Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.3/52-1.1.3.3-c

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**147**]. This is test number [52].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (147)	0.00 (0)
Mathematica	99.32 (146)	0.68 (1)
Maple	50.34 (74)	49.66 (73)
Fricas	49.66 (73)	50.34 (74)
Sympy	46.26 (68)	53.74 (79)
Mupad	44.90 (66)	55.10 (81)
Reduce	42.86 (63)	57.14 (84)
Maxima	28.57 (42)	71.43 (105)
Giac	26.53 (39)	73.47 (108)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

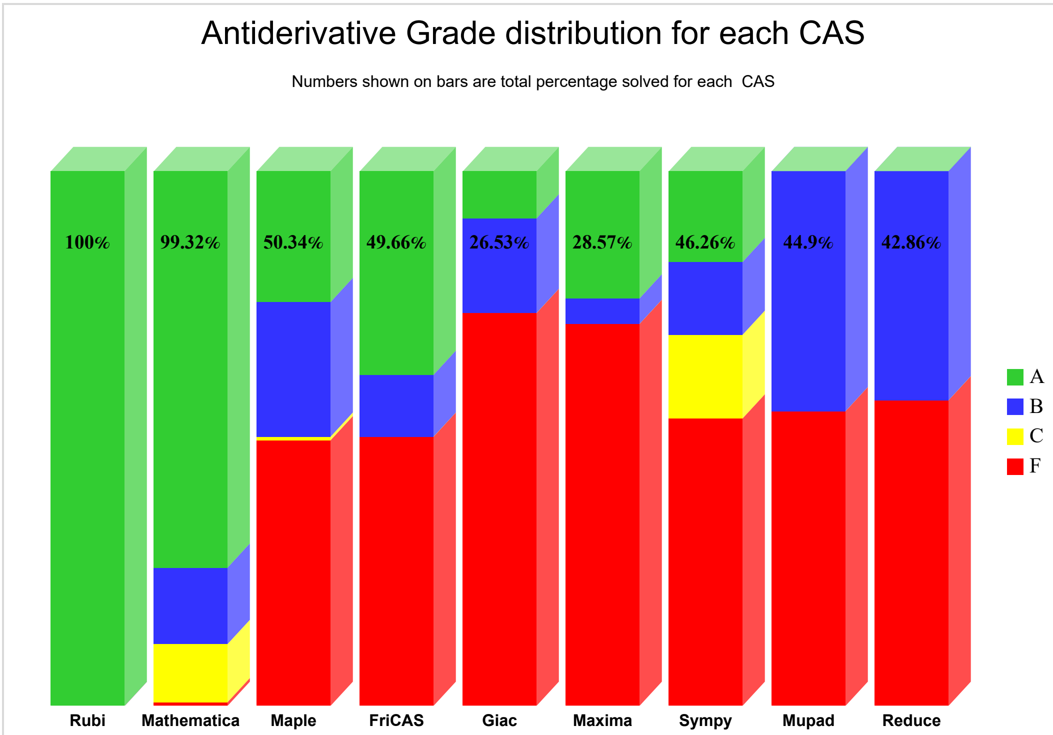
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

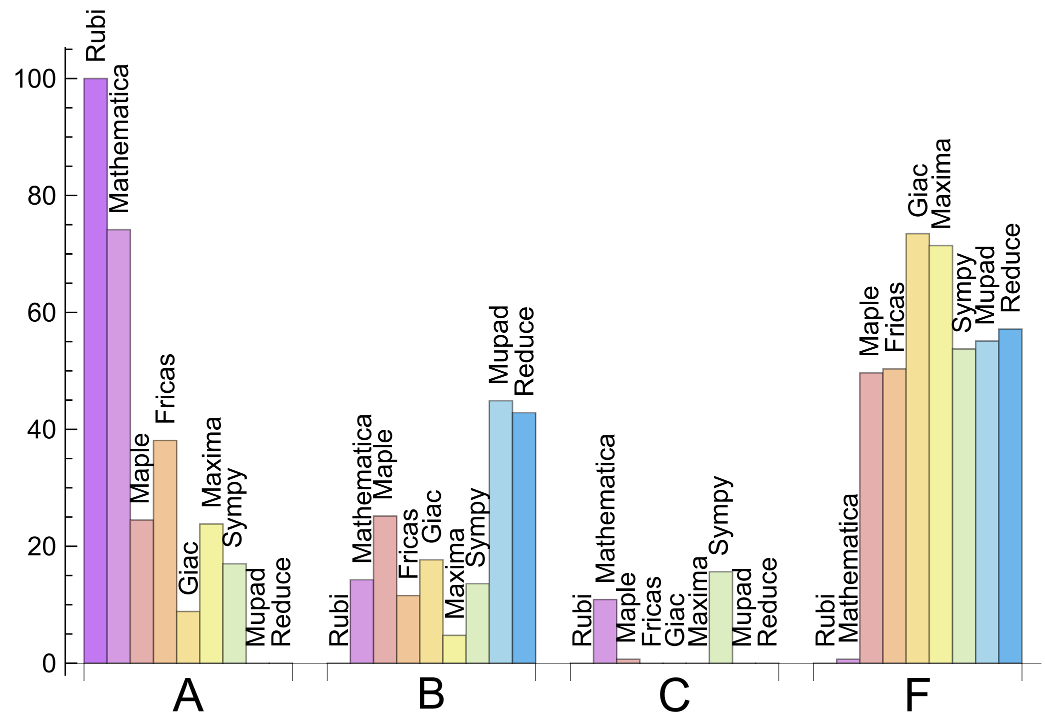
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	74.150	14.286	10.884	0.680
Fricas	38.095	11.565	0.000	50.340
Maple	24.490	25.170	0.680	49.660
Maxima	23.810	4.762	0.000	71.429
Sympy	17.007	13.605	15.646	53.741
Giac	8.844	17.687	0.000	73.469
Mupad	0.000	44.898	0.000	55.102
Reduce	0.000	42.857	0.000	57.143

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	73	100.00	0.00	0.00
Fricas	74	79.73	0.00	20.27
Sympy	79	65.82	13.92	20.25
Mupad	81	0.00	100.00	0.00
Reduce	84	100.00	0.00	0.00
Maxima	105	100.00	0.00	0.00
Giac	108	73.15	0.00	26.85

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Giac	0.15
Fricas	0.29
Rubi	0.49
Reduce	0.61
Mathematica	1.30
Maple	1.74
Mupad	2.32
Sympy	9.51

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	116.79	1.36	117.50	1.36
Rubi	126.07	1.07	96.00	1.00
Mathematica	206.10	2.06	113.00	0.98
Maple	288.82	2.02	171.00	1.72
Giac	373.05	2.61	232.00	2.51
Fricas	616.05	3.69	273.00	2.61
Sympy	618.38	5.53	183.50	2.18
Reduce	758.37	4.03	227.00	2.16
Mupad	898.71	4.97	107.50	1.44

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

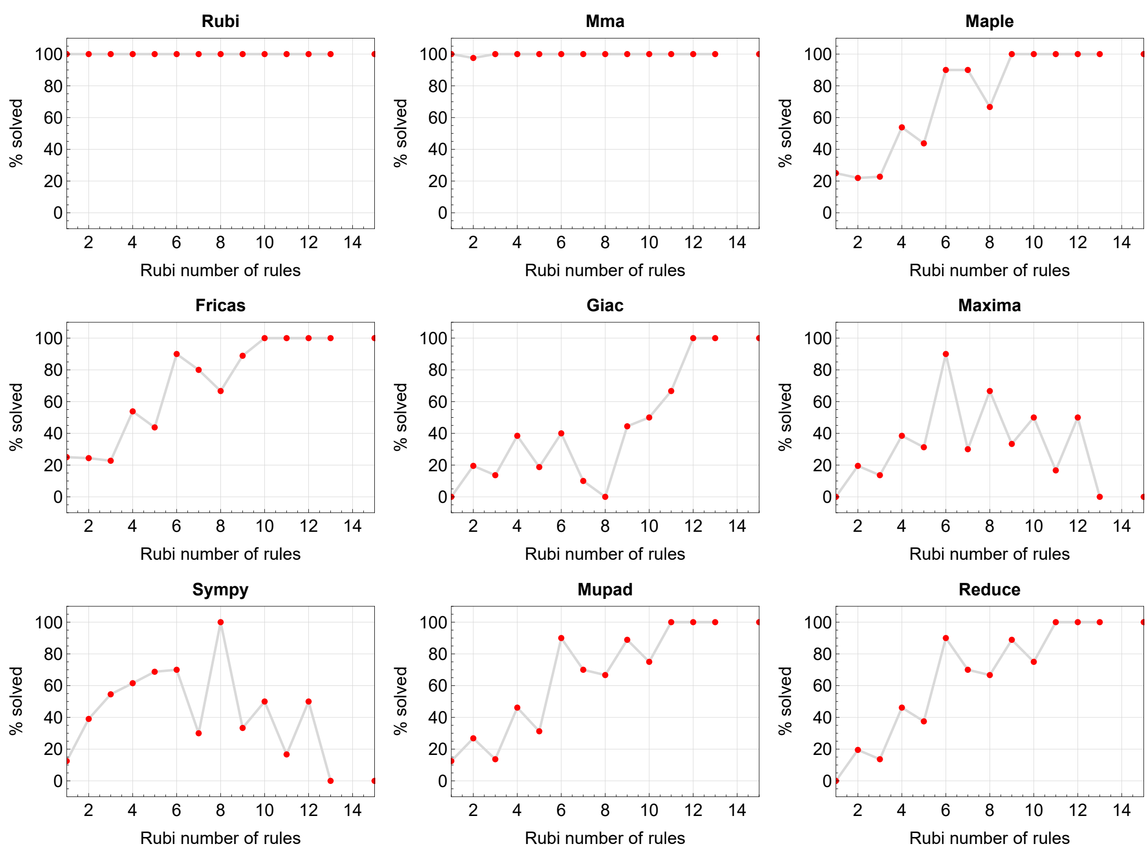


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

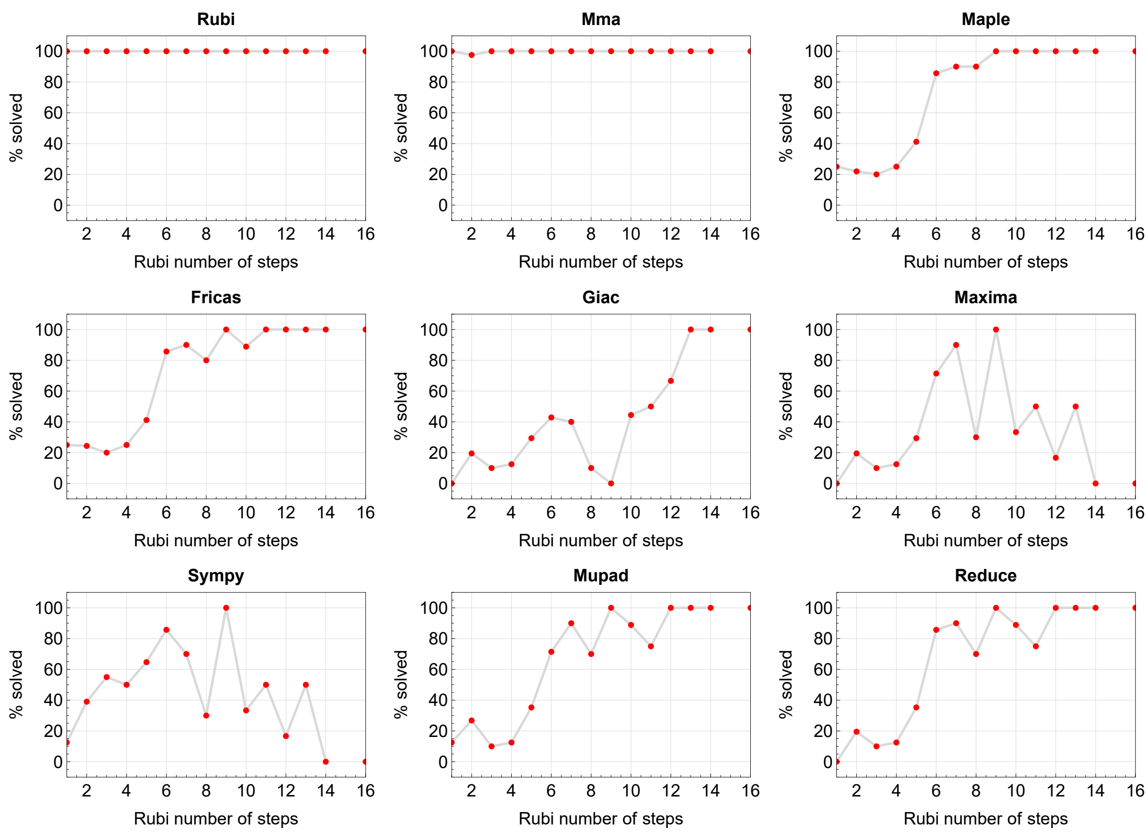


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

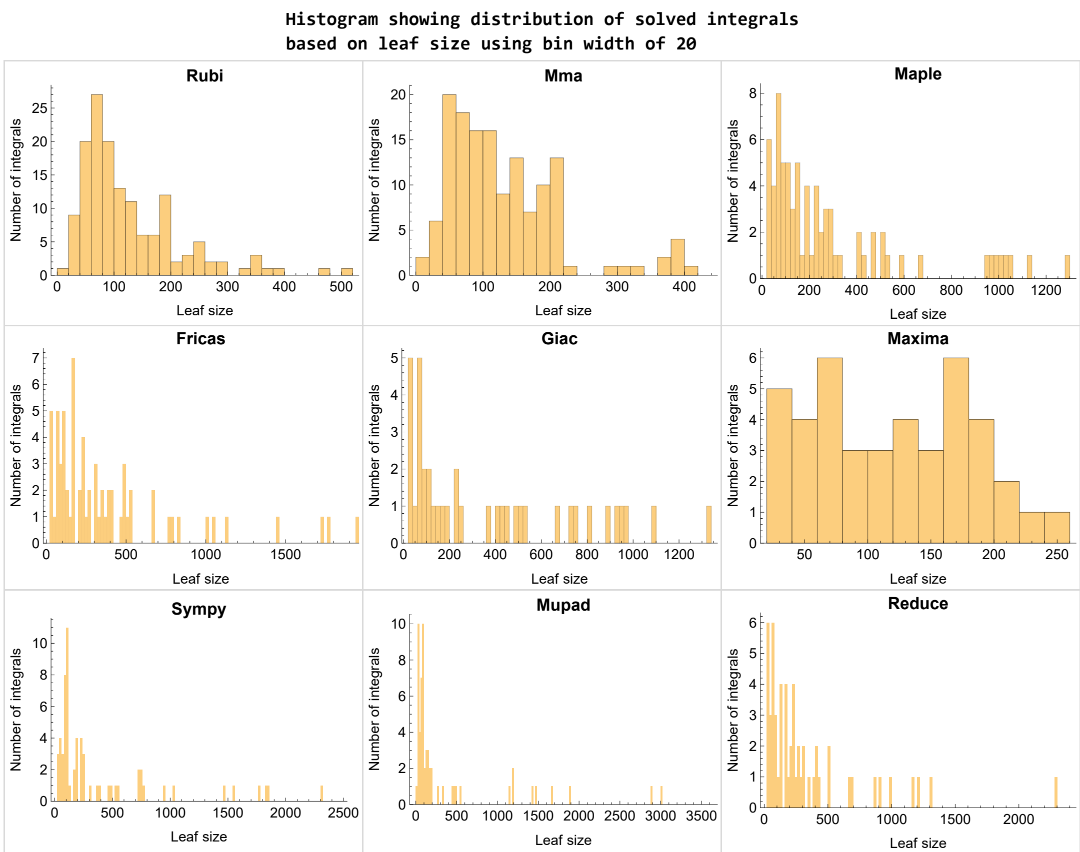


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

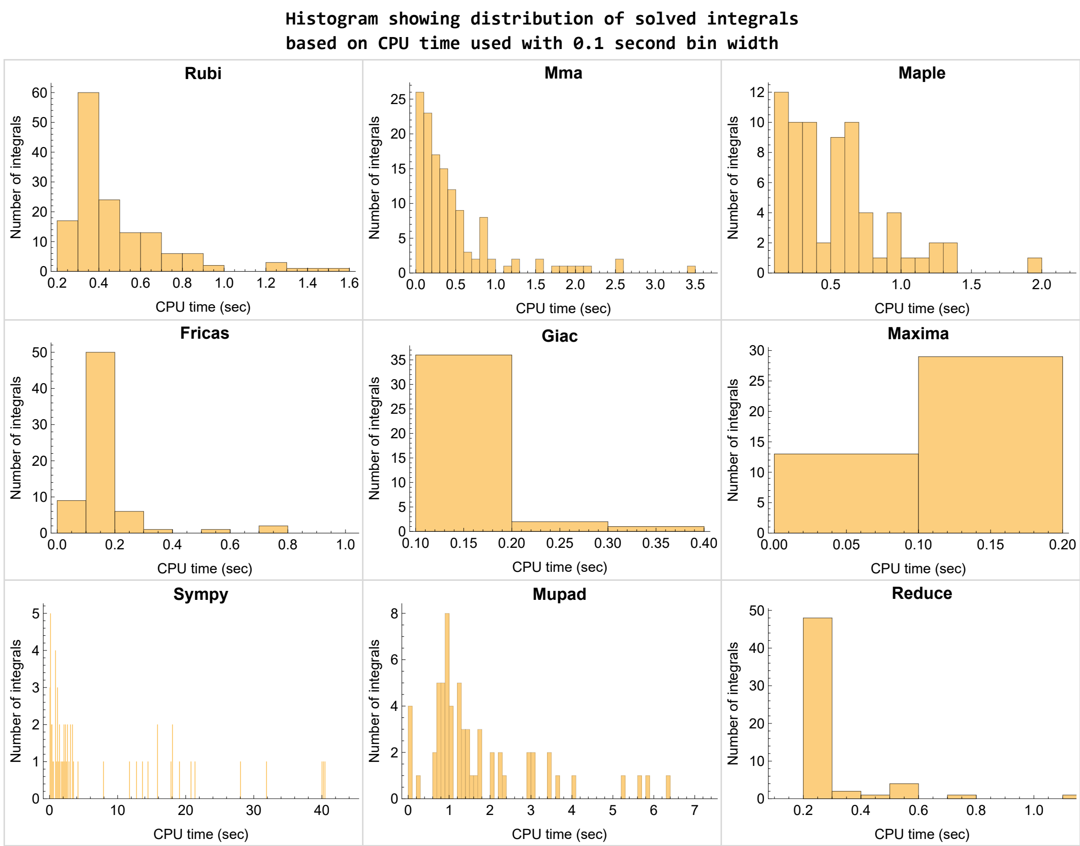


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

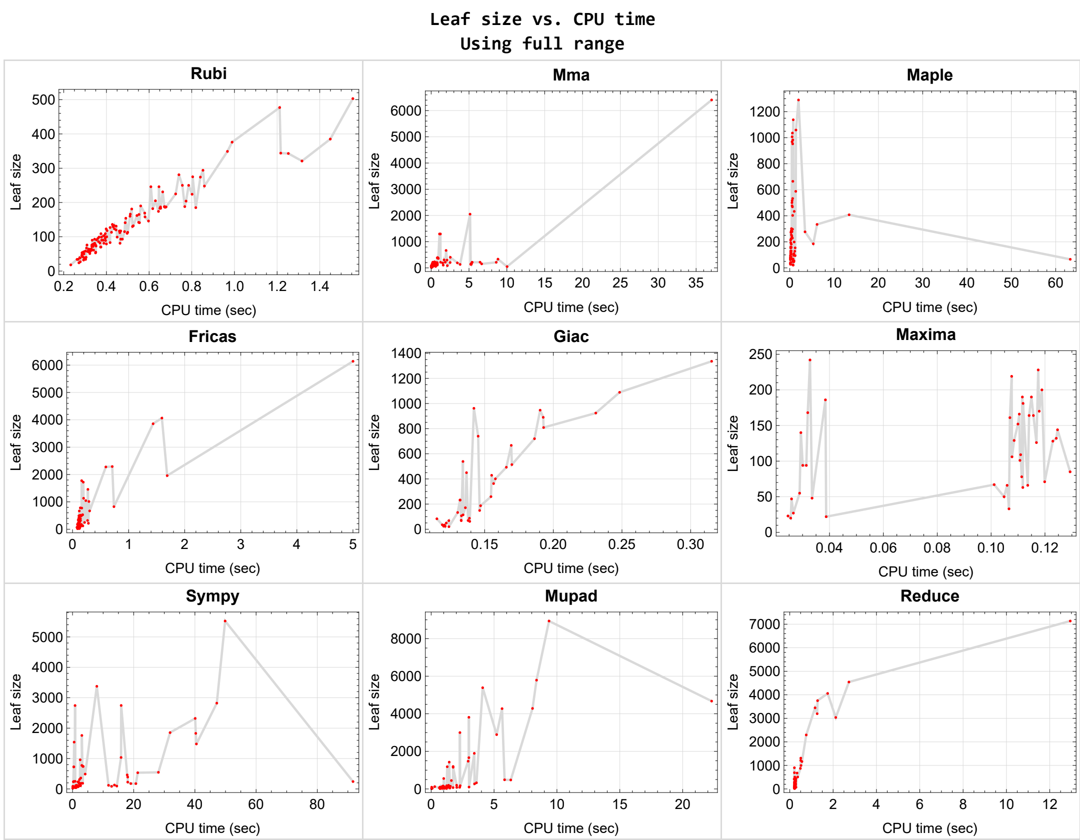


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{49, 89, 90, 97, 98, 101, 102, 105, 106, 109, 110, 113, 114, 117, 118, 123, 124, 125, 126, 140, 141, 142, 143, 144, 145, 146, 147\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

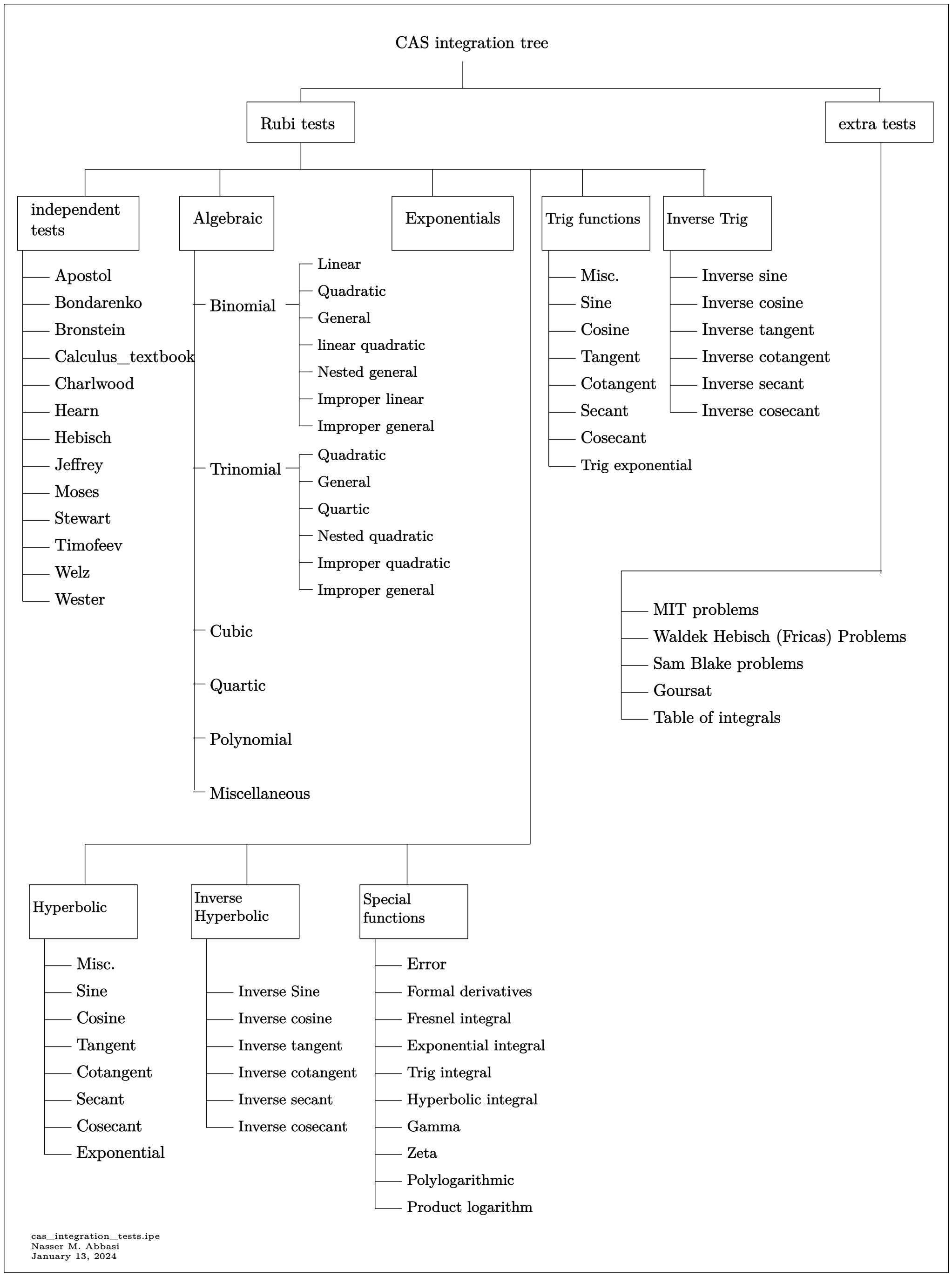
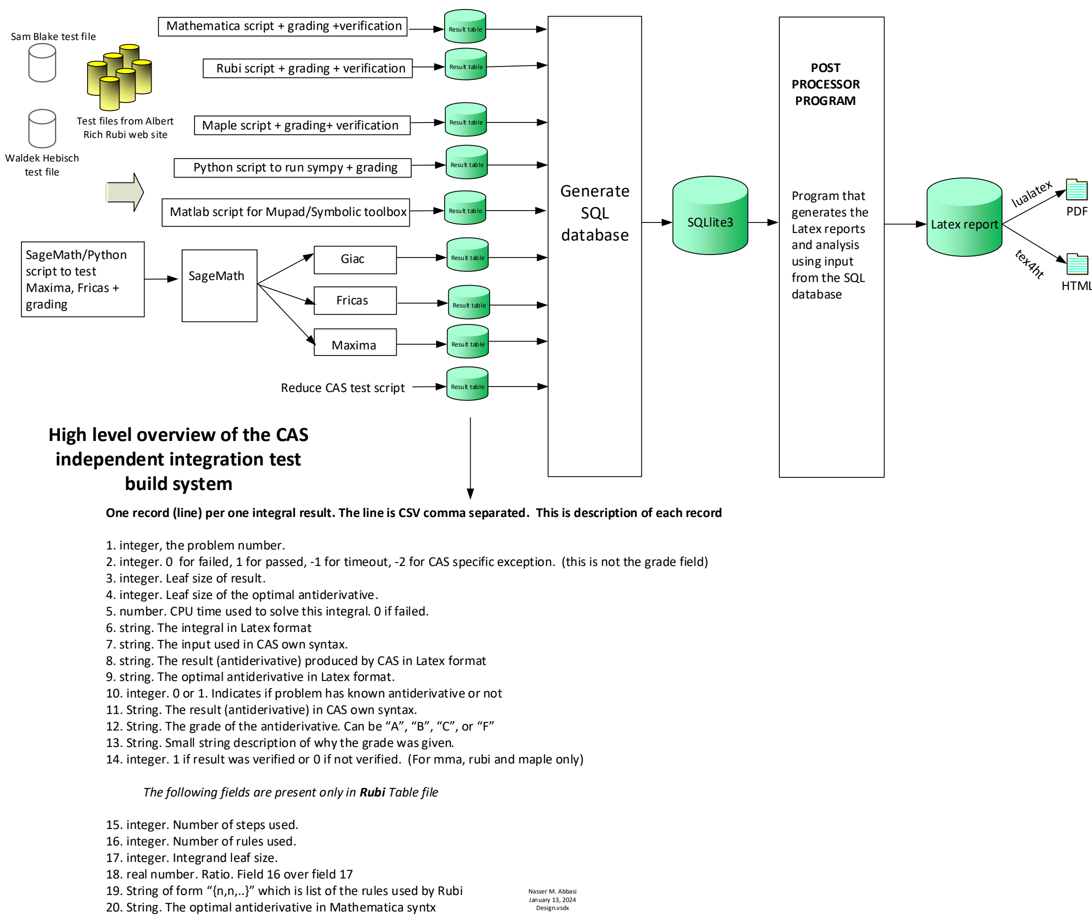


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 99, 100, 103, 104, 107, 108, 111, 112, 115, 116, 119, 120, 121, 122, 126, 127, 128, 130, 131, 132, 133, 135 }

B grade { 49, 97, 98, 101, 102, 105, 106, 109, 110, 113, 114, 117, 118, 123, 124, 125, 143, 144, 145, 146, 147 }

C grade { 1, 51, 53, 81, 82, 83, 89, 90, 129, 136, 137, 138, 139, 140, 141, 142 }

F normal fail { 134 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 11, 12, 13, 14, 18, 19, 20, 21, 25, 26, 27, 28, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 75, 76, 77 }

B grade { 7, 8, 9, 10, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 127, 128, 129, 130, 135, 136, 137 }

C grade { 55 }

F normal fail { 49, 54, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 41, 42, 46, 47, 48, 50, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 70, 128, 129, 130, 135, 136, 137, 138 }

B grade { 10, 17, 31, 37, 38, 39, 40, 43, 44, 45, 67, 68, 69, 75, 76, 77, 127 }

C grade { }

F normal fail { 49, 51, 54, 63, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 101, 102, 105, 106, 109, 110, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 64, 65, 95, 96, 99, 100, 103, 104, 107, 108, 111, 112, 115, 116 }

Maxima

A grade { 2, 3, 4, 5, 7, 11, 12, 13, 14, 18, 19, 20, 21, 25, 28, 35, 40, 41, 42, 50, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 75, 76, 77 }

B grade { 6, 26, 27, 32, 33, 34, 39 }

C grade { }

F normal fail { 1, 8, 9, 10, 15, 16, 17, 22, 23, 24, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 2, 3, 7, 28, 50, 55, 56, 57, 58, 59, 60, 61, 62 }

B grade { 9, 10, 16, 17, 23, 24, 27, 30, 31, 33, 34, 35, 38, 39, 40, 41, 42, 44, 45, 67, 68, 69, 70, 75, 76, 77 }

C grade { }

F normal fail { 1, 46, 47, 48, 49, 51, 52, 53, 54, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-1) timedout fail { }

F(-2) exception fail { 4, 5, 6, 8, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 25, 26, 29, 32, 36, 37, 43, 119, 120, 121, 127, 128, 129, 136, 137 }

Mupad

A grade { }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 68, 69, 70, 75, 76, 77, 122, 130, 138 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 48, 49, 51, 52, 53, 54, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-2) exception fail { }

Sympy

A grade { 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 25, 26, 27, 28, 35, 55, 56, 57, 58, 59, 60 }

B grade { 34, 41, 42, 50, 61, 62, 67, 68, 69, 70, 75, 76, 77, 127, 128, 129, 130, 136, 137, 138 }

C grade { 71, 72, 73, 78, 81, 82, 83, 84, 91, 95, 96, 99, 100, 104, 107, 108, 111, 112, 116, 119, 120, 121, 122 }

F normal fail { 1, 8, 9, 10, 15, 16, 22, 23, 29, 30, 32, 33, 36, 37, 39, 40, 43, 44, 46, 47, 48, 49, 51, 52, 53, 63, 64, 65, 66, 79, 80, 85, 88, 89, 90, 94, 97, 98, 101, 102, 103, 105, 106, 109, 110, 113, 114, 115, 117, 118, 125, 135 }

F(-1) timedout fail { 17, 24, 31, 38, 45, 54, 74, 144, 145, 146, 147 }

F(-2) exception fail { 86, 87, 92, 93, 123, 124, 126, 131, 132, 133, 134, 139, 140, 141, 142, 143 }

Reduce

A grade { }

B grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 75, 76, 77 }

C grade { }

F normal fail { 1, 49, 51, 52, 53, 54, 63, 64, 65, 66, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	503	48	408	0	0	0	0	172	0
N.S.	1	0.92	0.09	0.75	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.555	10.029	13.389	0.000	0.000	0.000	0.000	0.268	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	83	78	63	66	66	85	68	64	94
N.S.	1	1.41	1.32	1.07	1.12	1.12	1.44	1.15	1.08	1.59
time (sec)	N/A	0.417	0.026	0.670	0.114	0.080	0.130	0.133	0.221	0.690

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	81	78	63	66	66	85	68	64	94
N.S.	1	1.37	1.32	1.07	1.12	1.12	1.44	1.15	1.08	1.59
time (sec)	N/A	0.464	0.008	0.126	0.106	0.116	0.142	0.124	0.205	0.042

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	154	118	160	164	311	461	0	267	173
N.S.	1	1.28	0.98	1.33	1.37	2.59	3.84	0.00	2.22	1.44
time (sec)	N/A	0.491	0.352	0.198	0.116	0.130	17.862	0.000	0.233	2.307

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	99	84	115	126	213	129	0	173	99
N.S.	1	1.15	0.98	1.34	1.47	2.48	1.50	0.00	2.01	1.15
time (sec)	N/A	0.377	0.304	0.196	0.117	0.097	13.699	0.000	0.233	1.344

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	72	52	84	106	133	95	0	98	92
N.S.	1	1.18	0.85	1.38	1.74	2.18	1.56	0.00	1.61	1.51
time (sec)	N/A	0.332	0.178	0.173	0.108	0.139	14.466	0.000	0.240	1.415

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	72	50	104	42	62	37	58
N.S.	1	1.00	1.00	1.85	1.28	2.67	1.08	1.59	0.95	1.49
time (sec)	N/A	0.295	0.020	0.132	0.105	0.142	0.988	0.139	0.218	0.651

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	100	232	0	492	0	0	231	149
N.S.	1	1.09	0.96	2.23	0.00	4.73	0.00	0.00	2.22	1.43
time (sec)	N/A	0.500	0.335	0.513	0.000	0.113	0.000	0.000	0.257	0.961

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	162	122	477	0	773	0	401	996	1195
N.S.	1	1.10	0.83	3.24	0.00	5.26	0.00	2.73	6.78	8.13
time (sec)	N/A	0.543	0.597	0.521	0.000	0.164	0.000	0.158	0.518	1.746

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	250	189	969	0	1721	0	809	3447	1895
N.S.	1	1.17	0.89	4.55	0.00	8.08	0.00	3.80	16.18	8.90
time (sec)	N/A	0.756	1.520	0.510	0.000	0.192	0.000	0.193	1.167	3.431

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	166	159	214	190	385	1828	0	342	327
N.S.	1	1.14	1.09	1.47	1.30	2.64	12.52	0.00	2.34	2.24
time (sec)	N/A	0.512	0.422	0.188	0.115	0.110	40.279	0.000	0.271	3.602

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	115	153	152	273	546	0	227	197
N.S.	1	1.05	1.03	1.37	1.36	2.44	4.88	0.00	2.03	1.76
time (sec)	N/A	0.403	0.385	0.207	0.110	0.129	28.069	0.000	0.226	2.026

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	73	105	132	169	177	0	122	81
N.S.	1	1.07	0.86	1.24	1.55	1.99	2.08	0.00	1.44	0.95
time (sec)	N/A	0.353	0.275	0.201	0.124	0.133	19.048	0.000	0.218	2.035

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	46	78	63	105	92	0	59	34
N.S.	1	1.05	0.84	1.42	1.15	1.91	1.67	0.00	1.07	0.62
time (sec)	N/A	0.302	0.015	0.130	0.112	0.144	1.477	0.000	0.238	0.829

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	115	103	245	0	529	0	0	408	556
N.S.	1	1.08	0.97	2.31	0.00	4.99	0.00	0.00	3.85	5.25
time (sec)	N/A	0.485	0.433	0.457	0.000	0.180	0.000	0.000	0.269	0.999

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	188	144	501	0	781	0	514	873	448
N.S.	1	1.21	0.92	3.21	0.00	5.01	0.00	3.29	5.60	2.87
time (sec)	N/A	0.672	0.565	0.486	0.000	0.138	0.000	0.170	0.490	1.609

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	225	169	1007	0	1775	0	720	3196	1664
N.S.	1	1.08	0.81	4.82	0.00	8.49	0.00	3.44	15.29	7.96
time (sec)	N/A	0.724	0.704	0.526	0.000	0.163	0.000	0.186	1.264	2.998

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	190	201	273	219	499	5523	0	438	487
N.S.	1	1.07	1.13	1.53	1.23	2.80	31.03	0.00	2.46	2.74
time (sec)	N/A	0.560	0.539	0.253	0.108	0.125	49.877	0.000	0.276	5.837

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	136	145	194	181	355	1853	0	298	271
N.S.	1	0.99	1.05	1.41	1.31	2.57	13.43	0.00	2.16	1.96
time (sec)	N/A	0.428	0.428	0.217	0.112	0.133	31.883	0.000	0.228	3.448

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	109	94	129	161	227	534	0	177	99
N.S.	1	0.98	0.85	1.16	1.45	2.05	4.81	0.00	1.59	0.89
time (sec)	N/A	0.374	0.314	0.194	0.107	0.117	21.311	0.000	0.267	3.012

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	64	94	78	144	99	0	84	34
N.S.	1	1.03	0.86	1.27	1.05	1.95	1.34	0.00	1.14	0.46
time (sec)	N/A	0.309	0.064	0.141	0.111	0.145	2.163	0.000	0.223	0.996

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	146	117	278	0	669	0	0	677	1427
N.S.	1	1.09	0.87	2.07	0.00	4.99	0.00	0.00	5.05	10.65
time (sec)	N/A	0.597	0.442	0.506	0.000	0.305	0.000	0.000	0.336	1.435

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	186	146	517	0	1013	0	667	1310	1153
N.S.	1	1.12	0.88	3.11	0.00	6.10	0.00	4.02	7.89	6.95
time (sec)	N/A	0.675	0.652	0.531	0.000	0.292	0.000	0.169	0.508	1.748

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	274	205	1036	0	1457	0	947	3036	1476
N.S.	1	1.16	0.86	4.37	0.00	6.15	0.00	4.00	12.81	6.23
time (sec)	N/A	0.840	0.668	0.592	0.000	0.274	0.000	0.190	2.122	2.941

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	131	95	130	166	238	391	0	189	107
N.S.	1	1.25	0.90	1.24	1.58	2.27	3.72	0.00	1.80	1.02
time (sec)	N/A	0.446	0.345	0.328	0.110	0.120	18.035	0.000	0.219	1.054

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	82	66	105	129	163	119	0	122	63
N.S.	1	1.12	0.90	1.44	1.77	2.23	1.63	0.00	1.67	0.86
time (sec)	N/A	0.370	0.243	0.315	0.109	0.139	11.758	0.000	0.221	0.943

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	84	109	120	85	87	64	88
N.S.	1	1.00	1.00	1.65	2.14	2.35	1.67	1.71	1.25	1.73
time (sec)	N/A	0.322	0.153	0.300	0.111	0.184	12.770	0.139	0.260	1.355

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	103	44	69	38	66
N.S.	1	1.00	1.00	1.65	1.56	2.40	1.02	1.60	0.88	1.53
time (sec)	N/A	0.287	0.021	0.250	0.101	0.123	1.209	0.138	0.227	0.758

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	104	229	0	512	0	0	301	1183
N.S.	1	1.11	0.96	2.12	0.00	4.74	0.00	0.00	2.79	10.95
time (sec)	N/A	0.445	0.394	0.593	0.000	0.140	0.000	0.000	0.266	1.278

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	205	149	469	0	1131	0	493	1164	3813
N.S.	1	1.19	0.87	2.73	0.00	6.58	0.00	2.87	6.77	22.17
time (sec)	N/A	0.630	0.819	0.599	0.000	0.197	0.000	0.166	0.561	3.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	294	215	953	0	2277	0	890	3752	2890
N.S.	1	1.18	0.86	3.81	0.00	9.11	0.00	3.56	15.01	11.56
time (sec)	N/A	0.852	1.864	0.652	0.000	0.594	0.000	0.193	1.282	5.207

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	141	113	226	200	341	0	0	240	172
N.S.	1	1.36	1.09	2.17	1.92	3.28	0.00	0.00	2.31	1.65
time (sec)	N/A	0.488	0.340	0.356	0.119	0.112	0.000	0.000	0.263	1.234

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	100	89	182	164	277	0	187	172	120
N.S.	1	1.19	1.06	2.17	1.95	3.30	0.00	2.23	2.05	1.43
time (sec)	N/A	0.401	0.315	0.341	0.114	0.112	0.000	0.147	0.218	1.281

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	159	144	215	224	149	122	71
N.S.	1	1.00	0.92	2.09	1.89	2.83	2.95	1.96	1.61	0.93
time (sec)	N/A	0.349	0.220	0.328	0.125	0.283	18.035	0.146	0.208	1.741

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	67	57	116	85	161	71	114	68	34
N.S.	1	1.12	0.95	1.93	1.42	2.68	1.18	1.90	1.13	0.57
time (sec)	N/A	0.307	0.064	0.279	0.129	0.121	1.847	0.134	0.217	0.900

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	182	144	297	0	1045	0	0	507	3000
N.S.	1	1.24	0.98	2.02	0.00	7.11	0.00	0.00	3.45	20.41
time (sec)	N/A	0.617	0.733	0.628	0.000	0.237	0.000	0.000	0.366	2.287

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	275	215	533	0	2291	0	0	2294	4274
N.S.	1	1.23	0.96	2.38	0.00	10.23	0.00	0.00	10.24	19.08
time (sec)	N/A	0.804	1.239	0.648	0.000	0.708	0.000	0.000	0.760	5.637

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	376	298	983	0	4063	0	1089	4543	8936
N.S.	1	1.18	0.93	3.07	0.00	12.70	0.00	3.40	14.20	27.92
time (sec)	N/A	0.989	2.096	0.687	0.000	1.593	0.000	0.248	2.730	9.372

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	165	134	302	228	488	0	429	381	194
N.S.	1	1.32	1.07	2.42	1.82	3.90	0.00	3.43	3.05	1.55
time (sec)	N/A	0.553	0.371	0.373	0.117	0.156	0.000	0.155	0.261	1.386

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	112	287	190	412	0	363	304	144
N.S.	1	1.11	1.01	2.59	1.71	3.71	0.00	3.27	2.74	1.30
time (sec)	N/A	0.437	0.295	0.358	0.112	0.114	0.000	0.157	0.218	1.529

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	99	91	254	170	336	1479	259	225	87
N.S.	1	0.96	0.88	2.47	1.65	3.26	14.36	2.51	2.18	0.84
time (sec)	N/A	0.365	0.241	0.349	0.118	0.101	40.487	0.155	0.244	2.270

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	72	157	101	230	774	171	136	34
N.S.	1	1.14	0.91	1.99	1.28	2.91	9.80	2.16	1.72	0.43
time (sec)	N/A	0.333	0.046	0.297	0.111	0.110	3.030	0.136	0.255	1.059

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	250	200	403	0	1959	0	0	1212	5387
N.S.	1	1.24	1.00	2.00	0.00	9.75	0.00	0.00	6.03	26.80
time (sec)	N/A	0.785	0.803	0.648	0.000	1.688	0.000	0.000	0.504	4.096

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	349	305	665	0	3856	0	924	4057	5789
N.S.	1	1.22	1.06	2.32	0.00	13.44	0.00	3.22	14.14	20.17
time (sec)	N/A	0.967	1.705	0.690	0.000	1.436	0.000	0.231	1.747	8.379

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	477	404	1138	0	6140	0	1336	7135	4284
N.S.	1	1.17	0.99	2.78	0.00	15.01	0.00	3.27	17.44	10.47
time (sec)	N/A	1.212	2.530	0.783	0.000	5.001	0.000	0.315	12.946	8.059

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	176	220	0	824	0	0	236	4674
N.S.	1	1.05	1.43	1.79	0.00	6.70	0.00	0.00	1.92	38.00
time (sec)	N/A	0.432	0.546	0.266	0.000	0.738	0.000	0.000	0.238	22.301

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	113	155	0	247	0	0	102	478
N.S.	1	1.00	1.40	1.91	0.00	3.05	0.00	0.00	1.26	5.90
time (sec)	N/A	0.345	0.392	0.292	0.000	0.215	0.000	0.000	0.212	6.330

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	123	126	280	0	319	0	0	260	0
N.S.	1	1.01	1.03	2.30	0.00	2.61	0.00	0.00	2.13	0.00
time (sec)	N/A	0.399	0.495	0.307	0.000	0.268	0.000	0.000	0.212	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	96	206	0	0	0	0	0	0	0
N.S.	1	1.01	2.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.394	0.501	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	52	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	1.33	0.82
time (sec)	N/A	0.275	0.032	0.115	0.107	0.100	0.183	0.121	0.246	0.085

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	246	205	277	0	0	0	0	167	0
N.S.	1	1.06	0.88	1.19	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.646	2.505	3.438	0.000	0.000	0.000	0.000	0.458	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	246	86	94	0	116	0	0	28	0
N.S.	1	1.24	0.43	0.47	0.00	0.59	0.00	0.00	0.14	0.00
time (sec)	N/A	0.608	2.138	1.217	0.000	0.133	0.000	0.000	0.225	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	281	191	185	0	164	0	0	503	0
N.S.	1	1.20	0.82	0.79	0.00	0.70	0.00	0.00	2.15	0.00
time (sec)	N/A	0.740	3.461	5.302	0.000	0.160	0.000	0.000	0.591	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	419	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	5.30	0.00
time (sec)	N/A	0.400	0.268	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	130	129	42	128	390	71	133	163	123
N.S.	1	0.90	0.89	0.29	0.88	2.69	0.49	0.92	1.12	0.85
time (sec)	N/A	0.522	0.092	0.200	0.123	0.141	0.237	0.130	0.256	0.284

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	51	48	47	48	82	49	51	49
N.S.	1	1.08	1.04	0.98	0.96	0.98	1.67	1.00	1.04	1.00
time (sec)	N/A	0.349	0.055	0.671	0.026	0.128	0.187	0.122	0.213	0.705

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	32	26	21	20	20	24	20	20	20
N.S.	1	1.23	1.00	0.81	0.77	0.77	0.92	0.77	0.77	0.77
time (sec)	N/A	0.293	0.022	0.775	0.026	0.094	0.074	0.124	0.211	0.034

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	24	17	24	23	23	27	24	23	13
N.S.	1	1.41	1.00	1.41	1.35	1.35	1.59	1.41	1.35	0.76
time (sec)	N/A	0.271	0.026	0.683	0.025	0.122	0.093	0.120	0.225	0.706

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	104	66	71	71	102	71	70	90
N.S.	1	1.06	1.00	0.63	0.68	0.68	0.98	0.68	0.67	0.87
time (sec)	N/A	0.497	0.195	63.277	0.120	0.128	1.589	0.133	0.287	0.795

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	28	23	22	22	26	23	22	22
N.S.	1	1.13	0.93	0.77	0.73	0.73	0.87	0.77	0.73	0.73
time (sec)	N/A	0.302	0.027	0.689	0.039	0.087	0.087	0.121	0.230	0.045

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	54	55	99	246	107	82	53
N.S.	1	1.00	1.16	0.98	1.00	1.80	4.47	1.95	1.49	0.96
time (sec)	N/A	0.350	0.109	0.953	0.029	0.106	0.395	0.133	0.209	0.929

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	37	61	35	30	26
N.S.	1	1.00	1.00	1.00	1.00	1.37	2.26	1.30	1.11	0.96
time (sec)	N/A	0.277	0.059	0.149	0.027	0.103	0.167	0.119	0.229	1.224

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	24	35
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.63	0.92
time (sec)	N/A	0.276	0.135	0.000	0.000	0.000	0.000	0.000	0.216	1.204

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	366	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.63	0.00
time (sec)	N/A	0.340	0.430	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	146	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.92	0.00
time (sec)	N/A	0.332	0.137	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	165	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.29	0.00
time (sec)	N/A	0.337	0.076	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	128	186	527	2744	740	690	131
N.S.	1	1.00	0.83	0.97	1.41	3.99	20.79	5.61	5.23	0.99
time (sec)	N/A	0.526	0.692	1.130	0.038	0.178	0.821	0.145	0.213	0.916

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	96	140	319	1540	450	418	99
N.S.	1	1.00	0.91	0.97	1.41	3.22	15.56	4.55	4.22	1.00
time (sec)	N/A	0.450	0.246	0.870	0.029	0.143	0.520	0.137	0.201	0.837

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	214	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	3.06	1.01
time (sec)	N/A	0.373	0.164	0.717	0.030	0.156	0.429	0.132	0.221	0.814

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	39	48	69	236	83	75	38
N.S.	1	1.00	0.92	0.98	1.20	1.72	5.90	2.08	1.88	0.95
time (sec)	N/A	0.305	0.116	0.168	0.034	0.104	0.257	0.115	0.216	0.964

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	110	0	37	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	2.56	0.00	0.86	0.00
time (sec)	N/A	0.287	0.148	0.000	0.000	0.000	1.063	0.000	0.225	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	56	0	0	0	741	0	551	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	10.44	0.00	7.76	0.00
time (sec)	N/A	0.351	0.090	0.000	0.000	0.000	3.501	0.000	0.226	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	78	58	0	0	0	2319	0	0	0
N.S.	1	1.28	0.95	0.00	0.00	0.00	38.02	0.00	0.00	0.00
time (sec)	N/A	0.341	0.096	0.000	0.000	0.000	40.087	0.000	0.280	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	78	58	0	0	0	0	0	0	0
N.S.	1	1.28	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.111	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	149	154	242	667	3376	962	902	157
N.S.	1	1.00	0.94	0.97	1.53	4.22	21.37	6.09	5.71	0.99
time (sec)	N/A	0.581	0.822	1.212	0.033	0.121	7.935	0.142	0.213	0.941

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	105	109	168	370	1760	539	503	108
N.S.	1	1.00	0.94	0.97	1.50	3.30	15.71	4.81	4.49	0.96
time (sec)	N/A	0.463	0.816	0.932	0.032	0.128	3.045	0.134	0.232	0.821

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	214	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	3.06	1.01
time (sec)	N/A	0.374	0.165	0.717	0.031	0.088	0.389	0.132	0.264	0.782

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	82	0	0	0	235	0	156	0
N.S.	1	1.11	0.98	0.00	0.00	0.00	2.80	0.00	1.86	0.00
time (sec)	N/A	0.467	0.233	0.000	0.000	0.000	1.759	0.000	0.222	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	95	0	0	0	0	0	955	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	8.30	0.00
time (sec)	N/A	0.461	0.275	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	169	133	0	0	0	0	0	0	0
N.S.	1	1.01	0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.580	0.219	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	321	133	0	0	0	488	0	955	0
N.S.	1	1.35	0.56	0.00	0.00	0.00	2.05	0.00	4.01	0.00
time (sec)	N/A	1.317	3.822	0.000	0.000	0.000	4.136	0.000	0.219	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	185	104	0	0	0	360	0	444	0
N.S.	1	1.19	0.67	0.00	0.00	0.00	2.32	0.00	2.86	0.00
time (sec)	N/A	0.819	1.594	0.000	0.000	0.000	2.693	0.000	0.229	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	75	0	0	0	235	0	156	0
N.S.	1	1.11	0.89	0.00	0.00	0.00	2.80	0.00	1.86	0.00
time (sec)	N/A	0.475	0.525	0.000	0.000	0.000	1.954	0.000	0.227	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	110	0	37	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.62	0.00	0.88	0.00
time (sec)	N/A	0.287	0.137	0.000	0.000	0.000	1.159	0.000	0.259	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	28	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.333	0.105	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	141	121	0	0	0	0	0	56	0
N.S.	1	1.15	0.98	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.555	0.205	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	248	210	0	0	0	0	0	84	0
N.S.	1	1.18	1.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.859	0.293	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	344	217	0	0	0	0	0	0	0
N.S.	1	1.30	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.217	6.459	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	180	204	2050	0	0	0	0	0	1397	0
N.S.	1	1.13	11.39	0.00	0.00	0.00	0.00	0.00	7.76	0.00
time (sec)	N/A	0.773	5.137	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	666	0	0	0	0	0	955	0
N.S.	1	1.02	5.79	0.00	0.00	0.00	0.00	0.00	8.30	0.00
time (sec)	N/A	0.467	1.971	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	72	56	0	0	0	741	0	551	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	10.44	0.00	7.76	0.00
time (sec)	N/A	0.336	0.095	0.000	0.000	0.000	3.360	0.000	0.253	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	142	108	0	0	0	0	0	56	0
N.S.	1	1.16	0.89	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.551	0.246	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	224	147	0	0	0	0	0	103	0
N.S.	1	1.16	0.76	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.801	0.317	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	299	343	233	0	0	0	0	0	150	0
N.S.	1	1.15	0.78	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.253	0.444	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	214	0	0	0	185	0	1522	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.99	0.00	8.14	0.00
time (sec)	N/A	0.679	5.403	0.000	0.000	0.000	2.821	0.000	0.277	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	0	0	0	114	0	489	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	1.27	0.00	5.43	0.00
time (sec)	N/A	0.385	0.113	0.000	0.000	0.000	1.395	0.000	0.230	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	181	0	0	0	0	0	20	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.340	0.402	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	360	0	0	0	0	0	33	0
N.S.	1	1.00	5.90	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.323	0.943	0.000	0.000	0.000	0.000	0.000	0.376	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	141	0	0	0	182	0	871	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.99	0.00	4.76	0.00
time (sec)	N/A	0.650	5.265	0.000	0.000	0.000	2.358	0.000	0.262	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	0	112	0	206	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.30	0.00	2.40	0.00
time (sec)	N/A	0.363	0.117	0.000	0.000	0.000	1.200	0.000	0.227	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	181	0	0	0	0	0	37	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.330	0.161	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	391	0	0	0	0	0	65	0
N.S.	1	1.00	6.41	0.00	0.00	0.00	0.00	0.00	1.07	0.00
time (sec)	N/A	0.316	0.838	0.000	0.000	0.000	0.000	0.000	0.474	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	174	130	0	0	0	0	0	0	0
N.S.	1	0.98	0.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.644	5.258	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	103	77	0	0	0	112	0	706	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	1.20	0.00	7.59	0.00
time (sec)	N/A	0.386	0.116	0.000	0.000	0.000	2.374	0.000	0.275	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	382	0	0	0	0	0	65	0
N.S.	1	1.00	5.97	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.328	0.802	0.000	0.000	0.000	0.000	0.000	0.497	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	1294	0	0	0	0	0	112	0
N.S.	1	1.00	20.22	0.00	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.328	1.224	0.000	0.000	0.000	0.000	0.000	0.692	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	214	0	0	0	185	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.653	5.416	0.000	0.000	0.000	3.321	0.000	0.288	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	0	0	0	114	0	501	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	1.27	0.00	5.57	0.00
time (sec)	N/A	0.378	0.115	0.000	0.000	0.000	1.493	0.000	0.297	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	181	0	0	0	0	0	21	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.327	0.374	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	364	0	0	0	0	0	34	0
N.S.	1	1.00	5.97	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.322	0.935	0.000	0.000	0.000	0.000	0.000	0.361	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	146	0	0	0	182	0	56	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.97	0.00	0.30	0.00
time (sec)	N/A	0.672	5.260	0.000	0.000	0.000	2.130	0.000	0.242	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	83	0	0	0	112	0	31	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.24	0.00	0.34	0.00
time (sec)	N/A	0.382	0.139	0.000	0.000	0.000	1.178	0.000	0.230	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	181	0	0	0	0	0	30	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.331	0.544	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	391	0	0	0	0	0	52	0
N.S.	1	1.00	6.41	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.322	0.872	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	183	140	0	0	0	0	0	113	0
N.S.	1	0.98	0.75	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.654	5.313	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	103	77	0	0	0	112	0	69	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	1.20	0.00	0.74	0.00
time (sec)	N/A	0.389	0.146	0.000	0.000	0.000	2.561	0.000	0.211	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	382	0	0	0	0	0	64	0
N.S.	1	1.00	5.97	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.330	0.846	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	1294	0	0	0	0	0	110	0
N.S.	1	1.00	20.22	0.00	0.00	0.00	0.00	0.00	1.72	0.00
time (sec)	N/A	0.318	1.104	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	395	385	168	0	0	0	243	0	0	0
N.S.	1	0.97	0.43	0.00	0.00	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	1.449	5.317	0.000	0.000	0.000	91.679	0.000	0.316	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	188	140	0	0	0	175	0	0	0
N.S.	1	0.95	0.71	0.00	0.00	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.767	5.223	0.000	0.000	0.000	20.709	0.000	0.241	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	0	0	107	0	1336	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	1.20	0.00	15.01	0.00
time (sec)	N/A	0.396	0.083	0.000	0.000	0.000	2.618	0.000	0.211	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	46	0	96	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	2.09	1.02
time (sec)	N/A	0.284	0.003	0.000	0.000	0.000	0.893	0.000	0.217	1.490

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	21	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.326	0.490	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	34	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.324	0.387	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	47	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.321	0.500	0.000	0.000	0.000	0.000	0.000	0.291	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	56	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.345	0.178	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	160	218	1290	0	478	2822	0	461	0
N.S.	1	0.83	1.13	6.68	0.00	2.48	14.62	0.00	2.39	0.00
time (sec)	N/A	0.511	0.215	1.970	0.000	0.130	47.156	0.000	0.227	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	109	113	588	0	231	1035	0	269	0
N.S.	1	0.86	0.89	4.63	0.00	1.82	8.15	0.00	2.12	0.00
time (sec)	N/A	0.395	0.143	1.352	0.000	0.117	15.862	0.000	0.216	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	58	82	199	0	85	311	0	125	0
N.S.	1	0.81	1.14	2.76	0.00	1.18	4.32	0.00	1.74	0.00
time (sec)	N/A	0.301	0.178	0.992	0.000	0.087	2.272	0.000	0.236	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	47	0	31	42	0	34	75
N.S.	1	1.00	1.00	2.61	0.00	1.72	2.33	0.00	1.89	4.17
time (sec)	N/A	0.233	0.058	0.924	0.000	0.102	0.804	0.000	0.224	1.043

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	34	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.285	0.073	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	125	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	2.31	0.00
time (sec)	N/A	0.287	0.093	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	269	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	4.80	0.00
time (sec)	N/A	0.293	0.083	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	181	0	0	0	0	0	0	86	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.519	0.000	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	334	0	108	0	0	144	0
N.S.	1	1.00	0.96	5.86	0.00	1.89	0.00	0.00	2.53	0.00
time (sec)	N/A	0.312	0.597	6.151	0.000	0.136	0.000	0.000	0.226	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	231	136	1059	0	400	2746	0	341	0
N.S.	1	0.91	0.54	4.19	0.00	1.58	10.85	0.00	1.35	0.00
time (sec)	N/A	0.664	0.479	1.397	0.000	0.117	15.894	0.000	0.273	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	113	94	434	0	173	959	0	173	0
N.S.	1	0.89	0.74	3.42	0.00	1.36	7.55	0.00	1.36	0.00
time (sec)	N/A	0.424	0.223	1.004	0.000	0.093	2.492	0.000	0.218	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	250	0	58	64
N.S.	1	1.00	1.10	0.00	0.00	1.36	5.00	0.00	1.16	1.28
time (sec)	N/A	0.295	0.065	0.000	0.000	0.081	0.832	0.000	0.209	1.098

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	153	0	0	0	0	0	72	0
N.S.	1	1.01	1.63	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.360	6.671	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	212	0	0	0	0	0	58	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.417	8.595	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	131	333	0	0	0	0	0	173	0
N.S.	1	1.01	2.56	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.431	8.827	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	6405	0	0	0	0	0	341	0
N.S.	1	1.01	48.52	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.436	37.052	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.487	0.000	0.000	0.000	0.000	0.000	0.301	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	164	0	0	0	0	0	23	0
N.S.	1	1.00	4.82	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.267	0.283	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	164	0	0	0	0	0	23	0
N.S.	1	1.00	4.82	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.262	0.288	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	164	0	0	0	0	0	23	0
N.S.	1	1.00	2.93	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.323	0.457	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	164	0	0	0	0	0	23	0
N.S.	1	1.00	2.93	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.317	0.355	0.000	0.000	0.000	0.000	0.000	0.256	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [45] had the largest ratio of [.7142859999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	9	0.92	21	0.429
2	A	10	9	1.41	13	0.692
3	A	13	12	1.37	19	0.632
4	A	9	8	1.28	21	0.381
5	A	8	7	1.15	21	0.333
6	A	6	5	1.18	19	0.263
7	A	5	4	1.00	11	0.364
8	A	8	7	1.09	21	0.333
9	A	11	10	1.10	21	0.476
10	A	13	12	1.17	21	0.571
11	A	10	9	1.14	21	0.429
12	A	9	8	1.05	21	0.381
13	A	7	6	1.07	19	0.316
14	A	6	5	1.05	11	0.455
15	A	8	7	1.08	21	0.333
16	A	10	9	1.21	21	0.429
17	A	12	11	1.08	21	0.524
18	A	11	10	1.07	21	0.476
19	A	10	9	0.99	21	0.429
20	A	8	7	0.98	19	0.368
21	A	7	6	1.03	11	0.545

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	10	9	1.09	21	0.429
23	A	10	9	1.12	21	0.429
24	A	12	11	1.16	21	0.524
25	A	7	6	1.25	21	0.286
26	A	7	6	1.12	21	0.286
27	A	5	4	1.00	19	0.211
28	A	5	4	1.00	11	0.364
29	A	8	7	1.11	21	0.333
30	A	10	9	1.19	21	0.429
31	A	12	11	1.18	21	0.524
32	A	7	6	1.36	21	0.286
33	A	7	6	1.19	21	0.286
34	A	6	5	1.00	19	0.263
35	A	6	5	1.12	11	0.455
36	A	10	9	1.24	21	0.429
37	A	12	11	1.23	21	0.524
38	A	14	13	1.18	21	0.619
39	A	7	6	1.32	21	0.286
40	A	8	7	1.11	21	0.333
41	A	7	6	0.96	19	0.316
42	A	7	6	1.14	11	0.545
43	A	12	11	1.24	21	0.524
44	A	14	13	1.22	21	0.619
45	A	16	15	1.17	21	0.714
46	A	8	7	1.05	23	0.304
47	A	5	4	1.00	23	0.174
48	A	6	5	1.01	23	0.217
49	A	4	3	1.01	19	0.158
50	A	3	3	1.00	17	0.176
51	A	8	7	1.06	23	0.304
52	A	8	7	1.24	23	0.304
53	A	11	10	1.20	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
54	A	5	4	1.00	19	0.211
55	A	11	10	0.90	17	0.588
56	A	4	3	1.08	21	0.143
57	A	5	4	1.23	17	0.235
58	A	6	5	1.41	17	0.294
59	A	12	11	1.06	17	0.647
60	A	5	4	1.13	17	0.235
61	A	3	3	1.00	23	0.130
62	A	2	2	1.00	19	0.105
63	A	2	2	1.00	23	0.087
64	A	3	3	1.00	24	0.125
65	A	3	3	1.00	24	0.125
66	A	3	3	1.00	20	0.150
67	A	2	2	1.00	17	0.118
68	A	2	2	1.00	17	0.118
69	A	2	2	1.00	17	0.118
70	A	2	2	1.00	15	0.133
71	A	2	2	1.00	17	0.118
72	A	2	2	1.03	17	0.118
73	A	2	2	1.28	17	0.118
74	A	2	2	1.28	17	0.118
75	A	2	2	1.00	19	0.105
76	A	2	2	1.00	19	0.105
77	A	2	2	1.00	17	0.118
78	A	4	4	1.11	19	0.211
79	A	3	3	1.02	19	0.158
80	A	3	3	1.01	19	0.158
81	A	8	8	1.35	19	0.421
82	A	6	6	1.19	19	0.316
83	A	4	4	1.11	19	0.211
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.15	19	0.158
87	A	4	4	1.18	19	0.211
88	A	5	5	1.30	19	0.263
89	A	4	4	1.13	19	0.211
90	A	3	3	1.02	19	0.158
91	A	2	2	1.01	17	0.118
92	A	3	3	1.16	19	0.158
93	A	4	4	1.16	19	0.211
94	A	5	5	1.15	19	0.263
95	A	5	5	1.00	21	0.238
96	A	3	3	1.00	19	0.158
97	A	2	2	1.00	21	0.095
98	A	2	2	1.00	21	0.095
99	A	5	5	1.00	21	0.238
100	A	3	3	1.00	19	0.158
101	A	2	2	1.00	21	0.095
102	A	2	2	1.00	21	0.095
103	A	5	5	0.98	21	0.238
104	A	3	3	1.11	19	0.158
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	5	5	1.00	21	0.238
108	A	3	3	1.00	19	0.158
109	A	2	2	1.00	21	0.095
110	A	2	2	1.00	21	0.095
111	A	5	5	1.00	21	0.238
112	A	3	3	1.00	19	0.158
113	A	2	2	1.00	21	0.095
114	A	2	2	1.00	21	0.095
115	A	5	5	0.98	21	0.238
116	A	3	3	1.11	19	0.158
117	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	21	0.095
119	A	7	7	0.97	19	0.368
120	A	5	5	0.95	19	0.263
121	A	3	3	1.00	17	0.176
122	A	2	2	1.00	9	0.222
123	A	2	2	1.00	19	0.105
124	A	2	2	1.00	19	0.105
125	A	2	2	1.00	19	0.105
126	A	1	1	1.00	28	0.036
127	A	4	4	0.83	25	0.160
128	A	3	3	0.86	25	0.120
129	A	2	2	0.81	23	0.087
130	A	1	1	1.00	15	0.067
131	A	1	1	1.00	23	0.043
132	A	1	1	1.00	25	0.040
133	A	1	1	1.00	25	0.040
134	A	2	2	1.10	28	0.071
135	A	1	1	1.00	69	0.014
136	A	5	5	0.91	25	0.200
137	A	3	3	0.89	23	0.130
138	A	2	2	1.00	15	0.133
139	A	2	2	1.01	25	0.080
140	A	2	2	1.00	23	0.087
141	A	2	2	1.01	25	0.080
142	A	2	2	1.01	25	0.080
143	A	3	3	1.00	19	0.158
144	A	1	1	1.00	21	0.048
145	A	1	1	1.00	21	0.048
146	A	2	2	1.00	21	0.095
147	A	2	2	1.00	21	0.095

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 545

$$\begin{aligned}
\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx
&= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,\sqrt[5]{bc-ad}\,x}{\sqrt[5]{c}\,\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}+\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,\sqrt[5]{bc-ad}\,x}{\sqrt[5]{c}\,\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&-\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x\,\sqrt[5]{a+bx^5}-\sqrt{5}\,\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x\,\sqrt[5]{a+bx^5}+2c^{2/5}\left(a+bx^5\right)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\
&+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2(bc-ad)^{2/5}x^2+\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x\,\sqrt[5]{a+bx^5}+\sqrt{5}\,\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x\,\sqrt[5]{a+bx^5}+2c^{2/5}\left(a+bx^5\right)^{2/5}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}
\end{aligned}$$

output
```
1/10*(10+2*5^(1/2))^(1/2)*arctan(-1/5*(25-10*5^(1/2))^(1/2)+2*2^(1/2)/(5+5
^(1/2))^(1/2)*(-a*d+b*c)^(1/5)*x/c^(1/5)/(b*x^5+a)^(1/5))/c^(4/5)/(-a*d+b*
c)^(1/5)+1/10*(10-2*5^(1/2))^(1/2)*arctan(1/5*(25+10*5^(1/2))^(1/2)+1/5*(5
0+10*5^(1/2))^(1/2)*(-a*d+b*c)^(1/5)*x/c^(1/5)/(b*x^5+a)^(1/5))/c^(4/5)/(-
a*d+b*c)^(1/5)-1/5*ln(c^(1/5)-(-a*d+b*c)^(1/5)*x/(b*x^5+a)^(1/5))/c^(4/5)/
(-a*d+b*c)^(1/5)+1/20*(-5^(1/2)+1)*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*
d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)-5^(1/2)*c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a
)^(1/5)+2*c^(2/5)*(b*x^5+a)^(2/5))/(b*x^5+a)^(2/5))/c^(4/5)/(-a*d+b*c)^(1/
5)+1/20*(5^(1/2)+1)*ln((2*(-a*d+b*c)^(2/5)*x^2+c^(1/5)*(-a*d+b*c)^(1/5)*x*
(b*x^5+a)^(1/5)+5^(1/2)*c^(1/5)*(-a*d+b*c)^(1/5)*x*(b*x^5+a)^(1/5)+2*c^(2/
5)*(b*x^5+a)^(2/5))/(b*x^5+a)^(2/5))/c^(4/5)/(-a*d+b*c)^(1/5)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{5},1,\frac{6}{5},\frac{(bc-ad)x^5}{c(a+bx^5)}\right)}{c\sqrt[5]{a+bx^5}}$$

input
```
Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

output
```
(x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(
a + b*x^5)^(1/5))
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {902, 752, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

$$\Big\downarrow \text{ 902}$$

$$\int \frac{1}{c-\frac{x^5(bc-ad)}{a+bx^5}}d\frac{x}{\sqrt[5]{a+bx^5}}$$

$$\Big\downarrow \text{ 752}$$

$$\frac{\int \frac{1}{\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{2\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+$$

$$\frac{2\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+$$

$$\frac{2\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{2\left(\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}\right)}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\frac{\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}+\frac{\int \frac{\frac{\left(1+\sqrt{5}\right)\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+4\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}}{5c^{4/5}}-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$\Big\downarrow$ 1142

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\left(1-\sqrt{5}\right)\int \frac{\sqrt[5]{bc-ad}\left(\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)\right.}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}}}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}$$

$$\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\left(1+\sqrt{5}\right)\int \frac{\sqrt[5]{bc-ad}\left(\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)\right.}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}}}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-}}{\sqrt[5]{bx^5+a}}}}{5c^{4/5}}$$

$$\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-}}{\sqrt[5]{bx^5+a}}}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \ 1083$$

$$\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}-\left(5+\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{-\frac{x^2}{\left(bx^5+a\right)^{2/5}}-2\left(5+\sqrt{5}\right)c^{2/5}(bc-ad)}}{5c^{4/5}}$$

$$\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}-\left(5-\sqrt{5}\right)\sqrt[5]{c}\int \frac{1}{-\frac{x^2}{\left(bx^5+a\right)^{2/5}}-2\left(5-\sqrt{5}\right)c^{2/5}(bc-ad)}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{4}(1-\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}}{5c^{4/5}}$$

$$\frac{\frac{1}{4}(1+\sqrt{5})\int \frac{\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}}{\frac{2(bc-ad)^{2/5}x^2}{\left(bx^5+a\right)^{2/5}}+\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{bx^5+a}}+2c^{2/5}}d\frac{x}{\sqrt[5]{bx^5+a}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}}{5c^{4/5}}$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

$\downarrow$ 1103

$$\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}+\frac{2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}+2c^{2/5}\right)}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}+$$

$$\frac{\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{\frac{4x(bc-ad)^{2/5}}{\sqrt[5]{a+bx^5}}+\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{c}\sqrt[5]{bc-ad}}\right)}{\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{\left(1+\sqrt{5}\right)\sqrt[5]{c}x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}+\frac{2x^2(bc-ad)^{2/5}}{\left(a+bx^5\right)^{2/5}}+2c^{2/5}\right)}{4\sqrt[5]{bc-ad}}}{5c^{4/5}}-$$

$$\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

input
```
Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

output
```
-1/5*Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)]/(c^(4/5)*(b*c
- a*d)^(1/5)) + ((Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*c^(1/5)*(b*c
 - a*d)^(1/5) + (4*(b*c - a*d)^(2/5)*x)/(a + b*x^5)^(1/5))/(Sqrt[2*(5 + Sq
rt[5])]*c^(1/5)*(b*c - a*d)^(1/5))])/(b*c - a*d)^(1/5) + ((1 - Sqrt[5])*Lo
g[2*c^(2/5) + (2*(b*c - a*d)^(2/5)*x^2)/(a + b*x^5)^(2/5) + ((1 - Sqrt[5])
*c^(1/5)*(b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)])/(4*(b*c - a*d)^(1/5)))/(
5*c^(4/5)) + ((Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 + Sqrt[5])*c^(1/5)*(b*c -
a*d)^(1/5) + (4*(b*c - a*d)^(2/5)*x)/(a + b*x^5)^(1/5))/(Sqrt[2*(5 - Sqrt[
5])]*c^(1/5)*(b*c - a*d)^(1/5))])/(b*c - a*d)^(1/5) + ((1 + Sqrt[5])*Log[2
*c^(2/5) + (2*(b*c - a*d)^(2/5)*x^2)/(a + b*x^5)^(2/5) + ((1 + Sqrt[5])*c^
(1/5)*(b*c - a*d)^(1/5)*x)/(a + b*x^5)^(1/5)])/(4*(b*c - a*d)^(1/5)))/(5*c
^(4/5))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 752
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n
)   Int[1/(r - s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; F
reeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

rule 902
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 13.39 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\sqrt{5}\left(-\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}+1\right)\ln\left(\frac{2\left(\frac{ad-bc}{c}\right)^{\frac{2}{5}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{5}}\left(b\,x^{5}+a\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)x+2\left(b\,x^{5}+a\right)^{\frac{2}{5}}}{x^{2}}\right)}{4}+\frac{\sqrt{5+\sqrt{5}}\,\sqrt{5-\sqrt{5}}\,\left(\sqrt{5}-1\right)\ln}{}\right.$

input
```
int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x,method=_RETURNVERBOSE)
```

output
```
1/50*5^(1/2)/((a*d-b*c)/c)^(1/5)*(-1/4*(5+5^(1/2))^(1/2)*(5-5^(1/2))^(1/2)
*(5^(1/2)+1)*ln((2*((a*d-b*c)/c)^(2/5)*x^2-((a*d-b*c)/c)^(1/5)*(b*x^5+a)^(
1/5)*(5^(1/2)+1)*x+2*(b*x^5+a)^(2/5))/x^2)+1/4*(5+5^(1/2))^(1/2)*(5-5^(1/2
))^(1/2)*(5^(1/2)-1)*ln((2*((a*d-b*c)/c)^(2/5)*x^2+((a*d-b*c)/c)^(1/5)*(b*
x^5+a)^(1/5)*(5^(1/2)-1)*x+2*(b*x^5+a)^(2/5))/x^2)-1/2*2^(1/2)*(5+5^(1/2))
^(1/2)*(5^(1/2)-5)*arctan(1/2*2^(1/2)*(((a*d-b*c)/c)^(1/5)*(5^(1/2)+1)*x-4
*(b*x^5+a)^(1/5))/(5-5^(1/2))^(1/2)/((a*d-b*c)/c)^(1/5)/x)+(5-5^(1/2))^(1/
2)*(-1/2*2^(1/2)*(5+5^(1/2))*arctan(1/2*2^(1/2)*(((a*d-b*c)/c)^(1/5)*(5^(1
/2)-1)*x+4*(b*x^5+a)^(1/5))/(5+5^(1/2))^(1/2)/((a*d-b*c)/c)^(1/5)/x)+(5+5^
(1/2))^(1/2)*ln((((a*d-b*c)/c)^(1/5)*x+(b*x^5+a)^(1/5))/x)))/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (trace 0)
```

Sympy [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$

input
```
integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

output
```
Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(bx^5+a)^{\frac{1}{5}}(dx^5+c)}\,dx$$

input
```
integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

output
```
integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx = \int \frac{1}{(b\,x^5+a)^{1/5}\,(d\,x^5+c)}\,dx$$

input
```
int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
```

output
```
int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,(c+dx^5)}\,dx$$
$$= \frac{(b\,x^5+a)^{\frac{1}{5}}\left(\int \frac{1}{(b\,x^5+a)^{\frac{1}{5}}\,ac+(b\,x^5+a)^{\frac{1}{5}}\,ad\,x^5+(b\,x^5+a)^{\frac{1}{5}}\,bc\,x^5+(b\,x^5+a)^{\frac{1}{5}}\,bd\,x^{10}}dx\right)a^2d-(b\,x^5+a)^{\frac{1}{5}}\left(\int \frac{}{(b\,x^5+a)^{\frac{1}{5}}\,ac+(b\,x}\right.}{(b\,x^5+a)^{\frac{1}{5}}\,ad}$$

input
```
int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

output
```
((a + b*x**5)**(1/5)*int(1/((a + b*x**5)**(1/5)*a*c + (a + b*x**5)**(1/5)*
a*d*x**5 + (a + b*x**5)**(1/5)*b*c*x**5 + (a + b*x**5)**(1/5)*b*d*x**10),x
)*a**2*d - (a + b*x**5)**(1/5)*int(1/((a + b*x**5)**(1/5)*a*c + (a + b*x**
5)**(1/5)*a*d*x**5 + (a + b*x**5)**(1/5)*b*c*x**5 + (a + b*x**5)**(1/5)*b*
d*x**10),x)*a*b*c + b*x)/((a + b*x**5)**(1/5)*a*d)
```

3.2 $\int \frac{1+x^6}{-1+x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1+x^6}{-1+x^6}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}-\frac{1}{3}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)-2/3*arctanh(x)-1/3*arctanh(x/(x^2+1))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int \frac{1+x^6}{-1+x^6}\,dx = \frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)\right.$$
$$\left.-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(1 + x^6)/(-1 + x^6),x]
```

output
```
(6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2
])/6
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {913, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+1}{x^6-1} \, dx$$

$$\Big\downarrow \text{913}$$

$$2 \int \frac{1}{x^6-1}dx + x$$

$$\Big\downarrow \text{754}$$

$$2 \left(-\frac{1}{3} \int \frac{1}{1-x^2}dx - \frac{1}{3} \int \frac{2-x}{2 \left(x^2-x+1\right)}dx - \frac{1}{3} \int \frac{x+2}{2 \left(x^2+x+1\right)}dx\right) + x$$

$$\Big\downarrow \text{27}$$

$$2 \left(-\frac{1}{3} \int \frac{1}{1-x^2}dx - \frac{1}{6} \int \frac{2-x}{x^2-x+1}dx - \frac{1}{6} \int \frac{x+2}{x^2+x+1}dx\right) + x$$

$$\Big\downarrow \text{219}$$

$$2 \left(-\frac{1}{6} \int \frac{2-x}{x^2-x+1}dx - \frac{1}{6} \int \frac{x+2}{x^2+x+1}dx - \frac{\text{arctanh}(x)}{3}\right) + x$$

$$\Big\downarrow \text{1142}$$

$$2 \left(\frac{1}{6} \left(\frac{1}{2} \int -\frac{1-2x}{x^2-x+1}dx - \frac{3}{2} \int \frac{1}{x^2-x+1}dx\right) + \frac{1}{6} \left(-\frac{3}{2} \int \frac{1}{x^2+x+1}dx - \frac{1}{2} \int \frac{2x+1}{x^2+x+1}dx\right) - \frac{\text{arctanh}(x)}{3}\right) + x$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{x^2-x+1}dx-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(-\frac{3}{2}\int\frac{1}{x^2+x+1}dx-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\right)-\frac{\text{arctanh}(x}{3}\right.$$
$$x$$

$$\Big\downarrow \ 1083$$

$$2\left(\frac{1}{6}\left(3\int\frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(3\int\frac{1}{-(2x+1)^2-3}d(2x+1)-\frac{1}{2}\int\frac{2x+1}{x^2+x+}\right.\right.$$
$$x$$

$$\Big\downarrow \ 217$$

$$2\left(\frac{1}{6}\left(-\frac{1}{2}\int\frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{\text{arcta}}{}\right.$$
$$x$$

$$\Big\downarrow \ 1103$$

$$2\left(\frac{1}{6}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(-\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2+x+1\right)\right)-\frac{\text{arctanh}}{3}\right.$$
$$x$$

input
```
Int[(1 + x^6)/(-1 + x^6),x]
```

output
```
x + 2*(-1/3*ArcTanh[x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]]) + Log[1 -
x + x^2]/2)/6 + (-(Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]) - Log[1 + x + x^2]/2
)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

method	result
risch	$x+\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x}{2+(x}\right.\right.}{6(x^6)^{\frac{1}{6}}}$

input
```
int((x^6+1)/(x^6-1),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/3
*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{1+x^6}{-1+x^6}\,dx = x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log(x^2-x+1)}{6}-\frac{\log(x^2+x+1)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**6+1)/(x**6-1),x)
```

output
```
x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/
6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3
 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{1}{3}\log(x-1)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(|x+1|\right) + \frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(a
bs(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{1+x^6}{-1+x^6}\,dx = x + \frac{\mathrm{atan}(x\,1\mathrm{i})\,2\mathrm{i}}{3} - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}} - \frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} - \frac{1}{3}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}} + \frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} + \frac{1}{3}\mathrm{i}\right)$$

input
```
int((x^6 + 1)/(x^6 - 1),x)
```

output
```
x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(
3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) +
(32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{1+x^6}{-1+x^6}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + x$$

input
```
int((x^6+1)/(x^6-1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ log(x**2 - x + 1) - log(x**2 + x + 1) + 2*log(x - 1) - 2*log(x + 1) + 6*
x)/6
```

3.3 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2\text{arctanh}(x)}{3}-\frac{1}{3}\text{arctanh}\left(\frac{x}{1+x^2}\right)$$

output
```
x+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)-2/3*arctanh(x)-1/3*arctanh(x/(x^2+1))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = \frac{1}{6}\left(6x-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

output
```
(6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/S
qrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2
])/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2027, 10, 25, 913, 754, 27, 219, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+\frac{1}{x^3}}{x^3-\frac{1}{x^3}} \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^3\left(x^3+\frac{1}{x^3}\right)}{x^6-1}dx$$

$$\Big\downarrow \text{10}$$

$$\int -\frac{x^6+1}{1-x^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^6+1}{1-x^6}dx$$

$$\Big\downarrow \text{913}$$

$$x-2\int \frac{1}{1-x^6}dx$$

$$\Big\downarrow \text{754}$$

$$x-2\left(\frac{1}{3}\int \frac{1}{1-x^2}dx+\frac{1}{3}\int \frac{2-x}{2\left(x^2-x+1\right)}dx+\frac{1}{3}\int \frac{x+2}{2\left(x^2+x+1\right)}dx\right)$$

$$\Big\downarrow \text{27}$$

$$x-2\left(\frac{1}{3}\int \frac{1}{1-x^2}dx+\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx\right)$$

$$\Big\downarrow \text{ 219}$$

$$x-2\left(\frac{1}{6}\int \frac{2-x}{x^2-x+1}dx+\frac{1}{6}\int \frac{x+2}{x^2+x+1}dx+\frac{\text{arctanh}(x)}{3}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$x-$$
$$2\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{\text{arctanh}(x)}{3}\right.$$

$$\Big\downarrow \text{ 25}$$

$$x-$$
$$2\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{x^2-x+1}dx+\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx\right)+\frac{\text{arctanh}(x)}{3}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$x-$$
$$2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-3\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-3\int \frac{1}{-(2x+1)^2-3}d(2x\right.\right.$$

$$\Big\downarrow \text{ 217}$$

$$x-$$
$$2\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx+\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)+\frac{\text{arctanh}}{3}\right.$$

$$\Big\downarrow \text{ 1103}$$

$$x-$$
$$2\left(\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(x^2-x+1\right)\right)+\frac{1}{6}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+x+1\right)\right)+\frac{\text{arctanh}(}{3}\right.$$

input

```
Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

output

```
x - 2*(ArcTanh[x]/3 + (Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - Log[1 - x + x^
2]/2)/6 + (Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2)/6)
```

Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2  Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)  Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

method	result	si
risch	$x+\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	63
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2x)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	67

input
```
int((1/x^3+x^3)/(-1/x^3+x^3),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/3
*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(2/3*(x-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x+1\right) + \frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x + \frac{\log\left(x-1\right)}{3} - \frac{\log\left(x+1\right)}{3} + \frac{\log\left(x^2-x+1\right)}{6} - \frac{\log\left(x^2+x+1\right)}{6} - \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

output
```
x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/
6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3
 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(x+1\right) + \frac{1}{3}\log\left(x-1\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x
 + 1) + 1/3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log\left(|x+1|\right) + \frac{1}{3}\log\left(|x-1|\right)$$

input
```
integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(a
bs(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = x+\frac{\mathrm{atan}(x\,1\mathrm{i})\ 2\mathrm{i}}{3} - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

input
```
int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

output
```
x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(
3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) +
(32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}-\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3}+\frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6}+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+x$$

input
```
int((1/x^3+x^3)/(-1/x^3+x^3),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ log(x**2 - x + 1) - log(x**2 + x + 1) + 2*log(x - 1) - 2*log(x + 1) + 6*
x)/6
```

3.4 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -6c^2d\sqrt{a+\frac{b}{x}}-\frac{2d^2(3bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}-\frac{2d^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^2} + c^3\sqrt{a+\frac{b}{x}}x+\frac{c^2(bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-6*c^2*d*(a+b/x)^(1/2)-2/3*d^2*(-a*d+3*b*c)*(a+b/x)^(3/2)/b^2-2/5*d^3*(a+b
/x)^(5/2)/b^2+c^3*(a+b/x)^(1/2)*x+c^2*(6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^
(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$
$$= \frac{\sqrt{a+\frac{b}{x}}(4a^2d^3x^2-2abd^2x(d+15cx)-3b^2(2d^3+10cd^2x+30c^2dx^2-5c^3x^3))}{15b^2x^2}$$
$$+ \frac{c^2(bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 +
10*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {899, 108, 27, 170, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$
$$\Big\downarrow \text{ 899}$$
$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$
$$\Big\downarrow \text{ 108}$$

$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\int \frac{\left(c+\frac{d}{x}\right)^2\left(bc+6ad+\frac{7bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \textbf{27}$$

$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{1}{2}\int \frac{\left(c+\frac{d}{x}\right)^2\left(bc+6ad+\frac{7bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \textbf{170}$$

$$\frac{1}{2}\left(-\frac{2\int \frac{b\left(c+\frac{d}{x}\right)\left(\frac{d(33bc+2ad)}{x}+5c(bc+6ad)\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{5b}-\frac{14}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\right)+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{2}\left(-\frac{1}{5}\int \frac{\left(c+\frac{d}{x}\right)\left(\frac{d(33bc+2ad)}{x}+5c(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{14}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\right)+ x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \textbf{164}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-5c^2(6ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\sqrt{a-\right.$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(-\frac{10c^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\right.$$
$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{1}{5}\left(\frac{10c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)}{\sqrt{a}}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{3b^2}\right)-\frac{14}{5}d\sqrt{\right.$$

$$x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3$$

input
```
Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

output
```
Sqrt[a + b/x]*(c + d/x)^3*x + ((-14*d*Sqrt[a + b/x]*(c + d/x)^2)/5 + ((-2*
d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c +
2*a*d))/x))/(3*b^2) + (10*c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/Sqrt[a])/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.33

method	result
risch	$\frac{\left(15 b^{2} c^{3} x^{3}+4 a^{2} d^{3} x^{2}-30 a b c\, d^{2} x^{2}-90 b^{2} c^{2} d\, x^{2}-2 a b\, d^{3} x -30 b^{2} c\, d^{2} x -6 b^{2} d^{3}\right) \sqrt{\frac{a x +b}{x}}}{15 x^{2} b^{2}}+\frac{\left(6 a d +b c \right) c^{2} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a\, x^{2}+b x}\right) \sqrt{\frac{a x +}{x}}}{2 \sqrt{a}\, \left(a x +b \right)}$
default	$\frac{\sqrt{\frac{a x +b}{x}}\, \left(180 \sqrt{a\, x^{2}+b x}\, a^{\frac{3}{2}} b\, c^{2} d\, x^{4}+30 \sqrt{a\, x^{2}+b x}\, \sqrt{a}\, b^{2} c^{3} x^{4}+90 \ln \left(\frac{2 \sqrt{a\, x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a\, b^{2} c^{2} d\, x^{4}+15 \ln \left(\frac{2 \sqrt{a\, x^{2}+b x}\, \sqrt{a}+2 a x +}{2 \sqrt{a}}\right.}{30 x^{3} \sqrt{x \left(a x +b \right)}\, \sqrt{a}}$

input
```
int((a+b/x)^(1/2)*(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*b^2*c^3*x^3+4*a^2*d^3*x^2-30*a*b*c*d^2*x^2-90*b^2*c^2*d*x^2-2*a*b
*d^3*x-30*b^2*c*d^2*x-6*b^2*d^3)/x^2/b^2*((a*x+b)/x)^(1/2)+1/2*(6*a*d+b*c)
*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.59

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$$
$$= \left[\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a}{30\,ab^2x^2}\right.$$
$$-\frac{15\,(b^3c^3+6\,ab^2c^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(15\,ab^2c^3x^3-6\,ab^2d^3-2\,(45\,ab^2c^2d+15\,a^2bcd^2-}{15\,ab^2x^2}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2
*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*s
qrt((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a
)*x^2*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (15*a*b^2*c^3*x^3 -
 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15
*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.84

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1} - 3c^2d\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + 3cd^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right) + \frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2)*(c+d/x)**3,x)
```

output
```
4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b*
*(7/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)
*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(1
5*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(b)*c**3*sqrt(
x)*sqrt(a*x/b + 1) - 3*c**2*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/
sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 3*c*d**2
*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b*
c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3 - 3\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\sqrt{a+\frac{b}{x}}\right)c^2d - \frac{2}{15}d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2} - \frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^2}\right) - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}cd^2}{b}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/
b^2 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.44

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx=\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right)+\sqrt{a+\frac{b}{x}}\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)+c^3\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{5/2}}{5\,b^2}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,(6\,a\,d+b\,c)\,1\mathrm{i}}{\sqrt{a}}$$

input
```
int((a + b/x)^(1/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a +
 b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d -
 b*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)
^(5/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i
)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.22

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3\,dx$$
$$=\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^3d^3x^2-120\sqrt{x}\,\sqrt{ax+b}\,a^2bc\,d^2x^2-8\sqrt{x}\,\sqrt{ax+b}\,a^2b\,d^3x+60\sqrt{x}\,\sqrt{ax+b}\,a\,b^2c^3x^3-3}{}$$

input
```
int((a+b/x)^(1/2)*(c+d/x)^3,x)
```

output
```
(16*sqrt(x)*sqrt(a*x + b)*a**3*d**3*x**2 - 120*sqrt(x)*sqrt(a*x + b)*a**2*
b*c*d**2*x**2 - 8*sqrt(x)*sqrt(a*x + b)*a**2*b*d**3*x + 60*sqrt(x)*sqrt(a*
x + b)*a*b**2*c**3*x**3 - 360*sqrt(x)*sqrt(a*x + b)*a*b**2*c**2*d*x**2 - 1
20*sqrt(x)*sqrt(a*x + b)*a*b**2*c*d**2*x - 24*sqrt(x)*sqrt(a*x + b)*a*b**2
*d**3 + 360*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b**2*
c**2*d*x**3 + 60*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*
*3*c**3*x**3 - 16*sqrt(a)*a**3*d**3*x**3 - 24*sqrt(a)*a**2*b*c*d**2*x**3 +
 216*sqrt(a)*a*b**2*c**2*d*x**3 + 27*sqrt(a)*b**3*c**3*x**3)/(60*a*b**2*x*
*3)
```

3.5 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx=-4cd\sqrt{a+\frac{b}{x}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}+c^2\sqrt{a+\frac{b}{x}}x+\frac{c(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output

```
-4*c*d*(a+b/x)^(1/2)-2/3*d^2*(a+b/x)^(3/2)/b+c^2*(a+b/x)^(1/2)*x+c*(4*a*d+
b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx=\frac{\sqrt{a+\frac{b}{x}}(-2ad^2x+b(-2d^2-12cdx+3c^2x^2))}{3bx}+\frac{c(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
 (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$$

$$\Big\downarrow \ 899$$

$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 100$$

$$\frac{c^2 x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\int \frac{1}{2}\sqrt{a+\frac{b}{x}}\left(\frac{2ad^2}{x}+c(bc+4ad)\right)xd\frac{1}{x}}{a}$$

$$\Big\downarrow \ 27$$

$$\frac{c^2 x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\int \sqrt{a+\frac{b}{x}}\left(\frac{2ad^2}{x}+c(bc+4ad)\right)xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \ 90$$

$$\frac{c^2 x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow \ 60$$

$$\frac{c^2 x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow 73$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c(4ad+bc)\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

$$\Big\downarrow 221$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(4ad+bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}}{2a}$$

input
```
Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

output
```
(c^2*(a + b/x)^(3/2)*x)/a - ((4*a*d^2*(a + b/x)^(3/2))/(3*b) + c*(b*c + 4*
a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{\left(-3 b \,c^{2} x^{2}+2 x a \,d^{2}+12 b c d x +2 b \,d^{2}\right) \sqrt{\frac{a x +b}{x}}}{3 x b}+\frac{\left(4 a d +b c \right) c \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x \left(a x +b \right)}}{2 \sqrt{a}\, \left(a x +b \right)}$
default	$\frac{\sqrt{\frac{a x +b}{x}}\, \left(12 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a b c d \,x^{3}+3 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) b^{2} c^{2} x^{3}+24 a^{\frac{3}{2}} \sqrt{a \,x^{2}+b x}\, c d \,x^{3}+6 \sqrt{a}\, \sqrt{a \,x^{2}+b x}\, b \,c^{2} x^{3}}{6 x^{2} \sqrt{x \left(a x +b \right)}\, \sqrt{a}\, b}$

input
```
int((a+b/x)^(1/2)*(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b*c^2*x^2+2*a*d^2*x+12*b*c*d*x+2*b*d^2)/x/b*((a*x+b)/x)^(1/2)+1/2
*(4*a*d+b*c)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.48

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\,dx$$
$$=\left[\frac{3\,(b^2c^2+4\,abcd)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+}{x}}}{6\,abx}\right.$$
$$\left.-\frac{3\,(b^2c^2+4\,abcd)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(3\,abc^2x^2-2\,abd^2-2\,(6\,abcd+a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{3\,abx}\right]$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*s
qrt((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan
(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*
(6*a*b*c*d + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

Sympy [A] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.50

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1} - 2cd\left(\begin{cases} \frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0 \\ -\sqrt{a}\log{(x)} & \text{otherwise} \end{cases}\right) + d^2\left(\begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b=0 \\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) + \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2)*(c+d/x)**2,x)
```

output
```
sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*c*d*Piecewise((2*a*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), Tr
ue)) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b),
True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2 - 2\left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\sqrt{a+\frac{b}{x}}\right)cd - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3\,b}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^2,x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = \left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\sqrt{a+\frac{b}{x}}+c^2\,x\,\sqrt{a+\frac{b}{x}} - \frac{2\,d^2\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b}-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+b\,c\right)\,1\mathrm{i}}{\sqrt{a}}$$

input
```
int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

output
```
((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1
/2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2)
)*(4*a*d + b*c)*1i)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.01

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$$
$$= \frac{-4\sqrt{x}\,\sqrt{ax+b}\,a^2d^2x+6\sqrt{x}\,\sqrt{ax+b}\,ab\,c^2x^2-24\sqrt{x}\,\sqrt{ax+b}\,abcdx-4\sqrt{x}\,\sqrt{ax+b}\,ab\,d^2+24\sqrt{a}\log\Big(}{6ab\,x^2}$$

input
```
int((a+b/x)^(1/2)*(c+d/x)^2,x)
```

output
```
( - 4*sqrt(x)*sqrt(a*x + b)*a**2*d**2*x + 6*sqrt(x)*sqrt(a*x + b)*a*b*c**2
*x**2 - 24*sqrt(x)*sqrt(a*x + b)*a*b*c*d*x - 4*sqrt(x)*sqrt(a*x + b)*a*b*d
**2 + 24*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b*c*d*x*
*2 + 6*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2*c**2*x*
*2 - 4*sqrt(a)*a**2*d**2*x**2 + 8*sqrt(a)*a*b*c*d*x**2 + sqrt(a)*b**2*c**2
*x**2)/(6*a*b*x**2)
```

3.6 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) \, dx = -2d\sqrt{a+\frac{b}{x}}+c\sqrt{a+\frac{b}{x}}x+\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*d*(a+b/x)^(1/2)+c*(a+b/x)^(1/2)*x+(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(
1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) \, dx = \sqrt{a+\frac{b}{x}}(-2d+cx)+\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

output
```
Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {899, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) \, dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)}{2a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{(2ad+bc)\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)}{2a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(2ad+bc)}{2a}$$

input
```
Int[Sqrt[a + b/x]*(c + d/x),x]
```

output
```
(c*(a + b/x)^(3/2)*x)/a - ((b*c + 2*a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcT
anh[Sqrt[a + b/x]/Sqrt[a]]))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

method	result
risch	$(cx-2d)\sqrt{\frac{ax+b}{x}}+\frac{\left(ad+\frac{bc}{2}\right)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{\sqrt{a}\,(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(2\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd\,x^{2}+\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}c\,x^{2}+4a^{\frac{3}{2}}\sqrt{a\,x^{2}+bx}\,d\,x^{2}+2\sqrt{a}\,\sqrt{a\,x^{2}+bx}\,bc\,x^{2}-4\sqrt{a}\,(\right.}{2x\sqrt{x(ax+b)}\,b\sqrt{a}}$

input
```
int((a+b/x)^(1/2)*(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
(c*x-2*d)*((a*x+b)/x)^(1/2)+(a*d+1/2*b*c)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.18

$$\int\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \left[\frac{(bc+2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a},\; -\frac{(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{a}\right]$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
 + 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan
(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (a*c*x - 2*a*d)*sqrt((a*x + b)/
x))/a]
```

Sympy [A] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1} - d\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log\left(x\right) & \text{otherwise}\end{cases}\right) + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2)*(c+d/x),x)
```

output
```
sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - d*Piecewise((2*a*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) +
b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(51) = 102.

