots}{d}}{d}}{4cd}$$
$$\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int\frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx\ldots}{d}}{d}}{4cd}$$
$$\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{902}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4\ldots}}}{d}}{d}}{4cd}$$
$$\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\cdots\right)}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 218

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\cdots\right)}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

$\Big\downarrow$ 221

$$\frac{\frac{bx(a+bx^4)^{3/4}(2bc-ad)}{d}-\frac{b^2c(8bc-11ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)^2(3ad+8bc)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\cdots\right)}{d}}{4cd}$$

$$\frac{x(a+bx^4)^{7/4}(bc-ad)}{4cd(c+dx^4)}$$

input

```
Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(7/4))/(c*d*(c + d*x^4)) + ((b*(2*b*c - a*
d)*x*(a + b*x^4)^(3/4))/d - ((b^2*c*(8*b*c - 11*a*d)*(ArcTan[(b^(1/4)*x)/(
a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*
b^(1/4))))/d - ((b*c - a*d)^2*(8*b*c + 3*a*d)*(ArcTan[((b*c - a*d)^(1/4)*x
)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/
4))))/d)/d)/(4*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(232) = 464$.

Time = 2.47 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.71

method	result
pseudoelliptic	$-\frac{-8cd\left(2b^2c^2-2db\left(-\frac{b\,x^4}{2}+a\right)c+a^2d^2\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x\left(b\,x^4+a\right)^{\frac{3}{4}}+\left(d\,x^4+c\right)\left(-16\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^3\left(2\arctan\left(\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)-\right.\right.}{}$

input
```
int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/((a*d-b*c)/c)^(1/4)*(-8*c*d*(2*b^2*c^2-2*d*b*(-1/2*b*x^4+a)*c+a^2*d^
2)*((a*d-b*c)/c)^(1/4)*x*(b*x^4+a)^(3/4)+(d*x^4+c)*(-16*((a*d-b*c)/c)^(1/4
)*c^3*(2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1/4
))/(b^(1/4)*x-(b*x^4+a)^(1/4))))*b^(11/4)+22*((a*d-b*c)/c)^(1/4)*a*c^2*d*(
2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))-ln((-b^(1/4)*x-(b*x^4+a)^(1/4))/(b^(
1/4)*x-(b*x^4+a)^(1/4))))*b^(7/4)+2^(1/2)*(3*a*d+8*b*c)*(a*d-b*c)^2*(ln((-
((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x
^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^
(1/2)*x^2+(b*x^4+a)^(1/2)))+2*arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)
^(1/4)/x+1)-2*arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1))))/
d^3/c^2/(d*x^4+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.76 (sec) , antiderivative size = 2764, normalized size of antiderivative = 9.87

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/16*((c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464
*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^
12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*
a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5
*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a
^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*
c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4
+ c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2
- 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7
931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^
9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log((c
^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 -
37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 793
1*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*
b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3/4) - (512*b
^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{11}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{11/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2}\,dx = \text{too large to display}$$

input

```
int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

output

```
( - 12*(a + b*x**4)**(3/4)*a**2*b*d*x + 5*(a + b*x**4)**(3/4)*a*b**2*c*x +
 3*(a + b*x**4)**(3/4)*a*b**2*d*x**5 - 4*(a + b*x**4)**(3/4)*b**3*c*x**5 +
 36*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d
**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3
*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a
**5*c*d**3 + 36*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**
4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8
+ 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2
*x**12),x)*a**5*d**4*x**4 - 60*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*
a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a
*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8
- 4*b**2*c*d**2*x**12),x)*a**4*b*c**2*d**2 - 60*int((a + b*x**4)**(3/4)/(3
*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*
c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*
b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**4*b*c*d**3*x**4 + int((a + b
*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a
*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**
2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**3*b**2*c**3*
d + int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d
**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d...
```

3.116 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 230

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} \, dx = -\frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd(c+dx^4)} + \frac{b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} + \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(4bc+3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^4+a)^(3/4)/c/d/(d*x^4+c)+1/2*b^(7/4)*arctan(b^(1/4)
*x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctan((-a*d+b*
c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2+1/2*b^(7/4)*arctanh(b^(1/4
)*x/(b*x^4+a)^(1/4))/d^2-1/8*(-a*d+b*c)^(3/4)*(3*a*d+4*b*c)*arctanh((-a*d+
b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/d^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)d(bc-ad)x(a+bx^4)^{3/4}}{c(c+dx^4)} + (4-4i)b^{7/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - \frac{(4b^2c^2-abcd- \cdots}{\cdots} \cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

output
```
((1/16 + I/16)*(((-2 + 2*I)*d*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*(c + d*x
^4)) + (4 - 4*I)*b^(7/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c
^2 - a*b*c*d - 3*a^2*d^2)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)
*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4
))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4)) + (4 - 4*I)*b^(7/4)*ArcTanh[(b^(1/4
)*x)/(a + b*x^4)^(1/4)] - ((4*b^2*c^2 - a*b*c*d - 3*a^2*d^2)*ArcTanh[(((1
- I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)
*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(c^(7/4)*(b*c - a*d)^(1/4))
))/d^2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {930, 1026, 770, 756, 216, 219, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}dx$$

$$\Big\downarrow \text{ 930}$$

$$\frac{\int \frac{4b^2cx^4+a(bc+3ad)}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{ 1026}$$

$$\frac{\frac{4b^2c\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{\frac{4b^2c\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{\frac{4b^2c\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{4b^2c\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int \frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int\frac{1}{\sqrt[4]{bx^4+a}\,(dx^4+c)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 902$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\int\frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{b\cdots}}}{2\sqrt{c}}\cdots\right.}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\int\frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+b\cdots}}\right.}{2c^{3/4}\sqrt[4]{bc-ad}\cdots}\cdots\right.}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{4b^2c\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{d}-\frac{(bc-ad)(3ad+4bc)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(3/4))/(c*d*(c + d*x^4)) + ((4*b^2*c*(ArcT
an[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b
*x^4)^(1/4)]/(2*b^(1/4))))/d - ((b*c - a*d)*(4*b*c + 3*a*d)*(ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))
 + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(
b*c - a*d)^(1/4))))/d)/(4*c*d)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1026
```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Simp[f/d   Int[(a + b*x^n)^p, x], x] + Simp[(d*e -
 c*f)/d   Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f,
 p, n}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(186) = 372$.

Time = 2.45 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.64

method	result
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x(ad-bc)dc\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}+\left(d\,x^{4}+c\right)\left(4\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^{2}\left(\ln\left(\frac{-b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)\right)b^{\frac{7}{4}}+\cdots\right.}{16(\cdots}$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/((a*d-b*c)/c)^(1/4)*(4*(b*x^4+a)^(3/4)*x*(a*d-b*c)*d*c*((a*d-b*c)/c)^
(1/4)+(d*x^4+c)*(4*((a*d-b*c)/c)^(1/4)*c^2*(ln((-b^(1/4)*x-(b*x^4+a)^(1/4)
)/(b^(1/4)*x-(b*x^4+a)^(1/4)))-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b^(7
/4)+1/2*2^(1/2)*(3*a^2*d^2+a*b*c*d-4*b^2*c^2)*(2*arctan(-2^(1/2)/((a*d-b*c
)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-2*arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^
4+a)^(1/4)/x+1)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b
*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(
1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))))))/d^2/c^2/(d*x^4+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1440, normalized size of antiderivative = 6.26

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + (c*d^2*x^4 + c^2*d)*((256*b^7*c
^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((c^5
*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4
*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^
2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) - (c*d^2*x^
4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609
*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^
7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^
3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^
6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*
b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a
)^(1/4))/x) + (-I*c*d^2*x^4 - I*c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2
 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a
^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log((I*c^5*d^6*x*((256*b^7*c^7 -
 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5
*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^
5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c
*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) + (I*c*d^2*x^4 + I*c^2*d)*((25...
```

Sympy [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{7}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(7/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{7}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{7/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

output
```
( - (a + b*x**4)**(3/4)*a*b*x + 9*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d +
 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 +
2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x*
*8 - 4*b**2*c*d**2*x**12),x)*a**4*c*d**2 + 9*int((a + b*x**4)**(3/4)/(3*a*
*2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**
2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**
2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**4*d**3*x**4 - 21*int((a + b*x**
4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c
**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c*
*3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**3*b*c**2*d - 21*
int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*
x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**
12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**3*
b*c*d**2*x**4 + 12*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*
x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x*
*8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d
**2*x**12),x)*a**2*b**2*c**3 + 12*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d +
 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 +
2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x*
*8 - 4*b**2*c*d**2*x**12),x)*a**2*b**2*c**2*d*x**4 + 3*int(((a + b*x**4...
```