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.74

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx = \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c - \left(\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)d$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
sqrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a))) + 2*sqrt(a + b/x))*d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx=2\,\sqrt{a}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,d\,\sqrt{a+\frac{b}{x}}+c\,x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

input
```
int((a + b/x)^(1/2)*(c + d/x),x)
```

output
```
2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*
x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*
x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\,dx$$
$$= \frac{4\sqrt{x}\,\sqrt{ax+b}\,acx - 8\sqrt{x}\,\sqrt{ax+b}\,ad + 8\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)adx + 4\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)bcx - 8\sqrt{a}}{4ax}$$

input
```
int((a+b/x)^(1/2)*(c+d/x),x)
```

output
```
(4*sqrt(x)*sqrt(a*x + b)*a*c*x - 8*sqrt(x)*sqrt(a*x + b)*a*d + 8*sqrt(a)*l
og((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*d*x + 4*sqrt(a)*log((sqrt(
a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*c*x - 8*sqrt(a)*a*d*x - sqrt(a)*b*c
*x)/(4*a*x)
```

3.7 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
(a+b/x)^(1/2)*x+b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \sqrt{a+\frac{b}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\sqrt{a+\frac{b}{x}}-\frac{1}{2}b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$x\sqrt{a+\frac{b}{x}}-\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}$$

input
```
Int[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

method	result	size
risch	$\sqrt{\frac{ax+b}{x}}\, x+\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$	72
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x\left(2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\,\sqrt{a}}$	74

input
```
int((a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x+b)/x)^(1/2)*x+1/2*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2
)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.67

$$\int \sqrt{a+\frac{b}{x}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a}\right]$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*x*s
qrt((a*x + b)/x)/(a*x + b)))/a]
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqr
t(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \sqrt{a+\frac{b}{x}}\,dx = -\frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2+bx}\operatorname{sgn}(x)$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \sqrt{a+\frac{b}{x}}\,dx = x\sqrt{a\,x^2+b\,x}\sqrt{\frac{1}{x^2}} + \frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

input
```
int((a + b/x)^(1/2),x)
```

output
```
x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
 a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{a+\frac{b}{x}}\,dx=\frac{\sqrt{x}\,\sqrt{ax+b}\,a+\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b}{a}$$

input
```
int((a+b/x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a*x + b)*a + sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b)/a
```

3.8 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{c}+\frac{2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

output
```
(a+b/x)^(1/2)*x/c+2*d^(1/2)*(-a*d+b*c)^(1/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/
(-a*d+b*c)^(1/2))/c^2+(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)/
c^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx=\frac{c\sqrt{a+\frac{b}{x}}x+2\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

output
```
(c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/
x])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt
[a])/c^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {899, 110, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

$$\Big\downarrow \; 899$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \; 110$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(bc-2ad-\frac{bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(bc-2ad-\frac{bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{ 174}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2d(bc-ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4d(bc-ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c}}{2c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}-\frac{-\frac{4\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{\sqrt{a}c}}{2c}$$

input

```
Int[Sqrt[a + b/x]/(c + d/x),x]
```

output

```
(Sqrt[a + b/x]*x)/c - ((-4*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a
+ b/x])/Sqrt[b*c - a*d]])/c - (2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/(Sqrt[a]*c))/(2*c)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(86) = 172$.

Time = 0.51 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.23

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c} - \frac{\left(\frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2(ad-bc)d\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}}{2c(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2a^{\frac{3}{2}}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c-2adx+bcx-bd}{cx+d}\right)d^2-2\sqrt{x(ax+b)}\,c^2\sqrt{a}\sqrt{\frac{(ad-bc)d}{c^2}}-2\sqrt{a}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c}{cx+d}\right.\right.}{2\sqrt{x(ax+b)}\,c^3\sqrt{a}\sqrt{(}}$

input
```
int((a+b/x)^(1/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/c*x*((a*x+b)/x)^(1/2)-1/2/c*((2*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2
+b*x)^(1/2))/a^(1/2)+2*(a*d-b*c)*d/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-
b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^
2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))*((a*x+b)/x)^
(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.73

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right) + 2\,\sqrt{-bcd+ad^2}a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-b}}{cx+d}\right.}{2\,ac^2}\right.$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c -
2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2)
, 1/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c
*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b
)/x) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b
)) + sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a
*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 2*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/
(a*x + b)))/(a*c^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x),x)
```

output
```
Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \frac{x\sqrt{a+\frac{b}{x}}}{c} + \frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2} - \frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)(2\,a\,d-b\,c)}{2\,\sqrt{a}\,c^2} - \frac{\operatorname{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\sqrt{a\,d^2-b\,c\,d}\,2\mathrm{i}}{c^2}$$

input
```
int((a + b/x)^(1/2)/(c + d/x),x)
```

output
```
(x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/
2)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((
a + b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^
(1/2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = \frac{\sqrt{d}\,\sqrt{ad-bc}\log\left(\sqrt{c}\,\sqrt{ax+b}-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{x}\,\sqrt{c}\,\sqrt{a}\right)a+\sqrt{d}\,\sqrt{ad-bc}\log\left(\right.}{}$$

input
```
int((a+b/x)^(1/2)/(c+d/x),x)
```

output
```
(sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a + sqrt(d)*sq
rt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
- b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a - sqrt(d)*sqrt(a*d - b*
c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)
*c + 2*a*c*x + 2*a*d)*a + sqrt(x)*sqrt(a*x + b)*a*c - 2*sqrt(a)*log((sqrt(
a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*d + sqrt(a)*log((sqrt(a*x + b) + sq
rt(x)*sqrt(a))/sqrt(b))*b*c)/(a*c**2)
```

3.9 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} \, dx = \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

output
```
2*d*(a+b/x)^(1/2)/c^2/(c+d/x)+(a+b/x)^(1/2)*x/c/(c+d/x)+d^(1/2)*(-4*a*d+3*
b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/(-a*d+b*c)^(1/2)+(
-4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)/c^3
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} \, dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(2d+cx)}{d+cx} + \frac{\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcT
an[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {899, 110, 27, 168, 25, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2} \, dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 110}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)} - \frac{\int \frac{\left(bc-4ad-\frac{3bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\left(bc-4ad-\frac{3bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \; 168$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{-\frac{\int -\frac{(bc-ad)\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 25$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{(bc-ad)\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(bc-4ad-\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 174$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{(bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{d(3bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 73$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2d(3bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 218$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \; 221$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{-\frac{2\sqrt{d}(3bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}c}}{c}-\frac{4d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}}{2c}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*x)/(c*(c + d/x)) - ((-4*d*Sqrt[a + b/x])/(c*(c + d/x)) + ((
-2*Sqrt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (2*(b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(
Sqrt[a]*c))/c)/(2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(127) = 254$.

Time = 0.52 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.24

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{(4ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2d(3ad-2bc)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\right.}{c}}}{x+\frac{d}{c}}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(a}}{cx}\right.\right.}{}$

input
```
int((a+b/x)^(1/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*((4*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a
*x^2+b*x)^(1/2))/a^(1/2)+2/c^2*d*(3*a*d-2*b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln(
(2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x
+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+2*d^2
*(a*d-b*c)/c^3*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*
(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*
d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/
c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/
(x+1/c*d))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 773, normalized size of antiderivative = 5.26

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*
c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sq
rt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c
*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (
b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)
) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d
))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b
*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d
)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)) - (
b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*
x + a*c^3*d), -((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sq
rt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^
2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(d/(b*c - a*d))*sqrt((a*x
 + b)/x)) - (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d)
]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(127) = 254$.

Time = 0.16 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$
$$= \frac{\sqrt{ax^2+bx}\,\mathrm{sgn}(x)}{c^2} - \frac{\left(3\,bcd\,\mathrm{sgn}(x) - 4\,ad^2\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3}$$
$$- \frac{\left(bc\,\mathrm{sgn}(x) - 4\,ad\,\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,\sqrt{a}c^3}$$
$$- \frac{\left(6\,\sqrt{a}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 8\,a^{\frac{3}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - \sqrt{bcd-ad^2}\,bc\log\left(|b|\right) + 4\,\sqrt{bcd-ad^2}\,ad\,\mathrm{lo}\right.}{2\,\sqrt{bcd-ad^2}\,\sqrt{a}c^3}$$
$$- \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd\,\mathrm{sgn}(x) - 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^2\mathrm{sgn}(x) - \sqrt{a}bd^2\mathrm{sgn}(x)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d + bd\right)c^3}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*sgn(x)/c^2 - (3*b*c*d*sgn(x) - 4*a*d^2*sgn(x))*arctan(-(
(sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/(sqrt(
b*c*d - a*d^2)*c^3) - 1/2*(b*c*sgn(x) - 4*a*d*sgn(x))*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^3) - 1/2*(6*sqrt(a)*b*c*d*a
rctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(3/2)*d^2*arctan(sqrt(a)*d/sqrt
(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b*c*log(abs(b)) + 4*sqrt(b*c*d - a*
d^2)*a*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*d)*sgn(x)/(sqrt(b*c*d - a*d
^2)*sqrt(a)*c^3) - ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d*sgn(x) - 2*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*a*d^2*sgn(x) - sqrt(a)*b*d^2*sgn(x))/(((sqrt(a)
*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d
+ b*d)*c^3)
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 1195, normalized size of antiderivative = 8.13

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^2,x)
```

output
```
- ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((
a + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^
3*(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a
 + b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(
a^(1/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b
^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*
b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(
d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b
*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*
(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^
2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b
^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d
*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*
c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(
b*c^4 - a*c^3*d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/
c^6 - ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c
^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^
6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(
1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*
d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d...
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.78

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(1/2)/(c+d/x)^2,x)
```

output
```
(4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*c*d*x +
 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d**2 -
3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(
a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c**2*x -
3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(
a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c*d + 4*s
qrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*
sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*c*d*x + 4*s
qrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*
sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d**2 - 3*sq
rt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c**2*x - 3*sq
rt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c*d - 4*sqrt(
d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(
a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**2*c*d*x - 4*sqrt(d)*sqrt(a*d - b*
c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)
*c + 2*a*c*x + 2*a*d)*a**2*d**2 + 3*sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(...
```

3.10 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 213

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{(bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

output
```
3/2*d*(a+b/x)^(1/2)/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11*b*c)*(a+b/x)^(1/2)/c^3
/(-a*d+b*c)/(c+d/x)+(a+b/x)^(1/2)*x/c/(c+d/x)^2+1/4*d^(1/2)*(24*a^2*d^2-40
*a*b*c*d+15*b^2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-
a*d+b*c)^(3/2)+(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)/c^4
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$
$$=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(-2ad(6d^2+9cdx+2c^2x^2)+bc(11d^2+17cdx+4c^2x^2))}{(bc-ad)(d+cx)^2}+\frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{4(bc-6ad)\text{arctan}}{\sqrt{a}}}{4c^4}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-2*a*d*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + b*c*(11*d^2 +
17*c*d*x + 4*c^2*x^2)))/((b*c - a*d)*(d + c*x)^2) + (Sqrt[d]*(15*b^2*c^2 -
 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/(b*c - a*d)^(3/2) + (4*(b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt
[a])/(4*c^4)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {899, 110, 27, 168, 25, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}dx$$
$$\Big\downarrow \ 899$$
$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$
$$\Big\downarrow \ 110$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(bc-6ad-\frac{5bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(bc-6ad-\frac{5bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$\downarrow$ 168

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{-\frac{\int -\frac{(bc-ad)\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 25

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{(bc-ad)\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{\left(2(bc-6ad)-\frac{9bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 168

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{-\frac{\int -\frac{\left(4(bc-6ad)(bc-ad)-\frac{bd(11bc-12ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 27

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\int \frac{\left(4(bc-6ad)(bc-ad)-\frac{bd(11bc-12ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 174

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{4(bc-6ad)(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{d\left(24a^2d^2-40abcd+15b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 73

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8(bc-6ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2d\left(24a^2d^2-40abcd+15b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 218

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8(bc-6ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 221

$$\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{-\frac{2\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)(bc-ad)}{\sqrt{a}c}}{2c(bc-ad)}-\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c}-\frac{3d\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - ((-3*d*Sqrt[a + b/x])/(c*(c + d/x)^2)
+ (-((d*(11*b*c - 12*a*d)*Sqrt[a + b/x])/(c*(b*c - a*d)*(c + d/x))) + ((-2
*Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (8*(b*c - 6*a*d)*(b*c - a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(2*c*(b*c - a*d)))/(2*c))/(2*
c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. $2(185) = 370$.

Time = 0.51 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.55

method	result
risch	$\frac{x\sqrt{\frac{ax+b}{x}}}{c^3} - \frac{\left(\frac{(6ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{6d(2ad-bc)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\right.}{c}}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{}$
default	Expression too large to display

input
```
int((a+b/x)^(1/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*x*((a*x+b)/x)^(1/2)-1/2/c^3*((6*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a
*x^2+b*x)^(1/2))/a^(1/2)+6/c^2*d*(2*a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln((2
*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1
/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+2/c^3*d
^2*(4*a*d-3*b*c)*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/
c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c
)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*
d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)
)/(x+1/c*d)))-2*d^3*(a*d-b*c)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+1/c*d)^2*(a*(x+
1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/
(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+
1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c
^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2
)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+
1/c*d)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c
^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d
-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))))*((a*x+b)/x)^(1/2)*(
x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(185) = 370$.

Time = 0.19 (sec) , antiderivative size = 1721, normalized size of antiderivative = 8.08

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(
a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^
2)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(
b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) -
 b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*
a*b*c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt(
(a*x + b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(
a*b*c^6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*
a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b
^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(
sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)) - 2*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*
a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a
*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b
)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c^2*d^2)
*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^5*d^2
- a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^2)*x)
, -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d
+ 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*s...
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx=\int \frac{x^3\sqrt{a+\frac{b}{x}}}{(cx+d)^3}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

output
```
Integral(x**3*sqrt(a + b/x)/(c*x + d)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 809 vs. $2(185) = 370$.

Time = 0.19 (sec) , antiderivative size = 809, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
-1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3
/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(
sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b))
 + 14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2
*d^2*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*
a^2*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a
^(3/2)*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c
*d - a*d^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*s
gn(x)/c^3 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d*sgn(x) - 32
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x - sq
rt(a*x^2 + b*x))^3*a^2*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*
sqrt(a)*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*
b*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^4*sgn(x) +
 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sq
rt(a*x^2 + b*x))*a*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a
^2*b*d^4*sgn(x) - 9*sqrt(a)*b^3*c*d^3*sgn(x) + 10*a^(3/2)*b^2*d^4*sgn(x))/
((b*c^5 - a*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - s
qrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn(x))*lo
g(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 1895, normalized size of antiderivative = 8.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

output
```
(log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c
*d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3
*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2
)*(12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*
a*d^3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2
+ 24*a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2
 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^
3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*
c)^3)^(1/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a
^2*d^2 + 15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5
*d^2 - 3*a*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241
*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*
d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c
^11*d^2 - 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^
2*c^11 + a^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*
b^5*c^11*d^2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*
d^5))/(16*a^(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*
c^4))*(6*a*d - b*c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2
*d^7 + 241*b^6*c^4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^
2*b^4*c^2*d^5))/(8*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b...
```

Reduce [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 3447, normalized size of antiderivative = 16.18

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(1/2)/(c+d/x)^3,x)
```

output
```
(96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**4*c**2*d
**3*x**2 + 192*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*
a**4*c*d**4*x + 96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**4*d**5 - 208*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sq
rt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt
(a))*a**3*b*c**3*d**2*x**2 - 416*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(
a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)
*sqrt(c)*sqrt(a))*a**3*b*c**2*d**3*x - 208*sqrt(d)*sqrt(a*d - b*c)*log(sqr
t(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
 + sqrt(x)*sqrt(c)*sqrt(a))*a**3*b*c*d**4 + 140*sqrt(d)*sqrt(a*d - b*c)*lo
g(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*b**2*c**4*d*x**2 + 280*sqrt(d)*sqrt(
a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b
*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*b**2*c**3*d**2*x + 140*
sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)
*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*b**2*c**2*
d**3 - 30*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sq...
```

3.11 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx = &-2c^2(bc+3ad)\sqrt{a+\frac{b}{x}} \\ &- 2c^2d\left(a+\frac{b}{x}\right)^{3/2} - \frac{2d^2(3bc-ad)\left(a+\frac{b}{x}\right)^{5/2}}{5b^2} - \frac{2d^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^2} \\ &+ ac^3\sqrt{a+\frac{b}{x}}x + 3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

output
```
-2*c^2*(3*a*d+b*c)*(a+b/x)^(1/2)-2*c^2*d*(a+b/x)^(3/2)-2/5*d^2*(-a*d+3*b*c
)*(a+b/x)^(5/2)/b^2-2/7*d^3*(a+b/x)^(7/2)/b^2+a*c^3*(a+b/x)^(1/2)*x+3*a^(1
/2)*c^2*(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = \frac{\sqrt{a+\frac{b}{x}}(4a^3d^3x^3 - 2a^2bd^2x^2(d+21cx) + ab^2x(-16d^3 - 84cd^2x - 280c^2dx^2 + 35c^3x^3) - 2b^3(5}{35b^2x^3} + 3\sqrt{a}c^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-1
6*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2
*x + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {899, 108, 27, 170, 27, 164, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx$$

$$\Big\downarrow \ 899$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 108$$

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\int \frac{3}{2}\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\left(bc+2ad+\frac{3bd}{x}\right)xd\frac{1}{x}$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{3}{2}\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2\left(bc+2ad+\frac{3bd}{x}\right)xd\frac{1}{x}$$

$\downarrow$ 170

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{2\int \frac{1}{2}b\sqrt{a+\frac{b}{x}}(c+\frac{d}{x})\left(7c(bc+2ad)+\frac{d(19bc+2ad)}{x}\right)xd\frac{1}{x}}{7b}+\frac{6}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\right)$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)\left(7c(bc+2ad)+\frac{d(19bc+2ad)}{x}\right)xd\frac{1}{x}+\frac{6}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\right)$$

$\downarrow$ 164

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2d(a+\frac{b}{x})^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{15b^2}\right)+\frac{6}{7}d\left(a+\frac{b}{x}\right.\right.$$

$\downarrow$ 60

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2d(a+\frac{b}{x})^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{15b^2}\right.\right.$$

$\downarrow$ 73

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\left(7c^2(2ad+bc)\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2d(a+\frac{b}{x})^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+\right.}{15b^2}\right.\right.$$

$$\Big\downarrow 221$$

$$x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-$$
$$\frac{3}{2}\left(\frac{1}{7}\left(7c^2\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)(2ad+bc)+\frac{2d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2a\right.}{15b^2}\right.\right.$$

input
```
Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

output
```
(a + b/x)^(3/2)*(c + d/x)^3*x - (3*((6*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 +
((2*d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c +
 2*a*d))/x))/(15*b^2) + 7*c^2*(b*c + 2*a*d)*(2*Sqrt[a + b/x] - 2*Sqrt[a]*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]]))/7))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.47

method	result
risch	$\frac{(35a\,b^2c^3x^4+4x^3a^3d^3-42x^3a^2bc\,d^2-280x^3a\,b^2c^2d-70x^3b^3c^3-2x^2a^2b\,d^3-84x^2a\,b^2c\,d^2-70x^2b^3c^2d-16a\,b^2d^3x-42b^3c\,d^2x-10b^3}{35x^3b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(420\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b\,c^2d\,x^5+210\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2c^3x^5+210\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^2c^2d\,x^5+105\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2}{2\sqrt{a}}\right.}{}$

input
```
int((a+b/x)^(3/2)*(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/35*(35*a*b^2*c^3*x^4+4*a^3*d^3*x^3-42*a^2*b*c*d^2*x^3-280*a*b^2*c^2*d*x^
3-70*b^3*c^3*x^3-2*a^2*b*d^3*x^2-84*a*b^2*c*d^2*x^2-70*b^3*c^2*d*x^2-16*a*
b^2*d^3*x-42*b^3*c*d^2*x-10*b^3*d^3)/x^3/b^2*((a*x+b)/x)^(1/2)+3/2*(2*a*d+
b*c)*a^(1/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/
2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.64

$$\int\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx=\left[\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3}{}\right.$$

$$-\frac{105\,(b^3c^3+2\,ab^2c^2d)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(35\,ab^2c^3x^4-10\,b^3d^3-2\,(35\,b^3c^3+140\,ab^2c^2d+21}{35\,b^2x^3}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*s
qrt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 +
 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x
 + b)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arc
tan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (35*a*b^2*c^3*x^4 - 10*b^3*d
^3 - 2*(35*b^3*c^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2
*(35*b^3*c^2*d + 42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b
^2*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 40.28 (sec) , antiderivative size = 1828, normalized size of antiderivative = 12.52

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

output
```
-16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(
13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d*
*3*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4
0*a**(13/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19
/2)*d**3*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2
)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 12*a**(11/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x
**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt
(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a
**(9/2)*b**(21/2)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3
15*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 6*a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/
2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx = -\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b\right)c^3 - \left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{b}{x}}a\right)c^2d - \frac{2}{35}\left(\frac{5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{b^2} - \frac{7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^2}\right)d^3$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

output
```
-6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*
c^3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
+ 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b
^2 - 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.24

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3\,dx = \left(a+\frac{b}{x}\right)^{5/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right) + \sqrt{a+\frac{b}{x}}\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\left(\frac{6\,a\,d^3-6}{b^2}\right.\right. + \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}-\frac{2\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{3\,b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{7/2}}{7\,b^2}+a\,c^3\,x\,\sqrt{a+\frac{b}{x}}-2$$

input
```
int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
 b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 -
 (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d
^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 -
6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)
/(3*b^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*
c^2*atan((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2
*d + 3*a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.34

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = \frac{4\sqrt{x}\,\sqrt{ax+b}\,a^3d^3x^3 - 42\sqrt{x}\,\sqrt{ax+b}\,a^2bc\,d^2x^3 - 2\sqrt{x}\,\sqrt{ax+b}\,a^2b\,d^3x^2 + 35\sqrt{x}\,\sqrt{ax+b}\,a\,b^2}{}$$

input
```
int((a+b/x)^(3/2)*(c+d/x)^3,x)
```

output
```
(4*sqrt(x)*sqrt(a*x + b)*a**3*d**3*x**3 - 42*sqrt(x)*sqrt(a*x + b)*a**2*b*
c*d**2*x**3 - 2*sqrt(x)*sqrt(a*x + b)*a**2*b*d**3*x**2 + 35*sqrt(x)*sqrt(a
*x + b)*a*b**2*c**3*x**4 - 280*sqrt(x)*sqrt(a*x + b)*a*b**2*c**2*d*x**3 -
84*sqrt(x)*sqrt(a*x + b)*a*b**2*c*d**2*x**2 - 16*sqrt(x)*sqrt(a*x + b)*a*b
**2*d**3*x - 70*sqrt(x)*sqrt(a*x + b)*b**3*c**3*x**3 - 70*sqrt(x)*sqrt(a*x
 + b)*b**3*c**2*d*x**2 - 42*sqrt(x)*sqrt(a*x + b)*b**3*c*d**2*x - 10*sqrt(
x)*sqrt(a*x + b)*b**3*d**3 + 210*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt
(a))/sqrt(b))*a*b**2*c**2*d*x**4 + 105*sqrt(a)*log((sqrt(a*x + b) + sqrt(x
)*sqrt(a))/sqrt(b))*b**3*c**3*x**4 - 4*sqrt(a)*a**3*d**3*x**4 - 18*sqrt(a)
*a**2*b*c*d**2*x**4 + 160*sqrt(a)*a*b**2*c**2*d*x**4 + 75*sqrt(a)*b**3*c**
3*x**4)/(35*b**2*x**4)
```

3.12 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx = -2c(bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{4}{3}cd\left(a+\frac{b}{x}\right)^{3/2} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}+ac^2\sqrt{a+\frac{b}{x}}x+\sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output

```
-2*c*(2*a*d+b*c)*(a+b/x)^(1/2)-4/3*c*d*(a+b/x)^(3/2)-2/5*d^2*(a+b/x)^(5/2)
/b+a*c^2*(a+b/x)^(1/2)*x+a^(1/2)*c*(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(
1/2))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}(-6a^2d^2x^2+abx(-12d^2-80cdx+15c^2x^2)-2b^2(3d^2+10cdx+15c^2x^2))}{15bx^2} + \sqrt{a}c(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-6*a^2*d^2*x^2 + a*b*x*(-12*d^2 - 80*c*d*x + 15*c^2*x^2) -
 2*b^2*(3*d^2 + 10*c*d*x + 15*c^2*x^2)))/(15*b*x^2) + Sqrt[a]*c*(3*b*c + 4
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {899, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\int \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2ad^2}{x}+c(3bc+4ad)\right)xd\frac{1}{x}}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\int \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2ad^2}{x}+c(3bc+4ad)\right)xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \; 90$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \; 73$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c(4ad+3bc)\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

$$\Big\downarrow \; 221$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)(4ad+3bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}}{2a}$$

input
```
Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

output
```
(c^2*(a + b/x)^(5/2)*x)/a - ((4*a*d^2*(a + b/x)^(5/2))/(5*b) + c*(3*b*c +
4*a*d)*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\left(-15ab\,c^{2}x^{3}+6a^{2}d^{2}x^{2}+80abcd\,x^{2}+30b^{2}c^{2}x^{2}+12xab\,d^{2}+20x\,b^{2}cd+6b^{2}d^{2}\right)\sqrt{\frac{ax+b}{x}}}{15x^{2}b}+\frac{(4ad+3bc)\sqrt{a}\,c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{}}{2ax+2b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-120\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}cd\,x^{4}-90\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}b\,c^{2}x^{4}-60\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}bcd\,x^{4}-45\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right.\right.}{30x^{3}b}$

input
```
int((a+b/x)^(3/2)*(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-15*a*b*c^2*x^3+6*a^2*d^2*x^2+80*a*b*c*d*x^2+30*b^2*c^2*x^2+12*a*b*
d^2*x+20*b^2*c*d*x+6*b^2*d^2)/x^2/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+3*b*c)*a^
(1/2)*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*
x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.44

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2 -}{30\,bx^2}\right.$$

$$-\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(15\,abc^2x^3-6\,b^2d^2-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^2}{15\,bx^2}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*
b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(
b*x^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*x*s
qrt((a*x + b)/x)/(a*x + b)) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2
+ 40*a*b*c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b
)/x))/(b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 28.07 (sec) , antiderivative size = 546, normalized size of antiderivative = 4.88

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx=\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\sqrt{a}bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-2acd\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for}\ b\neq 0\\ -\sqrt{a}\log{(x)} & \text{otherwise}\end{cases}\right)+ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-bc^2\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for}\ b\neq 0\\ -\sqrt{a}\log{(x)} & \text{otherwise}\end{cases}\right)+2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

output
```
4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2)
+ 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b +
1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b*
*(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**
4*x**(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(
x)/sqrt(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) + a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) -
 2*a*c*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a +
 b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + a*d**2*Piecewise((-sqrt(a)/x,
 Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - b*c**2*Piecewise((2*a*ata
n(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)
*log(x), True)) + 2*b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)*
*(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b\right)c^2 - \frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{b}{x}}a\right)cd$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((s
qrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
2 - 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))
) + 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.76

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \sqrt{a+\frac{b}{x}}\left(2a\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)-\frac{2(ad-bc)^2}{b}+\frac{2a^2d^2}{b}\right)+\left(\frac{4ad^2-4bcd}{3b}-\frac{4ad^2}{3b}\right)\left(a+\frac{b}{x}\right)^{3/2}-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}+ac^2x\sqrt{a+\frac{b}{x}}-2c\,\mathrm{atan}\left(\frac{2c\sqrt{a+\frac{b}{x}}\,(4ad+3bc)\sqrt{-\frac{a}{4}}}{4da^2c+3bac^2}\right)(4ad+3bc)\sqrt{-\frac{a}{4}}$$

input
```
int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

output
```
(a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c
)^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a
 + b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) -
2*c*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4
*a^2*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.03

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx = \frac{-24\sqrt{x}\,\sqrt{ax+b}\,a^2d^2x^2+60\sqrt{x}\,\sqrt{ax+b}\,ab\,c^2x^3-320\sqrt{x}\,\sqrt{ax+b}\,abcd\,x^2-48\sqrt{x}\,\sqrt{ax+b}\,a}{}$$

input
```
int((a+b/x)^(3/2)*(c+d/x)^2,x)
```

output
```
( - 24*sqrt(x)*sqrt(a*x + b)*a**2*d**2*x**2 + 60*sqrt(x)*sqrt(a*x + b)*a*b
*c**2*x**3 - 320*sqrt(x)*sqrt(a*x + b)*a*b*c*d*x**2 - 48*sqrt(x)*sqrt(a*x
+ b)*a*b*d**2*x - 120*sqrt(x)*sqrt(a*x + b)*b**2*c**2*x**2 - 80*sqrt(x)*sq
rt(a*x + b)*b**2*c*d*x - 24*sqrt(x)*sqrt(a*x + b)*b**2*d**2 + 240*sqrt(a)*
log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b*c*d*x**3 + 180*sqrt(a)*
log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2*c**2*x**3 - 24*sqrt(a)
*a**2*d**2*x**3 + 128*sqrt(a)*a*b*c*d*x**3 + 99*sqrt(a)*b**2*c**2*x**3)/(6
0*b*x**3)
```

3.13 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx = -2(bc+ad)\sqrt{a+\frac{b}{x}}-\frac{2}{3}d\left(a+\frac{b}{x}\right)^{3/2} + ac\sqrt{a+\frac{b}{x}}x+\sqrt{a}(3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*(a*d+b*c)*(a+b/x)^(1/2)-2/3*d*(a+b/x)^(3/2)+a*c*(a+b/x)^(1/2)*x+a^(1/2)
*(2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx = \frac{\sqrt{a+\frac{b}{x}}(ax(-8d+3cx)-2b(d+3cx))}{3x} + \sqrt{a}(3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

output
```
(Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*
b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {899, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$$

$$\Big\downarrow \ 899$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 87$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \ 60$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \ 60$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \ 73$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{(2ad+3bc)\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)}{2a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)(2ad+3bc)}{2a}$$

input
```
Int[(a + b/x)^(3/2)*(c + d/x),x]
```

output
```
(c*(a + b/x)^(5/2)*x)/a - ((3*b*c + 2*a*d)*((2*(a + b/x)^(3/2))/3 + a*(2*S
qrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\left(3 a c \,x^{2}-8 a d x -6 b c x -2 b d \right) \sqrt{\frac{a x +b}{x}}}{3 x}+\frac{\left(2 a d +3 b c \right) \sqrt{a}\, \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x \left(a x +b \right)}}{2 a x +2 b}$
default	$-\frac{\sqrt{\frac{a x +b}{x}}\, \left(-12 \sqrt{a \,x^{2}+b x}\, a^{\frac{5}{2}} d \,x^{3}-18 \sqrt{a \,x^{2}+b x}\, a^{\frac{3}{2}} b c \,x^{3}-6 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a^{2} b d \,x^{3}-9 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a \,b^{2} c}{6 x^{2} \sqrt{x \left(a x +b \right)}\, b \sqrt{a}}$

input
```
int((a+b/x)^(3/2)*(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a*c*x^2-8*a*d*x-6*b*c*x-2*b*d)/x*((a*x+b)/x)^(1/2)+1/2*(2*a*d+3*b*c
)*a^(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(
a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = \left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{6\,x}, -\frac{3\,(3\,bc+2\,ad)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

output
```
[1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x
) + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x,
 -1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a
*x + b)) - (3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 19.05 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx = \sqrt{a}bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1} - ad\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) - bc\left(\begin{cases}\frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(3/2)*(c+d/x),x)
```

output
```
sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/
b + 1) - a*d*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt
(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) - b*c*Piecewise((2*a*atan(s
qrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*lo
g(x), True)) + b*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/
(3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax-3\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}b\right)c-\frac{1}{3}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}a\right)d$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt
(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a
+ b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(
a + b/x)*a)*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx = 2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{2\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3} - 2\,a\,d\,\sqrt{a+\frac{b}{x}} - \frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x)^(3/2)*(c + d/x),x)
```

output
```
2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a
*d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -
(a*x)/b))/((a*x)/b + 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\,dx=\frac{6\sqrt{x}\,\sqrt{ax+b}\,ac\,x^2-16\sqrt{x}\,\sqrt{ax+b}\,adx-12\sqrt{x}\,\sqrt{ax+b}\,bcx-4\sqrt{x}\,\sqrt{ax+b}\,bd+12\sqrt{a}\log\Big(}{6x^2}$$

input
```
int((a+b/x)^(3/2)*(c+d/x),x)
```

output
```
(6*sqrt(x)*sqrt(a*x + b)*a*c*x**2 - 16*sqrt(x)*sqrt(a*x + b)*a*d*x - 12*sq
rt(x)*sqrt(a*x + b)*b*c*x - 4*sqrt(x)*sqrt(a*x + b)*b*d + 12*sqrt(a)*log((
sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*d*x**2 + 18*sqrt(a)*log((sqrt(
a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*c*x**2 + 5*sqrt(a)*b*c*x**2)/(6*x**
2)
```

3.14 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 55

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx=-2b\sqrt{a+\frac{b}{x}}+a\sqrt{a+\frac{b}{x}}x+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*b*(a+b/x)^(1/2)+a*(a+b/x)^(1/2)*x+3*a^(1/2)*b*arctanh((a+b/x)^(1/2)/a^(
1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx=\sqrt{a+\frac{b}{x}}(-2b+ax)+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2),x]
```

output
```
Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{73}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{221}$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)$$

input
```
Int[(a + b/x)^(3/2),x]
```

output
```
(a + b/x)^(3/2)*x - (3*b*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

method	result	size
risch	$(ax-2b)\sqrt{\frac{ax+b}{x}}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	78
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-6\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}-3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}+4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2x\sqrt{x(ax+b)}\,\sqrt{a}}$	100

input
```
int((a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-2*b)*((a*x+b)/x)^(1/2)+3/2*a^(1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int\left(a+\frac{b}{x}\right)^{3/2}dx=\left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},-3\,\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b
)*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*
x + b)) + (a*x - 2*b)*sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = 3\sqrt{a}b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + \frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}} - \frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate((a+b/x)**(3/2),x)
```

output
```
3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a
*x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(
a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}ax - \frac{3}{2}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}b$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -\frac{2\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x)^(3/2),x)
```

output
```
-(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1
)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \frac{4\sqrt{x}\,\sqrt{ax+b}\,ax-8\sqrt{x}\,\sqrt{ax+b}\,b+12\sqrt{a}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)bx-9\sqrt{a}\,bx}{4x}$$

input
```
int((a+b/x)^(3/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x + b)*a*x - 8*sqrt(x)*sqrt(a*x + b)*b + 12*sqrt(a)*log(
(sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*x - 9*sqrt(a)*b*x)/(4*x)
```

3.15 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}} dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}} dx = \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

output
```
a*(a+b/x)^(1/2)*x/c-2*(-a*d+b*c)^(3/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+
b*c)^(1/2))/c^2/d^(1/2)+a^(1/2)*(-2*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/
2))/c^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\frac{ac\sqrt{a+\frac{b}{x}}x-\frac{2(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}-\sqrt{a}(-3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

output
```
(a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/Sqrt[d] - Sqrt[a]*(-3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]])/c^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {899, 109, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int -\frac{\left(a(3bc-2ad)+\frac{b(2bc-ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\int \frac{\left(a(3bc-2ad)+\frac{b(2bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}$$

$\Big\downarrow$ 174

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2(bc-ad)^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{a(3bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c}$$

$\Big\downarrow$ 73

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{4(bc-ad)^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a(3bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}$$

$\Big\downarrow$ 218

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{2a(3bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{4(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{2c}$$

$\Big\downarrow$ 221

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c}-\frac{\frac{4(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-2ad)}{c}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x),x]
```

output
```
(a*Sqrt[a + b/x]*x)/c - ((4*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c)/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.

Time = 0.46 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.31

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c}-\frac{\left(\frac{\sqrt{a}\,(2ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c}+\frac{2\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2}}{x+\frac{d}{c}}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}}{2c(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{a\,x^2+bx}\,\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,b\,c^3+\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,b^2c^3+2a^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{x(ax+b)}\,c^2d-2\sqrt{a}\,\sqrt{(}\right.}{}$

input
```
int((a+b/x)^(3/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/c*x*a*((a*x+b)/x)^(1/2)-1/2/c*(a^(1/2)*(2*a*d-3*b*c)/c*ln((1/2*b+a*x)/a^
(1/2)+(a*x^2+b*x)^(1/2))+2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/((a*d-b*c)*d/c^
2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)
^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1
/c*d)))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.99

$$\int\frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}}{2\,c^2}\right.$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2
*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x
+ d)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqr
t(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(
c*x + d)))/c^2, 1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c -
 a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3
*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2,
 (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*x*sqr
t((a*x + b)/x)/(a*x + b)) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sq
rt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)))/c^2]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x),x)
```

output
```
Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 556, normalized size of antiderivative = 5.25

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c} - \frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}+\frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d}+\frac{12\,a^{7/2}}{12\,a^4\,b^4\,d^4-46\,a^3\,b^5\,c\,d}\right.}{c^2} + \frac{2\,\mathrm{atanh}\left(\frac{12\,a^2\,b^4\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{12\,a^4\,b^4\,d^4-40\,a^3\,b^5\,c\,d^3+44\,a^2\,b^6\,c^2\,d^2-16\,a\,b^7\,c^3\,d}+\frac{16\,a\,b^5\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{40\,a^3\,b^5\,d^3-44\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+16\,a\,b^7\,c^2\,d}\right)\sqrt{d\,(a}{c^2\,d}$$

input
```
int((a + b/x)^(3/2)/(c + d/x),x)
```

output
```
(a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/
2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)
/c^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*
c*d^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/
x)^(1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b
^7*c^3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^
7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 +
 (2*atanh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c
^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2
*b^6*c^2*d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^
3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2
*b^6*c*d^2 - (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2)
)/(c^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.85

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = \frac{\sqrt{d}\,\sqrt{ad-bc}\,\log\left(\sqrt{c}\,\sqrt{ax+b}-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{x}\,\sqrt{c}\,\sqrt{a}\right)ad-}{}$$

input
```
int((a+b/x)^(3/2)/(c+d/x),x)
```

output
```
(sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*d - sqrt(d)*
sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c + sqrt(d)*sqrt(a*d
- b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
- 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*d - sqrt(d)*sqrt(a*d - b*c)*lo
g(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c - sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x +
 2*a*d)*a*d + sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c
) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*b*c + sqrt(x)*sqr
t(a*x + b)*a*c*d - 2*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*a*d**2 + 3*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*c*d)
/(c**2*d)
```

3.16 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} dx = -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

output
```
-(-2*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)+a*(a+b/x)^(1/2)*x/c/(c+d/x)-(-4*a*
d+b*c)*(-a*d+b*c)^(1/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3
/d^(1/2)+a^(1/2)*(-4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(-bc+2ad+acx)}{d+cx}-\frac{(b^2c^2-5abcd+4a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}-\sqrt{a}(-3bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*
b*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]) - Sqrt[a]*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/c^3
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {899, 109, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\left(a(3bc-4ad)+\frac{b(2bc-3ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\left(a(3bc-4ad)+\frac{b(2bc-3ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{\left(a(3bc-4ad)(bc-ad)+\frac{b(bc-2ad)(bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(a(3bc-4ad)(bc-ad)+\frac{b(bc-2ad)(bc-ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{174}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{(bc-4ad)(bc-ad)^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{a(3bc-4ad)(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)(bc-ad)^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a(3bc-4ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a(3bc-4ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-4ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-4ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)(bc-ad)}{c}}{c(bc-ad)}+\frac{2\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

output
```
(a*Sqrt[a + b/x]*x)/(c*(c + d/x)) - ((2*(b*c - 2*a*d)*Sqrt[a + b/x])/(c*(c
 + d/x)) + ((2*(b*c - 4*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/
x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*Sqrt[a]*(3*b*c - 4*a*d)*(b*c - a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(c*(b*c - a*d)))/(2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(136) = 272$.

Time = 0.49 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.21

method	result
risch	$\frac{xa\sqrt{\frac{ax+b}{x}}}{c^2} - \frac{\left(\frac{\sqrt{a}\,(4ad-3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c} + \frac{2\left(3a^2d^2-4abcd+b^2c^2\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)\ldots}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\ldots\right.}{\ldots}$
default	$-\frac{\left(4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c-2adx+bcx-bd}{cx+d}\right)c\,d^3x+2a^{\frac{5}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c^4x^2+4a^{\frac{7}{2}}\ln\left(\frac{2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{x(ax+b)}\,c-2ad\ldots}{cx+d}\right.\ldots\right.}{\ldots}$

input

```
int((a+b/x)^(3/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output

```
1/c^2*x*a*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(1/2)*(4*a*d-3*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))+2/c^2*(3*a^2*d^2-4*a*b*c*d+b^2*c^2)/((a*d-b*c
)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*
d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)
)/(x+1/c*d))+2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*(-1/(a*d-b*c)/d*c^2/(x+1/
c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*
a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*
d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*
(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))))*((a*x+b)/x)^(1/2)*(x*(a*x+b
))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 781, normalized size of antiderivative = 5.01

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*
sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqr
t((a*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*
d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/
(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a
*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)
*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*
c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b))
+ (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*
sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d))
 - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d),
 -((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*x*
sqrt((a*x + b)/x)/(a*x + b)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqr
t((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*
d)) - (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)
]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{(cx+d)^2}\,dx$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*(a + b/x)**(3/2)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(136) = 272$.

Time = 0.17 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\sqrt{ax^2+bx}\,a\,\mathrm{sgn}(x)}{c^2} + \frac{\left(b^2c^2\mathrm{sgn}(x) - 5\,abcd\,\mathrm{sgn}(x) + 4\,a^2d^2\mathrm{sgn}(x)\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^3} - \frac{\left(3\,abc\,\mathrm{sgn}(x) - 4\,a^2d\,\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\sqrt{a}c^3} + \frac{\left(2\sqrt{a}b^2c^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 10\,a^{\frac{3}{2}}bcd\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 8\,a^{\frac{5}{2}}d^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) + 3\sqrt{bcd-ad^2}\,ab}{2\sqrt{bcd-ad^2}\sqrt{a}c^3} + \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)b^2c^2\mathrm{sgn}(x) - 3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)abcd\,\mathrm{sgn}(x) + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)a^2d^2\mathrm{sgn}(x)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d + bd\right)c^3}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a*sgn(x)/c^2 + (b^2*c^2*sgn(x) - 5*a*b*c*d*sgn(x) + 4*a^
2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt
(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3) - 1/2*(3*a*b*c*sgn(x) - 4*a^2*d
*sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)
*c^3) + 1/2*(2*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 10*
a^(3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 8*a^(5/2)*d^2*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 3*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b))
- 4*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a*b*c -
2*sqrt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3) + ((
sqrt(a)*x - sqrt(a*x^2 + b*x))*b^2*c^2*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2
+ b*x))*a*b*c*d*sgn(x) + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*d^2*sgn(x)
- sqrt(a)*b^2*c*d*sgn(x) + a^(3/2)*b*d^2*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)*c^3)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\frac{\mathrm{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-b\right.}{c^3\,d}$$
$$-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{6\,\sqrt{a}\,b^7\,d\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,d-\frac{14\,a^2\,b^6\,d^2}{c}+\frac{8\,a^3\,b^5\,d^3}{c^2}}-\frac{14\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,c\,d-14\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}+\frac{8\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{8\,a^3\,b^5\,d^3-14\,a^2\,b^6\,c\,d^2+6\,a\,b^7\,c^2\,d}\right)\,\left(4\,a\,d-3\,b\,c\right.}{c^3}$$
$$-\frac{\frac{2\,\left(a\,b^2\,c-a^2\,b\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2}+\frac{b\,{\left(a+\frac{b}{x}\right)}^{3/2}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,{\left(a+\frac{b}{x}\right)}^2-a^2\,d+a\,b\,c}$$

input
```
int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

output
```
(atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^
3 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2
- b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*
d - b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(
a + b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (
14*a^(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3
*b^5*d^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2
*b^6*c*d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d
)*(a + b/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)
*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 873, normalized size of antiderivative = 5.60

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(3/2)/(c+d/x)^2,x)
```

output
```
(4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*c*d*x + 4*
sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)
*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*d**2 - sqrt(d
)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(
a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c**2*x - sqrt(d)*sq
rt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
- b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c*d + 4*sqrt(d)*sqrt(a*
d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c
) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*c*d*x + 4*sqrt(d)*sqrt(a*d -
 b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) -
 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*d**2 - sqrt(d)*sqrt(a*d - b*c)*
log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d
 + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c**2*x - sqrt(d)*sqrt(a*d - b*c)*log(
sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b
*c) + sqrt(x)*sqrt(c)*sqrt(a))*b*c*d - 4*sqrt(d)*sqrt(a*d - b*c)*log(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x
 + 2*a*d)*a*c*d*x - 4*sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a
*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a*d**2 +
sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(...
```

3.17 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 209

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx = -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3(b^2c^2-8abcd+8a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}} + \frac{3\sqrt{a}(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

output
```
-1/2*(-3*a*d+b*c)*(a+b/x)^(1/2)/c^2/(c+d/x)^2-3/4*(-4*a*d+b*c)*(a+b/x)^(1/
2)/c^3/(c+d/x)+a*(a+b/x)^(1/2)*x/c/(c+d/x)^2-3/4*(8*a^2*d^2-8*a*b*c*d+b^2*
c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(1/2)/(-a*d+b*c)
^(1/2)+3*a^(1/2)*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^4
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(-bc(3d+5cx)+2a\left(6d^2+9cdx+2c^2x^2\right)\right)}{(d+cx)^2}-\frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}-12\sqrt{a}(-bc+$$

$$4c^4$$

input
```
Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x
^2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) - 12*Sqrt[a]*(-(
b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.524$, Rules used = {899, 109, 27, 168, 27, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\left(\frac{b(2bc-5ad)}{x}+3a(bc-2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\left(\frac{b(2bc-5ad)}{x}+3a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int -\frac{3(bc-ad)\left(\frac{b(bc-3ad)}{x}+2a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\int \frac{\left(\frac{b(bc-3ad)}{x}+2a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \text{168}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{(bc-ad)\left(\frac{b(bc-4ad)}{x}+4a(bc-2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{3\left(\frac{\int \frac{\left(\frac{b(bc-4ad)}{x}+4a(bc-2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4a(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 73

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8a(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 218

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{8a(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}\sqrt{bc-ad}}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 221

$$\frac{\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}-\frac{3\left(\frac{\frac{2\left(8a^2d^2-8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}\sqrt{bc-ad}}-\frac{8\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{c}}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-4ad)}{c\left(c+\frac{d}{x}\right)}\right)}{2c}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{c\left(c+\frac{d}{x}\right)^2}}{2c}$$

input
```
Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

output
```
(a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (((b*c - 3*a*d)*Sqrt[a + b/x])/(c*(c
 + d/x)^2) + (3*(((b*c - 4*a*d)*Sqrt[a + b/x])/(c*(c + d/x)) + ((2*(b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[d]*Sqrt[b*c - a*d]) - (8*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c)/(2*c)))/(2*c))/(2*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(181) = 362$.

Time = 0.53 (sec) , antiderivative size = 1007, normalized size of antiderivative = 4.82

method	result	size
risch	Expression too large to display	1007
default	Expression too large to display	1817

input
```
int((a+b/x)^(3/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*x*a*((a*x+b)/x)^(1/2)-1/2/c^3*(2/c^2*(6*a^2*d^2-6*a*b*c*d+b^2*c^2)/(
(a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a
*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^
2)^(1/2))/(x+1/c*d))+3*a^(1/2)*(2*a*d-b*c)/c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2
+b*x)^(1/2))+4/c^3*d*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)*(-1/(a*d-b*c)/d*c^2/(x+
1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(
2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*
a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/
c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))-2*d^2*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+1/c*d)^2*(a*(x+1/c*d)^2-(2*a*d-b*c)/c
*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b
*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2
)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b
*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2
-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))+1/2*a/(a*d-b*
c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/
c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d
-b*c)*d/c^2)^(1/2))/(x+1/c*d))))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+
b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(181) = 362$.

Time = 0.16 (sec) , antiderivative size = 1775, normalized size of antiderivative = 8.49

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*
d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x
)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2
- 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b
*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sq
rt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b
*c^6*d^2 - a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5
+ (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b
*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a
*x + b)) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3
*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*s
qrt(-b*c*d + a*d^2)*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*
sqrt((a*x + b)/x))/(c*x + d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^
2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^
2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7
*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 -
8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(181) = 362$.

Time = 0.19 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) +
8*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/
sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2
*d*sgn(x))*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(
a)*c^4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2
4*a^(3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arc
tan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b
)) - 12*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*
c - 10*sqrt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4)
 + 1/4*(5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*sgn(x) - 24*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^3*a*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^3*a^2*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2
*d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^2*sgn(x) +
40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^3*sgn(x) + 3*(sqrt(a)*x - s
qrt(a*x^2 + b*x))*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*
b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^3*sgn(x) - 5
*sqrt(a)*b^3*c*d^2*sgn(x) + 10*a^(3/2)*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*
c^4)
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 1664, normalized size of antiderivative = 7.96

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(3/2)/(c + d/x)^3,x)
```

output
```
- ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
 - ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (
3*b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*
b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a
^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a
+ b/x)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*
b^6*d^2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a
*d - b*c))/c^4 - (atan((((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5
 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6)
- (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (
3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/
2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a
^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^
(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((
((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 172
8*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)
*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*
b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 -
8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c
*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*...
```

Reduce [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 3196, normalized size of antiderivative = 15.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(3/2)/(c+d/x)^3,x)
```

output

```
(96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*c**2*d
**3*x**2 + 192*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*
a**3*c*d**4*x + 96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**3*d**5 - 144*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sq
rt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt
(a))*a**2*b*c**3*d**2*x**2 - 288*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(
a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)
*sqrt(c)*sqrt(a))*a**2*b*c**2*d**3*x - 144*sqrt(d)*sqrt(a*d - b*c)*log(sqr
t(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
 + sqrt(x)*sqrt(c)*sqrt(a))*a**2*b*c*d**4 + 60*sqrt(d)*sqrt(a*d - b*c)*log
(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**4*d*x**2 + 120*sqrt(d)*sqrt(a*d
- b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
- 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**3*d**2*x + 60*sqrt(d)*
sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**2*d**3 - 6*sq
rt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a...
```

3.18 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx &= -2ac^2(2bc+3ad)\sqrt{a+\frac{b}{x}} \\ &- \frac{2}{3}c^2(bc+3ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{6}{5}c^2d\left(a+\frac{b}{x}\right)^{5/2} - \frac{2d^2(3bc-ad)\left(a+\frac{b}{x}\right)^{7/2}}{7b^2} \\ &- \frac{2d^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^2} + a^2c^3\sqrt{a+\frac{b}{x}}x + a^{3/2}c^2(5bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

output
```
-2*a*c^2*(3*a*d+2*b*c)*(a+b/x)^(1/2)-2/3*c^2*(3*a*d+b*c)*(a+b/x)^(3/2)-6/5
*c^2*d*(a+b/x)^(5/2)-2/7*d^2*(-a*d+3*b*c)*(a+b/x)^(7/2)/b^2-2/9*d^3*(a+b/x
)^(9/2)/b^2+a^2*c^3*(a+b/x)^(1/2)*x+a^(3/2)*c^2*(6*a*d+5*b*c)*arctanh((a+b
/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx = \frac{\sqrt{a+\frac{b}{x}}(20a^4d^3x^4-10a^3bd^2x^3(d+27cx)-3a^2b^2x^2(50d^3+270cd^2x+966c^2dx^2-105c^3x^3)-\cdots}{315\cdots} + a^{3/2}c^2(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

output
```
(Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2
*x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3
+ 135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d
^2*x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c +
 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {899, 108, 27, 170, 27, 164, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 dx$$

$$\Big\downarrow \; 899$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 x^2 d\frac{1}{x}$$

$$\Big\downarrow \; 108$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\int \frac{1}{2}\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\left(5bc+6ad+\frac{11bd}{x}\right)xd\frac{1}{x}$$

$$\Big\downarrow \; 27$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{1}{2}\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2\left(5bc+6ad+\frac{11bd}{x}\right)xd\frac{1}{x}$$

$$\Big\downarrow \; 170$$

$$\frac{1}{2}\left(-\frac{2\int \frac{1}{2}b\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\left(9c(5bc+6ad)+\frac{d(89bc+10ad)}{x}\right)xd\frac{1}{x}}{9b}-\frac{22}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\right)+ x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)\left(9c(5bc+6ad)+\frac{d(89bc+10ad)}{x}\right)xd\frac{1}{x}-\frac{22}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2\right)+ x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \; 164$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right.}{35b^2}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right.\right.}{35b^2}\right.\right.$$
$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\left(a\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+1\right.\right.}{}\right.\right.$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(\frac{1}{9}\left(-9c^2(6ad+5bc)\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d\right.\right.}{}\right.\right.$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{1}{9}\left(-\frac{2d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{35b^2}-9c^2\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\right.\right.\right.\right.$$

$$x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

output
```
(a + b/x)^(5/2)*(c + d/x)^3*x + ((-22*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 + (
(-2*d*(a + b/x)^(5/2)*(2*(469*b^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d
*(89*b*c + 10*a*d))/x))/(35*b^2) - 9*c^2*(5*b*c + 6*a*d)*((2*(a + b/x)^(3/
2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/9
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.53

method	result
risch	$\frac{(315a^2b^2c^3x^5+20a^4d^3x^4-270a^3bc\,d^2x^4-2898a^2b^2c^2d\,x^4-1470a\,b^3c^3x^4-10a^3b\,d^3x^3-810a^2b^2c\,d^2x^3-1386a\,b^3c^2d\,x^3-210b^4c^3x^3}{315x^4b^2}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(3780\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}b\,c^2d\,x^6+3150\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b^2c^3x^6+1890\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c^2d\,x^6+1575\ln\left(\frac{2\sqrt{a\,x^2+bx}}{2\sqrt{}}\right.\right.}{}$

input
```
int((a+b/x)^(5/2)*(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/315*(315*a^2*b^2*c^3*x^5+20*a^4*d^3*x^4-270*a^3*b*c*d^2*x^4-2898*a^2*b^2
*c^2*d*x^4-1470*a*b^3*c^3*x^4-10*a^3*b*d^3*x^3-810*a^2*b^2*c*d^2*x^3-1386*
a*b^3*c^2*d*x^3-210*b^4*c^3*x^3-150*a^2*b^2*d^3*x^2-810*a*b^3*c*d^2*x^2-37
8*b^4*c^2*d*x^2-190*a*b^3*d^3*x-270*b^4*c*d^2*x-70*b^4*d^3)/x^4/b^2*((a*x+
b)/x)^(1/2)+1/2*(6*a*d+5*b*c)*a^(3/2)*c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.80

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx = \left[\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{a}x^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(315\,a^2b^2c^3x^5-70\,b^4d^3-2\right.$$

$$-\frac{315\,(5\,ab^3c^3+6\,a^2b^2c^2d)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(315\,a^2b^2c^3x^5-70\,b^4d^3-2\,(735\,ab^3c^3+1449\,a^2}{}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(73
5*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(
105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(
63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 +
19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 +
6*a^2*b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b
)) - (315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c
^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*
d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d
^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x +
 b)/x))/(b^2*x^4)]
```

Sympy [A] (verification not implemented)

Time = 49.88 (sec) , antiderivative size = 5523, normalized size of antiderivative = 31.03

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

output
```
32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a*
*(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**
(29/2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**
(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b
**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**
(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*
sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(
19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) +
4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(
9/2)*b**21*x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(
315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(
17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/
2)) + 210*a**(21/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**
15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(1
7/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1
890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2
)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 3...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.23

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = -\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7\,b}$$
$$+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^3$$
$$-\frac{1}{5}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}a^2\right)c^2d$$
$$-\frac{2}{63}\left(\frac{7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{b^2}-\frac{9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{b^2}\right)d^3$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

output
```
-6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2
)*b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*
a + 30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x
)^(7/2)*a/b^2)*d^3
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.74

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3\,dx=\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{7\,b^2}-\frac{4\,a\,d^3}{7\,b^2}\right)$$
$$-\sqrt{a+\frac{b}{x}}\left(a^2\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-2\,a\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\right.\right.\right.$$
$$+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,(a\,d-b\,c)^3}{3\,b^2}+\frac{2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)}{3}-\frac{a^2\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a}{b}\right.}{3}\right.$$

input
```
int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

output
```
(a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
 b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(
a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2
*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 +
 (2*a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a
 + b/x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2
)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (
a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*
a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*
b^2) + (2*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*
(a + b/x)^(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d +
5*b*c)*1i
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.46

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = \frac{80\sqrt{x}\,\sqrt{ax+b}\,a^4d^3x^4 - 1080\sqrt{x}\,\sqrt{ax+b}\,a^3bc\,d^2x^4 - 40\sqrt{x}\,\sqrt{ax+b}\,a^3b\,d^3x^3 + 1260\sqrt{x}\,\sqrt{ax}$$

input
```
int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

output
```
(80*sqrt(x)*sqrt(a*x + b)*a**4*d**3*x**4 - 1080*sqrt(x)*sqrt(a*x + b)*a**3
*b*c*d**2*x**4 - 40*sqrt(x)*sqrt(a*x + b)*a**3*b*d**3*x**3 + 1260*sqrt(x)*
sqrt(a*x + b)*a**2*b**2*c**3*x**5 - 11592*sqrt(x)*sqrt(a*x + b)*a**2*b**2*
c**2*d*x**4 - 3240*sqrt(x)*sqrt(a*x + b)*a**2*b**2*c*d**2*x**3 - 600*sqrt(
x)*sqrt(a*x + b)*a**2*b**2*d**3*x**2 - 5880*sqrt(x)*sqrt(a*x + b)*a*b**3*c
**3*x**4 - 5544*sqrt(x)*sqrt(a*x + b)*a*b**3*c**2*d*x**3 - 3240*sqrt(x)*sq
rt(a*x + b)*a*b**3*c*d**2*x**2 - 760*sqrt(x)*sqrt(a*x + b)*a*b**3*d**3*x -
 840*sqrt(x)*sqrt(a*x + b)*b**4*c**3*x**3 - 1512*sqrt(x)*sqrt(a*x + b)*b**
4*c**2*d*x**2 - 1080*sqrt(x)*sqrt(a*x + b)*b**4*c*d**2*x - 280*sqrt(x)*sqr
t(a*x + b)*b**4*d**3 + 7560*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/
sqrt(b))*a**2*b**2*c**2*d*x**5 + 6300*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)
*sqrt(a))/sqrt(b))*a*b**3*c**3*x**5 - 80*sqrt(a)*a**4*d**3*x**5 - 600*sqrt
(a)*a**3*b*c*d**2*x**5 + 6552*sqrt(a)*a**2*b**2*c**2*d*x**5 + 5425*sqrt(a)
*a*b**3*c**3*x**5)/(1260*b**2*x**5)
```

3.19 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx &= -4ac(bc+ad)\sqrt{a+\frac{b}{x}} \\ &\quad -\frac{2}{3}c(bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}-\frac{4}{5}cd\left(a+\frac{b}{x}\right)^{5/2}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} \\ &\quad +a^2c^2\sqrt{a+\frac{b}{x}}x+a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\end{aligned}$$

output
```
-4*a*c*(a*d+b*c)*(a+b/x)^(1/2)-2/3*c*(2*a*d+b*c)*(a+b/x)^(3/2)-4/5*c*d*(a+
b/x)^(5/2)-2/7*d^2*(a+b/x)^(7/2)/b+a^2*c^2*(a+b/x)^(1/2)*x+a^(3/2)*c*(4*a*
d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx = \frac{\sqrt{a+\frac{b}{x}}(-30a^3d^2x^3-2b^3(15d^2+42cdx+35c^2x^2)+a^2bx^2(-90d^2-644cdx+105c^2x^2)-2ab^2x(45d^2+154cdx+245c^2x^2))}{105bx^3} + a^{3/2}c(5bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

output
```
(Sqrt[a + b/x]*(-30*a^3*d^2*x^3 - 2*b^3*(15*d^2 + 42*c*d*x + 35*c^2*x^2) +
 a^2*b*x^2*(-90*d^2 - 644*c*d*x + 105*c^2*x^2) - 2*a*b^2*x*(45*d^2 + 154*c
*d*x + 245*c^2*x^2)))/(105*b*x^3) + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 100, 27, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\int \frac{1}{2}\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{2ad^2}{x}+c(5bc+4ad)\right)xd\frac{1}{x}}{a}$$

$\big\downarrow$ 27

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\int \left(a+\frac{b}{x}\right)^{5/2}\left(\frac{2ad^2}{x}+c(5bc+4ad)\right)xd\frac{1}{x}}{2a}$$

$\big\downarrow$ 90

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\int \left(a+\frac{b}{x}\right)^{5/2}xd\frac{1}{x}+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$\big\downarrow$ 60

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$\big\downarrow$ 60

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$\big\downarrow$ 60

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$\big\downarrow$ 73

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c(4ad+5bc)\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

$\big\downarrow$ 221

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)(4ad+5bc)+\frac{4ad^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}}{2a}$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

output
```
(c^2*(a + b/x)^(7/2)*x)/a - ((4*a*d^2*(a + b/x)^(7/2))/(7*b) + c*(5*b*c +
4*a*d)*((2*(a + b/x)^(5/2))/5 + a*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b
/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\left(-105a^{2}b\,c^{2}x^{4}+30a^{3}d^{2}x^{3}+644a^{2}bcd\,x^{3}+490a\,b^{2}c^{2}x^{3}+90x^{2}a^{2}b\,d^{2}+308x^{2}a\,b^{2}cd+70x^{2}b^{3}c^{2}+90a\,b^{2}d^{2}x+84b^{3}cdx+30b^{3}d^{2}\right)\sqrt{a}}{105x^{3}b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-840\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}cd\,x^{5}-1050\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}b\,c^{2}x^{5}-420\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}bcd\,x^{5}-525\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2}{2\sqrt{a}}\right.}{}$

input
```
int((a+b/x)^(5/2)*(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-105*a^2*b*c^2*x^4+30*a^3*d^2*x^3+644*a^2*b*c*d*x^3+490*a*b^2*c^2*
x^3+90*a^2*b*d^2*x^2+308*a*b^2*c*d*x^2+70*b^3*c^2*x^2+90*a*b^2*d^2*x+84*b^
3*c*d*x+30*b^3*d^2)/x^3/b*((a*x+b)/x)^(1/2)+1/2*(4*a*d+5*b*c)*a^(3/2)*c*ln
((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2
)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.57

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \left[\frac{105\,(5\,ab^2c^2+4\,a^2bcd)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(105\,a^2bc^2x^4-30\,b^3d^2-2\,(2\right.$$

$$-\frac{105\,(5\,ab^2c^2+4\,a^2bcd)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(105\,a^2bc^2x^4-30\,b^3d^2-2\,(245\,ab^2c^2+322\,a^2bcd}{105\,bx^3}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*
x*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^
2*c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d +
45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b
*x^3), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a
)*x*sqrt((a*x + b)/x)/(a*x + b)) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(24
5*a*b^2*c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*
c*d + 45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/
x))/(b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 31.88 (sec) , antiderivative size = 1853, normalized size of antiderivative = 13.43

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

output
```
-16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(
13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a*
*(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d*
*2*x**4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4
0*a**(13/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21
/2)*d**2*x**2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2
)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2
)) + 8*a**(11/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x
/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9
/2)*b**(23/2)*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a
**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*
x**(7/2)) + 4*a**(9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sq...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx=-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^2}{7b}$$
$$+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^2$$
$$-\frac{2}{15}\left(15a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}a^2\right)cd$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log
((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a
 + 30*sqrt(a + b/x)*a^2)*c*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.96

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2a\left(\frac{4a d^2-4b c d}{b}-\frac{4a d^2}{b}\right)}{3}-\frac{2(a d-b c)^2}{3b}+\frac{2a^2 d^2}{3b}\right) + \left(\frac{4a d^2-4b c d}{5b}-\frac{4a d^2}{5b}\right)\left(a+\frac{b}{x}\right)^{5/2} - \sqrt{a+\frac{b}{x}}\left(a^2\left(\frac{4a d^2-4b c d}{b}-\frac{4a d^2}{b}\right)-2a\left(2a\left(\frac{4a d^2-4b c d}{b}-\frac{4a d^2}{b}\right)-\frac{2(a d-b c)^2}{b}+\frac{2a^2 d^2}{b}\right)\right)$$

input
```
int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

output
```
(a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
 b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^
2)/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b -
(4*a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d -
b*c)^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a
 + b/x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*
c)*1i
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.16

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = \frac{-30\sqrt{x}\,\sqrt{ax+b}\,a^3d^2x^3+105\sqrt{x}\,\sqrt{ax+b}\,a^2bc^2x^4-644\sqrt{x}\,\sqrt{ax+b}\,a^2bcd\,x^3-90\sqrt{x}\,\sqrt{ax+}}{}$$

input
```
int((a+b/x)^(5/2)*(c+d/x)^2,x)
```

output
```
( - 30*sqrt(x)*sqrt(a*x + b)*a**3*d**2*x**3 + 105*sqrt(x)*sqrt(a*x + b)*a*
*2*b*c**2*x**4 - 644*sqrt(x)*sqrt(a*x + b)*a**2*b*c*d*x**3 - 90*sqrt(x)*sq
rt(a*x + b)*a**2*b*d**2*x**2 - 490*sqrt(x)*sqrt(a*x + b)*a*b**2*c**2*x**3
- 308*sqrt(x)*sqrt(a*x + b)*a*b**2*c*d*x**2 - 90*sqrt(x)*sqrt(a*x + b)*a*b
**2*d**2*x - 70*sqrt(x)*sqrt(a*x + b)*b**3*c**2*x**2 - 84*sqrt(x)*sqrt(a*x
 + b)*b**3*c*d*x - 30*sqrt(x)*sqrt(a*x + b)*b**3*d**2 + 420*sqrt(a)*log((s
qrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**2*b*c*d*x**4 + 525*sqrt(a)*log
((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b**2*c**2*x**4 - 30*sqrt(a)*
a**3*d**2*x**4 + 284*sqrt(a)*a**2*b*c*d*x**4 + 385*sqrt(a)*a*b**2*c**2*x**
4)/(105*b*x**4)
```

3.20 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) \, dx = -2a(2bc+ad)\sqrt{a+\frac{b}{x}}-\frac{2}{3}(bc+ad)\left(a+\frac{b}{x}\right)^{3/2} -\frac{2}{5}d\left(a+\frac{b}{x}\right)^{5/2}+a^2c\sqrt{a+\frac{b}{x}}x+a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*a*(a*d+2*b*c)*(a+b/x)^(1/2)-2/3*(a*d+b*c)*(a+b/x)^(3/2)-2/5*d*(a+b/x)^(
5/2)+a^2*c*(a+b/x)^(1/2)*x+a^(3/2)*(2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(
1/2))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx = \frac{\sqrt{a+\frac{b}{x}}(-2b^2(3d+5cx)+a^2x^2(-46d+15cx)-2abx(11d+35cx))}{15x^2} + a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

output
```
(Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*
(11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {899, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\int\left(a+\frac{b}{x}\right)^{5/2}xd\frac{1}{x}}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\int\left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\int\sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \; 60$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\left(a\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \; 73$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{(2ad+5bc)\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)}{2a}$$

$$\Big\downarrow \; 221$$

$$\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{\left(a\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)+\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}\right)(2ad+5bc)}{2a}$$

input
```
Int[(a + b/x)^(5/2)*(c + d/x),x]
```

output
```
(c*(a + b/x)^(7/2)*x)/a - ((5*b*c + 2*a*d)*((2*(a + b/x)^(5/2))/5 + a*((2*
(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]))))/(2*a)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(15a^2c\,x^3-46a^2d\,x^2-70abc\,x^2-22axdb-10b^2cx-6d\,b^2)\sqrt{\frac{ax+b}{x}}}{15x^2}+\frac{(2ad+5bc)a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2ax+2b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-60\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}d\,x^4-150\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}bc\,x^4-30\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3bd\,x^4-75\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right.}{30x^3b\sqrt{x}}$

input
```
int((a+b/x)^(5/2)*(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/15*(15*a^2*c*x^3-46*a^2*d*x^2-70*a*b*c*x^2-22*a*b*d*x-10*b^2*c*x-6*b^2*d
)/x^2*((a*x+b)/x)^(1/2)+1/2*(2*a*d+5*b*c)*a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(
a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.05

$$\int\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx=\left[\frac{15\,(5\,abc+2\,a^2d)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a\right.}{30\,x^2}$$

$$-\frac{15\,(5\,abc+2\,a^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(15\,a^2cx^3-6\,b^2d-2\,(35\,abc+23\,a^2d)x^2-2\,(5\,b^2c+1}{15\,x^2}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

output
```
[1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 -
 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*
a^2*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (15*a
^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*
x)*sqrt((a*x + b)/x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.81

$$\int\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx=\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^2\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-a^2d\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)-2abc\left(\begin{cases}\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{a+\frac{b}{x}} & \text{for } b\neq 0\\ -\sqrt{a}\log(x) & \text{otherwise}\end{cases}\right)+2abd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b/x)**(5/2)*(c+d/x),x)
```

output
```
4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 1
5*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2
)*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/
2)) - 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) -
 4*a**6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(
5/2)) - 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b*
*4*x**(5/2)) + a**2*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - a**2*d*Piecewise((
2*a*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-
sqrt(a)*log(x), True)) - 2*a*b*c*Piecewise((2*a*atan(sqrt(a + b/x)/sqrt(-a
))/sqrt(-a) + 2*sqrt(a + b/x), Ne(b, 0)), (-sqrt(a)*log(x), True)) + 2*a*b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) +
b**2*c*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.45

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c-\frac{1}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}a^2\right)d$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

output
```
1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(s
qrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b - 24*sqrt(a + b/x)*a*b)*c -
 1/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
 + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 30*sqrt(a + b/x)*a^2)*d
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)\,dx=-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{5/2}}{5}-2\,a^2\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int((a + b/x)^(5/2)*(c + d/x),x)
```

output
```
- (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
 b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)
^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx = \frac{60\sqrt{x}\,\sqrt{ax+b}\,a^2 c\,x^3 - 184\sqrt{x}\,\sqrt{ax+b}\,a^2 d\,x^2 - 280\sqrt{x}\,\sqrt{ax+b}\,abc\,x^2 - 88\sqrt{x}\,\sqrt{ax+b}\,abdx}{}$$

input
```
int((a+b/x)^(5/2)*(c+d/x),x)
```

output
```
(60*sqrt(x)*sqrt(a*x + b)*a**2*c*x**3 - 184*sqrt(x)*sqrt(a*x + b)*a**2*d*x
**2 - 280*sqrt(x)*sqrt(a*x + b)*a*b*c*x**2 - 88*sqrt(x)*sqrt(a*x + b)*a*b*
d*x - 40*sqrt(x)*sqrt(a*x + b)*b**2*c*x - 24*sqrt(x)*sqrt(a*x + b)*b**2*d
+ 120*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**2*d*x**3 +
 300*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b*c*x**3 + 4
0*sqrt(a)*a**2*d*x**3 + 163*sqrt(a)*a*b*c*x**3)/(60*x**3)
```

3.21 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -4ab\sqrt{a+\frac{b}{x}} - \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} + a^2\sqrt{a+\frac{b}{x}}x + 5a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-4*a*b*(a+b/x)^(1/2)-2/3*b*(a+b/x)^(3/2)+a^2*(a+b/x)^(1/2)*x+5*a^(3/2)*b*a
rctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \frac{\sqrt{a+\frac{b}{x}}(-2b^2-14abx+3a^2x^2)}{3x} + 5a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {773, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2),x]
```

output
```
(a + b/x)^(5/2)*x - (5*b*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*S
qrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{\left(3a^{2}x^{2}-14abx-2b^{2}\right)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	94
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-30\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x^{3}-15\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b\,x^{3}+24\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x+4b\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{6x^{2}\sqrt{x(ax+b)}\,\sqrt{a}}$	120

input
```
int((a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^2*x^2-14*a*b*x-2*b^2)/x*((a*x+b)/x)^(1/2)+5/2*a^(3/2)*b*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+
b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.95

$$\int\left(a+\frac{b}{x}\right)^{5/2}dx=\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^{2}x^{2}-14\,abx-2\,b^{2})\sqrt{\frac{ax+b}{x}}}{6\,x},\right.$$
$$\left.-\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(3\,a^{2}x^{2}-14\,abx-2\,b^{2})\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3
*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x
*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (3*a^2*x^2 - 14*a*b*x -
2*b^2)*sqrt((a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}} - \frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2} + 5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{2\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

input
```
integrate((a+b/x)**(5/2),x)
```

output
```
a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3
/2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)
*b**2*sqrt(1 + b/(a*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \sqrt{a+\frac{b}{x}}a^2x - \frac{5}{2}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 4\sqrt{a+\frac{b}{x}}ab$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=-\frac{2\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

input
```
int((a + b/x)^(5/2),x)
```

output
```
-(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
 + 1)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x}\right)^{5/2}\,dx=\frac{6\sqrt{x}\,\sqrt{ax+b}\,a^2x^2-28\sqrt{x}\,\sqrt{ax+b}\,abx-4\sqrt{x}\,\sqrt{ax+b}\,b^2+30\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)ab\,x^2 \cdots}{6x^2}$$

input
```
int((a+b/x)^(5/2),x)
```

output
```
(6*sqrt(x)*sqrt(a*x + b)*a**2*x**2 - 28*sqrt(x)*sqrt(a*x + b)*a*b*x - 4*sq
rt(x)*sqrt(a*x + b)*b**2 + 30*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a)
)/sqrt(b))*a*b*x**2 + 5*sqrt(a)*a*b*x**2)/(6*x**2)
```

3.22 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd}+\frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}+\frac{a^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

output
```
-b*(a*d+2*b*c)*(a+b/x)^(1/2)/c/d+a*(a+b/x)^(3/2)*x/c+2*(-a*d+b*c)^(5/2)*ar
ctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(3/2)+a^(3/2)*(-2*a*d+5
*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}\left(-2b^2c+a^2dx\right)}{d}+\frac{2(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}-a^{3/2}(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

output
```
((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(S
qrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 2*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {899, 109, 27, 171, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$$

$$\Big\downarrow \ 899$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{c+\frac{d}{x}}d\frac{1}{x}$$

$$\Big\downarrow \ 109$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-2ad)+\frac{b(2bc+ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

$$\Big\downarrow \ 27$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-2ad)+\frac{b(2bc+ad)}{x}\right)x}{c+\frac{d}{x}}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{171}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{2\int \frac{\left(a^2d(5bc-2ad)-\frac{b\left(2b^2c^2-6abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\int \frac{\left(a^2d(5bc-2ad)-\frac{b\left(2b^2c^2-6abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{174}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{a^2d(5bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2(bc-ad)^3\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{2a^2d(5bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4(bc-ad)^3\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{\frac{2a^2d(5bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

$$\Big\downarrow \text{221}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}-\frac{\frac{-\frac{2a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-2ad)}{c}-\frac{4(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{d}+\frac{2b\sqrt{a+\frac{b}{x}}(ad+2bc)}{d}}{2c}$$

input
```
Int[(a + b/x)^(5/2)/(c + d/x),x]
```

output
```
(a*(a + b/x)^(3/2)*x)/c - ((2*b*(2*b*c + a*d)*Sqrt[a + b/x])/d + ((-4*(b*c
 - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d])
 - (2*a^(3/2)*d*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/d)/(2*c
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(114) = 228$.

Time = 0.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.07

method	result
risch	$\frac{(a^2dx-2b^2c)\sqrt{\frac{ax+b}{x}}}{dc}-\frac{\left(\frac{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\right.}{c}}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2cd(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x(ax+b)}\,a^{\frac{5}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d^2x^2-4\sqrt{x(ax+b)}\,a^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,bc^3dx^2+2\sqrt{x(ax+b)}\,\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,b^2c^4x^2+8\sqrt{a\,x^2+b}\right.}{}$

input
```
int((a+b/x)^(5/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
(a^2*d*x-2*b^2*c)/d/c*((a*x+b)/x)^(1/2)-1/2/c/d*(2*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(
2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c
)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+d*a^(3/2)*(2*a*d-5*b*c)/c
*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2)))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^
(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.99

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \left[-\frac{(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc}{}}}{2\,c^2d}\right.$$

$$-\frac{(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-}{cx+d}\right.}{c^2d}$$

$$-\frac{4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)+(5\,abcd-2\,a^2d^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+}{x}}\right.}{2\,c^2d}$$

$$-\frac{(5\,abcd-2\,a^2d^2)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)}{c^2d}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((
2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x
 + d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*
d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c
- a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d
*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)
/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^
2*d), -((5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/
x)/(a*x + b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arct
an(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (a^2*c*d*x - 2*
b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d)]
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x),x)
```

output
```
Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 1427, normalized size of antiderivative = 10.65

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(5/2)/(c + d/x),x)
```

output
```
(atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 -
10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*
a^3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (1
6*a*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a
+ b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5
 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5
*b^6*d^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (
340*a^6*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5
*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2
*d^6 - 5*a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a
^4*b^7*c^3*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d
^2 - (128*a^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^
3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*
d^7)^(1/2)*16i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2
 + 448*a^3*b^8*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a
*b^10*c^4*d))*(d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(
1/2))/d + (atan((b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3
- 790*a^5*b^6*d^3 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d
^4)/c - (60*a^7*b^4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b...
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 677, normalized size of antiderivative = 5.05

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = \frac{4\sqrt{d}\,\sqrt{ad-bc}\log\left(\sqrt{c}\,\sqrt{ax+b}-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{x}\,\sqrt{c}\,\sqrt{a}\right)a^2d^2}{}$$

input
```
int((a+b/x)^(5/2)/(c+d/x),x)
```

output
```
(4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d**2*x
- 8*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c*d*x +
 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b**2*c**2*x
+ 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d**2*x
 - 8*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c*d*x
+ 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b**2*c**2*x
 - 4*sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqr
t(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**2*d**2*x + 8*sqrt(d)*sq
rt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sq
rt(a*x + b)*c + 2*a*c*x + 2*a*d)*a*b*c*d*x - 4*sqrt(d)*sqrt(a*d - b*c)*log
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2
*a*c*x + 2*a*d)*b**2*c**2*x + 4*sqrt(x)*sqrt(a*x + b)*a**2*c*d**2*x - 8*sq
rt(x)*sqrt(a*x + b)*b**2*c**2*d - 8*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*s
qrt(a))/sqrt(b))*a**2*d**3*x + 20*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*...
```

3.23 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2} dx = \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

output
```
(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)+a*(a+b/x)^(3/2)*x/c/(c
+d/x)-(-a*d+b*c)^(3/2)*(4*a*d+b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)
^(1/2))/c^3/d^(3/2)+a^(3/2)*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/
c^3
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(b^2c^2-2abcd+a^2d(2d+cx))}{d(d+cx)} - \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - a^{3/2}(-5bc+4ad)\text{arcta}}{c^3}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

output
```
((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x
)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[
b*c - a*d]])/d^(3/2) - a^(3/2)*(-5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/c^3
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 109, 27, 166, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{109}$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-4ad)+\frac{b(2bc-ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-4ad)+\frac{b(2bc-ad)}{x}\right)x}{\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{166}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}-\frac{\int -\frac{\left(d(5bc-4ad)a^2+\frac{b\left(b^2c^2+2abdc-2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{cd}}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\int \frac{\left(d(5bc-4ad)a^2+\frac{b\left(b^2c^2+2abdc-2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \text{174}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{a^2d(5bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}+\frac{(bc-ad)^2(4ad+bc)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \text{73}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a^2d(5bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^2(4ad+bc)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2a^2d(5bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}-\frac{\frac{\frac{2(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{2a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-4ad)}{c}}{cd}+\frac{2\sqrt{a+\frac{b}{x}}\left(-\frac{2a^2d}{c}+3ab-\frac{b^2c}{d}\right)}{c+\frac{d}{x}}}{2c}}{}$$

input
```
Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

output
```
(a*(a + b/x)^(3/2)*x)/(c*(c + d/x)) - ((2*(3*a*b - (b^2*c)/d - (2*a^2*d)/c
)*Sqrt[a + b/x])/(c + d/x) + ((2*(b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(S
qrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (2*a^(3/2)*d*(5*b*c
- 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(c*d))/(2*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(146) = 292$.

Time = 0.53 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.11

method	result
risch	$\frac{a^{2} x \sqrt{\frac{ax+b}{x}}}{c^{2}} - \left(\frac{a^{\frac{3}{2}} (4ad-5bc) \ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}} + \sqrt{a\,x^{2}+bx}\right)}{c} + \frac{2\left(a^{3} d^{3} - 3 a^{2} b c\, d^{2} + 3 a\, b^{2} c^{2} d - b^{3} c^{3}\right) \left(-\frac{c^{2} \sqrt{a\left(x+\frac{d}{c}\right)^{2} - \frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c} + \frac{(ad-bc}{c^{2}}}}{(ad-bc) d \left(x+\frac{d}{c}\right)}\right.}{} \right.$
default	Expression too large to display

input
```
int((a+b/x)^(5/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*a^2*x*((a*x+b)/x)^(1/2)-1/2/c^2*(a^(3/2)*(4*a*d-5*b*c)/c*ln((1/2*b+a
*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)/c^3*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c
*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)
^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(
1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c
*d)))+6*a/c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a
*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*
d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))*((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1013, normalized size of antiderivative = 6.10

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 -
 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)
*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x
)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)
*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3
+ (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)
/x)/(a*x + b)) + (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c
^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d
)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2
 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x +
c^3*d^2), 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c
^2*d - 4*a^2*c*d^2)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*s
qrt((a*x + b)/x)/(b*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d -
4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2
*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/
x))/(c^4*d*x + c^3*d^2), -((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (b^2*
c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b...
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(cx+d\right)^2}\,dx$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2*(a + b/x)**(5/2)/(c*x + d)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(146) = 292$.

Time = 0.17 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^2 - 1/2*(5*a^2*b*c*sgn(x) - 4*a^3*d*sgn(x))
*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^3) +
(b^3*c^3*sgn(x) + 2*a*b^2*c^2*d*sgn(x) - 7*a^2*b*c*d^2*sgn(x) + 4*a^3*d^3*
sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d
 - a*d^2))/(sqrt(b*c*d - a*d^2)*c^3*d) + 1/2*(2*sqrt(a)*b^3*c^3*arctan(sqr
t(a)*d/sqrt(b*c*d - a*d^2)) + 4*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*
c*d - a*d^2)) - 14*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) +
 8*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 5*sqrt(b*c*d - a*d^
2)*a^2*b*c*d*log(abs(b)) - 4*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - 2*s
qrt(b*c*d - a*d^2)*a*b^2*c^2 + 4*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 2*sqrt(b*
c*d - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^3*d) - ((sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^3*sgn(x) - 4*(sqrt(a)*x - sqrt(a*x
^2 + b*x))*a^(3/2)*b^2*c^2*d*sgn(x) + 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^
(5/2)*b*c*d^2*sgn(x) - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3*sgn(x
) - a*b^3*c^2*d*sgn(x) + 2*a^2*b^2*c*d^2*sgn(x) - a^3*b*d^3*sgn(x))/(((sqr
t(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a
)*d + b*d)*sqrt(a)*c^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 1153, normalized size of antiderivative = 6.95

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

output
```
(((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*
(a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*
a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/
2)*(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (
130*a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(
a^3)^(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*
b^6*d^2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3
)^(1/2))/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6
*d^2)/c - (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(
1/2))/(32*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5
*d^3)/c - 132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/
(32*a^3*b^8*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 +
 (10*a^2*b^9*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^
6*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5
)^(1/2))/(14*a^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*
c*d^2 + (2*a*b^10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/
2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b
^10*c^3 - 82*a^4*b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b
^6*d^4)/c - (40*a^6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 -
 b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 +...
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1310, normalized size of antiderivative = 7.89

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(5/2)/(c+d/x)^2,x)
```

output
```
(4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*c*d**2*
x + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*s
qrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d**3
 - 3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c**2*d
*x - 3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c*d*
*2 - sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b**2*c**3*
x - sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*b**2*c**2*d
 + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*c*d**
2*x + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d*
*3 - 3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b*c**2
*d*x - 3*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b...
```

3.24 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} dx = \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}(b^2c^2+8abcd-24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}} + \frac{a^{3/2}(5bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

output
```
1/2*(-3*a*d+b*c)*(-a*d+b*c)*(a+b/x)^(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2
+7*a*b*c*d+b^2*c^2)*(a+b/x)^(1/2)/c^3/d/(c+d/x)+a*(a+b/x)^(3/2)*x/c/(c+d/x
)^2-1/4*(-a*d+b*c)^(1/2)*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctan(d^(1/2)*(a
+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/d^(3/2)+a^(3/2)*(-6*a*d+5*b*c)*arctanh((
a+b/x)^(1/2)/a^(1/2))/c^4
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(b^2c^2(-d+cx)-abcd(7d+11cx)+2a^2d(6d^2+9cdx+2c^2x^2))}{d(d+cx)^2}-\frac{(b^3c^3+7ab^2c^2d-32a^2bcd^2+24a^3d^3)\arctan\left(\right.}{d^{3/2}\sqrt{bc-ad}}}{4c^4}$$

input
```
Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

output
```
((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d
*(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - ((b^3*c^3 + 7*a*b^2*c^2
*d - 32*a^2*b*c*d^2 + 24*a^3*d^3)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(d^(3/2)*Sqrt[b*c - a*d]) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(4*c^4)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {899, 109, 27, 166, 25, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}x^2}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int -\frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-6ad)+\frac{b(2bc-3ad)}{x}\right)x}{2\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{c}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\int \frac{\sqrt{a+\frac{b}{x}}\left(a(5bc-6ad)+\frac{b(2bc-3ad)}{x}\right)x}{\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2c}$$

$\Big\downarrow$ 166

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}-\frac{\int -\frac{\left(2d(5bc-6ad)a^2+\frac{b\left(b^2c^2+6abdc-9a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2cd}}{2c}$$

$\Big\downarrow$ 25

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\int \frac{\left(2d(5bc-6ad)a^2+\frac{b\left(b^2c^2+6abdc-9a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 168

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}-\frac{\int -\frac{\left(4d(5bc-6ad)(bc-ad)a^2+\frac{b(bc-ad)\left(b^2c^2+7abdc-12a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\Big\downarrow$ 27

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\int \frac{\left(4d(5bc-6ad)(bc-ad)a^2+\frac{b(bc-ad)\left(b^2c^2+7abdc-12a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 174

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{(bc-ad)^2\left(-24a^2d^2+8abcd+b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4a^2d(5bc-6ad)(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 73

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{2(bc-ad)^2\left(-24a^2d^2+8abcd+b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8a^2d(5bc-6ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 218

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\frac{\frac{8a^2d(5bc-6ad)(bc-ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^{3/2}\left(-24a^2d^2+8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}}{2c(bc-ad)}+\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}}{2cd}+\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}}{2c}$$

$\downarrow$ 221

$$\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}-\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-\frac{3a^2d}{c}+4ab-\frac{b^2c}{d}\right)}{\left(c+\frac{d}{x}\right)^2}+\frac{\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{c\left(c+\frac{d}{x}\right)}+\frac{\frac{2(bc-ad)^{3/2}\left(-24a^2d^2+8abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{d}}-\frac{8a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c}}{2c(bc-ad)}}{2cd}}{2c}$$

input

```
Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

output

```
(a*(a + b/x)^(3/2)*x)/(c*(c + d/x)^2) - (((4*a*b - (b^2*c)/d - (3*a^2*d)/c
)*Sqrt[a + b/x])/(c + d/x)^2 + (((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a
 + b/x])/(c*(c + d/x)) + ((2*(b*c - a*d)^(3/2)*(b^2*c^2 + 8*a*b*c*d - 24*a
^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[d]) - (8*
a^(3/2)*d*(5*b*c - 6*a*d)*(b*c - a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c)/(
2*c*(b*c - a*d)))/(2*c*d))/(2*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(209) = 418.

Time = 0.59 (sec) , antiderivative size = 1036, normalized size of antiderivative = 4.37

method	result	size
risch	Expression too large to display	1036
default	Expression too large to display	1640

input
```
int((a+b/x)^(5/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*a^2*x*((a*x+b)/x)^(1/2)-1/2/c^3*(a^(3/2)*(6*a*d-5*b*c)/c*ln((1/2*b+a
*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+2/c^3*(4*a^3*d^3-9*a^2*b*c*d^2+6*a*b^2*c^2*
d-b^3*c^3)*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1
/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^
2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)
^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1
/c*d)))+6*a/c^2*(2*a^2*d^2-3*a*b*c*d+b^2*c^2)/((a*d-b*c)*d/c^2)^(1/2)*ln((
2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+
1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))-2*d*(a
^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+1
/c*d)^2*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)+3/4*
(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*
a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/
((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((
a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c
^2)^(1/2))/(x+1/c*d)))+1/2*a/(a*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2
*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1
/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))))*((a*x
+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1457, normalized size of antiderivative = 6.15

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*
(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b
*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x
) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11
*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d
^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c
^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^
2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/
d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b
*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2
- 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) +
 (4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c
^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*
c^5*d^2*x + c^4*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*
a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*
d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*
c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(209) = 418$.

Time = 0.19 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))
*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))/(sqrt(a)*c^4) +
1/4*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^
3*d^3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt
(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqr
t(b*c*d - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2
)) + 24*a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d
- a*d^2)*a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)
) - sqrt(b*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*
sqrt(b*c*d - a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) -
1/4*((sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b
*x))^3*a^(7/2)*c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^
3*d*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) +
48*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt
(a)*b^4*c^3*d*sgn(x) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*
d^2*sgn(x) + 52*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x...
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 1476, normalized size of antiderivative = 6.23

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

output
```
(atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*
d)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a
+ b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b
^7*d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)
^(1/2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) +
 (45*a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/
(4*(8*a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*
b^7*c*d)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4
*c^3 - 24*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a
 + b/x)^(5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(
1/2)*(12*a^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*
d))/((a + b/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*
a*b*c*d) - d^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (
5*a^2*b^9*c^6 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 -
 59*a^4*b^7*c^4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9
*d) - ((((a + b/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5
- 15*a^2*b^6*c^4*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b
^7*c^5*d))/(8*c^6*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^
3*b^3*c^8*d^4)/(16*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x
)^(1/2)*(d^3*(a*d - b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8...
```

Reduce [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 3036, normalized size of antiderivative = 12.81

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/x)^(5/2)/(c+d/x)^3,x)
```

output
```
(96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*c**2*d
**3*x**2 + 192*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*
a**3*c*d**4*x + 96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**3*d**5 - 80*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**2*b*c**3*d**2*x**2 - 160*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a
*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*
sqrt(c)*sqrt(a))*a**2*b*c**2*d**3*x - 80*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(
c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
 sqrt(x)*sqrt(c)*sqrt(a))*a**2*b*c*d**4 + 12*sqrt(d)*sqrt(a*d - b*c)*log(s
qrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*
c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**4*d*x**2 + 24*sqrt(d)*sqrt(a*d - b
*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2
*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**3*d**2*x + 12*sqrt(d)*sqr
t(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d -
 b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*b**2*c**2*d**3 + 2*sqrt(
d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*s...
```

3.25 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}} \, dx = -\frac{2d^2(3bc-ad)\sqrt{a+\frac{b}{x}}}{b^2} - \frac{2d^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^2} + \frac{c^3\sqrt{a+\frac{b}{x}}x}{a} - \frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2*d^2*(-a*d+3*b*c)*(a+b/x)^(1/2)/b^2-2/3*d^3*(a+b/x)^(3/2)/b^2+c^3*(a+b/x
)^(1/2)*x/a-c^2*(-6*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,(4a^2d^3x+3b^2c^3x^2-2abd^2(d+9cx))}{3ab^2x} + \frac{c^2(-bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a
*b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {899, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^3x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 109}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(bc-6ad)-\frac{d(3bc+2ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(bc-6ad)-\frac{d(3bc+2ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \; 164$$

$$\frac{c^2(bc-6ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2c^2(bc-6ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{2d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3b^2}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-6ad)}{\sqrt{a}}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

input
```
Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*(c + d/x)^2*x)/a + ((-2*d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9
*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*a*d))/x))/(3*b^2) - (2*c^2*(b*c -
6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(ax+b)\left(3b^2c^3x^2+4a^2d^3x-18abc\,d^2x-2a\,d^3b\right)}{3b^2x^2a\sqrt{\frac{ax+b}{x}}}+\frac{(6ad-bc)c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\left(-6\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}d^3x^3+18\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}bc\,d^2x^3+18\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2c^2d\,x^3+3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b\,d^3x^3-9\ln\left(\right.\right.$

input
```
int((c+1/x*d)^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x+b)*(3*b^2*c^3*x^2+4*a^2*d^3*x-18*a*b*c*d^2*x-2*a*b*d^3)/b^2/x^2/a
/((a*x+b)/x)^(1/2)+1/2*(6*a*d-b*c)*c^2/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x
^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.27

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \left[-\frac{3\,(b^3c^3 - 6\,ab^2c^2d)\sqrt{a}x\log\left(2\,ax + 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2\,(3\,ab^2c^3x^2 - 2\,a^2bd^3 - 2\,(9\,a^2bcd^2 - 2\,a^3d^3}{6\,a^2b^2x}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 -
2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^
2*d)*sqrt(-a)*x*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (3*a*b^2*
c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x)
)/(a^2*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.72

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-3c^2d\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + 3cd^2\left(\begin{cases}-\frac{1}{\sqrt{a}x} & \text{for } b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right) + \frac{\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

output
```
4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) +
3*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*
a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d
**3*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3
/2)) - 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b*
*4*x**(3/2)) - 3*c**2*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
, Ne(b, 0)), (-log(x)/sqrt(a), True)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x)
, Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b
 + 1)/a - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.58

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}\,c^3\left(\frac{2\,\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2}{3}\,d^3\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{a+\frac{b}{x}}a}{b^2}\right) - \frac{3\,c^2d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{6\,\sqrt{a+\frac{b}{x}}cd^2}{b}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2
- 3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,\left(a+\frac{b}{x}\right)^{3/2}}{3\,b^2} + \frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1\mathrm{i}}{a^{3/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a +
b/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/
2)*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.80

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3d^3x-36\sqrt{x}\,\sqrt{ax+b}\,a^2bc\,d^2x-4\sqrt{x}\,\sqrt{ax+b}\,a^2b\,d^3+6\sqrt{x}\,\sqrt{ax+b}\,a\,b^2c^3x^2+36\sqrt{a}\,\mathrm{lo}}{6a^2}$$

input
```
int((c+d/x)^3/(a+b/x)^(1/2),x)
```

output
```
(8*sqrt(x)*sqrt(a*x + b)*a**3*d**3*x - 36*sqrt(x)*sqrt(a*x + b)*a**2*b*c*d
**2*x - 4*sqrt(x)*sqrt(a*x + b)*a**2*b*d**3 + 6*sqrt(x)*sqrt(a*x + b)*a*b*
*2*c**3*x**2 + 36*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a
*b**2*c**2*d*x**2 - 6*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b
))*b**3*c**3*x**2 - 8*sqrt(a)*a**3*d**3*x**2 + 12*sqrt(a)*a**2*b*c*d**2*x*
*2 + sqrt(a)*b**3*c**3*x**2)/(6*a**2*b**2*x**2)
```

3.26 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}+\frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2*d^2*(a+b/x)^(1/2)/b+c^2*(a+b/x)^(1/2)*x/a-c*(-4*a*d+b*c)*arctanh((a+b/x
)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}(-2ad^2+bc^2x)}{ab}+\frac{c(-bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[S
qrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \; 899$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \; 100$$

$$\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{\int -\frac{\left(c(bc-4ad)-\frac{2ad^2}{x}\right)x}{2\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(c(bc-4ad)-\frac{2ad^2}{x}\right)x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \; 90$$

$$\frac{c(bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2c(bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow 221$$

$$\frac{-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-4ad)}{\sqrt{a}}-\frac{4ad^2\sqrt{a+\frac{b}{x}}}{b}}{2a}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}$$

input
```
Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

output
```
(c^2*Sqrt[a + b/x]*x)/a + ((-4*a*d^2*Sqrt[a + b/x])/b - (2*c*(b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

method	result
risch	$-\frac{(ax+b)\left(-bx\,c^{2}+2a\,d^{2}\right)}{bax\sqrt{\frac{ax+b}{x}}}+\frac{(4ad-bc)c\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-2\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}d^{2}x^{2}-4\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}bcd\,x^{2}-\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b\,d^{2}x^{2}-2\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^{2}c\right.}{}$

input
```
int((c+1/x*d)^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+b)*(-b*c^2*x+2*a*d^2)/b/a/x/((a*x+b)/x)^(1/2)+1/2*(4*a*d-b*c)*c/a^(3
/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+
b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.23

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \left[-\frac{(b^2c^2-4\,abcd)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(abc^2x-2\,a^2d^2)\sqrt{\frac{ax+b}{x}}}{2\,a^2b}, \frac{(b^2c^2-4\,abcd)\sqrt{-a}\,\mathrm{ar}\ldots}{}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b
)/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^
2 - 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (
a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 11.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = -2cd\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)+d^2\left(\begin{cases}-\frac{1}{\sqrt{a}x} & \text{for } b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)$$
$$+\frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

output
```
-2*c*d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-lo
g(x)/sqrt(a), True)) + d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt
(a + b/x)/b, True)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - b*c**2*asin
h(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}c^2\left(\frac{2\sqrt{a+\frac{b}{x}}b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{a+\frac{b}{x}}d^2}{b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sq
rt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{c^2\,x\,\sqrt{a+\frac{b}{x}}}{a} - \frac{2\,d^2\,\sqrt{a+\frac{b}{x}}}{b} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-b\,c)}{a^{3/2}}$$

input
```
int((c + d/x)^2/(a + b/x)^(1/2),x)
```

output
```
(c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)
^(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \frac{-8\sqrt{x}\,\sqrt{ax+b}\,a^2d^2 + 4\sqrt{x}\,\sqrt{ax+b}\,ab\,c^2x + 16\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)abcdx - 4\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2}{4a^2bx}$$

input
```
int((c+d/x)^2/(a+b/x)^(1/2),x)
```

output
```
( - 8*sqrt(x)*sqrt(a*x + b)*a**2*d**2 + 4*sqrt(x)*sqrt(a*x + b)*a*b*c**2*x
 + 16*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b*c*d*x - 4
*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2*c**2*x - 8*sq
rt(a)*a**2*d**2*x - sqrt(a)*b**2*c**2*x)/(4*a**2*b*x)
```

3.27 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
c*(a+b/x)^(1/2)*x/a-(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx = \frac{c\sqrt{a+\frac{b}{x}}x}{a} + \frac{(-bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*x)/a + ((-(b*c) + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {899, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{3/2}}$$

input
```
Int[(c + d/x)/Sqrt[a + b/x],x]
```

output
```
(c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(
3/2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.65

method	result
risch	$\frac{c(ax+b)}{a\sqrt{\frac{ax+b}{x}}}+\frac{(2ad-bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{x(ax+b)}\,a^{\frac{3}{2}}d-2\sqrt{x(ax+b)}\,\sqrt{a}\,bc-2\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}d-\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)abd-\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)c}{2\sqrt{x(ax+b)}\,b\,a^{\frac{3}{2}}}$

input
```
int((c+1/x*d)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2*(2*a*d-b*c)/a^(3/2)*ln((1/2*b+a*x)/a^(
1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.35

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{acx\sqrt{\frac{ax+b}{x}} + (bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-}}{}\right.}{a^2}\right.$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt
(a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d
)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)))/a^2]
```

Sympy [A] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -d\left(\begin{cases} \frac{2\,\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0 \\ -\frac{\log(x)}{\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{bc\,\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate((c+d/x)/(a+b/x)**(1/2),x)
```

output
```
-d*Piecewise((2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(x)
/sqrt(a), True)) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - b*c*asinh(sqrt(a)
*sqrt(x)/sqrt(b))/a**(3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{2}\,c\left(\frac{2\sqrt{a+\frac{b}{x}}b}{(a+\frac{b}{x})a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) - \frac{d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))
/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{(bc\log(|b|) - 2\,ad\log(|b|))\mathrm{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{\sqrt{ax^2+bx}c}{a\mathrm{sgn}(x)} + \frac{(bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}$$

input
```
integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*c*log(abs(b)) - 2*a*d*log(abs(b)))*sgn(x)/a^(3/2) + sqrt(a*x^2 + b
*x)*c/(a*sgn(x)) + 1/2*(b*c - 2*a*d)*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a) + b))/(a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{2\,c\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

input
```
int((c + d/x)/(a + b/x)^(1/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x
)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3
/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{x}\,\sqrt{ax+b}\,ac+2\sqrt{a}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)ad-\sqrt{a}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)bc}{a^2}$$

input
```
int((c+d/x)/(a+b/x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a*x + b)*a*c + 2*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a
))/sqrt(b))*a*d - sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b
*c)/a**2
```

3.28 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(a+b/x)^(1/2)*x/a-b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[1/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2\sqrt{x(ax+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\, a^{\frac{3}{2}}}$	71
risch	$\frac{ax+b}{a\sqrt{\frac{ax+b}{x}}}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}\, x\sqrt{\frac{ax+b}{x}}}$	75

input
```
int(1/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a^2}\right]$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*x
*sqrt((a*x + b)/x)/(a*x + b)))/a^2]
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3
/2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}b}{(a+\frac{b}{x})a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/
(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{b\log(|b|)\,\mathrm{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)} + \frac{\sqrt{ax^2+bx}}{a\mathrm{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) + b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x} + \frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

input
```
int(1/(a + b/x)^(1/2),x)
```

output
```
(2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)
*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{x}\,\sqrt{ax+b}\,a-\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b}{a^2}$$

input
```
int(1/(a+b/x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a*x + b)*a - sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b)/a**2
```

3.29 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} dx = \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

output
```
(a+b/x)^(1/2)*x/a/c-2*d^(3/2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2
))/c^2/(-a*d+b*c)^(1/2)-(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
/c^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x}{a}-\frac{2d^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^2}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

output
```
((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/a^(3/2))/c^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 114, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(bc+2ad+\frac{bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(bc+2ad+\frac{bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{(2ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2ad^2\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2(2ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4ad^2\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{2(2ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{4ad^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2ad+bc)}{\sqrt{a}c}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c) + ((-4*a*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c + 2*a*d)*ArcTanh[Sqrt[a + b/
x]/Sqrt[a]])/(Sqrt[a]*c))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(90) = 180$.

Time = 0.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.12

method	result
default	$\frac{\left(2\sqrt{x(ax+b)}\,c^2\sqrt{a}\,\sqrt{\frac{(ad-bc)d}{c^2}}-2\sqrt{\frac{(ad-bc)d}{c^2}}\,\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)acd-\sqrt{\frac{(ad-bc)d}{c^2}}\,\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b\,c^2-2a^{\frac{3}{2}}\ln \cdots\right.}{2a^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^3\sqrt{x(ax+b)}}$
risch	$\frac{ax+b}{ac\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{2a\,d^2\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+\cdots}}{x+\frac{d}{c}}\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\right.}{2acx\sqrt{\frac{ax+b}{x}}}$

input
```
int(1/(a+b/x)^(1/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*(x*(a*x+b))^(1/2)*c^2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)-2*((a*d-b*c)*
d/c^2)^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*c*d-(
(a*d-b*c)*d/c^2)^(1/2)*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*b*c^2-2*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*(x*(a*x+b))^(1/2)*c-2*a*d
*x+b*c*x-b*d)/(c*x+d))*d^2)*x*((a*x+b)/x)^(1/2)/a^(3/2)/((a*d-b*c)*d/c^2)^
(1/2)/c^3/(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 512, normalized size of antiderivative = 4.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

$$= \left[\frac{2\,a^2 d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,acx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\sqrt{a}\log\left(2\,ax-\right.}{2\,a^2c^2}\right.$$

$$-\frac{4\,a^2 d\sqrt{\frac{d}{bc-ad}}\arctan\left(\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}\right)-2\,acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2c^2},$$

$$\left.-\frac{2\,a^2 d\sqrt{\frac{d}{bc-ad}}\arctan\left(\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}\right)-acx\sqrt{\frac{ax+b}{x}}-(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a^2c^2}\right]$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((
a*x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqr
t(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) +
a*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a
*x + b)/x)/(a*x + b)))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan
(sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)) - 2*a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a^2*c^2
), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(sqrt(d/(b*c - a*d))*sqrt((a*x + b)
/x)) - a*c*x*sqrt((a*x + b)/x) - (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*x*
sqrt((a*x + b)/x)/(a*x + b)))/(a^2*c^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

input
```
integrate(1/(a+b/x)**(1/2)/(c+d/x),x)
```

output
```
Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 1183, normalized size of antiderivative = 10.95

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)),x)
```

output
```
(x*(a + b/x)^(1/2))/(a*c) - (atan((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4
*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1
/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3
)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d
^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2
*d) - (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*
b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/
(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) +
(2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2
))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2
 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d
^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*
(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2
*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(
b*c^3 - a*c^2*d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b
^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3
*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*
(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3
)^(1/2))/(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

$$= \frac{\sqrt{d}\,\sqrt{ad-bc}\log\left(\sqrt{c}\,\sqrt{ax+b}-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{x}\,\sqrt{c}\,\sqrt{a}\right)a^2d+\sqrt{d}\,\sqrt{ad-bc}\log}{}$$

input
```
int(1/(a+b/x)^(1/2)/(c+d/x),x)
```

output
```
(sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d + sqrt(
d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt
(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*d - sqrt(d)*sqr
t(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqr
t(a*x + b)*c + 2*a*c*x + 2*a*d)*a**2*d + sqrt(x)*sqrt(a*x + b)*a**2*c*d -
sqrt(x)*sqrt(a*x + b)*a*b*c**2 - 2*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sq
rt(a))/sqrt(b))*a**2*d**2 + sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/
sqrt(b))*a*b*c*d + sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*
b**2*c**2)/(a**2*c**2*(a*d - b*c))
```

3.30 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}$$

output
```
d*(-2*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)+(a+b/x)^(1/2)*x/a/c/
(c+d/x)-d^(3/2)*(-4*a*d+5*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/
2))/c^3/(-a*d+b*c)^(3/2)-(4*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2
)/c^3
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$
$$=\frac{\frac{c\sqrt{a+\frac{b}{x}}x(-bc(d+cx)+ad(2d+cx))}{a(-bc+ad)(d+cx)}+\frac{d^{3/2}(-5bc+4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{c^3}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-(b*c*(d + c*x)) + a*d*(2*d + c*x)))/(a*(-(b*c) + a*d
)*(d + c*x)) + (d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sq
rt[b*c - a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]])/a^(3/2))/c^3
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 114, 27, 168, 25, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$
$$\Big\downarrow \text{899}$$
$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$
$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(bc+4ad+\frac{3bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(bc+4ad+\frac{3bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 168

$$\frac{\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{bd(bc-2ad)}{x}+(bc-ad)(bc+4ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(\frac{bd(bc-2ad)}{x}+(bc-ad)(bc+4ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{(bc-ad)(4ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{ad^2(5bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^2(5bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{2(bc-ad)(4ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 221

$$\frac{\frac{-\frac{2ad^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)(4ad+bc)}{\sqrt{a}c}}{c(bc-ad)}+\frac{2d\sqrt{a+\frac{b}{x}}(bc-2ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c*(c + d/x)) + ((2*d*(b*c - 2*a*d)*Sqrt[a + b/x])/(c*
(b*c - a*d)*(c + d/x)) + ((-2*a*d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)*(b*c +
 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(c*(b*c - a*d)))/(2*a
*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(152) = 304$.

Time = 0.60 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.73

method	result
risch	$\frac{ax+b}{a\,c^2\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(4ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{6a\,d^2\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}\cdots}}{x+\frac{d}{c}}\cdots\right.}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(1/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a*((4*a*d+b*c)/c*ln((1/2*b+a*x)/
a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+6*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln(
(2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x
+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+2*a*d
^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d
)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(
1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/
2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d
))))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1131, normalized size of antiderivative = 6.58

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^
2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^
2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*
d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (
b*c - 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2
*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5
- a^3*c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a
^3*c*d^2)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(d/(b*c - a*d))*sqrt((a*x + b)
/x)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*((
a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))
/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x), 1/2*(2*(b^2*c^2*
d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqr
t(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (5*a^2*b*c*d^2 - 4*
a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b
*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)
*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x
), ((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*
c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (5*...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\,(cx+d)^2}\,dx$$

input
```
integrate(1/(a+b/x)**(1/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2/(sqrt(a + b/x)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(152) = 304$.

Time = 0.17 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$
$$= \frac{\left(10\,a^{\frac{3}{2}}bcd^2\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - 8\,a^{\frac{5}{2}}d^3\arctan\left(\frac{\sqrt{a}d}{\sqrt{bcd-ad^2}}\right) - \sqrt{bcd-ad^2}b^2c^2\log\left(|b|\right) - 3\sqrt{bcd-ad^2}abc\cdots}{2\left(\sqrt{bcd-ad^2}a^{\frac{3}{2}}bc^4 - \sqrt{bcd-ad^2}a^{\frac{5}{2}}c^3\cdots\right.}$$
$$+ \frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(-\frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)c+\sqrt{a}d}{\sqrt{bcd-ad^2}}\right)}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\sqrt{bcd-ad^2}}$$
$$+ \frac{\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)bcd^2 - 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)ad^3 - \sqrt{a}bd^3}{\left(bc^4\mathrm{sgn}\left(x\right)-ac^3d\mathrm{sgn}\left(x\right)\right)\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2c + 2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}d + bd\right)}$$
$$+ \frac{\sqrt{ax^2+bx}}{ac^2\mathrm{sgn}\left(x\right)} + \frac{\left(bc+4\,ad\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}c^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
1/2*(10*a^(3/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 8*a^(5/2)*
d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - sqrt(b*c*d - a*d^2)*b^2*c^2*lo
g(abs(b)) - 3*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) + 4*sqrt(b*c*d - a*d
^2)*a^2*d^2*log(abs(b)) + 2*sqrt(b*c*d - a*d^2)*a^2*d^2)*sgn(x)/(sqrt(b*c*
d - a*d^2)*a^(3/2)*b*c^4 - sqrt(b*c*d - a*d^2)*a^(5/2)*c^3*d) + (5*b*c*d^2
 - 4*a*d^3)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b
*c*d - a*d^2))/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*sqrt(b*c*d - a*d^2)) + ((s
qrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d^2 - 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*
a*d^3 - sqrt(a)*b*d^3)/((b*c^4*sgn(x) - a*c^3*d*sgn(x))*((sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)) +
 sqrt(a*x^2 + b*x)/(a*c^2*sgn(x)) + 1/2*(b*c + 4*a*d)*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(3/2)*c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 3813, normalized size of antiderivative = 22.17

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)
```

output
```
(((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b
*c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a +
b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4
*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^
5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d +
 b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5
*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d
)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/
2)) + (((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4
 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
a^3*b*c^5*d) - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4
 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a +
b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*
b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*
d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1
i)/(2*c^3*(a^3)^(1/2))))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*
d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((
2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*...
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1164, normalized size of antiderivative = 6.77

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(1/2)/(c+d/x)^2,x)
```

output
```
(4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*c*d**2*
x + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*s
qrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*d**3
 - 5*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*b*c**
2*d*x - 5*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**2*
b*c*d**2 + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sq
rt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a*
*3*c*d**2*x + 4*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2
*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))
*a**3*d**3 - 5*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*
a**2*b*c**2*d*x - 5*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sq
rt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt
(a))*a**2*b*c*d**2 - 4*sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(
a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**3*c*d
**2*x - 4*sqrt(d)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) +
2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**3*d**3 + 5*sqrt...
```

3.31 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx &= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} \\ &+ \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} \\ &- \frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}} \\ &- \frac{(bc+6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}\end{aligned}$$

output
```
1/2*d*(-3*a*d+2*b*c)*(a+b/x)^(1/2)/a/c^2/(-a*d+b*c)/(c+d/x)^2+1/4*d*(-4*a*
d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^(1/2)/a/c^3/(-a*d+b*c)^2/(c+d/x)+(a+b/x)^(1/
2)*x/a/c/(c+d/x)^2-1/4*d^(3/2)*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*arctan(d
^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(5/2)-(6*a*d+b*c)*ar
ctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)/c^4
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(4b^2c^2(d+cx)^2+2a^2d^2(6d^2+9cdx+2c^2x^2)-abcd(19d^2+29cdx+8c^2x^2))}{a(bc-ad)^2(d+cx)^2}-\frac{d^{3/2}(35b^2c^2-56abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}-}{4c^4}$$

input
```
Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x +
2*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/(a*(b*c - a*d)^2*(d
 + c*x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[
d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(4*c^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {899, 114, 27, 168, 25, 168, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,d\frac{1}{x}$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\int \frac{\left(bc+6ad+\frac{5bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(bc+6ad+\frac{5bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 168

$$\frac{\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{3bd(2bc-3ad)}{x}+2(bc-ad)(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(\frac{3bd(2bc-3ad)}{x}+2(bc-ad)(bc+6ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 168

$$\frac{\frac{\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(4(bc+6ad)(bc-ad)^2+\frac{bd(bc-4ad)(4bc-3ad)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\left(4(bc+6ad)(bc-ad)^2+\frac{bd(bc-4ad)(4bc-3ad)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{\frac{4(bc-ad)^2(6ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{ad^2\left(24a^2d^2-56abcd+35b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{\frac{8(bc-ad)^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^2\left(24a^2d^2-56abcd+35b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{8(bc-ad)^2(6ad+bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{-\frac{2ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad+bc)(bc-ad)^2}{\sqrt{a}c}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{c\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{a+\frac{b}{x}}(2bc-3ad)}{c\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

input
```
Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

output
```
(Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) + ((d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(
c*(b*c - a*d)*(c + d/x)^2) + ((d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/
x])/(c*(b*c - a*d)*(c + d/x)) + ((-2*a*d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d +
24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c -
 a*d]) - (8*(b*c - a*d)^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(S
qrt[a]*c))/(2*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(222) = 444.

Time = 0.65 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.81

method	result
risch	$\frac{ax+b}{a\,c^3\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(6ad+bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{12a\,d^2\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{x+\frac{d}{c}}\right)}{c^2\sqrt{\frac{(ad-bc)d}{c^2}}} \right.}{}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(1/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a/c^3*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^3/a*((6*a*d+b*c)/c*ln((1/2*b+a*x)/
a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+12*a*d^2/c^2/((a*d-b*c)*d/c^2)^(1/2)*ln
((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(
x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+8*a*
d^3/c^3*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*
d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^
(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1
/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*
d)))-2*a*d^4/c^4*(-1/2/(a*d-b*c)/d*c^2/(x+1/c*d)^2*(a*(x+1/c*d)^2-(2*a*d-b
*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)+3/4*(2*a*d-b*c)*c/(a*d-b*c)/d*(-1/(
a*d-b*c)/d*c^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*
d/c^2)^(1/2)-1/2*(2*a*d-b*c)*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(
a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c
*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))+1/2*a/(a
*d-b*c)/d*c^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*
(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)
+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(222) = 444$.

Time = 0.59 (sec) , antiderivative size = 2277, normalized size of antiderivative = 9.11

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
[1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3
*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*
d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x
 - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d
^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x
^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(
b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) -
 b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3
*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 +
(4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/
(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*
c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^
3)*x), 1/8*(8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5
+ (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^
3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*ar
ctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (35*a^2*b^2*c^2*d^3 - 56*a^
3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2
*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sq
rt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x +
b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b/x)**(1/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(222) = 444$.

Time = 0.19 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
1/4*(35*a^(3/2)*b^2*c^2*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 56*a^(
5/2)*b*c*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(7/2)*d^4*arctan
(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^3*c^3*log(abs(b)
) - 8*sqrt(b*c*d - a*d^2)*a*b^2*c^2*d*log(abs(b)) + 22*sqrt(b*c*d - a*d^2)
*a^2*b*c*d^2*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^3*log(abs(b)) + 13
*sqrt(b*c*d - a*d^2)*a^2*b*c*d^2 - 10*sqrt(b*c*d - a*d^2)*a^3*d^3)*sgn(x)/
(sqrt(b*c*d - a*d^2)*a^(3/2)*b^2*c^6 - 2*sqrt(b*c*d - a*d^2)*a^(5/2)*b*c^5
*d + sqrt(b*c*d - a*d^2)*a^(7/2)*c^4*d^2) + 1/4*(35*b^2*c^2*d^2 - 56*a*b*c
*d^3 + 24*a^2*d^4)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)
/sqrt(b*c*d - a*d^2))/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*
sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(13*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*b
^2*c^3*d^2 - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^3 + 24*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^4 + 7*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2
*sqrt(a)*b^2*c^2*d^3 - 56*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^
4 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^5 + 11*(sqrt(a)*x - sqr
t(a*x^2 + b*x))*b^3*c^2*d^3 - 60*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a*b^2*c*d
^4 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^2*b*d^5 - 13*sqrt(a)*b^3*c*d^4 +
 10*a^(3/2)*b^2*d^5)/((b^2*c^6*sgn(x) - 2*a*b*c^5*d*sgn(x) + a^2*c^4*d^2*s
gn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 +
b*x))*sqrt(a)*d + b*d)^2) + sqrt(a*x^2 + b*x)/(a*c^3*sgn(x)) + 1/2*(b*c...
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 2890, normalized size of antiderivative = 11.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
```

output
```
(log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a
*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b
^2*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*
b^3*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(1
2*a^2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a +
 b/x)^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d)
)/(4*a*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 5
6*a^2*b^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((
a + b/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d) - d^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d
 - b*c)^5)^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*
b*c*d))/(8*(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 -
 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^
6*b^2*d^9 + 16*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129
*a^2*b^6*c^4*d^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b
^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*
d^2)) - (((4*a*b^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 7
4*a^4*b^5*c^10*d^5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^1
3 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*...
```

Reduce [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 3752, normalized size of antiderivative = 15.01

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(1/2)/(c+d/x)^3,x)
```

output
```
(96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*c**2*d
**4*x**2 + 192*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*
a**5*c*d**5*x + 96*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**5*d**6 - 272*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sq
rt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt
(a))*a**4*b*c**3*d**3*x**2 - 544*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(
a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)
*sqrt(c)*sqrt(a))*a**4*b*c**2*d**4*x - 272*sqrt(d)*sqrt(a*d - b*c)*log(sqr
t(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
 + sqrt(x)*sqrt(c)*sqrt(a))*a**4*b*c*d**5 + 252*sqrt(d)*sqrt(a*d - b*c)*lo
g(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*b**2*c**4*d**2*x**2 + 504*sqrt(d)*sq
rt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
- b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*b**2*c**3*d**3*x + 2
52*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**3*b**2*c*
*2*d**4 - 70*sqrt(d)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2...
```

3.32 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2(bc-ad)^3}{a^2b^2\sqrt{a+\frac{b}{x}}} - \frac{2d^3\sqrt{a+\frac{b}{x}}}{b^2} + \frac{c^3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3c^2(bc-2ad)\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output

```
2*(-a*d+b*c)^3/a^2/b^2/(a+b/x)^(1/2)-2*d^3*(a+b/x)^(1/2)/b^2+c^3*(a+b/x)^(
1/2)*x/a^2-3*c^2*(-2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\left(3b^3c^3x-4a^3d^3x-2a^2bd^2(d-3cx)+ab^2c^2x(-6d+cx)\right)}{a^2b^2(b+ax)}+\frac{3c^2(-bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*(3*b^3*c^3*x - 4*a^3*d^3*x - 2*a^2*b*d^2*(d - 3*c*x) + a*b^
2*c^2*x*(-6*d + c*x)))/(a^2*b^2*(b + a*x)) + (3*c^2*(-(b*c) + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 109, 27, 163, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 899$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^3x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \; 109$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(3c(bc-2ad)-\frac{d(bc+2ad)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(3c(bc-2ad)-\frac{d(bc+2ad)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 163$$

$$\frac{\frac{3c^2(bc-2ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{6c^2(bc-2ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2\left((bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}\right)}{ab^2\sqrt{a+\frac{b}{x}}}-\frac{6c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-2ad)}{a^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

output
```
(c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) + ((2*((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*
b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d))/x))/(a*b^2*Sqrt[a + b/x]) - (
6*c^2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(92) = 184.

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{(ax+b)\left(-b^2x\,c^3+2a^2d^3\right)}{b^2a^2x\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{2\left(-2a^3d^3+6a^2bc\,d^2-6a\,b^2c^2d+2b^3c^3\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}-\frac{3b^2c^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+6\sqrt{a}\,b\,c\right.}{2a^2bx\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,b^3c\,d^2x^2+12a^{\frac{3}{2}}\sqrt{x(ax+b)}\,b^4c^2d\,x^2+24a^{\frac{5}{2}}\sqrt{x(ax+b)}\,b^3c^2d\,x^3-6\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b^3c\,d^2x^2-6\sqrt{a\,x^2+bx}\right.}{}$

input
```
int((c+1/x*d)^3/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*x+b)*(-b^2*c^3*x+2*a^2*d^3)/b^2/a^2/x/((a*x+b)/x)^(1/2)+1/2/a^2/b*(2*(
-2*a^3*d^3+6*a^2*b*c*d^2-6*a*b^2*c^2*d+2*b^3*c^3)/a/b/(x+b/a)*(a*(x+b/a)^2
-b*(x+b/a))^(1/2)-3*b^2*c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1
/2)+6*a^(1/2)*b*c^2*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2)))/x/((a*x+b
)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.28

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\left[-\frac{3\left(b^4c^3-2\,ab^3c^2d+(ab^3c^3-2\,a^2b^2c^2d)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2}{2\,(a^4b^2x+a^3b^3}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(
a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2
*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x
)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d +
(a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)
/x)/(a*x + b)) + (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2
*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3
*b^3)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

output
```
Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(92) = 184.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx&=\frac{1}{2}\,c^3\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)\\&\quad-3\,c^2d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)\\&\quad-2\,d^3\left(\frac{\sqrt{a+\frac{b}{x}}}{b^2}+\frac{a}{\sqrt{a+\frac{b}{x}}b^2}\right)+\frac{6\,cd^2}{\sqrt{a+\frac{b}{x}}b}\end{aligned}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^
3) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2))
 - 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/
2) + 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^
2)) + 6*c*d^2/(sqrt(a + b/x)*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\frac{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\,\left(2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{a^2}}{b^2\,{\left(a+\frac{b}{x}\right)}^{3/2}-a\,b^2\,\sqrt{a+\frac{b}{x}}}-\frac{2\,d^3\,\sqrt{a+\frac{b}{x}}}{b^2}+\frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(2\,a\,d-b\,c\right)}{a^{5/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(3/2),x)
```

output
```
((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2
*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)
^(3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*ata
nh((a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.31

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{6\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,b^2c^2dx-3\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3c^3x-4\sqrt{}$$

input
```
int((c+d/x)^3/(a+b/x)^(3/2),x)
```

output
```
(6*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*
b**2*c**2*d*x - 3*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(
a))/sqrt(b))*b**3*c**3*x - 4*sqrt(a)*sqrt(a*x + b)*a**3*d**3*x + 6*sqrt(a)
*sqrt(a*x + b)*a**2*b*c*d**2*x - 6*sqrt(a)*sqrt(a*x + b)*a*b**2*c**2*d*x +
 2*sqrt(a)*sqrt(a*x + b)*b**3*c**3*x - 4*sqrt(x)*a**4*d**3*x + 6*sqrt(x)*a
**3*b*c*d**2*x - 2*sqrt(x)*a**3*b*d**3 + sqrt(x)*a**2*b**2*c**3*x**2 - 6*s
qrt(x)*a**2*b**2*c**2*d*x + 3*sqrt(x)*a*b**3*c**3*x)/(sqrt(a*x + b)*a**3*b
**2*x)
```

3.33 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2(bc-ad)^2}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a^2}-\frac{c(3bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
2*(-a*d+b*c)^2/a^2/b/(a+b/x)^(1/2)+c^2*(a+b/x)^(1/2)*x/a^2-c*(-4*a*d+3*b*c
)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x(3b^2c^2+2a^2d^2+abc(-4d+cx))}{a^2b(b+ax)}+\frac{c(-3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x)))/(a^2*b*(b +
 a*x)) + (c*(-3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {899, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}-\frac{\int -\frac{\left(c(3bc-4ad)-\frac{2ad^2}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(c(3bc-4ad)-\frac{2ad^2}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\frac{c(3bc-4ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2c(3bc-4ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2\left(\frac{c(3bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{\sqrt{a+\frac{b}{x}}}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(3bc-4ad)}{a^{3/2}}}{2a}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

output
```
(c^2*x)/(a*Sqrt[a + b/x]) + ((2*((2*a*d^2)/b + (c*(3*b*c - 4*a*d))/a))/Sqr
t[a + b/x] - (2*c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))
/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.17

method	result
risch	$\frac{c^2(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{2\left(2a^2d^2-4abcd+2b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)}-\frac{3bc^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}+4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\right)\sqrt{x(a}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-4\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2cd\,x^2-8(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}bcd-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b^2c^2x^2+4(x(ax+b))^{\frac{3}{2}}a^{\frac{3}{2}}b^2c^2-2\ln\left(\right.}{}$

input
```
int((c+1/x*d)^2/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(2*(2*a^2*d^2-4*a*b*c*d+2*b^2*c^
2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-3*b*c^2*ln((1/2*b+a*x)/a^(1/2
)+(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b
*x)^(1/2)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.30

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^3c^2-4\,ab^2cd+(3\,ab^2c^2-4\,a^2bcd)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2b\cdots}{2\,(a^4bx+a^3b^2)}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*l
og(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^
2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)
, ((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2
 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

output
```
Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(74) = 148$.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.95

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}c^2\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - 2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)+\frac{2\,d^2}{\sqrt{a+\frac{b}{x}}b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^
3) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2))
 - 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(74) = 148$.

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.23

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^2\mathrm{sgn}\left(x\right)} + \frac{\left(3\,bc^2-4\,acd\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} - \frac{\left(3\,b^2c^2\log\left(|b|\right)-4\,abcd\log\left(|b|\right)+4\,b^2c^2-8\,abcd+4\,a^2d^2\right)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}b} + \frac{2\left(\sqrt{a}b^2c^2-2\,a^{\frac{3}{2}}bcd+a^{\frac{5}{2}}d^2\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^2/(a^2*sgn(x)) + 1/2*(3*b*c^2 - 4*a*c*d)*log(abs(2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) - 1/2*(3*b^2*c
^2*log(abs(b)) - 4*a*b*c*d*log(abs(b)) + 4*b^2*c^2 - 8*a*b*c*d + 4*a^2*d^2
)*sgn(x)/(a^(5/2)*b) + 2*(sqrt(a)*b^2*c^2 - 2*a^(3/2)*b*c*d + a^(5/2)*d^2)
/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,(4\,a\,d-3\,b\,c)}{a^{5/2}} - \frac{\frac{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{a}-\frac{\left(a+\frac{b}{x}\right)\,(2\,a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2)}{a^2}}{b\,\left(a+\frac{b}{x}\right)^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

input
```
int((c + d/x)^2/(a + b/x)^(3/2),x)
```

output
```
(c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d)
)/a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.05

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{16\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)abcd-12\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2c^2+8\sqrt{a}\,\cdots}{\cdots}$$

input
```
int((c+d/x)^2/(a+b/x)^(3/2),x)
```

output
```
(16*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a
*b*c*d - 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sq
rt(b))*b**2*c**2 + 8*sqrt(a)*sqrt(a*x + b)*a**2*d**2 - 16*sqrt(a)*sqrt(a*x
 + b)*a*b*c*d + 9*sqrt(a)*sqrt(a*x + b)*b**2*c**2 + 8*sqrt(x)*a**3*d**2 +
4*sqrt(x)*a**2*b*c**2*x - 16*sqrt(x)*a**2*b*c*d + 12*sqrt(x)*a*b**2*c**2)/
(4*sqrt(a*x + b)*a**3*b)
```

3.34 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2(bc-ad)}{a^2\sqrt{a+\frac{b}{x}}}+\frac{c\sqrt{a+\frac{b}{x}}\,x}{a^2}-\frac{(3bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
2*(-a*d+b*c)/a^2/(a+b/x)^(1/2)+c*(a+b/x)^(1/2)*x/a^2-(-2*a*d+3*b*c)*arctan
h((a+b/x)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x(3bc-2ad+acx)}{a^2(b+ax)}+\frac{(-3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b*c - 2*a*d + a*c*x))/(a^2*(b + a*x)) + ((-3*b*c + 2*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {899, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(3bc-2ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(3bc-2ad)\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)(3bc-2ad)}{2a}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(c + d/x)/(a + b/x)^(3/2),x]
```

output
```
(c*x)/(a*Sqrt[a + b/x]) + ((3*b*c - 2*a*d)*(2/(a*Sqrt[a + b/x]) - (2*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.09

method	result
risch	$\frac{c(ax+b)}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{3bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}-\frac{4(ad-bc)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{2a^2x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3bd\,x^2-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b^2c\,x^2-4a^{\frac{7}{2}}\sqrt{x(ax+b)}\,d\,x^2+6a^{\frac{5}{2}}\sqrt{x(ax+b)}\,bc\,x^2\right.}{}$

input
```
int((c+1/x*d)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^2*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1
/2)+(a*x^2+b*x)^(1/2))-3*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(
1/2)-4*(a*d-b*c)/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1
/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.83

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[-\frac{(3\,b^2c-2\,abd+(3\,abc-2\,a^2d)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(a^2cx^2+(3\right.}{2\,(a^4x+a^3b)}$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sq
rt((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d
)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (a^2*c*x^2
+ (3*a*b*c - 2*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(66) = 132$.

Time = 18.04 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.95

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right)$$
$$+d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right.$$
$$\left.-\frac{a^2b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

input
```
integrate((c+d/x)/(a+b/x)**(3/2),x)
```

output
```
c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x
/b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqr
t(1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x
 + a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2
)*b) - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1
 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{2}\,c\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) - d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}a}\right)$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3)
 + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(
sqrt(a + b/x)*a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(66) = 132$.

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.96

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,bc\log\left(|b|\right)-2\,ad\log\left(|b|\right)+4\,bc-4\,ad)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{\sqrt{ax^2+bx}c}{a^2\mathrm{sgn}\left(x\right)} + \frac{(3\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} + \frac{2\left(\sqrt{a}b^2c-a^{\frac{3}{2}}bd\right)}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*c*log(abs(b)) - 2*a*d*log(abs(b)) + 4*b*c - 4*a*d)*sgn(x)/a^(5/2
) + sqrt(a*x^2 + b*x)*c/(a^2*sgn(x)) + 1/2*(3*b*c - 2*a*d)*log(abs(2*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + 2*(sqrt(a)*b^2
*c - a^(3/2)*b*d)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^3*sgn(x
))
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}} + \frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int((c + d/x)/(a + b/x)^(3/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
 (2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a +
b/x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{8\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)ad-12\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)bc-8\sqrt{a}\,\sqrt{ax\ldots}}{4\sqrt{ax+b}\,a^{3}}$$

input
```
int((c+d/x)/(a+b/x)^(3/2),x)
```

output
```
(8*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*
d - 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*b*c - 8*sqrt(a)*sqrt(a*x + b)*a*d + 9*sqrt(a)*sqrt(a*x + b)*b*c + 4*sqrt
(x)*a**2*c*x - 8*sqrt(x)*a**2*d + 12*sqrt(x)*a*b*c)/(4*sqrt(a*x + b)*a**3)
```

3.35 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}-\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/x)^(1/2)-3*b*arctanh((a+b/x)^(1/2)/a^(1/2))
/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(3b+ax)}{a^2(b+ax)}-\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(a + b/x)^(-3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3b\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(a + b/x)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b/x]) + (3*b*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

method	result
risch	$\frac{ax+b}{a^{2}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{5}{2}}}+\frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{a^{3}\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}x^{2}+3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b\,x^{2}+4a^{\frac{3}{2}}\left(x(ax+b)\right)^{\frac{3}{2}}-12\sqrt{x(ax+b)}\,a^{\frac{3}{2}}bx+6\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}}{2\sqrt{a}}\right.}{2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,(ax+b)^{2}}$

input
```
int(1/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/2*b/a^(5/2)*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))+2*b/a^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.68

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\left[\frac{3\left(abx+b^{2}\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^{2}x^{2}+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^{4}x+a^{3}b\right)},\ \frac{3\left(abx+b^{2}\right)}{}\right.$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
b) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x +
 b^2)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (a^2*x^2 +
 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)**(3/2),x)
```

output
```
x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b
+ 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*
log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,b\log\left(|b|\right)+4\,b)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a^2\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*log(abs(b)) + 4*b)*sgn(x)/a^(5/2) + 3/2*b*log(abs(2*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a^
2*sgn(x)) + 2*b^2/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^(5/2)*s
gn(x))
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,x\,\left(\frac{a\,x}{b}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(a + b/x)^(3/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x
)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{-12\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b+9\sqrt{a}\,\sqrt{ax+b}\,b+4\sqrt{x}\,a^2x+12\sqrt{x}\,ab}{4\sqrt{ax+b}\,a^3}$$

input
```
int(1/(a+b/x)^(3/2),x)
```

output
```
( - 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*b + 9*sqrt(a)*sqrt(a*x + b)*b + 4*sqrt(x)*a**2*x + 12*sqrt(x)*a*b)/(4*sq
rt(a*x + b)*a**3)
```

3.36 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}-\frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

output
```
b*(-a*d+3*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(1/2)+2*d^(5/2
)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(3/2)-(2*a
*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^2
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-3b^2c+a^2dx+ab(d-cx))}{a^2(-bc+ad)(b+ax)}+\frac{2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}}{c^2}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-3*b^2*c + a^2*d*x + a*b*(d - c*x)))/(a^2*(-(b*c) + a
*d)*(b + a*x)) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]
])/(b*c - a*d)^(3/2) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^
(5/2))/c^2
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {899, 114, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(3bc+2ad+\frac{3bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3bc+2ad+\frac{3bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{169}$$

$$\frac{\frac{2\int \frac{\left(\frac{bd(3bc-ad)}{x}+(bc-ad)(3bc+2ad)\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(\frac{bd(3bc-ad)}{x}+(bc-ad)(3bc+2ad)\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{174}$$

$$\frac{\frac{\frac{2a^2d^3\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{(bc-ad)(2ad+3bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{4a^2d^3\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{2(bc-ad)(2ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{4a^2d^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)(2ad+3bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(3bc-ad)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

output
```
x/(a*c*Sqrt[a + b/x]) + ((2*b*(3*b*c - a*d))/(a*(b*c - a*d)*Sqrt[a + b/x])
 + ((4*a^2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqr
t[b*c - a*d]) - (2*(b*c - a*d)*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[
a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(127) = 254$.

Time = 0.63 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.02

method	result
risch	$\frac{ax+b}{a^2c\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+3bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}+\frac{4c\,b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{(ad-bc)a\left(x+\frac{b}{a}\right)}+\frac{2a^2d^3\ln\left(\frac{\frac{2(ad-bc)d}{c^2}-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}+2\sqrt{\frac{(ad-bc)d}{c^2}}\cdots}{x+\cdots}\right)}{c^2(ad-bc)\sqrt{\cdots}}\cdots\right)}{2a^2cx\sqrt{\frac{ax+b}{x}}\cdots}$
default	$\frac{\left(2\sqrt{x(ax+b)}\,a^{\frac{7}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,c^2d\,x^2-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,b\,c^3x^2-2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}\,a^4c\,d^2x^2-\ln\left(2\sqrt{\cdots}\right.\cdots\right.}{\cdots}$

input
```
int(1/(a+b/x)^(3/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^2/c*((2*a*d+3*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+4*c*b^2/(a*d-b*c)/a/(x+b/a)*(a*(x+b/a
)^2-b*(x+b/a))^(1/2)+2/c^2*a^2*d^3/(a*d-b*c)/((a*d-b*c)*d/c^2)^(1/2)*ln((2
*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1
/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))/x/((a*
x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1045, normalized size of antiderivative = 7.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^
4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b
*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2
*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a
^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2
*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arc
tan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (a^4*d^2*x + a^3*b*d^2)*sqrt
(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)
/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*
a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d +
(a^4*b*c^3 - a^5*c^2*d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)) + (3*b^3*c^2 - a*b^2*c*
d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*log(2*a
*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2 + (
3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d
+ (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a
*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x
+ b)/x)/(a*x + b)) + 2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(
sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (...
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)}\,dx$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

output
```
Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 3000, normalized size of antiderivative = 20.41

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)),x)
```

output
```
(atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 -
66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5
*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10
) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2
- 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a
^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c
^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10
*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/
(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5
*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c
^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2
*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a
*d - b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b
^7*c^10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*
d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56
*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a
^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d -
b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^
5 - 96*a^7*b^7*c^6*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10
*b^4*c^3*d^9 + 8*a^11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b...
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.45

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = \frac{4\sqrt{d}\,\sqrt{ax+b}\,\sqrt{ad-bc}\log\left(\sqrt{c}\,\sqrt{ax+b}-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc+\cdots}\right.}{}$$

input
```
int(1/(a+b/x)^(3/2)/(c+d/x),x)
```

output
```
(4*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(
2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a)
)*a**3*d**2 + 4*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x
 + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sq
rt(c)*sqrt(a))*a**3*d**2 - 4*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(2*s
qrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c
*x + 2*a*d)*a**3*d**2 - 8*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(
x)*sqrt(a))/sqrt(b))*a**3*d**3 + 4*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b
) + sqrt(x)*sqrt(a))/sqrt(b))*a**2*b*c*d**2 + 16*sqrt(a)*sqrt(a*x + b)*log
((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b**2*c**2*d - 12*sqrt(a)*sqr
t(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**3*c**3 + sqrt
(a)*sqrt(a*x + b)*a**2*b*c*d**2 - 10*sqrt(a)*sqrt(a*x + b)*a*b**2*c**2*d +
 9*sqrt(a)*sqrt(a*x + b)*b**3*c**3 + 4*sqrt(x)*a**4*c*d**2*x - 8*sqrt(x)*a
**3*b*c**2*d*x + 4*sqrt(x)*a**3*b*c*d**2 + 4*sqrt(x)*a**2*b**2*c**3*x - 16
*sqrt(x)*a**2*b**2*c**2*d + 12*sqrt(x)*a*b**3*c**3)/(4*sqrt(a*x + b)*a**3*
c**2*(a**2*d**2 - 2*a*b*c*d + b**2*c**2))
```

3.37 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{b(3b^2c^2-2abcd+2a^2d^2)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}} - \frac{(3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}$$

output
```
b*(2*a^2*d^2-2*a*b*c*d+3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(1/2)+d*(-2
*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(1/2)/(c+d/x)+x/a/c/(a+b/x)^(1/2)/(c+d/
x)+d^(5/2)*(-4*a*d+7*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c
^3/(-a*d+b*c)^(5/2)-(4*a*d+3*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c
^3
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}\,x(3b^3c^2(d+cx)+a^3d^2x(2d+cx)+a^2bd(2d^2-cdx-2c^2x^2)+ab^2c(-2d^2-cdx+c^2x^2))}{a^2(bc-ad)^2(b+ax)(d+cx)}+\frac{d^{5/2}(7bc-}{}}{c^3}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(3*b^3*c^2*(d + c*x) + a^3*d^2*x*(2*d + c*x) + a^2*b*d
*(2*d^2 - c*d*x - 2*c^2*x^2) + a*b^2*c*(-2*d^2 - c*d*x + c^2*x^2)))/(a^2*(
b*c - a*d)^2*(b + a*x)*(d + c*x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[
d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - ((3*b*c + 4*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/c^3
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 114, 27, 168, 25, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \ 899$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \ 114$$

$$\frac{\int \frac{\left(3bc+4ad+\frac{5bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(3bc+4ad+\frac{5bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 168$$

$$\frac{\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{3bd(bc-2ad)}{x}+(bc-ad)(3bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(\frac{3bd(bc-2ad)}{x}+(bc-ad)(3bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 169$$

$$\frac{\frac{\frac{2\int \frac{\left((3bc+4ad)(bc-ad)^2+\frac{bd\left(3b^2c^2-2abdc+2a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{\left((3bc+4ad)(bc-ad)^2+\frac{bd\left(3b^2c^2-2abdc+2a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \ 174$$

$$\frac{\frac{\frac{\frac{a^2d^3(7bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{(bc-ad)^2(4ad+3bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{2ac}{x}}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{2a^2d^3(7bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2(bc-ad)^2(4ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{2ac}{x}}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(4ad+3bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a^2d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{2ac}{x}}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{2a^2d^{5/2}(7bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^2(4ad+3bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{2ac}{x}}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

input
```
Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

output
```
x/(a*c*Sqrt[a + b/x]*(c + d/x)) + ((2*d*(b*c - 2*a*d))/(c*(b*c - a*d)*Sqrt
[a + b/x]*(c + d/x)) + ((2*b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a*(b*c
- a*d)*Sqrt[a + b/x]) + ((2*a^2*d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)^2*(3*b
*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/
(c*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(202) = 404$.

Time = 0.65 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.38

method	result
risch	$\frac{ax+b}{a^2c^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^3}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^2}+\frac{2b^3\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3(ad-bc)^2\left(x+\frac{b}{a}\right)}+\frac{d^3\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{c^3(ad-bc)^2\left(x+\frac{d}{c}\right)}\right.}{}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(3/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-2/a^(3/2)/c^3*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b+2/a^3*b^3/(a*d-b*c)^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+1/c^3*
d^3/(a*d-b*c)^2/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)
*d/c^2)^(1/2)-2*a/c^4*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b
*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2
-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+7/2/c^3*d^3/(a
*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1
/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*
d-b*c)*d/c^2)^(1/2))/(x+1/c*d))*b)/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(202) = 404$.

Time = 0.71 (sec) , antiderivative size = 2291, normalized size of antiderivative = 10.23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
[1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*
a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*
c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a
)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^
4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4
*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-
d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(
(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c
^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d
^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d
^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^
3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3
*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^
2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(sqrt(
d/(b*c - a*d))*sqrt((a*x + b)/x)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2
*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^
2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*
b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 -
 a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*...
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}(cx+d)^2}\,dx$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2/((a + b/x)**(3/2)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 4274, normalized size of antiderivative = 19.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx=\text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
```

output
```
((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b
*c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3
*b^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/
x)^(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^
11*c^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i
- a^10*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*
1694i - a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x
)^(1/2)*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*
(a + b/x)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*
c^4*d^10*(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i +
a^11*b^13*c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^
7*c^7*d^7 - 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d
^10 + 140*a^4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 169
4*a^2*b^10*c^10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^
3*b^14*c^14 + 189*a^4*b^13*c^13*d))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2))
 - (atan((((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*
a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^
9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^
12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768
*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 6...
```

Reduce [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 2294, normalized size of antiderivative = 10.24

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)
```

output

```
(16*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a
))*a**5*c*d**4*x + 16*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sq
rt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt
(x)*sqrt(c)*sqrt(a))*a**5*d**5 - 32*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*
log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d
 + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**4*b*c**2*d**3*x - 32*sqrt(d)*sqrt(a*
x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*
sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**4*b*c*d**4 +
7*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2
*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))
*a**3*b**2*c**3*d**2*x + 7*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(
c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
 sqrt(x)*sqrt(c)*sqrt(a))*a**3*b**2*c**2*d**3 + 16*sqrt(d)*sqrt(a*x + b)*s
qrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
 - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*c*d**4*x + 16*sqrt(
d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*d
**5 - 32*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) +
 sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c...
```

3.38 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{3d^{5/2}\left(21b^2c^2-24abcd+8a^2d^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}} - \frac{3(bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}$$

output
```
3/4*b*(-a*d+2*b*c)*(4*a^2*d^2-a*b*c*d+2*b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/(a+b
/x)^(1/2)+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(1/2)/(c+d/x)^2+1/
4*d*(12*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)^(1/2)/(c+
d/x)+x/a/c/(a+b/x)^(1/2)/(c+d/x)^2+3/4*d^(5/2)*(8*a^2*d^2-24*a*b*c*d+21*b^
2*c^2)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(7/2)
-3*(2*a*d+b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)/c^4
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x\left(-12b^4c^3(d+cx)^2-4ab^3c^2(-3d+cx)(d+cx)^2+2a^4d^3x\left(6d^2+9cdx+2c^2x^2\right)+a^3bd^2\left(12d^3-9cd^2x-\right.\right.}{a^2(-bc+ad)^3(b+ax)(d+cx)^2}}{}$$

input
```
Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-12*b^4*c^3*(d + c*x)^2 - 4*a*b^3*c^2*(-3*d + c*x)*(d
 + c*x)^2 + 2*a^4*d^3*x*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + a^3*b*d^2*(12*d^3
- 9*c*d^2*x - 37*c^2*d*x^2 - 12*c^3*x^3) + a^2*b^2*c*d*(-27*d^3 - 29*c*d^2
*x + 12*c^2*d*x^2 + 12*c^3*x^3)))/(a^2*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x
)^2) + (3*d^(5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sq
rt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (12*(b*c + 2*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2))/(4*c^4)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {899, 114, 27, 168, 25, 168, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(\frac{7bd}{x}+3(bc+2ad)\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(\frac{7bd}{x}+3(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 168

$$\frac{\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{5bd(2bc-3ad)}{x}+6(bc-ad)(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(\frac{5bd(2bc-3ad)}{x}+6(bc-ad)(bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 168

$$\frac{\frac{\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{3\left(4(bc+2ad)(bc-ad)^2+\frac{bd\left(4b^2c^2-21abdc+12a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{\left(4(bc+2ad)(bc-ad)^2+\frac{bd\left(4b^2c^2-21abdc+12a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\big\downarrow$ 169

$$\frac{\frac{3\left(\frac{2\int \frac{\left(4(bc+2ad)(bc-ad)^3+\frac{bd(2bc-ad)\left(2b^2c^2-abdc+4a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-\cdots}{c\sqrt{a+\frac{b}{x}}\left(c+\cdots\right.}\cdots$$

$$2ac$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{3\left(\frac{\int \frac{\left(4(bc+2ad)(bc-ad)^3+\frac{bd(2bc-ad)\left(2b^2c^2-abdc+4a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3\cdots}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\cdots\right.}\cdots$$

$$2ac$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$\big\downarrow$ 174

$$\frac{\frac{\frac{3\left(\frac{\frac{a^2d^3\left(8a^2d^2-24abcd+21b^2c^2\right)\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}+\frac{4(bc-ad)^3(2ad+bc)\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{3\left(\frac{\frac{2a^2d^3\left(8a^2d^2-24abcd+21b^2c^2\right)\int\frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{8(bc-ad)^3(2ad+bc)\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abcd\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{3\left(\frac{\frac{8(2ad+bc)(bc-ad)^3\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}+\frac{2a^2d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21abc\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{3\left(\frac{\frac{2a^2d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^3(2ad+bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\right)}{2c(bc-ad)}+\frac{d\left(12a^2d^2-21\cdots\right.}{c\sqrt{a+\frac{b}{x}}\left(c+\cdots\right.}}{2c(bc-ad)}}{2ac}$$

$$\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

input
```
Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

output
```
x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + ((d*(2*b*c - 3*a*d))/(c*(b*c - a*d)*Sq
rt[a + b/x]*(c + d/x)^2) + ((d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(c*(
b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + (3*((2*b*(2*b*c - a*d)*(2*b^2*c^2 -
a*b*c*d + 4*a^2*d^2))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((2*a^2*d^(5/2)*(21*
b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
- a*d]])/(c*Sqrt[b*c - a*d]) - (8*(b*c - a*d)^3*(b*c + 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d))))/(2*c*(b*c - a*d)))/(2*c
*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(288) = 576$.

Time = 0.69 (sec) , antiderivative size = 983, normalized size of antiderivative = 3.07

method	result
risch	$\frac{ax+b}{a^2c^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)d}{a^{\frac{3}{2}}c^4}-\frac{3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)b}{2a^{\frac{5}{2}}c^3}-\frac{2b^4\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3(ad-bc)^3\left(x+\frac{b}{a}\right)}-\frac{d^4\sqrt{a\left(x+\frac{d}{c}\right)^2-\frac{(2ad-bc)\left(x+\frac{d}{c}\right)}{c}}}{2c^5(ad-bc)^2\left(x+\frac{d}{c}\right)^2}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(3/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(3/2)/c^4*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-3/2/a^(5/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b-2/a^3*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^
5*d^4/(a*d-b*c)^2/(x+1/c*d)^2*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-
b*c)*d/c^2)^(1/2)+5/2*a/c^4*d^4/(a*d-b*c)^3/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*
d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-17/4/c^3*d^3/(a*d-b*c)^3/(x+1/c*
d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2
/c^5*d^5/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-
b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x
+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+19/2*a/c^4*d^4/(a*d-b*c)^3/((a*
d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-
b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^
(1/2))/(x+1/c*d))*b-63/8/c^3*d^3/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2
*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1
/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))*b^2+1/2
*a/c^5*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*
d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*
(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+
b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(288) = 576$.

Time = 1.59 (sec) , antiderivative size = 4063, normalized size of antiderivative = 12.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(288) = 576$.

Time $= 0.25$ (sec) , antiderivative size $= 1089$, normalized size of antiderivative $= 3.40$

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
2*sqrt(a)*b^5/((a^3*b^3*c^3*sgn(x) - 3*a^4*b^2*c^2*d*sgn(x) + 3*a^5*b*c*d^
2*sgn(x) - a^6*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))
- 1/4*(63*a^(5/2)*b^2*c^2*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 72*a
^(7/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(9/2)*d^5*arct
an(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*b^4*c^4*log(abs(
b)) - 6*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 18*sqrt(b*c*d - a*d^
2)*a^2*b^2*c^2*d^2*log(abs(b)) + 30*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3*log(ab
s(b)) - 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) + 8*sqrt(b*c*d - a*d^2)
*b^4*c^4 + 17*sqrt(b*c*d - a*d^2)*a^3*b*c*d^3 - 10*sqrt(b*c*d - a*d^2)*a^4
*d^4)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(5/2)*b^3*c^7 - 3*sqrt(b*c*d - a*d^2)*
a^(7/2)*b^2*c^6*d + 3*sqrt(b*c*d - a*d^2)*a^(9/2)*b*c^5*d^2 - sqrt(b*c*d -
 a*d^2)*a^(11/2)*c^4*d^3) - 3/4*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5
)*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b^3*c^7*sgn(x) - 3*a*b^2*c^6*d*sgn(x) + 3*a^2*b*c^5*d^2*sgn(x) - a^
3*c^4*d^3*sgn(x))*sqrt(b*c*d - a*d^2)) - 1/4*(17*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^3*b^2*c^3*d^3 - 48*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a*b*c^2*d^4 +
24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^5 + 11*(sqrt(a)*x - sqrt(a*x^
2 + b*x))^2*sqrt(a)*b^2*c^2*d^4 - 72*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(
3/2)*b*c*d^5 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^(5/2)*d^6 + 15*(sqrt
(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^4 - 76*(sqrt(a)*x - sqrt(a*x^2 + b...
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 8936, normalized size of antiderivative = 27.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)
```

output
```
((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*
b^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(
a*d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^
3 - 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)
^2*(24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72
*a^3*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(
3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d
^2*(a + b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
+ (atan((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25
*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*
b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 -
65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a
^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^1
5*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 588
16512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*
c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^
2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^1
8*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 944
9472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13
*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 +...
```

Reduce [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 4543, normalized size of antiderivative = 14.20

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)
```

output
```
(192*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**6*c**2*d**5*x**2 + 384*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sq
rt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c
) + sqrt(x)*sqrt(c)*sqrt(a))*a**6*c*d**6*x + 192*sqrt(d)*sqrt(a*x + b)*sqr
t(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d -
 b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**6*d**7 - 648*sqrt(d)*sq
rt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*b*c**3
*d**4*x**2 - 1296*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a
*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*
sqrt(c)*sqrt(a))*a**5*b*c**2*d**5*x - 648*sqrt(d)*sqrt(a*x + b)*sqrt(a*d -
 b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) -
 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*b*c*d**6 + 720*sqrt(d)*sqrt(
a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**4*b**2*c**4
*d**3*x**2 + 1440*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a
*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*
sqrt(c)*sqrt(a))*a**4*b**2*c**3*d**4*x + 720*sqrt(d)*sqrt(a*x + b)*sqrt(a*
d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - ...
```

3.39 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{2(bc-ad)^3}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2(bc-ad)^2(2bc+ad)}{a^3b^2\sqrt{a+\frac{b}{x}}}+\frac{c^3\sqrt{a+\frac{b}{x}}\,x}{a^3}-\frac{c^2(5bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
2/3*(-a*d+b*c)^3/a^2/b^2/(a+b/x)^(3/2)+2*(-a*d+b*c)^2*(a*d+2*b*c)/a^3/b^2/
(a+b/x)^(1/2)+c^3*(a+b/x)^(1/2)*x/a^3-c^2*(-6*a*d+5*b*c)*arctanh((a+b/x)^(
1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^4c^3+4a^4d^3x+3a^2b^2c^2x(-8d+cx)+6a^3bd^2(d+cx)+2ab^3c^2(-9d+10}{3a^3b^2(b+ax)^2}$$
$$+\frac{c^2(-5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^4*c^3 + 4*a^4*d^3*x + 3*a^2*b^2*c^2*x*(-8*d + c*x)
+ 6*a^3*b*d^2*(d + c*x) + 2*a*b^3*c^2*(-9*d + 10*c*x)))/(3*a^3*b^2*(b + a*
x)^2) + (c^2*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {899, 109, 27, 162, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 899}$$
$$-\int \frac{\left(c+\frac{d}{x}\right)^3x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 109}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(5bc-6ad)+\frac{d(bc-2ad)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\left(c+\frac{d}{x}\right)\left(c(5bc-6ad)+\frac{d(bc-2ad)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \textbf{162}$$

$$\frac{\frac{c^2(5bc-6ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a^2}+\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{\frac{2c^2(5bc-6ad)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a^2b}+\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{2\left(\frac{3b(bc-ad)\left(-2a^2d^2-abcd+5b^2c^2\right)}{x}+2a(bc-ad)\left(-2a^2d^2-5abcd+10b^2c^2\right)\right)}{3a^2b^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2c^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(5bc-6ad)}{a^{5/2}}}{2a}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

output
```
(c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((2*(2*a*(b*c - a*d)*(10*b^2*c^2 -
 5*a*b*c*d - 2*a^2*d^2) + (3*b*(b*c - a*d)*(5*b^2*c^2 - a*b*c*d - 2*a^2*d^
2))/x))/(3*a^2*b^2*(a + b/x)^(3/2)) - (2*c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/a^(5/2))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(111) = 222$.

Time = 0.37 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.42

method	result
risch	$\frac{c^3(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{\left(2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3\right)\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{a^2}-\frac{5c^3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}-\cdots\right)}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	Expression too large to display

input
```
int((c+1/x*d)^3/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^3/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*((2*a^3*d^3-6*a^2*b*c*d^2+6*a*b^
2*c^2*d-2*b^3*c^3)/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*
a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))-5*c^3*b*ln((1/2*b+a*x)/a^(1/2
)+(a*x^2+b*x)^(1/2))/a^(1/2)+6*a^(1/2)*c^2*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2
+b*x)^(1/2))+12*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(
x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.90

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^5c^3-6\,ab^4c^2d+(5\,a^2b^3c^3-6\,a^3b^2c^2d)x^2+2\,(5\,ab^4c^3-6\,a^2b^3c^2d)x)\sqrt{a}\log\left(2\right.}{}\right.$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^
2 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*s
qrt((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*
b^2*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^
2*d + 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*
b^4), 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d
)*x^2 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*x*sq
rt((a*x + b)/x)/(a*x + b)) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a
^3*b^2*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3
*c^2*d + 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a
^4*b^4)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

output
```
Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.82

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{1}{6}\,c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-c^2d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2}{3}\,d^3\left(\frac{3}{\sqrt{a+\frac{b}{x}}b^2}-\frac{a}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}\right)+\frac{2\,cd^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2
)*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt
(a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2
/3*d^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b
/x)^(3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(111) = 222$.

Time = 0.16 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.43

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{ax^2+bx}c^3}{a^3\mathrm{sgn}\left(x\right)} + \frac{\left(5\,bc^3-6\,ac^2d\right)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} - \frac{\left(15\,b^3c^3\log\left(\left|b\right|\right)-18\,ab^2c^2d\log\left(\left|b\right|\right)+28\,b^3c^3-48\,ab^2c^2d+12\,a^2bcd^2+8\,a^3d^3\right)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b^2} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^3-18\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bc^2d+9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3cd^2+15\left(\sqrt{\right.\right.}{}$$

input
```
integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^3/(a^3*sgn(x)) + 1/2*(5*b*c^3 - 6*a*c^2*d)*log(abs(2*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^
3*c^3*log(abs(b)) - 18*a*b^2*c^2*d*log(abs(b)) + 28*b^3*c^3 - 48*a*b^2*c^2
*d + 12*a^2*b*c*d^2 + 8*a^3*d^3)*sgn(x)/(a^(7/2)*b^2) + 2/3*(9*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^2*a*b^2*c^3 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^
2*b*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*c*d^2 + 15*(sqrt(a)*x
- sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^3 - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x))*
a^(3/2)*b^2*c^2*d + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*c*d^2 + 3*
(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(7/2)*d^3 + 7*b^4*c^3 - 12*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 + 2*a^3*b*d^3)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)
+ b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.55

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3a}+\frac{\left(a+\frac{b}{x}\right)^2\left(2a^3d^3-6ab^2c^2d+5b^3c^3\right)}{a^3}-\frac{2\left(a+\frac{b}{x}\right)\left(4a^3d^3-3a^2bcd^2-6a\right.}{3a^2}}{b^2\left(a+\frac{b}{x}\right)^{5/2}-a\,b^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\left(6\,a\,d-5\,b\,c\right)}{a^{7/2}}$$

input
```
int((c + d/x)^3/(a + b/x)^(5/2),x)
```

output
```
((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x
)^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3
 + 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/
2) - a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d -
 5*b*c))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.05

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{36\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a^2b^2c^2dx - 30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,b^3c^3x\cdots}{}$$

input
```
int((c+d/x)^3/(a+b/x)^(5/2),x)
```

output
```
(36*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a
**2*b**2*c**2*d*x - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*
sqrt(a))/sqrt(b))*a*b**3*c**3*x + 36*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x +
 b) + sqrt(x)*sqrt(a))/sqrt(b))*a*b**3*c**2*d - 30*sqrt(a)*sqrt(a*x + b)*l
og((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**4*c**3 - 8*sqrt(a)*sqrt(a
*x + b)*a**4*d**3*x + 12*sqrt(a)*sqrt(a*x + b)*a**3*b*c*d**2*x - 8*sqrt(a)
*sqrt(a*x + b)*a**3*b*d**3 + 12*sqrt(a)*sqrt(a*x + b)*a**2*b**2*c*d**2 - 5
*sqrt(a)*sqrt(a*x + b)*a*b**3*c**3*x - 5*sqrt(a)*sqrt(a*x + b)*b**4*c**3 +
 8*sqrt(x)*a**5*d**3*x + 12*sqrt(x)*a**4*b*c*d**2*x + 12*sqrt(x)*a**4*b*d*
*3 + 6*sqrt(x)*a**3*b**2*c**3*x**2 - 48*sqrt(x)*a**3*b**2*c**2*d*x + 40*sq
rt(x)*a**2*b**3*c**3*x - 36*sqrt(x)*a**2*b**3*c**2*d + 30*sqrt(x)*a*b**4*c
**3)/(6*sqrt(a*x + b)*a**4*b**2*(a*x + b))
```

3.40 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}} \, dx = \frac{2(bc-ad)^2}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}} + \frac{4c(bc-ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{c(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
2/3*(-a*d+b*c)^2/a^2/b/(a+b/x)^(3/2)+4*c*(-a*d+b*c)/a^3/(a+b/x)^(1/2)+c^2*
(a+b/x)^(1/2)*x/a^3-c*(-4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2
)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,x\left(15b^3c^2+2a^3d^2x+a^2bcx(-16d+3cx)+4ab^2c(-3d+5cx)\right)}{3a^3b(b+ax)^2} + \frac{c(-5bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^3*c^2 + 2*a^3*d^2*x + a^2*b*c*x*(-16*d + 3*c*x)) + 4
*a*b^2*c*(-3*d + 5*c*x)))/(3*a^3*b*(b + a*x)^2) + (c*(-5*b*c + 4*a*d)*ArcT
anh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {899, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)^2 x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{\int -\frac{\left(c(5bc-4ad)-\frac{2ad^2}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(c(5bc-4ad)-\frac{2ad^2}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{c(5bc-4ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{c(5bc-4ad)\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{c(5bc-4ad)\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{c\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)(5bc-4ad)}{a}+\frac{2\left(\frac{c(5bc-4ad)}{a}+\frac{2ad^2}{b}\right)}{3\left(a+\frac{b}{x}\right)^{3/2}}}{2a}+\frac{c^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

output
```
(c^2*x)/(a*(a + b/x)^(3/2)) + ((2*((2*a*d^2)/b + (c*(5*b*c - 4*a*d))/a))/(
3*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a*d)*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[
Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/a)/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(97) = 194$.

Time = 0.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.59

method	result
risch	$\frac{c^2(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}} + \frac{\left(\frac{2\left(2a^2d^2-8abcd+6b^2c^2\right)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{ab\left(x+\frac{b}{a}\right)} - \frac{5b\,c^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}} + 4\sqrt{a}\,cd\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right) - \frac{2\left(a^2d^2 \cdots\right.}{\cdots}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}\cdots}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-24\sqrt{x(ax+b)}\,a^{\frac{9}{2}}cd\,x^3+30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}b\,c^2x^3+12\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^4bcd\,x^3-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right.\cdots\right.}{\cdots}$

input
```
int((c+1/x*d)^2/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
c^2/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(2*(2*a^2*d^2-8*a*b*c*d+6*b^2*c^
2)/a/b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-5*b*c^2*ln((1/2*b+a*x)/a^(1/2
)+(a*x^2+b*x)^(1/2))/a^(1/2)+4*a^(1/2)*c*d*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b
*x)^(1/2))-2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)
^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)))/x/((
a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(97) = 194$.

Time = 0.11 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.71

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^4c^2-4\,ab^3cd+(5\,a^2b^2c^2-4\,a^3bcd)x^2+2\,(5\,ab^3c^2-4\,a^2b^2cd)x)\sqrt{a}\log\left(2\,ax\right.}{6\,(a}\right.$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*
(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4
*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x
^2 + 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*
c^2 - 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b
^2*c^2 - 8*a^3*b*c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*s
qrt((a*x + b)/x))/(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

Sympy [F]

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

output
```
Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{1}{6}\,c^2\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{2}{3}\,cd\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2\,d^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2
)*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sq
rt(a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) +
 2/3*d^2/((a + b/x)^(3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(97) = 194$.

Time = 0.16 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.27

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{ax^2+bx}c^2}{a^3\mathrm{sgn}(x)} + \frac{(5\,bc^2-4\,acd)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}(x)} - \frac{(15\,b^2c^2\log(|b|)-12\,abcd\log(|b|)+28\,b^2c^2-32\,abcd+4\,a^2d^2)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}b} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c^2-12\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bcd+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^3d^2+15\left(\sqrt{a}\right.\right.}{3\left(\left(\sqrt{a}x-\sqrt{a}\right.\right.}$$

input
```
integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(a*x^2 + b*x)*c^2/(a^3*sgn(x)) + 1/2*(5*b*c^2 - 4*a*c*d)*log(abs(2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) - 1/6*(15*b^2*
c^2*log(abs(b)) - 12*a*b*c*d*log(abs(b)) + 28*b^2*c^2 - 32*a*b*c*d + 4*a^2
*d^2)*sgn(x)/(a^(7/2)*b) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*
c^2 - 12*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d + 3*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^2*a^3*d^2 + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*
c^2 - 18*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*c*d + 3*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*a^(5/2)*b*d^2 + 7*b^4*c^2 - 8*a*b^3*c*d + a^2*b^2*d^2)/
(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2\left(a+\frac{b}{x}\right)(a^2\,d^2+4\,a\,b\,c\,d-5\,b^2\,c^2)}{3\,a^2}-\frac{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{3\,a}+\frac{b\left(a+\frac{b}{x}\right)^2(5\,b\,c^2-4\,a\,c\,d)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-a\,b\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4\,a\,d-5\,b\,c)}{a^{7/2}}$$

input
```
int((c + d/x)^2/(a + b/x)^(5/2),x)
```

output
```
((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(
a + b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*
(4*a*d - 5*b*c))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.74

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{24\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a^2bcdx - 30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,b^2c^2x +}{}$$

input
```
int((c+d/x)^2/(a+b/x)^(5/2),x)
```

output
```
(24*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a
**2*b*c*d*x - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a
))/sqrt(b))*a*b**2*c**2*x + 24*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) +
sqrt(x)*sqrt(a))/sqrt(b))*a*b**2*c*d - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(
a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**3*c**2 + 4*sqrt(a)*sqrt(a*x + b)*a
**3*d**2*x + 4*sqrt(a)*sqrt(a*x + b)*a**2*b*d**2 - 5*sqrt(a)*sqrt(a*x + b)
*a*b**2*c**2*x - 5*sqrt(a)*sqrt(a*x + b)*b**3*c**2 + 4*sqrt(x)*a**4*d**2*x
 + 6*sqrt(x)*a**3*b*c**2*x**2 - 32*sqrt(x)*a**3*b*c*d*x + 40*sqrt(x)*a**2*
b**2*c**2*x - 24*sqrt(x)*a**2*b**2*c*d + 30*sqrt(x)*a*b**3*c**2)/(6*sqrt(a
*x + b)*a**4*b*(a*x + b))
```

3.41 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{2(bc-ad)}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2(2bc-ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c\sqrt{a+\frac{b}{x}}x}{a^3} - \frac{(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
2/3*(-a*d+b*c)/a^2/(a+b/x)^(3/2)+2*(-a*d+2*b*c)/a^3/(a+b/x)^(1/2)+c*(a+b/x
)^(1/2)*x/a^3-(-2*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^2c+a^2x(-8d+3cx)+ab(-6d+20cx))}{3a^3(b+ax)^2} + \frac{(-5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2*c + a^2*x*(-8*d + 3*c*x) + a*b*(-6*d + 20*c*x)))/
(3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^
(7/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {899, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\left(c+\frac{d}{x}\right)x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(5bc-2ad)\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(5bc-2ad)\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{(5bc-2ad)\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(5bc-2ad)\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)(5bc-2ad)}{2a}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(c + d/x)/(a + b/x)^(5/2),x]
```

output
```
(c*x)/(a*(a + b/x)^(3/2)) + ((5*b*c - 2*a*d)*(2/(3*a*(a + b/x)^(3/2)) + (2
/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(89) = 178$.

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.47

method	result
risch	$\frac{c(ax+b)}{a^3\sqrt{\frac{ax+b}{x}}}+\frac{\left(2\sqrt{a}\,d\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)-\frac{5bc\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{\sqrt{a}}-\frac{4(2ad-3bc)\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a\left(x+\frac{b}{a}\right)}+\frac{2(ad-bc)b^2\left(\frac{2\sqrt{a\left(x+\right.}}{3b}\right.}{}\right.}{2a^3x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(6\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^4bd\,x^3-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b^2c\,x^3-12a^{\frac{9}{2}}\sqrt{x(ax+b)}\,d\,x^3+30a^{\frac{7}{2}}\sqrt{x(ax+b)}\,bc\right.}{}$

input
```
int((c+1/x*d)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*c*(a*x+b)/((a*x+b)/x)^(1/2)+1/2/a^3*(2*a^(1/2)*d*ln((1/2*b+a*x)/a^(1
/2)+(a*x^2+b*x)^(1/2))-5*b*c*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(
1/2)-4*(2*a*d-3*b*c)/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+2*(a*d-b*c)*b
^2/a^2*(2/3/b/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a
*(x+b/a)^2-b*(x+b/a))^(1/2)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.26

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^3c-2\,ab^2d+(5\,a^2bc-2\,a^3d)x^2+2\,(5\,ab^2c-2\,a^2bd)x)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\right.}{6\,(a^6x^2+2\,a^5bx}\right.$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
 2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*
(3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*
sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*
b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*
arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (3*a^3*c*x^3 + 4*(5*a^2*b
*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x
^2 + 2*a^5*b*x + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. $2(92) = 184$.

Time = 40.49 (sec) , antiderivative size = 1479, normalized size of antiderivative = 14.36

$$\int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c+d/x)/(a+b/x)**(5/2),x)
```

output
```
c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*
x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(
33/2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2
)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(s
qrt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a*
*(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**
3) + 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x*
*2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt
(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 4
5*a**14*b**3*x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)
) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a
**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/
2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sq
rt(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x + 6*a**(33/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.65

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2\,a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) - \frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*
a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
 + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(89) = 178.