3.117 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} \, dx = \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)} + \frac{3a\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

output
```
1/4*x*(b*x^4+a)^(3/4)/c/(d*x^4+c)+3/8*a*arctan((-a*d+b*c)^(1/4)*x/c^(1/4)/
(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)+3/8*a*arctanh((-a*d+b*c)^(1/4)*x
/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(1/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} \, dx = \frac{4c^{3/4}\sqrt[4]{bc-ad}x(a+bx^4)^{3/4} + (3+3i)a(c+dx^4)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}} - \frac{(1+i)\sqrt[4]{c}\sqrt[4]{}}{\sqrt[4]{bc-}}}{2x}\right.}{16c^{7/4}\sqrt[4]{bc-ad}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

output
```
(4*c^(3/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(3/4) + (3 + 3*I)*a*(c + d*x^4)
*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) - ((1
 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)] + (3 + 3*I)*a*(
c + d*x^4)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(
1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(16
*c^(7/4)*(b*c - a*d)^(1/4)*(c + d*x^4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {903, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{3a\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

$$\Big\downarrow \text{902}$$

$$\frac{3a\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

$$\Big\downarrow \text{756}$$

$$\frac{3a\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-adx^2}}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-adx^2}}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3a\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

$$\Big\downarrow \text{221}$$

$$\frac{3a\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c}+\frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

input
```
Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*(ArcTan[((b*c - a*d)^(1/4)*
x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)) + ArcTanh[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1
/4))))/(4*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(107) = 214$.

Time = 1.57 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.15

method	result
pseudoelliptic	$-\frac{3\left(-\frac{8\left(b\,x^{4}+a\right)^{\frac{3}{4}}xc\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}}{3}+\sqrt{2}\,a\left(d\,x^{4}+c\right)\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)+2\arctan\left(\frac{\sqrt{2}\,(\cdots}{}\right.\right.\right.}{32\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}c^{2}\left(d\,x^{4}+c\right)}$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/32/((a*d-b*c)/c)^(1/4)*(-8/3*(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/4)+2^
(1/2)*a*(d*x^4+c)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*
d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*
2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+2*arctan((2^(1/2)*(b*x
^4+a)^(1/4)+((a*d-b*c)/c)^(1/4)*x)/((a*d-b*c)/c)^(1/4)/x)-2*arctan((-2^(1/
2)*(b*x^4+a)^(1/4)+((a*d-b*c)/c)^(1/4)*x)/((a*d-b*c)/c)^(1/4)/x))/c^2/(d*
x^4+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{3}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\int \frac{(b\,x^4+a)^{3/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx=\frac{-(b\,x^4+a)^{\frac{3}{4}}\,bx+9\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{3ab\,d^3x^{12}-4b^2c\,d^2x^{12}+3a^2d^3x^8+2abc\,d^2x^8-8b^2c^2d\,x^8+6a^2c\,d^2x^4-5ab\,c^2d\,x^4-4b^2c\cdots}\right.}{\cdots}$$

input
```
int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

output
```
( - (a + b*x**4)**(3/4)*b*x + 9*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6
*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*
a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8
 - 4*b**2*c*d**2*x**12),x)*a**3*c*d**2 + 9*int((a + b*x**4)**(3/4)/(3*a**2
*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*
d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*
c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**3*d**3*x**4 - 21*int((a + b*x**4)
**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**
3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3
*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**2*b*c**2*d - 21*in
t((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x*
*8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12
 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**2*x**12),x)*a**2*b*
c*d**2*x**4 + 12*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x*
*4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b*c*d**2*x**8
 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 - 4*b**2*c*d**
2*x**12),x)*a*b**2*c**3 + 12*int((a + b*x**4)**(3/4)/(3*a**2*c**2*d + 6*a*
*2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 4*a*b*c**3 - 5*a*b*c**2*d*x**4 + 2*a*b
*c*d**2*x**8 + 3*a*b*d**3*x**12 - 4*b**2*c**3*x**4 - 8*b**2*c**2*d*x**8 -
4*b**2*c*d**2*x**12),x)*a*b**2*c**2*d*x**4)/(3*a*c*d + 3*a*d**2*x**4 - ...
```

3.118 $\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = -\frac{dx(a+bx^4)^{3/4}}{4c(bc-ad)\,(c+dx^4)} + \frac{(4bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}$$

output
```
-1/4*d*x*(b*x^4+a)^(3/4)/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*arctan(
(-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(5/4)+1/8*(
-3*a*d+4*b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/
(-a*d+b*c)^(5/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\frac{(2-2i)c^{3/4}dx(a+bx^4)^{3/4}}{(bc-ad)(c+dx^4)}+\frac{(4bc-3ad)\arctan\left(\frac{\frac{(1-i)\sqrt[4]{bc-ad}x^2}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}-\frac{(1+i)\sqrt[4]{c}\sqrt[4]{a+bx^4}}{\sqrt[4]{bc-ad}}}{2x}\right)}{(bc-ad)^{5/4}}+\frac{(4bc-3ad)\text{arctan}}{}\right.}{c^{7/4}}$$

input

```
Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

output

```
((1/16 + I/16)*(((-2 + 2*I)*c^(3/4)*d*x*(a + b*x^4)^(3/4))/((b*c - a*d)*(c
 + d*x^4)) + ((4*b*c - 3*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(
1/4)*(a + b*x^4)^(1/4)) - ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^
(1/4))/(2*x)])/(b*c - a*d)^(5/4) + ((4*b*c - 3*a*d)*ArcTanh[(((1 - I)*(b*c
 - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x
^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*d)^(5/4)))/c^(7/4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {907, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{907}$$

$$\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{902}$$

$$\frac{(4bc-3ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{756}$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{(4bc-3ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{4c(bc-ad)}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

output

```
-1/4*(d*x*(a + b*x^4)^(3/4))/(c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d
)*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b
*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4
))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(134) = 268.

Time = 1.56 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.92

method	result
pseudoelliptic	$\frac{-\frac{3\left(d\,x^{4}+c\right)\left(ad-\frac{4bc}{3}\right)\sqrt{2}\,\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^{2}+\sqrt{b\,x^{4}+a}}\right)}{32}+\frac{3\left(d\,x^{4}+c\right)\left(ad-\frac{4bc}{3}\right)\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(b\,x^{4}\right.}{\left(\frac{ad-bc}{c}\right.}\right.}{16}}{(ad-bc)c^{2}(d\,x^{4}+c)\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}}$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
3/16*(-1/2*(d*x^4+c)*(a*d-4/3*b*c)*2^(1/2)*ln((-((a*d-b*c)/c)^(1/4)*(b*x^4
+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c
)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))
)+(d*x^4+c)*(a*d-4/3*b*c)*2^(1/2)*arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x
^4+a)^(1/4)/x+1)-(d*x^4+c)*(a*d-4/3*b*c)*2^(1/2)*arctan(2^(1/2)/((a*d-b*c)
/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)+4/3*d*(b*x^4+a)^(3/4)*x*c*((a*d-b*c)/c)^(1/
4))/((a*d-b*c)/c)^(1/4)/(a*d-b*c)/c^2/(d*x^4+c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,c^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,cd\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,d^2x^8}dx$$

input
```
int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(1/4)*c**2 + 2*(a + b*x**4)**(1/4)*c*d*x**4 + (a + b*
x**4)**(1/4)*d**2*x**8),x)
```

3.119 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\,(c+dx^4)} - \frac{d(8bc-3ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}} - \frac{d(8bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}$$

output
```
1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a*d+b*c)/
(b*x^4+a)^(1/4)/(d*x^4+c)-1/8*d*(-3*a*d+8*b*c)*arctan((-a*d+b*c)^(1/4)*x/c
^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)-1/8*d*(-3*a*d+8*b*c)*arct
anh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(9/4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(\frac{(2-2i)c^{3/4}x(a^2d^2+abd^2x^4+4b^2c(c+dx^4))}{a(bc-ad)^2\sqrt[4]{a+bx^4}(c+dx^4)}+\frac{d(-8bc+3ad)\arctan\left(\frac{(1-i)\sqrt[4]{bc-}}{\sqrt[4]{c}\sqrt[4]{a+}}\right.}{(bc-ad}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

output
```
((1/16 + I/16)*(((2 - 2*I)*c^(3/4)*x*(a^2*d^2 + a*b*d^2*x^4 + 4*b^2*c*(c +
 d*x^4)))/(a*(b*c - a*d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) + (d*(-8*b*c + 3
*a*d)*ArcTan[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c^(1/4)*(a + b*x^4)^(1/4))
- ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d)^(1/4))/(2*x)])/(b*c - a*
d)^(9/4) + (d*(-8*b*c + 3*a*d)*ArcTanh[(((1 - I)*(b*c - a*d)^(1/4)*x^2)/(c
^(1/4)*(a + b*x^4)^(1/4)) + ((1 + I)*c^(1/4)*(a + b*x^4)^(1/4))/(b*c - a*d
)^(1/4))/(2*x)])/(b*c - a*d)^(9/4)))/c^(7/4)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {931, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 931$$

$$\frac{\int \frac{-4bdx^4+4bc-3ad}{(bx^4+a)^{5/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 1024$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 902$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{bx(ad+4bc)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{d(8bc-3ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{dx}{4c\sqrt[4]{a+bx^4}\,(c+dx^4)\,(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