Time = 0.15 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.51

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,bc\log(|b|)-6\,ad\log(|b|)+28\,bc-16\,ad)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{\sqrt{ax^2+bx}c}{a^3\mathrm{sgn}(x)} + \frac{(5\,bc-2\,ad)\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}(x)} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2c-6\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a^2bd+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3c-9\left(\sqrt{a}x\right.}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}(x)}$$

input
```
integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b*c*log(abs(b)) - 6*a*d*log(abs(b)) + 28*b*c - 16*a*d)*sgn(x)/a^(
7/2) + sqrt(a*x^2 + b*x)*c/(a^3*sgn(x)) + 1/2*(5*b*c - 2*a*d)*log(abs(2*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + 2/3*(9*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c - 6*(sqrt(a)*x - sqrt(a*x^2 + b*x))^
2*a^2*b*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c - 9*(sqrt(a)*
x - sqrt(a*x^2 + b*x))*a^(3/2)*b^2*d + 7*b^4*c - 4*a*b^3*d)/(((sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2\,d}{3\,a}+\frac{2\,d\,\left(a+\frac{b}{x}\right)}{a^2}}{\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\,c\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,\left(a+\frac{b}{x}\right)^{5/2}}$$

input
```
int((c + d/x)/(a + b/x)^(5/2),x)
```

output
```
(2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/
x))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2]
, 9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.18

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{12\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a^2dx - 30\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)abcx + 12\sqrt{}}{}$$

input
```
int((c+d/x)/(a+b/x)^(5/2),x)
```

output
```
(12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a
**2*d*x - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*a*b*c*x + 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sq
rt(a))/sqrt(b))*a*b*d - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt
(x)*sqrt(a))/sqrt(b))*b**2*c - 5*sqrt(a)*sqrt(a*x + b)*a*b*c*x - 5*sqrt(a)
*sqrt(a*x + b)*b**2*c + 6*sqrt(x)*a**3*c*x**2 - 16*sqrt(x)*a**3*d*x + 40*s
qrt(x)*a**2*b*c*x - 12*sqrt(x)*a**2*b*d + 30*sqrt(x)*a*b**2*c)/(6*sqrt(a*x
 + b)*a**4*(a*x + b))
```

3.42 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5b}{a^3\sqrt{a+\frac{b}{x}}} + \frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
5/3*b/a^2/(a+b/x)^(3/2)+5*b/a^3/(a+b/x)^(1/2)+x/a/(a+b/x)^(3/2)-5*b*arctan
h((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}} dx = \frac{\sqrt{a+\frac{b}{x}}x(15b^2+20abx+3a^2x^2)}{3a^3(b+ax)^2} - \frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(a + b/x)^(-5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2) - (5
*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(a + b/x)^(-5/2),x]
```

output
```
x/(a*(a + b/x)^(3/2)) + (5*b*(2/(3*a*(a + b/x)^(3/2)) + (2/(a*Sqrt[a + b/x
]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result
risch	$\frac{ax+b}{a^{3}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{7}{2}}}+\frac{14b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{4}\left(x+\frac{b}{a}\right)}-\frac{2b^{2}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{5}\left(x+\frac{b}{a}\right)^{2}}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}x^{3}+15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}b\,x^{3}+24(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}x-90\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b\,x^{2}+45\ln\left(\frac{2\sqrt{x(a}\right.\right.}{}$

input
```
int(1/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-5/2/a^(7/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))+14/3/a^4*b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-2/3/a^5*b
^2/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{15\,(a^2bx^2+2\,ab^2x+b^3)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^3x^3+20\,a^2bx^2+1}{6\,(a^6x^2+2\,a^5bx+a^4b^2)}\right.$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x
+ b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x +
 b^3)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (3*a^3*x^3
 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^
4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(66) = 132.

Time = 3.03 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(5/2),x)
```

output
```
6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))
/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b
*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt
(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3)
+ 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1
+ b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x
**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a
**14*b**3*x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) +
 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(
33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*
b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(
1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b
**2*x + 6*a**(33/2)*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{15\left(a+\frac{b}{x}\right)^{2}b-10\left(a+\frac{b}{x}\right)ab-2\,a^{2}b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
 (a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a)))/a^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(65) = 130$.

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,b\log\left(|b|\right)+28\,b)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a^3\mathrm{sgn}\left(x\right)} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3+7\,b^4\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b*log(abs(b)) + 28*b)*sgn(x)/a^(7/2) + 5/2*b*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(
a^3*sgn(x)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2 + 15*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,\left(a+\frac{b}{x}\right)^{5/2}}$$

input
```
int(1/(a + b/x)^(5/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x
)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{-30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)abx-30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2-5\sqrt{a}\,\sqrt{}}{6\sqrt{ax+b}\,a^4\,(ax+b}$$

input
```
int(1/(a+b/x)^(5/2),x)
```

output
```
( - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*a*b*x - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b**2 - 5*sqrt(a)*sqrt(a*x + b)*a*b*x - 5*sqrt(a)*sqrt(a*x + b)*b**
2 + 6*sqrt(x)*a**3*x**2 + 40*sqrt(x)*a**2*b*x + 30*sqrt(x)*a*b**2)/(6*sqrt
(a*x + b)*a**4*(a*x + b))
```

3.43 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b(5b^2c^2-8abcd+a^2d^2)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}-\frac{(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}$$

output
```
1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^(3/2)+b*(a^2*d^2-8*a*b*c*d+5
*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^(1/2)+x/a/c/(a+b/x)^(3/2)-2*d^(7/2)*a
rctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^2/(-a*d+b*c)^(5/2)-(2*a*d+
5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^2
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\frac{\frac{c\sqrt{a+\frac{b}{x}}\,x\left(15b^4c^2+3a^4d^2x^2+6a^3bdx(d-cx)+4ab^3c(-6d+5cx)+a^2b^2\left(3d^2-32cdx+3c^2x^2\right)\right)}{a^3(bc-ad)^2(b+ax)^2}-\frac{6d^{7/2}\arctan\ldots}{(bc-\ldots}}{3c^2}$$

input

```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

output

```
((c*Sqrt[a + b/x]*x*(15*b^4*c^2 + 3*a^4*d^2*x^2 + 6*a^3*b*d*x*(d - c*x) +
4*a*b^3*c*(-6*d + 5*c*x) + a^2*b^2*(3*d^2 - 32*c*d*x + 3*c^2*x^2)))/(a^3*(
b*c - a*d)^2*(b + a*x)^2) - (6*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(5/2) - (3*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(7/2))/(3*c^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.24, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {899, 114, 27, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(5bc+2ad+\frac{5bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(5bc+2ad+\frac{5bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{2\int \frac{3\left(\frac{bd(5bc-3ad)}{x}+(bc-ad)(5bc+2ad)\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(\frac{bd(5bc-3ad)}{x}+(bc-ad)(5bc+2ad)\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{\frac{2\int \frac{\left((5bc+2ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-8abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left((5bc+2ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-8abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 174$$

$$\frac{\frac{\frac{\frac{(bc-ad)^2(2ad+5bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{2a^3d^4\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(2ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4a^3d^4\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^2(2ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{4a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{-\frac{4a^3d^{7/2}\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^2(2ad+5bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b\left(a^2d^2-8abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b(5bc-3ad)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)) + ((2*b*(5*b*c - 3*a*d))/(3*a*(b*c - a*d)*(a + b/x
)^(3/2)) + ((2*b*(5*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a*(b*c - a*d)*Sqrt[a
+ b/x]) + ((-4*a^3*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (2*(b*c - a*d)^2*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(177) = 354$.

Time = 0.65 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.00

method	result
risch	$\frac{ax+b}{a^3c\sqrt{\frac{ax+b}{x}}}-\frac{\left(\frac{(2ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}}-\frac{2c\,b^4\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)a^2}+\frac{4c\,b^2(4ad-3bc)\sqrt{a\left(x+\cdots\right.}}{(ad-bc)^2a\left(x\cdots\right.}\cdots\right.}{2a^3\cdots}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(5/2)/(c+1/x*d),x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/a^3/c*((2*a*d+5*b*c)/c*ln((1/2*b+a*x
)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)-2*c*b^4/(a*d-b*c)/a^2*(2/3/b/(x+b/a)^
2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^
(1/2))+4*c*b^2*(4*a*d-3*b*c)/(a*d-b*c)^2/a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))
^(1/2)+2/c^2*a^3*d^4/(a*d-b*c)^2/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d
/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a
*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))/x/((a*x+b)/x)^(1/2
)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(177) = 354$.

Time = 1.69 (sec) , antiderivative size = 1959, normalized size of antiderivative = 9.75

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

output
```
[1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^
2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^
3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*
b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 -
 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3
*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*s
qrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*
b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^
3*d + a^7*b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2
 + 2*a^3*b^2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*
d^3)*x^2 + 2*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)
*x)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + 3*(a^6*d^3*x
^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)
*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x +
d)) + (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3
 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*
d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d +
a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(...
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx=\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

output
```
Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 5387, normalized size of antiderivative = 26.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

output
```
- ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2
*d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*
(a^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (ata
n(((((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858
*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 -
5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 7
50*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*
a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14
*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b
^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^1
8*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^2
1*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1
/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^
17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 228
0*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*
a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*
b*c)*1i)/(2*c^2*(a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 4
60*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6
+ 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*...
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1212, normalized size of antiderivative = 6.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(5/2)/(c+d/x),x)
```

output
```
(6*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(
2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a)
)*a**5*d**3*x + 6*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a
*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*
sqrt(c)*sqrt(a))*a**4*b*d**3 + 6*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log
(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*d**3*x + 6*sqrt(d)*sqrt(a*x + b)*sqrt
(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) + sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d -
b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**4*b*d**3 - 6*sqrt(d)*sqr
t(a*x + b)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(
x)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**5*d**3*x - 6*sqrt(d)*sqrt
(a*x + b)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(x
)*sqrt(a)*sqrt(a*x + b)*c + 2*a*c*x + 2*a*d)*a**4*b*d**3 - 12*sqrt(a)*sqrt
(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**5*d**4*x + 6*s
qrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**4*b
*c*d**3*x - 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))
/sqrt(b))*a**4*b*d**4 + 54*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt
(x)*sqrt(a))/sqrt(b))*a**3*b**2*c**2*d**2*x + 6*sqrt(a)*sqrt(a*x + b)*log(
(sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**3*b**2*c*d**3 - 78*sqrt(a)*s
qrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a**2*b**3*c...
```

3.44 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 287

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx &= \frac{b(5b^2c^2-6abcd+6a^2d^2)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} \\ &+ \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} \\ &+ \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} \\ &- \frac{d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}} - \frac{(5bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}\end{aligned}$$

output
```
1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+b
*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+b/x)^(1/
2)+d*(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/
2)/(c+d/x)-d^(7/2)*(-4*a*d+9*b*c)*arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^
(1/2))/c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*arctanh((a+b/x)^(1/2)/a^(1/2))/a
^(7/2)/c^3
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \frac{\frac{c\sqrt{a+\frac{b}{x}}x(-15b^5c^3(d+cx)+3a^5d^3x^2(2d+cx)+ab^4c^2(33d^2+13cdx-20c^2x^2)-3a^4bd^2x(-4d^2+cdx+3c^2x^2)}{a^3(-bc+ad)^3(b+ax)^2(d+c}}{}$$

input
```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

output
```
((c*Sqrt[a + b/x]*x*(-15*b^5*c^3*(d + c*x) + 3*a^5*d^3*x^2*(2*d + c*x) + a
*b^4*c^2*(33*d^2 + 13*c*d*x - 20*c^2*x^2) - 3*a^4*b*d^2*x*(-4*d^2 + c*d*x
+ 3*c^2*x^2) + a^2*b^3*c*(-9*d^3 + 35*c*d^2*x + 41*c^2*d*x^2 - 3*c^3*x^3)
+ 3*a^3*b^2*d*(2*d^3 - 5*c*d^2*x - 3*c^2*d*x^2 + 3*c^3*x^3)))/(a^3*(-(b*c)
 + a*d)^3*(b + a*x)^2*(d + c*x)) + (3*d^(7/2)*(-9*b*c + 4*a*d)*ArcTan[(Sqr
t[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2) - (3*(5*b*c + 4*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(3*c^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {899, 114, 27, 168, 25, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(5bc+4ad+\frac{7bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(5bc+4ad+\frac{7bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 168$$

$$\frac{\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(\frac{5bd(bc-2ad)}{x}+(bc-ad)(5bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(\frac{5bd(bc-2ad)}{x}+(bc-ad)(5bc+4ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{\frac{2\int \frac{3\left((5bc+4ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-6abdc+6a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left((5bc+4ad)(bc-ad)^2+\frac{bd\left(5b^2c^2-6abdc+6a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 169

$$\frac{\frac{\frac{\frac{2\int \frac{\left((5bc+4ad)(bc-ad)^3+\frac{bd(bc-2ad)\left(5b^2c^2-abdc+a^2d^2\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(\ldots}\ldots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\left((5bc+4ad)(bc-ad)^3+\frac{bd(bc-2ad)\left(5b^2c^2-abdc+a^2d^2\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc\ldots}\ldots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{\frac{\frac{(bc-ad)^3(4ad+5bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{a^3d^4(9bc-4ad)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{2d(bc-2\ldots}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\ldots\right.}\ldots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{\frac{\frac{2(bc-ad)^3(4ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^4(9bc-4ad)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{\ldots}{c\left(a+\ldots\right.}\ldots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{2(bc-ad)^3(4ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{c(bc-ad)}+\frac{\cdots}{c\left(a+\cdots\right.}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{\frac{-\frac{2a^3d^{7/2}(9bc-4ad)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(bc-ad)^3(4ad+5bc)}{\sqrt{a}c}}{a(bc-ad)}+\frac{2b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}}{c(bc-ad)}+\frac{\cdots}{c\cdots}}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)*(c + d/x)) + ((2*d*(b*c - 2*a*d))/(c*(b*c - a*d)*(a
 + b/x)^(3/2)*(c + d/x)) + ((2*b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a
*(b*c - a*d)*(a + b/x)^(3/2)) + ((2*b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d +
 a^2*d^2))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((-2*a^3*d^(7/2)*(9*b*c - 4*a*d
)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c*Sqrt[b*c - a*d]) - (
2*(b*c - a*d)^3*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]*c
))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(c*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(261) = 522$.

Time = 0.69 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.32

method	result
risch	$\frac{ax+b}{a^3c^2\sqrt{\frac{ax+b}{x}}} - \frac{\left(\frac{(4ad+5bc)\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{c\sqrt{a}} + \frac{2c^2b^5\left(\frac{2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b\left(x+\frac{b}{a}\right)^2}+\frac{4a\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3b^2\left(x+\frac{b}{a}\right)}\right)}{(ad-bc)^2a^2} + \frac{2a^3d^5\left(-\frac{c^2\sqrt{a\left(x+\frac{d}{c}\right.}}{\dots}\right.}{\dots}\right.}{\dots}$
default	Expression too large to display

input
```
int(1/(a+b/x)^(5/2)/(c+1/x*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c^2*(a*x+b)/((a*x+b)/x)^(1/2)-1/2/c^2/a^3*((4*a*d+5*b*c)/c*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)+2*c^2*b^5/(a*d-b*c)^2/a^2*(2/3/b/
(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/3*a/b^2/(x+b/a)*(a*(x+b/a)^2-b*(
x+b/a))^(1/2))+2/c^3*a^3*d^5/(a*d-b*c)^2*(-1/(a*d-b*c)/d*c^2/(x+1/c*d)*(a*
(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-1/2*(2*a*d-b*c)
*c/(a*d-b*c)/d/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c
*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d
)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d)))-4*c^2*b^3*(5*a*d-3*b*c)/(a*d-b*c)^3/
a/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+2/c^2*a^3*d^4*(3*a*d-5*b*c)/(a*d-b
*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d
)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*
c)*d/c^2)^(1/2))/(x+1/c*d)))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(261) = 522$.

Time = 1.44 (sec) , antiderivative size = 3856, normalized size of antiderivative = 13.44

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)^2}\,dx$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

output
```
Integral(x**2/((a + b/x)**(5/2)*(c*x + d)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^2}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(261) = 522$.

Time = 0.23 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.22

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

output
```
1/6*(54*a^(7/2)*b*c*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(9/2)
*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 15*sqrt(b*c*d - a*d^2)*b^4*c^
4*log(abs(b)) + 33*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d*log(abs(b)) - 9*sqrt(b*
c*d - a*d^2)*a^2*b^2*c^2*d^2*log(abs(b)) - 21*sqrt(b*c*d - a*d^2)*a^3*b*c*
d^3*log(abs(b)) + 12*sqrt(b*c*d - a*d^2)*a^4*d^4*log(abs(b)) - 28*sqrt(b*c
*d - a*d^2)*b^4*c^4 + 52*sqrt(b*c*d - a*d^2)*a*b^3*c^3*d + 6*sqrt(b*c*d -
a*d^2)*a^4*d^4)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^3*c^6 - 3*sqrt(b*c*d
 - a*d^2)*a^(9/2)*b^2*c^5*d + 3*sqrt(b*c*d - a*d^2)*a^(11/2)*b*c^4*d^2 - s
qrt(b*c*d - a*d^2)*a^(13/2)*c^3*d^3) + (9*b*c*d^4 - 4*a*d^5)*arctan(-((sqr
t(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/((b^3*c^6*
sgn(x) - 3*a*b^2*c^5*d*sgn(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x
))*sqrt(b*c*d - a*d^2)) + ((sqrt(a)*x - sqrt(a*x^2 + b*x))*b*c*d^4 - 2*(sq
rt(a)*x - sqrt(a*x^2 + b*x))*a*d^5 - sqrt(a)*b*d^5)/((b^3*c^6*sgn(x) - 3*a
*b^2*c^5*d*sgn(x) + 3*a^2*b*c^4*d^2*sgn(x) - a^3*c^3*d^3*sgn(x))*((sqrt(a)
*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d
+ b*d)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*c - 15*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^2*a^2*b^4*d + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqr
t(a)*b^6*c - 27*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^5*d + 7*b^7*c -
13*a*b^6*d)/((a^(7/2)*b^3*c^3*sgn(x) - 3*a^(9/2)*b^2*c^2*d*sgn(x) + 3*a^(1
1/2)*b*c*d^2*sgn(x) - a^(13/2)*d^3*sgn(x))*((sqrt(a)*x - sqrt(a*x^2 + b...
```

Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 5789, normalized size of antiderivative = 20.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

output
```
((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d
- a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2
- 58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/
x)^3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*
c)^3))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5
/2)*(2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^1
7*c^17*d^2*(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*377
76i + a^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a +
b/x)^(1/2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12
*c^12*d^7*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*1845
63i + a^24*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a +
b/x)^(1/2)*107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c
^7*d^12*(a + b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i
+ a^29*b^5*c^5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(
1/2)*4515i + a^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a
+ b/x)^(1/2)*1700i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107
740*a*b^9*c^9*d^10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 528
36*a^4*b^6*c^6*d^13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420
*a^7*b^3*c^3*d^16) + 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a
^2*b^15*c^15*d^4 - 126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 +...
```

Reduce [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 4057, normalized size of antiderivative = 14.14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)
```

output
```
(96*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a
))*a**7*c*d**5*x**2 + 96*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)
*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + s
qrt(x)*sqrt(c)*sqrt(a))*a**7*d**6*x - 192*sqrt(d)*sqrt(a*x + b)*sqrt(a*d -
 b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) -
 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**6*b*c**2*d**4*x**2 - 96*sqrt(d
)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**6*b*
c*d**5*x + 96*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x +
 b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt
(c)*sqrt(a))*a**6*b*d**6 - 54*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sq
rt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c
) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*b**2*c**3*d**3*x**2 - 246*sqrt(d)*sqrt(a
*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)
*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**5*b**2*c**2*
d**4*x - 192*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x +
b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(
c)*sqrt(a))*a**5*b**2*c*d**5 - 54*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*lo
g(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*...
```

3.45 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{b(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4)}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d(4b^2c^2-23abcd+12a^2d^2)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{d^{7/2}(99b^2c^2-88abcd+24a^2d^2)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}} - \frac{(5bc+6ad)\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}$$

output
```
1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*
d+b*c)^3/(a+b/x)^(3/2)+1/4*b*(12*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2
-56*a*b^3*c^3*d+20*b^4*c^4)/a^3/c^3/(-a*d+b*c)^4/(a+b/x)^(1/2)+1/2*d*(-3*a
*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*
b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3
/2)/(c+d/x)^2-1/4*d^(7/2)*(24*a^2*d^2-88*a*b*c*d+99*b^2*c^2)*arctan(d^(1/2
)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^4/(-a*d+b*c)^(9/2)-(6*a*d+5*b*c)*arcta
nh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^4
```

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \frac{c\sqrt{a+\frac{b}{x}}\,x\left(60b^6c^4(d+cx)^2+8ab^5c^3(d+cx)^2(-21d+10cx)+6a^6d^4x^2\left(6d^2+9cdx+2c^2x^2\right)+4a^2b^4c^2(d+cx)^2\left(1\right.\right.}{}$$

input
```
Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

output
```
((c*Sqrt[a + b/x]*x*(60*b^6*c^4*(d + c*x)^2 + 8*a*b^5*c^3*(d + c*x)^2*(-21
*d + 10*c*x) + 6*a^6*d^4*x^2*(6*d^2 + 9*c*d*x + 2*c^2*x^2) + 4*a^2*b^4*c^2
*(d + c*x)^2*(18*d^2 - 56*c*d*x + 3*c^2*x^2) + 3*a^5*b*d^3*x*(24*d^3 + c*d
^2*x - 45*c^2*d*x^2 - 16*c^3*x^3) + 6*a^4*b^2*d^2*(6*d^4 - 26*c*d^3*x - 39
*c^2*d^2*x^2 + 8*c^3*d*x^3 + 12*c^4*x^4) - 3*a^3*b^3*c*d*(35*d^4 + 5*c*d^3
*x - 64*c^2*d^2*x^2 - 16*c^3*d*x^3 + 16*c^4*x^4)))/(a^3*(b*c - a*d)^4*(b +
 a*x)^2*(d + c*x)^2) - (3*d^(7/2)*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*A
rcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(9/2) - (12*(5
*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2))/(12*c^4)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {899, 114, 27, 168, 25, 168, 27, 169, 27, 169, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{114}$$

$$\frac{\int \frac{\left(5bc+6ad+\frac{9bd}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(5bc+6ad+\frac{9bd}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}d\frac{1}{x}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}-\frac{\int -\frac{\left(\frac{7bd(2bc-3ad)}{x}+2(bc-ad)(5bc+6ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(\frac{7bd(2bc-3ad)}{x}+2(bc-ad)(5bc+6ad)\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}d\frac{1}{x}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}-\frac{\int -\frac{\left(4(5bc+6ad)(bc-ad)^2+\frac{5bd\left(4b^2c^2-23abdc+12a^2d^2\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^2+\frac{5bd\left(4b^2c^2-23abdc+12a^2d^2\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(2bc-3ad)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{\frac{\frac{2\int \frac{3\left(4(5bc+6ad)(bc-ad)^3+\frac{bd\left(20b^3c^3-36ab^2dc^2+87a^2bd^2c-36a^3d^3\right)}{x}\right)x}{2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{3a(bc-ad)}+\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+{}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^3+\frac{bd\left(20b^3c^3-36ab^2dc^2+87a^2bd^2c-36a^3d^3\right)}{x}\right)x}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}}{2c(bc-ad)}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}}{2c(bc-ad)}+{}}{2ac}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 169$$

$$\frac{\frac{\frac{\frac{\frac{2\int \frac{\left(4(5bc+6ad)(bc-ad)^4+\frac{bd\left(20b^4c^4-56ab^3dc^3+24a^2b^2d^2c^2-35a^3bd^3c+12a^4d^4\right)}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(-\cdots\right.}{\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{\frac{\int \frac{\left(4(5bc+6ad)(bc-ad)^4+\frac{bd\left(20b^4c^4-56ab^3dc^3+24a^2b^2d^2c^2-35a^3bd^3c+12a^4d^4\right)}{x}\right)x}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(-36\cdots\right.}{\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 174

$$\frac{\frac{\frac{\frac{\frac{\frac{4(bc-ad)^4(6ad+5bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{c}-\frac{a^3d^4\left(24a^2d^2-88abcd+99b^2c^2\right)\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}d\frac{1}{x}}{c}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}}{a(bc-ad)}+\frac{2b\left(-36\cdots\right.}{\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2ac}\cdots$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{8(bc-ad)^4(6ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^4\left(24a^2d^2-88abcd+99b^2c^2\right)\int \frac{1}{c-\frac{ad}{b}+\frac{d}{bx^2}}d\sqrt{a+\frac{b}{x}}}{bc}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\cdots}{a(bc-ad)}$$

$$\frac{\cdots}{2c(bc-ad)} \qquad \frac{\cdots}{2c(bc-ad)} \qquad \frac{\cdots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{8(bc-ad)^4(6ad+5bc)\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{bc}-\frac{2a^3d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}}{a(bc-ad)}+\frac{2b\left(12a^4d^4-35a^3bcd^3+24a^2b^2c^2d^2-56ab^3c^3d+20b^4c^4\right)}{a\sqrt{a+\frac{b}{x}}(bc-ad)}\cdots}{a(bc-ad)}$$

$$\frac{\cdots}{2c(bc-ad)} \qquad \frac{\cdots}{2c(bc-ad)} \qquad \frac{\cdots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{\frac{2b\left(-36a^3d^3+87a^2bcd^2-36ab^2c^2d+20b^3c^3\right)}{3a\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{\frac{-\frac{2a^3d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\arctan\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c\sqrt{bc-ad}}-\frac{8\operatorname{arctanh}\left(\frac{\sqrt{a+\cdots}}{\sqrt{a}}\right)\cdots}{\cdots}}{a(bc-ad)}\cdots}{a(bc\cdots}}{2c(bc-ad)}}{2c(bc-ad)}$$

$$\frac{\cdots}{2ac}$$

$$\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

input
```
Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

output
```
x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) + ((d*(2*b*c - 3*a*d))/(c*(b*c - a*d)*
(a + b/x)^(3/2)*(c + d/x)^2) + ((d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/
(c*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)) + ((2*b*(20*b^3*c^3 - 36*a*b^2*c
^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(3*a*(b*c - a*d)*(a + b/x)^(3/2)) + (
(2*b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*b^2*c^2*d^2 - 35*a^3*b*c*d^3 +
12*a^4*d^4))/(a*(b*c - a*d)*Sqrt[a + b/x]) + ((-2*a^3*d^(7/2)*(99*b^2*c^2
- 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]
)/(c*Sqrt[b*c - a*d]) - (8*(b*c - a*d)^4*(5*b*c + 6*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/(Sqrt[a]*c))/(a*(b*c - a*d)))/(a*(b*c - a*d)))/(2*c*(b*c -
a*d)))/(2*c*(b*c - a*d)))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(373) = 746$.

Time = 0.78 (sec) , antiderivative size = 1138, normalized size of antiderivative = 2.78

method	result	size
risch	Expression too large to display	1138
default	Expression too large to display	7300

input
```
int(1/(a+b/x)^(5/2)/(c+1/x*d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/a^(5/2)/c^4*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*d-5/2/a^(7/2)/c^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(
1/2))*b+2/3/a^5*b^5/(a*d-b*c)^3/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2)+4/
3/a^4*b^4/(a*d-b*c)^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-1/2/c^5*d^5/(a
*d-b*c)^3/(x+1/c*d)^2*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c
^2)^(1/2)+5/2*a/c^4*d^5/(a*d-b*c)^4/(x+1/c*d)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c
*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)-21/4/c^3*d^4/(a*d-b*c)^4/(x+1/c*d)*(a*(x
+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2)*b-7/2*a^2/c^5*d^6
/(a*d-b*c)^4/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(
x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+
(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))+23/2*a/c^4*d^5/(a*d-b*c)^4/((a*d-b*c)*d
/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c
^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(
x+1/c*d))*b-99/8/c^3*d^4/(a*d-b*c)^4/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*
c)*d/c^2-(2*a*d-b*c)/c*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-
(2*a*d-b*c)/c*(x+1/c*d)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))*b^2+1/2*a/c^5*d
^5/(a*d-b*c)^3/((a*d-b*c)*d/c^2)^(1/2)*ln((2*(a*d-b*c)*d/c^2-(2*a*d-b*c)/c
*(x+1/c*d)+2*((a*d-b*c)*d/c^2)^(1/2)*(a*(x+1/c*d)^2-(2*a*d-b*c)/c*(x+1/c*d
)+(a*d-b*c)*d/c^2)^(1/2))/(x+1/c*d))-12/a^3*b^4/(a*d-b*c)^4/(x+b/a)*(a*(x+
b/a)^2-b*(x+b/a))^(1/2)*d+6/a^4*c*b^5/(a*d-b*c)^4/(x+b/a)*(a*(x+b/a)^2-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(373) = 746$.

Time = 5.00 (sec) , antiderivative size = 6140, normalized size of antiderivative = 15.01

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^3}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(373) = 746$.

Time = 0.32 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.27

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

output
```
1/12*(297*a^(7/2)*b^2*c^2*d^4*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 264*
a^(9/2)*b*c*d^5*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 72*a^(11/2)*d^6*ar
ctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 30*sqrt(b*c*d - a*d^2)*b^5*c^5*log(a
bs(b)) + 84*sqrt(b*c*d - a*d^2)*a*b^4*c^4*d*log(abs(b)) - 36*sqrt(b*c*d -
a*d^2)*a^2*b^3*c^3*d^2*log(abs(b)) - 96*sqrt(b*c*d - a*d^2)*a^3*b^2*c^2*d^
3*log(abs(b)) + 114*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4*log(abs(b)) - 36*sqrt(
b*c*d - a*d^2)*a^5*d^5*log(abs(b)) - 56*sqrt(b*c*d - a*d^2)*b^5*c^5 + 128*
sqrt(b*c*d - a*d^2)*a*b^4*c^4*d + 63*sqrt(b*c*d - a*d^2)*a^4*b*c*d^4 - 30*
sqrt(b*c*d - a*d^2)*a^5*d^5)*sgn(x)/(sqrt(b*c*d - a*d^2)*a^(7/2)*b^4*c^8 -
 4*sqrt(b*c*d - a*d^2)*a^(9/2)*b^3*c^7*d + 6*sqrt(b*c*d - a*d^2)*a^(11/2)*
b^2*c^6*d^2 - 4*sqrt(b*c*d - a*d^2)*a^(13/2)*b*c^5*d^3 + sqrt(b*c*d - a*d^
2)*a^(15/2)*c^4*d^4) + 1/4*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*ar
ctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))
/((b^4*c^8*sgn(x) - 4*a*b^3*c^7*d*sgn(x) + 6*a^2*b^2*c^6*d^2*sgn(x) - 4*a^
3*b*c^5*d^3*sgn(x) + a^4*c^4*d^4*sgn(x))*sqrt(b*c*d - a*d^2)) + 1/4*(21*(s
qrt(a)*x - sqrt(a*x^2 + b*x))^3*b^2*c^3*d^4 - 56*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^3*a*b*c^2*d^5 + 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^2*c*d^6 + 15
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*sqrt(a)*b^2*c^2*d^5 - 88*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^(3/2)*b*c*d^6 + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2
*a^(5/2)*d^7 + 19*(sqrt(a)*x - sqrt(a*x^2 + b*x))*b^3*c^2*d^5 - 92*(sqr...
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 4284, normalized size of antiderivative = 10.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)
```

output
```
((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*
d^2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^
3*(a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4
*d^2 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3
*(a^2*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5
*d + 496*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a
^4*b*c*d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*
(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) +
d^2*(a + b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
 + (atan((a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b
/x)^(1/2)*277440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^2
0*c^20*d^4*(a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8
371440i + a^21*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7
*(a + b/x)^(1/2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i +
 a^24*b^15*c^15*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b
/x)^(1/2)*66665280i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27
*b^12*c^12*d^12*(a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^
(1/2)*85652532i + a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9
*c^9*d^15*(a + b/x)^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 7135, normalized size of antiderivative = 17.44

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)
```

output
```
(576*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(
a))*a**8*c**2*d**6*x**3 + 1152*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(s
qrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*
c) + sqrt(x)*sqrt(c)*sqrt(a))*a**8*c*d**7*x**2 + 576*sqrt(d)*sqrt(a*x + b)
*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a
*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**8*d**8*x - 2184*sqr
t(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt
(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**7
*b*c**3*d**5*x**3 - 3792*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)
*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + s
qrt(x)*sqrt(c)*sqrt(a))*a**7*b*c**2*d**6*x**2 - 1032*sqrt(d)*sqrt(a*x + b)
*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a
*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a**7*b*c*d**7*x + 576*
sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*s
qrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*sqrt(c)*sqrt(a))*a
**7*b*d**8 + 2640*sqrt(d)*sqrt(a*x + b)*sqrt(a*d - b*c)*log(sqrt(c)*sqrt(a
*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(x)*
sqrt(c)*sqrt(a))*a**6*b**2*c**4*d**4*x**3 + 3096*sqrt(d)*sqrt(a*x + b)*sqr
t(a*d - b*c)*log(sqrt(c)*sqrt(a*x + b) - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*...
```

3.46 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x + \frac{(bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

output
```
(a+b/x)^(1/2)*(c+d/x)^(1/2)*x+(a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1
/2)/(c+d/x)^(1/2))/a^(1/2)/c^(1/2)-2*b^(1/2)*d^(1/2)*arctanh(d^(1/2)*(a+b/
x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left((bc+ad)\sqrt{b+ax}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)+\sqrt{a}\sqrt{c}\left((b+ax)\sqrt{d+cx}-2\sqrt{b}\sqrt{d}\sqrt{b+ax}\text{ar}\right.\right.}{\sqrt{a}\sqrt{c}(b+ax)\sqrt{d+cx}}$$

input
```
Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x*((b*c + a*d)*Sqrt[b + a*x]*ArcTanh[(Sqrt[a]
*Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])] + Sqrt[a]*Sqrt[c]*((b + a*x)*Sqrt
[d + c*x] - 2*Sqrt[b]*Sqrt[d]*Sqrt[b + a*x]*ArcTanh[(Sqrt[b]*Sqrt[d + c*x]
)/(Sqrt[d]*Sqrt[b + a*x])])))/(Sqrt[a]*Sqrt[c]*(b + a*x)*Sqrt[d + c*x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {899, 108, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{108}$$

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}-\int \frac{\left(bc+ad+\frac{2bd}{x}\right)x}{2\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}-\frac{1}{2}\int \frac{\left(bc+ad+\frac{2bd}{x}\right)x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(-2bd\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-(ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(-4bd\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-(ad+bc)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$\Big\downarrow$ 104

$$\frac{1}{2}\left(-2(ad+bc)\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-4bd\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{2(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-4\sqrt{b}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)\right)+x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}$$

input
```
Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

output
```
Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((2*(b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 4*Sqrt[b]*Sqrt[d]*ArcT
anh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 108
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(m + 1)))
, x] - Simp[1/(b*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*
x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2
*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(95) = 190$.

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x\sqrt{\frac{cx+d}{x}}\left(2bd\ln\left(\frac{adx+bcx+2\sqrt{bd}\,\sqrt{(ax+b)(cx+d)}+2bd}{x}\right)\sqrt{ac}-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)\sqrt{bd}\,ad-\ln\left(\frac{2acx+2 \cdots}{}\right)\right)}{2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\,\sqrt{bd}}$

input
```
int((a+b/x)^(1/2)*(c+1/x*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(2*b*d*ln((a*d*x+b*c*x+2*(b*d)^
(1/2)*((a*x+b)*(c*x+d))^(1/2)+2*b*d)/x)*(a*c)^(1/2)-ln(1/2*(2*a*c*x+2*((a*
x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*a*d-ln(1
/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b
*d)^(1/2)*b*c-2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/((a*x+b)*
(c*x+d))^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 824, normalized size of antiderivative = 6.70

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(
8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)
*x^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2
)*x)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d
 - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x +
 b)/x)*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(2*sqrt(-b*d)*x*sqrt((a*
x + b)/x)*sqrt((c*x + d)/x)/(2*b*d + (b*c + a*d)*x)) + sqrt(a*c)*(b*c + a*
d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*
c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a
^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt
(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d
*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b
^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a
*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(-b*d)*a*c*arctan(2*sqrt(
-b*d)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*b*d + (b*c + a*d)*x)) - sqr
t(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)
/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)**(1/2)*(c+d/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

input
```
integrate((a+b/x)^(1/2)*(c+d/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Mupad [B] (verification not implemented)

Time = 22.30 (sec) , antiderivative size = 4674, normalized size of antiderivative = 38.00

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \text{Too large to display}$$

input
```
int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

output
```
atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9
/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d
^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1
/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*
b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*
d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d +
 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*
c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^
(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a
^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^
5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7
))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2
) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2
)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/
2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7
*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^
9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5
 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^
(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^
9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.92

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = \frac{\sqrt{cx+d}\,\sqrt{ax+b}\,ac+\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{c}\,\sqrt{ax+b}+\sqrt{a}\,\sqrt{cx+d}}{\sqrt{ad-bc}}\right)ad+\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{c}\,\sqrt{ax+b}+\sqrt{a}\,\sqrt{cx+d}}{\sqrt{ad-bc}}\right)bc+\sqrt{d}\,\sqrt{b}\,\text{lo}}{}$$

input

```
int((a+b/x)^(1/2)*(c+d/x)^(1/2),x)
```

output

```
(sqrt(c*x + d)*sqrt(a*x + b)*a*c + sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x +
 b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*a*d + sqrt(c)*sqrt(a)*log((s
qrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*b*c + sqrt(
d)*sqrt(b)*log(sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d) - sqrt(2*sqrt
(d)*sqrt(c)*sqrt(b)*sqrt(a) + a*d + b*c))*a*c + sqrt(d)*sqrt(b)*log(sqrt(c
)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d) + sqrt(2*sqrt(d)*sqrt(c)*sqrt(b)*s
qrt(a) + a*d + b*c))*a*c - sqrt(d)*sqrt(b)*log(2*sqrt(c)*sqrt(a)*sqrt(c*x
+ d)*sqrt(a*x + b) + 2*sqrt(d)*sqrt(c)*sqrt(b)*sqrt(a) + 2*a*c*x)*a*c)/(a*
c)
```

3.47 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} + \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}$$

output
```
(a+b/x)^(1/2)*(c+d/x)^(1/2)*x/c+(-a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a
^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{d+cx}\left(\frac{\sqrt{b+ax}\sqrt{d+cx}}{c} + \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d+cx}}{\sqrt{c}\sqrt{b+ax}}\right)}{\sqrt{a}c^{3/2}}\right)}{\sqrt{c+\frac{d}{x}}\sqrt{b+ax}}$$

input
```
Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[d + c*x]*((Sqrt[b + a*x]*Sqrt[d + c*x])/c + ((b*c - a*
d)*ArcTanh[(Sqrt[a]*Sqrt[d + c*x])/(Sqrt[c]*Sqrt[b + a*x])])/(Sqrt[a]*c^(3
/2))))/(Sqrt[c + d/x]*Sqrt[b + a*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {899, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{105}$$

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}-\frac{(bc-ad)\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}}{2c}$$

$$\Big\downarrow \text{104}$$

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}-\frac{(bc-ad)\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

input
```
Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
 b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)ad-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)bc-2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{(ax+b)(cx+d)}\,c\sqrt{ac}}$

input
```
int((a+b/x)^(1/2)/(c+1/x*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln(1/2*(2*a*c*x+2*((a*x+b)*(c*
x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d-ln(1/2*(2*a*c*x+2*((a*x+
b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c-2*((a*x+b)*(c*x+d)
)^(1/2)*(a*c)^(1/2))/((a*x+b)*(c*x+d))^(1/2)/c/(a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$
$$= \left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\sqrt{ac}(bc-ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx^2+(bc+ad)x)\sqrt{a}\right.}{4\,ac^2}\right.$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*
log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c +
 a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*
c*d)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(
-a*c)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x
)/(2*a*c*x + b*c + a*d)))/(a*c^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 478, normalized size of antiderivative = 5.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$
$$= \frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4\,c\,\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\,\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{b^2}{4\,c\,d} + \frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\,\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^3}{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^3} + \frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)} - \frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\,\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{a+\frac{b}{x}}-\sqrt{a}\,\sqrt{c+\frac{d}{x}}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

output
```
(d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - ((((a
+ b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c +
d/x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^
2*d^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/
2))^2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*
((a + b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)
^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(
1/2))^2)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*
(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b
/x)^(1/2) - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1
/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))
*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = \frac{\sqrt{cx+d}\,\sqrt{ax+b}\,ac-\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{c}\,\sqrt{ax+b}+\sqrt{a}\,\sqrt{cx+d}}{\sqrt{ad-bc}}\right)ad+\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{c}\,\sqrt{ax+b}+\sqrt{a}\,\sqrt{cx+d}}{\sqrt{ad-bc}}\right)bc}{a\,c^2}$$

input
```
int((a+b/x)^(1/2)/(c+d/x)^(1/2),x)
```

output
```
(sqrt(c*x + d)*sqrt(a*x + b)*a*c - sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x +
 b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*a*d + sqrt(c)*sqrt(a)*log((s
qrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*b*c)/(a*c**
2)
```

3.48 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}x}{ac\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}$$

output
```
-(-3*a*d+b*c)*(a+b/x)^(1/2)/a/c^2/(c+d/x)^(1/2)+(a+b/x)^(3/2)*x/a/c/(c+d/x
)^(1/2)+(-3*a*d+b*c)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/
a^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x\left(\sqrt{a}\sqrt{c}\sqrt{b+ax}(3d+cx)+(bc-3ad)\sqrt{d+cx}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{b+ax}}{\sqrt{a}\sqrt{d+cx}}\right)\right)}{\sqrt{a}c^{5/2}\sqrt{b+ax}(d+cx)}$$

input
```
Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[c + d/x]*x*(Sqrt[a]*Sqrt[c]*Sqrt[b + a*x]*(3*d + c*x)
+ (b*c - 3*a*d)*Sqrt[d + c*x]*ArcTanh[(Sqrt[c]*Sqrt[b + a*x])/(Sqrt[a]*Sqr
t[d + c*x])]))/(Sqrt[a]*c^(5/2)*Sqrt[b + a*x]*(d + c*x))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {899, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}x^2}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 107}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\int \frac{\sqrt{a+\frac{b}{x}}x}{\left(c+\frac{d}{x}\right)^{3/2}}d\frac{1}{x}}{2ac}$$

$$\Big\downarrow \text{ 105}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{a\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}}{c}+\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}\right)}{2ac}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{2a\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}}{c}+\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}\right)}{2ac}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}-\frac{(bc-3ad)\left(\frac{2\sqrt{a+\frac{b}{x}}}{c\sqrt{c+\frac{d}{x}}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{c^{3/2}}\right)}{2ac}$$

input
```
Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

output
```
((a + b/x)^(3/2)*x)/(a*c*Sqrt[c + d/x]) - ((b*c - 3*a*d)*((2*Sqrt[a + b/x]
)/(c*Sqrt[c + d/x]) - (2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*
Sqrt[c + d/x])])/c^(3/2)))/(2*a*c)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(102) = 204$.

Time $= 0.31$ (sec) , antiderivative size $= 280$, normalized size of antiderivative $= 2.30$

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}}\left(-3\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)acdx+\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)b\,c^{2}x+2cx\sqrt{(ax+b)(cx}\right.}{2\sqrt{ac}\,(cx+d)\sqrt{(ax}}$

input
```
int((a+b/x)^(1/2)/(c+1/x*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(
c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*c*d*x+ln(1/2*(2*a*c*x+2*
((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c^2*x+2*c*x*((
a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x+d))^(1
/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((a*x+b)*(c*
x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^
(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \left[-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{ac}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\,(2\,acx\right.}{4\,(ac}\right.$$

$$\left.-\frac{(bcd-3\,ad^2+(bc^2-3\,acd)x)\sqrt{-ac}\arctan\left(\frac{2\sqrt{-ac}x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)-2\,(ac^2x^2+3\,acdx)\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(ac^4x+ac^3d)}\right]$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^
2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c
)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^
2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d
), -1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(
-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*
c^2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3
*d)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

output
```
Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$$

input
```
int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)
```

output
```
int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = \frac{4\sqrt{cx+d}\,\sqrt{ax+b}\,a\,c^2x+12\sqrt{cx+d}\,\sqrt{ax+b}\,acd-12\sqrt{c}\,\sqrt{a}\log\left(\frac{\sqrt{c}\,\sqrt{ax+b}+\sqrt{a}\,\sqrt{cx+d}}{\sqrt{ad-bc}}\right)}{}$$

input
```
int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
```

output
```
(4*sqrt(c*x + d)*sqrt(a*x + b)*a*c**2*x + 12*sqrt(c*x + d)*sqrt(a*x + b)*a
*c*d - 12*sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x +
d))/sqrt(a*d - b*c))*a*c*d*x - 12*sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x +
b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*a*d**2 + 4*sqrt(c)*sqrt(a)*lo
g((sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d))/sqrt(a*d - b*c))*b*c**2*
x + 4*sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d))/
sqrt(a*d - b*c))*b*c*d + 9*sqrt(c)*sqrt(a)*a*c*d*x + 9*sqrt(c)*sqrt(a)*a*d
**2 - sqrt(c)*sqrt(a)*b*c**2*x - sqrt(c)*sqrt(a)*b*c*d)/(4*a*c**3*(c*x + d
))
```

3.49 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,2,-q,2+p,1+\frac{b}{ax},-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad}\right)}{a^2(1+p)}$$

output
```
-b*(a+b/x)^(p+1)*(c+d/x)^q*AppellF1(p+1,-q,2,2+p,-d*(a+b/x)/(-a*d+b*c),1+b
/a/x)/a^2/(p+1)/((b*(c+d/x)/(-a*d+b*c))^q)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(95) = 190.

Time = 0.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.17

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \frac{bd(-2+p+q)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q x \,\text{AppellF}}{(-1+p+q)\left(-bd(-2+p+q)\text{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax}{b},-\frac{cx}{d}\right)+x\left(adp\,\text{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

output
```
(b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2
- p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*Appell
F1[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*Appel
lF1[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*Appel
lF1[2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {899, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 154}$$

$$-\left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \int \left(a+\frac{b}{x}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 153}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1} \left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

input
```
Int[(a + b/x)^p*(c + d/x)^q,x]
```

output
```
-((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
 b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d)
)^q))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{p} \left(c+\frac{d}{x}\right)^{q} dx$$

input
```
int((a+b/x)^p*(c+1/x*d)^q,x)
```

output
```
int((a+b/x)^p*(c+1/x*d)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)**p*(c+d/x)**q,x)
```

output
```
Integral((a + b/x)**p*(c + d/x)**q, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^p*(c + d/x)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x)^p*(c + d/x)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

input
```
int((a + b/x)^p*(c + d/x)^q,x)
```

output
```
int((a + b/x)^p*(c + d/x)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = \text{too large to display}$$

input
```
int((a+b/x)^p*(c+d/x)^q,x)
```

output

```
((c*x + d)**q*(a*x + b)**p*a*d*q*x + (c*x + d)**q*(a*x + b)**p*b*c*p*x - (
c*x + d)**q*(a*x + b)**p*b*d*p - (c*x + d)**q*(a*x + b)**p*b*d*q + x**(p +
 q)*int(((c*x + d)**q*(a*x + b)**p*x)/(x**(p + q)*a**2*c*d*q*x**2 + x**(p
+ q)*a**2*d**2*q*x + x**(p + q)*a*b*c**2*p*x**2 + x**(p + q)*a*b*c*d*p*x +
 x**(p + q)*a*b*c*d*q*x + x**(p + q)*a*b*d**2*q + x**(p + q)*b**2*c**2*p*x
 + x**(p + q)*b**2*c*d*p),x)*a**3*d**3*q**3 + 3*x**(p + q)*int(((c*x + d)*
*q*(a*x + b)**p*x)/(x**(p + q)*a**2*c*d*q*x**2 + x**(p + q)*a**2*d**2*q*x
+ x**(p + q)*a*b*c**2*p*x**2 + x**(p + q)*a*b*c*d*p*x + x**(p + q)*a*b*c*d
*q*x + x**(p + q)*a*b*d**2*q + x**(p + q)*b**2*c**2*p*x + x**(p + q)*b**2*
c*d*p),x)*a**2*b*c*d**2*p*q**2 + 3*x**(p + q)*int(((c*x + d)**q*(a*x + b)*
*p*x)/(x**(p + q)*a**2*c*d*q*x**2 + x**(p + q)*a**2*d**2*q*x + x**(p + q)*
a*b*c**2*p*x**2 + x**(p + q)*a*b*c*d*p*x + x**(p + q)*a*b*c*d*q*x + x**(p
+ q)*a*b*d**2*q + x**(p + q)*b**2*c**2*p*x + x**(p + q)*b**2*c*d*p),x)*a*b
**2*c**2*d*p**2*q + x**(p + q)*int(((c*x + d)**q*(a*x + b)**p*x)/(x**(p +
q)*a**2*c*d*q*x**2 + x**(p + q)*a**2*d**2*q*x + x**(p + q)*a*b*c**2*p*x**2
 + x**(p + q)*a*b*c*d*p*x + x**(p + q)*a*b*c*d*q*x + x**(p + q)*a*b*d**2*q
 + x**(p + q)*b**2*c**2*p*x + x**(p + q)*b**2*c*d*p),x)*b**3*c**3*p**3 - x
**(p + q)*int(((c*x + d)**q*(a*x + b)**p)/(x**(p + q)*a**2*c*d*q*x**3 + x*
*(p + q)*a**2*d**2*q*x**2 + x**(p + q)*a*b*c**2*p*x**3 + x**(p + q)*a*b*c*
d*p*x**2 + x**(p + q)*a*b*c*d*q*x**2 + x**(p + q)*a*b*d**2*q*x + x**(p ...
```

3.50 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

output
```
a*x/c+(-a*d+b*c)*arctan(c^(1/2)*x/d^(1/2))/c^(3/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} \, dx = \frac{ax}{c} - \frac{(-bc+ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

input
```
Integrate[(a + b/x^2)/(c + d/x^2),x]
```

output
```
(a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {898, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{ 898}$$

$$\int \frac{ax^2+b}{cx^2+d}dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(bc-ad)\int \frac{1}{cx^2+d}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

input
```
Int[(a + b/x^2)/(c + d/x^2),x]
```

output
```
(a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{ax}{c}+\frac{(-ad+bc)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{c\sqrt{cd}}$	34
risch	$\frac{ax}{c}-\frac{\ln(cx-\sqrt{-cd})ad}{2c\sqrt{-cd}}+\frac{\ln(cx-\sqrt{-cd})b}{2\sqrt{-cd}}+\frac{\ln(-cx-\sqrt{-cd})ad}{2c\sqrt{-cd}}-\frac{\ln(-cx-\sqrt{-cd})b}{2\sqrt{-cd}}$	106

input
```
int((a+b/x^2)/(c+1/x^2*d),x,method=_RETURNVERBOSE)
```

output
```
a*x/c+(-a*d+b*c)/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \left[\frac{2\,acdx+(bc-ad)\sqrt{-cd}\log\left(\frac{cx^2+2\,\sqrt{-cd}x-d}{cx^2+d}\right)}{2\,c^2d}, \frac{acdx+(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{c^2d}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/
(c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x
/d))/(c^2*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2),x)
```

output
```
a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 -
sqrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

output
```
a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

input
```
integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
```

output
```
a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{a\,x}{c} - \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

input
```
int((a + b/x^2)/(c + d/x^2),x)
```

output
```
(a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx = \frac{-\sqrt{d}\,\sqrt{c}\,atan\left(\frac{cx}{\sqrt{d}\,\sqrt{c}}\right)ad+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{cx}{\sqrt{d}\,\sqrt{c}}\right)bc+acdx}{c^2d}$$

input
```
int((a+b/x^2)/(c+d/x^2),x)
```

output
```
( - sqrt(d)*sqrt(c)*atan((c*x)/(sqrt(d)*sqrt(c)))*a*d + sqrt(d)*sqrt(c)*at
an((c*x)/(sqrt(d)*sqrt(c)))*b*c + a*c*d*x)/(c**2*d)
```

3.51 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 233

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{2b\sqrt{c+\frac{d}{x^2}}}{\sqrt{a+\frac{b}{x^2}}\,x}+\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x + \frac{2\sqrt{a}\sqrt{b}\sqrt{c+\frac{d}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}} - \frac{\sqrt{a}(bc+ad)\sqrt{c+\frac{d}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}}$$

output

```
-2*b*(c+d/x^2)^(1/2)/(a+b/x^2)^(1/2)/x+(a+b/x^2)^(1/2)*(c+d/x^2)^(1/2)*x+2
*a^(1/2)*b^(1/2)*(c+d/x^2)^(1/2)*EllipticE(1/(1+a*x^2/b)^(1/2),(1-a*d/b/c)
^(1/2))/(a+b/x^2)^(1/2)/(a*(c+d/x^2)/c/(a+b/x^2))^(1/2)-a^(1/2)*(a*d+b*c)*
(c+d/x^2)^(1/2)*InverseJacobiAM(arccot(a^(1/2)*x/b^(1/2)),(1-a*d/b/c)^(1/2
))/b^(1/2)/c/(a+b/x^2)^(1/2)/(a*(c+d/x^2)/c/(a+b/x^2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x\left(\sqrt{\frac{a}{b}}(b+ax^2)(d+cx^2)+2iadx\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+i(bc}{\sqrt{\frac{a}{b}}(b+ax^2)(d+cx^2)}$$

input
```
Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

output
```
-((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) +
(2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c
*x^2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b +
a*x^2)*(d + c*x^2)))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {899, 375, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

$$\Big\downarrow \text{899}$$

$$-\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{375}$$

$$x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-2\int \frac{bc+ad+\frac{2bd}{x^2}}{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\int \frac{bc+ad+\frac{2bd}{x^2}}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{406}$$

$$-(ad+bc)\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-2bd\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow \text{320}$$

$$-2bd\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+$$
$$x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow \text{388}$$

$$-2bd\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)-$$
$$\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

$$\Big\downarrow \text{313}$$

$$-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-$$
$$2bd\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}$$

input
```
Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

output
```
Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x - 2*b*d*(Sqrt[a + b/x^2]/(b*Sqrt[c + d/x
^2]*x) - (Sqrt[c]*Sqrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1
 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
 d/x^2])) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/
(Sqrt[c]*x)], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/
x^2))]*Sqrt[c + d/x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 375
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^p*((c + d*x^2)^q/(e*(m + 1)))
, x] - Simp[2/(e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^(p - 1)*(c + d*
x^2)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^2, x], x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0
] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(-\sqrt{-\frac{c}{d}}\,a c\,x^{4}+\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right) a d x-c b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,x\,\operatorname{EllipticF}\left(x\sqrt{-\frac{c}{d}},\right.\right.}{\left(a\,x^{4} c+a d\,x^{2}+x^{2} b c+b d\right)\sqrt{-\frac{c}{d}}}$
risch	$-x\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\sqrt{\frac{c\,x^{2}+d}{x^{2}}}+\frac{\left(\frac{a d\sqrt{1+\frac{c\,x^{2}}{d}}\,\sqrt{1+\frac{a\,x^{2}}{b}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^{4} c+a d\,x^{2}+x^{2} b c+b d}}+\frac{b c\sqrt{1+\frac{c\,x^{2}}{d}}\,\sqrt{1+\frac{a\,x^{2}}{b}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{-1+\right.}{\sqrt{-\frac{c}{d}}\,\sqrt{a\,x^{4} c+a d\,x^{2}+x^{2} b c+b d}}\right.}{}$

input
```
int((a+b/x^2)^(1/2)*(c+1/x^2*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*a*c*x^4+((c*x
^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2
))*a*d*x-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticF(x*(-c/d)^
(1/2),(a*d/b/c)^(1/2))+2*c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ell
ipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-(-c/d)^(1/2)*a*d*x^2-(-c/d)^(1/2)*b
*c*x^2-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

Sympy [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((a+b/x**2)**(1/2)*(c+d/x**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = \int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx=\int \sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

input
```
int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2),x)
```

output
```
int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$
$$=\frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}+2\left(\int \frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}}{ac\,x^6+ad\,x^4+bc\,x^4+bd\,x^2}dx\right)bdx+\left(\int \frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}}{ac\,x^4+ad\,x^2+bc\,x^2+bd}dx\right)adx+\left(\int \frac{\sqrt{c\,x^2+d}\,\sqrt{}}{ac\,x^4+ad\,x^2+}\right.}{x}$$

input
```
int((a+b/x^2)^(1/2)*(c+d/x^2)^(1/2),x)
```

output
```
(sqrt(c*x**2 + d)*sqrt(a*x**2 + b) + 2*int((sqrt(c*x**2 + d)*sqrt(a*x**2 +
 b))/(a*c*x**6 + a*d*x**4 + b*c*x**4 + b*d*x**2),x)*b*d*x + int((sqrt(c*x*
*2 + d)*sqrt(a*x**2 + b))/(a*c*x**4 + a*d*x**2 + b*c*x**2 + b*d),x)*a*d*x
+ int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b))/(a*c*x**4 + a*d*x**2 + b*c*x**2
+ b*d),x)*b*c*x)/x
```

3.52 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}} \, dx = \frac{\sqrt{a+\frac{b}{x^2}}x}{\sqrt{c+\frac{d}{x^2}}} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}}$$

output
```
(a+b/x^2)^(1/2)*x/(c+d/x^2)^(1/2)+d^(1/2)*(a+b/x^2)^(1/2)*EllipticE(1/(1+c
*x^2/d)^(1/2),(1-b*c/a/d)^(1/2))/c^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(
c+d/x^2)^(1/2)-b*c^(1/2)*(a+b/x^2)^(1/2)*InverseJacobiAM(arccot(c^(1/2)*x/
d^(1/2)),(1-b*c/a/d)^(1/2))/a/d^(1/2)/(c*(a+b/x^2)/a/(c+d/x^2))^(1/2)/(c+d
/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{d+cx^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{c}{d}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{b+ax^2}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*
d)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {899, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^2}}x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{377}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{\int \frac{b\sqrt{c+\frac{d}{x^2}}}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\int \frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{324}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(c\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}\right)}{c}$$

$$\Big\downarrow \text{320}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\left(\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)\right)}{c}$$

input
```
Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c - (b*(d*(Sqrt[a + b/x^2]/(b*Sqrt[c +
 d/x^2]*x) - (Sqrt[c]*Sqrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)
], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt
[c + d/x^2])) + (c^(3/2)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/(Sqrt[c]
*x)], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, \operatorname{EllipticE}\left(x \sqrt{-\frac{c}{d}}, \sqrt{\frac{a d}{b c}}\right) \sqrt{\frac{a\,x^{2}+b}{b}}\, \sqrt{\frac{c\,x^{2}+d}{d}}\, b}{\left(a\,x^{2}+b\right) \sqrt{-\frac{c}{d}}\, \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	94

input
```
int((a+b/x^2)^(1/2)/(c+1/x^2*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*
((a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/((c*x^2+d)/x^2)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}} - \sqrt{ac}b\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc}) + \sqrt{ac}(a+b)\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right) \mid \frac{ad}{bc})}{ac}$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
(a*x*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2) - sqrt(a*c)*b*sqrt(-b/a)*
elliptic_e(arcsin(sqrt(-b/a)/x), a*d/(b*c)) + sqrt(a*c)*(a + b)*sqrt(-b/a)
*elliptic_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)))/(a*c)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2),x)
```

output
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \int \frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}}{c\,x^2+d}dx$$

input
```
int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x)
```

output
```
int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b))/(c*x**2 + d),x)
```

3.53 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 234

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}}+\frac{2a\sqrt{c+\frac{d}{x^2}}\,x}{c^2\sqrt{a+\frac{b}{x^2}}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+\frac{d}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)|1-\frac{ad}{bc}\right)}{c^2\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}}-\frac{\sqrt{a}\sqrt{b}\sqrt{c+\frac{d}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right),1-\frac{ad}{bc}\right)}{c^2\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{a\left(c+\frac{d}{x^2}\right)}{c\left(a+\frac{b}{x^2}\right)}}}$$

output
```
-(a+b/x^2)^(1/2)*x/c/(c+d/x^2)^(1/2)+2*a*(c+d/x^2)^(1/2)*x/c^2/(a+b/x^2)^(
1/2)+2*a^(1/2)*b^(1/2)*(c+d/x^2)^(1/2)*EllipticE(1/(1+a*x^2/b)^(1/2),(1-a*
d/b/c)^(1/2))/c^2/(a+b/x^2)^(1/2)/(a*(c+d/x^2)/c/(a+b/x^2))^(1/2)-a^(1/2)*
b^(1/2)*(c+d/x^2)^(1/2)*InverseJacobiAM(arccot(a^(1/2)*x/b^(1/2)),(1-a*d/b
/c)^(1/2))/c^2/(a+b/x^2)^(1/2)/(a*(c+d/x^2)/c/(a+b/x^2))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{\frac{a}{b}}cx(b+ax^2)+2iad\sqrt{1+\frac{ax^2}{b}}\sqrt{1+\frac{cx^2}{d}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{a}{b}}x\right)|\frac{bc}{ad}\right)+i(bc-2ad)\sqrt{1+\frac{ax^2}{b}}\sqrt{1\cdots}\right.}{\sqrt{\frac{a}{b}}c^2\sqrt{c+\frac{d}{x^2}}\,(b+ax^2)}$$

input
```
Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

output
```
-((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)
/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I
*(b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh
[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {899, 371, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 899}$$
$$-\int \frac{\sqrt{a+\frac{b}{x^2}}x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \; 371$$

$$\frac{\int -\frac{\left(2a+\frac{b}{x^2}\right)x^2}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{\left(2a+\frac{b}{x^2}\right)x^2}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 445$$

$$-\frac{-\frac{\int -\frac{ab\left(c+\frac{2d}{x^2}\right)}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{ac}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{ab\left(c+\frac{2d}{x^2}\right)}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{ac}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{b\int \frac{c+\frac{2d}{x^2}}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 406$$

$$-\frac{\frac{b\left(c\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+2d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \; 320$$

$$-\frac{\frac{b\left(2d\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{388}$$

$$-\frac{\frac{b\left(2d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{c\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{b}\right)+\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{313}$$

$$-\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+2d\left(\frac{\sqrt{a+\frac{b}{x^2}}}{bx\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}E\left(\arctan\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}\right)\right)}{c}-\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}}{c}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

output
```
-((Sqrt[a + b/x^2]*x)/(c*Sqrt[c + d/x^2])) - ((-2*Sqrt[a + b/x^2]*Sqrt[c +
 d/x^2]*x)/c + (b*(2*d*(Sqrt[a + b/x^2]/(b*Sqrt[c + d/x^2]*x) - (Sqrt[c]*S
qrt[a + b/x^2]*EllipticE[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])) + (c^(3/2
)*Sqrt[a + b/x^2]*EllipticF[ArcTan[Sqrt[d]/(Sqrt[c]*x)], 1 - (b*c)/(a*d)])
/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 899

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 5.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

method	result
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(\sqrt{-\frac{c}{d}}\,a\,x^{3}+b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right)-2b\sqrt{\frac{c\,x^{2}+d}{d}}\,\sqrt{\frac{a\,x^{2}+b}{b}}\,\operatorname{EllipticE}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{a d}{b c}}\right)+\sqrt{-\frac{c}{d}}\right.}{x^{2}\left(a\,x^{2}+b\right)\sqrt{-\frac{c}{d}}\,c\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}}$

input

```
int((a+b/x^2)^(1/2)/(c+1/x^2*d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*((-c/d)^(1/2)*a*x^3+b*((c*x^2+d)/d)^(
1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-2*b*((c
*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1
/2))+(-c/d)^(1/2)*b*x)*(c*x^2+d)/(-c/d)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx =$$

$$-\frac{2\,(bcx^2+bd)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right)\mid\frac{ad}{bc})-((a+2\,b)cx^2+(a+2\,b)d)\sqrt{ac}\sqrt{-\frac{b}{a}}F(\arcsin\left(\frac{\sqrt{-\frac{b}{a}}}{x}\right)\mid}{ac^3x^2+ac^2d}$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
-(2*(b*c*x^2 + b*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(sqrt(-b/a)/x),
a*d/(b*c)) - ((a + 2*b)*c*x^2 + (a + 2*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic
_f(arcsin(sqrt(-b/a)/x), a*d/(b*c)) - (a*c*x^3 + 2*a*d*x)*sqrt((a*x^2 + b)
/x^2)*sqrt((c*x^2 + d)/x^2))/(a*c^3*x^2 + a*c^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

output
```
Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)
```

output
```
int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}\,bx+2\left(\int \frac{\sqrt{c\,x^2+d}\,\sqrt{a\,x^2+b}\,x^4}{a\,c^2x^6+2acd\,x^4+b\,c^2x^4+a\,d^2x^2+2bcd\,x^2+b\,d^2}dx\right)a^2cd\,x^2+2\left(\int \frac{}{a\,c\cdots}\right.}{}$$

input
```
int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x)
```

output
```
(sqrt(c*x**2 + d)*sqrt(a*x**2 + b)*b*x + 2*int((sqrt(c*x**2 + d)*sqrt(a*x*
*2 + b)*x**4)/(a*c**2*x**6 + 2*a*c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*
b*c*d*x**2 + b*d**2),x)*a**2*c*d*x**2 + 2*int((sqrt(c*x**2 + d)*sqrt(a*x**
2 + b)*x**4)/(a*c**2*x**6 + 2*a*c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*b
*c*d*x**2 + b*d**2),x)*a**2*d**2 - int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b)*
x**4)/(a*c**2*x**6 + 2*a*c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*b*c*d*x*
*2 + b*d**2),x)*a*b*c**2*x**2 - int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b)*x**
4)/(a*c**2*x**6 + 2*a*c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*b*c*d*x**2
+ b*d**2),x)*a*b*c*d - int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b))/(a*c**2*x**
6 + 2*a*c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*b*c*d*x**2 + b*d**2),x)*b
**2*c*d*x**2 - int((sqrt(c*x**2 + d)*sqrt(a*x**2 + b))/(a*c**2*x**6 + 2*a*
c*d*x**4 + a*d**2*x**2 + b*c**2*x**4 + 2*b*c*d*x**2 + b*d**2),x)*b**2*d**2
)/(2*a*d*(c*x**2 + d))
```

3.54 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b
/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-1+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {899, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 394}$$

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -
(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a+b/x^2)^p*(c+1/x^2*d)^q,x)
```

output
```
int((a+b/x^2)^p*(c+1/x^2*d)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \frac{(c\,x^2+d)^q\,(a\,x^2+b)^p\,x+2x^{2p+2q}\left(\int \frac{(c\,x^2+d)^q\,(a\,x^2+b)^p\,x^2}{x^{2p+2q}ac\,x^4+x^{2p+2q}ad\,x^2+x^{2p+2q}bc\,x^2+x^{2p+2q}bd}dx\right)adq+2x^{2p+2q}\left(\int \frac{}{x^{2p+2q}ac}\right.}{}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

output
```
((c*x**2 + d)**q*(a*x**2 + b)**p*x + 2*x**(2*p + 2*q)*int((((c*x**2 + d)**q
*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2
 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a*d*q + 2*x**(2*p + 2*q
)*int((((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x*
*(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b
*c*p + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2
*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p
 + 2*q)*b*d),x)*b*d*p + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)
**p)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b
*c*x**2 + x**(2*p + 2*q)*b*d),x)*b*d*q)/x**(2*p + 2*q)
```

3.55 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}} \, dx = \frac{ax}{c} - \frac{(bc-ad)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}$$

output
```
a*x/c-1/3*(-a*d+b*c)*arctan(1/3*(d^(1/3)-2*c^(1/3)*x)*3^(1/2)/d^(1/3))*3^(
1/2)/c^(4/3)/d^(2/3)+1/3*(-a*d+b*c)*ln(d^(1/3)+c^(1/3)*x)/c^(4/3)/d^(2/3)-
1/6*(-a*d+b*c)*ln(d^(2/3)-c^(1/3)*d^(1/3)*x+c^(2/3)*x^2)/c^(4/3)/d^(2/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$
$$= \frac{6a\sqrt[3]{c}d^{2/3}x - 2\sqrt{3}(bc-ad)\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right) - (bc-ad)\log\left(d^{2/3} - \sqrt[3]{\ }\right.}{6c^{4/3}d^{2/3}}$$

input
```
Integrate[(a + b/x^3)/(c + d/x^3),x]
```

output
```
(6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d
^(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Lo
g[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {898, 913, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$
$$\Big\downarrow \; 898$$
$$\int \frac{ax^3+b}{cx^3+d}dx$$
$$\Big\downarrow \; 913$$
$$\frac{(bc-ad)\int \frac{1}{cx^3+d}dx}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{750}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{c}x+\sqrt[3]{d}}dx}{3d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{(bc-ad)\left(\frac{\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{c}\left(\sqrt[3]{d}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{\int \frac{\sqrt[3]{c}\left(\sqrt[3]{d}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bc-ad)\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{(bc-ad)\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

$$\Big\downarrow \; 1103$$

$$\frac{(bc-ad)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{2\sqrt[3]{c}}}{3d^{2/3}}+\frac{\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3\sqrt[3]{c}d^{2/3}}\right)}{c}+\frac{ax}{c}$$

input
```
Int[(a + b/x^3)/(c + d/x^3),x]
```

output
```
(a*x)/c + ((b*c - a*d)*(Log[d^(1/3) + c^(1/3)*x]/(3*c^(1/3)*d^(2/3)) + (-(
(Sqrt[3]*ArcTan[(1 - (2*c^(1/3)*x)/d^(1/3))/Sqrt[3]])/c^(1/3)) - Log[d^(2/
3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/(3*d^(2/3))))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{ax}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^3+d\right)}\frac{(-ad+bc)\ln\left(x-\_R\right)}{\_R^2}}{3c^2}$	42
default	$\frac{ax}{c}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3c\left(\frac{d}{c}\right)^{\frac{2}{3}}}\right)(-ad+bc)}{c}$	110

input
```
int((a+b/x^3)/(c+d/x^3),x,method=_RETURNVERBOSE)
```

output
```
a*x/c+1/3/c^2*sum((-a*d+b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*c+d))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.69

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \left[\frac{6\,acd^2x - 3\sqrt{\frac{1}{3}}(bc^2d - acd^2)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^3+3\,(-cd^2)^{\frac{1}{3}}dx-d^2-3\sqrt{\frac{1}{3}}\left(2\,cdx^2+(-cd^2)^{\frac{2}{3}}x+(-cd^2)^{\frac{1}{3}}d\right)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}}}{cx^3+d}\right.}{6\,\ldots}\right.$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

output
```
[1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)
*log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-
c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) -
(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)
*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2),
 1/6*(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c
)*arctan(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^
(1/3)/c)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x
- (-c*d^2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2
/3)))/(c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \operatorname{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

input
```
integrate((a+b/x**3)/(c+d/x**3),x)
```

output
```
a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{ax}{c} + \frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2-x\left(\frac{d}{c}\right)^{\frac{1}{3}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} + \frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

output
```
a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/
c)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d
/c)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(
d/c)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = -\frac{\sqrt{3}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\left(-c^2d\right)^{\frac{2}{3}}} - \frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(-c^2d\right)^{\frac{2}{3}}} + \frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1
/3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2
/3))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d
/c)^(1/3)))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{a\,x}{c}-\frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} + \frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}} - \frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}$$

input
```
int((a + b/x^3)/(c + d/x^3),x)
```

output
```
(a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (lo
g(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d
- b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1
/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \frac{-2d^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2c^{\frac{1}{3}}x-d^{\frac{1}{3}}}{d^{\frac{1}{3}}\sqrt{3}}\right)a+2d^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2c^{\frac{1}{3}}x-d^{\frac{1}{3}}}{d^{\frac{1}{3}}\sqrt{3}}\right)bc+6c^{\frac{1}{3}}adx+d^{\frac{4}{3}}\log\left(c^{\frac{2}{3}}x^2-d^{\frac{1}{3}}c^{\frac{1}{3}}x+d^{\frac{2}{3}}\right)a-d^{\frac{1}{3}}\mathrm{lo}}{6c^{\frac{4}{3}}d}$$

input
```
int((a+b/x^3)/(c+d/x^3),x)
```

output
```
( - 2*d**(1/3)*sqrt(3)*atan((2*c**(1/3)*x - d**(1/3))/(d**(1/3)*sqrt(3)))*
a*d + 2*d**(1/3)*sqrt(3)*atan((2*c**(1/3)*x - d**(1/3))/(d**(1/3)*sqrt(3))
)*b*c + 6*c**(1/3)*a*d*x + d**(1/3)*log(c**(2/3)*x**2 - d**(1/3)*c**(1/3)*
x + d**(2/3))*a*d - d**(1/3)*log(c**(2/3)*x**2 - d**(1/3)*c**(1/3)*x + d**
(2/3))*b*c - 2*d**(1/3)*log(c**(1/3)*x + d**(1/3))*a*d + 2*d**(1/3)*log(c*
*(1/3)*x + d**(1/3))*b*c)/(6*c**(1/3)*c*d)
```

3.56 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} \, dx = -\frac{2(bc-ad)\sqrt{x}}{d^2} + \frac{bx}{d} + \frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

output
```
-2*(-a*d+b*c)*x^(1/2)/d^2+b*x/d+2*c*(-a*d+b*c)*ln(c+d*x^(1/2))/d^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} \, dx = \frac{\left(-2bc+2ad+bd\sqrt{x}\right)\sqrt{x}}{d^2} + \frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}$$

input
```
Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

output
```
((-2*b*c + 2*a*d + b*d*Sqrt[x])*Sqrt[x])/d^2 + (2*c*(b*c - a*d)*Log[c + d*
Sqrt[x]])/d^3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}dx$$

$$\Big\downarrow \text{900}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)\sqrt{x}}{c+d\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{86}$$

$$2\int \left(\frac{\sqrt{x}b}{d}+\frac{ad-bc}{d^2}+\frac{c(bc-ad)}{d^2\left(c+d\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{\sqrt{x}(bc-ad)}{d^2}+\frac{bx}{2d}\right)$$

input
```
Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

output
```
2*(-(((b*c - a*d)*Sqrt[x])/d^2) + (b*x)/(2*d) + (c*(b*c - a*d)*Log[c + d*S
qrt[x]])/d^3)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln\left(c+d\sqrt{x}\right)}{d^3}$	48
default	$\frac{bdx+2ad\sqrt{x}-2bc\sqrt{x}}{d^2}-\frac{2(ad-bc)c\ln\left(c+d\sqrt{x}\right)}{d^3}$	48

input
```
int((a+b*x^(1/2))/(c+d*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(1/2*b*d*x+a*d*x^(1/2)-b*c*x^(1/2))-2*(a*d-b*c)*c/d^3*ln(c+d*x^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\frac{bd^2x+2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)-2\left(bcd-ad^2\right)\sqrt{x}}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

output
```
(b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x
))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

output
```
Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c
/d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x*
*(3/2)/3)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2} + \frac{2\,(bc^2-acd)\log\left(d\sqrt{x}+c\right)}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

output
```
(b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)
/d^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx = \frac{bdx-2\,bc\sqrt{x}+2\,ad\sqrt{x}}{d^2} + \frac{2\,(bc^2-acd)\log\left(\left|d\sqrt{x}+c\right|\right)}{d^3}$$

input
```
integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

output
```
(b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*
sqrt(x) + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx=\sqrt{x}\left(\frac{2\,a}{d}-\frac{2\,b\,c}{d^2}\right)+\frac{\ln\left(c+d\,\sqrt{x}\right)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{d^3}+\frac{b\,x}{d}$$

input
```
int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

output
```
x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))
/d^3 + (b*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx =\frac{2\sqrt{x}\,a\,d^2-2\sqrt{x}\,bcd-2\log\left(\sqrt{x}\,d+c\right)acd+2\log\left(\sqrt{x}\,d+c\right)b\,c^2+b\,d^2x}{d^3}$$

input
```
int((a+b*x^(1/2))/(c+d*x^(1/2)),x)
```

output
```
(2*sqrt(x)*a*d**2 - 2*sqrt(x)*b*c*d - 2*log(sqrt(x)*d + c)*a*c*d + 2*log(s
qrt(x)*d + c)*b*c**2 + b*d**2*x)/d**3
```

3.57 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log \left(1+\sqrt[3]{x}\right)$$

output
```
6*x^(1/3)-3*x^(2/3)+x-6*ln(1+x^(1/3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx = 6\sqrt[3]{x}-3x^{2/3}+x-6\log \left(1+\sqrt[3]{x}\right)$$

input
```
Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

output
```
6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {900, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}-1}{\sqrt[3]{x}+1} \, dx$$

$$\Big\downarrow \; 900$$

$$3 \int -\frac{\left(1-\sqrt[3]{x}\right) x^{2/3}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \; 25$$

$$-3 \int \frac{\left(1-\sqrt[3]{x}\right) x^{2/3}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \; 86$$

$$-3 \int \left(-x^{2/3}+2\sqrt[3]{x}+\frac{2}{\sqrt[3]{x}+1}-2\right)d\sqrt[3]{x}$$

$$\Big\downarrow \; 2009$$

$$3\left(-x^{2/3}+\frac{x}{3}+2\sqrt[3]{x}-2\log\left(\sqrt[3]{x}+1\right)\right)$$

input
```
Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

output
```
3*(2*x^(1/3) - x^(2/3) + x/3 - 2*Log[1 + x^(1/3)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(x^{\frac{1}{3}}+1\right)$	21
default	$6x^{\frac{1}{3}}-3x^{\frac{2}{3}}+x-6\ln\left(x^{\frac{1}{3}}+1\right)$	21
trager	$-1+x+6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	32
meijerg	$\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}-6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(x^{\frac{1}{3}}+1\right)+\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}$	39

input
```
int((x^(1/3)-1)/(x^(1/3)+1),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/3)-3*x^(2/3)+x-6*ln(x^(1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = -3x^{\frac{2}{3}} + 6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right)$$

input
```
integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

output
```
-3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 3\,x^{\frac{2}{3}} + 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
```

output
```
x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = x - 6\,\ln\left(x^{1/3} + 1\right) + 6\,x^{1/3} - 3\,x^{2/3}$$

input
```
int((x^(1/3) - 1)/(x^(1/3) + 1),x)
```

output
```
x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx = -3x^{\frac{2}{3}} + 6x^{\frac{1}{3}} - 6\log\left(x^{\frac{1}{3}} + 1\right) + x$$

input
```
int((-1+x^(1/3))/(1+x^(1/3)),x)
```

output
```
- 3*x**(2/3) + 6*x**(1/3) - 6*log(x**(1/3) + 1) + x
```

3.58 $\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x-6\text{arctanh}\left(\sqrt[3]{x}\right)$$

output
```
6*x^(1/3)+x-6*arctanh(x^(1/3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x-6\text{arctanh}\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

output
```
6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {900, 25, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}+1}{x^{2/3}-1}dx$$

$$\Big\downarrow \text{900}$$

$$3\int -\frac{\left(x^{2/3}+1\right)x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{25}$$

$$-3\int \frac{\left(x^{2/3}+1\right)x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{363}$$

$$3\left(\frac{x}{3}-2\int \frac{x^{2/3}}{1-x^{2/3}}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{262}$$

$$3\left(\frac{x}{3}-2\left(\int \frac{1}{1-x^{2/3}}d\sqrt[3]{x}-\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$3\left(\frac{x}{3}-2\left(\text{arctanh}\left(\sqrt[3]{x}\right)-\sqrt[3]{x}\right)\right)$$

input
```
Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

output
```
3*(x/3 - 2*(-x^(1/3) + ArcTanh[x^(1/3)]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(x^{\frac{1}{3}}+1\right)$	24
default	$x+6x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)-3\ln\left(x^{\frac{1}{3}}+1\right)$	24
trager	$x-2+6x^{\frac{1}{3}}+3\ln\left(-\frac{2x^{\frac{2}{3}}-2x^{\frac{1}{3}}-x+1}{1+x}\right)$	34
meijerg	$-\frac{3i\left(2ix^{\frac{1}{3}}-2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}+\frac{3i\left(-\frac{2ix^{\frac{1}{3}}\left(5x^{\frac{2}{3}}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x^{\frac{1}{3}}\right)\right)}{2}$	43

input
```
int((1+x^(2/3))/(-1+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(x^(1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

input
```
integrate((1+x**(2/3))/(-1+x**(2/3)),x)
```

output
```
6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
```

output
```
x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = x-6\,\mathrm{atanh}\left(x^{1/3}\right)+6\,x^{1/3}$$

input
```
int((x^(2/3) + 1)/(x^(2/3) - 1),x)
```

output
```
x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 6x^{\frac{1}{3}}+3\log\left(x^{\frac{1}{3}}-1\right)-3\log\left(x^{\frac{1}{3}}+1\right)+x$$

input
```
int((1+x^(2/3))/(-1+x^(2/3)),x)
```

output
```
6*x**(1/3) + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1) + x
```

3.59 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\ 2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)$$

output
```
-128*x^(1/4)+x-256/3*2^(1/3)*arctan(1/6*(2^(1/3)-x^(1/4))*2^(2/3)*3^(1/2))
*3^(1/2)+256/3*2^(1/3)*ln(2*2^(1/3)+x^(1/4))-128/3*2^(1/3)*ln(4*2^(2/3)-2*
2^(1/3)*x^(1/4)+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\arctan\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}} + \frac{256}{3}\sqrt[3]{2}\log\left(4+2^{2/3}\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(-8+2\ 2^{2/3}\sqrt[4]{x}-\sqrt[3]{2}\sqrt{x}\right)$$

input
```
Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

output
```
-128*x^(1/4) + x - (256*2^(1/3)*ArcTan[1/Sqrt[3] - x^(1/4)/(2^(1/3)*Sqrt[3
])])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (128*2^(1/3)*Log
[-8 + 2*2^(2/3)*x^(1/4) - 2^(1/3)*Sqrt[x]])/3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {900, 25, 959, 843, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/4}-16}{x^{3/4}+16}dx$$

$$\Big\downarrow \text{ 900}$$

$$4\int -\frac{\left(16-x^{3/4}\right)x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}$$

$$\Big\downarrow \text{ 25}$$

$$-4\int \frac{\left(16-x^{3/4}\right)x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}$$

$$\Big\downarrow \text{ 959}$$

$$4\left(\frac{x}{4}-32\int \frac{x^{3/4}}{x^{3/4}+16}d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{ 843}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\int \frac{1}{x^{3/4}+16}d\sqrt[4]{x}\right)\right)$$

$$\Big\downarrow \text{ 750}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{1}{\sqrt[4]{x}+2\sqrt[3]{2}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\int \frac{4\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{4\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\sqrt[3]{2}\int \frac{1}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}-\frac{1}{2}\int -\frac{2\left(\sqrt[3]{2}-\sqrt[4]{x}\right)}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\sqrt[3]{2}\int \frac{1}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}+\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{3\int \frac{1}{-\sqrt{x}-3}d\left(1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}\right)+\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{\int \frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}}d\sqrt[4]{x}-\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}}{\sqrt{3}}\right)}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{x}{4}-32\left(\sqrt[4]{x}-16\left(\frac{-\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[4]{x}}{\sqrt[3]{2}}}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)}{12\ 2^{2/3}}+\frac{\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}\right)\right)\right)$$

input
```
Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

output
```
4*(x/4 - 32*(x^(1/4) - 16*(Log[2*2^(1/3) + x^(1/4)]/(12*2^(2/3)) + (-(Sqrt
[3]*ArcTan[(1 - x^(1/4)/2^(1/3))/Sqrt[3]]) - Log[4*2^(2/3) - 2*2^(1/3)*x^(
1/4) + Sqrt[x]]/2)/(12*2^(2/3)))))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 63.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
derivativedivides	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\Big(\frac{\sqrt{3}\,\big(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1}{3}}{3}$
default	$x-128x^{\frac{1}{4}}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\Big(\frac{\sqrt{3}\,\big(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1}{3}}{3}$
meijerg	$-\frac{128\,2^{\frac{1}{3}}\left(\frac{3\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}\left(\frac{2\,2^{\frac{1}{3}}\ln\left(1+\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}\right)}{x^{\frac{1}{4}}}-\frac{2^{\frac{1}{3}}\ln\left(1-\frac{2^{\frac{2}{3}}x^{\frac{1}{4}}}{4}+\frac{2^{\frac{1}{3}}\sqrt{x}}{8}\right)}{x^{\frac{1}{4}}}+\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}}{8-2^{\frac{2}{3}}x^{\frac{1}{4}}}\right)}{x^{\frac{1}{4}}}\right)}{4}\right)}{3}+\frac{128}{}$
trager	Expression too large to display

input
```
int((-16+x^(3/4))/(16+x^(3/4)),x,method=_RETURNVERBOSE)
```

output
```
x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)
-16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8
*16^(2/3)*x^(1/4)-1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx=\frac{256}{3}\,\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\,\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) -
128/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)
*log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = -128\sqrt[4]{x}+x+\frac{256\cdot\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\cdot\sqrt[3]{2}\right)}{3} - \frac{128\cdot\sqrt[3]{2}\log\left(-2\cdot\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\cdot\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

output
```
-128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/
3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(
3)*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*
log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

input
```
integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

output
```
256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*
log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = x+\frac{256\,2^{1/3}\,\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}-6144\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

input
```
int((x^(3/4) - 16)/(x^(3/4) + 16),x)
```

output
```
x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128
*2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1
))/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(
1/2)*1i + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx = \frac{256\,2^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{\left(x^{\frac{1}{4}}-2^{\frac{1}{3}}\right)2^{\frac{2}{3}}}{2\sqrt{3}}\right)}{3} - 128x^{\frac{1}{4}} + \frac{256\,2^{\frac{1}{3}}\log\left(x^{\frac{1}{4}}+2\,2^{\frac{1}{3}}\right)}{3} - \frac{128\,2^{\frac{1}{3}}\log\left(-2x^{\frac{1}{4}}2^{\frac{1}{3}}+\sqrt{x}+4\,2^{\frac{2}{3}}\right)}{3} + x$$

input
```
int((-16+x^(3/4))/(16+x^(3/4)),x)
```

output
```
(256*2**(1/3)*sqrt(3)*atan((x**(1/4) - 2**(1/3))/(2**(1/3)*sqrt(3))) - 384
*x**(1/4) + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3)) - 128*2**(1/3)*log( -
2*x**(1/4)*2**(1/3) + sqrt(x) + 4*2**(2/3)) + 3*x)/3
```