output

```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) + ((b*(4*b*c + a*
d)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*(8*b*c - 3*a*d)*(ArcTan[((b*c
 - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c - a*d)^(1/4)
) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*
(b*c - a*d)^(1/4))))/(b*c - a*d))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.58

method	result
pseudoelliptic	$\frac{\frac{a d \sqrt{2}\left(d x^4+c\right)(3 a d-8 b c)\left(2 \arctan\left(-\frac{\sqrt{2}\left(b x^4+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}+1\right)-2 \arctan\left(\frac{\sqrt{2}\left(b x^4+a\right)^{\frac{1}{4}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}} x}+1\right)-\ln\left(\frac{-\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a d-}{c}}}{\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a d-b}{c}}}\right.\right.}{2}}{16\left(\frac{a d-b c}{c}\right)^{\frac{1}{4}}\left(b x^4+a\right)^{\frac{1}{4}} c^2\left(d x^4+c\right)(a d-b c)^2}$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/((a*d-b*c)/c)^(1/4)*(1/2*a*d*2^(1/2)*(d*x^4+c)*(3*a*d-8*b*c)*(2*arcta
n(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-2*arctan(2^(1/2)/((a*d
-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/x+1)-ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/
4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)
*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2))))*(b*x
^4+a)^(1/4)+4*c*((a*d-b*c)/c)^(1/4)*(4*b^2*c^2+4*b^2*c*d*x^4+a*d^2*(b*x^4+
a))*x)/(b*x^4+a)^(1/4)/c^2/(d*x^4+c)/(a*d-b*c)^2/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{5}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{5/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,c^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,acd\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,a\,d^2x^8+(b\,x^4+a)^{\frac{1}{4}}}$$

input
```
int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*c**2 + 2*(a + b*x**4)**(1/4)*a*c*d*x**4 + (a
+ b*x**4)**(1/4)*a*d**2*x**8 + (a + b*x**4)**(1/4)*b*c**2*x**4 + 2*(a + b*
x**4)**(1/4)*b*c*d*x**8 + (a + b*x**4)**(1/4)*b*d**2*x**12),x)
```

3.120 $\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2} \, dx = \frac{b(4bc+5ad)x}{20ac(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(16b^2c^2-56abcd-5a^2d^2)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{5/4}(c+dx^4)} + \frac{3d^2(4bc-ad)\arctan\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}$$

output
```
1/20*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(5/4)+1/20*b*(-5*a^2*d^2
-56*a*b*c*d+16*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^4+a)^(1/4)-1/4*d*x/c/(-a
*d+b*c)/(b*x^4+a)^(5/4)/(d*x^4+c)+3/8*d^2*(-a*d+4*b*c)*arctan((-a*d+b*c)^(
1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c)^(13/4)+3/8*d^2*(-a*d+4*
b*c)*arctanh((-a*d+b*c)^(1/4)*x/c^(1/4)/(b*x^4+a)^(1/4))/c^(7/4)/(-a*d+b*c
)^(13/4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 14.53 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.57

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

output
```
-1/198900*(285532*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 + 933504*c^4*d*(b*c
- a*d)^2*x^12*(a + b*x^4)^2 + 891072*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4
)^2 + 282880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 + 9793836*c^6*(b*c -
 a*d)*x^4*(a + b*x^4)^3 + 27973296*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 + 2
5968384*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3 + 8146944*c^3*d^3*(b*c - a*
d)*x^16*(a + b*x^4)^3 - 23529870*c^7*(a + b*x^4)^4 - 65547495*c^6*d*x^4*(a
 + b*x^4)^4 - 60505380*c^5*d^2*x^8*(a + b*x^4)^4 - 18935280*c^4*d^3*x^12*(
a + b*x^4)^4 - 14499810*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3*Hypergeometric2F
1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 41082795*c^5*d*(b*c -
a*d)*x^8*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c
*(a + b*x^4))] - 38069460*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3*Hypergeom
etric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 11934000*c^3*d^
3*(b*c - a*d)*x^16*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*
d)*x^4)/(c*(a + b*x^4))] + 23529870*c^7*(a + b*x^4)^4*Hypergeometric2F1[1/
4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 65547495*c^6*d*x^4*(a + b*
x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] +
 60505380*c^5*d^2*x^8*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c -
 a*d)*x^4)/(c*(a + b*x^4))] + 18935280*c^4*d^3*x^12*(a + b*x^4)^4*Hypergeo
metric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 77760*c^3*(b*c
 - a*d)^4*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*...
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {931, 1024, 25, 1024, 27, 902, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-8bdx^4+4bc-3ad}{(bx^4+a)^{9/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}-\frac{\int -\frac{4bd(4bc+5ad)x^4+16b^2c^2+15a^2d^2-40abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4bd(4bc+5ad)x^4+16b^2c^2+15a^2d^2-40abcd}{\left(bx^4+a\right)^{5/4}\left(dx^4+c\right)}dx}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{1024}$$

$$\frac{\frac{\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}-\frac{\int -\frac{15a^2d^2(4bc-ad)}{\sqrt[4]{bx^4+a}\left(dx^4+c\right)}dx}{a(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{5/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}(dx^4+c)}dx}{bc-ad}+\frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 902$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\int \frac{1}{c-\frac{(bc-ad)x^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}+\frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 756$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{15ad^2(4bc-ad)\left(\frac{\int \frac{1}{\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}+\frac{bx(-5a^2d^2-56abcd+16b^2c^2)}{a\sqrt[4]{a+bx^4}(bc-ad)}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{a\sqrt[4]{a+bx^4}(bc-ad)}+\frac{15ad^2(4bc-ad)\left(\frac{\arctan\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\text{arctanh}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\right)}{bc-ad}}{5a(bc-ad)}+\frac{bx(5ad+4bc)}{5a(a+bx^4)^{5/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + ((b*(4*b*c + 5*
a*d)*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + ((b*(16*b^2*c^2 - 56*a*b*c*d
 - 5*a^2*d^2)*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) + (15*a*d^2*(4*b*c - a*
d)*(ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(
b*c - a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))]/(2*c^(3/4)*(b*c - a*d)^(1/4))))/(b*c - a*d))/(5*a*(b*c - a*d)))/(4*c*
(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$-\frac{\frac{3\left(b\,x^4+a\right)^{\frac{5}{4}} a^2 d^2 \left(d\,x^4+c\right)(ad-4bc)\left(\ln\left(\frac{-\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}\left(b\,x^4+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{ad-bc}{c}}\,x^2+\sqrt{b\,x^4+a}}\right)+2\arctan\left(\frac{\sqrt{2}\left(b\,x^4+a\right)^{\frac{1}{4}}}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{4}}x}+1\right)-}{8}}{4\left(\frac{ad-bc}{c}\right.}$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)/c)^(1/4)*(-3/8*(b*x^4+a)^(5/4)*a^2*d^2*(d*x^4+c)*(a*d-4*b*c
)*(ln((-((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-b*c)/c)^(1/2)*
x^2+(b*x^4+a)^(1/2))/(((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)*2^(1/2)*x+((a*d-
b*c)/c)^(1/2)*x^2+(b*x^4+a)^(1/2)))+2*arctan(2^(1/2)/((a*d-b*c)/c)^(1/4)*(
b*x^4+a)^(1/4)/x+1)-2*arctan(-2^(1/2)/((a*d-b*c)/c)^(1/4)*(b*x^4+a)^(1/4)/
x+1))*2^(1/2)+c*(a^4*d^3+2*x^4*a^3*b*d^3+12*d*(1/12*d^2*x^8+c*d*x^4+c^2)*b
^2*a^2-4*(d*x^4+c)*c*b^3*(-14/5*d*x^4+c)*a-16/5*b^4*c^2*x^4*(d*x^4+c))*((a
*d-b*c)/c)^(1/4)*x)/(b*x^4+a)^(5/4)/c^2/(d*x^4+c)/(a*d-b*c)^3/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{9}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{9}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{9/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2}\,dx = \int \frac{}{(b\,x^4+a)^{\frac{1}{4}}\,a^2c^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,a^2cd\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,a^2d^2x^8+2\,(b\,x^4+}$$

input
```
int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**2*c**2 + 2*(a + b*x**4)**(1/4)*a**2*c*d*x**4
 + (a + b*x**4)**(1/4)*a**2*d**2*x**8 + 2*(a + b*x**4)**(1/4)*a*b*c**2*x**
4 + 4*(a + b*x**4)**(1/4)*a*b*c*d*x**8 + 2*(a + b*x**4)**(1/4)*a*b*d**2*x*
*12 + (a + b*x**4)**(1/4)*b**2*c**2*x**8 + 2*(a + b*x**4)**(1/4)*b**2*c*d*
x**12 + (a + b*x**4)**(1/4)*b**2*d**2*x**16),x)
```

3.121 $\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 353

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} - \frac{\sqrt{a}b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd^2(a+bx^4)^{3/4}} - \frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2} - \frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d^2}$$

output
```
1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^(1/4)/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^(5/4
)/c/d/(d*x^4+c)-1/4*a^(1/2)*b^(3/2)*(-a*d+3*b*c)*(1+a/b/x^4)^(3/4)*x^3*Inv
erseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/c/d^2/(b*x^4+a)^(3/4
)-3/8*(-a*d+b*c)*(a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticP
i(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c
^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*Elli
pticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/
4)/c^2/d^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.54 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \frac{2b(-3b^2c^2+3abcd+a^2d^2)\,x^5\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c(-5a\cdots}{}}{}$$

input
```
Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

output
```
(2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1
[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2
 + a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -
((b*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c
*(3*c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((
c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)]
+ x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*
c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(
a + b*x^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {930, 1025, 27, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{\sqrt[4]{bx^4+a}(2b(3bc-ad)x^4+a(bc+3ad))}{dx^4+c}dx}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 1025

$$\frac{\frac{\int -\frac{2\left(2b\left(3b^2c^2-3abdc-a^2d^2\right)x^4+a\left(3b^2c^2-2abdc-3a^2d^2\right)\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{2d}+\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{\int \frac{2b\left(3b^2c^2-3abdc-a^2d^2\right)x^4+a\left(3b^2c^2-2abdc-3a^2d^2\right)}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 404

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-ab(3bc-ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 768

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 858

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\Big\downarrow$ 807

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 229

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3(bc-ad)(ad+2bc)\int\frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\left(a+bx^4\right)^{3/4}}}{d}}{4cd}-\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 923

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\,\text{E}\cdots}{\left(a+bx^4\right)\cdots}}{d}}{4cd}\cdots$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 925