3.60 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)$$

output
```
-6*x^(1/3)-3*x^(2/3)-x-6*ln(1-x^(1/3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(-1+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

output
```
-6*x^(1/3) - 3*x^(2/3) - x - 6*Log[-1 + x^(1/3)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {898, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{1}{\sqrt[3]{x}}+1}{\frac{1}{\sqrt[3]{x}}-1}dx$$

$$\Big\downarrow \text{ 898}$$

$$\int \frac{\sqrt[3]{x}+1}{1-\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{ 900}$$

$$3\int \frac{\left(\sqrt[3]{x}+1\right)x^{2/3}}{1-\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 86}$$

$$3\int \left(-x^{2/3}-2\sqrt[3]{x}-\frac{2}{\sqrt[3]{x}-1}-2\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-x^{2/3}-\frac{x}{3}-2\sqrt[3]{x}-2\log\left(1-\sqrt[3]{x}\right)\right)$$

input
```
Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

output
```
3*(-2*x^(1/3) - x^(2/3) - x/3 - 2*Log[1 - x^(1/3)])
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
default	$-x-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}-6\ln\left(x^{\frac{1}{3}}-1\right)$	23
trager	$-x+2-6x^{\frac{1}{3}}-3x^{\frac{2}{3}}-2\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	32
meijerg	$-\frac{x^{\frac{1}{3}}\left(4x^{\frac{2}{3}}+6x^{\frac{1}{3}}+12\right)}{4}-6\ln\left(1-x^{\frac{1}{3}}\right)-\frac{x^{\frac{1}{3}}\left(3x^{\frac{1}{3}}+6\right)}{2}$	41

input
```
int((1+1/x^(1/3))/(-1+1/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -3x^{\frac{2}{3}}-6\sqrt[3]{x}-x-6\log\left(\sqrt[3]{x}-1\right)$$

input
```
integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

output
```
-3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
```

output
```
-x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -x-6\,\ln\left(x^{1/3}-1\right)-6\,x^{1/3}-3\,x^{2/3}$$

input
```
int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
```

output
```
- x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx = -3x^{\frac{2}{3}} - 6x^{\frac{1}{3}} - 6\log\left(x^{\frac{1}{3}} - 1\right) - x$$

input
```
int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
```

output
```
- 3*x**(2/3) - 6*x**(1/3) - 6*log(x**(1/3) - 1) - x
```

3.61 $\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = a^4d^2x - \frac{2a^2b^2d^2x^{1+2n}}{1+2n} + \frac{b^4d^2x^{1+4n}}{1+4n}$$

output
```
a^4*d^2*x-2*a^2*b^2*d^2*x^(1+2*n)/(1+2*n)+b^4*d^2*x^(1+4*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = \frac{d^2x(a^4(1+6n+8n^2) - 2a^2b^2(1+4n)x^{2n} + b^4(1+2n)x^{4n})}{1+6n+8n^2}$$

input
```
Integrate[(a + b*x^n)^2*(a*d - b*d*x^n)^2,x]
```

output
```
(d^2*x*(a^4*(1 + 6*n + 8*n^2) - 2*a^2*b^2*(1 + 4*n)*x^(2*n) + b^4*(1 + 2*n
)*x^(4*n)))/(1 + 6*n + 8*n^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {780, 775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx$$

$$\Big\downarrow \text{ 780}$$

$$\int \left(a^2d-b^2dx^{2n}\right)^2 dx$$

$$\Big\downarrow \text{ 775}$$

$$\int \left(a^4d^2-2a^2b^2d^2x^{2n}+b^4d^2x^{4n}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4d^2x-\frac{2a^2b^2d^2x^{2n+1}}{2n+1}+\frac{b^4d^2x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n)^2*(a*d - b*d*x^n)^2,x]
```

output
```
a^4*d^2*x - (2*a^2*b^2*d^2*x^(1 + 2*n))/(1 + 2*n) + (b^4*d^2*x^(1 + 4*n))/
(1 + 4*n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 780
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

method	result
risch	$a^4d^2x+\frac{b^4d^2x\,x^{4n}}{1+4n}-\frac{2b^2a^2d^2x\,x^{2n}}{1+2n}$
norman	$a^4d^2x+\frac{b^4d^2x\,\mathrm{e}^{4n\ln(x)}}{1+4n}-\frac{2b^2a^2d^2x\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{2x\,x^{4n}b^4d^2n+b^4d^2x\,x^{4n}-8x\,x^{2n}a^2b^2d^2n-2b^2a^2d^2x\,x^{2n}+8x\,a^4d^2n^2+6x\,a^4d^2n+a^4d^2x}{(1+4n)(1+2n)}$
orering	$x(a+b\,x^n)^2\,(ad-bd\,x^n)^2-\frac{6x^2n\left(\frac{2(a+b\,x^n)(ad-bd\,x^n)^2b\,x^nn}{x}-\frac{2(a+b\,x^n)^2(ad-bd\,x^n)bd\,x^nn}{x}\right)}{8n^2+6n+1}+\frac{x^3\left(\frac{2b^2x^{2n}n^2(\cdots}{\cdots}\right.}{\cdots}$

input
```
int((a+b*x^n)^2*(a*d-b*d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^4*d^2*x+b^4*d^2/(1+4*n)*x*(x^n)^4-2*b^2*a^2*d^2/(1+2*n)*x*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.80

$$\int (a+bx^n)^2\,(ad-bdx^n)^2\;dx = \frac{(2\,b^4d^2n+b^4d^2)xx^{4\,n}-2\,(4\,a^2b^2d^2n+a^2b^2d^2)xx^{2\,n}+(8\,a^4d^2n^2+6\,a^4d^2n+a^4d^2)x}{8\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)^2*(a*d-b*d*x^n)^2,x, algorithm="fricas")
```

output
```
((2*b^4*d^2*n + b^4*d^2)*x*x^(4*n) - 2*(4*a^2*b^2*d^2*n + a^2*b^2*d^2)*x*x
^(2*n) + (8*a^4*d^2*n^2 + 6*a^4*d^2*n + a^4*d^2)*x)/(8*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(49) = 98$.

Time = 0.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 4.47

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = \begin{cases} a^4d^2x - 2a^2b^2d^2\log(x) - \frac{b^4d^2}{x} & \text{for } n = -\frac{1}{2} \\ a^4d^2x - 4a^2b^2d^2\sqrt{x} + b^4d^2\log(x) & \text{for } n = -\frac{1}{4} \\ \frac{8a^4d^2n^2x}{8n^2+6n+1} + \frac{6a^4d^2nx}{8n^2+6n+1} + \frac{a^4d^2x}{8n^2+6n+1} - \frac{8a^2b^2d^2nxx^{2n}}{8n^2+6n+1} - \frac{2a^2b^2d^2xx^{2n}}{8n^2+6n+1} + \frac{2b^4d^2nxx^{4n}}{8n^2+6n+1} + \frac{b^4d^2xx^{4n}}{8n^2+6n+1} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(a*d-b*d*x**n)**2,x)
```

output
```
Piecewise((a**4*d**2*x - 2*a**2*b**2*d**2*log(x) - b**4*d**2/x, Eq(n, -1/2
)), (a**4*d**2*x - 4*a**2*b**2*d**2*sqrt(x) + b**4*d**2*log(x), Eq(n, -1/4
)), (8*a**4*d**2*n**2*x/(8*n**2 + 6*n + 1) + 6*a**4*d**2*n*x/(8*n**2 + 6*n
 + 1) + a**4*d**2*x/(8*n**2 + 6*n + 1) - 8*a**2*b**2*d**2*n*x*x**(2*n)/(8*
n**2 + 6*n + 1) - 2*a**2*b**2*d**2*x*x**(2*n)/(8*n**2 + 6*n + 1) + 2*b**4*
d**2*n*x*x**(4*n)/(8*n**2 + 6*n + 1) + b**4*d**2*x*x**(4*n)/(8*n**2 + 6*n
+ 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = a^4d^2x + \frac{b^4d^2x^{4n+1}}{4n+1} - \frac{2a^2b^2d^2x^{2n+1}}{2n+1}$$

input
```
integrate((a+b*x^n)^2*(a*d-b*d*x^n)^2,x, algorithm="maxima")
```

output
```
a^4*d^2*x + b^4*d^2*x^(4*n + 1)/(4*n + 1) - 2*a^2*b^2*d^2*x^(2*n + 1)/(2*n
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = \frac{8 a^4 d^2 n^2 x + 2 b^4 d^2 n x x^{4 n} - 8 a^2 b^2 d^2 n x x^{2 n} + 6 a^4 d^2 n x + b^4 d^2 x x^{4 n} - 2 a^2 b^2 d^2 x x^{2 n} + a^4 d^2 x}{8 n^2 + 6 n + 1}$$

input
```
integrate((a+b*x^n)^2*(a*d-b*d*x^n)^2,x, algorithm="giac")
```

output
```
(8*a^4*d^2*n^2*x + 2*b^4*d^2*n*x*x^(4*n) - 8*a^2*b^2*d^2*n*x*x^(2*n) + 6*a
^4*d^2*n*x + b^4*d^2*x*x^(4*n) - 2*a^2*b^2*d^2*x*x^(2*n) + a^4*d^2*x)/(8*n
^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = a^4 d^2 x + \frac{b^4 d^2 x x^{4 n}}{4 n + 1} - \frac{2 a^2 b^2 d^2 x x^{2 n}}{2 n + 1}$$

input
```
int((a + b*x^n)^2*(a*d - b*d*x^n)^2,x)
```

output
```
a^4*d^2*x + (b^4*d^2*x*x^(4*n))/(4*n + 1) - (2*a^2*b^2*d^2*x*x^(2*n))/(2*n
 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int (a+bx^n)^2 (ad-bdx^n)^2 \, dx = \frac{d^2x(2x^{4n}b^4n+x^{4n}b^4-8x^{2n}a^2b^2n-2x^{2n}a^2b^2+8a^4n^2+6a^4n+a^4)}{8n^2+6n+1}$$

input
```
int((a+b*x^n)^2*(a*d-b*d*x^n)^2,x)
```

output
```
(d**2*x*(2*x**(4*n)*b**4*n + x**(4*n)*b**4 - 8*x**(2*n)*a**2*b**2*n - 2*x*
*(2*n)*a**2*b**2 + 8*a**4*n**2 + 6*a**4*n + a**4))/(8*n**2 + 6*n + 1)
```

3.62 $\int (a+bx^n)(ad-bdx^n)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int (a+bx^n)(ad-bdx^n)\,dx = a^2dx - \frac{b^2dx^{1+2n}}{1+2n}$$

output
```
a^2*d*x-b^2*d*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(ad-bdx^n)\,dx = d\left(a^2x - \frac{b^2x^{1+2n}}{1+2n}\right)$$

input
```
Integrate[(a + b*x^n)*(a*d - b*d*x^n),x]
```

output
```
d*(a^2*x - (b^2*x^(1 + 2*n))/(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {780, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(ad-bdx^n)\,dx$$

$$\Big\downarrow \text{780}$$

$$\int \left(a^2d-b^2dx^{2n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx-\frac{b^2dx^{2n+1}}{2n+1}$$

input
```
Int[(a + b*x^n)*(a*d - b*d*x^n),x]
```

output
```
a^2*d*x - (b^2*d*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 780
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
risch	$a^2 dx - \frac{d\,b^2 x\,x^{2n}}{1+2n}$	27
norman	$a^2 dx - \frac{d\,b^2 x\,e^{2n\ln(x)}}{1+2n}$	29
parallelrisch	$-\frac{d\,b^2 x\,x^{2n} - 2x\,a^2 dn - a^2 dx}{1+2n}$	37
orering	$x(a+b\,x^n)\,(ad-bd\,x^n) - \frac{x^2\left(\frac{b\,x^n n(ad-bd\,x^n)}{x} - \frac{(a+b\,x^n)bd\,x^n n}{x}\right)}{1+2n}$	73

input
```
int((a+b*x^n)*(a*d-b*d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x-d*b^2/(1+2*n)*x*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int (a+bx^n)\,(ad-bdx^n)\;dx = -\frac{b^2dxx^{2\,n} - (2\,a^2dn + a^2d)x}{2\,n+1}$$

input
```
integrate((a+b*x^n)*(a*d-b*d*x^n),x, algorithm="fricas")
```

output
```
-(b^2*d*x*x^(2*n) - (2*a^2*d*n + a^2*d)*x)/(2*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int (a+bx^n)\,(ad-bdx^n)\;dx = \begin{cases} \frac{2a^2dnx}{2n+1} + \frac{a^2dx}{2n+1} - \frac{b^2dxx^{2n}}{2n+1} & \text{for } n \neq -\frac{1}{2} \\ a^2dx - b^2d\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)*(a*d-b*d*x**n),x)
```

output
```
Piecewise((2*a**2*d*n*x/(2*n + 1) + a**2*d*x/(2*n + 1) - b**2*d*x*x**(2*n)
/(2*n + 1), Ne(n, -1/2)), (a**2*d*x - b**2*d*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a+bx^n)(ad-bdx^n)\,dx = a^2dx - \frac{b^2dx^{2n+1}}{2n+1}$$

input
```
integrate((a+b*x^n)*(a*d-b*d*x^n),x, algorithm="maxima")
```

output
```
a^2*d*x - b^2*d*x^(2*n + 1)/(2*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int (a+bx^n)(ad-bdx^n)\,dx = \frac{2a^2dnx - b^2dxx^{2n} + a^2dx}{2n+1}$$

input
```
integrate((a+b*x^n)*(a*d-b*d*x^n),x, algorithm="giac")
```

output
```
(2*a^2*d*n*x - b^2*d*x*x^(2*n) + a^2*d*x)/(2*n + 1)
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a+bx^n)(ad-bdx^n)\,dx=a^2\,d\,x-\frac{b^2\,d\,x\,x^{2\,n}}{2\,n+1}$$

input
```
int((a + b*x^n)*(a*d - b*d*x^n),x)
```

output
```
a^2*d*x - (b^2*d*x*x^(2*n))/(2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int (a+bx^n)(ad-bdx^n)\,dx=\frac{dx(-x^{2n}b^2+2a^2n+a^2)}{2n+1}$$

input
```
int((a+b*x^n)*(a*d-b*d*x^n),x)
```

output
```
(d*x*( - x**(2*n)*b**2 + 2*a**2*n + a**2))/(2*n + 1)
```

3.63 $\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{a^2d}$$

output
```
x*hypergeom([1, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)/a^2/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{a^2d}$$

input
```
Integrate[1/((a + b*x^n)*(a*d - b*d*x^n)),x]
```

output
```
(x*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), (b^2*x^(2*n))/a^2])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {780, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}dx$$
$$\Big\downarrow \text{780}$$
$$\int \frac{1}{a^2d-b^2dx^{2n}}dx$$
$$\Big\downarrow \text{778}$$
$$\frac{x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{a^2d}$$

input
```
Int[1/((a + b*x^n)*(a*d - b*d*x^n)),x]
```

output
```
(x*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/(a^2*
d)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 780
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)\,(ad-bd\,x^n)}dx$$

input
```
int(1/(a+b*x^n)/(a*d-b*d*x^n),x)
```

output
```
int(1/(a+b*x^n)/(a*d-b*d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(ad-bdx^n)}\,dx = \int -\frac{1}{(bdx^n-ad)(bx^n+a)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(a*d-b*d*x^n),x, algorithm="fricas")
```

output
```
integral(-1/(b^2*d*x^(2*n) - a^2*d), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)\,(ad-bdx^n)}\,dx = \frac{\int \frac{1}{a^2-b^2x^{2n}}\,dx}{d}$$

input
```
integrate(1/(a+b*x**n)/(a*d-b*d*x**n),x)
```

output
```
Integral(1/(a**2 - b**2*x**(2*n)), x)/d
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx = \int -\frac{1}{(bdx^n-ad)(bx^n+a)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(a*d-b*d*x^n),x, algorithm="maxima")
```

output
```
-integrate(1/((b*d*x^n - a*d)*(b*x^n + a)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx = \int -\frac{1}{(bdx^n-ad)(bx^n+a)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(a*d-b*d*x^n),x, algorithm="giac")
```

output
```
integrate(-1/((b*d*x^n - a*d)*(b*x^n + a)), x)
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx = \frac{x\,{}_2F_1\left(1,\frac{1}{2n};\ \frac{1}{2n}+1;\ \frac{b^2x^{2n}}{a^2}\right)}{a^2\,d}$$

input
```
int(1/((a + b*x^n)*(a*d - b*d*x^n)),x)
```

output
```
(x*hypergeom([1, 1/(2*n)], 1/(2*n) + 1, (b^2*x^(2*n))/a^2))/(a^2*d)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)(ad-bdx^n)}\,dx = -\frac{\int \frac{1}{x^{2n}b^2-a^2}dx}{d}$$

input
```
int(1/(a+b*x^n)/(a*d-b*d*x^n),x)
```

output
```
( - int(1/(x**(2*n)*b**2 - a**2),x))/d
```

3.64 $\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

output
```
a^2*x*(a-b*x^n)^(1/2)*(a+b*x^n)^(1/2)*hypergeom([-3/2, 1/2/n],[1+1/2/n],b^
2*x^(2*n)/a^2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

output
```
(a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1
+ 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$$

$$\Big\downarrow \; 785$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \left(a^2-b^2x^{2n}\right)^{3/2}dx}{\sqrt{a^2-b^2x^{2n}}}$$

$$\Big\downarrow \; 779$$

$$\frac{a^2\sqrt{a-bx^n}\sqrt{a+bx^n}\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

$$\Big\downarrow \; 778$$

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

output
```
(a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
 + n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int (a - b\,x^{n})^{\frac{3}{2}} (a + b\,x^{n})^{\frac{3}{2}}\,dx$$

input
```
int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

output
```
int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a - bx^n)^{3/2} (a + bx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (a-bx^n)^{\frac{3}{2}}(a+bx^n)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

output
```
Integral((a - b*x**n)**(3/2)*(a + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\int (a+b\,x^n)^{3/2}\,(a-b\,x^n)^{3/2}\,dx$$

input
```
int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2),x)
```

output
```
int((a + b*x^n)^(3/2)*(a - b*x^n)^(3/2), x)
```

Reduce [F]

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx=\frac{-x^{2n}\sqrt{x^nb+a}\,\sqrt{-x^nb+a}\,b^2nx-x^{2n}\sqrt{x^nb+a}\,\sqrt{-x^nb+a}\,b^2x+4\sqrt{x^nb+a}\,\sqrt{-x^nb+a}\,a^2}{}$$

input
```
int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

output
```
( - x**(2*n)*sqrt(x**n*b + a)*sqrt( - x**n*b + a)*b**2*n*x - x**(2*n)*sqrt
(x**n*b + a)*sqrt( - x**n*b + a)*b**2*x + 4*sqrt(x**n*b + a)*sqrt( - x**n*
b + a)*a**2*n*x + sqrt(x**n*b + a)*sqrt( - x**n*b + a)*a**2*x - 9*int((sqr
t(x**n*b + a)*sqrt( - x**n*b + a))/(3*x**(2*n)*b**2*n**2 + 4*x**(2*n)*b**2
*n + x**(2*n)*b**2 - 3*a**2*n**2 - 4*a**2*n - a**2),x)*a**4*n**4 - 12*int(
(sqrt(x**n*b + a)*sqrt( - x**n*b + a))/(3*x**(2*n)*b**2*n**2 + 4*x**(2*n)*
b**2*n + x**(2*n)*b**2 - 3*a**2*n**2 - 4*a**2*n - a**2),x)*a**4*n**3 - 3*i
nt((sqrt(x**n*b + a)*sqrt( - x**n*b + a))/(3*x**(2*n)*b**2*n**2 + 4*x**(2*
n)*b**2*n + x**(2*n)*b**2 - 3*a**2*n**2 - 4*a**2*n - a**2),x)*a**4*n**2)/(
3*n**2 + 4*n + 1)
```

3.65 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

output
```
x*(a-b*x^n)^(1/2)*(a+b*x^n)^(1/2)*hypergeom([-1/2, 1/2/n],[1+1/2/n],b^2*x^
(2*n)/a^2)/(1-b^2*x^(2*n)/a^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/
(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{ 785}$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \sqrt{a^2-b^2x^{2n}}dx}{\sqrt{a^2-b^2x^{2n}}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\sqrt{a-bx^n}\sqrt{a+bx^n}\int \sqrt{1-\frac{b^2x^{2n}}{a^2}}dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

input
```
Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n
^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{a-b\,x^{n}}\,\sqrt{a+b\,x^{n}}dx$$

input
```
int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

output
```
int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx=\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

input
```
integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

input
```
integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \int \sqrt{a+b\,x^n}\,\sqrt{a-b\,x^n}\,dx$$

input
```
int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2),x)
```

output
```
int((a + b*x^n)^(1/2)*(a - b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{\sqrt{x^n b+a}\,\sqrt{-x^n b+a}\,x-\left(\int \frac{\sqrt{x^n b+a}\,\sqrt{-x^n b+a}}{x^{2n}b^2n+x^{2n}b^2-a^2n-a^2}dx\right)a^2n^2-\left(\int \frac{\sqrt{x^n b+a}\,\sqrt{-x^n b+a}}{x^{2n}b^2n+x^{2n}b^2-a^2n-a^2}dx\right)a^2n}{n+1}$$

input
```
int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

output
```
(sqrt(x**n*b + a)*sqrt( - x**n*b + a)*x - int((sqrt(x**n*b + a)*sqrt( - x*
*n*b + a))/(x**(2*n)*b**2*n + x**(2*n)*b**2 - a**2*n - a**2),x)*a**2*n**2
- int((sqrt(x**n*b + a)*sqrt( - x**n*b + a))/(x**(2*n)*b**2*n + x**(2*n)*b
**2 - a**2*n - a**2),x)*a**2*n)/(n + 1)
```

3.66 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

output
```
x*(a-b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)
/((1-b^2*x^(2*n)/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-bx^n)^p (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{785}$$

$$(a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p} \int \left(a^2-b^2x^{2n}\right)^p dx$$

$$\Big\downarrow \text{779}$$

$$(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int (a-b\,x^{n})^{p} (a+b\,x^{n})^{p} \,dx$$

input
```
int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a-bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p(-bx^n+a)^p \,dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F]

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (a - bx^n)^p (a + bx^n)^p \, dx$$

input
```
integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (bx^n + a)^p (-bx^n + a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Giac [F]

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (bx^n + a)^p (-bx^n + a)^p \, dx$$

input
```
integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a - bx^n)^p (a + bx^n)^p \, dx = \int (a + b\,x^n)^p (a - b\,x^n)^p \, dx$$

input
```
int((a + b*x^n)^p*(a - b*x^n)^p,x)
```

output
```
int((a + b*x^n)^p*(a - b*x^n)^p, x)
```

Reduce [F]

$$\int (a - bx^n)^p (a + bx^n)^p \, dx$$
$$= \frac{(x^n b + a)^p (-x^n b + a)^p x - 4\left(\int \frac{(x^n b+a)^p(-x^n b+a)^p}{2x^{2n}b^2np+x^{2n}b^2-2a^2np-a^2}dx\right) a^2n^2p^2 - 2\left(\int \frac{(x^n b+a)^p(-x^n b+a)^p}{2x^{2n}b^2np+x^{2n}b^2-2a^2np-a^2}dx\right) a^2n}{2np + 1}$$

input
```
int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*( - x**n*b + a)**p*x - 4*int(((x**n*b + a)**p*( - x**n*b
+ a)**p)/(2*x**(2*n)*b**2*n*p + x**(2*n)*b**2 - 2*a**2*n*p - a**2),x)*a**2
*n**2*p**2 - 2*int(((x**n*b + a)**p*( - x**n*b + a)**p)/(2*x**(2*n)*b**2*n
*p + x**(2*n)*b**2 - 2*a**2*n*p - a**2),x)*a**2*n*p)/(2*n*p + 1)
```

3.67 $\int (a+bx^n)(c+dx^n)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int (a+bx^n)(c+dx^n)^4 \, dx = ac^4x + \frac{c^3(bc+4ad)x^{1+n}}{1+n} + \frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n} + \frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n} + \frac{d^3(4bc+ad)x^{1+4n}}{1+4n} + \frac{bd^4x^{1+5n}}{1+5n}$$

output
```
a*c^4*x+c^3*(4*a*d+b*c)*x^(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^(1+2*n)/(1+2
*n)+2*c*d^2*(2*a*d+3*b*c)*x^(1+3*n)/(1+3*n)+d^3*(a*d+4*b*c)*x^(1+4*n)/(1+4
*n)+b*d^4*x^(1+5*n)/(1+5*n)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \frac{bx(c+dx^n)^5 - (bc-ad(1+5n))x\left(c^4 + \frac{4c^3dx^n}{1+n} + \frac{6c^2d^2x^{2n}}{1+2n} + \frac{4cd^3x^{3n}}{1+3n} + \frac{d^4x^{4n}}{1+4n}\right)}{d+5dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

output
```
(b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n)
+ (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*
n))/(1 + 4*n)))/(d + 5*d*n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^4\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^3x^n(4ad+bc)+2c^2dx^{2n}(3ad+2bc)+d^3x^{4n}(ad+4bc)+2cd^2x^{3n}(2ad+3bc)+ac^4+bd^4x^{5n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

output
```
a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)
*x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n)
+ (d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n
)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^{4}x+\frac{b\,d^{4}x\,x^{5n}}{5n+1}+\frac{c^{3}(4ad+bc)x\,x^{n}}{1+n}+\frac{d^{3}(ad+4bc)x\,x^{4n}}{1+4n}+\frac{2c\,d^{2}(2ad+3bc)x\,x^{3n}}{1+3n}+\frac{2c^{2}d(3ad+2bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^{4}x+\frac{b\,d^{4}x\,e^{5n\ln(x)}}{5n+1}+\frac{c^{3}(4ad+bc)x\,e^{n\ln(x)}}{1+n}+\frac{d^{3}(ad+4bc)x\,e^{4n\ln(x)}}{1+4n}+\frac{2c\,d^{2}(2ad+3bc)x\,e^{3n\ln(x)}}{1+3n}+\frac{2c^{2}d(3ad+2b}{1+}$
parallelrisch	$a\,c^{4}x+616x\,x^{n}a\,c^{3}d\,n^{3}+480x\,x^{n}a\,c^{3}d\,n^{4}+284x\,x^{n}a\,c^{3}d\,n^{2}+56x\,x^{n}a\,c^{3}dn+24x\,x^{5n}b\,d^{4}n^{4}+50x\,x^{5n}b\,d^{4}n^{3}+30x\,x^{4n}a\,d^{4}n^{4}+$
orering	Expression too large to display

input
```
int((a+b*x^n)*(c+d*x^n)^4,x,method=_RETURNVERBOSE)
```

output
```
a*c^4*x+b*d^4/(5*n+1)*x*(x^n)^5+c^3*(4*a*d+b*c)/(1+n)*x*x^n+d^3*(a*d+4*b*c
)/(1+4*n)*x*(x^n)^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*(x^n)^3+2*c^2*d*(3*a*d
+2*b*c)/(1+2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(132) = 264.

Time = 0.18 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.99

$$\int (a+bx^n)\,(c+dx^n)^4\,dx$$
$$=\frac{(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4)xx^{5\,n}+(4\,bcd^3+ad^4+30\,(4\,bcd^3+ad^4)n^4+61\,(4\,bcd^3}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*
n) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d
^4)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n)
 + 2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b
*c^2*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2
*d^2 + 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*
d + 3*a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d +
 3*a*c^2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4
*a*c^3*d + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*
(b*c^4 + 4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5
 + 274*a*c^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(
120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2744 vs. $2(124) = 248$.

Time = 0.82 (sec) , antiderivative size = 2744, normalized size of antiderivative = 20.79

$$\int (a + bx^n) (c + dx^n)^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

output
```
Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**
2 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 -
4*b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*
sqrt(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*
sqrt(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*
d**4/(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c*
*2*d**2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/
3)/2 + 12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3)
- 3*b*d**4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3
+ 12*a*c**2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4
*x**(3/4)/3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*lo
g(x) - 4*b*d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) +
10*a*c**2*d**2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c
**4*x**(4/5)/4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c
*d**3*x**(1/5) + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5
+ 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 27
4*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^4\,dx = ac^4x + \frac{bd^4x^{5n+1}}{5n+1} + \frac{4bcd^3x^{4n+1}}{4n+1} + \frac{ad^4x^{4n+1}}{4n+1} + \frac{6bc^2d^2x^{3n+1}}{3n+1} + \frac{4acd^3x^{3n+1}}{3n+1} + \frac{4bc^3dx^{2n+1}}{2n+1} + \frac{6ac^2d^2x^{2n+1}}{2n+1} + \frac{bc^4x^{n+1}}{n+1} + \frac{4ac^3dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) +
a*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^
3*x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(132) = 264$.

Time = 0.15 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.61

$$\int (a + bx^n)(c + dx^n)^4\,dx$$
$$= \frac{120\,ac^4n^5x + 24\,bd^4n^4xx^{5\,n} + 120\,bcd^3n^4xx^{4\,n} + 30\,ad^4n^4xx^{4\,n} + 240\,bc^2d^2n^4xx^{3\,n} + 160\,acd^3n^4xx^{3\,n} + 2}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

output
```
(120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30
*a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(
3*n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4
*n^4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5
*n) + 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n
^3*x*x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642
*a*c^2*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 2
25*a*c^4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a
*d^4*n^2*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*
n) + 236*b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^
2*x*x^n + 284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) +
44*b*c*d^3*n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) +
 48*a*c*d^3*n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n
) + 14*b*c^4*n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n)
 + 4*b*c*d^3*x*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d
^3*x*x^(3*n) + 4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n +
 4*a*c^3*d*x*x^n + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n +
 1)
```

Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (a+bx^n)(c+dx^n)^4\,dx = a\,c^4\,x+\frac{x\,x^n\,(b\,c^4+4\,a\,d\,c^3)}{n+1}+\frac{x\,x^{4\,n}\,(a\,d^4+4\,b\,c\,d^3)}{4\,n+1}+\frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1}+\frac{2\,c^2\,d\,x\,x^{2\,n}\,(3\,a\,d+2\,b\,c)}{2\,n+1}+\frac{2\,c\,d^2\,x\,x^{3\,n}\,(2\,a\,d+3\,b\,c)}{3\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^4,x)
```

output
```
a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*
d^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d
+ 2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.23

$$\int (a+bx^n)(c+dx^n)^4\,dx$$
$$= \frac{x(120a\,c^4n^5+274a\,c^4n^4+225a\,c^4n^3+85a\,c^4n^2+15a\,c^4n+a\,c^4+24x^{5n}b\,d^4n^4+50x^{5n}b\,d^4n^3+35x^{5n}b}{}$$

input
```
int((a+b*x^n)*(c+d*x^n)^4,x)
```

output

```
(x*(24*x**(5*n)*b*d**4*n**4 + 50*x**(5*n)*b*d**4*n**3 + 35*x**(5*n)*b*d**4
*n**2 + 10*x**(5*n)*b*d**4*n + x**(5*n)*b*d**4 + 30*x**(4*n)*a*d**4*n**4 +
 61*x**(4*n)*a*d**4*n**3 + 41*x**(4*n)*a*d**4*n**2 + 11*x**(4*n)*a*d**4*n
+ x**(4*n)*a*d**4 + 120*x**(4*n)*b*c*d**3*n**4 + 244*x**(4*n)*b*c*d**3*n**
3 + 164*x**(4*n)*b*c*d**3*n**2 + 44*x**(4*n)*b*c*d**3*n + 4*x**(4*n)*b*c*d
**3 + 160*x**(3*n)*a*c*d**3*n**4 + 312*x**(3*n)*a*c*d**3*n**3 + 196*x**(3*
n)*a*c*d**3*n**2 + 48*x**(3*n)*a*c*d**3*n + 4*x**(3*n)*a*c*d**3 + 240*x**(
3*n)*b*c**2*d**2*n**4 + 468*x**(3*n)*b*c**2*d**2*n**3 + 294*x**(3*n)*b*c**
2*d**2*n**2 + 72*x**(3*n)*b*c**2*d**2*n + 6*x**(3*n)*b*c**2*d**2 + 360*x**
(2*n)*a*c**2*d**2*n**4 + 642*x**(2*n)*a*c**2*d**2*n**3 + 354*x**(2*n)*a*c*
*2*d**2*n**2 + 78*x**(2*n)*a*c**2*d**2*n + 6*x**(2*n)*a*c**2*d**2 + 240*x*
*(2*n)*b*c**3*d*n**4 + 428*x**(2*n)*b*c**3*d*n**3 + 236*x**(2*n)*b*c**3*d*
n**2 + 52*x**(2*n)*b*c**3*d*n + 4*x**(2*n)*b*c**3*d + 480*x**n*a*c**3*d*n*
*4 + 616*x**n*a*c**3*d*n**3 + 284*x**n*a*c**3*d*n**2 + 56*x**n*a*c**3*d*n
+ 4*x**n*a*c**3*d + 120*x**n*b*c**4*n**4 + 154*x**n*b*c**4*n**3 + 71*x**n*
b*c**4*n**2 + 14*x**n*b*c**4*n + x**n*b*c**4 + 120*a*c**4*n**5 + 274*a*c**
4*n**4 + 225*a*c**4*n**3 + 85*a*c**4*n**2 + 15*a*c**4*n + a*c**4))/(120*n*
*5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
```

3.68 $\int (a+bx^n)(c+dx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int (a+bx^n)(c+dx^n)^3\,dx = ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}$$

output
```
a*c^3*x+c^2*(3*a*d+b*c)*x^(1+n)/(1+n)+3*c*d*(a*d+b*c)*x^(1+2*n)/(1+2*n)+d^
2*(a*d+3*b*c)*x^(1+3*n)/(1+3*n)+b*d^3*x^(1+4*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int (a+bx^n)(c+dx^n)^3\,dx = \frac{bx(c+dx^n)^4-(bc-ad(1+4n))x\left(c^3+\frac{3c^2dx^n}{1+n}+\frac{3cd^2x^{2n}}{1+2n}+\frac{d^3x^{3n}}{1+3n}\right)}{d+4dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

output
```
(b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n)
+ (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^3\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(c^2x^n(3ad+bc)+d^2x^{3n}(ad+3bc)+3cdx^{2n}(ad+bc)+ac^3+bd^3x^{4n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

output
```
a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1
+ 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(
1 + 4*n))/(1 + 4*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^{3}x+\frac{b\,d^{3}x\,x^{4n}}{1+4n}+\frac{c^{2}(3ad+bc)x\,x^{n}}{1+n}+\frac{d^{2}(ad+3bc)x\,x^{3n}}{1+3n}+\frac{3cd(ad+bc)x\,x^{2n}}{1+2n}$
norman	$a\,c^{3}x+\frac{b\,d^{3}x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{c^{2}(3ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{d^{2}(ad+3bc)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{3cd(ad+bc)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{78x\,x^{n}a\,c^{2}d\,n^{2}+27x\,x^{n}a\,c^{2}dn+24x\,x^{n}b\,c^{3}n^{3}+26x\,x^{n}b\,c^{3}n^{2}+9x\,x^{n}b\,c^{3}n+3x\,x^{n}a\,c^{2}d+24xa\,c^{3}n^{4}+50xa\,c^{3}n^{3}+35xa\,c^{3}n^{2}+x}{}$
orering	Expression too large to display

input
```
int((a+b*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a*c^3*x+b*d^3/(1+4*n)*x*(x^n)^4+c^2*(3*a*d+b*c)/(1+n)*x*x^n+d^2*(a*d+3*b*c
)/(1+3*n)*x*(x^n)^3+3*c*d*(a*d+b*c)/(1+2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(99) = 198.

Time = 0.14 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.22

$$\int(a+bx^{n})(c+dx^{n})^{3}\,dx$$
$$=\frac{(6\,bd^{3}n^{3}+11\,bd^{3}n^{2}+6\,bd^{3}n+bd^{3})xx^{4\,n}+(3\,bcd^{2}+ad^{3}+8\,(3\,bcd^{2}+ad^{3})n^{3}+14\,(3\,bcd^{2}+ad^{3})n^{2}+7}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
 a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c
*d^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)
*n^3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (
b*c^3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^
2 + 9*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c
^3*n^2 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. $2(90) = 180$.

Time = 0.52 (sec) , antiderivative size = 1540, normalized size of antiderivative = 15.56

$$\int (a + bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
 b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(
n, -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqr
t(x) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/
x, Eq(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) +
 a*d**3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*lo
g(x) - 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) +
6*a*c*d**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*
sqrt(x) + 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n
**4*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 3
5*n**2 + 10*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + a*c**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x
*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 +
50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*
n**2 + 10*n + 1) + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 57*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 +
 10*n + 1) + 24*a*c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n
+ 1) + 3*a*c*d**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) +...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int (a+bx^n)(c+dx^n)^3\,dx = ac^3x + \frac{bd^3x^{4n+1}}{4n+1} + \frac{3\,bcd^2x^{3n+1}}{3n+1} + \frac{ad^3x^{3n+1}}{3n+1} + \frac{3\,bc^2dx^{2n+1}}{2n+1} + \frac{3\,acd^2x^{2n+1}}{2n+1} + \frac{bc^3x^{n+1}}{n+1} + \frac{3\,ac^2dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) +
a*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*
x^(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n +
 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.55

$$\int (a + bx^n)(c + dx^n)^3 \, dx$$
$$= \frac{24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3x}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d
^3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 2
4*b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x
*x^(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*
n^2*x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d
*n^2*x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n)
 + 7*a*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) +
 9*b*c^3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3
*b*c*d^2*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x
^(2*n) + b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^
2 + 10*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (a + bx^n)(c + dx^n)^3 \, dx = a\,c^3\,x + \frac{x\,x^n\,(b\,c^3 + 3\,a\,d\,c^2)}{n+1} + \frac{x\,x^{3\,n}\,(a\,d^3 + 3\,b\,c\,d^2)}{3\,n+1} + \frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1} + \frac{3\,c\,d\,x\,x^{2\,n}\,(a\,d + b\,c)}{2\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^3,x)
```

output
```
a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*
d^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*
c))/(2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.22

$$\int (a+bx^n)(c+dx^n)^3\,dx$$
$$= \frac{x(a\,c^3+24x^{3n}bc\,d^2n^3+42x^{3n}bc\,d^2n^2+21x^{3n}bc\,d^2n+36x^{2n}ac\,d^2n^3+57x^{2n}ac\,d^2n^2+24x^{2n}ac\,d^2n+36}{}$$

input
```
int((a+b*x^n)*(c+d*x^n)^3,x)
```

output
```
(x*(6*x**(4*n)*b*d**3*n**3 + 11*x**(4*n)*b*d**3*n**2 + 6*x**(4*n)*b*d**3*n
 + x**(4*n)*b*d**3 + 8*x**(3*n)*a*d**3*n**3 + 14*x**(3*n)*a*d**3*n**2 + 7*
x**(3*n)*a*d**3*n + x**(3*n)*a*d**3 + 24*x**(3*n)*b*c*d**2*n**3 + 42*x**(3
*n)*b*c*d**2*n**2 + 21*x**(3*n)*b*c*d**2*n + 3*x**(3*n)*b*c*d**2 + 36*x**(
2*n)*a*c*d**2*n**3 + 57*x**(2*n)*a*c*d**2*n**2 + 24*x**(2*n)*a*c*d**2*n +
3*x**(2*n)*a*c*d**2 + 36*x**(2*n)*b*c**2*d*n**3 + 57*x**(2*n)*b*c**2*d*n**
2 + 24*x**(2*n)*b*c**2*d*n + 3*x**(2*n)*b*c**2*d + 72*x**n*a*c**2*d*n**3 +
 78*x**n*a*c**2*d*n**2 + 27*x**n*a*c**2*d*n + 3*x**n*a*c**2*d + 24*x**n*b*
c**3*n**3 + 26*x**n*b*c**3*n**2 + 9*x**n*b*c**3*n + x**n*b*c**3 + 24*a*c**
3*n**4 + 50*a*c**3*n**3 + 35*a*c**3*n**2 + 10*a*c**3*n + a*c**3))/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1)
```

3.69 $\int (a+bx^n) (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n) (c+dx^n)^2 \, dx = ac^2x + \frac{c(bc+2ad)x^{1+n}}{1+n} + \frac{d(2bc+ad)x^{1+2n}}{1+2n} + \frac{bd^2x^{1+3n}}{1+3n}$$

output
```
a*c^2*x+c*(2*a*d+b*c)*x^(1+n)/(1+n)+d*(a*d+2*b*c)*x^(1+2*n)/(1+2*n)+b*d^2*
x^(1+3*n)/(1+3*n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n) (c+dx^n)^2 \, dx = \frac{bx(c+dx^n)^3 - (bc-ad(1+3n))x\left(c^2+\frac{2cdx^n}{1+n}+\frac{d^2x^{2n}}{1+2n}\right)}{d+3dn}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

output
```
(b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) +
(d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^2\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^{2n}(ad+2bc)+cx^n(2ad+bc)+ac^2+bd^2x^{3n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

output
```
a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*
n))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a\,c^{2} x +\frac{b\,d^{2} x\,x^{3 n}}{1+3 n}+\frac{c \left(2 a d +b c \right) x\,x^{n}}{1+n}+\frac{d \left(a d +2 b c \right) x\,x^{2 n}}{1+2 n}$
norman	$a\,c^{2} x +\frac{b\,d^{2} x\,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}+\frac{c \left(2 a d +b c \right) x\,{\mathrm e}^{n \ln \left(x \right)}}{1+n}+\frac{d \left(a d +2 b c \right) x\,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}$
parallelrisch	$\frac{2 x\,x^{3 n} b\,d^{2} n^{2}+3 x\,x^{3 n} b\,d^{2} n +3 x\,x^{2 n} a\,d^{2} n^{2}+6 x\,x^{2 n} b c d\,n^{2}+b\,d^{2} x\,x^{3 n}+4 x\,x^{2 n} a\,d^{2} n +8 x\,x^{2 n} b c d n +12 x\,x^{n} a c d\,n^{2}+6 x\,x^{n} b\,c^{2} n^{2}}{\left(1+3 n \right) \left(1+n \right) \left(1+\right.}$
orering	$x \left(a +b\,x^{n}\right) \left(c +d\,x^{n}\right)^{2}-\frac{x^{2} \left(11 n^{2}+1\right) \left(\frac{b\,x^{n} n \left(c +d\,x^{n}\right)^{2}}{x}+\frac{2 \left(a +b\,x^{n}\right) \left(c +d\,x^{n}\right) d\,x^{n} n}{x}\right)}{\left(2 n^{2}+3 n +1\right) \left(1+3 n \right)}+\frac{2 x^{3} \left(-1+3 n \right) \left(\frac{b\,x^{n} n^{2} \left(c +d\,x\right.}{x^{2}}\right.}{}$

input
```
int((a+b*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a*c^2*x+b*d^2/(1+3*n)*x*(x^n)^3+c*(2*a*d+b*c)/(1+n)*x*x^n+d*(a*d+2*b*c)/(1
+2*n)*x*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)\,(c+dx^n)^2\,dx$$
$$=\frac{(2\,bd^2n^2+3\,bd^2n+bd^2)xx^{3\,n}+(2\,bcd+ad^2+3\,(2\,bcd+ad^2)n^2+4\,(2\,bcd+ad^2)n)xx^{2\,n}+(bc^2+2\,acd}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c
*d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*
(b*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a
*c^2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.43 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a+bx^n)(c+dx^n)^2\,dx$$
$$= \begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{}{6n^3} \end{cases}$$

input

```
integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

output

```
Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/
x - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(
x) + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (
a*c**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*
b*c*d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 1
1*n**2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c*
*2*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1
) + 12*a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
 + 3*a*d**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x*
*(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2
+ 6*n + 1) + 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*
n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 +
6*n + 1) + 6*b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*
n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 3*b*d**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(
6*n**3 + 11*n**2 + 6*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)(c+dx^n)^2\,dx = ac^2x + \frac{bd^2x^{3n+1}}{3n+1} + \frac{2bcdx^{2n+1}}{2n+1} + \frac{ad^2x^{2n+1}}{2n+1} + \frac{bc^2x^{n+1}}{n+1} + \frac{2acdx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*
d^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n
 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(70) = 140.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)(c+dx^n)^2\,dx$$
$$= \frac{6ac^2n^3x + 2bd^2n^2xx^{3n} + 6bcdn^2xx^{2n} + 3ad^2n^2xx^{2n} + 6bc^2n^2xx^n + 12acdn^2xx^n + 11ac^2n^2x + 3bd^2n}{}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n
^2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3
*b*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n
*x*x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2
*n) + a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11
*n^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)\,(c+dx^n)^2\,dx = a\,c^2\,x + \frac{x\,x^{2\,n}\,(a\,d^2+2\,b\,c\,d)}{2\,n+1} + \frac{x\,x^n\,(b\,c^2+2\,a\,d\,c)}{n+1} + \frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n)^2,x)
```

output
```
a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*
d))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.06

$$\int (a+bx^n)\,(c+dx^n)^2\,dx$$
$$= \frac{x(2x^{3n}b\,d^2n^2 + 3x^{3n}b\,d^2n + x^{3n}b\,d^2 + 3x^{2n}a\,d^2n^2 + 4x^{2n}a\,d^2n + x^{2n}a\,d^2 + 6x^{2n}bcd\,n^2 + 8x^{2n}bcdn + 2x^{2n}}{6n^3 + }$$

input
```
int((a+b*x^n)*(c+d*x^n)^2,x)
```

output
```
(x*(2*x**(3*n)*b*d**2*n**2 + 3*x**(3*n)*b*d**2*n + x**(3*n)*b*d**2 + 3*x**
(2*n)*a*d**2*n**2 + 4*x**(2*n)*a*d**2*n + x**(2*n)*a*d**2 + 6*x**(2*n)*b*c
*d*n**2 + 8*x**(2*n)*b*c*d*n + 2*x**(2*n)*b*c*d + 12*x**n*a*c*d*n**2 + 10*
x**n*a*c*d*n + 2*x**n*a*c*d + 6*x**n*b*c**2*n**2 + 5*x**n*b*c**2*n + x**n*
b*c**2 + 6*a*c**2*n**3 + 11*a*c**2*n**2 + 6*a*c**2*n + a*c**2))/(6*n**3 +
11*n**2 + 6*n + 1)
```

3.70 $\int (a+bx^n)(c+dx^n)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int (a+bx^n)(c+dx^n)\, dx = acx + \frac{(bc+ad)x^{1+n}}{1+n} + \frac{bdx^{1+2n}}{1+2n}$$

output
```
a*c*x+(a*d+b*c)*x^(1+n)/(1+n)+b*d*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (a+bx^n)(c+dx^n)\, dx = x\left(ac + \frac{(bc+ad)x^n}{1+n} + \frac{bdx^{2n}}{1+2n}\right)$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n),x]
```

output
```
x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)\,dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(x^n(ad+bc)+ac+bdx^{2n}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

input
```
Int[(a + b*x^n)*(c + d*x^n),x]
```

output
```
a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
risch	$acx+\frac{(ad+bc)x\,x^{n}}{1+n}+\frac{bdx\,x^{2n}}{1+2n}$
norman	$acx+\frac{(ad+bc)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bdx\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{x\,x^{2n}bdn+bdx\,x^{2n}+2x\,x^{n}adn+2x\,x^{n}bcn+2xac\,n^{2}+x\,x^{n}ad+x\,x^{n}bc+3xacn+acx}{(1+n)(1+2n)}$
orering	$(c+d\,x^{n})\,x(a+b\,x^{n})-\frac{3n\,x^{2}\left(\frac{b\,x^{n}n(c+d\,x^{n})}{x}+\frac{(a+b\,x^{n})d\,x^{n}n}{x}\right)}{2n^{2}+3n+1}+\frac{x^{3}\left(\frac{b\,x^{n}n^{2}(c+d\,x^{n})}{x^{2}}-\frac{b\,x^{n}n(c+d\,x^{n})}{x^{2}}+\frac{2x^{2n}n^{2}bd}{x^{2}}\right.}{2n^{2}+3n+1}$

input
```
int((a+b*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+(a*d+b*c)/(1+n)*x*x^n+b*d/(1+2*n)*x*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{(bdn+bd)xx^{2n}+(bc+ad+2(bc+ad)n)xx^n+(2acn^2+3acn+ac)x}{2n^2+3n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^
2 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.90

$$\int (a + bx^n)(c + dx^n)\, dx = \begin{cases} acx + ad\log(x) + bc\log(x) - \frac{bd}{x} \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log(x) \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} \end{cases}$$

input
```
integrate((a+b*x**n)*(c+d*x**n),x)
```

output
```
Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2
*a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*
n**2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1)
+ 2*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*
c*n*x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x*
*(2*n)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a + bx^n)(c + dx^n)\, dx = acx + \frac{bdx^{2n+1}}{2n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/
(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{2\,acn^2x + bdnxx^{2\,n} + 2\,bcnxx^n + 2\,adnxx^n + 3\,acnx + bdxx^{2\,n} + bcxx^n + adxx^n + acx}{2\,n^2 + 3\,n + 1}$$

input
```
integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
 + b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (a+bx^n)(c+dx^n)\,dx = a\,c\,x + \frac{x\,x^n\,(a\,d+b\,c)}{n+1} + \frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

input
```
int((a + b*x^n)*(c + d*x^n),x)
```

output
```
a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int (a+bx^n)(c+dx^n)\,dx = \frac{x(x^{2n}bdn + x^{2n}bd + 2x^n adn + x^n ad + 2x^n bcn + x^n bc + 2ac\,n^2 + 3acn + ac)}{2n^2 + 3n + 1}$$

input
```
int((a+b*x^n)*(c+d*x^n),x)
```

output
```
(x*(x**(2*n)*b*d*n + x**(2*n)*b*d + 2*x**n*a*d*n + x**n*a*d + 2*x**n*b*c*n
 + x**n*b*c + 2*a*c*n**2 + 3*a*c*n + a*c))/(2*n**2 + 3*n + 1)
```

3.71 $\int \frac{a+bx^n}{c+dx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{bx}{d} - \frac{(bc-ad)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

output
```
b*x/d-(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{x\left(bc+(-bc+ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{cd}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n),x]
```

output
```
(x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n
)/c)]))/(c*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{c+dx^n}dx$$

$$\Big\downarrow \ 913$$

$$\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{dx^n+c}dx}{d}$$

$$\Big\downarrow \ 778$$

$$\frac{bx}{d}-\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}$$

input
```
Int[(a + b*x^n)/(c + d*x^n),x]
```

output
```
(b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n
)/c)])/(c*d)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{a+b\,x^{n}}{c+d\,x^{n}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n),x)
```

output
```
int((a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \int \frac{bx^n+a}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.56

$$\int \frac{a+bx^n}{c+dx^n}\,dx = \frac{ac^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{bc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)/(c+d*x**n),x)
```

output
```
a*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) - b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n)*x*ler
chphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(c*c**(
1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{a + bx^n}{c + dx^n} \, dx = \int \frac{bx^n + a}{dx^n + c} \, dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

Giac [F]

$$\int \frac{a + bx^n}{c + dx^n} \, dx = \int \frac{bx^n + a}{dx^n + c} \, dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + bx^n}{c + dx^n} \, dx = \int \frac{a + b\,x^n}{c + d\,x^n} \, dx$$

input
```
int((a + b*x^n)/(c + d*x^n),x)
```

output
```
int((a + b*x^n)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{a+bx^n}{c+dx^n}\,dx=\frac{\left(\int \frac{1}{x^n d+c}dx\right)ad-\left(\int \frac{1}{x^n d+c}dx\right)bc+bx}{d}$$

input
```
int((a+b*x^n)/(c+d*x^n),x)
```

output
```
(int(1/(x**n*d + c),x)*a*d - int(1/(x**n*d + c),x)*b*c + b*x)/d
```

3.72 $\int \frac{a+bx^n}{(c+dx^n)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{a+bx^n}{(c+dx^n)^2} \, dx = \frac{bx}{d(1-n)(c+dx^n)} - \frac{(bc-ad(1-n))x \, \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2d(1-n)}$$

output
```
b*x/d/(1-n)/(c+d*x^n)-(b*c-a*d*(1-n))*x*hypergeom([2, 1/n],[1+1/n],-d*x^n/
c)/c^2/d/(1-n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^n}{(c+dx^n)^2} \, dx = \frac{x\left(\frac{b}{c+dx^n} - \frac{(bc+ad(-1+n)) \, \text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

output
```
(x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(bc-ad(1-n))\int \frac{1}{dx^n+c}dx}{cdn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(bc-ad(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

output
```
-(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeome
tric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.50 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.44

$$\int \frac{a+bx^n}{(c+dx^n)^2} \, dx = \text{Too large to display}$$

input

```
integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

output

```
a*(c*c**(1/n)*c**(-2 - 1/n)*n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)
*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) + c*c**(1
/n)*c**(-2 - 1/n)*n*x*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamm
a(1 + 1/n)) - c*c**(1/n)*c**(-2 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c
, 1, 1/n)*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))
+ c**(1/n)*c**(-2 - 1/n)*d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1,
1/n)*gamma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n)) - c**
(1/n)*c**(-2 - 1/n)*d*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*ga
mma(1/n)/(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + b*(c*c**(
-3 - 1/n)*c**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/
n) + d*n**3*x**n*gamma(2 + 1/n)) - c*c**(-3 - 1/n)*c**(1 + 1/n)*n*x**(n +
1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*g
amma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n)) + c*c**(-3 - 1/n)*c**(1 + 1/n)
*n*x**(n + 1)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2
+ 1/n)) - c*c**(-3 - 1/n)*c**(1 + 1/n)*x**(n + 1)*lerchphi(d*x**n*exp_pola
r(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/n) + d*n**3*x**n
*gamma(2 + 1/n)) - c**(-3 - 1/n)*c**(1 + 1/n)*d*n*x**n*x**(n + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*n**3*gamma(2 + 1/
n) + d*n**3*x**n*gamma(2 + 1/n)) - c**(-3 - 1/n)*c**(1 + 1/n)*d*x**n*x**(n
 + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*...
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*
x/(c*d^2*n*x^n + c^2*d*n)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{bx^n+a}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{a+bx^n}{(c+dx^n)^2}\,dx$$
$$= \frac{-x^n \left(\int \frac{x^{2n}}{x^{2n}d^2n+x^{2n}d^2+2x^n cdn+2x^n cd+c^2n+c^2}dx\right) a\,d^3n^2 + x^n \left(\int \frac{x^{2n}}{x^{2n}d^2n+x^{2n}d^2+2x^n cdn+2x^n cd+c^2n+c^2}dx\right) a\,d^3 - x^n}{}$$

input
```
int((a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
( - x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*
x**n*c*d + c**2*n + c**2),x)*a*d**3*n**2 + x**n*int(x**(2*n)/(x**(2*n)*d**
2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*a*d**3
 - x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x
**n*c*d + c**2*n + c**2),x)*b*c*d**2*n - x**n*int(x**(2*n)/(x**(2*n)*d**2*
n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*b*c*d**2
 + x**n*a*d*n*x - x**n*a*d*x + x**n*b*c*x - int(x**(2*n)/(x**(2*n)*d**2*n
+ x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*a*c*d**2*n
**2 + int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**
n*c*d + c**2*n + c**2),x)*a*c*d**2 - int(x**(2*n)/(x**(2*n)*d**2*n + x**(2
*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*b*c**2*d*n - int(
x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c*
*2*n + c**2),x)*b*c**2*d + a*c*n*x + a*c*x)/(c**2*(x**n*d*n + x**n*d + c*n
 + c))
```

3.73 $\int \frac{a+bx^n}{(c+dx^n)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+bx^n}{(c+dx^n)^3} dx = \frac{bx}{d(1-2n)(c+dx^n)^2} + \frac{\left(a-\frac{bc}{d-2dn}\right) x \operatorname{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^3}$$

output
```
b*x/d/(1-2*n)/(c+d*x^n)^2+(a-b*c/(-2*d*n+d))*x*hypergeom([3, 1/n],[1+1/n],
-d*x^n/c)/c^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^n}{(c+dx^n)^3} dx = \frac{x\left(\frac{b}{(c+dx^n)^2} - \frac{(bc+ad(-1+2n)) \operatorname{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

output
```
(x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
 1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(bc-ad(1-2n))\int \frac{1}{(dx^n+c)^2}dx}{2cdn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

$$\Big\downarrow \; 778$$

$$\frac{x(bc-ad(1-2n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

output
```
-1/2*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hype
rgeometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{3}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx=\int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 40.09 (sec) , antiderivative size = 2319, normalized size of antiderivative = 38.02

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

output
```
a*(2*c**2*c**(1/n)*c**(-3 - 1/n)*n**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c,
 1, 1/n)*gamma(1/n)/(2*c**2*n**4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(1
+ 1/n) + 2*d**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 3*c**2*c**(1/n)*c**(-3 - 1
/n)*n**2*x*gamma(1/n)/(2*c**2*n**4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(
1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(1 + 1/n)) - 3*c**2*c**(1/n)*c**(-3 -
 1/n)*n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(2*c**2*n*
*4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(1 + 1/n) + 2*d**2*n**4*x**(2*n)*
gamma(1 + 1/n)) - c**2*c**(1/n)*c**(-3 - 1/n)*n*x*gamma(1/n)/(2*c**2*n**4*
gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(1 + 1/n) + 2*d**2*n**4*x**(2*n)*gam
ma(1 + 1/n)) + c**2*c**(1/n)*c**(-3 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1/n)*gamma(1/n)/(2*c**2*n**4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gam
ma(1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 4*c*c**(1/n)*c**(-3 -
 1/n)*d*n**2*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/
(2*c**2*n**4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(1 + 1/n) + 2*d**2*n**4
*x**(2*n)*gamma(1 + 1/n)) + 2*c*c**(1/n)*c**(-3 - 1/n)*d*n**2*x*x**n*gamma
(1/n)/(2*c**2*n**4*gamma(1 + 1/n) + 4*c*d*n**4*x**n*gamma(1 + 1/n) + 2*d**
2*n**4*x**(2*n)*gamma(1 + 1/n)) - 6*c*c**(1/n)*c**(-3 - 1/n)*d*n*x*x**n*le
rchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(2*c**2*n**4*gamma(1 +
 1/n) + 4*c*d*n**4*x**n*gamma(1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(1 + 1/
n)) - c*c**(1/n)*c**(-3 - 1/n)*d*n*x*x**n*gamma(1/n)/(2*c**2*n**4*gamma...
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3
*d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*
c^2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \int \frac{bx^n+a}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{a+bx^n}{(c+dx^n)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
(2*x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d
**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**
3),x)*a*d**4*n**3 - x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3
 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2
*d + c**3*n + c**3),x)*a*d**4*n**2 - 2*x**(2*n)*int(x**(2*n)/(x**(3*n)*d**
3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**
2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a*d**4*n + x**(2*n)*int(x**(2*n)
/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**
2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a*d**4 + x**(2*n)*
int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x*
*(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*b*c*d*
*3*n**2 - x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2
*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*
n + c**3),x)*b*c*d**3 + 4*x**n*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d*
*3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c*
*2*d + c**3*n + c**3),x)*a*c*d**3*n**3 - 2*x**n*int(x**(2*n)/(x**(3*n)*d**
3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**
2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a*c*d**3*n**2 - 4*x**n*int(x**(2
*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*
d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a*c*d**3*n +...
```

3.74 $\int \frac{a+bx^n}{(c+dx^n)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{a+bx^n}{(c+dx^n)^4} dx = \frac{bx}{d(1-3n)(c+dx^n)^3} + \frac{\left(a-\frac{bc}{d-3dn}\right) x \,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^4}$$

output
```
b*x/d/(1-3*n)/(c+d*x^n)^3+(a-b*c/(-3*d*n+d))*x*hypergeom([4, 1/n],[1+1/n],
-d*x^n/c)/c^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^n}{(c+dx^n)^4} dx = \frac{x\left(\frac{b}{(c+dx^n)^3} - \frac{(bc+ad(-1+3n))\,\text{Hypergeometric2F1}\left(4,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

input
```
Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

output
```
(x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1),
 1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(bc-ad(1-3n))\int \frac{1}{(dx^n+c)^3}dx}{3cdn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(bc-ad(1-3n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

input
```
Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

output
```
-1/3*((b*c - a*d)*x)/(c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hype
rgeometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{a+b\,x^{n}}{\left(c+d\,x^{n}\right)^{4}}dx$$

input
```
int((a+b*x^n)/(c+d*x^n)^4,x)
```

output
```
int((a+b*x^n)/(c+d*x^n)^4,x)
```

Fricas [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx=\int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) +
4*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^
3*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(
2*n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*
x^n - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n
^3*x^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

Giac [F]

$$\int \frac{a+bx^n}{(c+dx^n)^4}\,dx = \int \frac{bx^n+a}{(dx^n+c)^4}\,dx$$

input
```
integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^n}{\left(c+dx^n\right)^4}\,dx=\int \frac{a+b\,x^n}{\left(c+d\,x^n\right)^4}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^4,x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^4, x)
```

Reduce [F]

$$\int \frac{a+bx^n}{\left(c+dx^n\right)^4}\,dx=\text{too large to display}$$

input
```
int((a+b*x^n)/(c+d*x^n)^4,x)
```

output
```
(6*x**(3*n)*int(x**(2*n)/(x**(4*n)*d**4*n + x**(4*n)*d**4 + 4*x**(3*n)*c*d
**3*n + 4*x**(3*n)*c*d**3 + 6*x**(2*n)*c**2*d**2*n + 6*x**(2*n)*c**2*d**2
+ 4*x**n*c**3*d*n + 4*x**n*c**3*d + c**4*n + c**4),x)*a*d**5*n**3 + x**(3*
n)*int(x**(2*n)/(x**(4*n)*d**4*n + x**(4*n)*d**4 + 4*x**(3*n)*c*d**3*n + 4
*x**(3*n)*c*d**3 + 6*x**(2*n)*c**2*d**2*n + 6*x**(2*n)*c**2*d**2 + 4*x**n*
c**3*d*n + 4*x**n*c**3*d + c**4*n + c**4),x)*a*d**5*n**2 - 4*x**(3*n)*int(
x**(2*n)/(x**(4*n)*d**4*n + x**(4*n)*d**4 + 4*x**(3*n)*c*d**3*n + 4*x**(3*
n)*c*d**3 + 6*x**(2*n)*c**2*d**2*n + 6*x**(2*n)*c**2*d**2 + 4*x**n*c**3*d*
n + 4*x**n*c**3*d + c**4*n + c**4),x)*a*d**5*n + x**(3*n)*int(x**(2*n)/(x*
*(4*n)*d**4*n + x**(4*n)*d**4 + 4*x**(3*n)*c*d**3*n + 4*x**(3*n)*c*d**3 +
6*x**(2*n)*c**2*d**2*n + 6*x**(2*n)*c**2*d**2 + 4*x**n*c**3*d*n + 4*x**n*c
**3*d + c**4*n + c**4),x)*a*d**5 + 2*x**(3*n)*int(x**(2*n)/(x**(4*n)*d**4*
n + x**(4*n)*d**4 + 4*x**(3*n)*c*d**3*n + 4*x**(3*n)*c*d**3 + 6*x**(2*n)*c
**2*d**2*n + 6*x**(2*n)*c**2*d**2 + 4*x**n*c**3*d*n + 4*x**n*c**3*d + c**4
*n + c**4),x)*b*c*d**4*n**2 + x**(3*n)*int(x**(2*n)/(x**(4*n)*d**4*n + x**
(4*n)*d**4 + 4*x**(3*n)*c*d**3*n + 4*x**(3*n)*c*d**3 + 6*x**(2*n)*c**2*d**
2*n + 6*x**(2*n)*c**2*d**2 + 4*x**n*c**3*d*n + 4*x**n*c**3*d + c**4*n + c*
*4),x)*b*c*d**4*n - x**(3*n)*int(x**(2*n)/(x**(4*n)*d**4*n + x**(4*n)*d**4
 + 4*x**(3*n)*c*d**3*n + 4*x**(3*n)*c*d**3 + 6*x**(2*n)*c**2*d**2*n + 6*x*
*(2*n)*c**2*d**2 + 4*x**n*c**3*d*n + 4*x**n*c**3*d + c**4*n + c**4),x)*...
```

3.75 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{ad^2(2bd+3ae)x^{1+n}}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{1+2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{1+3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{1+4n}}{1+4n} + \frac{b^2e^3x^{1+5n}}{1+5n}$$

output
```
a^2*d^3*x+a*d^2*(3*a*e+2*b*d)*x^(1+n)/(1+n)+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2
)*x^(1+2*n)/(1+2*n)+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)*x^(1+3*n)/(1+3*n)+b*e^
2*(2*a*e+3*b*d)*x^(1+4*n)/(1+4*n)+b^2*e^3*x^(1+5*n)/(1+5*n)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = x\left(a^2d^3 + \frac{ad^2(2bd+3ae)x^n}{1+n} + \frac{d(b^2d^2+6abde+3a^2e^2)\,x^{2n}}{1+2n} + \frac{e(3b^2d^2+6abde+a^2e^2)\,x^{3n}}{1+3n} + \frac{be^2(3bd+2ae)x^{4n}}{1+4n} + \frac{b^2e^3x^{5n}}{1+5n}\right)$$

input
```
Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

output
```
x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
 + 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^
(3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^
(5*n))/(1 + 5*n))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)+ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)+a^2d^3+ad^2x^n(3ae+2bd)+be^2x^{4n}(2ae+3bd)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

input
```
Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

output
```
a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*
b*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^
2*e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4
*n) + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
risch	$a^2d^3x+\frac{d(3a^2e^2+6abde+b^2d^2)x\,x^{2n}}{1+2n}+\frac{e(a^2e^2+6abde+3b^2d^2)x\,x^{3n}}{1+3n}+\frac{e^3b^2x\,x^{5n}}{5n+1}+\frac{a\,d^2(3ae+2bd)x\,x^n}{1+n}+\frac{b\,e^2(2a$
norman	$a^2d^3x+\frac{d(3a^2e^2+6abde+b^2d^2)x\,e^{2n\ln(x)}}{1+2n}+\frac{e(a^2e^2+6abde+3b^2d^2)x\,e^{3n\ln(x)}}{1+3n}+\frac{e^3b^2x\,e^{5n\ln(x)}}{5n+1}+\frac{a\,d^2(3ae+2bd)x\,e}{1+n}$
parallelrisch	$120x\,a^2d^3n^5+274x\,a^2d^3n^4+225x\,a^2d^3n^3+85x\,a^2d^3n^2+15x\,a^2d^3n+462x\,x^n a^2d^2e\,n^3+308x\,x^n ab\,d^3n^3+213x\,x^n a^2d^2e\,n^2+$
orering	Expression too large to display

input
```
int((a+b*x^n)^2*(d+e*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^3*x+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/(1+2*n)*x*(x^n)^2+e*(a^2*e^2+6*a
*b*d*e+3*b^2*d^2)/(1+3*n)*x*(x^n)^3+e^3*b^2/(5*n+1)*x*(x^n)^5+a*d^2*(3*a*e
+2*b*d)/(1+n)*x*x^n+b*e^2*(2*a*e+3*b*d)/(1+4*n)*x*(x^n)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(158) = 316$.

Time = 0.12 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.22

$$\int (a + bx^n)^2 (d + ex^n)^3 \, dx = \frac{(24\, b^2e^3n^4 + 50\, b^2e^3n^3 + 35\, b^2e^3n^2 + 10\, b^2e^3n + b^2e^3)xx^{5\,n} + (3\, b^2de^2 + 2\, abe^3 + 30\, (3\, b^2de^2 + 2\, abe^3)n^4}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

output
```
((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^
3)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4
 + 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 1
1*(3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^
2*e^3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6
*a*b*d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 +
 12*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*
d^2*e + 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(
b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a
^2*d*e^2)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2
*a*b*d^3 + 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^
3 + 3*a^2*d^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 +
3*a^2*d^2*e)*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n
^3 + 85*a^2*d^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*
n^3 + 85*n^2 + 15*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3376 vs. $2(151) = 302$.

Time = 7.94 (sec) , antiderivative size = 3376, normalized size of antiderivative = 21.37

$$\int (a + bx^n)^2 (d + ex^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

output
```
Piecewise((a**2*d**3*x + 3*a**2*d**2*e*log(x) - 3*a**2*d*e**2/x - a**2*e**
3/(2*x**2) + 2*a*b*d**3*log(x) - 6*a*b*d**2*e/x - 3*a*b*d*e**2/x**2 - 2*a*
b*e**3/(3*x**3) - b**2*d**3/x - 3*b**2*d**2*e/(2*x**2) - b**2*d*e**2/x**3
- b**2*e**3/(4*x**4), Eq(n, -1)), (a**2*d**3*x + 6*a**2*d**2*e*sqrt(x) + 3
*a**2*d*e**2*log(x) - 2*a**2*e**3/sqrt(x) + 4*a*b*d**3*sqrt(x) + 6*a*b*d**
2*e*log(x) - 12*a*b*d*e**2/sqrt(x) - 2*a*b*e**3/x + b**2*d**3*log(x) - 6*b
**2*d**2*e/sqrt(x) - 3*b**2*d*e**2/x - 2*b**2*e**3/(3*x**(3/2)), Eq(n, -1/
2)), (a**2*d**3*x + 9*a**2*d**2*e*x**(2/3)/2 + 9*a**2*d*e**2*x**(1/3) + a*
*2*e**3*log(x) + 3*a*b*d**3*x**(2/3) + 18*a*b*d**2*e*x**(1/3) + 6*a*b*d*e*
*2*log(x) - 6*a*b*e**3/x**(1/3) + 3*b**2*d**3*x**(1/3) + 3*b**2*d**2*e*log
(x) - 9*b**2*d*e**2/x**(1/3) - 3*b**2*e**3/(2*x**(2/3)), Eq(n, -1/3)), (a*
*2*d**3*x + 4*a*d**2*x**(3/4)*(3*a*e + 2*b*d)/3 - 4*b**2*e**3/x**(1/4) + 4
*b*e**2*(2*a*e + 3*b*d)*log(x**(1/4)) + 2*d*sqrt(x)*(3*a**2*e**2 + 6*a*b*d
*e + b**2*d**2) + 4*e*x**(1/4)*(a**2*e**2 + 6*a*b*d*e + 3*b**2*d**2), Eq(n
, -1/4)), (a**2*d**3*x + 5*a*d**2*x**(4/5)*(3*a*e + 2*b*d)/4 + 5*b**2*e**3
*log(x**(1/5)) + 5*b*e**2*x**(1/5)*(2*a*e + 3*b*d) + 5*d*x**(3/5)*(3*a**2*
e**2 + 6*a*b*d*e + b**2*d**2)/3 + 5*e*x**(2/5)*(a**2*e**2 + 6*a*b*d*e + 3*
b**2*d**2)/2, Eq(n, -1/5)), (120*a**2*d**3*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a**2*d**3*n**4*x/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 225*a**2*d**3*n**3*x/(120*n**5 + 274...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.53

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2d^3x + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{3b^2de^2x^{4n+1}}{4n+1} + \frac{2abe^3x^{4n+1}}{4n+1} + \frac{3b^2d^2ex^{3n+1}}{3n+1} + \frac{6abde^2x^{3n+1}}{3n+1} + \frac{a^2e^3x^{3n+1}}{3n+1} + \frac{b^2d^3x^{2n+1}}{2n+1} + \frac{6abd^2ex^{2n+1}}{2n+1} + \frac{3a^2de^2x^{2n+1}}{2n+1} + \frac{2abd^3x^{n+1}}{n+1} + \frac{3a^2d^2ex^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

output
```
a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
 1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1)
+ 6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*
d^3*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^
2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n +
 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(158) = 316.

Time = 0.14 (sec) , antiderivative size = 962, normalized size of antiderivative = 6.09

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

output
```
(120*a^2*d^3*n^5*x + 24*b^2*e^3*n^4*x*x^(5*n) + 90*b^2*d*e^2*n^4*x*x^(4*n)
 + 60*a*b*e^3*n^4*x*x^(4*n) + 120*b^2*d^2*e*n^4*x*x^(3*n) + 240*a*b*d*e^2*
n^4*x*x^(3*n) + 40*a^2*e^3*n^4*x*x^(3*n) + 60*b^2*d^3*n^4*x*x^(2*n) + 360*
a*b*d^2*e*n^4*x*x^(2*n) + 180*a^2*d*e^2*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*
x^n + 360*a^2*d^2*e*n^4*x*x^n + 274*a^2*d^3*n^4*x + 50*b^2*e^3*n^3*x*x^(5*
n) + 183*b^2*d*e^2*n^3*x*x^(4*n) + 122*a*b*e^3*n^3*x*x^(4*n) + 234*b^2*d^2
*e*n^3*x*x^(3*n) + 468*a*b*d*e^2*n^3*x*x^(3*n) + 78*a^2*e^3*n^3*x*x^(3*n)
 + 107*b^2*d^3*n^3*x*x^(2*n) + 642*a*b*d^2*e*n^3*x*x^(2*n) + 321*a^2*d*e^2*
n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x^n + 462*a^2*d^2*e*n^3*x*x^n + 225*a^2*
d^3*n^3*x + 35*b^2*e^3*n^2*x*x^(5*n) + 123*b^2*d*e^2*n^2*x*x^(4*n) + 82*a*
b*e^3*n^2*x*x^(4*n) + 147*b^2*d^2*e*n^2*x*x^(3*n) + 294*a*b*d*e^2*n^2*x*x^
(3*n) + 49*a^2*e^3*n^2*x*x^(3*n) + 59*b^2*d^3*n^2*x*x^(2*n) + 354*a*b*d^2*
e*n^2*x*x^(2*n) + 177*a^2*d*e^2*n^2*x*x^(2*n) + 142*a*b*d^3*n^2*x*x^n + 21
3*a^2*d^2*e*n^2*x*x^n + 85*a^2*d^3*n^2*x + 10*b^2*e^3*n*x*x^(5*n) + 33*b^2
*d*e^2*n*x*x^(4*n) + 22*a*b*e^3*n*x*x^(4*n) + 36*b^2*d^2*e*n*x*x^(3*n) + 7
2*a*b*d*e^2*n*x*x^(3*n) + 12*a^2*e^3*n*x*x^(3*n) + 13*b^2*d^3*n*x*x^(2*n)
 + 78*a*b*d^2*e*n*x*x^(2*n) + 39*a^2*d*e^2*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n
 + 42*a^2*d^2*e*n*x*x^n + 15*a^2*d^3*n*x + b^2*e^3*x*x^(5*n) + 3*b^2*d*e^2
*x*x^(4*n) + 2*a*b*e^3*x*x^(4*n) + 3*b^2*d^2*e*x*x^(3*n) + 6*a*b*d*e^2*x*x
^(3*n) + a^2*e^3*x*x^(3*n) + b^2*d^3*x*x^(2*n) + 6*a*b*d^2*e*x*x^(2*n) ...
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = a^2 d^3 x + \frac{x x^{2n} (3 a^2 d e^2 + 6 a b d^2 e + b^2 d^3)}{2n+1} + \frac{x x^{3n} (a^2 e^3 + 6 a b d e^2 + 3 b^2 d^2 e)}{3n+1} + \frac{b^2 e^3 x x^{5n}}{5n+1} + \frac{a d^2 x x^n (3 a e + 2 b d)}{n+1} + \frac{b e^2 x x^{4n} (2 a e + 3 b d)}{4n+1}$$

input
```
int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

output
```
a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) +
(x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x
^(5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*
n)*(2*a*e + 3*b*d))/(4*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 902, normalized size of antiderivative = 5.71

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx$$

$$= \frac{x(240x^{3n}abd\,e^2n^4 + 468x^{3n}abd\,e^2n^3 + 294x^{3n}abd\,e^2n^2 + 72x^{3n}abd\,e^2n + 360x^{2n}ab\,d^2e\,n^4 + 642x^{2n}ab\,d^2e}{}$$

input
```
int((a+b*x^n)^2*(d+e*x^n)^3,x)
```

output
```
(x*(24*x**(5*n)*b**2*e**3*n**4 + 50*x**(5*n)*b**2*e**3*n**3 + 35*x**(5*n)*
b**2*e**3*n**2 + 10*x**(5*n)*b**2*e**3*n + x**(5*n)*b**2*e**3 + 60*x**(4*n
)*a*b*e**3*n**4 + 122*x**(4*n)*a*b*e**3*n**3 + 82*x**(4*n)*a*b*e**3*n**2 +
 22*x**(4*n)*a*b*e**3*n + 2*x**(4*n)*a*b*e**3 + 90*x**(4*n)*b**2*d*e**2*n*
*4 + 183*x**(4*n)*b**2*d*e**2*n**3 + 123*x**(4*n)*b**2*d*e**2*n**2 + 33*x*
*(4*n)*b**2*d*e**2*n + 3*x**(4*n)*b**2*d*e**2 + 40*x**(3*n)*a**2*e**3*n**4
 + 78*x**(3*n)*a**2*e**3*n**3 + 49*x**(3*n)*a**2*e**3*n**2 + 12*x**(3*n)*a
**2*e**3*n + x**(3*n)*a**2*e**3 + 240*x**(3*n)*a*b*d*e**2*n**4 + 468*x**(3
*n)*a*b*d*e**2*n**3 + 294*x**(3*n)*a*b*d*e**2*n**2 + 72*x**(3*n)*a*b*d*e**
2*n + 6*x**(3*n)*a*b*d*e**2 + 120*x**(3*n)*b**2*d**2*e*n**4 + 234*x**(3*n)
*b**2*d**2*e*n**3 + 147*x**(3*n)*b**2*d**2*e*n**2 + 36*x**(3*n)*b**2*d**2*
e*n + 3*x**(3*n)*b**2*d**2*e + 180*x**(2*n)*a**2*d*e**2*n**4 + 321*x**(2*n
)*a**2*d*e**2*n**3 + 177*x**(2*n)*a**2*d*e**2*n**2 + 39*x**(2*n)*a**2*d*e*
*2*n + 3*x**(2*n)*a**2*d*e**2 + 360*x**(2*n)*a*b*d**2*e*n**4 + 642*x**(2*n
)*a*b*d**2*e*n**3 + 354*x**(2*n)*a*b*d**2*e*n**2 + 78*x**(2*n)*a*b*d**2*e*
n + 6*x**(2*n)*a*b*d**2*e + 60*x**(2*n)*b**2*d**3*n**4 + 107*x**(2*n)*b**2
*d**3*n**3 + 59*x**(2*n)*b**2*d**3*n**2 + 13*x**(2*n)*b**2*d**3*n + x**(2*
n)*b**2*d**3 + 360*x**n*a**2*d**2*e*n**4 + 462*x**n*a**2*d**2*e*n**3 + 213
*x**n*a**2*d**2*e*n**2 + 42*x**n*a**2*d**2*e*n + 3*x**n*a**2*d**2*e + 240*
x**n*a*b*d**3*n**4 + 308*x**n*a*b*d**3*n**3 + 142*x**n*a*b*d**3*n**2 + ...
```

3.76 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{2ad(bd+ae)x^{1+n}}{1+n} + \frac{(b^2d^2+4abde+a^2e^2)\,x^{1+2n}}{1+2n} + \frac{2be(bd+ae)x^{1+3n}}{1+3n} + \frac{b^2e^2x^{1+4n}}{1+4n}$$

output
```
a^2*d^2*x+2*a*d*(a*e+b*d)*x^(1+n)/(1+n)+(a^2*e^2+4*a*b*d*e+b^2*d^2)*x^(1+2
*n)/(1+2*n)+2*b*e*(a*e+b*d)*x^(1+3*n)/(1+3*n)+b^2*e^2*x^(1+4*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = x\left(a^2d^2 + \frac{2ad(bd+ae)x^n}{1+n} + \frac{(b^2d^2+4abde+a^2e^2)\,x^{2n}}{1+2n} + \frac{2be(bd+ae)x^{3n}}{1+3n} + \frac{b^2e^2x^{4n}}{1+4n}\right)$$

input
```
Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

output
```
x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2
*e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^
2*x^(4*n))/(1 + 4*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$

$$\Big\downarrow \; 897$$

$$\int \left(x^{2n}\left(a^2e^2+4abde+b^2d^2\right)+a^2d^2+2bex^{3n}(ae+bd)+2adx^n(ae+bd)+b^2e^2x^{4n}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

input
```
Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

output
```
a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e
+ a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3
*n) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
risch	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,x^{2n}}{1+2n}+\frac{b^2e^2x\,x^{4n}}{1+4n}+\frac{2ad(ae+bd)x\,x^{n}}{1+n}+\frac{2be(ae+bd)x\,x^{3n}}{1+3n}$
norman	$a^2d^2x+\frac{(a^2e^2+4abde+b^2d^2)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2e^2x\,\mathrm{e}^{4n\ln(x)}}{1+4n}+\frac{2ad(ae+bd)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{2be(ae+bd)x\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{2x\,x^n a^2de+a^2d^2x+35x\,a^2d^2n^2+10x\,a^2d^2n+2x\,x^n ab\,d^2+52x\,x^n ab\,d^2n^2+18x\,x^n ab\,d^2n+18x\,x^n a^2den+48x\,x^n ab\,d^2n^3+52x}{}$
orering	Expression too large to display

input
```
int((a+b*x^n)^2*(d+e*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*(x^n)^2+b^2*e^2/(1+4*n)*x*
(x^n)^4+2*a*d*(a*e+b*d)/(1+n)*x*x^n+2*b*e*(a*e+b*d)/(1+3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(112) = 224.