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int\frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\cdots\right)}\cdots}{2c}\cdots\right.}{d}}{4cd}\cdots$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int\frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\cdots\right)}\cdots}{2\sqrt{c}}\cdots\right.}{d}}{4cd}\cdots$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \text{1542}$$

$$\frac{\frac{bx\sqrt[4]{a+bx^4}(3bc-ad)}{d}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\ldots\right.\right.}{2\sqrt[4]{b}c}\right.}{d}}{4cd}$$

$$\frac{x\left(a+bx^4\right)^{5/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(5/4))/(c*d*(c + d*x^4)) + ((b*(3*b*c - a*
d)*x*(a + b*x^4)^(1/4))/d - ((Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4)
)^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a + b*x^4)^(3/
4) + 3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(Elli
pticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4
)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])
, ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/d)/(4*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{9}{4}}}{(d\,x^4+c)^2}dx$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{9}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(9/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{9}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{9/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(9/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(9/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}\,dx = \text{too large to display}$$

input
```
int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

output

```
( - 6*(a + b*x**4)**(1/4)*a**2*b*d*x + 5*(a + b*x**4)**(1/4)*a*b**2*c*x +
3*(a + b*x**4)**(1/4)*a*b**2*d*x**5 - 2*(a + b*x**4)**(1/4)*b**3*c*x**5 +
18*int((a + b*x**4)**(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d*
*3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x*
*12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*x**12),x)*a**5
*c*d**3 + 18*int((a + b*x**4)**(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 +
 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a
*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*x**1
2),x)*a**5*d**4*x**4 - 6*int((a + b*x**4)**(1/4)/(3*a**2*c**2*d + 6*a**2*c
*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**
2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*x**8 - 2*b**2
*c*d**2*x**12),x)*a**4*b*c**2*d**2 - 6*int((a + b*x**4)**(1/4)/(3*a**2*c**
2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4
 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d
*x**8 - 2*b**2*c*d**2*x**12),x)*a**4*b*c*d**3*x**4 - 19*int((a + b*x**4)**
(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3
- a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**
4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*x**12),x)*a**3*b**2*c**3*d - 19*int
((a + b*x**4)**(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**
8 - 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12...
```

3.122 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 298

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd(a+bx^4)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\operatorname{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2d}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^4+a)^(1/4)/c/d/(d*x^4+c)+1/4*a^(1/2)*b^(3/2)*(1+a/b
/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/c
/d/(b*x^4+a)^(3/4)+1/8*(3*a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b
^(1/4)/c^2/d+1/8*(3*a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*Ellipti
cPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/
c^2/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.37 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \frac{x\left(2b(bc+ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{5c\left(-5ac(4a^2d-b^2cx^4+ab\right.}{(c+dx^4)}\right.}{}$$

input
```
Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

output
```
(x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4
)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*
x^4*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/
c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c
+ d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] +
x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*
AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*d*(a +
 b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {930, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{2b(bc+ad)x^4+a(bc+3ad)}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 404

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-ab\int \frac{1}{(bx^4+a)^{3/4}}dx}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 768

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}}}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 858

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}}}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 807

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}}}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 229

$$\frac{(3ad+2bc)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}}}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 923

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}}}{4cd}-\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\,(c+dx^4)}$$

$\Big\downarrow$ 925

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)+}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 27

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)+}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

$\big\downarrow$ 1542

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-\right.}{2\sqrt[4]{b}c}\right.}{4cd}$$

$$\frac{x\sqrt[4]{a+bx^4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

input
```
Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^4)^(1/4))/(c*d*(c + d*x^4)) + ((Sqrt[a]*b^(3/
2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(a + b*x^4)^(3/4) + (2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*
(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqr
t[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(4*c*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}{\left(d\,x^{4}+c\right)^{2}}d x$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(a+bx^4)^{\frac{5}{4}}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(5/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(5/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(5/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

output
```
( - 2*(a + b*x**4)**(1/4)*a*b*x + 9*int((a + b*x**4)**(1/4)/(3*a**2*c**2*d
 + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4 +
4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*x*
*8 - 2*b**2*c*d**2*x**12),x)*a**4*c*d**2 + 9*int((a + b*x**4)**(1/4)/(3*a*
*2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*
d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*
c**2*d*x**8 - 2*b**2*c*d**2*x**12),x)*a**4*d**3*x**4 - 6*int((a + b*x**4)*
*(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3
 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x*
*4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*x**12),x)*a**3*b*c**2*d - 6*int((a
 + b*x**4)**(1/4)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 -
 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b
**2*c**3*x**4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*x**12),x)*a**3*b*c*d**2
*x**4 - 3*int(((a + b*x**4)**(1/4)*x**8)/(3*a**2*c**2*d + 6*a**2*c*d**2*x*
*4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4 + 4*a*b*c*d**2*x**8 +
 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*x**8 - 2*b**2*c*d**2*
x**12),x)*a**2*b**2*c*d**2 - 3*int(((a + b*x**4)**(1/4)*x**8)/(3*a**2*c**2
*d + 6*a**2*c*d**2*x**4 + 3*a**2*d**3*x**8 - 2*a*b*c**3 - a*b*c**2*d*x**4
+ 4*a*b*c*d**2*x**8 + 3*a*b*d**3*x**12 - 2*b**2*c**3*x**4 - 4*b**2*c**2*d*
x**8 - 2*b**2*c*d**2*x**12),x)*a**2*b**2*d**3*x**4 - 4*int(((a + b*x**4...
```

3.123 $\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 308

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx &= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4c(bc-ad)\,(a+bx^4)^{3/4}} \\ &+ \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)} \\ &+ \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}\end{aligned}$$

output

```
1/4*x*(b*x^4+a)^(1/4)/c/(d*x^4+c)-1/4*a^(1/2)*b^(3/2)*(1+a/b/x^4)^(3/4)*x^
3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/c/(-a*d+b*c)/(b
*x^4+a)^(3/4)+1/8*(-3*a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*Ellip
ticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/
4)/c^2/(-a*d+b*c)+1/8*(-3*a*d+2*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*E
llipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^
(1/4)/c^2/(-a*d+b*c)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$
$$= \frac{x\left(\frac{2bx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}+\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\right.}{-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+x^4\left(4ad\,\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{bx^4}{a},\right.\right.}\right.}{c+dx^4}\right.}{20\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

output
```
(x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5
/4, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x
^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)
/c)]))))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {929, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \; 929$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}-\frac{\int -\frac{2bx^4+3a}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{4c}$$

$$\Big\downarrow \textbf{ 25}$$

$$\frac{\int \frac{2bx^4+3a}{\left(bx^4+a\right)^{3/4}\left(dx^4+c\right)}dx}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 404}$$

$$\frac{\frac{ab\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}+\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 768}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}+\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 858}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 807}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{abx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 229}$$

$$\frac{\frac{(2bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\left(a+bx^4\right)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$$\Big\downarrow \textbf{ 923}$$

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{(a+bx^4)^{3/4}(bc-ad)}}{4c}+\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\Big\downarrow$ 925

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}\cdots\right.}{\cdots}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}\cdots\right.}{\cdots}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{\sqrt{a}b^{3/2}x^3\cdots}{\cdots}}{4c}$$

$$\frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)}$$

input

```
Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) + (-((Sqrt[a]*b^(3/2)*(1 + a/(b*x^
4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/((b*c - a*d)*
(a + b*x^4)^(3/4))) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]
*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sq
rt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(b*c -
a*d))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{(d\,x^4+c)^2}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx$$

input
```
integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

output
```
Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^(1/4)/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^(1/4)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^{\frac{1}{4}}}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**4)**(1/4)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.124 $\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 330

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} dx = -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4\sqrt{a}c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2} - \frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$

output
```
-1/4*d*x*(b*x^4+a)^(1/4)/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^(3/2)*(-a*d+4*b*c)*(
1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/
2))/a^(1/2)/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-3/8*d*(-a*d+2*b*c)*(a/(b*x^4+a)
)^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d+b*c)^(
1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*c)*(a/(b*
x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),(-a*d+b
*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.41 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \frac{x\left(\frac{2bdx^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{-bc+ad}+\frac{c\left(25ac(-4bc+4ad+bdx^4)\,\text{AppellF1}\left(\frac{1}{4},\right.\right.}{(bc-ad)(c+dx^4)\left(-5ac\,\text{AppellF1}\right.}}{20}$$

input
```
Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

output
```
(x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*
AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^
4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*A
ppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c +
 d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x
^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*A
ppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*(a + b*
x^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {931, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{\int \frac{-2bdx^4+4bc-3ad}{(bx^4+a)^{3/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{404}$$

$$\frac{\frac{b(4bc-ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{bc-ad}-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}(bc-ad)}-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{858}$$

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{807}$$

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{229}$$

$$\frac{-\frac{3d(2bc-ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{923}$$

$$\frac{-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\cdots\right.}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1542$$

$$-\frac{\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\cdots}{\cdots}}{4c(bc-ad)}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)(bc-ad)}$$

input

```
Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

output

```
-1/4*(d*x*(a + b*x^4)^(1/4))/(c*(b*c - a*d)*(c + d*x^4)) + (-((b^(3/2)*(4*
b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (3*d*(2*b*c - a*d)*Sq
rt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*S
qrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c) + Ellip
ticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1
/4)], -1]/(2*b^(1/4)*c)))/(b*c - a*d))/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(d\,x^{4}+c\right)^{2}}dx$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^4\right)^{3/4} \left(c+dx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{3}{4}}(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{3/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{3/4}\left(d\,x^4+c\right)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(3/4)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/4}\left(c+dx^4\right)^2}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}c^2+2\left(b\,x^4+a\right)^{\frac{3}{4}}c d\,x^4+\left(b\,x^4+a\right)^{\frac{3}{4}}d^2x^8}dx$$

input
```
int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(3/4)*c**2 + 2*(a + b*x**4)**(3/4)*c*d*x**4 + (a + b*
x**4)**(3/4)*d**2*x**8),x)
```

3.125 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 390

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^4)^{3/4}} - \frac{dx}{4c(bc-ad)(a+bx^4)^{3/4}(c+dx^4)} - \frac{b^{3/2}(8b^2c^2-32abcd+3a^2d^2)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}c(bc-ad)^3(a+bx^4)^{3/4}} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right),-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

output
```
1/12*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^4+a)^(3/4)-1/4*d*x/c/(-a*d+b*
c)/(b*x^4+a)^(3/4)/(d*x^4+c)-1/12*b^(3/2)*(3*a^2*d^2-32*a*b*c*d+8*b^2*c^2)
*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(
1/2))/a^(3/2)/c/(-a*d+b*c)^3/(b*x^4+a)^(3/4)+1/8*d^2*(-3*a*d+10*b*c)*(a/(b
*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(1/4),-(-a*d
+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^3+1/8*d^2*(-3*a*d+10
*b*c)*(a/(b*x^4+a))^(1/2)*(b*x^4+a)^(1/2)*EllipticPi(b^(1/4)*x/(b*x^4+a)^(
1/4),(-a*d+b*c)^(1/2)/b^(1/2)/c^(1/2),I)/b^(1/4)/c^2/(-a*d+b*c)^3
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.64 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \frac{x\left(2bd(4bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{AppellF1}\left(\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+\frac{c(25a}{}\right.}{}$$

input
```
Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

output
```
(x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1,
9/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c
+ d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a),
 -((d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*A
ppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF
1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x
^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {931, 1024, 25, 404, 768, 858, 807, 229, 923, 925, 27, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{-6bdx^4+4bc-3ad}{(bx^4+a)^{7/4}(dx^4+c)}dx}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 1024}$$

$$\frac{\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}-\frac{\int -\frac{2bd(4bc+3ad)x^4+8b^2c^2+9a^2d^2-24abcd}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{2bd(4bc+3ad)x^4+8b^2c^2+9a^2d^2-24abcd}{(bx^4+a)^{3/4}(dx^4+c)}dx}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 404}$$

$$\frac{\frac{\frac{b(3a^2d^2-32abcd+8b^2c^2)\int \frac{1}{(bx^4+a)^{3/4}}dx}{bc-ad}+\frac{3ad^2(10bc-3ad)\int \frac{\sqrt[4]{bx^4+a}}{dx^4+c}dx}{bc-ad}}{3a(bc-ad)}+\frac{bx(3ad+4bc)}{3a(a+bx^4)^{3/4}(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(a+bx^4)^{3/4}(c+dx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{\frac{\frac{b x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \left(3 a^2 d^2-32 a b c d+8 b^2 c^2\right) \int \frac{1}{\left(\frac{a}{b x^4}+1\right)^{3/4} x^3}dx}{\left(a+b x^4\right)^{3/4} (b c-a d)}+\frac{3 a d^2 (10 b c-3 a d) \int \frac{\sqrt[4]{b x^4+a}}{d x^4+c}dx}{b c-a d}}{3 a (b c-a d)}+\frac{b x (3 a d+4 b c)}{3 a \left(a+b x^4\right)^{3/4} (b c-a d)}}{4 c (b c-a d)}-\frac{d x}{4 c \left(a+b x^4\right)^{3/4} \left(c+d x^4\right) (b c-a d)}$$

$$\Big\downarrow \; 858$$

$$\frac{\frac{\frac{3 a d^2 (10 b c-3 a d) \int \frac{\sqrt[4]{b x^4+a}}{d x^4+c}dx}{b c-a d}-\frac{b x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \left(3 a^2 d^2-32 a b c d+8 b^2 c^2\right) \int \frac{1}{\left(\frac{a}{b x^4}+1\right)^{3/4} x}d\frac{1}{x}}{\left(a+b x^4\right)^{3/4} (b c-a d)}}{3 a (b c-a d)}+\frac{b x (3 a d+4 b c)}{3 a \left(a+b x^4\right)^{3/4} (b c-a d)}}{4 c (b c-a d)}-\frac{d x}{4 c \left(a+b x^4\right)^{3/4} \left(c+d x^4\right) (b c-a d)}$$

$$\Big\downarrow \; 807$$

$$\frac{\frac{\frac{3 a d^2 (10 b c-3 a d) \int \frac{\sqrt[4]{b x^4+a}}{d x^4+c}dx}{b c-a d}-\frac{b x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \left(3 a^2 d^2-32 a b c d+8 b^2 c^2\right) \int \frac{1}{\left(\frac{a}{b x^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2 \left(a+b x^4\right)^{3/4} (b c-a d)}}{3 a (b c-a d)}+\frac{b x (3 a d+4 b c)}{3 a \left(a+b x^4\right)^{3/4} (b c-a d)}}{4 c (b c-a d)}-\frac{d x}{4 c \left(a+b x^4\right)^{3/4} \left(c+d x^4\right) (b c-a d)}$$

$$\Big\downarrow \; 229$$

$$\frac{\frac{\frac{3 a d^2 (10 b c-3 a d) \int \frac{\sqrt[4]{b x^4+a}}{d x^4+c}dx}{b c-a d}-\frac{b^{3/2} x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \left(3 a^2 d^2-32 a b c d+8 b^2 c^2\right) \text{EllipticF}\left(\frac{1}{2} \arctan\left(\frac{\sqrt{a}}{\sqrt{b} x^2}\right),2\right)}{\sqrt{a} \left(a+b x^4\right)^{3/4} (b c-a d)}}{3 a (b c-a d)}+\frac{b x (3 a d+4 b c)}{3 a \left(a+b x^4\right)^{3/4} (b c-a d)}}{4 c (b c-a d)}-\frac{d x}{4 c \left(a+b x^4\right)^{3/4} \left(c+d x^4\right) (b c-a d)}$$

$$\Big\downarrow \; 923$$

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(c-\frac{(bc-ad)x^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\right.}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 925

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}+\frac{\int \frac{\sqrt{c}}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+\right.}{}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\sqrt{c}-\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}+\frac{\int \frac{1}{\sqrt{1-\frac{bx^4}{bx^4+a}}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{bx^4+a}}+\sqrt{c}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+\right.}{}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1542

$$\frac{\frac{\frac{3ad^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\left(\frac{\text{EllipticPi}\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}+\frac{\text{EllipticPi}\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right),-1\right)}{2\sqrt[4]{b}c}\right)}{bc-ad}-\frac{b^{3/2}x^3\left(\frac{a}{bx}\right.}{}}{3a(bc-ad)}}{4c(bc-ad)}$$

$$\frac{dx}{4c\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) + ((b*(4*b*c + 3*
a*d)*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) + (-((b^(3/2)*(8*b^2*c^2 - 32*
a*b*c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(S
qrt[b]*x^2)]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) + (3*a*d^2*(1
0*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*(EllipticPi[-(Sqrt[b*c
- a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1]/(2*b
^(1/4)*c) + EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*
x)/(a + b*x^4)^(1/4)], -1]/(2*b^(1/4)*c)))/(b*c - a*d))/(3*a*(b*c - a*d)))
/(4*c*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 404
```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^4)^(3/4), x]
, x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(a + b*x^4)^(1/4)/(c + d*x^4), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 923
```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)]   Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c
- a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^{4}+a)^{\frac{7}{4}}\,(d\,x^{4}+c)^{2}}dx$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

output
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(a+bx^4)^{\frac{7}{4}}\,(c+dx^4)^2}\,dx$$

input
```
integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

output
```
Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \int \frac{1}{(bx^4+a)^{\frac{7}{4}}(dx^4+c)^2}\,dx$$

input
```
integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{7/4}\,(d\,x^4+c)^2}\,dx$$

input
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x)
```

output
```
int(1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{7/4}\,(c+dx^4)^2}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,c^2+2\,(b\,x^4+a)^{\frac{3}{4}}\,acd\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,a\,d^2x^8+(b\,x^4+a)^{\frac{3}{4}}}$$

input
```
int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*c**2 + 2*(a + b*x**4)**(3/4)*a*c*d*x**4 + (a
+ b*x**4)**(3/4)*a*d**2*x**8 + (a + b*x**4)**(3/4)*b*c**2*x**4 + 2*(a + b*
x**4)**(3/4)*b*c*d*x**8 + (a + b*x**4)**(3/4)*b*d**2*x**12),x)
```

3.126 $\int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}$$

output
```
1/4*arctan(1/2*x*2^(3/4)/(x^4+1)^(1/4))*2^(1/4)+1/4*arctanh(1/2*x*2^(3/4)/
(x^4+1)^(1/4))*2^(1/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)+\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}$$

input
```
Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

output
```
(ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))
])/(2*2^(3/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{2-\frac{x^4}{x^4+1}}d\frac{x}{\sqrt[4]{x^4+1}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}}+\sqrt{2}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{\sqrt{2}-\frac{x^2}{\sqrt{x^4+1}}}d\frac{x}{\sqrt[4]{x^4+1}}}{2\sqrt{2}}+\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

input
```
Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

output
```
ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 +
x^4)^(1/4))]/(2*2^(3/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{4}}\left(2\arctan\left(\frac{2^{\frac{1}{4}}\left(x^{4}+1\right)^{\frac{1}{4}}}{x}\right)-\ln\left(\frac{2^{\frac{3}{4}}x+2\left(x^{4}+1\right)^{\frac{1}{4}}}{-2^{\frac{3}{4}}x+2\left(x^{4}+1\right)^{\frac{1}{4}}}\right)\right)}{8}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}\right)\ln\left(\frac{2\sqrt{x^{4}+1}\,\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}\right)\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}x^{2}-2\left(x^{4}+1\right.}{}\right.}{}$

input
```
int(1/(x^4+1)^(1/4)/(x^4+2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*2^(1/4)*(2*arctan(1/x*2^(1/4)*(x^4+1)^(1/4))-ln((2^(3/4)*x+2*(x^4+1)^
(1/4))/(-2^(3/4)*x+2*(x^4+1)^(1/4))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(39) = 78$.