Time = 0.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.30

$$\int (a+bx^n)^2\,(d+ex^n)^2\,dx$$
$$=\frac{(6\,b^2e^2n^3+11\,b^2e^2n^2+6\,b^2e^2n+b^2e^2)xx^{4\,n}+2\,(b^2de+abe^2+8\,(b^2de+abe^2)n^3+14\,(b^2de+abe^2)n^2+}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

output
```
((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b
^2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2
+ 7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12
*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*
n^2 + 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*
e + 24*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 +
 a^2*d*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 1
0*a^2*d^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(104) = 208$.

Time = 3.04 (sec) , antiderivative size = 1760, normalized size of antiderivative = 15.71

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

output
```
Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(
x) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2
/(3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x
) + 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2
*log(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3
*a**2*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d
*e*x**(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x)
 - 3*b**2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e
+ b*d)/3 + 4*b**2*e**2*log(x**(1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - 2*sqrt
(x)*(-a**2*e**2 - 4*a*b*d*e - b**2*d**2), Eq(n, -1/4)), (24*a**2*d**2*n**4
*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d
*e*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2
*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**
3 + 35*n**2 + 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.50

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx = a^2d^2x + \frac{b^2e^2x^{4n+1}}{4n+1} + \frac{2b^2dex^{3n+1}}{3n+1} + \frac{2abe^2x^{3n+1}}{3n+1} + \frac{b^2d^2x^{2n+1}}{2n+1} + \frac{4abdex^{2n+1}}{2n+1} + \frac{a^2e^2x^{2n+1}}{2n+1} + \frac{2abd^2x^{n+1}}{n+1} + \frac{2a^2dex^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

output
```
a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1
) + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*
b*d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^
(n + 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(112) = 224$.

Time = 0.13 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.81

$$\int (a + bx^n)^2 (d + ex^n)^2 \, dx$$
$$= \frac{24\, a^2 d^2 n^4 x + 6\, b^2 e^2 n^3 x x^{4\,n} + 16\, b^2 d e n^3 x x^{3\,n} + 16\, a b e^2 n^3 x x^{3\,n} + 12\, b^2 d^2 n^3 x x^{2\,n} + 48\, a b d e n^3 x x^{2\,n} + 12\, a^2}{}$$

input
```
integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

output
```
(24*a^2*d^2*n^4*x + 6*b^2*e^2*n^3*x*x^(4*n) + 16*b^2*d*e*n^3*x*x^(3*n) + 1
6*a*b*e^2*n^3*x*x^(3*n) + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d*e*n^3*x*x^(2
*n) + 12*a^2*e^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 48*a^2*d*e*n^3*x*x
^n + 50*a^2*d^2*n^3*x + 11*b^2*e^2*n^2*x*x^(4*n) + 28*b^2*d*e*n^2*x*x^(3*n
) + 28*a*b*e^2*n^2*x*x^(3*n) + 19*b^2*d^2*n^2*x*x^(2*n) + 76*a*b*d*e*n^2*x
*x^(2*n) + 19*a^2*e^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 52*a^2*d*e*n^
2*x*x^n + 35*a^2*d^2*n^2*x + 6*b^2*e^2*n*x*x^(4*n) + 14*b^2*d*e*n*x*x^(3*n
) + 14*a*b*e^2*n*x*x^(3*n) + 8*b^2*d^2*n*x*x^(2*n) + 32*a*b*d*e*n*x*x^(2*n
) + 8*a^2*e^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 18*a^2*d*e*n*x*x^n + 10*a
^2*d^2*n*x + b^2*e^2*x*x^(4*n) + 2*b^2*d*e*x*x^(3*n) + 2*a*b*e^2*x*x^(3*n)
 + b^2*d^2*x*x^(2*n) + 4*a*b*d*e*x*x^(2*n) + a^2*e^2*x*x^(2*n) + 2*a*b*d^2
*x*x^n + 2*a^2*d*e*x*x^n + a^2*d^2*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (a + bx^n)^2 (d + ex^n)^2 \, dx = a^2 d^2 x + \frac{x\, x^{2\,n} \left(a^2 e^2 + 4\, a\, b\, d\, e + b^2 d^2\right)}{2\, n + 1} + \frac{b^2 e^2 x\, x^{4\,n}}{4\, n + 1} + \frac{2\, b\, e\, x\, x^{3\,n} \left(a\, e + b\, d\right)}{3\, n + 1} + \frac{2\, a\, d\, x\, x^n \left(a\, e + b\, d\right)}{n + 1}$$

input
```
int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

output
```
a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e
^2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d
*x*x^n*(a*e + b*d))/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 503, normalized size of antiderivative = 4.49

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$
$$= \frac{x(a^2d^2 + x^{2n}a^2e^2 + x^{4n}b^2e^2 + x^{2n}b^2d^2 + 6x^{4n}b^2e^2n^3 + 11x^{4n}b^2e^2n^2 + 6x^{4n}b^2e^2n + 2x^{3n}ab\,e^2 + 2x^{3n}b^2de}{}$$

input
```
int((a+b*x^n)^2*(d+e*x^n)^2,x)
```

output
```
(x*(6*x**(4*n)*b**2*e**2*n**3 + 11*x**(4*n)*b**2*e**2*n**2 + 6*x**(4*n)*b*
*2*e**2*n + x**(4*n)*b**2*e**2 + 16*x**(3*n)*a*b*e**2*n**3 + 28*x**(3*n)*a
*b*e**2*n**2 + 14*x**(3*n)*a*b*e**2*n + 2*x**(3*n)*a*b*e**2 + 16*x**(3*n)*
b**2*d*e*n**3 + 28*x**(3*n)*b**2*d*e*n**2 + 14*x**(3*n)*b**2*d*e*n + 2*x**
(3*n)*b**2*d*e + 12*x**(2*n)*a**2*e**2*n**3 + 19*x**(2*n)*a**2*e**2*n**2 +
 8*x**(2*n)*a**2*e**2*n + x**(2*n)*a**2*e**2 + 48*x**(2*n)*a*b*d*e*n**3 +
76*x**(2*n)*a*b*d*e*n**2 + 32*x**(2*n)*a*b*d*e*n + 4*x**(2*n)*a*b*d*e + 12
*x**(2*n)*b**2*d**2*n**3 + 19*x**(2*n)*b**2*d**2*n**2 + 8*x**(2*n)*b**2*d*
*2*n + x**(2*n)*b**2*d**2 + 48*x**n*a**2*d*e*n**3 + 52*x**n*a**2*d*e*n**2
+ 18*x**n*a**2*d*e*n + 2*x**n*a**2*d*e + 48*x**n*a*b*d**2*n**3 + 52*x**n*a
*b*d**2*n**2 + 18*x**n*a*b*d**2*n + 2*x**n*a*b*d**2 + 24*a**2*d**2*n**4 +
50*a**2*d**2*n**3 + 35*a**2*d**2*n**2 + 10*a**2*d**2*n + a**2*d**2))/(24*n
**4 + 50*n**3 + 35*n**2 + 10*n + 1)
```

3.77 $\int (a+bx^n)^2 (c+dx^n) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)^2 (c+dx^n) \, dx = a^2cx + \frac{a(2bc+ad)x^{1+n}}{1+n} + \frac{b(bc+2ad)x^{1+2n}}{1+2n} + \frac{b^2dx^{1+3n}}{1+3n}$$

output
```
a^2*c*x+a*(a*d+2*b*c)*x^(1+n)/(1+n)+b*(2*a*d+b*c)*x^(1+2*n)/(1+2*n)+b^2*d*
x^(1+3*n)/(1+3*n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2 (c+dx^n) \, dx = \frac{dx(a+bx^n)^3 - (ad - b(c+3cn))x\left(a^2 + \frac{2abx^n}{1+n} + \frac{b^2x^{2n}}{1+2n}\right)}{b+3bn}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

output
```
(d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) +
(b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n) \, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(a^2c+bx^{2n}(2ad+bc)+ax^n(ad+2bc)+b^2dx^{3n}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*
n))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a^2cx+\frac{a(ad+2bc)x\,x^n}{1+n}+\frac{b(2ad+bc)x\,x^{2n}}{1+2n}+\frac{d\,b^2x\,x^{3n}}{1+3n}$
norman	$a^2cx+\frac{a(ad+2bc)x\,e^{n\ln(x)}}{1+n}+\frac{b(2ad+bc)x\,e^{2n\ln(x)}}{1+2n}+\frac{d\,b^2x\,e^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{2x\,x^{3n}b^2d\,n^2+3x\,x^{3n}b^2dn+6x\,x^{2n}abd\,n^2+3x\,x^{2n}b^2c\,n^2+d\,b^2x\,x^{3n}+8x\,x^{2n}abdn+4x\,x^{2n}b^2cn+6x\,x^n a^2d\,n^2+12x\,x^n abc\,n^2+}{(1+n)(1+2n)(1+3}$
orering	$x(a+b\,x^n)^2(c+d\,x^n)-\frac{x^2(11n^2+1)\left(\frac{2(a+b\,x^n)(c+d\,x^n)b\,x^n n}{x}+\frac{(a+b\,x^n)^2d\,x^n n}{x}\right)}{(2n^2+3n+1)(1+3n)}+\frac{2x^3(-1+3n)\left(\frac{2b^2x^{2n}n^2(c+}{x^2}\right.}{}$

input
```
int((a+b*x^n)^2*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*x^n+b*(2*a*d+b*c)/(1+2*n)*x*(x^n)^2+d*b^2/(1
+3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)^2(c+dx^n)\,dx$$
$$=\frac{(2\,b^2dn^2+3\,b^2dn+b^2d)xx^{3\,n}+(b^2c+2\,abd+3\,(b^2c+2\,abd)n^2+4\,(b^2c+2\,abd)n)xx^{2\,n}+(2\,abc+a^2d}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

output
```
((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
 + 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*
(2*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a
^2*c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.39 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a+bx^n)^2 (c+dx^n)\, dx$$
$$= \begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{}{6n^3} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n),x)
```

output
```
Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/
x - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sq
rt(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (
a**2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b
**2*c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 1
1*n**2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2
*c*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1
) + 6*a**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
+ 12*a*b*c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6
*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) +
 6*a*b*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*
n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6
*n + 1) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c
*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 3*b**2*d*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(
6*n**3 + 11*n**2 + 6*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)^2 (c+dx^n)\,dx = a^2cx + \frac{b^2dx^{3n+1}}{3n+1} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2abdx^{2n+1}}{2n+1} + \frac{2abcx^{n+1}}{n+1} + \frac{a^2dx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*
b*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n
 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(70) = 140.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)^2 (c+dx^n)\,dx$$
$$= \frac{6a^2cn^3x + 2b^2dn^2xx^{3n} + 3b^2cn^2xx^{2n} + 6abdn^2xx^{2n} + 12abcn^2xx^n + 6a^2dn^2xx^n + 11a^2cn^2x + 3b^2dn}{}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

output
```
(6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n
^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3
*b^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*
n*x*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n
) + 2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11
*n^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^2 (c+dx^n) \, dx = a^2 \, c \, x + \frac{x \, x^{2\,n} \, (c \, b^2 + 2 \, a \, d \, b)}{2 \, n + 1} + \frac{x \, x^n \, (d \, a^2 + 2 \, b \, c \, a)}{n + 1} + \frac{b^2 \, d \, x \, x^{3\,n}}{3 \, n + 1}$$

input
```
int((a + b*x^n)^2*(c + d*x^n),x)
```

output
```
a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*
c))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.06

$$\int (a + bx^n)^2 (c + dx^n) \, dx$$
$$= \frac{x(2x^{3n}b^2d\,n^2 + 3x^{3n}b^2dn + x^{3n}b^2d + 6x^{2n}abd\,n^2 + 8x^{2n}abdn + 2x^{2n}abd + 3x^{2n}b^2c\,n^2 + 4x^{2n}b^2cn + x^{2n}b^2}{6n^3 + 11}$$

input
```
int((a+b*x^n)^2*(c+d*x^n),x)
```

output
```
(x*(2*x**(3*n)*b**2*d*n**2 + 3*x**(3*n)*b**2*d*n + x**(3*n)*b**2*d + 6*x**
(2*n)*a*b*d*n**2 + 8*x**(2*n)*a*b*d*n + 2*x**(2*n)*a*b*d + 3*x**(2*n)*b**2
*c*n**2 + 4*x**(2*n)*b**2*c*n + x**(2*n)*b**2*c + 6*x**n*a**2*d*n**2 + 5*x
**n*a**2*d*n + x**n*a**2*d + 12*x**n*a*b*c*n**2 + 10*x**n*a*b*c*n + 2*x**n
*a*b*c + 6*a**2*c*n**3 + 11*a**2*c*n**2 + 6*a**2*c*n + a**2*c))/(6*n**3 +
11*n**2 + 6*n + 1)
```

3.78 $\int \frac{(a+bx^n)^2}{c+dx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(a+bx^n)^2}{c+dx^n} \, dx = -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2 x \, \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

output
```
-b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*
x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c/d^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2}{c+dx^n} \, dx = \frac{a^2x}{c} - \frac{(bc-ad)^2x}{cd^2} + \frac{b^2x^{1+n}}{d(1+n)} + \frac{(-bc+ad)^2 x \, \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd^2}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

output
```
(a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(
b*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c
*d^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {933, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int -\frac{b(bc(n+1)-ad(2n+1))x^n+a(bc-ad(n+1))}{dx^n+c}dx}{d(n+1)}+\frac{bx(a+bx^n)}{d(n+1)}$$

$$\Big\downarrow \ 25$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\int \frac{b(bc(n+1)-ad(2n+1))x^n+a(bc-ad(n+1))}{dx^n+c}dx}{d(n+1)}$$

$$\Big\downarrow \ 913$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\frac{bx(bc(n+1)-ad(2n+1))}{d}-\frac{(n+1)(bc-ad)^2\int \frac{1}{dx^n+c}dx}{d}}{d(n+1)}$$

$$\Big\downarrow \ 778$$

$$\frac{bx(a+bx^n)}{d(n+1)}-\frac{\frac{bx(bc(n+1)-ad(2n+1))}{d}-\frac{(n+1)x(bc-ad)^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}}{d(n+1)}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n),x]
```

output
```
(b*x*(a + b*x^n))/(d*(1 + n)) - ((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/d - (
(b*c - a*d)^2*(1 + n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d))/(d*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^2}{c+d\,x^n}dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\begin{aligned}\int \frac{(a+bx^n)^2}{c+dx^n}\,dx &= \frac{a^2c^{\frac{1}{n}}c^{-1-\frac{1}{n}}x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} \\ &\quad - \frac{2abc^{-\frac{1}{n}}c^{1+\frac{1}{n}}d^{\frac{1}{n}}d^{-1-\frac{1}{n}}x\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)} \\ &\quad + \frac{2b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} \\ &\quad + \frac{b^2c^{-3-\frac{1}{n}}c^{2+\frac{1}{n}}x^{2n+1}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n),x)
```

output
```
a**2*c**(1/n)*c**(-1 - 1/n)*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) - 2*a*b*c**(1 + 1/n)*d**(1/n)*d**(-1 - 1/n
)*x*lerchphi(c*exp_polar(I*pi)/(d*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/
(c*c**(1/n)*n**2*gamma(1 + 1/n)) + 2*b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2
*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*g
amma(3 + 1/n)) + b**2*c**(-3 - 1/n)*c**(2 + 1/n)*x**(2*n + 1)*lerchphi(d*x
**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d
*x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{x^n b^2 dx+\left(\int \frac{1}{x^n d+c}dx\right)a^2d^2n+\left(\int \frac{1}{x^n d+c}dx\right)a^2d^2-2\left(\int \frac{1}{x^n d+c}dx\right)abcdn-2\left(\int \frac{1}{x^n d+c}dx\right)abcd+\left(\int \frac{1}{x^n d+c}\right.}{d^2(n+1)}$$

input
```
int((a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(x**n*b**2*d*x + int(1/(x**n*d + c),x)*a**2*d**2*n + int(1/(x**n*d + c),x)
*a**2*d**2 - 2*int(1/(x**n*d + c),x)*a*b*c*d*n - 2*int(1/(x**n*d + c),x)*a
*b*c*d + int(1/(x**n*d + c),x)*b**2*c**2*n + int(1/(x**n*d + c),x)*b**2*c*
*2 + 2*a*b*d*n*x + 2*a*b*d*x - b**2*c*n*x - b**2*c*x)/(d**2*(n + 1))
```

3.79 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2} \, dx = -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{(bc-ad)(ad(1-n)-bc(1+n))x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2d^2n}$$

output
```
-b*(a*d-b*c*(1+n))*x/c/d^2/n-(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)+(-a*d+
b*c)*(a*d*(1-n)-b*c*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^2/d^2/
n
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2} \, dx = \frac{x\left(\frac{c(-2abcd+a^2d^2+b^2c(c+cn+dnx^n))}{c+dx^n} - (bc-ad)(ad(-1+n)+bc(1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\right.\right.}{c^2d^2n}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

output
```
(x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (
b*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {930, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(n+1))x^n}{dx^n+c}dx}{cdn}-\frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(bc-ad)(ad(1-n)-bc(n+1))\int \frac{1}{dx^n+c}dx}{d}-\frac{bx(ad-bc(n+1))}{d}}{cdn}-\frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{cd}-\frac{bx(ad-bc(n+1))}{d}}{cdn}-\frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

output
```
-(((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n))) + (-((b*(a*d - b*c*(1 +
 n))*x)/d) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F1[
1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d))/(c*d*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{2}}{(c+d\,x^{n})^{2}}dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
x)
```

Sympy [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

output
```
Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
-(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n
+ c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d
^2)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = \frac{-x^n\left(\int \frac{x^{2n}}{x^{2n}d^2n+x^{2n}d^2+2x^n cdn+2x^n cd+c^2n+c^2}dx\right)a^2d^3n^2+x^n\left(\int \frac{x^{2n}}{x^{2n}d^2n+x^{2n}d^2+2x^n cdn+2x^n cd+c^2n+c^2}dx\right)a^2d^3-2\ldots}{\ldots}$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

output

```
( - x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*
x**n*c*d + c**2*n + c**2),x)*a**2*d**3*n**2 + x**n*int(x**(2*n)/(x**(2*n)*
d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*a**
2*d**3 - 2*x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d
*n + 2*x**n*c*d + c**2*n + c**2),x)*a*b*c*d**2*n - 2*x**n*int(x**(2*n)/(x*
*(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2)
,x)*a*b*c*d**2 + x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x*
*n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*b**2*c**2*d*n**2 + 2*x**n*int(x*
*(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2
*n + c**2),x)*b**2*c**2*d*n + x**n*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n
)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*b**2*c**2*d + x**n*
a**2*d*n*x - x**n*a**2*d*x + 2*x**n*a*b*c*x - int(x**(2*n)/(x**(2*n)*d**2*
n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*a**2*c*d
**2*n**2 + int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n +
2*x**n*c*d + c**2*n + c**2),x)*a**2*c*d**2 - 2*int(x**(2*n)/(x**(2*n)*d**2
*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*a*b*c**
2*d*n - 2*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + 2
*x**n*c*d + c**2*n + c**2),x)*a*b*c**2*d + int(x**(2*n)/(x**(2*n)*d**2*n +
 x**(2*n)*d**2 + 2*x**n*c*d*n + 2*x**n*c*d + c**2*n + c**2),x)*b**2*c**3*n
**2 + 2*int(x**(2*n)/(x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*n + ...
```

3.80 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x(a+bx^n)}{2cdn\,(c+dx^n)^2} - \frac{b(ad(1-n)-bc(1+n))x}{2cd^2(1-n)n\,(c+dx^n)} + \frac{(2abcd(1-n)-b^2c^2(1+n)-a^2d^2(1-3n+2n^2))\,x\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{2c^3d^2(1-n)n}$$

output
```
-1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2-1/2*b*(a*d*(1-n)-b*c*(1+n))*
x/c/d^2/(1-n)/n/(c+d*x^n)+1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*d^2*(2*n^
2-3*n+1))*x*hypergeom([2, 1/n],[1+1/n],-d*x^n/c)/c^3/d^2/(1-n)/n
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \frac{x\left(\frac{c^2(bc-ad)^2n}{(c+dx^n)^2} - \frac{c(bc-ad)(ad(-1+2n)+b(c+2cn))}{c+dx^n} + (2abcd(-1+n)+b^2c^2(1+n)+a^2d^2(1-3n+2n^2))\,\text{Hyper}\right.}{2c^3d^2n^2}$$

input
```
Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

output
```
(x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
 b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2
*d^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(2*c^3*d^2*n^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {930, 910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(n+1))x^n}{(dx^n+c)^2}dx}{2cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{2cdn\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 910$$

$$\frac{\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{cdn(c+dx^n)}-\frac{\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\int \frac{1}{dx^n+c}dx}{cdn}}{2cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{2cdn\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{cdn(c+dx^n)}-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2dn}}{2cdn}-\frac{x(bc-ad)\left(a+bx^n\right)}{2cdn\left(c+dx^n\right)^2}$$

input
```
Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

output
```
-1/2*((b*c - a*d)*x*(a + b*x^n))/(c*d*n*(c + d*x^n)^2) + (((b*c - a*d)*(a*
d*(1 - 2*n) - b*c*(1 + n))*x)/(c*d*n*(c + d*x^n)) - ((2*a*b*c*d*(1 - n) -
b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n))/(2*c*d*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(a + b\,x^n)^2}{(c + d\,x^n)^3} dx$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) +
3*c^2*d*x^n + c^3), x)
```

Sympy [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$$

input
```
integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

output
```
Integral((a + b*x**n)**2/(c + d*x**n)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrat
e(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^
2*d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n +
 1) - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c
^4*d^2*n^2)
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^2}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

output

```
(2*x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d
**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**
3),x)*a**2*d**4*n**3 - x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d
**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c
**2*d + c**3*n + c**3),x)*a**2*d**4*n**2 - 2*x**(2*n)*int(x**(2*n)/(x**(3*
n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x*
*n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a**2*d**4*n + x**(2*n)*int
(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2
*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*a**2*d**4
 + 2*x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c
*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c
**3),x)*a*b*c*d**3*n**2 - 2*x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3
*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x
**n*c**2*d + c**3*n + c**3),x)*a*b*c*d**3 + x**(2*n)*int(x**(2*n)/(x**(3*n
)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**
n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*b**2*c**2*d**2*n**2 + 2*x**
(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*n
+ 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*n + c**3),x)*
b**2*c**2*d**2*n + x**(2*n)*int(x**(2*n)/(x**(3*n)*d**3*n + x**(3*n)*d**3
+ 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*n + 3*x**n*c*...
```

3.81 $\int \frac{(c+dx^n)^4}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\begin{aligned}\int \frac{(c+dx^n)^4}{a+bx^n} dx &= -\frac{d(6ab^2c^2d(1+3n)-4a^2bcd^2(1+3n)+a^3d^3(1+3n)-3b^3c^3(1+4n))\,x}{b^4(1+3n)}\\ &\quad-\frac{d^2(4abcd(1+3n)-a^2d^2(1+3n)-3b^2c^2(1+5n))\,x^{1+n}}{b^3(1+n)(1+3n)}\\ &\quad-\frac{d^3(ad(1+3n)-b(c+6cn))x^{1+2n}}{b^2(1+5n+6n^2)}+\frac{dx(c+dx^n)^3}{b(1+3n)}\\ &\quad+\frac{(bc-ad)^4x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^4}\end{aligned}$$

output
```
-d*(6*a*b^2*c^2*d*(1+3*n)-4*a^2*b*c*d^2*(1+3*n)+a^3*d^3*(1+3*n)-3*b^3*c^3*
(1+4*n))*x/b^4/(1+3*n)-d^2*(4*a*b*c*d*(1+3*n)-a^2*d^2*(1+3*n)-3*b^2*c^2*(1
+5*n))*x^(1+n)/b^3/(1+n)/(1+3*n)-d^3*(a*d*(1+3*n)-b*(6*c*n+c))*x^(1+2*n)/b
^2/(6*n^2+5*n+1)+d*x*(c+d*x^n)^3/b/(1+3*n)+(-a*d+b*c)^4*x*hypergeom([1, 1/
n],[1+1/n],-b*x^n/a)/a/b^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$
$$= \frac{x\left(4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+d^4x^{4n}\Phi\left(-\frac{bx^n}{a},1\right.\right.}{an}$$

input
```
Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

output
```
(x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x
^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurw
itzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((
b*x^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))
/(a*n)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {933, 25, 1025, 25, 1025, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int -\frac{(dx^n+c)^2(d(ad(3n+1)-b(6nc+c))x^n+c(ad-b(3nc+c)))}{bx^n+a}dx}{b(3n+1)}+\frac{dx(c+dx^n)^3}{b(3n+1)}$$
$$\Big\downarrow \text{25}$$
$$\frac{dx(c+dx^n)^3}{b(3n+1)}-\frac{\int \frac{(dx^n+c)^2(d(ad(3n+1)-b(6nc+c))x^n+c(ad-b(3nc+c)))}{bx^n+a}dx}{b(3n+1)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} - \frac{\frac{\int -\frac{(dx^n+c)\left(c\left(b^2\left(6n^2+5n+1\right)c^2-2abd(4n+1)c+a^2d^2(3n+1)\right)-d\left(-b^2\left(18n^2+7n+1\right)c^2+2abd(3n+1)^2c-a^2d^2\left(6n^2+5n+1\right)\right)x^n\right)}{bx^n+a}dx}{b(2n+1)} + \frac{dx(c+dx^n)^2(ad(3 \ldots}{b(2n \ldots}}{b(3n+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} - \frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{\int \frac{(dx^n+c)\left(c\left(b^2\left(6n^2+5n+1\right)c^2-2abd(4n+1)c+a^2d^2(3n+1)\right)-d\left(-b^2\left(18n^2+7n+1\right)c^2+2abd(3n+1)^2c-a^2d^2\left(6n^2 \ldots\right.\right.\right.}{bx^n+a}}{b(2n+1)} \ldots}{b(3n+1)}$$

$$\Big\downarrow \text{1025}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} - \frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{\int -\frac{d\left(-b^3\left(24n^3+18n^2+7n+1\right)c^3+ab^2d\left(36n^3+45n^2+20n+3\right)c^2-a^2bd^2\left(24n^3+38n^2+19n+3\right)c+a^3d^3\left(6n^3+11n^2 \ldots\right.\right.}{bx \ldots}}{b(n \ldots} \ldots}{\ldots}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} - \frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\int \frac{d\left(-b^3\left(24n^3+18n^2+7n+1\right)c^3+ab^2d\left(36n^3+45n^2+20n+3\right)c^2-a^2bd^2\left(24n^3+38n^2+19n+3\right)c+a^3d^3\left(6n^3+11n^2 \ldots\right.\right.}{bx \ldots}}{b(\ldots} \ldots}{\ldots}$$

$$\Big\downarrow \text{913}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} - \frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(24n^3+38n^2+19n+3\right)+ab^2c^2d\left(36n^3+45n^2+20n+3\right)-b^3c^3\left(24n^3+18n^2+ \ldots\right.\right.}{b}}{b(n+1)} \ldots}{b(3n+1)}$$

$$\Big\downarrow \text{778}$$

$$\frac{dx(c+dx^n)^3}{b(3n+1)} -$$

$$\frac{\frac{dx(c+dx^n)^2(ad(3n+1)-b(6cn+c))}{b(2n+1)} - \frac{-\frac{dx(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b(n+1)} - \frac{dx\left(a^3d^3\left(6n^3+11n^2+6n+1\right)-a^2bcd^2\left(\right.\right.}{}}{}}{b(3n+}$$

input
```
Int[(c + d*x^n)^4/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n)^3)/(b*(1 + 3*n)) - ((d*(a*d*(1 + 3*n) - b*(c + 6*c*n))*x*
(c + d*x^n)^2)/(b*(1 + 2*n)) - (-((d*(2*a*b*c*d*(1 + 3*n)^2 - a^2*d^2*(1 +
 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x^n))/(b*(1 + n))) -
((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 4
5*n^2 + 36*n^3))*x)/b - ((b*c - a*d)^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*Hyper
geometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(b*(1 + n)))/(b*
(1 + 2*n)))/(b*(1 + 3*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^{n})^{4}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^n)^4/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^4/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n
+ c^4)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{4a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n\Gamma\left(5+\frac{1}{n}\right)} + \frac{a^{-5-\frac{1}{n}}a^{4+\frac{1}{n}}d^4x^{4n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,4+\frac{1}{n}\right)\Gamma\left(4+\frac{1}{n}\right)}{n^2\Gamma\left(5+\frac{1}{n}\right)} + \frac{12a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)} + \frac{4a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}cd^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)} + \frac{12a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}c^2d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} - \frac{4a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**4/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 4*a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n*ga
mma(5 + 1/n)) + a**(-5 - 1/n)*a**(4 + 1/n)*d**4*x**(4*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(n**2*gamma(5 + 1/n)) + 1
2*a**(-4 - 1/n)*a**(3 + 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*gamma(4 + 1/n)) + 4*a**(-4 - 1/n)*
a**(3 + 1/n)*c*d**3*x**(3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 +
 1/n)*gamma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 12*a**(-3 - 1/n)*a**(2 + 1/n)
*c**2*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gam
ma(2 + 1/n)/(n*gamma(3 + 1/n)) + 6*a**(-3 - 1/n)*a**(2 + 1/n)*c**2*d**2*x*
*(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(
n**2*gamma(3 + 1/n)) - 4*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c**3*d*x*lerc
hphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1
/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx=\int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*in
tegrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (
4*(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (
6*(6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2
 + 5*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6
*(6*n^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a
^2*b*c*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n
 + 1)*b^4)
```

Giac [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(dx^n+c)^4}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^4/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^4}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^4/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^4/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \text{Too large to display}$$

input
```
int((c+d*x^n)^4/(a+b*x^n),x)
```

output

```
(2*x**(3*n)*b**3*d**4*n**2*x + 3*x**(3*n)*b**3*d**4*n*x + x**(3*n)*b**3*d*
*4*x - 3*x**(2*n)*a*b**2*d**4*n**2*x - 4*x**(2*n)*a*b**2*d**4*n*x - x**(2*
n)*a*b**2*d**4*x + 12*x**(2*n)*b**3*c*d**3*n**2*x + 16*x**(2*n)*b**3*c*d**
3*n*x + 4*x**(2*n)*b**3*c*d**3*x + 6*x**n*a**2*b*d**4*n**2*x + 5*x**n*a**2
*b*d**4*n*x + x**n*a**2*b*d**4*x - 24*x**n*a*b**2*c*d**3*n**2*x - 20*x**n*
a*b**2*c*d**3*n*x - 4*x**n*a*b**2*c*d**3*x + 36*x**n*b**3*c**2*d**2*n**2*x
 + 30*x**n*b**3*c**2*d**2*n*x + 6*x**n*b**3*c**2*d**2*x + 6*int(1/(x**n*b
+ a),x)*a**4*d**4*n**3 + 11*int(1/(x**n*b + a),x)*a**4*d**4*n**2 + 6*int(1
/(x**n*b + a),x)*a**4*d**4*n + int(1/(x**n*b + a),x)*a**4*d**4 - 24*int(1/
(x**n*b + a),x)*a**3*b*c*d**3*n**3 - 44*int(1/(x**n*b + a),x)*a**3*b*c*d**
3*n**2 - 24*int(1/(x**n*b + a),x)*a**3*b*c*d**3*n - 4*int(1/(x**n*b + a),x
)*a**3*b*c*d**3 + 36*int(1/(x**n*b + a),x)*a**2*b**2*c**2*d**2*n**3 + 66*i
nt(1/(x**n*b + a),x)*a**2*b**2*c**2*d**2*n**2 + 36*int(1/(x**n*b + a),x)*a
**2*b**2*c**2*d**2*n + 6*int(1/(x**n*b + a),x)*a**2*b**2*c**2*d**2 - 24*in
t(1/(x**n*b + a),x)*a*b**3*c**3*d*n**3 - 44*int(1/(x**n*b + a),x)*a*b**3*c
**3*d*n**2 - 24*int(1/(x**n*b + a),x)*a*b**3*c**3*d*n - 4*int(1/(x**n*b +
a),x)*a*b**3*c**3*d + 6*int(1/(x**n*b + a),x)*b**4*c**4*n**3 + 11*int(1/(x
**n*b + a),x)*b**4*c**4*n**2 + 6*int(1/(x**n*b + a),x)*b**4*c**4*n + int(1
/(x**n*b + a),x)*b**4*c**4 - 6*a**3*d**4*n**3*x - 11*a**3*d**4*n**2*x - 6*
a**3*d**4*n*x - a**3*d**4*x + 24*a**2*b*c*d**3*n**3*x + 44*a**2*b*c*d**...
```

3.82 $\int \frac{(c+dx^n)^3}{a+bx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{(c+dx^n)^3}{a+bx^n} \, dx = -\frac{d(3abcd(1+2n)-a^2d^2(1+2n)-2b^2c^2(1+3n))\,x}{b^3(1+2n)} - \frac{d^2(ad(1+2n)-b(c+4cn))x^{1+n}}{b^2(1+n)(1+2n)} + \frac{dx(c+dx^n)^2}{b(1+2n)} + \frac{(bc-ad)^3 x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^3}$$

output
```
-d*(3*a*b*c*d*(1+2*n)-a^2*d^2*(1+2*n)-2*b^2*c^2*(1+3*n))*x/b^3/(1+2*n)-d^2
*(a*d*(1+2*n)-b*(4*c*n+c))*x^(1+n)/b^2/(1+n)/(1+2*n)+d*x*(c+d*x^n)^2/b/(1+
2*n)+(-a*d+b*c)^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^n)^3}{a+bx^n} \, dx = \frac{x\left(3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

input
```
Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

output
```
(x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(
2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLer
chPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1,
n^(-1)]))/(a*n)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {933, 25, 1025, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int -\frac{(dx^n+c)(d(ad(2n+1)-b(4nc+c))x^n+c(ad-b(2nc+c)))}{bx^n+a}dx}{b(2n+1)}+\frac{dx(c+dx^n)^2}{b(2n+1)}$$

$$\Big\downarrow \ 25$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\int \frac{(dx^n+c)(d(ad(2n+1)-b(4nc+c))x^n+c(ad-b(2nc+c)))}{bx^n+a}dx}{b(2n+1)}$$

$$\Big\downarrow \ 1025$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{\int -\frac{d\left(b^2\left(6n^2+4n+1\right)c^2-abd\left(6n^2+7n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n+c\left(b^2\left(2n^2+3n+1\right)c^2-abd(5n+2)c+a^2d^2(2n+1)\right)}{bx^n+a}dx}{b(n+1)}+\frac{dx(c+dx^n)(ad(2n+1)-b(4c\ldots}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \ 25$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)}-\frac{\int \frac{d\left(b^2\left(6n^2+4n+1\right)c^2-abd\left(6n^2+7n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n+c\left(b^2\left(2n^2+3n+1\right)c^2-abd(5n+2)c+a^2d^2(2n+1)\right)}{bx^n+a}}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \ 913$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)}-\frac{\frac{(n+1)(2n+1)(bc-ad)^3\int \frac{1}{bx^n+a}dx}{b}+\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b}}{b(n+1)}}{b(2n+1)}$$

$$\Big\downarrow \ 778$$

$$\frac{dx(c+dx^n)^2}{b(2n+1)}-\frac{\frac{dx(c+dx^n)(ad(2n+1)-b(4cn+c))}{b(n+1)}-\frac{\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b}+\frac{(n+1)(2n+1)x(bc-ad)^3\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1\right.}{ab}}{b(n+1)}}{b(2n+1)}$$

input
```
Int[(c + d*x^n)^3/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n)^2)/(b*(1 + 2*n)) - ((d*(a*d*(1 + 2*n) - b*(c + 4*c*n))*x*
(c + d*x^n))/(b*(1 + n)) - ((d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4
*n + 6*n^2) - a*b*c*d*(2 + 7*n + 6*n^2))*x)/b + ((b*c - a*d)^3*(1 + n)*(1
+ 2*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(b
*(1 + n)))/(b*(1 + 2*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(c + d\,x^n)^3}{a + b\,x^n}dx$$

input
```
int((c+d*x^n)^3/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^3/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx=\int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \frac{(c+dx^n)^3}{a+bx^n}\,dx=&\ \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}\\&+\frac{3a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}\\&+\frac{a^{-4-\frac{1}{n}}a^{3+\frac{1}{n}}d^3x^{3n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,3+\frac{1}{n}\right)\Gamma\left(3+\frac{1}{n}\right)}{n^2\Gamma\left(4+\frac{1}{n}\right)}\\&+\frac{6a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}\\&+\frac{3a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}cd^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)}\\&-\frac{3a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((c+d*x**n)**3/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 3*a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n*ga
mma(4 + 1/n)) + a**(-4 - 1/n)*a**(3 + 1/n)*d**3*x**(3*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(n**2*gamma(4 + 1/n)) + 6
*a**(-3 - 1/n)*a**(2 + 1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*gamma(3 + 1/n)) + 3*a**(-3 - 1/n)*a
**(2 + 1/n)*c*d**2*x**(2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 +
1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 3*a**(1 + 1/n)*b**(1/n)*b**(-1
 - 1/n)*c**2*d*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n
)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx=\int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
 a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^
3*(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*
a*b*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

Giac [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx=\int \frac{(dx^n+c)^3}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^3/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^3}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^3/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^3/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{3b^2c^2dx+3\left(\int \frac{1}{x^nb+a}dx\right)a^2bc\,d^2-3\left(\int \frac{1}{x^nb+a}dx\right)a\,b^2c^2d-2x^nab\,d^3nx+6x^nb^2c\,d^2nx-6abc\,d^2n^2x-9ab}{}$$

input
```
int((c+d*x^n)^3/(a+b*x^n),x)
```

output
```
(x**(2*n)*b**2*d**3*n*x + x**(2*n)*b**2*d**3*x - 2*x**n*a*b*d**3*n*x - x**
n*a*b*d**3*x + 6*x**n*b**2*c*d**2*n*x + 3*x**n*b**2*c*d**2*x - 2*int(1/(x*
*n*b + a),x)*a**3*d**3*n**2 - 3*int(1/(x**n*b + a),x)*a**3*d**3*n - int(1/
(x**n*b + a),x)*a**3*d**3 + 6*int(1/(x**n*b + a),x)*a**2*b*c*d**2*n**2 + 9
*int(1/(x**n*b + a),x)*a**2*b*c*d**2*n + 3*int(1/(x**n*b + a),x)*a**2*b*c*
d**2 - 6*int(1/(x**n*b + a),x)*a*b**2*c**2*d*n**2 - 9*int(1/(x**n*b + a),x
)*a*b**2*c**2*d*n - 3*int(1/(x**n*b + a),x)*a*b**2*c**2*d + 2*int(1/(x**n*
b + a),x)*b**3*c**3*n**2 + 3*int(1/(x**n*b + a),x)*b**3*c**3*n + int(1/(x*
*n*b + a),x)*b**3*c**3 + 2*a**2*d**3*n**2*x + 3*a**2*d**3*n*x + a**2*d**3*
x - 6*a*b*c*d**2*n**2*x - 9*a*b*c*d**2*n*x - 3*a*b*c*d**2*x + 6*b**2*c**2*
d*n**2*x + 9*b**2*c**2*d*n*x + 3*b**2*c**2*d*x)/(b**3*(2*n**2 + 3*n + 1))
```

3.83 $\int \frac{(c+dx^n)^2}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)} + \frac{dx(c+dx^n)}{b(1+n)} + \frac{(bc-ad)^2 x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2}$$

output
```
-d*(a*d*(1+n)-b*(2*c*n+c))*x/b^2/(1+n)+d*x*(c+d*x^n)/b/(1+n)+(-a*d+b*c)^2*
x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^n)^2}{a+bx^n} dx = \frac{x\left(2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)\right)}{an}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

output
```
(x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*H
urwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n
)/a), 1, n^(-1)]))/(a*n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {933, 25, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$$

$$\Big\downarrow \; 933$$

$$\frac{\int -\frac{d(ad(n+1)-b(2nc+c))x^n+c(ad-bc(n+1))}{bx^n+a}dx}{b(n+1)}+\frac{dx(c+dx^n)}{b(n+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\int \frac{d(ad(n+1)-b(2nc+c))x^n+c(ad-bc(n+1))}{bx^n+a}dx}{b(n+1)}$$

$$\Big\downarrow \; 913$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\frac{dx(ad(n+1)-b(2cn+c))}{b}-\frac{(n+1)(bc-ad)^2\int \frac{1}{bx^n+a}dx}{b}}{b(n+1)}$$

$$\Big\downarrow \; 778$$

$$\frac{dx(c+dx^n)}{b(n+1)}-\frac{\frac{dx(ad(n+1)-b(2cn+c))}{b}-\frac{(n+1)x(bc-ad)^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}}{b(n+1)}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n),x]
```

output
```
(d*x*(c + d*x^n))/(b*(1 + n)) - ((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/b - (
(b*c - a*d)^2*(1 + n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b))/(b*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c + d\,x^n)^2}{a + b\,x^n}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.80

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{a^{-3-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{n^2\Gamma\left(3+\frac{1}{n}\right)} - \frac{2a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
amma(1/n)/(n**2*gamma(1 + 1/n)) + 2*a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*
n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n*ga
mma(3 + 1/n)) + a**(-3 - 1/n)*a**(2 + 1/n)*d**2*x**(2*n + 1)*lerchphi(b*x*
*n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(n**2*gamma(3 + 1/n)) - 2
*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c*d*x*lerchphi(a*exp_polar(I*pi)/(b*x
**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2
*x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \int \frac{(c+d\,x^n)^2}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \frac{x^n b\,d^2x + \left(\int \frac{1}{x^n b+a}dx\right)a^2d^2n + \left(\int \frac{1}{x^n b+a}dx\right)a^2d^2 - 2\left(\int \frac{1}{x^n b+a}dx\right)abcdn - 2\left(\int \frac{1}{x^n b+a}dx\right)abcd + \left(\int \frac{1}{x^n b+a}\right.}{b^2\,(n+1)}$$

input
```
int((c+d*x^n)^2/(a+b*x^n),x)
```

output
```
(x**n*b*d**2*x + int(1/(x**n*b + a),x)*a**2*d**2*n + int(1/(x**n*b + a),x)
*a**2*d**2 - 2*int(1/(x**n*b + a),x)*a*b*c*d*n - 2*int(1/(x**n*b + a),x)*a
*b*c*d + int(1/(x**n*b + a),x)*b**2*c**2*n + int(1/(x**n*b + a),x)*b**2*c*
*2 - a*d**2*n*x - a*d**2*x + 2*b*c*d*n*x + 2*b*c*d*x)/(b**2*(n + 1))
```

3.84 $\int \frac{c+dx^n}{a+bx^n} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{c+dx^n}{a+bx^n} \, dx = \frac{dx}{b} + \frac{(bc-ad)x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

output
```
d*x/b+(-a*d+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^n}{a+bx^n} \, dx = \frac{x\left(ad+(bc-ad) \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{ab}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n),x]
```

output
```
(x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a
)]))/(a*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{a+bx^n}dx$$

$$\Big\downarrow \text{ 913}$$

$$\frac{(bc-ad)\int \frac{1}{bx^n+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

input
```
Int[(c + d*x^n)/(a + b*x^n),x]
```

output
```
(d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{c+d\,x^n}{a+b\,x^n}dx$$

input
```
int((c+d*x^n)/(a+b*x^n),x)
```

output
```
int((c+d*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.62

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) - a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*d*x*ler
chphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(
1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
(b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

Giac [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\int \frac{dx^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{a+bx^n}\,dx=\int \frac{c+d\,x^n}{a+b\,x^n}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n),x)
```

output
```
int((c + d*x^n)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{c+dx^n}{a+bx^n}\,dx = \frac{-\left(\int \frac{1}{x^n b+a}dx\right)ad + \left(\int \frac{1}{x^n b+a}dx\right)bc + dx}{b}$$

input
```
int((c+d*x^n)/(a+b*x^n),x)
```

output
```
( - int(1/(x**n*b + a),x)*a*d + int(1/(x**n*b + a),x)*b*c + d*x)/b
```

3.85 $\int \frac{1}{(a+bx^n)(c+dx^n)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)} \, dx = \frac{bx \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

output
```
b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/
n],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)(c+dx^n)} \, dx = \frac{x\left(-bc \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d)
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {917, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

$$\Big\downarrow \text{917}$$

$$\frac{b\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d
)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c
- a*d))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{x^{2n}bd+x^nad+x^nbc+ac}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.86 $\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx = -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2 x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{d(bc(1-2n)-ad(1-n))x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^2 n}$$

output
```
-d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a
/(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-d*x^
n/c)/c^2/(-a*d+b*c)^2/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2} \, dx = \frac{x\left(b^2c^2n(c+dx^n) \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right) + ad\big(c(-bc+ad) + (ad(-1+n) + b(c-2cn\right.}{ac^2(bc-ad)^2n(c+dx^n)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
(x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^
n)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c -
a*d)^2*n*(c + d*x^n))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}dx$$

$$\Big\downarrow \text{ 931}$$

$$\frac{\int \frac{bd(1-n)x^n+bcn+a(d-dn)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

$$\Big\downarrow \text{ 1020}$$

$$\frac{\frac{b^2cn\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d(ad(1-n)-b(c-2cn))\int \frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{\frac{b^2cnx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx(ad(1-n)-b(c-2cn))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}-\frac{dx}{cn(bc-ad)(c+dx^n)}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + ((b^2*c*n*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)) - (d*(a*d*(1 - n) - b*(
c - 2*c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(c*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})^{2}}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2
*a*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b
^2*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n
+ a^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*
n)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{1}{x^{3n}b\,d^2+x^{2n}a\,d^2+2x^{2n}bcd+2x^n acd+x^n b\,c^2+a\,c^2}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int(1/(x**(3*n)*b*d**2 + x**(2*n)*a*d**2 + 2*x**(2*n)*b*c*d + 2*x**n*a*c*d
 + x**n*b*c**2 + a*c**2),x)
```

3.87 $\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 210

$$
\begin{aligned}
\int \frac{1}{(a+bx^n)\,(c+dx^n)^3}\,dx = &-\frac{dx}{2c(bc-ad)n\,(c+dx^n)^2} \\
&-\frac{d(ad(1-2n)-b(c-4cn))x}{2c^2(bc-ad)^2n^2\,(c+dx^n)}+\frac{b^3x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^3} \\
&-\frac{d(a^2d^2(1-3n+2n^2)-2abcd(1-4n+3n^2)+b^2c^2(1-5n+6n^2))\,x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\right.}{2c^3(bc-ad)^3n^2}
\end{aligned}
$$

output
```
-1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2
/(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(
-a*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(
6*n^2-5*n+1))*x*hypergeom([1, 1/n],[1+1/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/n^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$$
$$= \frac{x\left(-ac^2d(bc-ad)^2n+acd(bc-ad)(ad(-1+2n)+b(c-4cn))\,(c+dx^n)+2b^3c^3n^2(c+dx^n)^2\,\text{Hyperge}\right.}{}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

output
```
(x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c
- 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*
c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2
*(c + d*x^n)^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {931, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$$
$$\Big\downarrow \; 931$$
$$\frac{\int \frac{bd(1-2n)x^n+2bcn+a(d-2dn)}{(bx^n+a)(dx^n+c)^2}dx}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)\,(c+dx^n)^2}$$
$$\Big\downarrow \; 1024$$

$$\frac{\frac{\int \frac{-bd(bc(1-4n)-ad(1-2n))(1-n)x^n+2b^2c^2n^2+a^2d^2\left(2n^2-3n+1\right)-abcd\left(4n^2-5n+1\right)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{dx(bc(1-4n)-ad(1-2n))}{cn(bc-ad)(c+dx^n)}}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

$\Big\downarrow$ 1020

$$\frac{\frac{\frac{2b^3c^2n^2\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\int \frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}+\frac{dx(bc(1-4n)-ad(1-2n))}{cn(bc-ad)(c+dx^n)}}{2cn(bc-ad)}-\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

$\Big\downarrow$ 778

$$\frac{\frac{\frac{2b^3c^2n^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}+}{2cn(bc-ad)}$$

$$\frac{dx}{2cn(bc-ad)\left(c+dx^n\right)^2}$$

input

```
Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

output

```
-1/2*(d*x)/(c*(b*c - a*d)*n*(c + d*x^n)^2) + ((d*(b*c*(1 - 4*n) - a*d*(1 -
 2*n))*x)/(c*(b*c - a*d)*n*(c + d*x^n)) + ((2*b^3*c^2*n^2*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)) - (d*(a^2*d^2*(1
 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))
*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c - a*d))
)/(c*(b*c - a*d)*n))/(2*c*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^3}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2
*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
-b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
 (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^
2 - 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)
*a^2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^
2*n^2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c
^3*d^3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(
2*n - 1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2
- 2*a*b*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2
 + a^2*c^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c
^3*d^3*n^2)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx=\int \frac{1}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx=\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^3}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^3),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}\,dx$$
$$= \int \frac{1}{x^{4n}b\,d^3 + x^{3n}a\,d^3 + 3x^{3n}bc\,d^2 + 3x^{2n}ac\,d^2 + 3x^{2n}b\,c^2d + 3x^n a\,c^2d + x^n b\,c^3 + a\,c^3}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int(1/(x**(4*n)*b*d**3 + x**(3*n)*a*d**3 + 3*x**(3*n)*b*c*d**2 + 3*x**(2*n
)*a*c*d**2 + 3*x**(2*n)*b*c**2*d + 3*x**n*a*c**2*d + x**n*b*c**3 + a*c**3)
,x)
```

3.88 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 264

$$\begin{aligned}\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx &= -\frac{d(3b^3c^3-6ab^2c^2d(1+n)+4a^2bcd^2(1+2n)-a^3d^3(1+3n))\,x}{ab^4n} \\ &\quad -\frac{d^2(3b^2c^2(1+n)-4abcd(1+2n)+a^2d^2(1+3n))\,x^{1+n}}{ab^3n(1+n)} \\ &\quad +\frac{d^3(ad(1+3n)-b(c+2cn))x^{1+2n}}{ab^2n(1+2n)}+\frac{(bc-ad)x(c+dx^n)^3}{abn\,(a+bx^n)} \\ &\quad -\frac{(bc-ad)^3(bc(1-n)-ad(1+3n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^4n}\end{aligned}$$

output
```
-d*(3*b^3*c^3-6*a*b^2*c^2*d*(1+n)+4*a^2*b*c*d^2*(1+2*n)-a^3*d^3*(1+3*n))*x
/a/b^4/n-d^2*(3*b^2*c^2*(1+n)-4*a*b*c*d*(1+2*n)+a^2*d^2*(1+3*n))*x^(1+n)/a
/b^3/n/(1+n)+d^3*(a*d*(1+3*n)-b*(2*c*n+c))*x^(1+2*n)/a/b^2/n/(1+2*n)+(-a*d
+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(b*c*(1-n)-a*d*(1+3*n))*x
*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^4/n
```

Mathematica [A] (verified)

Time = 6.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$
$$= \frac{x\left(\frac{4ab^3c^3d-6a^2b^2c^2d^2+4a^3bcd^3-a^4d^4+b^4c^4(-1+n)}{a^2n}+\frac{(-bc+ad)^3(bc(-1+n)+ad(1+3n))}{a^2n}+\frac{2bd^3(2bc-ad)x^n}{1+n}+\frac{b^2d^4x^{2n}}{1+2n}+\frac{(bc-ad)}{an(a+bx}\right.}{b^4}$$

input
```
Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

output
```
(x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4
*(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^
2*n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) +
 (b*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 +
 3*n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b
^4
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {930, 1025, 1025, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 930$$

$$\frac{\int \frac{(dx^n+c)^2(d(ad(3n+1)-b(2nc+c))x^n+c(ad-bc(1-n)))}{bx^n+a}dx}{abn}+\frac{x(bc-ad)(c+dx^n)^3}{abn(a+bx^n)}$$

$$\Big\downarrow \; 1025$$

$$\int \frac{(dx^n+c)\left(c\left(-b^2\left(-2n^2+n+1\right)c^2+2abd(2n+1)c-a^2d^2(3n+1)\right)-d\left(b^2\left(2n^2+3n+1\right)c^2-2abd\left(5n^2+4n+1\right)c+a^2d^2\left(6n^2+5n+1\right)\right)x^n\right)}{bx^n+a}dx \Big/ b(2n+1) + \frac{dx(c+dx^n)^2(ad}{b(2} \quad \Big/ \; abn$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\downarrow$ 1025

$$\int \frac{c\left(-b^3\left(-2n^3-n^2+2n+1\right)c^3+3ab^2d\left(2n^2+3n+1\right)c^2-a^2bd^2\left(13n^2+12n+3\right)c+a^3d^3\left(6n^2+5n+1\right)\right)-d\left(b^3\left(2n^2+3n+1\right)c^3-ab^2d\left(12n^3+17n^2+12n+3\right)c^2+a^2b}{bx^n+a} \Big/ b(n+1) \Big/ b(2n+1)$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\downarrow$ 913

$$-\frac{\left(2n^2+3n+1\right)(bc-ad)^3(bc(1-n)-ad(3n+1))\int \frac{1}{bx^n+a}dx}{b}-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c}{b} \Big/ b(n+1) \Big/ b(2n+1) \Big/ abn$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

$\downarrow$ 778

$$-\frac{dx\left(-a^3d^3\left(6n^3+11n^2+6n+1\right)+a^2bcd^2\left(16n^3+26n^2+15n+3\right)-ab^2c^2d\left(12n^3+17n^2+12n+3\right)+b^3c^3\left(2n^2+3n+1\right)\right)}{b}-\left(2n^2+3n+1\right)x(bc-ad)^3(bc(1-n)-ad(3n \Big/ b(n+1) \Big/ b(2n+1)$$

$$\frac{x(bc-ad)\left(c+dx^n\right)^3}{abn\left(a+bx^n\right)}$$

input

```
Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) + ((d*(a*d*(1 + 3*n) - b
*(c + 2*c*n))*x*(c + d*x^n)^2)/(b*(1 + 2*n)) + (-((d*(b^2*c^2*(1 + 3*n + 2
*n^2) - 2*a*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*
x^n))/(b*(1 + n))) + (-((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n +
 11*n^2 + 6*n^3) - a*b^2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*
(3 + 15*n + 26*n^2 + 16*n^3))*x)/b) - ((b*c - a*d)^3*(1 + 3*n + 2*n^2)*(b*
c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b
*x^n)/a)])/(a*b))/(b*(1 + n)))/(b*(1 + 2*n)))/(a*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n
+ c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

input
```
integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**4/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1)
- b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n),
x) + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2
+ n)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*
n^3 + 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((
2*n^2 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*
n^2 + 4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 +
 (6*n^3 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (
2*n^3 + 3*n^2 + n)*a^2*b^4)
```

Giac [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx=\int \frac{(c+d\,x^n)^4}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^4/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^4/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx=\text{too large to display}$$

input
```
int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

output

```
(x**(3*n)*a**2*b*d**4*n*x + x**(3*n)*a**2*b*d**4*x - 3*x**(2*n)*a**3*d**4*
n*x - x**(2*n)*a**3*d**4*x + 8*x**(2*n)*a**2*b*c*d**3*n*x + 4*x**(2*n)*a**
2*b*c*d**3*x + 6*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x*
*n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**4*b*d**4*n**3 + 11*x**n*int(x
**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**
2*n + a**2),x)*a**4*b*d**4*n**2 + 6*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x
**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**4*b*d**4*n
 + x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x
**n*a*b + a**2*n + a**2),x)*a**4*b*d**4 - 16*x**n*int(x**(2*n)/(x**(2*n)*b
**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3
*b**2*c*d**3*n**3 - 32*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2
+ 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3*b**2*c*d**3*n**2 - 20
*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**
n*a*b + a**2*n + a**2),x)*a**3*b**2*c*d**3*n - 4*x**n*int(x**(2*n)/(x**(2*
n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*
a**3*b**2*c*d**3 + 12*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 +
 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**2*b**3*c**2*d**2*n**3 +
30*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x
**n*a*b + a**2*n + a**2),x)*a**2*b**3*c**2*d**2*n**2 + 24*x**n*int(x**(2*n
)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n...
```

3.89 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 180

$$\begin{aligned}\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx &= -\frac{d(2b^2c^2-3abcd(1+n)+a^2d^2(1+2n))\,x}{ab^3n} \\ &\quad -\frac{d^2(bc(1+n)-ad(1+2n))x^{1+n}}{ab^2n(1+n)}+\frac{(bc-ad)x(c+dx^n)^2}{abn\,(a+bx^n)} \\ &\quad -\frac{(bc-ad)^2(bc(1-n)-ad(1+2n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^3n}\end{aligned}$$

output
```
-d*(2*b^2*c^2-3*a*b*c*d*(1+n)+a^2*d^2*(1+2*n))*x/a/b^3/n-d^2*(b*c*(1+n)-a*
d*(1+2*n))*x^(1+n)/a/b^2/n/(1+n)+(-a*d+b*c)*x*(c+d*x^n)^2/a/b/n/(a+b*x^n)-
(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a
)/a^2/b^3/n
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 5.14 (sec) , antiderivative size = 2050, normalized size of antiderivative = 11.39

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

output
```
(x*(3*a*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + n)^4 + 3*c^2*d*(1
+ 4*n + 6*n^2 + 2*n^3 + n^4)*x^n + 3*c*d^2*(1 + n)^4*x^(2*n) + d^3*(1 + n)
^4*x^(3*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] - 3*a*(1 + 10*n +
 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + 2*n)^4 + 3*c^2*d*(1 + 2*n)^4*x^n + 3*
c*d^2*(1 + 8*n + 24*n^2 + 34*n^3 + 18*n^4)*x^(2*n) + d^3*(1 + 2*n)^4*x^(3*
n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + a*c^3*HurwitzLerchPhi[-
((b*x^n)/a), 1, 3 + n^(-1)] + 22*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1,
3 + n^(-1)] + 209*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
 1118*a*c^3*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3675*a*c^3*
n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7578*a*c^3*n^5*HurwitzL
erchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 9531*a*c^3*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, 3 + n^(-1)] + 6642*a*c^3*n^7*HurwitzLerchPhi[-((b*x^n)/a), 1,
 3 + n^(-1)] + 1944*a*c^3*n^8*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)]
 + 3*a*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c^2*d
*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c^2*d*n^2*x^n*
HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c^2*d*n^3*x^n*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 11025*a*c^2*d*n^4*x^n*HurwitzLerc
hPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 22734*a*c^2*d*n^5*x^n*HurwitzLerchPhi[
-((b*x^n)/a), 1, 3 + n^(-1)] + 28593*a*c^2*d*n^6*x^n*HurwitzLerchPhi[-((b*
x^n)/a), 1, 3 + n^(-1)] + 19926*a*c^2*d*n^7*x^n*HurwitzLerchPhi[-((b*x^...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {930, 1025, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{(dx^n+c)(c(ad-bc(1-n))-d(bc(n+1)-ad(2n+1))x^n)}{bx^n+a}dx}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 1025$$

$$\frac{\frac{\int \frac{c\left(-b^2\left(1-n^2\right)c^2+2abd(n+1)c-a^2d^2(2n+1)\right)-d\left(b^2(n+1)c^2-abd\left(3n^2+4n+2\right)c+a^2d^2\left(2n^2+3n+1\right)\right)x^n}{bx^n+a}dx}{b(n+1)}-\frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{b(n+1)}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 913$$

$$\frac{\frac{-\frac{(n+1)(bc-ad)^2(bc(1-n)-ad(2n+1))\int \frac{1}{bx^n+a}dx}{b}-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{b}}{b(n+1)}-\frac{dx(c+dx^n)(bc(n+1)-ad(2n+1))}{b(n+1)}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{b}-\frac{(n+1)x(bc-ad)^2(bc(1-n)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}}{b(n+1)}-\frac{dx(c+dx^n)(bc(}{b(}}{abn}$$
$$\frac{x(bc-ad)\left(c+dx^n\right)^2}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) + (-((d*(b*c*(1 + n) - a
*d*(1 + 2*n))*x*(c + d*x^n))/(b*(1 + n))) + (-((d*(b^2*c^2*(1 + n) + a^2*d
^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^2))*x)/b) - ((b*c - a*d)^2*(
1 + n)*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^
(-1), -((b*x^n)/a)])/(a*b))/(b*(1 + n)))/(a*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^3}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n)
+ 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$$

input
```
integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**3/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2
*d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (
3*(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1
)*a^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*
d*(n + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

Giac [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^3}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^3}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^3/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^3/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

output
```
(x**(2*n)*a**2*d**3*x - 2*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b*
*2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3*b*d**3*n**2 - 3*x*
*n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a
*b + a**2*n + a**2),x)*a**3*b*d**3*n - x**n*int(x**(2*n)/(x**(2*n)*b**2*n
+ x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3*b*d**
3 + 3*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n +
2*x**n*a*b + a**2*n + a**2),x)*a**2*b**2*c*d**2*n**2 + 6*x**n*int(x**(2*n)
/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a
**2),x)*a**2*b**2*c*d**2*n + 3*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*
n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**2*b**2*c*d**2 -
 3*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x
**n*a*b + a**2*n + a**2),x)*a*b**3*c**2*d*n - 3*x**n*int(x**(2*n)/(x**(2*n
)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a
*b**3*c**2*d - x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n
*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*b**4*c**3*n**2 + x**n*int(x**(2*n)
/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a
**2),x)*b**4*c**3 + 3*x**n*a*b*c**2*d*x + x**n*b**2*c**3*n*x - x**n*b**2*c
**3*x - 2*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2
*x**n*a*b + a**2*n + a**2),x)*a**4*d**3*n**2 - 3*int(x**(2*n)/(x**(2*n)*b*
*2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a*...
```

3.90 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx = -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x(c+dx^n)}{abn(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x \text{ Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b^2n}$$

output
```
-d*(b*c-a*d*(1+n))*x/a/b^2/n+(-a*d+b*c)*x*(c+d*x^n)/a/b/n/(a+b*x^n)-(-a*d+
b*c)*(b*c*(1-n)-a*d*(1+n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b^2/
n
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.97 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.79

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx = \frac{x\left(-2a(1+6n+11n^2+6n^3)\left(c^2(1+n)^3+2cd(1+3n+4n^2+n^3)x^n+d^2(1+n)^3x^{2n}\right)\Phi\left(-\frac{bx^n}{a},1,1+\right.}{}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

output
```
(x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^
2 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 +
 n^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)
^3*x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)
/a), 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a
*c^2*n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1
, n^(-1)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c
^2*n^5*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerc
hPhi[-((b*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*
a*c*d*n^2*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*
HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-
((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 11*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 6*a*d^2*n^3*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*
n^6*x^n*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*
x^n)/a)] - 4*b*c*d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1
, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2
, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 ...
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {930, 913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}dx$$

$$\Big\downarrow \ 930$$

$$\frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(n+1))x^n}{bx^n+a}dx}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \text{913}$$

$$\frac{-\frac{(bc-ad)(bc(1-n)-ad(n+1))\int \frac{1}{bx^n+a}dx}{b}-\frac{dx(bc-ad(n+1))}{b}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

$$\Big\downarrow \text{778}$$

$$\frac{-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}-\frac{dx(bc-ad(n+1))}{b}}{abn}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^n))/(a*b*n*(a + b*x^n)) + (-((d*(b*c - a*d*(1 + n)
)*x)/b) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b))/(a*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c + d\,x^n)^2}{(a + b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2}\,dx = \int \frac{(dx^n + c)^2}{(bx^n + a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2),
x)
```

Sympy [F]

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2}\,dx = \int \frac{(c + dx^n)^2}{(a + bx^n)^2}\,dx$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

output
```
Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n
+ a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c
*d)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$
$$= \frac{x^n\left(\int \frac{x^{2n}}{x^{2n}b^2n+x^{2n}b^2+2x^nabn+2x^nab+a^2n+a^2}dx\right)a^2b\,d^2n^2+2x^n\left(\int \frac{x^{2n}}{x^{2n}b^2n+x^{2n}b^2+2x^nabn+2x^nab+a^2n+a^2}dx\right)a^2b\,d^2n\ \cdots}{\cdots}$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

output
```
(x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**
n*a*b + a**2*n + a**2),x)*a**2*b*d**2*n**2 + 2*x**n*int(x**(2*n)/(x**(2*n)
*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a*
*2*b*d**2*n + x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*
a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**2*b*d**2 - 2*x**n*int(x**(2*n)/(
x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**
2),x)*a*b**2*c*d*n - 2*x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2
+ 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a*b**2*c*d - x**n*int(x**(
2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n
 + a**2),x)*b**3*c**2*n**2 + x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)
*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*b**3*c**2 + 2*x**n*a
*c*d*x + x**n*b*c**2*n*x - x**n*b*c**2*x + int(x**(2*n)/(x**(2*n)*b**2*n +
 x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3*d**2*n
**2 + 2*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x
**n*a*b + a**2*n + a**2),x)*a**3*d**2*n + int(x**(2*n)/(x**(2*n)*b**2*n +
x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**3*d**2 -
2*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*
b + a**2*n + a**2),x)*a**2*b*c*d*n - 2*int(x**(2*n)/(x**(2*n)*b**2*n + x**
(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**2*b*c*d - in
t(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b...
```

3.91 $\int \frac{c+dx^n}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{c+dx^n}{(a+bx^n)^2} \, dx = \frac{dx}{b(1-n)\,(a+bx^n)} - \frac{(ad-bc(1-n))x \operatorname{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b(1-n)}$$

output
```
d*x/b/(1-n)/(a+b*x^n)-(a*d-b*c*(1-n))*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/
a)/a^2/b/(1-n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^n}{(a+bx^n)^2} \, dx = \frac{x\left(\frac{d}{a+bx^n} - \frac{(ad+bc(-1+n)) \operatorname{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
```

output
```
(x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad-bc(1-n))\int \frac{1}{bx^n+a}dx}{abn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(ad-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

input
```
Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

output
```
((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{c+d\,x^{n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.44

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

output
```
c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)
*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1
/n)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamm
a(1 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
+ a**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**
(1/n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*(a*a**(
-3 - 1/n)*a**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*n*x**(n +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*g
amma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n)) + a*a**(-3 - 1/n)*a**(1 + 1/n)
*n*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2
+ 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*x**(n + 1)*lerchphi(b*x**n*exp_pola
r(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n
*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*n*x**n*x**(n + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*x**n*x**(n
 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*...
```

Maxima [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*
x/(a*b^2*n*x^n + a^2*b*n)
```

Giac [F]

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\int \frac{dx^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{(a+bx^n)^2}\,dx=\int \frac{c+d\,x^n}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{c + dx^n}{(a + bx^n)^2} \, dx$$
$$= \frac{-x^n \left(\int \frac{x^{2n}}{x^{2n}b^2n + x^{2n}b^2 + 2x^n abn + 2x^n ab + a^2n + a^2} dx\right) a\, b^2 dn - x^n \left(\int \frac{x^{2n}}{x^{2n}b^2n + x^{2n}b^2 + 2x^n abn + 2x^n ab + a^2n + a^2} dx\right) a\, b^2 d - x^n}{}$$

input

```
int((c+d*x^n)/(a+b*x^n)^2,x)
```

output

```
( - x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*
x**n*a*b + a**2*n + a**2),x)*a*b**2*d*n - x**n*int(x**(2*n)/(x**(2*n)*b**2
*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a*b**2*
d - x**n*int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*
x**n*a*b + a**2*n + a**2),x)*b**3*c*n**2 + x**n*int(x**(2*n)/(x**(2*n)*b**
2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*b**3*c
 + x**n*a*d*x + x**n*b*c*n*x - x**n*b*c*x - int(x**(2*n)/(x**(2*n)*b**2*n
+ x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a**2*b*d*n
 - int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a
*b + a**2*n + a**2),x)*a**2*b*d - int(x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)
*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a**2*n + a**2),x)*a*b**2*c*n**2 + int(
x**(2*n)/(x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*n + 2*x**n*a*b + a*
*2*n + a**2),x)*a*b**2*c + a*c*n*x + a*c*x)/(a**2*(x**n*b*n + x**n*b + a*n
 + a))
```

3.92 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx}{a(bc-ad)n(a+bx^n)} \\ &\quad + \frac{b(ad(1-2n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} \\ &\quad + \frac{d^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}\end{aligned}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n
],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n)
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{bd(1-n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)}-\frac{-\frac{ad^2n\int \frac{1}{dx^n+c}dx}{bc-ad}-\frac{b(ad(1-2n)-bc(1-n))\int \frac{1}{bx^n+a}dx}{bc-ad}}{an(bc-ad)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)}-\frac{-\frac{ad^2nx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{bx(ad(1-2n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-((b*(a*d*(1 - 2*n) - b*c*(1 - n))*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)))
 - (a*d^2*n*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{2}\,(c+d\,x^{n})}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^{n})^{2}\,(c+dx^{n})}\,dx = \int \frac{1}{(bx^{n}+a)^{2}(dx^{n}+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^{n})^{2}\,(c+dx^{n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a
^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n
+ a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{1}{x^{3n}b^2d+2x^{2n}abd+x^{2n}b^2c+x^na^2d+2x^nabc+a^2c}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n*a**2*d
+ 2*x**n*a*b*c + a**2*c),x)
```

3.93 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{d(bc+ad)x}{ac(bc-ad)^2n(c+dx^n)} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)} + \frac{b^2(ad(1-3n)-b(c-cn))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^3n} - \frac{d^2(bc(1-3n)-ad(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2(bc-ad)^3n}$$

output
```
d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c
+d*x^n)+b^2*(a*d*(1-3*n)-b*(-c*n+c))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a
)/a^2/(-a*d+b*c)^3/n-d^2*(b*c*(1-3*n)-a*d*(1-n))*x*hypergeom([1, 1/n],[1+1
/n],-d*x^n/c)/c^2/(-a*d+b*c)^3/n
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} \, dx$$
$$= \frac{x \left(\frac{b^2(bc-ad)}{a(a+bx^n)} + \frac{d^2(bc-ad)}{c(c+dx^n)} + \frac{b^2(ad(1-3n)+bc(-1+n)) \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2} + \frac{d^2(-ad(-1+n)+bc(-1+3n)) \text{Hyperge}}{c^2}\right.}{(bc-ad)^3 n}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output
```
(x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n))
+ (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeo
metric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {931, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} \, dx$$
$$\Big\downarrow \ 931$$
$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)} - \frac{\int \frac{bd(1-2n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)^2}dx}{an(bc-ad)}$$
$$\Big\downarrow \ 1024$$

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)}-\frac{\frac{\int \frac{bd(bc+ad)(1-n)nx^n+n\left(b^2(1-n)c^2+2abdnc+a^2d^2(1-n)\right)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

$\Big\downarrow$ 1020

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)}-\frac{\frac{-\frac{b^2cn(ad(1-3n)-bc(1-n))\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{ad^2n(ad(1-n)-b(c-3cn))\int \frac{1}{dx^n+c}dx}{bc-ad}}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

$\Big\downarrow$ 778

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)}-\frac{\frac{-\frac{b^2cnx(ad(1-3n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{ad^2nx(ad(1-n)-b(c-3cn))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}}{cn(bc-ad)}-\frac{dx(ad+bc)}{c(bc-ad)(c+dx^n)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)) - (-((d*(b*c + a*d)*x)/(c*
(b*c - a*d)*(c + d*x^n))) + (-((b^2*c*(a*d*(1 - 3*n) - b*c*(1 - n))*n*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d))) - (
a*d^2*n*(a*d*(1 - n) - b*(c - 3*c*n))*x*Hypergeometric2F1[1, n^(-1), 1 + n
^(-1), -((d*x^n)/c)])/(c*(b*c - a*d)))/(c*(b*c - a*d)*n))/(a*(b*c - a*d)*n
)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{2}\,(c+d\,x^{n})^{2}}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)^2} dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b
^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2} dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)^2} dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^
2*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n
 + 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*
(n - 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a
^3*c^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*
c*d^4*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a
^2*b^2*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^
2*c^2*d^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^
3*b*c^2*d^2*n + a^4*c*d^3*n)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx=\int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx=\int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^2}\,dx$$
$$=\int \frac{1}{x^{4n}b^2d^2+2x^{3n}ab\,d^2+2x^{3n}b^2cd+x^{2n}a^2d^2+4x^{2n}abcd+x^{2n}b^2c^2+2x^n a^2cd+2x^n ab\,c^2+a^2c^2}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int(1/(x**(4*n)*b**2*d**2 + 2*x**(3*n)*a*b*d**2 + 2*x**(3*n)*b**2*c*d + x*
*(2*n)*a**2*d**2 + 4*x**(2*n)*a*b*c*d + x**(2*n)*b**2*c**2 + 2*x**n*a**2*c
*d + 2*x**n*a*b*c**2 + a**2*c**2),x)
```

3.94 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3} dx = \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d(abcd(1-6n)-a^2d^2(1-2n)-2b^2c^2n)x}{2ac^2(bc-ad)^3n^2(c+dx^n)} + \frac{b^3(ad(1-4n)-bc(1-n))x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^4n} + \frac{d^2(a^2d^2(1-3n+2n^2)-2abcd(1-5n+4n^2)+b^2c^2(1-7n+12n^2))x \,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1-}{2c^3(bc-ad)^4n^2}$$

output
```
1/2*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b
*x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/
c^2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1,
 1/n],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-
2*a*b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n],[1+1/
n],-d*x^n/c)/c^3/(-a*d+b*c)^4/n^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$$
$$= \frac{x\left(\frac{2b^3(bc-ad)n}{a(a+bx^n)}+\frac{d^2(bc-ad)^2n}{c(c+dx^n)^2}+\frac{d^2(-bc+ad)(ad(-1+2n)+b(c-6cn))}{c^2(c+dx^n)}+\frac{2b^3(ad(1-4n)+bc(-1+n))n\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},\cdots\right.}{a^2}\cdots\right.}{2(bc-ad)^4n^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output
```
(x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c +
d*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c
+ d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) -
2*a*b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {931, 1024, 1024, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$$
$$\Big\downarrow \; 931$$
$$\frac{bx}{an(bc-ad)(a+bx^n)(c+dx^n)^2}-\frac{\int \frac{bd(1-3n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)^3}dx}{an(bc-ad)}$$
$$\Big\downarrow \; 1024$$

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}-\frac{\frac{\int \frac{bd(2bc+ad)(1-2n)nx^n+n\left(2b^2(1-n)c^2+4abdnc+a^2d^2(1-2n)\right)}{(bx^n+a)(dx^n+c)^2}dx}{2cn(bc-ad)}-\frac{dx(ad+2bc)}{2c(bc-ad)(c+dx^n)^2}}{an(bc-ad)}$$

$\Big\downarrow$ 1024

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}-\frac{\frac{\frac{\int \frac{n\left(2b^3(1-n)nc^3+6ab^2dn^2c^2-a^2bd^2\left(6n^2-7n+1\right)c+a^3d^3\left(2n^2-3n+1\right)\right)-bd(1-n)n\left(-2b^2nc^2+abd(1-6n)c-a^2d^2(1-2n)\right)x^n}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{dx\left(-a^2d^2(1-2n)+abcd(1\cdots\right.}{c(bc-ad)(c+d\cdots}}{2cn(bc-ad)}\cdots}{an(bc-ad)}$$

$\Big\downarrow$ 1020

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}-\frac{\frac{\frac{-\frac{ad^2n\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\int\frac{1}{dx^n+c}dx}{bc-ad}-\frac{2b^3c^2n^2(ad(1-4n)-bc(1-n))\int\frac{1}{bx^n+a}dx}{bc-ad}}{cn(bc-ad)}+\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6\cdots\right.}{c(bc-ad)(c+dx^n)\cdots}}{2cn(bc-ad)}\cdots}{an(bc-ad)}$$

$\Big\downarrow$ 778

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}-\frac{\frac{\frac{dx\left(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n\right)}{c(bc-ad)(c+dx^n)}+\frac{-\frac{ad^2nx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{2b\cdots}{\cdots}}{cn(bc-ad)}}{2cn(bc-ad)}\cdots}{an(bc-ad)}$$

input

```
Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output

```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (-1/2*(d*(2*b*c + a*d)
*x)/(c*(b*c - a*d)*(c + d*x^n)^2) + ((d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 -
2*n) - 2*b^2*c^2*n)*x)/(c*(b*c - a*d)*(c + d*x^n)) + ((-2*b^3*c^2*(a*d*(1
- 4*n) - b*c*(1 - n))*n^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*(b*c - a*d)) - (a*d^2*n*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*
d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c - a*d)))/(c*(b*c - a*d)*n))/(2*c
*(b*c - a*d)*n))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{2} \left(c+d\,x^{n}\right)^{3}}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n)
+ (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d +
 3*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx = \int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

output
```
Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 -
 3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*
b^2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 -
 4*a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c
^2*d^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^
2*b^4*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n +
a^6*d^4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4
*b^2*c*d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b
*d^4*(2*n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a
^3*d^4*(2*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2
*(7*n - 1) - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^
3*b^2*c^6*d*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n
^2 - 3*a^2*b^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^
(3*n) + (2*a*b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2
 + a^4*b*c^3*d^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3
*c^6*d*n^2 - 3*a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n
^2)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3} dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)^3} dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3} dx = \int \frac{1}{(a+b x^n)^2 (c+d x^n)^3} dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)^3}\,dx$$
$$= \int \frac{1}{x^{5n}b^2d^3 + 2x^{4n}ab\,d^3 + 3x^{4n}b^2c\,d^2 + x^{3n}a^2d^3 + 6x^{3n}abc\,d^2 + 3x^{3n}b^2c^2d + 3x^{2n}a^2c\,d^2 + 6x^{2n}ab\,c^2d + x^2$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int(1/(x**(5*n)*b**2*d**3 + 2*x**(4*n)*a*b*d**3 + 3*x**(4*n)*b**2*c*d**2 +
 x**(3*n)*a**2*d**3 + 6*x**(3*n)*a*b*c*d**2 + 3*x**(3*n)*b**2*c**2*d + 3*x
**(2*n)*a**2*c*d**2 + 6*x**(2*n)*a*b*c**2*d + x**(2*n)*b**2*c**3 + 3*x**n*
a**2*c**2*d + 2*x**n*a*b*c**3 + a**2*c**3),x)
```

3.95 $\int \sqrt{a+bx^n}(c+dx^n)^2\, dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \sqrt{a+bx^n}(c+dx^n)^2\, dx = -\frac{2d(2ad(1+n)-bc(2+7n))x(a+bx^n)^{3/2}}{b^2(2+3n)(2+5n)}+\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(2+5n)} + \frac{(4a^2d^2(1+n)-4abcd(2+5n)+b^2c^2(4+16n+15n^2))\,x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n}\right.}{b^2(2+3n)(2+5n)\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*d*(2*a*d*(1+n)-b*c*(2+7*n))*x*(a+b*x^n)^(3/2)/b^2/(2+3*n)/(2+5*n)+2*d*x
*(a+b*x^n)^(3/2)*(c+d*x^n)/b/(2+5*n)+(4*a^2*d^2*(1+n)-4*a*b*c*d*(2+5*n)+b^
2*c^2*(15*n^2+16*n+4))*x*(a+b*x^n)^(1/2)*hypergeom([-1/2, 1/n],[1+1/n],-b*
x^n/a)/b^2/(2+3*n)/(2+5*n)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \sqrt{a+bx^n}(c+dx^n)^2 \, dx$$
$$= \frac{2x(a+bx^n)\left(-2a^2d^2n(1+n)+abdn(2c(2+5n)+d(2+n)x^n)+b^2(c^2(4+16n+15n^2)+2cd(4+12n\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^n]*(c + d*x^n)^2,x]
```

output
```
(2*x*(a + b*x^n)*(-2*a^2*d^2*n*(1 + n) + a*b*d*n*(2*c*(2 + 5*n) + d*(2 + n
)*x^n) + b^2*(c^2*(4 + 16*n + 15*n^2) + 2*c*d*(4 + 12*n + 5*n^2)*x^n + d^2
*(4 + 8*n + 3*n^2)*x^(2*n))) + a*n*(4*a^2*d^2*(1 + n) - 4*a*b*c*d*(2 + 5*n
) + b^2*c^2*(4 + 16*n + 15*n^2))*x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1
/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b^2*(2 + n)*(2 + 3*n)*(2 + 5*n)*Sq
rt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n}(c+dx^n)^2 \, dx$$
$$\Big\downarrow \ 933$$
$$\frac{2\int -\frac{1}{2}\sqrt{bx^n+a}(d(2ad(n+1)-bc(7n+2))x^n+c(2ad-bc(5n+2)))dx}{b(5n+2)}+\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(5n+2)}$$
$$\Big\downarrow \ 27$$

$$\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(5n+2)} - \frac{\int \sqrt{bx^n+a}(d(2ad(n+1)-bc(7n+2))x^n+c(2ad-bc(5n+2)))\,dx}{b(5n+2)}$$

$\Big\downarrow$ 913

$$\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(5n+2)} - \frac{\frac{2dx(a+bx^n)^{3/2}(2ad(n+1)-bc(7n+2))}{b(3n+2)} - \frac{(4a^2d^2(n+1)-4abcd(5n+2)+b^2c^2(15n^2+16n+4))\int \sqrt{bx^n+a}dx}{b(3n+2)}}{b(5n+2)}$$

$\Big\downarrow$ 779

$$\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(5n+2)} - \frac{\frac{2dx(a+bx^n)^{3/2}(2ad(n+1)-bc(7n+2))}{b(3n+2)} - \frac{\sqrt{a+bx^n}(4a^2d^2(n+1)-4abcd(5n+2)+b^2c^2(15n^2+16n+4))\int \sqrt{\frac{bx^n}{a}+1}dx}{b(3n+2)\sqrt{\frac{bx^n}{a}+1}}}{b(5n+2)}$$

$\Big\downarrow$ 778

$$\frac{2dx(a+bx^n)^{3/2}(c+dx^n)}{b(5n+2)} - \frac{\frac{2dx(a+bx^n)^{3/2}(2ad(n+1)-bc(7n+2))}{b(3n+2)} - \frac{x\sqrt{a+bx^n}(4a^2d^2(n+1)-4abcd(5n+2)+b^2c^2(15n^2+16n+4))\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(3n+2)\sqrt{\frac{bx^n}{a}+1}}}{b(5n+2)}$$

input
```
Int[Sqrt[a + b*x^n]*(c + d*x^n)^2,x]
```

output
```
(2*d*x*(a + b*x^n)^(3/2)*(c + d*x^n))/(b*(2 + 5*n)) - ((2*d*(2*a*d*(1 + n)
 - b*c*(2 + 7*n))*x*(a + b*x^n)^(3/2))/(b*(2 + 3*n)) - ((4*a^2*d^2*(1 + n)
 - 4*a*b*c*d*(2 + 5*n) + b^2*c^2*(4 + 16*n + 15*n^2))*x*Sqrt[a + b*x^n]*Hy
pergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(2 + 3*n)*Sqrt
[1 + (b*x^n)/a]))/(b*(2 + 5*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}}\,(c+d\,x^{n})^{2}\,dx$$

input
```
int((a+b*x^n)^(1/2)*(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^(1/2)*(c+d*x^n)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \frac{a^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}c^2x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{2a^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}cdx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(c+d*x**n)**2,x)
```

output
```
a**(1/n)*a**(1/2 - 1/n)*c**2*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-3/2 - 1/n)*a**(2 + 1/n)
*d**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-1/2, 2 + 1/n), (3 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + 2*a**(-1/2 - 1/n)*a**(1 + 1/n)*c
*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \int \sqrt{bx^n+a}(dx^n+c)^2\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*(d*x^n + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \int \sqrt{bx^n+a}(dx^n+c)^2\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \int \sqrt{a+b\,x^n}\,(c+d\,x^n)^2\,dx$$

input
```
int((a + b*x^n)^(1/2)*(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^(1/2)*(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \sqrt{a+bx^n}(c+dx^n)^2\,dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^(1/2)*(c+d*x^n)^2,x)
```

output

```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*d**2*n**2*x + 16*x**(2*n)*sqrt(x**n*b +
a)*b**2*d**2*n*x + 8*x**(2*n)*sqrt(x**n*b + a)*b**2*d**2*x + 2*x**n*sqrt(x
**n*b + a)*a*b*d**2*n**2*x + 4*x**n*sqrt(x**n*b + a)*a*b*d**2*n*x + 20*x**
n*sqrt(x**n*b + a)*b**2*c*d*n**2*x + 48*x**n*sqrt(x**n*b + a)*b**2*c*d*n*x
 + 16*x**n*sqrt(x**n*b + a)*b**2*c*d*x - 4*sqrt(x**n*b + a)*a**2*d**2*n**2
*x - 4*sqrt(x**n*b + a)*a**2*d**2*n*x + 20*sqrt(x**n*b + a)*a*b*c*d*n**2*x
 + 8*sqrt(x**n*b + a)*a*b*c*d*n*x + 30*sqrt(x**n*b + a)*b**2*c**2*n**2*x +
 32*sqrt(x**n*b + a)*b**2*c**2*n*x + 8*sqrt(x**n*b + a)*b**2*c**2*x + 60*i
nt(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**
n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*d**2*n**5 + 244*int(sq
rt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b +
 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*d**2*n**4 + 328*int(sqrt(x*
*n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a
*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*d**2*n**3 + 176*int(sqrt(x**n*b
+ a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a*n**3
 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*d**2*n**2 + 32*int(sqrt(x**n*b + a)/(
15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*
a*n**2 + 36*a*n + 8*a),x)*a**3*d**2*n - 300*int(sqrt(x**n*b + a)/(15*x**n*
b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*a*n**2 +
 36*a*n + 8*a),x)*a**2*b*c*d*n**5 - 1040*int(sqrt(x**n*b + a)/(15*x**n*...
```

3.96 $\int \sqrt{a+bx^n}(c+dx^n)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \sqrt{a+bx^n}(c+dx^n)\,dx = \frac{2dx(a+bx^n)^{3/2}}{b(2+3n)} + \frac{\left(c-\frac{2ad}{2b+3bn}\right)x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*d*x*(a+b*x^n)^(3/2)/b/(2+3*n)+(c-2*a*d/(3*b*n+2*b))*x*(a+b*x^n)^(1/2)*hy
pergeom([-1/2, 1/n],[1+1/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \sqrt{a+bx^n}(c+dx^n)\,dx = \frac{x\sqrt{a+bx^n}\left(2d(a+bx^n)\sqrt{1+\frac{bx^n}{a}}+(-2ad+bc(2+3n))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(2+3n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^n]*(c + d*x^n),x]
```

output
```
(x*Sqrt[a + b*x^n]*(2*d*(a + b*x^n)*Sqrt[1 + (b*x^n)/a] + (-2*a*d + b*c*(2
 + 3*n))*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(b*(2
 + 3*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n}(c+dx^n)\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{2ad}{3bn+2b}\right)\int \sqrt{bx^n+a}dx+\frac{2dx(a+bx^n)^{3/2}}{b(3n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt{a+bx^n}\left(c-\frac{2ad}{3bn+2b}\right)\int \sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2dx(a+bx^n)^{3/2}}{b(3n+2)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt{a+bx^n}\left(c-\frac{2ad}{3bn+2b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2dx(a+bx^n)^{3/2}}{b(3n+2)}$$

input
```
Int[Sqrt[a + b*x^n]*(c + d*x^n),x]
```

output
```
(2*d*x*(a + b*x^n)^(3/2))/(b*(2 + 3*n)) + ((c - (2*a*d)/(2*b + 3*b*n))*x*S
qrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/
Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}}\,(c+d\,x^{n})\,dx$$

input
```
int((a+b*x^n)^(1/2)*(c+d*x^n),x)
```

output
```
int((a+b*x^n)^(1/2)*(c+d*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx = \frac{a^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(c+d*x**n),x)
```

output
```
a**(1/n)*a**(1/2 - 1/n)*c*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-1/2 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx = \int \sqrt{bx^n+a}(dx^n+c)\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*(d*x^n + c), x)
```

Giac [F]

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx = \int \sqrt{bx^n+a}(dx^n+c)\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx = \int \sqrt{a+b\,x^n}\,(c+d\,x^n)\ dx$$

input
```
int((a + b*x^n)^(1/2)*(c + d*x^n),x)
```

output
```
int((a + b*x^n)^(1/2)*(c + d*x^n), x)
```

Reduce [F]

$$\int \sqrt{a+bx^n}(c+dx^n)\ dx$$
$$= \frac{2x^n\sqrt{x^nb+a}\,bdnx+4x^n\sqrt{x^nb+a}\,bdx+2\sqrt{x^nb+a}\,adnx+6\sqrt{x^nb+a}\,bcnx+4\sqrt{x^nb+a}\,bcx-6\left(\int \frac{}{3}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(c+d*x^n),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*d*n*x + 4*x**n*sqrt(x**n*b + a)*b*d*x + 2*sqrt(
x**n*b + a)*a*d*n*x + 6*sqrt(x**n*b + a)*b*c*n*x + 4*sqrt(x**n*b + a)*b*c*
x - 6*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n*
*2 + 8*a*n + 4*a),x)*a**2*d*n**3 - 16*int(sqrt(x**n*b + a)/(3*x**n*b*n**2
+ 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*d*n**2 - 8*int(s
qrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n
+ 4*a),x)*a**2*d*n + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n +
4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a*b*c*n**4 + 30*int(sqrt(x**n*b + a)
/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a*b*c
*n**3 + 28*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3
*a*n**2 + 8*a*n + 4*a),x)*a*b*c*n**2 + 8*int(sqrt(x**n*b + a)/(3*x**n*b*n*
*2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a*b*c*n)/(b*(3*n**
2 + 8*n + 4))
```

3.97 $\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\frac{x\sqrt{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt{1+\frac{bx^n}{a}}}$$

output
```
x*(a+b*x^n)^(1/2)*AppellF1(1/n,-1/2,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/(1+b*x^n/
a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(61) = 122.

Time = 0.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx$$
$$=\frac{2ac(1+n)x\sqrt{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},1,1+\frac{1}{n},-\right.}{(c+dx^n)\left(-2adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\right.\right.}$$

input
```
Integrate[Sqrt[a + b*x^n]/(c + d*x^n),x]
```

output
```
(2*a*c*(1 + n)*x*Sqrt[a + b*x^n]*AppellF1[n^(-1), -1/2, 1, 1 + n^(-1), -((
b*x^n)/a), -((d*x^n)/c)])/((c + d*x^n)*(-2*a*d*n*x^n*AppellF1[1 + n^(-1),
-1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1 +
n^(-1), 1/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 2*a*c*(1 + n)*Ap
pellF1[n^(-1), -1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{a+bx^n}\int \frac{\sqrt{\frac{bx^n}{a}+1}}{dx^n+c}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^n]/(c + d*x^n),x]
```

output
```
(x*Sqrt[a + b*x^n]*AppellF1[n^(-1), -1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((
d*x^n)/c)])/(c*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}}{c+d\,x^{n}}dx$$

input
```
int((a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^(1/2)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{bx^n+a}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(d*x^n + c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx$$

input
```
integrate((a+b*x**n)**(1/2)/(c+d*x**n),x)
```

output
```
Integral(sqrt(a + b*x**n)/(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{bx^n+a}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{bx^n+a}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{a+b\,x^n}}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^(1/2)/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^(1/2)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{c+dx^n}\,dx=\int \frac{\sqrt{x^nb+a}}{x^nd+c}dx$$

input
```
int((a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*d + c),x)
```

3.98 $\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx=\frac{x\sqrt{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
x*(a+b*x^n)^(1/2)*AppellF1(1/n,-1/2,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/(1+b*x^
n/a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 360 vs. 2(61) = 122.