Time = 4.02 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.34

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \frac{1}{32}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{8^{\frac{3}{4}}(x^4+1)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{3}{4}}x}{2\,(x^4+2)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2+8^{\frac{3}{4}}(3\,x^4+2)+16\,(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}(x^4+1)^{\frac{1}{4}}x^3-8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}x^2-8^{\frac{3}{4}}(3\,x^4+2)+16\,(x^4+1)^{\frac{3}{4}}x}{x^4+2}\right)$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

output
```
1/32*8^(3/4)*arctan(1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1)
^(3/4)*x)/(x^4 + 2)) + 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8
*8^(1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(
x^4 + 2)) - 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sq
rt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{\sqrt[4]{x^4+1}\,(x^4+2)}\,dx$$

input
```
integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

output
```
Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+1)^{1/4}\,(x^4+2)}\,dx$$

input
```
int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
```

output
```
int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \int \frac{1}{(x^4+1)^{\frac{1}{4}}\,x^4+2\,(x^4+1)^{\frac{1}{4}}}dx$$

input
```
int(1/(x^4+1)^(1/4)/(x^4+2),x)
```

output
```
int(1/((x**4 + 1)**(1/4)*x**4 + 2*(x**4 + 1)**(1/4)),x)
```

3.127 $\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

output
```
1/2*arctan(a^(1/4)*x/(b*x^4+a)^(1/4))/a^(5/4)+1/2*arctanh(a^(1/4)*x/(b*x^4
+a)^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

input
```
Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

output
```
(ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(
1/4)])/(2*a^(5/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {902, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4(a-b))\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{902}$$

$$\int \frac{1}{a-\frac{x^4(ab-a(b-a))}{a+bx^4}}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}+\frac{\int \frac{1}{\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{a}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2a}+\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

input
```
Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

output
```
ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*a^(5/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x}\right)+\ln\left(\frac{-a^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}\,x-\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{4a^{\frac{5}{4}}}$	65

input
```
int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*arctan(1/a^(1/4)/x*(b*x^4+a)^(1/4))+ln((-a^(1/4)*x-(b*x^4+a)^(1/4)
)/(a^(1/4)*x-(b*x^4+a)^(1/4))))/a^(5/4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

input
```
integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

output
```
-Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(
a + b*x**4)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx=\int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int -\frac{1}{((a-b)x^4-a)(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{1/4}\,(a-x^4\,(a-b))}\,dx$$

input
```
int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
```

output
```
int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

Reduce [F]

$$\int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}}\,dx$$
$$= -\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^4-(b\,x^4+a)^{\frac{1}{4}}\,a-(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)$$

input
```
int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

output
```
- int(1/((a + b*x**4)**(1/4)*a*x**4 - (a + b*x**4)**(1/4)*a - (a + b*x**4
)**(1/4)*b*x**4),x)
```

3.128 $\int (a+bx^4)^p (c+dx^4)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

output
```
x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^4/a,-d*x^4/c)/((1+b*
x^4/a)^p)/((1+d*x^4/c)^q)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^4)^p (c+dx^4)^q \, dx = \frac{5acx(a+bx^4)^p (c+dx^4)^q \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ac \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(bcp \operatorname{AppellF1}\left(\frac{5}{4},1-p,-q,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adq \operatorname{Appell}\right.}$$

input
```
Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
(5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/
a), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^
4)/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx$$

$$\Big\downarrow \text{ 937}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \left(\frac{bx^4}{a}+1\right)^p \left(dx^4+c\right)^q dx$$

$$\Big\downarrow \text{ 937}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^q \left(\frac{dx^4}{c}+1\right)^{-q} \int \left(\frac{bx^4}{a}+1\right)^p \left(\frac{dx^4}{c}+1\right)^q dx$$

$$\Big\downarrow \text{ 936}$$

$$x\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^q \left(\frac{dx^4}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

input
```
Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
(x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)^{q}d x$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

output
```
int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx=\int \left(bx^{4}+a\right)^{p} \left(dx^{4}+c\right)^{q}\,dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^q \,dx$$

input
```
int((a + b*x^4)^p*(c + d*x^4)^q,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^4)^q, x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

output

```
((c + d*x**4)**q*(a + b*x**4)**p*x + 16*int(((c + d*x**4)**q*(a + b*x**4)*
*p*x**4)/(4*a*c*p + 4*a*c*q + a*c + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4
 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 + 4*b*d*p*x**8 + 4*b*d*q*x**8 +
b*d*x**8),x)*a*d*p**2 + 16*int(((c + d*x**4)**q*(a + b*x**4)**p*x**4)/(4*a
*c*p + 4*a*c*q + a*c + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + 4*b*c*p*x*
*4 + 4*b*c*q*x**4 + b*c*x**4 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*
a*d*p*q + 4*int(((c + d*x**4)**q*(a + b*x**4)**p*x**4)/(4*a*c*p + 4*a*c*q
+ a*c + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + 4*b*c*p*x**4 + 4*b*c*q*x*
*4 + b*c*x**4 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*a*d*p + 16*int(
((c + d*x**4)**q*(a + b*x**4)**p*x**4)/(4*a*c*p + 4*a*c*q + a*c + 4*a*d*p*
x**4 + 4*a*d*q*x**4 + a*d*x**4 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 +
4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*b*c*p*q + 16*int(((c + d*x**4)*
*q*(a + b*x**4)**p*x**4)/(4*a*c*p + 4*a*c*q + a*c + 4*a*d*p*x**4 + 4*a*d*q
*x**4 + a*d*x**4 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 + 4*b*d*p*x**8 +
 4*b*d*q*x**8 + b*d*x**8),x)*b*c*q**2 + 4*int(((c + d*x**4)**q*(a + b*x**4
)**p*x**4)/(4*a*c*p + 4*a*c*q + a*c + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x*
*4 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 + 4*b*d*p*x**8 + 4*b*d*q*x**8
+ b*d*x**8),x)*b*c*q + 16*int(((c + d*x**4)**q*(a + b*x**4)**p)/(4*a*c*p +
 4*a*c*q + a*c + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + 4*b*c*p*x**4 + 4
*b*c*q*x**4 + b*c*x**4 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*a*c...
```

3.129 $\int (a+bx^4)^p (c+dx^4)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 282

$$\int (a+bx^4)^p (c+dx^4)^3 \, dx = \frac{3d(15a^2d^2-5abcd(13+4p)+b^2c^2(117+88p+16p^2))\,x(a+bx^4)^{1+p}}{b^3(5+4p)(9+4p)(13+4p)} - \frac{3d^2(3ad-bc(13+4p))x^5(a+bx^4)^{1+p}}{b^2(9+4p)(13+4p)} + \frac{d^3x^9(a+bx^4)^{1+p}}{b(13+4p)} + \frac{\left(b^3c^3(117+88p+16p^2)-\frac{3ad(15a^2d^2-5abcd(13+4p)+b^2c^2(117+88p+16p^2))}{5+4p}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hyperge}}{b^3(9+4p)(13+4p)}$$

output
```
3*d*(15*a^2*d^2-5*a*b*c*d*(13+4*p)+b^2*c^2*(16*p^2+88*p+117))*x*(b*x^4+a)^
(p+1)/b^3/(5+4*p)/(9+4*p)/(13+4*p)-3*d^2*(3*a*d-b*c*(13+4*p))*x^5*(b*x^4+a
)^(p+1)/b^2/(9+4*p)/(13+4*p)+d^3*x^9*(b*x^4+a)^(p+1)/b/(13+4*p)+(b^3*c^3*(
16*p^2+88*p+117)-3*a*d*(15*a^2*d^2-5*a*b*c*d*(13+4*p)+b^2*c^2*(16*p^2+88*p
+117))/(5+4*p))*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/b^3/(9+4
*p)/(13+4*p)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 5.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.49

$$\int (a+bx^4)^p (c+dx^4)^3 \, dx$$
$$= \frac{1}{195} x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(195c^3 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + dx^4\left(117c^2 \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) +5dx^4\left(13c \text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^4}{a}\right)+3dx^4 \text{Hypergeometric2F1}\left(\frac{13}{4},-p,\frac{17}{4},-\frac{bx^4}{a}\right)\right)\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(c + d*x^4)^3,x]
```

output
```
(x*(a + b*x^4)^p*(195*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] +
d*x^4*(117*c^2*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*d*x^4*(13
*c*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^4)/a)] + 3*d*x^4*Hypergeometric
2F1[13/4, -p, 17/4, -((b*x^4)/a)]))))/(195*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^4)^3 (a+bx^4)^p \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -(bx^4+a)^p (dx^4+c) \left(d(9ad-bc(4p+21))x^4+c(ad-bc(4p+13))\right) dx}{b(4p+13)} + \frac{dx(c+dx^4)^2 (a+bx^4)^{p+1}}{b(4p+13)}$$

$$\Big\downarrow \ 25$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\int\left(bx^4+a\right)^p\left(dx^4+c\right)\left(d(9ad-bc(4p+21))x^4+c(ad-bc(4p+13))\right)dx}{b(4p+13)}$$

$$\Big\downarrow \ 1025$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\frac{\int-\left(bx^4+a\right)^p\left(d\left(b^2\left(16p^2+104p+201\right)c^2-6abd(4p+25)c+45a^2d^2\right)x^4+c\left(b^2\left(16p^2+88p+117\right)c^2-2abd(4p+15)c+9a^2d^2\right)\right)dx}{b(4p+9)}+\frac{dx\left(c+dx^4\right)\left(a+b\right.}{}}{b(4p+13)}$$

$$\Big\downarrow \ 25$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}(9ad-bc(4p+21))}{b(4p+9)}-\frac{\int\left(bx^4+a\right)^p\left(d\left(b^2\left(16p^2+104p+201\right)c^2-6abd(4p+25)c+45a^2d^2\right)x^4+c\left(b^2\left(16p^2+88p+117\right)c^2-2abc\right.\right.}{b(4p+9)}}{b(4p+13)}$$

$$\Big\downarrow \ 913$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}(9ad-bc(4p+21))}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}\left(45a^2d^2-6abcd(4p+25)+b^2c^2\left(16p^2+104p+201\right)\right)}{b(4p+5)}-\frac{\left(45a^3d^3-15a^2bcd^2(4p+13)+3ab^2c^2d\left(1\right.\right.}{}}{b(4p+9)}}{b(4p+13)}$$

$$\Big\downarrow \ 779$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}(9ad-bc(4p+21))}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}\left(45a^2d^2-6abcd(4p+25)+b^2c^2\left(16p^2+104p+201\right)\right)}{b(4p+5)}-\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(45a^3d^3-15a^2b\right.}{}}{b(4p+9)}}{b(4p+13)}$$

$$\Big\downarrow \ 778$$

$$\frac{dx\left(c+dx^4\right)^2\left(a+bx^4\right)^{p+1}}{b(4p+13)}-\frac{\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}(9ad-bc(4p+21))}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}\left(45a^2d^2-6abcd(4p+25)+b^2c^2\left(16p^2+104p+201\right)\right)}{b(4p+5)}-\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(45a^3d^3-15a^2\right.}{}}{b(4\ldots}}{b(4p+13)}$$

input
```
Int[(a + b*x^4)^p*(c + d*x^4)^3,x]
```

output
```
(d*x*(a + b*x^4)^(1 + p)*(c + d*x^4)^2)/(b*(13 + 4*p)) - ((d*(9*a*d - b*c*
(21 + 4*p))*x*(a + b*x^4)^(1 + p)*(c + d*x^4))/(b*(9 + 4*p)) - ((d*(45*a^2
*d^2 - 6*a*b*c*d*(25 + 4*p) + b^2*c^2*(201 + 104*p + 16*p^2))*x*(a + b*x^4
)^(1 + p))/(b*(5 + 4*p)) - ((45*a^3*d^3 - 15*a^2*b*c*d^2*(13 + 4*p) + 3*a*
b^2*c^2*d*(117 + 88*p + 16*p^2) - b^3*c^3*(585 + 908*p + 432*p^2 + 64*p^3)
)*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(b*(5 + 4
*p)*(1 + (b*x^4)/a)^p))/(b*(9 + 4*p)))/(b*(13 + 4*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int (b\,x^{4}+a)^{p}\left(d\,x^{4}+c\right)^{3}dx$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^3,x)
```

output
```
int((b*x^4+a)^p*(d*x^4+c)^3,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p\left(c+dx^4\right)^3\,dx=\int \left(dx^4+c\right)^3\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^12 + 3*c*d^2*x^8 + 3*c^2*d*x^4 + c^3)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p*(d*x**4+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int \left(dx^4+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^3*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int \left(dx^4+c\right)^3 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^3*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx=\int \left(b\,x^4+a\right)^p\,\left(d\,x^4+c\right)^3 \,dx$$

input
```
int((a + b*x^4)^p*(c + d*x^4)^3,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^4)^3, x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx=\text{too large to display}$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^3,x)
```

output
```
(180*(a + b*x**4)**p*a**3*d**3*p*x - 240*(a + b*x**4)**p*a**2*b*c*d**2*p**
2*x - 780*(a + b*x**4)**p*a**2*b*c*d**2*p*x - 144*(a + b*x**4)**p*a**2*b*d
**3*p**2*x**5 - 36*(a + b*x**4)**p*a**2*b*d**3*p*x**5 + 192*(a + b*x**4)**
p*a*b**2*c**2*d*p**3*x + 1056*(a + b*x**4)**p*a*b**2*c**2*d*p**2*x + 1404*
(a + b*x**4)**p*a*b**2*c**2*d*p*x + 192*(a + b*x**4)**p*a*b**2*c*d**2*p**3
*x**5 + 672*(a + b*x**4)**p*a*b**2*c*d**2*p**2*x**5 + 156*(a + b*x**4)**p*
a*b**2*c*d**2*p*x**5 + 64*(a + b*x**4)**p*a*b**2*d**3*p**3*x**9 + 96*(a +
b*x**4)**p*a*b**2*d**3*p**2*x**9 + 20*(a + b*x**4)**p*a*b**2*d**3*p*x**9 +
 64*(a + b*x**4)**p*b**3*c**3*p**3*x + 432*(a + b*x**4)**p*b**3*c**3*p**2*
x + 908*(a + b*x**4)**p*b**3*c**3*p*x + 585*(a + b*x**4)**p*b**3*c**3*x +
192*(a + b*x**4)**p*b**3*c**2*d*p**3*x**5 + 1104*(a + b*x**4)**p*b**3*c**2
*d*p**2*x**5 + 1668*(a + b*x**4)**p*b**3*c**2*d*p*x**5 + 351*(a + b*x**4)*
*p*b**3*c**2*d*x**5 + 192*(a + b*x**4)**p*b**3*c*d**2*p**3*x**9 + 912*(a +
 b*x**4)**p*b**3*c*d**2*p**2*x**9 + 996*(a + b*x**4)**p*b**3*c*d**2*p*x**9
 + 195*(a + b*x**4)**p*b**3*c*d**2*x**9 + 64*(a + b*x**4)**p*b**3*d**3*p**
3*x**13 + 240*(a + b*x**4)**p*b**3*d**3*p**2*x**13 + 236*(a + b*x**4)**p*b
**3*d**3*p*x**13 + 45*(a + b*x**4)**p*b**3*d**3*x**13 - 46080*int((a + b*x
**4)**p/(256*a*p**4 + 1792*a*p**3 + 4064*a*p**2 + 3248*a*p + 585*a + 256*b
*p**4*x**4 + 1792*b*p**3*x**4 + 4064*b*p**2*x**4 + 3248*b*p*x**4 + 585*b*x
**4),x)*a**4*d**3*p**5 - 322560*int((a + b*x**4)**p/(256*a*p**4 + 1792*...
```

3.130 $\int (a+bx^4)^p (c+dx^4)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 173

$$\int (a+bx^4)^p (c+dx^4)^2 \, dx = -\frac{d(5ad-2bc(9+4p))x(a+bx^4)^{1+p}}{b^2(5+4p)(9+4p)} + \frac{d^2x^5(a+bx^4)^{1+p}}{b(9+4p)} + \frac{(5a^2d^2-2abcd(9+4p)+b^2c^2(45+56p+16p^2))\, x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p\right.}{b^2(5+4p)(9+4p)}$$

output
```
-d*(5*a*d-2*b*c*(9+4*p))*x*(b*x^4+a)^(p+1)/b^2/(5+4*p)/(9+4*p)+d^2*x^5*(b*
x^4+a)^(p+1)/b/(9+4*p)+(5*a^2*d^2-2*a*b*c*d*(9+4*p)+b^2*c^2*(16*p^2+56*p+4
5))*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/b^2/(5+4*p)/(9+4*p)/
((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int (a+bx^4)^p (c+dx^4)^2\,dx = \frac{1}{45}x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(45c^2 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+dx^4\left(18c\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+5dx^4\,\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^4}{a}\right)\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^p*(45*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d
*x^4*(18*c*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*d*x^4*Hyperge
ometric2F1[9/4, -p, 13/4, -((b*x^4)/a)])))/(45*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^4)^2 (a+bx^4)^p\,dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -(bx^4+a)^p \left(d(5ad-bc(4p+13))x^4+c(ad-bc(4p+9))\right)dx}{b(4p+9)}+\frac{dx(c+dx^4)(a+bx^4)^{p+1}}{b(4p+9)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}}{b(4p+9)}-\frac{\int\left(bx^4+a\right)^p\left(d(5ad-bc(4p+13))x^4+c(ad-bc(4p+9))\right)dx}{b(4p+9)}$$

$$\Big\downarrow \textbf{913}$$

$$\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}(5ad-bc(4p+13))}{b(4p+5)}-\frac{\left(5a^2d^2-2abcd(4p+9)+b^2c^2\left(16p^2+56p+45\right)\right)\int\left(bx^4+a\right)^pdx}{b(4p+5)}}{b(4p+9)}$$

$$\Big\downarrow \textbf{779}$$

$$\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}(5ad-bc(4p+13))}{b(4p+5)}-\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(5a^2d^2-2abcd(4p+9)+b^2c^2\left(16p^2+56p+45\right)\right)\int\left(\frac{bx^4}{a}+1\right)^pdx}{b(4p+5)}}{b(4p+9)}$$

$$\Big\downarrow \textbf{778}$$

$$\frac{dx\left(c+dx^4\right)\left(a+bx^4\right)^{p+1}}{b(4p+9)}-\frac{\frac{dx\left(a+bx^4\right)^{p+1}(5ad-bc(4p+13))}{b(4p+5)}-\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(5a^2d^2-2abcd(4p+9)+b^2c^2\left(16p^2+56p+45\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)}{b(4p+5)}}{b(4p+9)}$$

input
```
Int[(a + b*x^4)^p*(c + d*x^4)^2,x]
```

output
```
(d*x*(a + b*x^4)^(1 + p)*(c + d*x^4))/(b*(9 + 4*p)) - ((d*(5*a*d - b*c*(13
 + 4*p))*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) - ((5*a^2*d^2 - 2*a*b*c*d*(9
 + 4*p) + b^2*c^2*(45 + 56*p + 16*p^2))*x*(a + b*x^4)^p*Hypergeometric2F1[
1/4, -p, 5/4, -((b*x^4)/a)])/(b*(5 + 4*p)*(1 + (b*x^4)/a)^p))/(b*(9 + 4*p)
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)^{2}dx$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^p*(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \int \left(dx^4+c\right)^2 \left(bx^4+a\right)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 81.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.69

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p c d x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{a^p d^2 x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{9}{4}, -p \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(d*x**4+c)**2,x)
```