Time = 0.94 (sec) , antiderivative size = 360, normalized size of antiderivative = 5.90

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx$$
$$=\frac{x\left(\frac{b(-2+n)x^n\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+n}+\frac{2c\left(2adnx^n(a+bx^n)\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcnx^n(a\cdots\right.}{(c+dx^n)\left(2adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+\cdots\right.}\right.}{2c^2n\sqrt{a+bx^n}}$$

input
```
Integrate[Sqrt[a + b*x^n]/(c + d*x^n)^2,x]
```

output
```
(x*((b*(-2 + n)*x^n*Sqrt[1 + (b*x^n)/a]*AppellF1[1 + n^(-1), 1/2, 1, 2 + n
^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(1 + n) + (2*c*(2*a*d*n*x^n*(a + b*x^n
)*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b
*c*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)] - 2*a*c*(1 + n)*(a*n + b*x^n)*AppellF1[n^(-1), 1/2, 1, 1 +
 n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))/((c + d*x^n)*(2*a*d*n*x^n*AppellF1[
1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*Ap
pellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*c
*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
)))/(2*c^2*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2} dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{a+bx^n}\int \frac{\sqrt{\frac{bx^n}{a}+1}}{(dx^n+c)^2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^n]/(c + d*x^n)^2,x]
```

output
```
(x*Sqrt[a + b*x^n]*AppellF1[n^(-1), -1/2, 2, 1 + n^(-1), -((b*x^n)/a), -((
d*x^n)/c)])/(c^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}}{\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int((a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx=\int \frac{\sqrt{bx^n+a}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx$$

input
```
integrate((a+b*x**n)**(1/2)/(c+d*x**n)**2,x)
```

output
```
Integral(sqrt(a + b*x**n)/(c + d*x**n)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{\sqrt{bx^n+a}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{\sqrt{bx^n+a}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{\sqrt{a+b\,x^n}}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^(1/2)/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^(1/2)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{2n}d^2+2x^n cd+c^2}dx$$

input
```
int((a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*d**2 + 2*x**n*c*d + c**2),x)
```

3.99 $\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx = -\frac{2d(2ad(1+n)-bc(2+5n))x\sqrt{a+bx^n}}{b^2(2+n)(2+3n)} + \frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(2+3n)} + \frac{(4a^2d^2(1+n)-4abcd(2+3n)+b^2c^2(4+8n+3n^2))\,x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{b}{}\right.}{b^2(2+n)(2+3n)\sqrt{a+bx^n}}$$

output
```
-2*d*(2*a*d*(1+n)-b*c*(2+5*n))*x*(a+b*x^n)^(1/2)/b^2/(2+n)/(2+3*n)+2*d*x*(
a+b*x^n)^(1/2)*(c+d*x^n)/b/(2+3*n)+(4*a^2*d^2*(1+n)-4*a*b*c*d*(2+3*n)+b^2*
c^2*(3*n^2+8*n+4))*x*(1+b*x^n/a)^(1/2)*hypergeom([1/2, 1/n],[1+1/n],-b*x^n
/a)/b^2/(2+n)/(2+3*n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx = \frac{2dx(a+bx^n)\left(-2ad(1+n)+bc(4+6n)+bd(2+n)x^n\right)+\left(4a^2d^2(1+n)-4abcd(2+3n)+b^2c^2(4+8\right.}{b^2(2+n)(2+3n)\sqrt{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)^2/Sqrt[a + b*x^n],x]
```

output
```
(2*d*x*(a + b*x^n)*(-2*a*d*(1 + n) + b*c*(4 + 6*n) + b*d*(2 + n)*x^n) + (4
*a^2*d^2*(1 + n) - 4*a*b*c*d*(2 + 3*n) + b^2*c^2*(4 + 8*n + 3*n^2))*x*Sqrt
[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/
(b^2*(2 + n)*(2 + 3*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{933}$$

$$\frac{2\int -\frac{d(2ad(n+1)-bc(5n+2))x^n+c(2ad-bc(3n+2))}{2\sqrt{bx^n+a}}dx}{b(3n+2)}+\frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(3n+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(3n+2)}-\frac{\int \frac{d(2ad(n+1)-bc(5n+2))x^n+c(2ad-bc(3n+2))}{\sqrt{bx^n+a}}dx}{b(3n+2)}$$

$$\Big\downarrow \text{913}$$

$$\frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(3n+2)}-\frac{\frac{2dx\sqrt{a+bx^n}(2ad(n+1)-bc(5n+2))}{b(n+2)}-\frac{(4a^2d^2(n+1)-4abcd(3n+2)+b^2c^2(3n^2+8n+4))\int \frac{1}{\sqrt{bx^n+a}}dx}{b(n+2)}}{b(3n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(3n+2)}-\frac{\frac{2dx\sqrt{a+bx^n}(2ad(n+1)-bc(5n+2))}{b(n+2)}-\frac{\sqrt{\frac{bx^n}{a}+1}\left(4a^2d^2(n+1)-4abcd(3n+2)+b^2c^2\left(3n^2+8n+4\right)\right)\int\frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{b(n+2)\sqrt{a+bx^n}}}{b(3n+2)}}{}$$

$\Big\downarrow$ 778

$$\frac{2dx\sqrt{a+bx^n}(c+dx^n)}{b(3n+2)}-\frac{\frac{2dx\sqrt{a+bx^n}(2ad(n+1)-bc(5n+2))}{b(n+2)}-\frac{x\sqrt{\frac{bx^n}{a}+1}\left(4a^2d^2(n+1)-4abcd(3n+2)+b^2c^2\left(3n^2+8n+4\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(n+2)\sqrt{a+bx^n}}}{b(3n+2)}$$

input
```
Int[(c + d*x^n)^2/Sqrt[a + b*x^n],x]
```

output
```
(2*d*x*Sqrt[a + b*x^n]*(c + d*x^n))/(b*(2 + 3*n)) - ((2*d*(2*a*d*(1 + n) -
 b*c*(2 + 5*n))*x*Sqrt[a + b*x^n])/(b*(2 + n)) - ((4*a^2*d^2*(1 + n) - 4*a
*b*c*d*(2 + 3*n) + b^2*c^2*(4 + 8*n + 3*n^2))*x*Sqrt[1 + (b*x^n)/a]*Hyperg
eometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(2 + n)*Sqrt[a + b*
x^n]))/(b*(2 + 3*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c+d\,x^{n})^{2}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(1/2),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{1}{2}-\frac{1}{n}}c^2x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{5}{2}-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{2a^{-\frac{3}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}cdx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**(1/2),x)
```

output
```
a**(1/n)*a**(-1/2 - 1/n)*c**2*x*gamma(1/n)*hyper((1/2, 1/n), (1 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-5/2 - 1/n)*a**(2 + 1/n)
*d**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((1/2, 2 + 1/n), (3 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + 2*a**(-3/2 - 1/n)*a**(1 + 1/n)*c*
d*x**(n + 1)*gamma(1 + 1/n)*hyper((1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx = \int \frac{(dx^n+c)^2}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx=\int \frac{(dx^n+c)^2}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx=\int \frac{(c+d\,x^n)^2}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^(1/2),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{\sqrt{a+bx^n}}\,dx$$
$$=\frac{2x^n\sqrt{x^nb+a}\,b\,d^2nx+4x^n\sqrt{x^nb+a}\,b\,d^2x-4\sqrt{x^nb+a}\,a\,d^2nx-4\sqrt{x^nb+a}\,a\,d^2x+12\sqrt{x^nb+a}\,bcdnx}{}$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(1/2),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*d**2*n*x + 4*x**n*sqrt(x**n*b + a)*b*d**2*x - 4
*sqrt(x**n*b + a)*a*d**2*n*x - 4*sqrt(x**n*b + a)*a*d**2*x + 12*sqrt(x**n*
b + a)*b*c*d*n*x + 8*sqrt(x**n*b + a)*b*c*d*x + 12*int(sqrt(x**n*b + a)/(3
*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*d**
2*n**3 + 44*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b +
3*a*n**2 + 8*a*n + 4*a),x)*a**2*d**2*n**2 + 48*int(sqrt(x**n*b + a)/(3*x**
n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*d**2*n
+ 16*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**
2 + 8*a*n + 4*a),x)*a**2*d**2 - 36*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8
*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a*b*c*d*n**3 - 120*int(s
qrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n
+ 4*a),x)*a*b*c*d*n**2 - 112*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*
b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a*b*c*d*n - 32*int(sqrt(x**n*b
 + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*
a*b*c*d + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b +
3*a*n**2 + 8*a*n + 4*a),x)*b**2*c**2*n**4 + 48*int(sqrt(x**n*b + a)/(3*x**
n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*b**2*c**2*n*
*3 + 88*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*
n**2 + 8*a*n + 4*a),x)*b**2*c**2*n**2 + 64*int(sqrt(x**n*b + a)/(3*x**n*b*
n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*b**2*c**2*n +...
```

3.100 $\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx=\frac{2dx\sqrt{a+bx^n}}{b(2+n)}+\frac{\left(c-\frac{2ad}{b(2+n)}\right)x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

output
```
2*d*x*(a+b*x^n)^(1/2)/b/(2+n)+(c-2*a*d/b/(2+n))*x*(1+b*x^n/a)^(1/2)*hyperg
eom([1/2, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx=\frac{x\left(2d(a+bx^n)+(-2ad+bc(2+n))\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)/Sqrt[a + b*x^n],x]
```

output
```
(x*(2*d*(a + b*x^n) + (-2*a*d + b*c*(2 + n))*Sqrt[1 + (b*x^n)/a]*Hypergeom
etric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(b*(2 + n)*Sqrt[a + b*x^
n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 913}$$

$$\left(c-\frac{2ad}{b(n+2)}\right)\int \frac{1}{\sqrt{bx^n+a}}dx+\frac{2dx\sqrt{a+bx^n}}{b(n+2)}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(c-\frac{2ad}{b(n+2)}\right)\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2dx\sqrt{a+bx^n}}{b(n+2)}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\left(c-\frac{2ad}{b(n+2)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}+\frac{2dx\sqrt{a+bx^n}}{b(n+2)}$$

input
```
Int[(c + d*x^n)/Sqrt[a + b*x^n],x]
```

output
```
(2*d*x*Sqrt[a + b*x^n])/(b*(2 + n)) + ((c - (2*a*d)/(b*(2 + n)))*x*Sqrt[1
+ (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/Sqr
t[a + b*x^n]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{c+d\,x^{n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{1}{2}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(1/n)*a**(-1/2 - 1/n)*c*x*gamma(1/n)*hyper((1/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-3/2 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{\sqrt{a+bx^n}}\,dx = \int \frac{dx^n+c}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{c + dx^n}{\sqrt{a + bx^n}}\, dx = \int \frac{dx^n + c}{\sqrt{bx^n + a}}\, dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^n}{\sqrt{a + bx^n}}\, dx = \int \frac{c + d\,x^n}{\sqrt{a + b\,x^n}}\, dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^(1/2),x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{c + dx^n}{\sqrt{a + bx^n}}\, dx = \frac{2\sqrt{x^n b + a}\, dx - 2\left(\int \frac{\sqrt{x^n b + a}}{x^n bn + 2x^n b + an + 2a}\, dx\right) adn - 4\left(\int \frac{\sqrt{x^n b + a}}{x^n bn + 2x^n b + an + 2a}\, dx\right) ad + \left(\int \frac{\sqrt{x^n b + a}}{x^n bn + 2x^n b + an + 2a}\, dx\right) bc}{b\,(n + 2)}$$

input
```
int((c+d*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*d*x - 2*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*
n + 2*a),x)*a*d*n - 4*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*
a),x)*a*d + int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*a),x)*b*c*
n**2 + 4*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*a),x)*b*c*n +
 4*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*a),x)*b*c)/(b*(n +
2))
```

3.101 $\int \frac{1}{\sqrt{a+bx^n}(c+dx^n)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)} \, dx = \frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/(a+b*x^n
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(61) = 122.

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)} \, dx =$$

$$-\frac{2ac(1+n)x\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\right.}{\sqrt{a+bx^n}\,(c+dx^n)\left(2adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{3}{2},1\right.\right.}$$

input
```
Integrate[1/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(-2*a*c*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*
x^n)/c)])/(Sqrt[a + b*x^n]*(c + d*x^n)*(2*a*d*n*x^n*AppellF1[1 + n^(-1), 1
/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1 + n^
(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*c*(1 + n)*Appe
llF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}(dx^n+c)}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt{a+bx^n}}$$

input
```
Int[1/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a),
-((d*x^n)/c)])/(c*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}}\,\left(c+d\,x^{n}\right)}dx$$

input
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)**(1/2)/(c+d*x**n),x)
```

output
```
Integral(1/(sqrt(a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx=\int \frac{1}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(1/((a + b*x^n)^(1/2)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^(1/2)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx=\int \frac{\sqrt{x^nb+a}}{x^{2n}bd+x^nad+x^nbc+ac}dx$$

input
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.102 $\int \frac{1}{\sqrt{a+bx^n}(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx=\frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/(a+b*x
^n)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 391 vs. 2(61) = 122.

Time = 0.84 (sec) , antiderivative size = 391, normalized size of antiderivative = 6.41

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx$$
$$=\frac{x\left(\frac{bd(-2+n)x^n\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(-bc+ad)(1+n)}-\frac{2c\left(2ad^2nx^n(a+bx^n)\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcdnx\right.}{(bc-ad)(c+dx^n)\left(2adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx}{a}\right.\right.}\right.}{2c^2n\sqrt{a+bx}}$$

input
```
Integrate[1/(Sqrt[a + b*x^n]*(c + d*x^n)^2),x]
```

output
```
(x*((b*d*(-2 + n)*x^n*Sqrt[1 + (b*x^n)/a]*AppellF1[1 + n^(-1), 1/2, 1, 2 +
 n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/((-(b*c) + a*d)*(1 + n)) - (2*c*(2*a
*d^2*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)] + b*c*d*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 3/2, 1, 2
 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*c*(1 + n)*(-(b*c*n) + a*d*n +
 b*d*x^n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]
))/((b*c - a*d)*(c + d*x^n)*(2*a*d*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 +
n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 3/2,
1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*c*(1 + n)*AppellF1[n^(-1)
, 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))))/(2*c^2*n*Sqrt[a + b*
x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}(dx^n+c)^2}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt{a+bx^n}}$$

input
```
Int[1/(Sqrt[a + b*x^n]*(c + d*x^n)^2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 2, 1 + n^(-1), -((b*x^n)/a),
-((d*x^n)/c)])/(c^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}}\,\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n
) + (b*c^2 + 2*a*c*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx$$

input
```
integrate(1/(a+b*x**n)**(1/2)/(c+d*x**n)**2,x)
```

output
```
Integral(1/(sqrt(a + b*x**n)*(c + d*x**n)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a)*(d*x^n + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{\sqrt{bx^n+a}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx=\int \frac{1}{\sqrt{a+b\,x^n}\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)^(1/2)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^(1/2)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^n}\,(c+dx^n)^2}\,dx=\int \frac{\sqrt{x^n b+a}}{x^{3n}b\,d^2+x^{2n}a\,d^2+2x^{2n}bcd+2x^n acd+x^n b\,c^2+a\,c^2}dx$$

input
```
int(1/(a+b*x^n)^(1/2)/(c+d*x^n)^2,x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b*d**2 + x**(2*n)*a*d**2 + 2*x**(2*n)*b*c*d
 + 2*x**n*a*c*d + x**n*b*c**2 + a*c**2),x)
```

3.103 $\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = -\frac{2d\left(c-\frac{2ad(1+n)}{b(2+n)}\right)x\sqrt{a+bx^n}}{abn}+\frac{2(bc-ad)x(c+dx^n)}{abn\sqrt{a+bx^n}}$$
$$-\frac{\left(4a^2d^2(1+n)-4abcd(2+n)+b^2c^2(4-n^2)\right)x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2n(2+n)\sqrt{a+bx^n}}$$

output
```
-2*d*(c-2*a*d*(1+n)/b/(2+n))*x*(a+b*x^n)^(1/2)/a/b/n+2*(-a*d+b*c)*x*(c+d*x
^n)/a/b/n/(a+b*x^n)^(1/2)-(4*a^2*d^2*(1+n)-4*a*b*c*d*(2+n)+b^2*c^2*(-n^2+4
))*x*(1+b*x^n/a)^(1/2)*hypergeom([1/2, 1/n],[1+1/n],-b*x^n/a)/a/b^2/n/(2+n
)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \frac{2x((bc-ad)^2(2+n)+ad^2n(a+bx^n))-(4a^2d^2(1+n)-4abcd(2+n)-b^2c^2(-4+}{ab^2n(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n)^(3/2),x]
```

output
```
(2*x*((b*c - a*d)^2*(2 + n) + a*d^2*n*(a + b*x^n)) - (4*a^2*d^2*(1 + n) -
4*a*b*c*d*(2 + n) - b^2*c^2*(-4 + n^2))*x*Sqrt[1 + (b*x^n)/a]*Hypergeometr
ic2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^2*n*(2 + n)*Sqrt[a + b*
x^n])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {930, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{2\int \frac{d(2ad(n+1)-bc(n+2))x^n+c(2ad-bc(2-n))}{2\sqrt{bx^n+a}}dx}{abn}+\frac{2x(bc-ad)(c+dx^n)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(2ad(n+1)-bc(n+2))x^n+c(2ad-bc(2-n))}{\sqrt{bx^n+a}}dx}{abn}+\frac{2x(bc-ad)(c+dx^n)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{913}$$

$$\frac{-\frac{(4a^2d^2(n+1)-4abcd(n+2)+b^2c^2(4-n^2))\int \frac{1}{\sqrt{bx^n+a}}dx}{b(n+2)}-2dx\sqrt{a+bx^n}\left(c-\frac{2ad(n+1)}{b(n+2)}\right)}{abn}+$$
$$\frac{2x(bc-ad)(c+dx^n)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{779}$$

$$\frac{-\frac{\sqrt{\frac{bx^n}{a}+1}\left(4a^2d^2(n+1)-4abcd(n+2)+b^2c^2\left(4-n^2\right)\right)\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{b(n+2)\sqrt{a+bx^n}}-2dx\sqrt{a+bx^n}\left(c-\frac{2ad(n+1)}{b(n+2)}\right)}{abn}+\frac{2x(bc-ad)\left(c+dx^n\right)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \ 778$$

$$\frac{-\frac{x\sqrt{\frac{bx^n}{a}+1}\left(4a^2d^2(n+1)-4abcd(n+2)+b^2c^2\left(4-n^2\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(n+2)\sqrt{a+bx^n}}-2dx\sqrt{a+bx^n}\left(c-\frac{2ad(n+1)}{b(n+2)}\right)}{abn}+\frac{2x(bc-ad)\left(c+dx^n\right)}{abn\sqrt{a+bx^n}}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x*(c + d*x^n))/(a*b*n*Sqrt[a + b*x^n]) + (-2*d*(c - (2*a*d*
(1 + n))/(b*(2 + n)))*x*Sqrt[a + b*x^n] - ((4*a^2*d^2*(1 + n) - 4*a*b*c*d*
(2 + n) + b^2*c^2*(4 - n^2))*x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2,
n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(2 + n)*Sqrt[a + b*x^n]))/(a*b*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(3/2),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \int \frac{(c+dx^n)^2}{(a+bx^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**(3/2),x)
```

output
```
Integral((c + d*x**n)**2/(a + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^(3/2),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(3/2),x)
```

output

```
(2*x**n*sqrt(x**n*b + a)*b*d**2*n*x - 4*x**n*sqrt(x**n*b + a)*b*d**2*x + 4
*sqrt(x**n*b + a)*a*d**2*n*x + 4*sqrt(x**n*b + a)*a*d**2*x - 4*sqrt(x**n*b
 + a)*b*c*d*n*x - 8*sqrt(x**n*b + a)*b*c*d*x - 4*x**n*int(sqrt(x**n*b + a)
/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a*
*2*n**2 - 4*a**2),x)*a**2*b*d**2*n**3 - 4*x**n*int(sqrt(x**n*b + a)/(x**(2
*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a**2*n**2
 - 4*a**2),x)*a**2*b*d**2*n**2 + 16*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b*
*2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a**2*n**2 - 4*a
**2),x)*a**2*b*d**2*n + 16*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n**2 -
 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a**2*n**2 - 4*a**2),x)*a
**2*b*d**2 + 4*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*
b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a**2*n**2 - 4*a**2),x)*a*b**2*c*d*n*
*3 + 8*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2
*x**n*a*b*n**2 - 8*x**n*a*b + a**2*n**2 - 4*a**2),x)*a*b**2*c*d*n**2 - 16*
x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a
*b*n**2 - 8*x**n*a*b + a**2*n**2 - 4*a**2),x)*a*b**2*c*d*n - 32*x**n*int(s
qrt(x**n*b + a)/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 -
8*x**n*a*b + a**2*n**2 - 4*a**2),x)*a*b**2*c*d + x**n*int(sqrt(x**n*b + a)
/(x**(2*n)*b**2*n**2 - 4*x**(2*n)*b**2 + 2*x**n*a*b*n**2 - 8*x**n*a*b + a*
*2*n**2 - 4*a**2),x)*b**3*c**2*n**4 - 8*x**n*int(sqrt(x**n*b + a)/(x**(...
```

3.104 $\int \frac{c+dx^n}{(a+bx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}} \, dx = \frac{2dx}{b(2-n)\sqrt{a+bx^n}} + \frac{\left(\frac{c}{a}-\frac{2d}{b(2-n)}\right) x \sqrt{1+\frac{bx^n}{a}} \, \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

output
```
2*d*x/b/(2-n)/(a+b*x^n)^(1/2)+(c/a-2*d/b/(2-n))*x*(1+b*x^n/a)^(1/2)*hyperg
eom([3/2, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}} \, dx = \frac{-2adx + (2ad+bc(-2+n))x\sqrt{1+\frac{bx^n}{a}} \, \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab(-2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n)^(3/2),x]
```

output
```
(-2*a*d*x + (2*a*d + b*c*(-2 + n))*x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1
[3/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b*(-2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(2ad-bc(2-n))\int \frac{1}{\sqrt{bx^n+a}}dx}{abn}+\frac{2x(bc-ad)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}(2ad-bc(2-n))\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{abn\sqrt{a+bx^n}}+\frac{2x(bc-ad)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}(2ad-bc(2-n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{abn\sqrt{a+bx^n}}+\frac{2x(bc-ad)}{abn\sqrt{a+bx^n}}$$

input
```
Int[(c + d*x^n)/(a + b*x^n)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x)/(a*b*n*Sqrt[a + b*x^n]) + ((2*a*d - b*c*(2 - n))*x*Sqrt[
1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a*b*n*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{c+d\,x^{n}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{3}{2}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{5}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
a**(1/n)*a**(-3/2 - 1/n)*c*x*gamma(1/n)*hyper((3/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-5/2 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((3/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \int \frac{dx^n+c}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \int \frac{dx^n+c}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \int \frac{c+d\,x^n}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^(3/2),x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{3/2}}\,dx = \frac{-2\sqrt{x^nb+a}\,dx+2x^n\left(\int \frac{\sqrt{x^nb+a}}{x^{2n}b^2n-2x^{2n}b^2+2x^nabn-4x^nab+a^2n-2a^2}dx\right)abdn-4x^n\left(\int \frac{}{x^{2n}b^2n-}\right.}{}$$

input
```
int((c+d*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(x**n*b + a)*d*x + 2*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n
- 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*a*b*d*
n - 4*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**
n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*a*b*d + x**n*int(sqrt(x**n*b +
a)/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n
 - 2*a**2),x)*b**2*c*n**2 - 4*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n -
 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*b**2*c*
n + 4*x**n*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**
n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*b**2*c + 2*int(sqrt(x**n*b + a)
/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n -
 2*a**2),x)*a**2*d*n - 4*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n - 2*x**(2*n
)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*a**2*d + int(sqrt
(x**n*b + a)/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*
b + a**2*n - 2*a**2),x)*a*b*c*n**2 - 4*int(sqrt(x**n*b + a)/(x**(2*n)*b**2
*n - 2*x**(2*n)*b**2 + 2*x**n*a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*a*b
*c*n + 4*int(sqrt(x**n*b + a)/(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 2*x**n*
a*b*n - 4*x**n*a*b + a**2*n - 2*a**2),x)*a*b*c)/(b*(x**n*b*n - 2*x**n*b +
a*n - 2*a))
```

3.105 $\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} \, dx = \frac{x\sqrt{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{3}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac\sqrt{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,3/2,1,1+1/n,-b*x^n/a,-d*x^n/c)/a/c/(a+b*x
^n)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 382 vs. 2(64) = 128.

Time = 0.80 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.97

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} \, dx =$$
$$-\frac{x\left(\frac{bd(-2+n)x^n\sqrt{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(1+n)}+\frac{2\left(2abdnx^n(c+dx^n)\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+b^2cnx^n \cdots\right.}{(c+dx^n)\left(2adnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\cdots\right.\right.}\cdots\right.}{a(-bc+ad)n\sqrt{a+\cdots}}$$

input
```
Integrate[1/((a + b*x^n)^(3/2)*(c + d*x^n)),x]
```

output
```
-((x*((b*d*(-2 + n)*x^n*Sqrt[1 + (b*x^n)/a]*AppellF1[1 + n^(-1), 1/2, 1, 2
 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c*(1 + n)) + (2*(2*a*b*d*n*x^n*(c
 + d*x^n)*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)
/c)] + b^2*c*n*x^n*(c + d*x^n)*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -(
(b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*(a*d*n - b*(c*n + 2*d*x^n))*Appell
F1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))/((c + d*x^n)*
(2*a*d*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x
^n)/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a),
 -((d*x^n)/c)] - 2*a*c*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x
^n)/a), -((d*x^n)/c)]))))/(a*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{3/2}(dx^n+c)}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\operatorname{AppellF1}\left(\frac{1}{n},\frac{3}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac\sqrt{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(3/2)*(c + d*x^n)),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 3/2, 1, 1 + n^(-1), -((b*x^n)/a),
-((d*x^n)/c)])/(a*c*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{3}{2}} \left(c+d\,x^{n}\right)}d x$$

input
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^{3/2}\,(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n
) + (2*a*b*c + a^2*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} dx = \int \frac{1}{(a+bx^n)^{\frac{3}{2}}(c+dx^n)} dx$$

input
```
integrate(1/(a+b*x**n)**(3/2)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)**(3/2)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)} dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)} dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)} dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^{3/2}\,(c+dx^n)}\,dx=\int \frac{1}{(a+b\,x^n)^{3/2}\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(1/((a + b*x^n)^(3/2)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^(3/2)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{3/2}\,(c+dx^n)}\,dx=\int \frac{\sqrt{x^n b+a}}{x^{3n}b^2d+2x^{2n}abd+x^{2n}b^2c+x^n a^2d+2x^n abc+a^2c}dx$$

input
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c
 + x**n*a**2*d + 2*x**n*a*b*c + a**2*c),x)
```

3.106 $\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2} dx = \frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac^2\sqrt{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,3/2,2,1+1/n,-b*x^n/a,-d*x^n/c)/a/c^2/(a+b
*x^n)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1294 vs. $2(64) = 128$.

Time = 1.22 (sec) , antiderivative size = 1294, normalized size of antiderivative = 20.22

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2} dx = \text{Too large to display}$$

input
```
Integrate[1/((a + b*x^n)^(3/2)*(c + d*x^n)^2),x]
```

output
```
(x*(8*a*b^2*c^4*(1 + n)^2*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((d*x^n)/c)] + 4*a^3*c^2*d^2*(1 + n)^2*AppellF1[n^(-1), 1/2, 1, 1 + n^
(-1), -((b*x^n)/a), -((d*x^n)/c)] - 4*a*b^2*c^4*n*(1 + n)^2*AppellF1[n^(-1
), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 8*a^2*b*c^3*d*n*(1 +
n)^2*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 4*
a^3*c^2*d^2*n*(1 + n)^2*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a),
 -((d*x^n)/c)] + 2*c*(1 + n)*(a*d^2*(a + b*x^n) + 2*b^2*c*(c + d*x^n))*(2*
a*d*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)
/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -(
(d*x^n)/c)] - 2*a*c*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)]) - 4*b^2*c*d*x^n*Sqrt[1 + (b*x^n)/a]*(c + d*x^n)*Appell
F1[1 + n^(-1), 1/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]*(2*a*d*n*x^
n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b
*c*n*x^n*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/
c)] - 2*a*c*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((
d*x^n)/c)]) - 2*a*b*d^2*x^n*Sqrt[1 + (b*x^n)/a]*(c + d*x^n)*AppellF1[1 + n
^(-1), 1/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]*(2*a*d*n*x^n*Appell
F1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n
*AppellF1[1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*
a*c*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n...
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{3/2}(dx^n+c)^2}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{3}{2},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac^2\sqrt{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(3/2)*(c + d*x^n)^2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 3/2, 2, 1 + n^(-1), -((b*x^n)/a),
-((d*x^n)/c)])/(a*c^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2
)*x^(3*n) + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d
)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{1}{(a+bx^n)^{\frac{3}{2}}(c+dx^n)^2}\,dx$$

input
```
integrate(1/(a+b*x**n)**(3/2)/(c+d*x**n)**2,x)
```

output
```
Integral(1/((a + b*x**n)**(3/2)*(c + d*x**n)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*(d*x^n + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)^{3/2}\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)^(3/2)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^(3/2)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{3/2}(c+dx^n)^2}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{4n}b^2d^2+2x^{3n}ab\,d^2+2x^{3n}b^2cd+x^{2n}a^2d^2+4x^{2n}abcd+x^{2n}b^2c^2+2x^n a}$$

input
```
int(1/(a+b*x^n)^(3/2)/(c+d*x^n)^2,x)
```

output
```
int(sqrt(x**n*b + a)/(x**(4*n)*b**2*d**2 + 2*x**(3*n)*a*b*d**2 + 2*x**(3*n
)*b**2*c*d + x**(2*n)*a**2*d**2 + 4*x**(2*n)*a*b*c*d + x**(2*n)*b**2*c**2
+ 2*x**n*a**2*c*d + 2*x**n*a*b*c**2 + a**2*c**2),x)
```

3.107 $\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = -\frac{3d(3ad(1+n)-bc(3+10n))x(a+bx^n)^{4/3}}{b^2(3+4n)(3+7n)}+\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(3+7n)} + \frac{(9a^2d^2(1+n)-6abcd(3+7n)+b^2c^2(9+33n+28n^2))\,x\sqrt[3]{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{n},1+\frac{1}{n}\right.}{b^2(3+4n)(3+7n)\sqrt[3]{1+\frac{bx^n}{a}}}$$

output

```
-3*d*(3*a*d*(1+n)-b*c*(3+10*n))*x*(a+b*x^n)^(4/3)/b^2/(3+4*n)/(3+7*n)+3*d*
x*(a+b*x^n)^(4/3)*(c+d*x^n)/b/(3+7*n)+(9*a^2*d^2*(1+n)-6*a*b*c*d*(3+7*n)+b
^2*c^2*(28*n^2+33*n+9))*x*(a+b*x^n)^(1/3)*hypergeom([-1/3, 1/n],[1+1/n],-b
*x^n/a)/b^2/(3+4*n)/(3+7*n)/(1+b*x^n/a)^(1/3)
```

Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx$$
$$= \frac{3x(a+bx^n)\left(-3a^2d^2n(1+n)+abdn(2c(3+7n)+d(3+n)x^n)+b^2(c^2(9+33n+28n^2)+2cd(9+24n\right.}{}$$

input
```
Integrate[(a + b*x^n)^(1/3)*(c + d*x^n)^2,x]
```

output
```
(3*x*(a + b*x^n)*(-3*a^2*d^2*n*(1 + n) + a*b*d*n*(2*c*(3 + 7*n) + d*(3 + n
)*x^n) + b^2*(c^2*(9 + 33*n + 28*n^2) + 2*c*d*(9 + 24*n + 7*n^2)*x^n + d^2
*(9 + 15*n + 4*n^2)*x^(2*n))) + a*n*(9*a^2*d^2*(1 + n) - 6*a*b*c*d*(3 + 7*
n) + b^2*c^2*(9 + 33*n + 28*n^2))*x*(1 + (b*x^n)/a)^(2/3)*Hypergeometric2F
1[2/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b^2*(3 + n)*(3 + 4*n)*(3 + 7*n)
*(a + b*x^n)^(2/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx$$
$$\Big\downarrow \text{933}$$
$$\frac{3\int -\frac{1}{3}\sqrt[3]{bx^n+a}(d(3ad(n+1)-bc(10n+3))x^n+c(3ad-bc(7n+3)))\,dx}{b(7n+3)}+\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(7n+3)}$$
$$\Big\downarrow \text{27}$$

$$\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(7n+3)}-\frac{\int \sqrt[3]{bx^n+a}(d(3ad(n+1)-bc(10n+3))x^n+c(3ad-bc(7n+3)))\,dx}{b(7n+3)}$$

$\Big\downarrow$ 913

$$\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(7n+3)}-\frac{\frac{3dx(a+bx^n)^{4/3}(3ad(n+1)-bc(10n+3))}{b(4n+3)}-\frac{(9a^2d^2(n+1)-6abcd(7n+3)+b^2c^2(28n^2+33n+9))\int \sqrt[3]{bx^n+a}dx}{b(4n+3)}}{b(7n+3)}$$

$\Big\downarrow$ 779

$$\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(7n+3)}-\frac{\frac{3dx(a+bx^n)^{4/3}(3ad(n+1)-bc(10n+3))}{b(4n+3)}-\frac{\sqrt[3]{a+bx^n}(9a^2d^2(n+1)-6abcd(7n+3)+b^2c^2(28n^2+33n+9))\int \sqrt[3]{\frac{bx^n}{a}+1}dx}{b(4n+3)\sqrt[3]{\frac{bx^n}{a}+1}}}{b(7n+3)}$$

$\Big\downarrow$ 778

$$\frac{3dx(a+bx^n)^{4/3}(c+dx^n)}{b(7n+3)}-\frac{\frac{3dx(a+bx^n)^{4/3}(3ad(n+1)-bc(10n+3))}{b(4n+3)}-\frac{x\sqrt[3]{a+bx^n}(9a^2d^2(n+1)-6abcd(7n+3)+b^2c^2(28n^2+33n+9))\text{ Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{n},1+\frac{1}{n},\right.}{b(4n+3)\sqrt[3]{\frac{bx^n}{a}+1}}}{b(7n+3)}$$

input
```
Int[(a + b*x^n)^(1/3)*(c + d*x^n)^2,x]
```

output
```
(3*d*x*(a + b*x^n)^(4/3)*(c + d*x^n))/(b*(3 + 7*n)) - ((3*d*(3*a*d*(1 + n)
 - b*c*(3 + 10*n))*x*(a + b*x^n)^(4/3))/(b*(3 + 4*n)) - ((9*a^2*d^2*(1 + n
) - 6*a*b*c*d*(3 + 7*n) + b^2*c^2*(9 + 33*n + 28*n^2))*x*(a + b*x^n)^(1/3)
*Hypergeometric2F1[-1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(3 + 4*n)*(
1 + (b*x^n)/a)^(1/3)))/(b*(3 + 7*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{1}{3}} (c+d\,x^{n})^{2}\,dx$$

input
```
int((a+b*x^n)^(1/3)*(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^(1/3)*(c+d*x^n)^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \frac{a^{\frac{1}{n}}a^{\frac{1}{3}-\frac{1}{n}}c^2x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{5}{3}-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{2a^{-\frac{2}{3}-\frac{1}{n}}a^{1+\frac{1}{n}}cdx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/3)*(c+d*x**n)**2,x)
```

output
```
a**(1/n)*a**(1/3 - 1/n)*c**2*x*gamma(1/n)*hyper((-1/3, 1/n), (1 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-5/3 - 1/n)*a**(2 + 1/n)
*d**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-1/3, 2 + 1/n), (3 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + 2*a**(-2/3 - 1/n)*a**(1 + 1/n)*c
*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-1/3, 1 + 1/n), (2 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \int (bx^n+a)^{\frac{1}{3}}(dx^n+c)^2\,dx$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(1/3)*(d*x^n + c)^2, x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \int (bx^n+a)^{\frac{1}{3}}(dx^n+c)^2\,dx$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(1/3)*(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \int (a+b\,x^n)^{1/3}\,(c+d\,x^n)^2\,dx$$

input
```
int((a + b*x^n)^(1/3)*(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^(1/3)*(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)^2\,dx = \text{too large to display}$$

input
```
int((a+b*x^n)^(1/3)*(c+d*x^n)^2,x)
```

output

```
(12*x**(2*n)*(x**n*b + a)**(1/3)*b**2*d**2*n**2*x + 45*x**(2*n)*(x**n*b +
a)**(1/3)*b**2*d**2*n*x + 27*x**(2*n)*(x**n*b + a)**(1/3)*b**2*d**2*x + 3*
x**n*(x**n*b + a)**(1/3)*a*b*d**2*n**2*x + 9*x**n*(x**n*b + a)**(1/3)*a*b*
d**2*n*x + 42*x**n*(x**n*b + a)**(1/3)*b**2*c*d*n**2*x + 144*x**n*(x**n*b
+ a)**(1/3)*b**2*c*d*n*x + 54*x**n*(x**n*b + a)**(1/3)*b**2*c*d*x - 9*(x**
n*b + a)**(1/3)*a**2*d**2*n**2*x - 9*(x**n*b + a)**(1/3)*a**2*d**2*n*x + 4
2*(x**n*b + a)**(1/3)*a*b*c*d*n**2*x + 18*(x**n*b + a)**(1/3)*a*b*c*d*n*x
+ 84*(x**n*b + a)**(1/3)*b**2*c**2*n**2*x + 99*(x**n*b + a)**(1/3)*b**2*c*
*2*n*x + 27*(x**n*b + a)**(1/3)*b**2*c**2*x + 252*int((x**n*b + a)**(1/3)/
(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n + 27*x**n*b + 28*a*n**3 +
 117*a*n**2 + 108*a*n + 27*a),x)*a**3*d**2*n**5 + 1305*int((x**n*b + a)**(
1/3)/(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n + 27*x**n*b + 28*a*n
**3 + 117*a*n**2 + 108*a*n + 27*a),x)*a**3*d**2*n**4 + 2025*int((x**n*b +
a)**(1/3)/(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n + 27*x**n*b + 2
8*a*n**3 + 117*a*n**2 + 108*a*n + 27*a),x)*a**3*d**2*n**3 + 1215*int((x**n
*b + a)**(1/3)/(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n + 27*x**n*
b + 28*a*n**3 + 117*a*n**2 + 108*a*n + 27*a),x)*a**3*d**2*n**2 + 243*int((
x**n*b + a)**(1/3)/(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n + 27*x
**n*b + 28*a*n**3 + 117*a*n**2 + 108*a*n + 27*a),x)*a**3*d**2*n - 1176*int
((x**n*b + a)**(1/3)/(28*x**n*b*n**3 + 117*x**n*b*n**2 + 108*x**n*b*n +...
```

3.108 $\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx = \frac{3dx(a+bx^n)^{4/3}}{b(3+4n)}+\frac{\left(c-\frac{3ad}{3b+4bn}\right)x\sqrt[3]{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt[3]{1+\frac{bx^n}{a}}}$$

output
```
3*d*x*(a+b*x^n)^(4/3)/b/(3+4*n)+(c-3*a*d/(4*b*n+3*b))*x*(a+b*x^n)^(1/3)*hy
pergeom([-1/3, 1/n],[1+1/n],-b*x^n/a)/(1+b*x^n/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx = \frac{x\sqrt[3]{a+bx^n}\left(3d(a+bx^n)\sqrt[3]{1+\frac{bx^n}{a}}+(-3ad+bc(3+4n))\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(3+4n)\sqrt[3]{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(1/3)*(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^(1/3)*(3*d*(a + b*x^n)*(1 + (b*x^n)/a)^(1/3) + (-3*a*d + b*
c*(3 + 4*n))*Hypergeometric2F1[-1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(
b*(3 + 4*n)*(1 + (b*x^n)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{3ad}{4bn+3b}\right)\int \sqrt[3]{bx^n+a}dx+\frac{3dx(a+bx^n)^{4/3}}{b(4n+3)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^n}\left(c-\frac{3ad}{4bn+3b}\right)\int \sqrt[3]{\frac{bx^n}{a}+1}dx}{\sqrt[3]{\frac{bx^n}{a}+1}}+\frac{3dx(a+bx^n)^{4/3}}{b(4n+3)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^n}\left(c-\frac{3ad}{4bn+3b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt[3]{\frac{bx^n}{a}+1}}+\frac{3dx(a+bx^n)^{4/3}}{b(4n+3)}$$

input
```
Int[(a + b*x^n)^(1/3)*(c + d*x^n),x]
```

output
```
(3*d*x*(a + b*x^n)^(4/3))/(b*(3 + 4*n)) + ((c - (3*a*d)/(3*b + 4*b*n))*x*(
a + b*x^n)^(1/3)*Hypergeometric2F1[-1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)]
)/(1 + (b*x^n)/a)^(1/3)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{1}{3}}\,(c+d\,x^{n})\,dx$$

input
```
int((a+b*x^n)^(1/3)*(c+d*x^n),x)
```

output
```
int((a+b*x^n)^(1/3)*(c+d*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx = \frac{a^{\frac{1}{n}}a^{\frac{1}{3}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{2}{3}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/3)*(c+d*x**n),x)
```

output
```
a**(1/n)*a**(1/3 - 1/n)*c*x*gamma(1/n)*hyper((-1/3, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-2/3 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((-1/3, 1 + 1/n), (2 + 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\ dx = \int (bx^n+a)^{\frac{1}{3}}(dx^n+c)\, dx$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(1/3)*(d*x^n + c), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\ dx = \int (bx^n+a)^{\frac{1}{3}}(dx^n+c)\, dx$$

input
```
integrate((a+b*x^n)^(1/3)*(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(1/3)*(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\ dx = \int {\left(a+b\,x^n\right)}^{1/3}\,\left(c+d\,x^n\right) \,d x$$

input
```
int((a + b*x^n)^(1/3)*(c + d*x^n),x)
```

output
```
int((a + b*x^n)^(1/3)*(c + d*x^n), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^n}(c+dx^n)\,dx$$
$$= \frac{3x^n(x^nb+a)^{\frac{1}{3}}bdnx + 9x^n(x^nb+a)^{\frac{1}{3}}bdx + 3(x^nb+a)^{\frac{1}{3}}adnx + 12(x^nb+a)^{\frac{1}{3}}bcnx + 9(x^nb+a)^{\frac{1}{3}}bcx -}{}$$

input
```
int((a+b*x^n)^(1/3)*(c+d*x^n),x)
```

output
```
(3*x**n*(x**n*b + a)**(1/3)*b*d*n*x + 9*x**n*(x**n*b + a)**(1/3)*b*d*x + 3
*(x**n*b + a)**(1/3)*a*d*n*x + 12*(x**n*b + a)**(1/3)*b*c*n*x + 9*(x**n*b
+ a)**(1/3)*b*c*x - 12*int((x**n*b + a)**(1/3)/(4*x**n*b*n**2 + 15*x**n*b*
n + 9*x**n*b + 4*a*n**2 + 15*a*n + 9*a),x)*a**2*d*n**3 - 45*int((x**n*b +
a)**(1/3)/(4*x**n*b*n**2 + 15*x**n*b*n + 9*x**n*b + 4*a*n**2 + 15*a*n + 9*
a),x)*a**2*d*n**2 - 27*int((x**n*b + a)**(1/3)/(4*x**n*b*n**2 + 15*x**n*b*
n + 9*x**n*b + 4*a*n**2 + 15*a*n + 9*a),x)*a**2*d*n + 16*int((x**n*b + a)*
*(1/3)/(4*x**n*b*n**2 + 15*x**n*b*n + 9*x**n*b + 4*a*n**2 + 15*a*n + 9*a),
x)*a*b*c*n**4 + 72*int((x**n*b + a)**(1/3)/(4*x**n*b*n**2 + 15*x**n*b*n +
9*x**n*b + 4*a*n**2 + 15*a*n + 9*a),x)*a*b*c*n**3 + 81*int((x**n*b + a)**(
1/3)/(4*x**n*b*n**2 + 15*x**n*b*n + 9*x**n*b + 4*a*n**2 + 15*a*n + 9*a),x)
*a*b*c*n**2 + 27*int((x**n*b + a)**(1/3)/(4*x**n*b*n**2 + 15*x**n*b*n + 9*
x**n*b + 4*a*n**2 + 15*a*n + 9*a),x)*a*b*c*n)/(b*(4*n**2 + 15*n + 9))
```

3.109 $\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n} \, dx = \frac{x\sqrt[3]{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt[3]{1+\frac{bx^n}{a}}}$$

output
```
x*(a+b*x^n)^(1/3)*AppellF1(1/n,-1/3,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/(1+b*x^n/
a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. $2(61) = 122$.

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n} \, dx$$
$$= \frac{3ac(1+n)x\sqrt[3]{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{3},1,1+\frac{1}{n},-\right.}{(c+dx^n)\left(-3adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},-\frac{1}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{2}{3},1,2+\frac{1}{n},-\right.\right.}$$

input
```
Integrate[(a + b*x^n)^(1/3)/(c + d*x^n),x]
```

output
```
(3*a*c*(1 + n)*x*(a + b*x^n)^(1/3)*AppellF1[n^(-1), -1/3, 1, 1 + n^(-1), -
((b*x^n)/a), -((d*x^n)/c)])/((c + d*x^n)*(-3*a*d*n*x^n*AppellF1[1 + n^(-1)
, -1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1
+ n^(-1), 2/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 3*a*c*(1 + n)*
AppellF1[n^(-1), -1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{a+bx^n}\int \frac{\sqrt[3]{\frac{bx^n}{a}+1}}{dx^n+c}dx}{\sqrt[3]{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt[3]{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(1/3)/(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^(1/3)*AppellF1[n^(-1), -1/3, 1, 1 + n^(-1), -((b*x^n)/a), -
((d*x^n)/c)])/(c*(1 + (b*x^n)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{1}{3}}}{c+d\,x^n}dx$$

input
```
int((a+b*x^n)^(1/3)/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^(1/3)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx = \int \frac{(bx^n+a)^{\frac{1}{3}}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(1/3)/(d*x^n + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx = \int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx$$

input
```
integrate((a+b*x**n)**(1/3)/(c+d*x**n),x)
```

output
```
Integral((a + b*x**n)**(1/3)/(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx = \int \frac{(bx^n+a)^{\frac{1}{3}}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(1/3)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx = \int \frac{(bx^n+a)^{\frac{1}{3}}}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(1/3)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx=\int \frac{{\left(a+b\,x^n\right)}^{1/3}}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^(1/3)/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^(1/3)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{c+dx^n}\,dx=\int \frac{(x^n b+a)^{\frac{1}{3}}}{x^n d+c}dx$$

input
```
int((a+b*x^n)^(1/3)/(c+d*x^n),x)
```

output
```
int((x**n*b + a)**(1/3)/(x**n*d + c),x)
```

3.110 $\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx=\frac{x\sqrt[3]{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^n}{a}}}$$

output
```
x*(a+b*x^n)^(1/3)*AppellF1(1/n,-1/3,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/(1+b*x^
n/a)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(61) = 122.

Time = 0.93 (sec) , antiderivative size = 364, normalized size of antiderivative = 5.97

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx$$
$$=\frac{x\left(\frac{b(-3+2n)x^n\left(1+\frac{bx^n}{a}\right)^{2/3}\text{AppellF1}\left(1+\frac{1}{n},\frac{2}{3},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+n}+\frac{3c\left(3adnx^n(a+bx^n)\,\text{AppellF1}\left(1+\frac{1}{n},\frac{2}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+2bc\cdots}{(c+dx^n)\left(3adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{2}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx}{c}\cdots\right.\right.}\cdots\right.}{3c^2n\,(a+bx^n)^{2/3}}$$

input
```
Integrate[(a + b*x^n)^(1/3)/(c + d*x^n)^2,x]
```

output
```
(x*((b*(-3 + 2*n)*x^n*(1 + (b*x^n)/a)^(2/3)*AppellF1[1 + n^(-1), 2/3, 1, 2
 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(1 + n) + (3*c*(3*a*d*n*x^n*(a + b
*x^n)*AppellF1[1 + n^(-1), 2/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]
 + 2*b*c*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 5/3, 1, 2 + n^(-1), -((b*x
^n)/a), -((d*x^n)/c)] - 3*a*c*(1 + n)*(a*n + b*x^n)*AppellF1[n^(-1), 2/3,
1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))/((c + d*x^n)*(3*a*d*n*x^n*App
ellF1[1 + n^(-1), 2/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 2*b*c*
n*x^n*AppellF1[1 + n^(-1), 5/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]
 - 3*a*c*(1 + n)*AppellF1[n^(-1), 2/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x
^n)/c)]))))/(3*c^2*n*(a + b*x^n)^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}dx$$

$$\Big\downarrow \ 937$$

$$\frac{\sqrt[3]{a+bx^n}\int \frac{\sqrt[3]{\frac{bx^n}{a}+1}}{(dx^n+c)^2}dx}{\sqrt[3]{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{x\sqrt[3]{a+bx^n}\,\text{AppellF1}\left(\frac{1}{n},-\frac{1}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt[3]{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(1/3)/(c + d*x^n)^2,x]
```

output
```
(x*(a + b*x^n)^(1/3)*AppellF1[n^(-1), -1/3, 2, 1 + n^(-1), -((b*x^n)/a), -
((d*x^n)/c)])/(c^2*(1 + (b*x^n)/a)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a + b\,x^n)^{\frac{1}{3}}}{(c + d\,x^n)^2}dx$$

input
```
int((a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx=\int \frac{(bx^n+a)^{\frac{1}{3}}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(1/3)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx=\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx$$

input
```
integrate((a+b*x**n)**(1/3)/(c+d*x**n)**2,x)
```

output
```
Integral((a + b*x**n)**(1/3)/(c + d*x**n)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx=\int \frac{(bx^n+a)^{\frac{1}{3}}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(1/3)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^{\frac{1}{3}}}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(1/3)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^{1/3}}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^(1/3)/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^(1/3)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{(c+dx^n)^2}\,dx = \int \frac{(x^n b+a)^{\frac{1}{3}}}{x^{2n}d^2+2x^n cd+c^2}dx$$

input
```
int((a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

output
```
int((x**n*b + a)**(1/3)/(x**(2*n)*d**2 + 2*x**n*c*d + c**2),x)
```

3.111 $\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = -\frac{3d(3ad(1+n)-bc(3+8n))x(a+bx^n)^{2/3}}{b^2(3+2n)(3+5n)} + \frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(3+5n)} + \frac{(9a^2d^2(1+n)-6abcd(3+5n)+b^2c^2(9+21n+10n^2))\,x\sqrt[3]{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n}\right.}{b^2(3+2n)(3+5n)\sqrt[3]{a+bx^n}}$$

output
```
-3*d*(3*a*d*(1+n)-b*c*(3+8*n))*x*(a+b*x^n)^(2/3)/b^2/(3+2*n)/(3+5*n)+3*d*x
*(a+b*x^n)^(2/3)*(c+d*x^n)/b/(3+5*n)+(9*a^2*d^2*(1+n)-6*a*b*c*d*(3+5*n)+b^
2*c^2*(10*n^2+21*n+9))*x*(1+b*x^n/a)^(1/3)*hypergeom([1/3, 1/n],[1+1/n],-b
*x^n/a)/b^2/(3+2*n)/(3+5*n)/(a+b*x^n)^(1/3)
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx$$
$$= \frac{3dx(a+bx^n)\left(-3ad(1+n)+2bc(3+5n)+bd(3+2n)x^n\right)+\left(9a^2d^2(1+n)-6abcd(3+5n)+b^2c^2(9+\right.}{b^2(3+2n)(3+5n)\sqrt[3]{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^n)*(-3*a*d*(1 + n) + 2*b*c*(3 + 5*n) + b*d*(3 + 2*n)*x^n)
+ (9*a^2*d^2*(1 + n) - 6*a*b*c*d*(3 + 5*n) + b^2*c^2*(9 + 21*n + 10*n^2))*
x*(1 + (b*x^n)/a)^(1/3)*Hypergeometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/(b^2*(3 + 2*n)*(3 + 5*n)*(a + b*x^n)^(1/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {933, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx$$
$$\Big\downarrow \; 933$$
$$\frac{3\int -\frac{d(3ad(n+1)-bc(8n+3))x^n+c(3ad-bc(5n+3))}{3\sqrt[3]{bx^n+a}}dx}{b(5n+3)}+\frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(5n+3)}$$
$$\Big\downarrow \; 27$$
$$\frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(5n+3)}-\frac{\int \frac{d(3ad(n+1)-bc(8n+3))x^n+c(3ad-bc(5n+3))}{\sqrt[3]{bx^n+a}}dx}{b(5n+3)}$$

$$\Big\downarrow \; 913$$

$$\frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(5n+3)}-\frac{\frac{3dx(a+bx^n)^{2/3}(3ad(n+1)-bc(8n+3))}{b(2n+3)}-\frac{(9a^2d^2(n+1)-6abcd(5n+3)+b^2c^2(10n^2+21n+9))\int\frac{1}{\sqrt[3]{bx^n+a}}dx}{b(2n+3)}}{b(5n+3)}$$

$$\Big\downarrow \; 779$$

$$\frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(5n+3)}-\frac{\frac{3dx(a+bx^n)^{2/3}(3ad(n+1)-bc(8n+3))}{b(2n+3)}-\frac{\sqrt[3]{\frac{bx^n}{a}+1}(9a^2d^2(n+1)-6abcd(5n+3)+b^2c^2(10n^2+21n+9))\int\frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}}dx}{b(2n+3)\sqrt[3]{a+bx^n}}}{b(5n+3)}$$

$$\Big\downarrow \; 778$$

$$\frac{3dx(a+bx^n)^{2/3}(c+dx^n)}{b(5n+3)}-\frac{\frac{3dx(a+bx^n)^{2/3}(3ad(n+1)-bc(8n+3))}{b(2n+3)}-\frac{x\sqrt[3]{\frac{bx^n}{a}+1}(9a^2d^2(n+1)-6abcd(5n+3)+b^2c^2(10n^2+21n+9))\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\right.}{b(2n+3)\sqrt[3]{a+bx^n}}}{b(5n+3)}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^n)^(2/3)*(c + d*x^n))/(b*(3 + 5*n)) - ((3*d*(3*a*d*(1 + n)
 - b*c*(3 + 8*n))*x*(a + b*x^n)^(2/3))/(b*(3 + 2*n)) - ((9*a^2*d^2*(1 + n)
 - 6*a*b*c*d*(3 + 5*n) + b^2*c^2*(9 + 21*n + 10*n^2))*x*(1 + (b*x^n)/a)^(1
/3)*Hypergeometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(3 + 2*n)
*(a + b*x^n)^(1/3)))/(b*(3 + 5*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c + d\,x^n)^2}{(a + b\,x^n)^{\frac{1}{3}}}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(1/3),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + dx^n)^2}{\sqrt[3]{a + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{1}{3}-\frac{1}{n}}c^2x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{7}{3}-\frac{1}{n}}a^{2+\frac{1}{n}}d^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{2a^{-\frac{4}{3}-\frac{1}{n}}a^{1+\frac{1}{n}}cdx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**(1/3),x)
```

output
```
a**(1/n)*a**(-1/3 - 1/n)*c**2*x*gamma(1/n)*hyper((1/3, 1/n), (1 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-7/3 - 1/n)*a**(2 + 1/n)
*d**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((1/3, 2 + 1/n), (3 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + 2*a**(-4/3 - 1/n)*a**(1 + 1/n)*c*
d*x**(n + 1)*gamma(1 + 1/n)*hyper((1/3, 1 + 1/n), (2 + 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^{1/3}}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^(1/3),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{\sqrt[3]{a+bx^n}}\,dx = \left(\int \frac{x^{2n}}{(x^nb+a)^{\frac{1}{3}}}dx\right)d^2 + 2\left(\int \frac{x^n}{(x^nb+a)^{\frac{1}{3}}}dx\right)cd + \left(\int \frac{1}{(x^nb+a)^{\frac{1}{3}}}dx\right)c^2$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(1/3),x)
```

output
```
int(x**(2*n)/(x**n*b + a)**(1/3),x)*d**2 + 2*int(x**n/(x**n*b + a)**(1/3),
x)*c*d + int(1/(x**n*b + a)**(1/3),x)*c**2
```

3.112 $\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}} \, dx = \frac{3dx(a+bx^n)^{2/3}}{b(3+2n)} + \frac{\left(c-\frac{3ad}{3b+2bn}\right) x \sqrt[3]{1+\frac{bx^n}{a}} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt[3]{a+bx^n}}$$

output
```
3*d*x*(a+b*x^n)^(2/3)/b/(3+2*n)+(c-3*a*d/(2*b*n+3*b))*x*(1+b*x^n/a)^(1/3)*
hypergeom([1/3, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}} \, dx = \frac{x\left(3d(a+bx^n)+(-3ad+bc(3+2n)) \sqrt[3]{1+\frac{bx^n}{a}} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(3+2n)\sqrt[3]{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n)^(1/3),x]
```

output
```
(x*(3*d*(a + b*x^n) + (-3*a*d + b*c*(3 + 2*n))*(1 + (b*x^n)/a)^(1/3)*Hyper
geometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(b*(3 + 2*n)*(a + b*
x^n)^(1/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{3ad}{2bn+3b}\right)\int \frac{1}{\sqrt[3]{bx^n+a}}dx+\frac{3dx(a+bx^n)^{2/3}}{b(2n+3)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}\left(c-\frac{3ad}{2bn+3b}\right)\int \frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}}dx}{\sqrt[3]{a+bx^n}}+\frac{3dx(a+bx^n)^{2/3}}{b(2n+3)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\left(c-\frac{3ad}{2bn+3b}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt[3]{a+bx^n}}+\frac{3dx(a+bx^n)^{2/3}}{b(2n+3)}$$

input
```
Int[(c + d*x^n)/(a + b*x^n)^(1/3),x]
```

output
```
(3*d*x*(a + b*x^n)^(2/3))/(b*(3 + 2*n)) + ((c - (3*a*d)/(3*b + 2*b*n))*x*(
1 + (b*x^n)/a)^(1/3)*Hypergeometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/
a)])/(a + b*x^n)^(1/3)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{c + d\,x^{n}}{\left(a + b\,x^{n}\right)^{\frac{1}{3}}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^(1/3),x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{1}{3}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{4}{3}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**(1/3),x)
```

output
```
a**(1/n)*a**(-1/3 - 1/n)*c*x*gamma(1/n)*hyper((1/3, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-4/3 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((1/3, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{dx^n+c}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(1/3), x)
```

Giac [F]

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{dx^n+c}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{c+d\,x^n}{(a+b\,x^n)^{1/3}}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^(1/3),x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^(1/3), x)
```

Reduce [F]

$$\int \frac{c+dx^n}{\sqrt[3]{a+bx^n}}\,dx = \left(\int \frac{x^n}{(x^n b+a)^{\frac{1}{3}}}dx\right)d + \left(\int \frac{1}{(x^n b+a)^{\frac{1}{3}}}dx\right)c$$

input

```
int((c+d*x^n)/(a+b*x^n)^(1/3),x)
```

output

```
int(x**n/(x**n*b + a)**(1/3),x)*d + int(1/(x**n*b + a)**(1/3),x)*c
```

3.113 $\int \frac{1}{\sqrt[3]{a+bx^n}(c+dx^n)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx=\frac{x\sqrt[3]{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{1}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt[3]{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/3)*AppellF1(1/n,1/3,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/(a+b*x^n
)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(61) = 122.

Time = 0.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx =$$
$$-\frac{3ac(1+n)x\operatorname{AppellF1}\left(\frac{1}{n},\frac{1}{3},1,1+\frac{1}{n},-\right.}{\sqrt[3]{a+bx^n}\,(c+dx^n)\left(3adnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+bcnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{4}{3},1\right.\right.}$$

input
```
Integrate[1/((a + b*x^n)^(1/3)*(c + d*x^n)),x]
```

output
```
(-3*a*c*(1 + n)*x*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*
x^n)/c)])/((a + b*x^n)^(1/3)*(c + d*x^n)*(3*a*d*n*x^n*AppellF1[1 + n^(-1),
 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1 +
n^(-1), 4/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*c*(1 + n)*Ap
pellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{937, 936\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}(dx^n+c)}dx}{\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\operatorname{AppellF1}\left(\frac{1}{n},\frac{1}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c\sqrt[3]{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(1/3)*(c + d*x^n)),x]
```

output
```
(x*(1 + (b*x^n)/a)^(1/3)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c*(a + b*x^n)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{\frac{1}{3}}\,(c+d\,x^{n})}dx$$

input
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(2/3)/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)**(1/3)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)**(1/3)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(1/3)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(1/3)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)^{1/3}\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)^(1/3)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^(1/3)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{1}{x^n\,(x^nb+a)^{\frac{1}{3}}\,d+(x^nb+a)^{\frac{1}{3}}\,c}dx$$

input
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n),x)
```

output
```
int(1/(x**n*(x**n*b + a)**(1/3)*d + (x**n*b + a)**(1/3)*c),x)
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^n}(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx=\frac{x\sqrt[3]{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt[3]{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/3)*AppellF1(1/n,1/3,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/(a+b*x
^n)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 391 vs. $2(61) = 122$.

Time = 0.87 (sec) , antiderivative size = 391, normalized size of antiderivative = 6.41

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx$$
$$=\frac{x\Bigg(\frac{bd(-3+n)x^n\sqrt[3]{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(-bc+ad)(1+n)}-\frac{3c\left(3ad^2nx^n(a+bx^n)\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+\cdots\right.}{(bc-ad)(c+dx^n)\left(3adnx^n\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},2,2+\frac{1}{n}\cdots\right.\right.}\cdots}{3c^2n\sqrt[3]{a+b\cdots}}$$

input
```
Integrate[1/((a + b*x^n)^(1/3)*(c + d*x^n)^2),x]
```

output
```
(x*((b*d*(-3 + n)*x^n*(1 + (b*x^n)/a)^(1/3)*AppellF1[1 + n^(-1), 1/3, 1, 2
 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/((-(b*c) + a*d)*(1 + n)) - (3*c*(3
*a*d^2*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n
)/a), -((d*x^n)/c)] + b*c*d*n*x^n*(a + b*x^n)*AppellF1[1 + n^(-1), 4/3, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*c*(1 + n)*(-(b*c*n) + a*d*n
 + b*d*x^n)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c
)]))/((b*c - a*d)*(c + d*x^n)*(3*a*d*n*x^n*AppellF1[1 + n^(-1), 1/3, 2, 2
+ n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 4/3
, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*c*(1 + n)*AppellF1[n^(-
1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))))/(3*c^2*n*(a + b*x^
n)^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}(dx^n+c)^2}dx}{\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2\sqrt[3]{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(1/3)*(c + d*x^n)^2),x]
```

output
```
(x*(1 + (b*x^n)/a)^(1/3)*AppellF1[n^(-1), 1/3, 2, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(c^2*(a + b*x^n)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{1}{3}}\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(2/3)/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2
*n) + (b*c^2 + 2*a*c*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx$$

input
```
integrate(1/(a+b*x**n)**(1/3)/(c+d*x**n)**2,x)
```

output
```
Integral(1/((a + b*x**n)**(1/3)*(c + d*x**n)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(1/3)*(d*x^n + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx=\int \frac{1}{(bx^n+a)^{\frac{1}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(1/3)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx=\int \frac{1}{(a+b\,x^n)^{1/3}\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)^(1/3)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^(1/3)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^n}\,(c+dx^n)^2}\,dx=\int \frac{1}{x^{2n}\,(x^n b+a)^{\frac{1}{3}}\,d^2+2x^n\,(x^n b+a)^{\frac{1}{3}}\,cd+(x^n b+a)^{\frac{1}{3}}\,c^2}dx$$

input
```
int(1/(a+b*x^n)^(1/3)/(c+d*x^n)^2,x)
```

output
```
int(1/(x**(2*n)*(x**n*b + a)**(1/3)*d**2 + 2*x**n*(x**n*b + a)**(1/3)*c*d
+ (x**n*b + a)**(1/3)*c**2),x)
```

3.115 $\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx = -\frac{3d\left(c-\frac{3ad(1+n)}{b(3+2n)}\right)x(a+bx^n)^{2/3}}{abn} + \frac{3(bc-ad)x(c+dx^n)}{abn\sqrt[3]{a+bx^n}} - \frac{(9a^2d^2(1+n)-6abcd(3+2n)+b^2c^2(9+3n-2n^2))\,x\sqrt[3]{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab^2n(3+2n)\sqrt[3]{a+bx^n}}$$

output
```
-3*d*(c-3*a*d*(1+n)/b/(3+2*n))*x*(a+b*x^n)^(2/3)/a/b/n+3*(-a*d+b*c)*x*(c+d
*x^n)/a/b/n/(a+b*x^n)^(1/3)-(9*a^2*d^2*(1+n)-6*a*b*c*d*(3+2*n)+b^2*c^2*(-2
*n^2+3*n+9))*x*(1+b*x^n/a)^(1/3)*hypergeom([1/3, 1/n],[1+1/n],-b*x^n/a)/a/
b^2/n/(3+2*n)/(a+b*x^n)^(1/3)
```

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx = \frac{3x((bc-ad)^2(3+2n)+ad^2n(a+bx^n))-(9a^2d^2(1+n)-6abcd(3+2n)+b^2c^2(9+}{ab^2n(3+2n)\sqrt[3]{a+bx}}$$

input
```
Integrate[(c + d*x^n)^2/(a + b*x^n)^(4/3),x]
```

output
```
(3*x*((b*c - a*d)^2*(3 + 2*n) + a*d^2*n*(a + b*x^n)) - (9*a^2*d^2*(1 + n)
- 6*a*b*c*d*(3 + 2*n) + b^2*c^2*(9 + 3*n - 2*n^2))*x*(1 + (b*x^n)/a)^(1/3)
*Hypergeometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b^2*n*(3 + 2
*n)*(a + b*x^n)^(1/3))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {930, 27, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}dx$$

$$\Big\downarrow \text{930}$$

$$\frac{3\int \frac{d(3ad(n+1)-bc(2n+3))x^n+c(3ad-bc(3-n))}{3\sqrt[3]{bx^n+a}}dx}{abn}+\frac{3x(bc-ad)(c+dx^n)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(3ad(n+1)-bc(2n+3))x^n+c(3ad-bc(3-n))}{\sqrt[3]{bx^n+a}}dx}{abn}+\frac{3x(bc-ad)(c+dx^n)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{913}$$

$$\frac{-\frac{\left(9a^2d^2(n+1)-6abcd(2n+3)+b^2c^2\left(-2n^2+3n+9\right)\right)\int \frac{1}{\sqrt[3]{bx^n+a}}dx}{b(2n+3)}-3dx(a+bx^n)^{2/3}\left(c-\frac{3ad(n+1)}{b(2n+3)}\right)}{abn}+\frac{3x(bc-ad)\left(c+dx^n\right)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{ 779}$$

$$\frac{-\frac{\sqrt[3]{\frac{bx^n}{a}+1}\left(9a^2d^2(n+1)-6abcd(2n+3)+b^2c^2\left(-2n^2+3n+9\right)\right)\int \frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}}dx}{b(2n+3)\sqrt[3]{a+bx^n}}-3dx(a+bx^n)^{2/3}\left(c-\frac{3ad(n+1)}{b(2n+3)}\right)}{abn}+\frac{3x(bc-ad)\left(c+dx^n\right)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{-\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\left(9a^2d^2(n+1)-6abcd(2n+3)+b^2c^2\left(-2n^2+3n+9\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{b(2n+3)\sqrt[3]{a+bx^n}}-3dx(a+bx^n)^{2/3}\left(c-\frac{3ad}{b(2\ldots}\right.}{abn}+\frac{3x(bc-ad)\left(c+dx^n\right)}{abn\sqrt[3]{a+bx^n}}$$

input
```
Int[(c + d*x^n)^2/(a + b*x^n)^(4/3),x]
```

output
```
(3*(b*c - a*d)*x*(c + d*x^n))/(a*b*n*(a + b*x^n)^(1/3)) + (-3*d*(c - (3*a*
d*(1 + n))/(b*(3 + 2*n)))*x*(a + b*x^n)^(2/3) - ((9*a^2*d^2*(1 + n) - 6*a*
b*c*d*(3 + 2*n) + b^2*c^2*(9 + 3*n - 2*n^2))*x*(1 + (b*x^n)/a)^(1/3)*Hyper
geometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(b*(3 + 2*n)*(a + b*x
^n)^(1/3)))/(a*b*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^{\frac{4}{3}}}dx$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(4/3),x)
```

output
```
int((c+d*x^n)^2/(a+b*x^n)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(4/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx = \int \frac{(c+dx^n)^2}{(a+bx^n)^{\frac{4}{3}}}\,dx$$

input
```
integrate((c+d*x**n)**2/(a+b*x**n)**(4/3),x)
```

output
```
Integral((c + d*x**n)**2/(a + b*x**n)**(4/3), x)
```

Maxima [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx = \int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx=\int \frac{(dx^n+c)^2}{(bx^n+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((c+d*x^n)^2/(a+b*x^n)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^2/(b*x^n + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx=\int \frac{(c+d\,x^n)^2}{(a+b\,x^n)^{4/3}}\,dx$$

input
```
int((c + d*x^n)^2/(a + b*x^n)^(4/3),x)
```

output
```
int((c + d*x^n)^2/(a + b*x^n)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^{4/3}}\,dx=\left(\int \frac{x^{2n}}{x^n\,(x^nb+a)^{\frac{1}{3}}\,b+(x^nb+a)^{\frac{1}{3}}\,a}dx\right)d^2+2\left(\int \frac{x^n}{x^n\,(x^nb+a)^{\frac{1}{3}}\,b+(x^nb+a)^{\frac{1}{3}}\,a}dx\right)cd+\left(\int \frac{1}{x^n\,(x^nb+a)^{\frac{1}{3}}\,b+(x^nb+a)^{\frac{1}{3}}\,a}dx\right)c^2$$

input
```
int((c+d*x^n)^2/(a+b*x^n)^(4/3),x)
```

output

```
int(x**(2*n)/(x**n*(x**n*b + a)**(1/3)*b + (x**n*b + a)**(1/3)*a),x)*d**2
+ 2*int(x**n/(x**n*(x**n*b + a)**(1/3)*b + (x**n*b + a)**(1/3)*a),x)*c*d +
 int(1/(x**n*(x**n*b + a)**(1/3)*b + (x**n*b + a)**(1/3)*a),x)*c**2
```

3.116 $\int \frac{c+dx^n}{(a+bx^n)^{4/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}} \, dx = \frac{3dx}{b(3-n)\sqrt[3]{a+bx^n}} + \frac{\left(\frac{c}{a}-\frac{3d}{b(3-n)}\right)x\sqrt[3]{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt[3]{a+bx^n}}$$

output
```
3*d*x/b/(3-n)/(a+b*x^n)^(1/3)+(c/a-3*d/b/(3-n))*x*(1+b*x^n/a)^(1/3)*hyperg
eom([4/3, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}} \, dx = \frac{-3adx+(3ad+bc(-3+n))x\sqrt[3]{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab(-3+n)\sqrt[3]{a+bx^n}}$$

input
```
Integrate[(c + d*x^n)/(a + b*x^n)^(4/3),x]
```

output
```
(-3*a*d*x + (3*a*d + b*c*(-3 + n))*x*(1 + (b*x^n)/a)^(1/3)*Hypergeometric2
F1[4/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b*(-3 + n)*(a + b*x^n)^(1/3)
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(3ad-bc(3-n))\int \frac{1}{\sqrt[3]{bx^n+a}}dx}{abn}+\frac{3x(bc-ad)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}(3ad-bc(3-n))\int \frac{1}{\sqrt[3]{\frac{bx^n}{a}+1}}dx}{abn\sqrt[3]{a+bx^n}}+\frac{3x(bc-ad)}{abn\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}(3ad-bc(3-n))\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{abn\sqrt[3]{a+bx^n}}+\frac{3x(bc-ad)}{abn\sqrt[3]{a+bx^n}}$$

input
```
Int[(c + d*x^n)/(a + b*x^n)^(4/3),x]
```

output
```
(3*(b*c - a*d)*x)/(a*b*n*(a + b*x^n)^(1/3)) + ((3*a*d - b*c*(3 - n))*x*(1
+ (b*x^n)/a)^(1/3)*Hypergeometric2F1[1/3, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b*n*(a + b*x^n)^(1/3))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{c+d\,x^{n}}{\left(a+b\,x^{n}\right)^{\frac{4}{3}}}dx$$

input
```
int((c+d*x^n)/(a+b*x^n)^(4/3),x)
```

output
```
int((c+d*x^n)/(a+b*x^n)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(4/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{4}{3}-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{7}{3}-\frac{1}{n}}a^{1+\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)/(a+b*x**n)**(4/3),x)
```

output
```
a**(1/n)*a**(-4/3 - 1/n)*c*x*gamma(1/n)*hyper((4/3, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(-7/3 - 1/n)*a**(1 + 1/n)*d*
x**(n + 1)*gamma(1 + 1/n)*hyper((4/3, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx = \int \frac{dx^n+c}{(bx^n+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(4/3), x)
```

Giac [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx=\int \frac{dx^n+c}{(bx^n+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((c+d*x^n)/(a+b*x^n)^(4/3),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/(b*x^n + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx=\int \frac{c+d\,x^n}{(a+b\,x^n)^{4/3}}\,dx$$

input
```
int((c + d*x^n)/(a + b*x^n)^(4/3),x)
```

output
```
int((c + d*x^n)/(a + b*x^n)^(4/3), x)
```

Reduce [F]

$$\int \frac{c+dx^n}{(a+bx^n)^{4/3}}\,dx=\left(\int \frac{x^n}{x^n\,(x^nb+a)^{\frac{1}{3}}\,b+(x^nb+a)^{\frac{1}{3}}\,a}dx\right)d+\left(\int \frac{1}{x^n\,(x^nb+a)^{\frac{1}{3}}\,b+(x^nb+a)^{\frac{1}{3}}\,a}dx\right)c$$

input
```
int((c+d*x^n)/(a+b*x^n)^(4/3),x)
```

output
```
int(x**n/(x**n*(x**n*b + a)**(1/3)*b + (x**n*b + a)**(1/3)*a),x)*d + int(1
/(x**n*(x**n*b + a)**(1/3)*b + (x**n*b + a)**(1/3)*a),x)*c
```

3.117 $\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx = \frac{x\sqrt[3]{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{4}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac\sqrt[3]{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/3)*AppellF1(1/n,4/3,1,1+1/n,-b*x^n/a,-d*x^n/c)/a/c/(a+b*x
^n)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 382 vs. 2(64) = 128.

Time = 0.85 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.97

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx =$$

$$-\frac{x\left(\frac{bd(-3+n)x^n\sqrt[3]{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(1+n)}+\frac{3\left(3abdnx^n(c+dx^n)\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+b^2\cdots\right.}{(c+dx^n)\left(3adnx^n\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{3},2,2+\frac{1}{n},-\frac{bx^n}{a}\cdots\right.\right.}\right.}{a(-bc+ad)n\sqrt[3]{a\cdots}}$$

input
```
Integrate[1/((a + b*x^n)^(4/3)*(c + d*x^n)),x]
```

output
```
-((x*((b*d*(-3 + n)*x^n*(1 + (b*x^n)/a)^(1/3)*AppellF1[1 + n^(-1), 1/3, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/(c*(1 + n)) + (3*(3*a*b*d*n*x^n*
(c + d*x^n)*AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)] + b^2*c*n*x^n*(c + d*x^n)*AppellF1[1 + n^(-1), 4/3, 1, 2 + n^(-1),
-((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*(a*d*n - b*(c*n + 3*d*x^n))*Appe
llF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))/((c + d*x^n
)*(3*a*d*n*x^n*AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d
*x^n)/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 4/3, 1, 2 + n^(-1), -((b*x^n)/a
), -((d*x^n)/c)] - 3*a*c*(1 + n)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b
*x^n)/a), -((d*x^n)/c)]))))/(a*(-(b*c) + a*d)*n*(a + b*x^n)^(1/3)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}dx$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{4/3}(dx^n+c)}dx}{a\sqrt[3]{a+bx^n}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\operatorname{AppellF1}\left(\frac{1}{n},\frac{4}{3},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac\sqrt[3]{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(4/3)*(c + d*x^n)),x]
```

output
```
(x*(1 + (b*x^n)/a)^(1/3)*AppellF1[n^(-1), 4/3, 1, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(a*c*(a + b*x^n)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{4}{3}}\left(c+d\,x^{n}\right)}dx$$

input
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(2/3)/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2
*n) + (2*a*b*c + a^2*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx=\int \frac{1}{(a+bx^n)^{\frac{4}{3}}(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)**(4/3)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)**(4/3)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(4/3)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(4/3)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)^{4/3}\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)^(4/3)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^(4/3)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)}\,dx = \int \frac{1}{x^{2n}(x^nb+a)^{\frac{1}{3}}bd + x^n(x^nb+a)^{\frac{1}{3}}ad + x^n(x^nb+a)^{\frac{1}{3}}bc + (x^nb+a)^{\frac{1}{3}}ac}$$

input
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n),x)
```

output
```
int(1/(x**(2*n)*(x**n*b + a)**(1/3)*b*d + x**n*(x**n*b + a)**(1/3)*a*d + x
**n*(x**n*b + a)**(1/3)*b*c + (x**n*b + a)**(1/3)*a*c),x)
```

3.118 $\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx$

Optimal result

Integrand size $=21$, antiderivative size $=64$

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx=\frac{x\sqrt[3]{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{4}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac^2\sqrt[3]{a+bx^n}}$$

output
```
x*(1+b*x^n/a)^(1/3)*AppellF1(1/n,4/3,2,1+1/n,-b*x^n/a,-d*x^n/c)/a/c^2/(a+b
*x^n)^(1/3)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1294 vs. $2(64)=128$.

Time $=1.10$ (sec), antiderivative size $=1294$, normalized size of antiderivative $=20.22$

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx=\text{Too large to display}$$

input
```
Integrate[1/((a + b*x^n)^(4/3)*(c + d*x^n)^2),x]
```

output
```
(x*(27*a*b^2*c^4*(1 + n)^2*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)] + 9*a^3*c^2*d^2*(1 + n)^2*AppellF1[n^(-1), 1/3, 1, 1 + n
^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 9*a*b^2*c^4*n*(1 + n)^2*AppellF1[n^(-
1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + 18*a^2*b*c^3*d*n*(1
+ n)^2*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] -
9*a^3*c^2*d^2*n*(1 + n)^2*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a
), -((d*x^n)/c)] + 3*c*(1 + n)*(a*d^2*(a + b*x^n) + 3*b^2*c*(c + d*x^n))*(
3*a*d*n*x^n*AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)] + b*c*n*x^n*AppellF1[1 + n^(-1), 4/3, 1, 2 + n^(-1), -((b*x^n)/a),
-((d*x^n)/c)] - 3*a*c*(1 + n)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)]) - 9*b^2*c*d*x^n*(1 + (b*x^n)/a)^(1/3)*(c + d*x^n)*Ap
pellF1[1 + n^(-1), 1/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]*(3*a*d*
n*x^n*AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]
 + b*c*n*x^n*AppellF1[1 + n^(-1), 4/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x
^n)/c)] - 3*a*c*(1 + n)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a),
 -((d*x^n)/c)]) - 3*a*b*d^2*x^n*(1 + (b*x^n)/a)^(1/3)*(c + d*x^n)*AppellF1
[1 + n^(-1), 1/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]*(3*a*d*n*x^n*
AppellF1[1 + n^(-1), 1/3, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + b*c
*n*x^n*AppellF1[1 + n^(-1), 4/3, 1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)
] - 3*a*c*(1 + n)*AppellF1[n^(-1), 1/3, 1, 1 + n^(-1), -((b*x^n)/a), -(...
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}dx$$
$$\Big\downarrow \ 937$$
$$\frac{\sqrt[3]{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{4/3}(dx^n+c)^2}dx}{a\sqrt[3]{a+bx^n}}$$
$$\Big\downarrow \ 936$$

$$\frac{x\sqrt[3]{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{4}{3},2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ac^2\sqrt[3]{a+bx^n}}$$

input
```
Int[1/((a + b*x^n)^(4/3)*(c + d*x^n)^2),x]
```

output
```
(x*(1 + (b*x^n)/a)^(1/3)*AppellF1[n^(-1), 4/3, 2, 1 + n^(-1), -((b*x^n)/a)
, -((d*x^n)/c)])/(a*c^2*(a + b*x^n)^(1/3))
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{4}{3}}\left(c+d\,x^{n}\right)^{2}}dx$$

input
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^(2/3)/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d
^2)*x^(3*n) + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c
*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx = \int \frac{1}{(a+bx^n)^{\frac{4}{3}}(c+dx^n)^2}\,dx$$

input
```
integrate(1/(a+b*x**n)**(4/3)/(c+d*x**n)**2,x)
```

output
```
Integral(1/((a + b*x**n)**(4/3)*(c + d*x**n)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{4/3}(c+dx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(4/3)*(d*x^n + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{4/3}\,(c+dx^n)^2}\,dx=\int \frac{1}{(bx^n+a)^{\frac{4}{3}}(dx^n+c)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(4/3)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^{4/3}\,(c+dx^n)^2}\,dx=\int \frac{1}{(a+b\,x^n)^{4/3}\,(c+d\,x^n)^2}\,dx$$

input
```
int(1/((a + b*x^n)^(4/3)*(c + d*x^n)^2),x)
```

output
```
int(1/((a + b*x^n)^(4/3)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{4/3}\,(c+dx^n)^2}\,dx=\int \frac{1}{x^{3n}(x^nb+a)^{\frac{1}{3}}\,b\,d^2+x^{2n}(x^nb+a)^{\frac{1}{3}}\,a\,d^2+2x^{2n}(x^nb+a)^{\frac{1}{3}}\,bcd+2x^{n}(x}$$

input
```
int(1/(a+b*x^n)^(4/3)/(c+d*x^n)^2,x)
```

output
```
int(1/(x**(3*n)*(x**n*b + a)**(1/3)*b*d**2 + x**(2*n)*(x**n*b + a)**(1/3)*
a*d**2 + 2*x**(2*n)*(x**n*b + a)**(1/3)*b*c*d + 2*x**n*(x**n*b + a)**(1/3)
*a*c*d + x**n*(x**n*b + a)**(1/3)*b*c**2 + (x**n*b + a)**(1/3)*a*c**2),x)
```

3.119 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 395

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \frac{d(a^2d^2(1+3n+2n^2)-3abcd(1+n)(1+n(3+p))+2b^2c^2(1+2n(3+p)+n^2(8+6p+p^2)))\,x(a+bx}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))} - \frac{d^2(ad(1+2n)-b(c+cn(5+p)))x^{1+n}(a+bx^n)^{1+p}}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} - \frac{(a^3d^3(1+3n+2n^2)-3a^2bcd^2(1+n)(1+n(3+p))+3ab^2c^2d(1+n(5+2p)+n^2(6+5p+p^2))-b}{b^3(1+}$$

output
```
d*(a^2*d^2*(2*n^2+3*n+1)-3*a*b*c*d*(1+n)*(1+n*(3+p))+2*b^2*c^2*(1+2*n*(3+p
)+n^2*(p^2+6*p+8)))*x*(a+b*x^n)^(p+1)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p)
)-d^2*(a*d*(1+2*n)-b*(c+c*n*(5+p)))*x^(1+n)*(a+b*x^n)^(p+1)/b^2/(1+n*(2+p)
)/(1+n*(3+p))+d*x*(a+b*x^n)^(p+1)*(c+d*x^n)^2/b/(1+n*(3+p))-(a^3*d^3*(2*n^
2+3*n+1)-3*a^2*b*c*d^2*(1+n)*(1+n*(3+p))+3*a*b^2*c^2*d*(1+n*(5+2*p)+n^2*(p
^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11)+n^3*(p^3+6*p^2+11*p+6)
))*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/(1+n*
(2+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.43

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{3c^2dx^n \operatorname{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+n} + \frac{3cd^2x^{2n} \operatorname{Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+2n} + \frac{d^3x^{3n} \operatorname{Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+3n} + c^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1
), -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
 -p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1
[3 + n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric
2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^3 (a+bx^n)^p \, dx$$

$$\Big\downarrow \ 933$$

$$\frac{\int -(bx^n+a)^p (dx^n+c) (d(ad(2n+1)-b(n(p+5)c+c))x^n+c(ad-b(n(p+3)c+c)))\,dx}{b(n(p+3)+1)}+\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}$$

$\downarrow$ 25

$$\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}-\frac{\int (bx^n+a)^p (dx^n+c) (d(ad(2n+1)-b(n(p+5)c+c))x^n+c(ad-b(n(p+3)c+c)))\,dx}{b(n(p+3)+1)}$$

$\downarrow$ 1025

$$\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{\int -(bx^n+a)^p (d(b^2((p^2+6p+11)n^2+2(p+3)n+1)c^2-abd((p+7)n^2+(2p+9)n+2)c+a^2d^2(2n^2+3n+1))x^n+c(b^2((p^2+5p+6)n^2+(2p+5)n+1)c}{b(n(p+2)+1)}}{b(n(p+3)+1)}$$

$\downarrow$ 25

$$\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\int (bx^n+a)^p (d(b^2((p^2+6p+11)n^2+2(p+3)n+1)c^2-abd((p+7)n^2+(2p+9)n+2)c+a^2d^2(}{b(n(}}{b(n(p+3)+1)}$$

$\downarrow$ 913

$$\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\frac{dx(a+bx^n)^{p+1}(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3}{b(np+n+1)}}{}}{b(}$$

$\downarrow$ 779

$$\frac{dx(c+dx^n)^2 (a+bx^n)^{p+1}}{b(n(p+3)+1)}-$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)}-\frac{\frac{dx(a+bx^n)^{p+1}(a^2d^2(2n^2+3n+1)-abcd(n^2(p+7)+n(2p+9)+2)+b^2c^2(n^2(p^2+6p+11)+2n(p+3}{b(np+n+1)}}{}$$

$\downarrow$ 778

$$\frac{dx(c+dx^n)^2(a+bx^n)^{p+1}}{b(n(p+3)+1)} -$$

$$\frac{\frac{dx(c+dx^n)(a+bx^n)^{p+1}(ad(2n+1)-b(cn(p+5)+c))}{b(n(p+2)+1)} - \frac{dx(a+bx^n)^{p+1}\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(n^2(p+7)+n(2p+9)+2\right)+b^2c^2\left(n^2\left(p^2+6p+11\right)+2n(p+3\right.\right.}{b(np+n+1)}}{}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))) - ((d*(a*d*(1
 + 2*n) - b*(c + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n
*(9 + 2*p)) + b^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x
^n)^(1 + p))/(b*(1 + n + n*p)) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d
^2*(1 + n)*(1 + n*(3 + p)) + 3*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p
 + p^2)) - b^3*c^3*(1 + 3*n*(2 + p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 1
1*p + 6*p^2 + p^3)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(
-1), -((b*x^n)/a)])/(b*(1 + n + n*p)*(1 + (b*x^n)/a)^p))/(b*(1 + n*(2 + p)
)))/(b*(1 + n*(3 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int (a + b\,x^n)^p\,(c + d\,x^n)^3\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (dx^n+c)^3 (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 91.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.62

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \frac{a^{\frac{1}{n}} a^{p-\frac{1}{n}} c^3 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^{1+\frac{1}{n}} a^{p-1-\frac{1}{n}} c^2 d x^{n+1} \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{3a^{2+\frac{1}{n}} a^{p-2-\frac{1}{n}} c d^2 x^{2n+1} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{a^{3+\frac{1}{n}} a^{p-3-\frac{1}{n}} d^3 x^{3n+1} \Gamma\left(3+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 3+\frac{1}{n} \\ 4+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
a**(1/n)*a**(p - 1/n)*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 3*a**(1 + 1/n)*a**(p - 1 - 1/n)*
c**2*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**(2 + 1/n)*a**(p - 2 - 1/n)*c*d
**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**(3 + 1/n)*a**(p - 3 - 1/n)*d**3*x
**(3*n + 1)*gamma(3 + 1/n)*hyper((-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(4 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (dx^n+c)^3(bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,3,4,3,0]%%%}+%%
%{6,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n)^3\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^3, x)
```

Reduce [F]

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

output

```
(x**(3*n)*(x**n*b + a)**p*b**3*d**3*n**3*p**3*x + 3*x**(3*n)*(x**n*b + a)*
*p*b**3*d**3*n**3*p**2*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*d**3*n**3*p*x +
 3*x**(3*n)*(x**n*b + a)**p*b**3*d**3*n**2*p**2*x + 6*x**(3*n)*(x**n*b + a
)**p*b**3*d**3*n**2*p*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*d**3*n**2*x + 3*
x**(3*n)*(x**n*b + a)**p*b**3*d**3*n*p*x + 3*x**(3*n)*(x**n*b + a)**p*b**3
*d**3*n*x + x**(3*n)*(x**n*b + a)**p*b**3*d**3*x + x**(2*n)*(x**n*b + a)**
p*a*b**2*d**3*n**3*p**3*x + x**(2*n)*(x**n*b + a)**p*a*b**2*d**3*n**3*p**2
*x + 2*x**(2*n)*(x**n*b + a)**p*a*b**2*d**3*n**2*p**2*x + x**(2*n)*(x**n*b
 + a)**p*a*b**2*d**3*n**2*p*x + x**(2*n)*(x**n*b + a)**p*a*b**2*d**3*n*p*x
 + 3*x**(2*n)*(x**n*b + a)**p*b**3*c*d**2*n**3*p**3*x + 12*x**(2*n)*(x**n*
b + a)**p*b**3*c*d**2*n**3*p**2*x + 9*x**(2*n)*(x**n*b + a)**p*b**3*c*d**2
*n**3*p*x + 9*x**(2*n)*(x**n*b + a)**p*b**3*c*d**2*n**2*p**2*x + 24*x**(2*
n)*(x**n*b + a)**p*b**3*c*d**2*n**2*p*x + 9*x**(2*n)*(x**n*b + a)**p*b**3*
c*d**2*n**2*x + 9*x**(2*n)*(x**n*b + a)**p*b**3*c*d**2*n*p*x + 12*x**(2*n)
*(x**n*b + a)**p*b**3*c*d**2*n*x + 3*x**(2*n)*(x**n*b + a)**p*b**3*c*d**2*
x - 2*x**n*(x**n*b + a)**p*a**2*b*d**3*n**3*p**2*x - x**n*(x**n*b + a)**p*
a**2*b*d**3*n**2*p**2*x - 2*x**n*(x**n*b + a)**p*a**2*b*d**3*n**2*p*x - x*
*n*(x**n*b + a)**p*a**2*b*d**3*n*p*x + 3*x**n*(x**n*b + a)**p*a*b**2*c*d**
2*n**3*p**3*x + 9*x**n*(x**n*b + a)**p*a*b**2*c*d**2*n**3*p**2*x + 6*x**n*
(x**n*b + a)**p*a*b**2*c*d**2*n**2*p**2*x + 9*x**n*(x**n*b + a)**p*a*b*...
```

3.120 $\int (a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{d(ad(1+n)-b(c+cn(3+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(ad-b(c+cn(2+p))) - \frac{ad(ad(1+n)-b(c+cn(3+p)))}{b(1+n+np)}\right) x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},\right.}{b(1+n(2+p))}$$

output
```
-d*(a*d*(1+n)-b*(c+c*n*(3+p)))*x*(a+b*x^n)^(p+1)/b^2/(n*p+n+1)/(1+n*(2+p))
+d*x*(a+b*x^n)^(p+1)*(c+d*x^n)/b/(1+n*(2+p))-(c*(a*d-b*(c+c*n*(2+p)))-a*d*
(a*d*(1+n)-b*(c+c*n*(3+p)))/b/(n*p+n+1))*x*(a+b*x^n)^p*hypergeom([-p, 1/n]
,[1+1/n],-b*x^n/a)/b/(1+n*(2+p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.71

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx$$
$$= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(2cd(1+2n)x^n \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)+(1+n)\left(d^2x^{2n}\right.\right.}{(1+n)(}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2
+ n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1
), -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1),
 -p, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^2 (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{ 933}$$

$$\frac{\int -(bx^n+a)^p \left(d(ad(n+1)-b(n(p+3)c+c))x^n+c(ad-b(n(p+2)c+c))\right)dx}{b(n(p+2)+1)} + \frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{\int (bx^n+a)^p (d(ad(n+1)-b(n(p+3)c+c))x^n + c(ad-b(n(p+2)c+c)))\,dx}{b(n(p+2)+1)}$$

$\Big\downarrow$ 913

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)} + acd - bc(cn(p+2)+c)\right)\int (bx^n+a)^p\,dx + \frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}}{b(n(p+2)+1)}$$

$\Big\downarrow$ 779

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)} + acd - bc(cn(p+2)+c)\right)\int \left(\frac{bx^n}{a}+1\right)^p dx + \frac{dx(a+bx^n)^{p+1}(ad(}{b(np}}{b(n(p+2)+1)}$$

$\Big\downarrow$ 778

$$\frac{dx(c+dx^n)(a+bx^n)^{p+1}}{b(n(p+2)+1)} - \frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-\frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)} + acd - bc(cn(p+2)+c)\right)\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\right.}{b(n(p+2)+1)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n*(2 + p))) - ((d*(a*d*(1 +
n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((a*c
*d - b*c*(c + c*n*(2 + p)) - (a*d*(a*d*(1 + n) - b*(c + c*n*(3 + p))))/(b*
(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
-((b*x^n)/a)])/(1 + (b*x^n)/a)^p)/(b*(1 + n*(2 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(a +b \,x^{n}\right)^{p} \left(c +d \,x^{n}\right)^{2}d x$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (dx^n+c)^2 (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.71 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \frac{a^{\frac{1}{n}} a^{p-\frac{1}{n}} c^2 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^{1+\frac{1}{n}} a^{p-1-\frac{1}{n}} c d x^{n+1} \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{a^{2+\frac{1}{n}} a^{p-2-\frac{1}{n}} d^2 x^{2n+1} \Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
a**(1/n)*a**(p - 1/n)*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 2*a**(1 + 1/n)*a**(p - 1 - 1/n)*
c*d*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**(2 + 1/n)*a**(p - 2 - 1/n)*d**2*x**
(2*n + 1)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \int (dx^n+c)^2 (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p\,(c+dx^n)^2\;dx=\int (a+b\,x^n)^p\,(c+d\,x^n)^2\,dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^2, x)
```

Reduce [F]

$$\int (a+bx^n)^p\,(c+dx^n)^2\;dx=\text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b**2*d**2*n**2*p**2*x + x**(2*n)*(x**n*b + a)**p
*b**2*d**2*n**2*p*x + 2*x**(2*n)*(x**n*b + a)**p*b**2*d**2*n*p*x + x**(2*n
)*(x**n*b + a)**p*b**2*d**2*n*x + x**(2*n)*(x**n*b + a)**p*b**2*d**2*x + x
**n*(x**n*b + a)**p*a*b*d**2*n**2*p**2*x + x**n*(x**n*b + a)**p*a*b*d**2*n
*p*x + 2*x**n*(x**n*b + a)**p*b**2*c*d*n**2*p**2*x + 4*x**n*(x**n*b + a)**
p*b**2*c*d*n**2*p*x + 4*x**n*(x**n*b + a)**p*b**2*c*d*n*p*x + 4*x**n*(x**n
*b + a)**p*b**2*c*d*n*x + 2*x**n*(x**n*b + a)**p*b**2*c*d*x - (x**n*b + a)
**p*a**2*d**2*n**2*p*x - (x**n*b + a)**p*a**2*d**2*n*p*x + 2*(x**n*b + a)*
*p*a*b*c*d*n**2*p**2*x + 4*(x**n*b + a)**p*a*b*c*d*n**2*p*x + 2*(x**n*b +
a)**p*a*b*c*d*n*p*x + (x**n*b + a)**p*b**2*c**2*n**2*p**2*x + 3*(x**n*b +
a)**p*b**2*c**2*n**2*p*x + 2*(x**n*b + a)**p*b**2*c**2*n**2*x + 2*(x**n*b
+ a)**p*b**2*c**2*n*p*x + 3*(x**n*b + a)**p*b**2*c**2*n*x + (x**n*b + a)**
p*b**2*c**2*x + int((x**n*b + a)**p/(x**n*b*n**3*p**3 + 3*x**n*b*n**3*p**2
 + 2*x**n*b*n**3*p + 3*x**n*b*n**2*p**2 + 6*x**n*b*n**2*p + 2*x**n*b*n**2
+ 3*x**n*b*n*p + 3*x**n*b*n + x**n*b + a*n**3*p**3 + 3*a*n**3*p**2 + 2*a*n
**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n**2 + 3*a*n*p + 3*a*n + a),x)*a*
*3*d**2*n**5*p**4 + 3*int((x**n*b + a)**p/(x**n*b*n**3*p**3 + 3*x**n*b*n**
3*p**2 + 2*x**n*b*n**3*p + 3*x**n*b*n**2*p**2 + 6*x**n*b*n**2*p + 2*x**n*b
*n**2 + 3*x**n*b*n*p + 3*x**n*b*n + x**n*b + a*n**3*p**3 + 3*a*n**3*p**2 +
 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n**2 + 3*a*n*p + 3*a*n +...
```

3.121 $\int (a+bx^n)^p (c+dx^n) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int (a+bx^n)^p (c+dx^n) \, dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c - \frac{ad}{b+bn+bnp}\right) x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)$$

output
```
d*x*(a+b*x^n)^(p+1)/b/(n*p+n+1)+(c-a*d/(b*n*p+b*n+b))*x*(a+b*x^n)^p*hyperg
eom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int (a+bx^n)^p (c+dx^n) \, dx$$
$$= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(d(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p + (-ad+bc(1+n+np)) \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1\right.\right.}{b(1+n+np)}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
 n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n
 + n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{bnp+bn+b}\right)\int (bx^n+a)^p\,dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\int \left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p
))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]
)/(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int (a+bx^n)^p (c+dx^n)\,dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}cx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}dx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(1/n)*a**(p - 1/n)*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(1 + 1/n)*a**(p - 1 - 1/n)*d*x**
(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)\,dx = \int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p\,(c+dx^n)\;dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p\,(c+dx^n)\;dx = \int (a+b\,x^n)^p\,(c+d\,x^n)\;dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (a+bx^n)^p\,(c+dx^n)\;dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**n*(x**n*b + a)**p*b*d*n*p*x + x**n*(x**n*b + a)**p*b*d*x + (x**n*b + a
)**p*a*d*n*p*x + (x**n*b + a)**p*b*c*n*p*x + (x**n*b + a)**p*b*c*n*x + (x*
*n*b + a)**p*b*c*x - int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p
 + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a
*n + a),x)*a**2*d*n**3*p**3 - int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n
*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*
a*n*p + a*n + a),x)*a**2*d*n**3*p**2 - 2*int((x**n*b + a)**p/(x**n*b*n**2*
p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*
n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n**2*p**2 - int((x**n*b + a)**p/(x**
n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*
p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n**2*p - int((x**n*b + a)**
p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a
*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n*p + int((x**n*b + a
)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b
+ a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n**4*p**4 + 2*int((
x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n
 + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n**4*p**3
 + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p +
x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n
**4*p**2 + 3*int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*...
```

3.122 $\int (a+bx^n)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Integrate[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p\, dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Int[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^n)^p\,dx$$

input
```
int((a+b*x^n)^p,x)
```

output
```
int((a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a+bx^n)^p\;dx = \int (bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p\;dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p,x)
```

output
```
a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p, x)
```

Giac [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int (a+bx^n)^p \, dx = \frac{x\,(a+b\,x^n)^p\;{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\;\frac{1}{n}+1;\;-\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

input
```
int((a + b*x^n)^p,x)
```

output
```
(x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1
)^p
```

Reduce [F]

$$\int (a+bx^n)^p\,dx$$
$$=\frac{(x^n b+a)^p x+\left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right)a\,n^2p^2+\left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right)anp}{np+1}$$

input
```
int((a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*x + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a)
,x)*a*n**2*p**2 + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a),x)
*a*n*p)/(n*p + 1)
```

3.123 $\int \frac{(a+bx^n)^p}{c+dx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{c+dx^n} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,1,1+1/n,-b*x^n/a,-d*x^n/c)/c/((1+b*x^n/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.