output
```
a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + a**p*c*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*
exp_polar(I*pi)/a)/(2*gamma(9/4)) + a**p*d**2*x**9*gamma(9/4)*hyper((9/4,
-p), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int (a+bx^4)^p (c+dx^4)^2 \, dx = \int (dx^4+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a+bx^4)^p (c+dx^4)^2 \, dx = \int (dx^4+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^4)^p (c+dx^4)^2 \, dx = \int (b\,x^4+a)^p \, (d\,x^4+c)^2 \, dx$$

input
```
int((a + b*x^4)^p*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int (a + bx^4)^p (c + dx^4)^2 \, dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^p*(d*x^4+c)^2,x)
```

output
```
( - 20*(a + b*x**4)**p*a**2*d**2*p*x + 32*(a + b*x**4)**p*a*b*c*d*p**2*x +
 72*(a + b*x**4)**p*a*b*c*d*p*x + 16*(a + b*x**4)**p*a*b*d**2*p**2*x**5 +
4*(a + b*x**4)**p*a*b*d**2*p*x**5 + 16*(a + b*x**4)**p*b**2*c**2*p**2*x +
56*(a + b*x**4)**p*b**2*c**2*p*x + 45*(a + b*x**4)**p*b**2*c**2*x + 32*(a
+ b*x**4)**p*b**2*c*d*p**2*x**5 + 80*(a + b*x**4)**p*b**2*c*d*p*x**5 + 18*
(a + b*x**4)**p*b**2*c*d*x**5 + 16*(a + b*x**4)**p*b**2*d**2*p**2*x**9 + 2
4*(a + b*x**4)**p*b**2*d**2*p*x**9 + 5*(a + b*x**4)**p*b**2*d**2*x**9 + 12
80*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*p**
3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**3*d**2*p**4 + 4
800*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*p*
*3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**3*d**2*p**3 +
4720*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*p
**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**3*d**2*p**2 +
 900*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*p
**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**3*d**2*p - 20
48*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*p**
3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**2*b*c*d*p**5 -
12288*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + 64*b*
p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**2*b*c*d*p**4
 - 24832*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a +...
```

3.131 $\int (a+bx^4)^p (c+dx^4)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int (a+bx^4)^p (c+dx^4)\, dx = \frac{dx(a+bx^4)^{1+p}}{b(5+4p)} + \left(c - \frac{ad}{5b+4bp}\right) x (a+bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right)$$

output
```
d*x*(b*x^4+a)^(p+1)/b/(5+4*p)+(c-a*d/(4*b*p+5*b))*x*(b*x^4+a)^p*hypergeom(
[1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int (a+bx^4)^p (c+dx^4)\, dx = \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(d(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p + (-ad+bc(5+4p))\,\text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\right.\right.}{b(5+4p)}$$

input
```
Integrate[(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(x*(a + b*x^4)^p*(d*(a + b*x^4)*(1 + (b*x^4)/a)^p + (-(a*d) + b*c*(5 + 4*p
))*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)]))/(b*(5 + 4*p)*(1 + (b*x^
4)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)\left(a+bx^4\right)^p dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{4bp+5b}\right)\int \left(bx^4+a\right)^p dx+\frac{dx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad}{4bp+5b}\right)\int \left(\frac{bx^4}{a}+1\right)^p dx+\frac{dx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad}{4bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{dx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

input
```
Int[(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(d*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c - (a*d)/(5*b + 4*b*p))*x*(a
+ b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^
p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)dx$$

input
```
int((b*x^4+a)^p*(d*x^4+c),x)
```

output
```
int((b*x^4+a)^p*(d*x^4+c),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)\, dx = \int \left(dx^{4}+c\right)\left(bx^{4}+a\right)^{p}\, dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int (a+bx^4)^p (c+dx^4)\ dx = \frac{a^p cx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p dx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(d*x**4+c),x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int (a+bx^4)^p (c+dx^4)\ dx = \int (dx^4+c)(bx^4+a)^p\ dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)\, dx = \int \left(dx^4+c\right)\left(bx^4+a\right)^p dx$$

input
```
integrate((b*x^4+a)^p*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)\, dx = \int {\left(b\,x^4+a\right)}^p\,\left(d\,x^4+c\right) \,d x$$

input
```
int((a + b*x^4)^p*(c + d*x^4),x)
```

output
```
int((a + b*x^4)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \left(c+dx^4\right)\, dx$$
$$= \frac{4(b\,x^4+a)^p\,adpx+4(b\,x^4+a)^p\,bcpx+5(b\,x^4+a)^p\,bcx+4(b\,x^4+a)^p\,bdp\,x^5+(b\,x^4+a)^p\,bd\,x^5-64\Big(\int}{}$$

input
```
int((b*x^4+a)^p*(d*x^4+c),x)
```

output

```
(4*(a + b*x**4)**p*a*d*p*x + 4*(a + b*x**4)**p*b*c*p*x + 5*(a + b*x**4)**p
*b*c*x + 4*(a + b*x**4)**p*b*d*p*x**5 + (a + b*x**4)**p*b*d*x**5 - 64*int(
(a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 +
 5*b*x**4),x)*a**2*d*p**3 - 96*int((a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5
*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*p**2 - 20*int((a +
 b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b
*x**4),x)*a**2*d*p + 256*int((a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a + 1
6*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a*b*c*p**4 + 704*int((a + b*x**
4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4)
,x)*a*b*c*p**3 + 560*int((a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*
p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a*b*c*p**2 + 100*int((a + b*x**4)**
p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*
a*b*c*p)/(b*(16*p**2 + 24*p + 5))
```

3.132 $\int (a+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Integrate[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p \, dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \left(\frac{bx^4}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((b*x^4+a)^p,x)
```

output
```
int((b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p\,dx = \int \left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^4\right)^p\,dx = \frac{a^p x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, -p\\ \frac{5}{4}\end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**p,x)
```

output
```
a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(5/4))
```

Maxima [F]

$$\int (a+bx^4)^p\,dx = \int (bx^4+a)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a+bx^4)^p\,dx = \int (bx^4+a)^p\,dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^4)^p\,dx = \frac{x\,(b\,x^4+a)^p\,{}_2F_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^p}$$

input
```
int((a + b*x^4)^p,x)
```

output
```
(x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^4\right)^p\,dx$$
$$=\frac{\left(b\,x^4+a\right)^p x+16\left(\int \frac{\left(b\,x^4+a\right)^p}{4bp\,x^4+b\,x^4+4ap+a}dx\right)a\,p^2+4\left(\int \frac{\left(b\,x^4+a\right)^p}{4bp\,x^4+b\,x^4+4ap+a}dx\right)ap}{4p+1}$$

input
```
int((b*x^4+a)^p,x)
```

output
```
((a + b*x**4)**p*x + 16*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x*
*4),x)*a*p**2 + 4*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x**4),x)
*a*p)/(4*p + 1)
```

3.133 $\int \frac{(a+bx^4)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,-p,1,5/4,-b*x^4/a,-d*x^4/c)/c/((1+b*x^4/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx =$$
$$-\frac{5acx(a+bx^4)^p\operatorname{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)\left(-5ac\operatorname{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+4x^4\left(-bcp\operatorname{AppellF1}\left(\frac{5}{4},1-p,1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x^4),x]
```

output
```
(-5*a*c*x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), -((d*x^4)/
c)])/((c + d*x^4)*(-5*a*c*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + 4*x^4*(-(b*c*p*AppellF1[5/4, 1 - p, 1, 9/4, -((b*x^4)/a), -((d*x^4
)/c)]) + a*d*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^4}{a}+1\right)^p}{dx^4+c}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c}$$

input
```
Int[(a + b*x^4)^p/(c + d*x^4),x]
```

output
```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(c
*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^p/(d*x^4+c),x)
```

output
```
int((b*x^4+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(bx^4+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(bx^4+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(bx^4+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^p}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x^4),x)
```

output
```
int((a + b*x^4)^p/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(b\,x^4+a)^p}{d\,x^4+c}dx$$

input
```
int((b*x^4+a)^p/(d*x^4+c),x)
```

output
```
int((a + b*x**4)**p/(c + d*x**4),x)
```

3.134 $\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx=\frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,-p,2,5/4,-b*x^4/a,-d*x^4/c)/c^2/((1+b*x^4/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.52 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx=$$

$$-\frac{5acx(a+bx^4)^p\,\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{(c+dx^4)^2\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4x^4\left(bcp\,\text{AppellF1}\left(\frac{5}{4},1-p,2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-\right.\right.}$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x^4)^2,x]
```

output
```
(-5*a*c*x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), -((d*x^4)/
c)])/((c + d*x^4)^2*(-5*a*c*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] - 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, 2, 9/4, -((b*x^4)/a), -((d*x^4
)/c)] - 2*a*d*AppellF1[5/4, -p, 3, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2} dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^4}{a}+1\right)^p}{(dx^4+c)^2} dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^4)^p/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(c
^2*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{(d\,x^4+c)^2}dx$$

input
```
int((b*x^4+a)^p/(d*x^4+c)^2,x)
```

output
```
int((b*x^4+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2} \, dx = \int \frac{(bx^4+a)^p}{(dx^4+c)^2} \, dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2} \, dx = \int \frac{(bx^4+a)^p}{(dx^4+c)^2} \, dx$$

input
```
integrate((b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^4)^p/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^4+a)^p}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int((b*x^4+a)^p/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**4)**p/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file