Time = 0.49 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{c+dx^n} \, dx = \frac{ac(1+n)x(a+bx^n)^p \text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},\right.}{(c+dx^n)\left(bcnpx^n \text{AppellF1}\left(1+\frac{1}{n},1-p,1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^n \text{AppellF1}\left(1+\frac{1}{n},-p,2,2+\frac{1}{n}\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
1, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1)
, -p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \; 936$$

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(x^nb+a)^p}{x^nd+c}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((x**n*b + a)**p/(x**n*d + c),x)
```

3.124 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,2,1+1/n,-b*x^n/a,-d*x^n/c)/c^2/((1+b*x^n/a)^
p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. $2(59) = 118$.

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$$

$$= \frac{ac(1+n)x(a+bx^n)^p \text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},\right.}{(c+dx^n)^2 \left(bcnpx^n \text{AppellF1}\left(1+\frac{1}{n},1-p,2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^n \text{AppellF1}\left(1+\frac{1}{n},-p,3,2+\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^
(-1), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF
1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx$$

$$\Big\downarrow \ 937$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \ 936$$

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{n},-p,2,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^2}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx=\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^2,x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx=\int \frac{(x^nb+a)^p}{x^{2n}d^2+2x^ncd+c^2}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int((x**n*b + a)**p/(x**(2*n)*d**2 + 2*x**n*c*d + c**2),x)
```

3.125 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

output
```
x*(a+b*x^n)^p*AppellF1(1/n,-p,3,1+1/n,-b*x^n/a,-d*x^n/c)/c^3/((1+b*x^n/a)^
p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(59) = 118.

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx = \frac{ac(1+n)x(a+bx^n)^p \text{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},\right.}{(c+dx^n)^3 \left(bcnpx^n \text{AppellF1}\left(1+\frac{1}{n},1-p,3,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-3adnx^n \text{AppellF1}\left(1+\frac{1}{n},-p,4,2+\right.\right.}$$

input
```
Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)
/a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^
(-1), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF
1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \; 936$$

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{n},-p,3,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

output
```
(x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^
n)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)
, x)
```

Sympy [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

input
```
integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p}{(dx^n+c)^3}\,dx$$

input
```
integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^3,x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(x^nb+a)^p}{x^{3n}d^3+3x^{2n}c\,d^2+3x^nc^2d+c^3}dx$$

input
```
int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int((x**n*b + a)**p/(x**(3*n)*d**3 + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d + c
**3),x)
```

3.126 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

output
```
x*(a+b*x^n)^p*(c+d*x^n)^(-1/n-p)*hypergeom([-p, 1/n],[1+1/n],-(-a*d+b*c)*x
^n/a/(c+d*x^n))/c/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^{-\frac{1+np}{n}} \left(1+\frac{dx^n}{c}\right)^p \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{c}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

output
```
(x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1
), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)
^((1 + n*p)/n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p-1} \, dx$$

$$\Big\downarrow \; 905$$

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

output
```
(x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(
c + d*x^n)))^p)
```

Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^{-1-\frac{1}{n}-p}\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^{-1-\frac{1}{n}-p}\,dx=\int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-1}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p\,(c+dx^n)^{-1-\frac{1}{n}-p}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p} \, dx = \int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+1}} \, dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 1), x)
```

Reduce [F]

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \int \frac{(x^nb+a)^p}{x^n(x^nd+c)^{\frac{np+1}{n}}d+(x^nd+c)^{\frac{np+1}{n}}c}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

output
```
int((x**n*b + a)**p/(x**n*(x**n*d + c)**((n*p + 1)/n)*d + (x**n*d + c)**((
n*p + 1)/n)*c),x)
```

3.127 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2 (1+5n+6n^2)} - \frac{6a^2(bc-ad)n^2x(c+dx^n)^{-1-\frac{1}{n}}}{c^3d(1+n)(1+2n)(1+3n)} + \frac{6a^2n^2(bc+adn)x(c+dx^n)^{-1/n}}{c^4d(1+n)(1+2n)(1+3n)}$$

output
```
x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^(
-2-1/n)/c^2/(6*n^2+5*n+1)-6*a^2*(-a*d+b*c)*n^2*x*(c+d*x^n)^(-1-1/n)/c^3/d/
(1+n)/(1+2*n)/(1+3*n)+6*a^2*n^2*(a*d*n+b*c)*x/c^4/d/(1+n)/(1+2*n)/(1+3*n)/
((c+d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx$$
$$= \frac{x(c+dx^n)^{-3-\frac{1}{n}} \left(b^3c^3(1+3n+2n^2)\, x^{3n} + 3ab^2c^2(1+n)x^{2n}(c+3cn+dnx^n) + 3a^2bcx^n(c^2(1+5n+6n\right.}{c^4}$$

input
```
Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*
c^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n +
6*n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n +
11*n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*
x^(2*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-4}\, dx$$

$$\Big\downarrow \text{ 903}$$

$$\frac{3an\int (bx^n+a)^2 (dx^n+c)^{-3-\frac{1}{n}}dx}{c(3n+1)} + \frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{ 903}$$

$$\frac{3an\left(\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)} + \frac{x(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{903}$$

$$\frac{3an\left(\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{3an\left(\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

input
```
Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*((x*(a
+ b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*((x*(a + b*x^
n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c + d*x^
n)^n^(-1))))/(c*(1 + 2*n))))/(c*(1 + 3*n))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(193) = 386$.

Time = 1.97 (sec) , antiderivative size = 1290, normalized size of antiderivative = 6.68

method	result	size
parallelrisch	Expression too large to display	1290

input
```
int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

output
```
(2*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d*n^2+24*x*(x^n)^3*(c+d*x^n)^(
-(1+4*n)/n)*a^3*c*d^3*n^3+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^3*d*n+6
*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^3*c*d^3*n^2+36*x*(x^n)^2*(c+d*x^n)^(-(
1+4*n)/n)*a^3*c^2*d^2*n^3+21*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d^2*
n^2+9*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^4*n^2+24*x*x^n*(c+d*x^n)^(-
(1+4*n)/n)*a^3*c^3*d*n^3+3*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^3*d+3*
x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^2*d^2*n+12*x*(x^n)^2*(c+d*x^n)^(-(1
+4*n)/n)*a*b^2*c^4*n+26*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^3*d*n^2+6*x*(x^
n)^4*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c*d^3*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n
)/n)*a*b^2*c^2*d^2*n^2+3*x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^2*d^2*n+
24*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^2*d^2*n^2+12*x*(x^n)^3*(c+d*x^
n)^(-(1+4*n)/n)*a*b^2*c^3*d*n^2+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c
^2*d^2*n+15*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b^2*c^3*d*n+6*x*(x^n)^4*(c+
d*x^n)^(-(1+4*n)/n)*a^3*d^4*n^3+2*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^4
*n^2+36*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^3*d*n^2+21*x*(x^n)^2*(c+d
*x^n)^(-(1+4*n)/n)*a^2*b*c^3*d*n+18*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^4
*n^2+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^3*d+9*x*x^n*(c+d*x^n)^(-(1
+4*n)/n)*a^3*c^3*d*n+15*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*b*c^4*n+x*(c+d*x^
n)^(-(1+4*n)/n)*a^3*c^4+x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^3*c^4+6*x*(c+d*
x^n)^(-(1+4*n)/n)*a^3*c^4*n^3+11*x*(c+d*x^n)^(-(1+4*n)/n)*a^3*c^4*n^2+6...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(193) = 386.

Time = 0.13 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.48

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{(6a^3d^4n^3+b^3c^3d+(2b^3c^3d+3ab^2c^2d^2+6a^2bcd^3)n^2+3(b^3c^3d+ab^2c^2d^2)n)xx^{4n}+(24a^3cd^3n^3+b^3c^4}{}$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

output
```
((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d
^3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 +
 b^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 +
 3*a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(
3*n) + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12
*a^2*b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2
*d^2)*n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^
2*b*c^4 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*
a^3*c^4*n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*
c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2822 vs. 2(168) = 336.

Time = 47.16 (sec) , antiderivative size = 2822, normalized size of antiderivative = 14.62

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

output
```
6*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
1)**(1/n)*gamma(4 + 1/n)) + 11*a**3*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma
(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*
d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**
(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)
*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**3*c**3*c**(1
/n)*c**(-4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)
*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamm
a(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma
(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
1/n)) + a**3*c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c
/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**3*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x*
*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3...
```

Maxima [F]

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx=\int (bx^n+a)^3(dx^n+c)^{-\frac{1}{n}-4}\,dx$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%
{54,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}} dx = \int \frac{(a+b\,x^n)^3}{(c+d\,x^n)^{\frac{1}{n}+4}} dx$$

input
```
int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
```

output
```
int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

Reduce [F]

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$$
$$= \left(\int \frac{x^{3n}}{x^{4n}(x^nd+c)^{\frac{1}{n}}d^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}cd^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}c^2d^2+4x^n(x^nd+c)^{\frac{1}{n}}c^3d+(x^nd+c)^{\frac{1}{n}}\right.$$
$$+3\left(\int \frac{x^{2n}}{x^{4n}(x^nd+c)^{\frac{1}{n}}d^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}cd^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}c^2d^2+4x^n(x^nd+c)^{\frac{1}{n}}c^3d+(x^nd+\right.$$
$$+3\left(\int \frac{x^{n}}{x^{4n}(x^nd+c)^{\frac{1}{n}}d^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}cd^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}c^2d^2+4x^n(x^nd+c)^{\frac{1}{n}}c^3d+(x^nd+\right.$$
$$+\left(\int \frac{1}{x^{4n}(x^nd+c)^{\frac{1}{n}}d^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}cd^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}c^2d^2+4x^n(x^nd+c)^{\frac{1}{n}}c^3d+(x^nd+c\right.$$

input
```
int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

output
```
int(x**(3*n)/(x**(4*n)*(x**n*d + c)**(1/n)*d**4 + 4*x**(3*n)*(x**n*d + c)*
*(1/n)*c*d**3 + 6*x**(2*n)*(x**n*d + c)**(1/n)*c**2*d**2 + 4*x**n*(x**n*d
+ c)**(1/n)*c**3*d + (x**n*d + c)**(1/n)*c**4),x)*b**3 + 3*int(x**(2*n)/(x
**(4*n)*(x**n*d + c)**(1/n)*d**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*c*d**3 +
 6*x**(2*n)*(x**n*d + c)**(1/n)*c**2*d**2 + 4*x**n*(x**n*d + c)**(1/n)*c**
3*d + (x**n*d + c)**(1/n)*c**4),x)*a*b**2 + 3*int(x**n/(x**(4*n)*(x**n*d +
 c)**(1/n)*d**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*c*d**3 + 6*x**(2*n)*(x**n
*d + c)**(1/n)*c**2*d**2 + 4*x**n*(x**n*d + c)**(1/n)*c**3*d + (x**n*d + c
)**(1/n)*c**4),x)*a**2*b + int(1/(x**(4*n)*(x**n*d + c)**(1/n)*d**4 + 4*x*
*(3*n)*(x**n*d + c)**(1/n)*c*d**3 + 6*x**(2*n)*(x**n*d + c)**(1/n)*c**2*d*
*2 + 4*x**n*(x**n*d + c)**(1/n)*c**3*d + (x**n*d + c)**(1/n)*c**4),x)*a**3
```

3.128 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 127

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)} - \frac{2a(bc-ad)nx(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{2an(bc+adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}$$

output
```
x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c/(1+2*n)-2*a*(-a*d+b*c)*n*x*(c+d*x^n)^(-
1-1/n)/c^2/d/(1+n)/(1+2*n)+2*a*n*(a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x
^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \frac{x(c+dx^n)^{-2-\frac{1}{n}} (b^2c^2(1+n)x^{2n} + 2abcx^n(c+2cn+dnx^n) + a^2(c^2(1+3n+2n^2) + 2cdn(1+2n)x^n +}{c^3(1+n)(1+2n)}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2
*c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d
^2*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-3} \, dx$$

$$\Big\downarrow \text{ 903}$$

$$\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

$$\Big\downarrow \text{ 903}$$

$$\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*((x*(a
+ b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c
+ d*x^n)^n^(-1))))/(c*(1 + 2*n))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(127) = 254$.

Time = 1.35 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.63

method	result
parallelrisch	$\frac{2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}\,a^{2}d^{3}n^{2}+2x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}\,abc\,d^{2}n+x\,x^{3n}(c+d\,x^{n})^{-\frac{1+3n}{n}}\,b^{2}c^{2}dn+6x\,x^{2n}(c+d\,x^{n})^{-\frac{1+3n}{n}}\,a^{2}c\,d^{2}}{}$

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)
```

output
```
(2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a^2*d^3*n^2+2*x*(x^n)^3*(c+d*x^n)^(-(1
+3*n)/n)*a*b*c*d^2*n+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^2*d*n+6*x*(x^n
)^2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*
b^2*c^2*d+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a^2*c*d^2*n+6*x*(x^n)^2*(c+d*
x^n)^(-(1+3*n)/n)*a*b*c^2*d*n+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3*n+6
*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d*n^2+2*x*(x^n)^2*(c+d*x^n)^(-(1+3*n
)/n)*a*b*c^2*d+x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b^2*c^3+5*x*x^n*(c+d*x^n)^
(-(1+3*n)/n)*a^2*c^2*d*n+4*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*b*c^3*n+2*x*(c+d
*x^n)^(-(1+3*n)/n)*a^2*c^3*n^2+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^2*d+2*x*
x^n*(c+d*x^n)^(-(1+3*n)/n)*a*b*c^3+3*x*(c+d*x^n)^(-(1+3*n)/n)*a^2*c^3*n+x*
(c+d*x^n)^(-(1+3*n)/n)*a^2*c^3)/(1+n)/(1+2*n)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.82

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\, dx = \frac{(2\,a^2d^3n^2 + b^2c^2d + (b^2c^2d + 2\,abcd^2)n)xx^{3\,n} + (6\,a^2cd^2n^2 + b^2c^3 + 2\,abc^2d + (b^2c^3 + 6\,abc^2d + 2\,a^2cd^2)n}{(2\,c^3n^2 + 3\,c^3n + c^3)(}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

output
```
((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*
a^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d
^2)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 +
 5*a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^
3*n^2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(109) = 218$.

Time = 15.86 (sec) , antiderivative size = 1035, normalized size of antiderivative = 8.15

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

output
```
2*a**2*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3
 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a**
2*c**2*c**(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1
/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)
+ d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a**2*c**2*c**
(1/n)*c**(-3 - 2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3
+ 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*
x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a**2*c*c**(1/n)*c**(-3
- 2/n)*d*n**2*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 +
 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x
**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c*c**(1/n)*c**(-3 -
 2/n)*d*n*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n
) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2
*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a**2*c**(1/n)*c**(-3 - 2/n)*
d**2*n**2*x*x**(2*n)*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 +
 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x
**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*b*c*c**(-3 - 1/n)*c**(
1 + 1/n)*n*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**
2*gamma(3 + 1/n) + d*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + 2*a*b*c*c...
```

Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \int (bx^n+a)^2 (dx^n+c)^{-\frac{1}{n}-3}\,dx$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6
,[1,0,4,1
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+3}}\,dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

Reduce [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\, dx$$
$$= \left(\int \frac{x^{2n}}{x^{3n} (x^n d+c)^{\frac{1}{n}} d^3+3x^{2n} (x^n d+c)^{\frac{1}{n}} c\, d^2+3x^n (x^n d+c)^{\frac{1}{n}} c^2 d+(x^n d+c)^{\frac{1}{n}} c^3}dx\right) b^2$$
$$+2\left(\int \frac{x^{n}}{x^{3n} (x^n d+c)^{\frac{1}{n}} d^3+3x^{2n} (x^n d+c)^{\frac{1}{n}} c\, d^2+3x^n (x^n d+c)^{\frac{1}{n}} c^2 d+(x^n d+c)^{\frac{1}{n}} c^3}dx\right) ab$$
$$+\left(\int \frac{1}{x^{3n} (x^n d+c)^{\frac{1}{n}} d^3+3x^{2n} (x^n d+c)^{\frac{1}{n}} c\, d^2+3x^n (x^n d+c)^{\frac{1}{n}} c^2 d+(x^n d+c)^{\frac{1}{n}} c^3}dx\right) a^2$$

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

output
```
int(x**(2*n)/(x**(3*n)*(x**n*d + c)**(1/n)*d**3 + 3*x**(2*n)*(x**n*d + c)*
*(1/n)*c*d**2 + 3*x**n*(x**n*d + c)**(1/n)*c**2*d + (x**n*d + c)**(1/n)*c*
*3),x)*b**2 + 2*int(x**n/(x**(3*n)*(x**n*d + c)**(1/n)*d**3 + 3*x**(2*n)*(
x**n*d + c)**(1/n)*c*d**2 + 3*x**n*(x**n*d + c)**(1/n)*c**2*d + (x**n*d +
c)**(1/n)*c**3),x)*a*b + int(1/(x**(3*n)*(x**n*d + c)**(1/n)*d**3 + 3*x**(
2*n)*(x**n*d + c)**(1/n)*c*d**2 + 3*x**n*(x**n*d + c)**(1/n)*c**2*d + (x**
n*d + c)**(1/n)*c**3),x)*a**2
```

3.129 $\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx=-\frac{(bc-ad)x(c+dx^n)^{-1-\frac{1}{n}}}{cd(1+n)}+\frac{(bc+adn)x(c+dx^n)^{-1/n}}{c^2d(1+n)}$$

output
```
-(-a*d+b*c)*x*(c+d*x^n)^(-1-1/n)/c/d/(1+n)+(a*d*n+b*c)*x/c^2/d/(1+n)/((c+d
*x^n)^(1/n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(bcx^n+a(1+n)(c+dx^n)\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{c^2(1+n)}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2
F1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n
)^((1 + n)/n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)\left(c+dx^n\right)^{-\frac{1}{n}-2}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c(n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{x(a+bx^n)\left(c+dx^n\right)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 +
n)*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(72) = 144$.

Time = 0.99 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

method	result
parallelrisch	$\frac{x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}\,a\,d^2n+x\,x^{2n}(c+d\,x^n)^{-\frac{1+2n}{n}}\,bcd+2x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}\,acdn+x\,x^n(c+d\,x^n)^{-\frac{1+2n}{n}}\,acd+x\,x^n(c+d\,x^n)}{c^2(1+n)}$

input
```
int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x,method=_RETURNVERBOSE)
```

output
```
(x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)*a*d^2*n+x*(x^n)^2*(c+d*x^n)^(-(1+2*n)/n)
*b*c*d+2*x*x^n*(c+d*x^n)^(-(1+2*n)/n)*a*c*d*n+x*x^n*(c+d*x^n)^(-(1+2*n)/n)
*a*c*d+x*x^n*(c+d*x^n)^(-(1+2*n)/n)*b*c^2+x*(c+d*x^n)^(-(1+2*n)/n)*a*c^2*n
+x*(c+d*x^n)^(-(1+2*n)/n)*a*c^2)/c^2/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int (a+bx^n)\,(c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{(ad^2n+bcd)xx^{2\,n}+(2\,acdn+bc^2+acd)xx^n+(ac^2n+ac^2)x}{(c^2n+c^2)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

output
```
((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*
n + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(56) = 112$.

Time = 2.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.32

$$\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{acc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{ac^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{bc^{-2-\frac{1}{n}}c^{1+\frac{1}{n}}d^{-1-\frac{1}{n}}\left(\frac{cx^{-n}}{d}+1\right)^{-1-\frac{1}{n}}\Gamma\left(1+\frac{1}{n}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

output
```
a*c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)*
*(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma
(2 + 1/n)) + a*c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*
x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**
(1/n)*gamma(2 + 1/n)) + a*c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d*
*(1/n)*n**2*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*
(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n)) + b*c**(-2 - 1/n)*c**(1 + 1/n)*d**
(-1 - 1/n)*(c/(d*x**n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-2} \, dx$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,
[0,0,2,1,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n) (c+dx^n)^{-2-\frac{1}{n}} \, dx = \int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^{\frac{1}{n}+2}} \,d x$$

input
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

Reduce [F]

$$\int (a+bx^n)\,(c+dx^n)^{-2-\frac{1}{n}}\,dx$$
$$=\left(\int \frac{x^n}{x^{2n}\,(x^n d+c)^{\frac{1}{n}}\,d^2+2x^n\,(x^n d+c)^{\frac{1}{n}}\,cd+(x^n d+c)^{\frac{1}{n}}\,c^2}dx\right)b+\left(\int \frac{1}{x^{2n}\,(x^n d+c)^{\frac{1}{n}}\,d^2+2x^n\,(x^n d+c)^{\frac{1}{n}}\,cd+(x^n d+c)^{\frac{1}{n}}\,c^2}dx\right)a$$

input
```
int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

output
```
int(x**n/(x**(2*n)*(x**n*d + c)**(1/n)*d**2 + 2*x**n*(x**n*d + c)**(1/n)*c
*d + (x**n*d + c)**(1/n)*c**2),x)*b + int(1/(x**(2*n)*(x**n*d + c)**(1/n)*
d**2 + 2*x**n*(x**n*d + c)**(1/n)*c*d + (x**n*d + c)**(1/n)*c**2),x)*a
```

3.130 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

output
```
x/c/((c+d*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

input
```
Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

output
```
x/(c*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^{-\frac{1}{n}-1}\,dx$$

$$\Big\downarrow \text{746}$$

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

input
```
Int[(c + d*x^n)^(-1 - n^(-1)),x]
```

output
```
x/(c*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.

Time = 0.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

method	result	size
parallelrisch	$\frac{x\,x^{n}(c+d\,x^{n})^{-\frac{1+n}{n}}d+x(c+d\,x^{n})^{-\frac{1+n}{n}}c}{c}$	47
norman	$x\,\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}+\frac{xd\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-1-\frac{1}{n}\right)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}}{c}$	53

input
```
int((c+d*x^n)^(-1-1/n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*(c+d*x^n)^(-(1+n)/n)*d+x*(c+d*x^n)^(-(1+n)/n)*c)/c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int (c+dx^n)^{-1-\frac{1}{n}}\, dx = \frac{dxx^n + cx}{(dx^n + c)^{\frac{n+1}{n}}c}$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

output
```
(d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int (c+dx^n)^{-1-\frac{1}{n}}\, dx = \frac{c^{\frac{1}{n}}c^{-1-\frac{1}{n}}d^{-\frac{1}{n}}\left(\frac{cx^{-n}}{d}+1\right)^{-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**(-1-1/n),x)
```

output
```
c**(1/n)*c**(-1 - 1/n)*gamma(1/n)/(d**(1/n)*n*(c/(d*x**n) + 1)**(1/n)*gamm
a(1 + 1/n))
```

Maxima [F]

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-1}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 1), x)
```

Giac [F]

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-1}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.17

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \frac{d\,x^{n+1}\left(\frac{c}{d\,x^n}-\left(\frac{c}{d\,x^n}+1\right)^{\frac{n+1}{n}}+1\right)}{c\,n\,\left(\frac{n+1}{n}-1\right)\,\left(c+d\,x^n\right)^{\frac{n+1}{n}}}$$

input
```
int(1/(c + d*x^n)^(1/n + 1),x)
```

output
```
(d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/
n - 1)*(c + d*x^n)^((n + 1)/n))
```

Reduce [F]

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx = \int \frac{1}{x^n (x^n d+c)^{\frac{1}{n}} d + (x^n d+c)^{\frac{1}{n}} c}dx$$

input
```
int((c+d*x^n)^(-1-1/n),x)
```

output
```
int(1/(x**n*(x**n*d + c)**(1/n)*d + (x**n*d + c)**(1/n)*c),x)
```

3.131 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx = \frac{x(c+dx^n)^{-1/n} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

output

```
x*hypergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a/((c+d*x^n)^(1/
n))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx = \frac{x(c+dx^n)^{-1/n} \operatorname{Hypergeometric2F1}\left(1, \frac{1}{n}, 1+\frac{1}{n}, \frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a}$$

input

```
Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

output

```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d
*x^n))])/(a*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{904}$$

$$\frac{x(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

output
```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*
x^n)))])/(a*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{a+b\,x^n}dx$$

input
```
int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{1/n}}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n)), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx = \int \frac{1}{x^n\left(x^n d+c\right)^{\frac{1}{n}} b+\left(x^n d+c\right)^{\frac{1}{n}} a}dx$$

input
```
int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(x**n*(x**n*d + c)**(1/n)*b + (x**n*d + c)**(1/n)*a),x)
```

3.132 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

output
```
c*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/((c+d*x^n)
^(1/n))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} \text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, \frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

input
```
Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

output
```
(c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c +
 d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \text{904}$$

$$\frac{cx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

input
```
Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

output
```
(c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
d*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}dx$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2,x)
```

output
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$$
$$= \left(\int \frac{x^n}{x^{2n}(x^nd+c)^{\frac{1}{n}}b^2+2x^n(x^nd+c)^{\frac{1}{n}}ab+(x^nd+c)^{\frac{1}{n}}a^2}dx\right)d$$
$$+ \left(\int \frac{1}{x^{2n}(x^nd+c)^{\frac{1}{n}}b^2+2x^n(x^nd+c)^{\frac{1}{n}}ab+(x^nd+c)^{\frac{1}{n}}a^2}dx\right)c$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

output
```
int(x**n/(x**(2*n)*(x**n*d + c)**(1/n)*b**2 + 2*x**n*(x**n*d + c)**(1/n)*a
*b + (x**n*d + c)**(1/n)*a**2),x)*d + int(1/(x**(2*n)*(x**n*d + c)**(1/n)*
b**2 + 2*x**n*(x**n*d + c)**(1/n)*a*b + (x**n*d + c)**(1/n)*a**2),x)*c
```

3.133 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx=\frac{c^2x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

output
```
c^2*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^
n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx=\frac{c^2x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

input
```
Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
 + d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} \, dx$$

$$\Big\downarrow \text{904}$$

$$\frac{c^2x(c+dx^n)^{-1/n}\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

input
```
Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c
+ d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}dx$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*
b*x^n + a^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

input
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3,x)
```

output
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$$
$$= \left(\int \frac{x^{2n}}{x^{3n}(x^n d+c)^{\frac{1}{n}} b^3+3x^{2n}(x^n d+c)^{\frac{1}{n}} a b^2+3x^n(x^n d+c)^{\frac{1}{n}} a^2 b+(x^n d+c)^{\frac{1}{n}} a^3}dx\right) d^2$$
$$+2\left(\int \frac{x^{n}}{x^{3n}(x^n d+c)^{\frac{1}{n}} b^3+3x^{2n}(x^n d+c)^{\frac{1}{n}} a b^2+3x^n(x^n d+c)^{\frac{1}{n}} a^2 b+(x^n d+c)^{\frac{1}{n}} a^3}dx\right) cd$$
$$+\left(\int \frac{1}{x^{3n}(x^n d+c)^{\frac{1}{n}} b^3+3x^{2n}(x^n d+c)^{\frac{1}{n}} a b^2+3x^n(x^n d+c)^{\frac{1}{n}} a^2 b+(x^n d+c)^{\frac{1}{n}} a^3}dx\right) c^2$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

output
```
int(x**(2*n)/(x**(3*n)*(x**n*d + c)**(1/n)*b**3 + 3*x**(2*n)*(x**n*d + c)*
*(1/n)*a*b**2 + 3*x**n*(x**n*d + c)**(1/n)*a**2*b + (x**n*d + c)**(1/n)*a*
*3),x)*d**2 + 2*int(x**n/(x**(3*n)*(x**n*d + c)**(1/n)*b**3 + 3*x**(2*n)*(
x**n*d + c)**(1/n)*a*b**2 + 3*x**n*(x**n*d + c)**(1/n)*a**2*b + (x**n*d +
c)**(1/n)*a**3),x)*c*d + int(1/(x**(3*n)*(x**n*d + c)**(1/n)*b**3 + 3*x**(
2*n)*(x**n*d + c)**(1/n)*a*b**2 + 3*x**n*(x**n*d + c)**(1/n)*a**2*b + (x**
n*d + c)**(1/n)*a**3),x)*c**2
```

3.134 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 164

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx = \frac{x(a+bx^n)^{1+p} (c+dx^n)^{-1-\frac{1}{n}-p}}{ac} + \frac{(adn(1+p)-bc(1+n+np))x^{1+n}(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-1-\frac{1}{n}-p} \text{Hypergeometric2F1}\left(1+\right.}{ac^2(1+n)}$$

output
```
x*(a+b*x^n)^(p+1)*(c+d*x^n)^(-1-1/n-p)/a/c+(a*d*n*(p+1)-b*c*(n*p+n+1))*x^(
1+n)*(a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p)*hypergeom([-p, 1+1/n],[2+1/n],-(-a*d
+b*c)*x^n/a/(c+d*x^n))/a/c^2/(1+n)/((c*(a+b*x^n)/a/(c+d*x^n))^p)
```

Mathematica [F]

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx = \int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

output
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p), x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {907, 905}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p-2} \, dx$$

$$\Big\downarrow \text{907}$$

$$\frac{\left(\frac{bc}{n(p+1)(bc-ad)}+1\right)\int (bx^n+a)^{p+1}(dx^n+c)^{-p-\frac{1}{n}-2}dx}{a}-\frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

$$\Big\downarrow \text{905}$$

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{bc}{n(p+1)(bc-ad)}+1\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1}\text{Hypergeometric2F1}\left(\frac{1}{n},-p-1,1+\frac{1}{n},-\frac{(bc}{a(}\right.}{ac}$$
$$\frac{bx(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}}{an(p+1)(bc-ad)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

output
```
-((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n
*(1 + p))) + ((1 + (b*c)/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((
c*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyp
ergeometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*c)
```

Defintions of rubi rules used

rule 905
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*((a + b*x^n)^p/(c*(c*((a + b*x^n)/(a*(c + d*x^n))))^p*(c + d*x^n)
^(1/n + p)))*Hypergeometric2F1[1/n, -p, 1 + 1/n, (-(b*c - a*d))*(x^n/(a*(c
+ d*x^n)))], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [F]

$$\int (a+b\,x^n)^p\,(c+d\,x^n)^{-2-\frac{1}{n}-p}\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)^{-2-\frac{1}{n}-p}\,dx=\int (bx^n+a)^p(dx^n+c)^{-p-\frac{1}{n}-2}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Giac [F]

$$\int (a+bx^n)^p \left(c+dx^n\right)^{-2-\frac{1}{n}-p} \, dx = \int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx=\int \frac{(a+b\,x^n)^p}{(c+d\,x^n)^{p+\frac{1}{n}+2}}\,dx$$

input
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2),x)
```

output
```
int((a + b*x^n)^p/(c + d*x^n)^(p + 1/n + 2), x)
```

Reduce [F]

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx$$
$$=\int \frac{(x^nb+a)^p}{x^{2n}(x^nd+c)^{\frac{np+1}{n}}d^2+2x^n(x^nd+c)^{\frac{np+1}{n}}cd+(x^nd+c)^{\frac{np+1}{n}}c^2}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

output
```
int((x**n*b + a)**p/(x**(2*n)*(x**n*d + c)**((n*p + 1)/n)*d**2 + 2*x**n*(x
**n*d + c)**((n*p + 1)/n)*c*d + (x**n*d + c)**((n*p + 1)/n)*c**2),x)
```

3.135 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$

Optimal result

Integrand size = 69, antiderivative size = 57

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}} (c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

output
```
x*(c+d*x^n)^(a*d/(-a*d+b*c)/n)/a/c/((a+b*x^n)^(b*c/(-a*d+b*c)/n))
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}} (c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

input
```
Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^
((a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

output
```
(x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a
*d*n)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{\frac{adn-bc(n+1)}{n(bc-ad)}}(c+dx^n)^{\frac{adn+ad-bcn}{bcn-adn}}dx$$

$$\Big\downarrow \text{ 906}$$

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

input
```
Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d
 - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

output
```
(x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*
d)*n)))
```

Defintions of rubi rules used

rule 906
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(58) = 116.

Time = 6.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 5.86

method	result
parallelrisch	$\frac{x\,x^{2n}(a+b\,x^{n})^{\frac{-adn+bc(1+n)}{n(ad-bc)}}(c+d\,x^{n})^{-\frac{adn-bcn+ad}{n(ad-bc)}}b^{2}d^{2}+x\,x^{n}(a+b\,x^{n})^{\frac{-adn+bc(1+n)}{n(ad-bc)}}(c+d\,x^{n})^{-\frac{adn-bcn+ad}{n(ad-bc)}}ab\,d^{2}+x\,x^{n}(c}{abcd}$

input
```
int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d
)/(-a*d*n+b*c*n)),x,method=_RETURNVERBOSE)
```

output
```
(x*(x^n)^2*(a+b*x^n)^((-a*d*n+b*c*(1+n))/n/(a*d-b*c))*(c+d*x^n)^(-(a*d*n-b
*c*n+a*d)/n/(a*d-b*c))*b^2*d^2+x*x^n*(a+b*x^n)^((-a*d*n+b*c*(1+n))/n/(a*d-
b*c))*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/n/(a*d-b*c))*a*b*d^2+x*x^n*(a+b*x^n)^(
(-a*d*n+b*c*(1+n))/n/(a*d-b*c))*(c+d*x^n)^(-(a*d*n-b*c*n+a*d)/n/(a*d-b*c))
*b^2*c*d+x*(a+b*x^n)^((-a*d*n+b*c*(1+n))/n/(a*d-b*c))*(c+d*x^n)^(-(a*d*n-b
*c*n+a*d)/n/(a*d-b*c))*a*b*c*d)/a/b/c/d
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx = \frac{(bdxx^{2n}+acx+(bc+ad)xx^n)(dx^n+c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n+a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

output
```
(b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d
)*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))
*a*c)
```

Sympy [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int (a+bx^n)^{\frac{adn-bc(n+1)}{n(-ad+bc)}}(c+dx^n)^{\frac{adn+ad-bcn}{-adn+bcn}}\,dx$$

input
```
integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n
-b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

output
```
Integral((a + b*x**n)**((a*d*n - b*c*(n + 1))/(n*(-a*d + b*c)))*(c + d*x**
n)**((a*d*n + a*d - b*c*n)/(-a*d*n + b*c*n)), x)
```

Maxima [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n +
c)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Giac [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\int \frac{1}{(bx^n+a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^n+c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

input
```
integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c
*n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n +
c)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx = \int \frac{1}{(a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(n+1)}{n\,(a\,d-b\,c)}}\,(c+d\,x^n)^{\frac{a\,d+a\,d\,n-b\,c\,n}{a\,d\,n-b\,c\,n}}}\,dx$$

input
```
int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a
*d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
```

output
```
int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a
*d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

Reduce [F]

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}}(c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx = \int \frac{(x^nb+a)^{\frac{bc}{adn-bcn}}}{x^{2n}(x^nd+c)^{\frac{ad}{adn-bcn}}bd+x^n(x^nd+c)^{\frac{ad}{adn-bcn}}ad+x^n(x^nd+c)^{\frac{ad}{adn-bcn}}bc+(x^nd+c)^{\frac{ad}{adn-bcn}}ac}dx$$

input
```
int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d
)/(-a*d*n+b*c*n)),x)
```

output
```
int((x**n*b + a)**((b*c)/(a*d*n - b*c*n))/(x**(2*n)*(x**n*d + c)**((a*d)/(
a*d*n - b*c*n))*b*d + x**n*(x**n*d + c)**((a*d)/(a*d*n - b*c*n))*a*d + x**
n*(x**n*d + c)**((a*d)/(a*d*n - b*c*n))*b*c + (x**n*d + c)**((a*d)/(a*d*n
- b*c*n))*a*c),x)
```

3.136 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 253

$$\begin{aligned}\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = &-\frac{dx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(bc-ad)(1+3n)}\\ &-\frac{(3adn-b(c+3cn))x(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2(bc-ad)(1+2n)(1+3n)}\\ &+\frac{2an(3adn-b(c+3cn))x(c+dx^n)^{-1-\frac{1}{n}}}{c^3d(1+n)(1+2n)(1+3n)}\\ &+\frac{2an(bc+3bcn-3adn)(bc+adn)x(c+dx^n)^{-1/n}}{c^4d(bc-ad)(1+n)(1+2n)(1+3n)}\end{aligned}$$

output
```
-d*x*(a+b*x^n)^3*(c+d*x^n)^(-3-1/n)/c/(-a*d+b*c)/(1+3*n)-(3*a*d*n-b*(3*c*n
+c))*x*(a+b*x^n)^2*(c+d*x^n)^(-2-1/n)/c^2/(-a*d+b*c)/(1+2*n)/(1+3*n)+2*a*n
*(3*a*d*n-b*(3*c*n+c))*x*(c+d*x^n)^(-1-1/n)/c^3/d/(1+n)/(1+2*n)/(1+3*n)+2*
a*n*(-3*a*d*n+3*b*c*n+b*c)*(a*d*n+b*c)*x/c^4/d/(-a*d+b*c)/(1+n)/(1+2*n)/(1
+3*n)/((c+d*x^n)^(1/n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.54

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$$
$$= \frac{x(c+dx^n)^{-1/n} \left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}} \left(b^2c^2 \text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)-(bc-ad)\left(2bc \text{Hypergeo}\right.\right.}{c^4d^2}$$

input
```
Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
 + n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1)
, n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 +
 n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {907, 903, 903, 903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (c+dx^n)^{-\frac{1}{n}-4} \, dx$$

$$\Big\downarrow \text{907}$$

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\int (bx^n+a)^3 (dx^n+c)^{-4-\frac{1}{n}}dx}{3a}-\frac{bx(a+bx^n)^3 (c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$$\Big\downarrow \text{903}$$

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\int (bx^n+a)^2(dx^n+c)^{-3-\frac{1}{n}}dx}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$\big\downarrow$ 903

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\int (bx^n+a)(dx^n+c)^{-2-\frac{1}{n}}dx}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$\big\downarrow$ 903

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\left(\frac{an\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

$\big\downarrow$ 746

$$\frac{\left(\frac{bc}{bcn-adn}+3\right)\left(\frac{3an\left(\frac{2an\left(\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}\right)}{c(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}\right)}{c(3n+1)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}\right)}{3a}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

input
```
Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

output
```
-1/3*(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(a*(b*c - a*d)*n) + ((3
 + (b*c)/(b*c*n - a*d*n))*((x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*
(1 + 3*n)) + (3*a*n*((x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2
*n)) + (2*a*n*((x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*
n*x)/(c^2*(1 + n)*(c + d*x^n)^n^(-1))))/(c*(1 + 2*n))))/(c*(1 + 3*n))))/(3
*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(253) = 506$.

Time = 1.40 (sec) , antiderivative size = 1059, normalized size of antiderivative = 4.19

method	result	size
parallelrisch	Expression too large to display	1059

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x,method=_RETURNVERBOSE)
```

output

```
(3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n+26*x*x^n*(c+d*x^n)^(-(1+
4*n)/n)*a^2*c^3*d*n^2+12*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^4*n^2+2*x*(x^n
)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d+9*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^
3*d*n+10*x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^4*n+4*x*(x^n)^4*(c+d*x^n)^(-(1
+4*n)/n)*a*b*c*d^3*n^2+16*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n^2
+4*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^2*d^2*n+24*x*(x^n)^2*(c+d*x^n)^(
-(1+4*n)/n)*a*b*c^3*d*n^2+14*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a*b*c^3*d*n+
24*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*c*d^3*n^3+x*(x^n)^4*(c+d*x^n)^(-(1
+4*n)/n)*b^2*c^2*d^2*n+6*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*a^2*c*d^3*n^2+4*
x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^3*d*n^2+36*x*(x^n)^2*(c+d*x^n)^(-(1
+4*n)/n)*a^2*c^2*d^2*n^3+5*x*(x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^3*d*n+21
*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^2*d^2*n^2+24*x*x^n*(c+d*x^n)^(-(1+
4*n)/n)*a^2*c^3*d*n^3+x*(x^n)^4*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^2*d^2*n^2+x*(
x^n)^3*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^3*d+4*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)
*b^2*c^4*n+x*x^n*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^3*d+2*x*x^n*(c+d*x^n)^(-(1+4
*n)/n)*a*b*c^4+x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4+6*x*(c+d*x^n)^(-(1+4*n)/n)
*a^2*c^4*n^3+x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^4+11*x*(c+d*x^n)^(-(1+
4*n)/n)*a^2*c^4*n^2+6*x*(c+d*x^n)^(-(1+4*n)/n)*a^2*c^4*n+6*x*(x^n)^4*(c+d*
x^n)^(-(1+4*n)/n)*a^2*d^4*n^3+3*x*(x^n)^2*(c+d*x^n)^(-(1+4*n)/n)*b^2*c^4*n
^2)/(2*n^2+3*n+1)/(1+3*n)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.58

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$$
$$= \frac{(6\,a^2d^4n^3 + b^2c^2d^2n + (b^2c^2d^2 + 4\,abcd^3)n^2)xx^{4\,n} + (24\,a^2cd^3n^3 + b^2c^3d + 2\,(2\,b^2c^3d + 8\,abc^2d^2 + 3\,a^2cd^3}{}$$

input

```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

output
```
((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*
n) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^
2*c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^
2*n^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*
n^2 + (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^
3*d*n^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a
*b*c^4 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c
^4*n + a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^(
(4*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(219) = 438$.

Time = 15.89 (sec) , antiderivative size = 2746, normalized size of antiderivative = 10.85

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

output
```
6*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**3*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/
(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**
n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
1)**(1/n)*gamma(4 + 1/n)) + 11*a**2*c**3*c**(1/n)*c**(-4 - 1/n)*n**2*gamma
(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*
d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**
(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)
*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n)) + 6*a**2*c**3*c**(1
/n)*c**(-4 - 1/n)*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)
*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamm
a(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x**n) + 1)**(1/n)*gamma
(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) + 1)**(1/n)*gamma(4 +
1/n)) + a**2*c**3*c**(1/n)*c**(-4 - 1/n)*gamma(1/n)/(c**3*d**(1/n)*n**4*(c
/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x
**n) + 1)**(1/n)*gamma(4 + 1/n) + 3*c*d**2*d**(1/n)*n**4*x**(2*n)*(c/(d*x*
*n) + 1)**(1/n)*gamma(4 + 1/n) + d**3*d**(1/n)*n**4*x**(3*n)*(c/(d*x**n) +
 1)**(1/n)*gamma(4 + 1/n)) + 18*a**2*c**2*c**(1/n)*c**(-4 - 1/n)*d*n**3*x*
*n*gamma(1/n)/(c**3*d**(1/n)*n**4*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) +
 3*c**2*d*d**(1/n)*n**4*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(4 + 1/n) + 3...
```

Maxima [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx = \int (bx^n+a)^2 (dx^n+c)^{-\frac{1}{n}-4} \, dx$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{
9,[1,0,4,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx = \int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+4}} \, dx$$

input
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
```

output
```
int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

Reduce [F]

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}} \, dx$$
$$= \left(\int \frac{x^{2n}}{x^{4n} (x^n d+c)^{\frac{1}{n}} d^4 + 4x^{3n} (x^n d+c)^{\frac{1}{n}} c\,d^3 + 6x^{2n} (x^n d+c)^{\frac{1}{n}} c^2 d^2 + 4x^n (x^n d+c)^{\frac{1}{n}} c^3 d + (x^n d+c)^{\frac{1}{n}}\right.$$
$$+2\left(\int \frac{x^n}{x^{4n} (x^n d+c)^{\frac{1}{n}} d^4 + 4x^{3n} (x^n d+c)^{\frac{1}{n}} c\,d^3 + 6x^{2n} (x^n d+c)^{\frac{1}{n}} c^2 d^2 + 4x^n (x^n d+c)^{\frac{1}{n}} c^3 d + (x^n d+\right.$$
$$+\left(\int \frac{1}{x^{4n} (x^n d+c)^{\frac{1}{n}} d^4 + 4x^{3n} (x^n d+c)^{\frac{1}{n}} c\,d^3 + 6x^{2n} (x^n d+c)^{\frac{1}{n}} c^2 d^2 + 4x^n (x^n d+c)^{\frac{1}{n}} c^3 d + (x^n d+c\right.$$

input
```
int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

output
```
int(x**(2*n)/(x**(4*n)*(x**n*d + c)**(1/n)*d**4 + 4*x**(3*n)*(x**n*d + c)*
*(1/n)*c*d**3 + 6*x**(2*n)*(x**n*d + c)**(1/n)*c**2*d**2 + 4*x**n*(x**n*d
+ c)**(1/n)*c**3*d + (x**n*d + c)**(1/n)*c**4),x)*b**2 + 2*int(x**n/(x**(4
*n)*(x**n*d + c)**(1/n)*d**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*c*d**3 + 6*x
**(2*n)*(x**n*d + c)**(1/n)*c**2*d**2 + 4*x**n*(x**n*d + c)**(1/n)*c**3*d
+ (x**n*d + c)**(1/n)*c**4),x)*a*b + int(1/(x**(4*n)*(x**n*d + c)**(1/n)*d
**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*c*d**3 + 6*x**(2*n)*(x**n*d + c)**(1/
n)*c**2*d**2 + 4*x**n*(x**n*d + c)**(1/n)*c**3*d + (x**n*d + c)**(1/n)*c**
4),x)*a**2
```

3.137 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)x(c+dx^n)^{-1/n}}{c^3d(1+n)(1+2n)}$$

output
```
-(-a*d+b*c)*x*(c+d*x^n)^(-2-1/n)/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^(-1
-1/n)/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)
^(1/n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$$

$$= \frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\left(bc\,\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)+(-bc+ad)\,\text{Hypergeometr}\right.}{c^3d}$$

input
```
Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n
^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1)
, 1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {910, 777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(c+dx^n)^{-\frac{1}{n}-3}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2adn+bc)\int (dx^n+c)^{-2-\frac{1}{n}}dx}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

$$\Big\downarrow \text{777}$$

$$\frac{(2adn+bc)\left(\frac{n\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}\right)(2adn+bc)}{cd(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

input
```
Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

output
```
-(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a
*d*n)*((x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c +
 d*x^n)^n^(-1))))/(c*d*(1 + 2*n))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(127) = 254$.

Time = 1.00 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.42

method	result
parallelrisch	$\frac{2x\,x^{3n}(c+d\,x^n)^{-\frac{1+3n}{n}}a\,d^3n^2+x\,x^{3n}(c+d\,x^n)^{-\frac{1+3n}{n}}bc\,d^2n+6x\,x^{2n}(c+d\,x^n)^{-\frac{1+3n}{n}}ac\,d^2n^2+2x\,x^{2n}(c+d\,x^n)^{-\frac{1+3n}{n}}ac\,d^2n\cdots}{}$

input
```
int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x,method=_RETURNVERBOSE)
```

output
```
(2*x*(x^n)^3*(c+d*x^n)^(-(1+3*n)/n)*a*d^3*n^2+x*(x^n)^3*(c+d*x^n)^(-(1+3*n
)/n)*b*c*d^2*n+6*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n^2+2*x*(x^n)^2*
(c+d*x^n)^(-(1+3*n)/n)*a*c*d^2*n+3*x*(x^n)^2*(c+d*x^n)^(-(1+3*n)/n)*b*c^2*
d*n+6*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n^2+x*(x^n)^2*(c+d*x^n)^(-(1+3*
n)/n)*b*c^2*d+5*x*x^n*(c+d*x^n)^(-(1+3*n)/n)*a*c^2*d*n+2*x*x^n*(c+d*x^n)^(
-(1+3*n)/n)*b*c^3*n+2*x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3*n^2+x*x^n*(c+d*x^n)^(
-(1+3*n)/n)*a*c^2*d+x*x^n*(c+d*x^n)^(-(1+3*n)/n)*b*c^3+3*x*(c+d*x^n)^(-(1+
3*n)/n)*a*c^3*n+x*(c+d*x^n)^(-(1+3*n)/n)*a*c^3)/(1+n)/(1+2*n)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int (a+bx^n)\,(c+dx^n)^{-3-\frac{1}{n}}\,dx = \frac{(2\,ad^3n^2+bcd^2n)xx^{3\,n}+(6\,acd^2n^2+bc^2d+(3\,bc^2d+2\,acd^2)n)xx^{2\,n}+(6\,ac^2dn^2+bc^3+ac^2d+(2\,bc^3+}{(2\,c^3n^2+3\,c^3n+c^3)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

output
```
((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2
*d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3
 + 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2
+ 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. 2(105) = 210.

Time = 2.49 (sec) , antiderivative size = 959, normalized size of antiderivative = 7.55

$$\int (a+bx^n)\,(c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

output
```
2*a*c**2*c**(1/n)*c**(-3 - 2/n)*n**2*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c
)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 +
1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 3*a*c**2
*c**(1/n)*c**(-3 - 2/n)*n*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*ga
mma(3 + 1/n) + 2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2
*n**3*x**(2*n)*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + a*c**2*c**(1/n)*c**
(-3 - 2/n)*x*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) +
2*c*d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*
(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 4*a*c*c**(1/n)*c**(-3 - 2/n)*d*n**
2*x*x**n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*
d*n**3*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 +
 d*x**n/c)**(1/n)*gamma(3 + 1/n)) + 2*a*c*c**(1/n)*c**(-3 - 2/n)*d*n*x*x**
n*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3*
x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**n
/c)**(1/n)*gamma(3 + 1/n)) + 2*a*c**(1/n)*c**(-3 - 2/n)*d**2*n**2*x*x**(2*
n)*gamma(1/n)/(c**2*n**3*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + 2*c*d*n**3
*x**n*(1 + d*x**n/c)**(1/n)*gamma(3 + 1/n) + d**2*n**3*x**(2*n)*(1 + d*x**
n/c)**(1/n)*gamma(3 + 1/n)) + 2*b*c*c**(-3 - 1/n)*c**(1 + 1/n)*n*(c/(d*x**
n) + 1)**(-1 - 1/n)*gamma(1 + 1/n)/(c*d**(1 + 1/n)*n**2*gamma(3 + 1/n) + d
*d**(1 + 1/n)*n**2*x**n*gamma(3 + 1/n)) + b*c*c**(-3 - 1/n)*c**(1 + 1/n...
```

Maxima [F]

$$\int (a+bx^n)\left(c+dx^n\right)^{-3-\frac{1}{n}}\,dx=\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,
[0,0,2,1,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = \int \frac{a+b\,x^n}{(c+d\,x^n)^{\frac{1}{n}+3}}\,dx$$

input
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)
```

output
```
int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)
```

Reduce [F]

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$$
$$= \left(\int \frac{x^n}{x^{3n}(x^n d+c)^{\frac{1}{n}} d^3+3x^{2n}(x^n d+c)^{\frac{1}{n}} c\,d^2+3x^n(x^n d+c)^{\frac{1}{n}} c^2 d+(x^n d+c)^{\frac{1}{n}} c^3}dx\right) b$$
$$+ \left(\int \frac{1}{x^{3n}(x^n d+c)^{\frac{1}{n}} d^3+3x^{2n}(x^n d+c)^{\frac{1}{n}} c\,d^2+3x^n(x^n d+c)^{\frac{1}{n}} c^2 d+(x^n d+c)^{\frac{1}{n}} c^3}dx\right) a$$

input
```
int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

output
```
int(x**n/(x**(3*n)*(x**n*d + c)**(1/n)*d**3 + 3*x**(2*n)*(x**n*d + c)**(1/
n)*c*d**2 + 3*x**n*(x**n*d + c)**(1/n)*c**2*d + (x**n*d + c)**(1/n)*c**3),
x)*b + int(1/(x**(3*n)*(x**n*d + c)**(1/n)*d**3 + 3*x**(2*n)*(x**n*d + c)*
*(1/n)*c*d**2 + 3*x**n*(x**n*d + c)**(1/n)*c**2*d + (x**n*d + c)**(1/n)*c*
*3),x)*a
```

3.138 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}$$

output
```
x*(c+d*x^n)^(-1-1/n)/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^(1/n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx =\frac{x(c+dx^n)^{-1/n}\left(1+\frac{dx^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c^2}$$

input
```
Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1)
, -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {777, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)^{-\frac{1}{n}-2}dx$$

$$\Big\downarrow \text{777}$$

$$\frac{n\int (dx^n+c)^{-1-\frac{1}{n}}dx}{c(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

$$\Big\downarrow \text{746}$$

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

input
```
Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

output
```
(x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)
^n^(-1))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 777
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
 0] && NeQ[p, -1]
```

Maple [F]

$$\int (c + d\,x^n)^{-2-\frac{1}{n}}\,dx$$

input
```
int((c+d*x^n)^(-2-1/n),x)
```

output
```
int((c+d*x^n)^(-2-1/n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int (c + dx^n)^{-2-\frac{1}{n}}\,dx = \frac{d^2nxx^{2n} + (2\,cdn + cd)xx^n + (c^2n + c^2)x}{(c^2n + c^2)(dx^n + c)^{\frac{2\,n+1}{n}}}$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

output
```
(d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)
*(d*x^n + c)^((2*n + 1)/n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(39) = 78$.

Time = 0.83 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.00

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{cc^{\frac{1}{n}}c^{-2-\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)} + \frac{c^{\frac{1}{n}}c^{-2-\frac{1}{n}}dnx^n\Gamma\left(\frac{1}{n}\right)}{cd^{\frac{1}{n}}n^2\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)+dd^{\frac{1}{n}}n^2x^n\left(\frac{cx^{-n}}{d}+1\right)^{\frac{1}{n}}\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**n)**(-2-1/n),x)
```

output
```
c*c**(1/n)*c**(-2 - 1/n)*n*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n) + 1)**(
1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n)*gamma(2
 + 1/n)) + c*c**(1/n)*c**(-2 - 1/n)*gamma(1/n)/(c*d**(1/n)*n**2*(c/(d*x**n
) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*x**n) + 1)**(1/n
)*gamma(2 + 1/n)) + c**(1/n)*c**(-2 - 1/n)*d*n*x**n*gamma(1/n)/(c*d**(1/n)
*n**2*(c/(d*x**n) + 1)**(1/n)*gamma(2 + 1/n) + d*d**(1/n)*n**2*x**n*(c/(d*
x**n) + 1)**(1/n)*gamma(2 + 1/n))
```

Maxima [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx = \int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 2), x)
```

Giac [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\int (dx^n+c)^{-\frac{1}{n}-2}\,dx$$

input
```
integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 2), x)
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=-\frac{x^{1-2\,n}\,\left(\frac{c}{d\,x^n}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,\left(c+d\,x^n\right)^{1/n}}$$

input
```
int(1/(c + d*x^n)^(1/n + 2),x)
```

output
```
-(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n))
)/(2*d^2*n*(c + d*x^n)^(1/n))
```

Reduce [F]

$$\int (c+dx^n)^{-2-\frac{1}{n}}\,dx=\int \frac{1}{x^{2n}\,(x^nd+c)^{\frac{1}{n}}\,d^2+2x^n\,(x^nd+c)^{\frac{1}{n}}\,cd+(x^nd+c)^{\frac{1}{n}}\,c^2}dx$$

input
```
int((c+d*x^n)^(-2-1/n),x)
```

output
```
int(1/(x**(2*n)*(x**n*d + c)**(1/n)*d**2 + 2*x**n*(x**n*d + c)**(1/n)*c*d
+ (x**n*d + c)**(1/n)*c**2),x)
```

3.139 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-1/n}}{ac} - \frac{bx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{ax^{-n}(c+dx^n)}{bc-ad}\right)}{a(bc-ad)}$$

output
```
x/a/c/((c+d*x^n)^(1/n))-b*x*hypergeom([1, -1/n],[-(1-n)/n],-a*(c+d*x^n)/(-
a*d+b*c)/(x^n))/a/(-a*d+b*c)/((c+d*x^n)^(1/n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 6.67 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \frac{x(c+dx^n)^{-\frac{1+n}{n}}\left(\frac{a(c+dx^n)}{c(a+bx^n)}+\frac{bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)}{a}+\frac{b(-bc+ad)nx^{2n}\,\text{Hypergeometric2F1}\left(2,2+\frac{1}{n},3+\frac{1}{n},\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{a^2(1+2n)(c+dx^n)}\right)}{a}$$

input
```
Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

output
```
(x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*
d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*H
ypergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d
*x^n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-\frac{1}{n}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{b\int \frac{(dx^n+c)^{-1/n}}{bx^n+a}dx}{bc-ad}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

$$\Big\downarrow \; 904$$

$$\frac{bx(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

input
```
Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^
(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c
 + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(c+d\,x^{n})^{-1-\frac{1}{n}}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

output
```
int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx=\int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)^{\frac{1}{n}+1}}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx$$
$$= \int \frac{1}{x^{2n}\,(x^nd+c)^{\frac{1}{n}}\,bd + x^n\,(x^nd+c)^{\frac{1}{n}}\,ad + x^n\,(x^nd+c)^{\frac{1}{n}}\,bc + (x^nd+c)^{\frac{1}{n}}\,ac}dx$$

input
```
int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

output
```
int(1/(x**(2*n)*(x**n*d + c)**(1/n)*b*d + x**n*(x**n*d + c)**(1/n)*a*d + x
**n*(x**n*d + c)**(1/n)*b*c + (x**n*d + c)**(1/n)*a*c),x)
```

3.140 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx=\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n\,(a+bx^n)}-\frac{(adn+b(c-cn))x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)/((c+d*x^n)^((1-n)/n))-(a*d*n+b*(-c*n+c))*x*hy
pergeom([1, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^2/(-a*d+b*c)/n/((c
+d*x^n)^(1/n))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx=\frac{x(c+dx^n)^{\frac{-1+n}{n}}\left(bx^n\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+(an+b(-1+n)x^n)\,\Phi\left(\frac{(-b}{a(}\right.\right.}{n\,(a+bx^n)\left(-b(bc-ad)x^{2n}\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,1+\frac{1}{n}\right)+a\,(c+dx^n)\left(n\,(a+bx^n)-b\right.\right.}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

output
```
(x*(c + d*x^n)^((-1 + n)/n)*(b*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a
*(c + d*x^n)), 1, 1 + n^(-1)] + (a*n + b*(-1 + n)*x^n)*HurwitzLerchPhi[((-
(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)]))/(n*(a + b*x^n)*(-(b*(b*c -
 a*d)*x^(2*n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 +
 n^(-1)]) + a*(c + d*x^n)*(n*(a + b*x^n) - b*x^n*HurwitzLerchPhi[((-(b*c)
+ a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)}-\frac{(adn+bc(1-n))\int \frac{(dx^n+c)^{-1/n}}{bx^n+a}dx}{an(bc-ad)}$$

$$\Big\downarrow \; 904$$

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)}-\frac{x(c+dx^n)^{-1/n}(adn+bc(1-n))\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n
) + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/
(a*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{-\frac{1}{n}}}{(a+b\,x^n)^2}dx$$

input
```
int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

output
```
integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)^{1/n}}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2}\,dx = \int \frac{1}{x^{2n}\,(x^n d+c)^{\frac{1}{n}}\,b^2+2x^n\,(x^n d+c)^{\frac{1}{n}}\,ab+(x^n d+c)^{\frac{1}{n}}\,a^2}dx$$

input
```
int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

output
```
int(1/(x**(2*n)*(x**n*d + c)**(1/n)*b**2 + 2*x**n*(x**n*d + c)**(1/n)*a*b
+ (x**n*d + c)**(1/n)*a**2),x)
```

3.141 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\begin{aligned}\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx &= \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2} \\ &\quad - \frac{c\left(2ad-bc\left(2-\frac{1}{n}\right)\right)x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)}\end{aligned}$$

output
```
1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^2-1/2*c*(2*a*d-b*c*(2-1
/n))*x*hypergeom([2, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d+b
*c)/((c+d*x^n)^(1/n))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 8.83 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.56

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx =$$

$$-\frac{x(c+dx^n)^{2-\frac{1}{n}}\left((2an+b(-1+2n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)-bx\right.}{2n\,(a+bx^n)^2\left(-a\,(c+dx^n)\,(an+b(-1+n)x^n)\,\Phi\left(\frac{(-bc+ad)x^n}{a(c+dx^n)},1,-1+\frac{1}{n}\right)+x^n\left(-b(bc-ad)x^n\Phi\left(\right.\right.\right.}$$

input
```
Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
-1/2*(x*(c + d*x^n)^(2 - n^(-1))*((2*a*n + b*(-1 + 2*n)*x^n)*HurwitzLerchP
hi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)] - b*x^n*HurwitzLe
rchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)] - 2*(a*n + b*(
-1 + n)*x^n)*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-
1)]))/(n*(a + b*x^n)^2*(-(a*(c + d*x^n)*(a*n + b*(-1 + n)*x^n)*HurwitzLerc
hPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, -1 + n^(-1)]) + x^n*(-(b*(b*
c - a*d)*x^n*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, 1 +
n^(-1)]) + (a^2*d*n - b^2*c*(-1 + n)*x^n + a*b*(-(c*(1 + n)) + d*(-2 + n)*
x^n))*HurwitzLerchPhi[((-(b*c) + a*d)*x^n)/(a*(c + d*x^n)), 1, n^(-1)])))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \; 907$$

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2}-\frac{(2adn+bc(1-2n))\int\frac{(dx^n+c)^{-\frac{1-n}{n}}}{(bx^n+a)^2}dx}{2an(bc-ad)}$$

$$\Big\downarrow \text{904}$$

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2}-\frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

input
```
Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

output
```
(b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c
*(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^3}dx$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*
x^n + a^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+1}}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \int \frac{(c+d\,x^n)^{1-\frac{1}{n}}}{(a+b\,x^n)^3}\,dx$$

input
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3,x)
```

output
```
int((c + d*x^n)^(1 - 1/n)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$$
$$=\left(\int \frac{x^n}{x^{3n}\,(x^nd+c)^{\frac{1}{n}}\,b^3+3x^{2n}\,(x^nd+c)^{\frac{1}{n}}\,a\,b^2+3x^n\,(x^nd+c)^{\frac{1}{n}}\,a^2b+(x^nd+c)^{\frac{1}{n}}\,a^3}dx\right)d$$
$$+\left(\int \frac{1}{x^{3n}\,(x^nd+c)^{\frac{1}{n}}\,b^3+3x^{2n}\,(x^nd+c)^{\frac{1}{n}}\,a\,b^2+3x^n\,(x^nd+c)^{\frac{1}{n}}\,a^2b+(x^nd+c)^{\frac{1}{n}}\,a^3}dx\right)c$$

input
```
int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

output
```
int(x**n/(x**(3*n)*(x**n*d + c)**(1/n)*b**3 + 3*x**(2*n)*(x**n*d + c)**(1/
n)*a*b**2 + 3*x**n*(x**n*d + c)**(1/n)*a**2*b + (x**n*d + c)**(1/n)*a**3),
x)*d + int(1/(x**(3*n)*(x**n*d + c)**(1/n)*b**3 + 3*x**(2*n)*(x**n*d + c)*
*(1/n)*a*b**2 + 3*x**n*(x**n*d + c)**(1/n)*a**2*b + (x**n*d + c)**(1/n)*a*
*3),x)*c
```

3.142 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} \, dx = \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3} - \frac{c^2\left(3ad-bc\left(3-\frac{1}{n}\right)\right)x(c+dx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)}$$

output
```
1/3*b*x*(c+d*x^n)^(3-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^3-1/3*c^2*(3*a*d-b*c*(3
-1/n))*x*hypergeom([3, 1/n],[1+1/n],-(-a*d+b*c)*x^n/a/(c+d*x^n))/a^4/(-a*d
+b*c)/((c+d*x^n)^(1/n))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 37.05 (sec) , antiderivative size = 6405, normalized size of antiderivative = 48.52

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {907, 904}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx$$

$$\Big\downarrow \text{907}$$

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{(3adn+bc(1-3n))\int \frac{(dx^n+c)^{2-\frac{1}{n}}}{(bx^n+a)^3}dx}{3an(bc-ad)}$$

$$\Big\downarrow \text{904}$$

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

input
```
Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

output
```
(b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b
*c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c
 - a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 904
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*(x/(c^(p + 1)*(c + d*x^n)^(1/n)))*Hypergeometric2F1[1/n, -p, 1
+ 1/n, (-(b*c - a*d))*(x^n/(a*(c + d*x^n)))], x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

rule 907
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d))
  Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}
, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !
LtQ[q, -1]) && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(c + d\,x^n)^{2-\frac{1}{n}}}{(a + b\,x^n)^4}dx$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

output
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

Fricas [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*
b^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx = \int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Giac [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx=\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

input
```
integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx=\int \frac{(c+d\,x^n)^{2-\frac{1}{n}}}{(a+b\,x^n)^4}\,dx$$

input
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4,x)
```

output
```
int((c + d*x^n)^(2 - 1/n)/(a + b*x^n)^4, x)
```

Reduce [F]

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}\,dx$$
$$=\left(\int \frac{x^{2n}}{x^{4n}(x^nd+c)^{\frac{1}{n}}b^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}a\,b^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}a^2b^2+4x^n(x^nd+c)^{\frac{1}{n}}a^3b+(x^nd+c)^{\frac{1}{n}}\right.$$
$$+2\left(\int \frac{x^n}{x^{4n}(x^nd+c)^{\frac{1}{n}}b^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}a\,b^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}a^2b^2+4x^n(x^nd+c)^{\frac{1}{n}}a^3b+(x^nd+\right.$$
$$+\left(\int \frac{1}{x^{4n}(x^nd+c)^{\frac{1}{n}}b^4+4x^{3n}(x^nd+c)^{\frac{1}{n}}a\,b^3+6x^{2n}(x^nd+c)^{\frac{1}{n}}a^2b^2+4x^n(x^nd+c)^{\frac{1}{n}}a^3b+(x^nd+c\right.$$

input
```
int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

output
```
int(x**(2*n)/(x**(4*n)*(x**n*d + c)**(1/n)*b**4 + 4*x**(3*n)*(x**n*d + c)*
*(1/n)*a*b**3 + 6*x**(2*n)*(x**n*d + c)**(1/n)*a**2*b**2 + 4*x**n*(x**n*d
+ c)**(1/n)*a**3*b + (x**n*d + c)**(1/n)*a**4),x)*d**2 + 2*int(x**n/(x**(4
*n)*(x**n*d + c)**(1/n)*b**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*a*b**3 + 6*x
**(2*n)*(x**n*d + c)**(1/n)*a**2*b**2 + 4*x**n*(x**n*d + c)**(1/n)*a**3*b
+ (x**n*d + c)**(1/n)*a**4),x)*c*d + int(1/(x**(4*n)*(x**n*d + c)**(1/n)*b
**4 + 4*x**(3*n)*(x**n*d + c)**(1/n)*a*b**3 + 6*x**(2*n)*(x**n*d + c)**(1/
n)*a**2*b**2 + 4*x**n*(x**n*d + c)**(1/n)*a**3*b + (x**n*d + c)**(1/n)*a**
4),x)*c**2
```

3.143 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

output
```
x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n,-p,-q,1+1/n,-b*x^n/a,-d*x^n/c)/((1+
b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(81) = 162$.

Time = 0.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.35

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \frac{ac(1+n)x(a+bx^n)^p (c+dx^n)^q \operatorname{AppellF1}\left(\frac{1}{n},-p,-q,1 + \ldots\right.}{bcnpx^n \operatorname{AppellF1}\left(1+\frac{1}{n},1-p,-q,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right) + adnqx^n \operatorname{AppellF1}\left(1+\frac{1}{n},-p,1-q,2+\frac{1}{n},-\frac{b\ldots}{}\right.}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^
(-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p
, -q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n
^(-1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Ap
pellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \int \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{936}$$

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x
^n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{p} (c+d\,x^{n})^{q}\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int (a+bx^n)^p (c+dx^n)^q \,dx = \int (bx^n+a)^p (dx^n+c)^q \,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Giac [F]

$$\int (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + bx^n)^p (c + dx^n)^q \, dx = \int (a + b\,x^n)^p \,(c + d\,x^n)^q \, dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int (a + bx^n)^p (c + dx^n)^q \, dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
((x**n*d + c)**q*(x**n*b + a)**p*a*d*x + (x**n*d + c)**q*(x**n*b + a)**p*b
*c*x + int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*n*q + x**(
2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2
*n*q + x**n*a**2*d**2 + x**n*a*b*c*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c
*d + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*n*q + a**2*c*d + a*b*c
**2*n*p + a*b*c**2),x)*a**3*c*d**2*n**2*q**2 + int(((x**n*d + c)**q*(x**n*
b + a)**p)/(x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*
n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + x**n*a*b*c
*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*n*p + x**n*b**
2*c**2 + a**2*c*d*n*q + a**2*c*d + a*b*c**2*n*p + a*b*c**2),x)*a**3*c*d**2
*n*q + 2*int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*n*q + x*
*(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d*
*2*n*q + x**n*a**2*d**2 + x**n*a*b*c*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b
*c*d + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*n*q + a**2*c*d + a*b
*c**2*n*p + a*b*c**2),x)*a**2*b*c**2*d*n**2*p*q + int(((x**n*d + c)**q*(x*
*n*b + a)**p)/(x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c
*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + x**n*a*
b*c*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*n*p + x**n*
b**2*c**2 + a**2*c*d*n*q + a**2*c*d + a*b*c**2*n*p + a*b*c**2),x)*a**2*b*c
**2*d*n*p + int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*n*...
```

3.144 $\int (2+3x^n)^p (5+7x^n)^{-p} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (2+3x^n)^p (5+7x^n)^{-p} \, dx = \left(\frac{2}{5}\right)^p x \operatorname{AppellF1}\left(\frac{1}{n}, -p, p, 1+\frac{1}{n}, -\frac{3x^n}{2}, -\frac{7x^n}{5}\right)$$

output
```
(2/5)^p*x*AppellF1(1/n,p,-p,1+1/n,-7/5*x^n,-3/2*x^n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.82

$$\int (2+3x^n)^p (5+7x^n)^{-p} \, dx$$
$$= \frac{10(1+n)x(2+3x^n)^p (5+7x^n)^{-p} \operatorname{AppellF1}\left(\frac{1}{n}, -p, p, 1+\right.}{15npx^n \operatorname{AppellF1}\left(1+\frac{1}{n}, 1-p, p, 2+\frac{1}{n}, -\frac{3x^n}{2}, -\frac{7x^n}{5}\right) - 14npx^n \operatorname{AppellF1}\left(1+\frac{1}{n}, -p, 1+p, 2+\frac{1}{n}, -\frac{3x^n}{2}\right.}$$

input
```
Integrate[(2 + 3*x^n)^p/(5 + 7*x^n)^p,x]
```

output
```
(10*(1 + n)*x*(2 + 3*x^n)^p*AppellF1[n^(-1), -p, p, 1 + n^(-1), (-3*x^n)/2
, (-7*x^n)/5])/((5 + 7*x^n)^p*(15*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, p, 2
 + n^(-1), (-3*x^n)/2, (-7*x^n)/5] - 14*n*p*x^n*AppellF1[1 + n^(-1), -p, 1
 + p, 2 + n^(-1), (-3*x^n)/2, (-7*x^n)/5] + 10*(1 + n)*AppellF1[n^(-1), -p
, p, 1 + n^(-1), (-3*x^n)/2, (-7*x^n)/5]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^n+2)^p (7x^n+5)^{-p} \, dx$$

$$\Big\downarrow \text{936}$$

$$\left(\frac{2}{5}\right)^p x \,\text{AppellF1}\left(\frac{1}{n},-p,p,1+\frac{1}{n},-\frac{3x^n}{2},-\frac{7x^n}{5}\right)$$

input
```
Int[(2 + 3*x^n)^p/(5 + 7*x^n)^p,x]
```

output
```
(2/5)^p*x*AppellF1[n^(-1), -p, p, 1 + n^(-1), (-3*x^n)/2, (-7*x^n)/5]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Maple [F]

$$\int (2+3x^n)^p (5+7x^n)^{-p} \, dx$$

input
```
int((2+3*x^n)^p/((5+7*x^n)^p),x)
```

output
```
int((2+3*x^n)^p/((5+7*x^n)^p),x)
```

Fricas [F]

$$\int (2+3x^n)^p (5+7x^n)^{-p} \, dx = \int \frac{(3x^n+2)^p}{(7x^n+5)^p} \, dx$$

input
```
integrate((2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="fricas")
```

output
```
integral((3*x^n + 2)^p/(7*x^n + 5)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (2+3x^n)^p (5+7x^n)^{-p} \, dx = \text{Timed out}$$

input
```
integrate((2+3*x**n)**p/((5+7*x**n)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (2+3x^n)^p (5+7x^n)^{-p}\,dx = \int \frac{(3\,x^n+2)^p}{(7\,x^n+5)^p}\,dx$$

input
```
integrate((2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="maxima")
```

output
```
integrate((3*x^n + 2)^p/(7*x^n + 5)^p, x)
```

Giac [F]

$$\int (2+3x^n)^p (5+7x^n)^{-p}\,dx = \int \frac{(3\,x^n+2)^p}{(7\,x^n+5)^p}\,dx$$

input
```
integrate((2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="giac")
```

output
```
integrate((3*x^n + 2)^p/(7*x^n + 5)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+3x^n)^p (5+7x^n)^{-p}\,dx = \int \frac{(3\,x^n+2)^p}{(7\,x^n+5)^p}\,dx$$

input
```
int((3*x^n + 2)^p/(7*x^n + 5)^p,x)
```

output
```
int((3*x^n + 2)^p/(7*x^n + 5)^p, x)
```

Reduce [F]

$$\int (2+3x^n)^p (5+7x^n)^{-p}\,dx = \int \frac{(3x^n+2)^p}{(7x^n+5)^p}dx$$

input
```
int((2+3*x^n)^p/((5+7*x^n)^p),x)
```

output
```
int((3*x**n + 2)**p/(7*x**n + 5)**p,x)
```

3.145 $\int (5-7x^n)^{-p} (2+3x^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx = \left(\frac{2}{5}\right)^p x \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\frac{7x^n}{5},-\frac{3x^n}{2}\right)$$

output
```
(2/5)^p*x*AppellF1(1/n,-p,p,1+1/n,-3/2*x^n,7/5*x^n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.82

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx$$
$$= \frac{10(1+n)x(5-7x^n)^{-p} (2+3x^n)^p \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\right.}{15npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},p,1-p,2+\frac{1}{n},\frac{7x^n}{5},-\frac{3x^n}{2}\right)+14npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},1+p,-p,2+\frac{1}{n},\frac{7x^n}{5},-\right.}$$

input
```
Integrate[(2 + 3*x^n)^p/(5 - 7*x^n)^p,x]
```

output
```
(10*(1 + n)*x*(2 + 3*x^n)^p*AppellF1[n^(-1), p, -p, 1 + n^(-1), (7*x^n)/5,
 (-3*x^n)/2])/((5 - 7*x^n)^p*(15*n*p*x^n*AppellF1[1 + n^(-1), p, 1 - p, 2
+ n^(-1), (7*x^n)/5, (-3*x^n)/2] + 14*n*p*x^n*AppellF1[1 + n^(-1), 1 + p,
-p, 2 + n^(-1), (7*x^n)/5, (-3*x^n)/2] + 10*(1 + n)*AppellF1[n^(-1), p, -p
, 1 + n^(-1), (7*x^n)/5, (-3*x^n)/2]))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-7x^n)^{-p} (3x^n+2)^p \, dx$$

$$\Big\downarrow \text{936}$$

$$\left(\frac{2}{5}\right)^p x \,\text{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\frac{7x^n}{5},-\frac{3x^n}{2}\right)$$

input
```
Int[(2 + 3*x^n)^p/(5 - 7*x^n)^p,x]
```

output
```
(2/5)^p*x*AppellF1[n^(-1), p, -p, 1 + n^(-1), (7*x^n)/5, (-3*x^n)/2]
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Maple [F]

$$\int (2+3x^n)^p (5-7x^n)^{-p}\,dx$$

input
```
int((2+3*x^n)^p/((5-7*x^n)^p),x)
```

output
```
int((2+3*x^n)^p/((5-7*x^n)^p),x)
```

Fricas [F]

$$\int (5-7x^n)^{-p} (2+3x^n)^p\,dx = \int \frac{(3\,x^n+2)^p}{(-7\,x^n+5)^p}\,dx$$

input
```
integrate((2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="fricas")
```

output
```
integral((3*x^n + 2)^p/(-7*x^n + 5)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (5-7x^n)^{-p} (2+3x^n)^p\,dx = \text{Timed out}$$

input
```
integrate((2+3*x**n)**p/((5-7*x**n)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx = \int \frac{(3\,x^n+2)^p}{(-7\,x^n+5)^p}\, dx$$

input
```
integrate((2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="maxima")
```

output
```
integrate((3*x^n + 2)^p/(-7*x^n + 5)^p, x)
```

Giac [F]

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx = \int \frac{(3\,x^n+2)^p}{(-7\,x^n+5)^p}\, dx$$

input
```
integrate((2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="giac")
```

output
```
integrate((3*x^n + 2)^p/(-7*x^n + 5)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx = \int \frac{(3\,x^n+2)^p}{(5-7\,x^n)^p}\, dx$$

input
```
int((3*x^n + 2)^p/(5 - 7*x^n)^p,x)
```

output
```
int((3*x^n + 2)^p/(5 - 7*x^n)^p, x)
```

Reduce [F]

$$\int (5-7x^n)^{-p} (2+3x^n)^p \, dx = \int \frac{(3x^n+2)^p}{(-7x^n+5)^p} dx$$

input
```
int((2+3*x^n)^p/((5-7*x^n)^p),x)
```

output
```
int((3*x**n + 2)**p/( - 7*x**n + 5)**p,x)
```

3.146 $\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx = 5^{-p}x\left(1-\frac{3x^n}{2}\right)^{-p} (-2+3x^n)^p \operatorname{AppellF1}\left(\frac{1}{n},-p,p,1+\frac{1}{n},\frac{3x^n}{2},-\frac{7x^n}{5}\right)$$

output
```
x*(-2+3*x^n)^p*AppellF1(1/n,p,-p,1+1/n,-7/5*x^n,3/2*x^n)/(5^p)/((1-3/2*x^n
)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(56) = 112.

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.93

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx$$
$$= \frac{10(1+n)x(-2+3x^n)^p (5+7x^n)^{-p} \operatorname{AppellF1}\left(\frac{1}{n},-p,p,1+\right.}{-15npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},1-p,p,2+\frac{1}{n},\frac{3x^n}{2},-\frac{7x^n}{5}\right)-14npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},-p,1+p,2+\frac{1}{n},\frac{3x^n}{2},\right.}$$

input
```
Integrate[(-2 + 3*x^n)^p/(5 + 7*x^n)^p,x]
```

output
```
(10*(1 + n)*x*(-2 + 3*x^n)^p*AppellF1[n^(-1), -p, p, 1 + n^(-1), (3*x^n)/2
, (-7*x^n)/5])/((5 + 7*x^n)^p*(-15*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, p,
2 + n^(-1), (3*x^n)/2, (-7*x^n)/5] - 14*n*p*x^n*AppellF1[1 + n^(-1), -p, 1
 + p, 2 + n^(-1), (3*x^n)/2, (-7*x^n)/5] + 10*(1 + n)*AppellF1[n^(-1), -p,
 p, 1 + n^(-1), (3*x^n)/2, (-7*x^n)/5]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^n-2)^p (7x^n+5)^{-p}\,dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-\frac{3x^n}{2}\right)^{-p}(3x^n-2)^p\int \left(1-\frac{3x^n}{2}\right)^p (7x^n+5)^{-p}dx$$

$$\Big\downarrow \text{936}$$

$$5^{-p}x\left(1-\frac{3x^n}{2}\right)^{-p}(3x^n-2)^p \operatorname{AppellF1}\left(\frac{1}{n},-p,p,1+\frac{1}{n},\frac{3x^n}{2},-\frac{7x^n}{5}\right)$$

input
```
Int[(-2 + 3*x^n)^p/(5 + 7*x^n)^p,x]
```

output
```
(x*(-2 + 3*x^n)^p*AppellF1[n^(-1), -p, p, 1 + n^(-1), (3*x^n)/2, (-7*x^n)/
5])/(5^p*(1 - (3*x^n)/2)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx$$

input
```
int((-2+3*x^n)^p/((5+7*x^n)^p),x)
```

output
```
int((-2+3*x^n)^p/((5+7*x^n)^p),x)
```

Fricas [F]

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx = \int \frac{(3\,x^n-2)^p}{(7\,x^n+5)^p} \, dx$$

input
```
integrate((-2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="fricas")
```

output
```
integral((3*x^n - 2)^p/(7*x^n + 5)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (-2+3x^n)^p (5+7x^n)^{-p}\, dx = \text{Timed out}$$

input
```
integrate((-2+3*x**n)**p/((5+7*x**n)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (-2+3x^n)^p (5+7x^n)^{-p}\, dx = \int \frac{(3\,x^n-2)^p}{(7\,x^n+5)^p}\, dx$$

input
```
integrate((-2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="maxima")
```

output
```
integrate((3*x^n - 2)^p/(7*x^n + 5)^p, x)
```

Giac [F]

$$\int (-2+3x^n)^p (5+7x^n)^{-p}\, dx = \int \frac{(3\,x^n-2)^p}{(7\,x^n+5)^p}\, dx$$

input
```
integrate((-2+3*x^n)^p/((5+7*x^n)^p),x, algorithm="giac")
```

output
```
integrate((3*x^n - 2)^p/(7*x^n + 5)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx = \int \frac{(3\,x^n-2)^p}{(7\,x^n+5)^p} \, dx$$

input
```
int((3*x^n - 2)^p/(7*x^n + 5)^p,x)
```

output
```
int((3*x^n - 2)^p/(7*x^n + 5)^p, x)
```

Reduce [F]

$$\int (-2+3x^n)^p (5+7x^n)^{-p} \, dx = \int \frac{(3x^n-2)^p}{(7x^n+5)^p} dx$$

input
```
int((-2+3*x^n)^p/((5+7*x^n)^p),x)
```

output
```
int((3*x**n - 2)**p/(7*x**n + 5)**p,x)
```

3.147 $\int (5-7x^n)^{-p} (-2+3x^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int (5-7x^n)^{-p} (-2+3x^n)^p \, dx = 5^{-p} x \left(1-\frac{3x^n}{2}\right)^{-p} (-2+3x^n)^p \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\frac{7x^n}{5},\frac{3x^n}{2}\right)$$

output
```
x*(-2+3*x^n)^p*AppellF1(1/n,-p,p,1+1/n,3/2*x^n,7/5*x^n)/(5^p)/((1-3/2*x^n)
^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 164 vs. 2(56) = 112.

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.93

$$\int (5-7x^n)^{-p} (-2+3x^n)^p \, dx = \frac{10(1+n)x(5-7x^n)^{-p}(-2+3x^n)^p \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\right.}{-15npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},p,1-p,2+\frac{1}{n},\frac{7x^n}{5},\frac{3x^n}{2}\right)+14npx^n \operatorname{AppellF1}\left(1+\frac{1}{n},1+p,-p,2+\frac{1}{n},\frac{7x^n}{5},\frac{3x}{2}\right.}$$

input
```
Integrate[(-2 + 3*x^n)^p/(5 - 7*x^n)^p,x]
```

output
```
(10*(1 + n)*x*(-2 + 3*x^n)^p*AppellF1[n^(-1), p, -p, 1 + n^(-1), (7*x^n)/5
, (3*x^n)/2])/((5 - 7*x^n)^p*(-15*n*p*x^n*AppellF1[1 + n^(-1), p, 1 - p, 2
 + n^(-1), (7*x^n)/5, (3*x^n)/2] + 14*n*p*x^n*AppellF1[1 + n^(-1), 1 + p,
-p, 2 + n^(-1), (7*x^n)/5, (3*x^n)/2] + 10*(1 + n)*AppellF1[n^(-1), p, -p,
 1 + n^(-1), (7*x^n)/5, (3*x^n)/2]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5-7x^n)^{-p} (3x^n-2)^p \, dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-\frac{3x^n}{2}\right)^{-p} (3x^n-2)^p \int (5-7x^n)^{-p} \left(1-\frac{3x^n}{2}\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$5^{-p} x \left(1-\frac{3x^n}{2}\right)^{-p} (3x^n-2)^p \operatorname{AppellF1}\left(\frac{1}{n},p,-p,1+\frac{1}{n},\frac{7x^n}{5},\frac{3x^n}{2}\right)$$

input
```
Int[(-2 + 3*x^n)^p/(5 - 7*x^n)^p,x]
```

output
```
(x*(-2 + 3*x^n)^p*AppellF1[n^(-1), p, -p, 1 + n^(-1), (7*x^n)/5, (3*x^n)/2
])/(5^p*(1 - (3*x^n)/2)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (-2+3x^n)^p (5-7x^n)^{-p} \, dx$$

input
```
int((-2+3*x^n)^p/((5-7*x^n)^p),x)
```

output
```
int((-2+3*x^n)^p/((5-7*x^n)^p),x)
```

Fricas [F]

$$\int (5-7x^n)^{-p} (-2+3x^n)^p \, dx = \int \frac{(3\,x^n-2)^p}{(-7\,x^n+5)^p} \, dx$$

input
```
integrate((-2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="fricas")
```

output
```
integral((3*x^n - 2)^p/(-7*x^n + 5)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (5-7x^n)^{-p}(-2+3x^n)^p\,dx = \text{Timed out}$$

input
```
integrate((-2+3*x**n)**p/((5-7*x**n)**p),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (5-7x^n)^{-p}(-2+3x^n)^p\,dx = \int \frac{(3\,x^n-2)^p}{(-7\,x^n+5)^p}\,dx$$

input
```
integrate((-2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="maxima")
```

output
```
integrate((3*x^n - 2)^p/(-7*x^n + 5)^p, x)
```

Giac [F]

$$\int (5-7x^n)^{-p}(-2+3x^n)^p\,dx = \int \frac{(3\,x^n-2)^p}{(-7\,x^n+5)^p}\,dx$$

input
```
integrate((-2+3*x^n)^p/((5-7*x^n)^p),x, algorithm="giac")
```

output
```
integrate((3*x^n - 2)^p/(-7*x^n + 5)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (5-7x^n)^{-p}(-2+3x^n)^p\,dx=\int \frac{(3\,x^n-2)^p}{(5-7\,x^n)^p}\,dx$$

input
```
int((3*x^n - 2)^p/(5 - 7*x^n)^p,x)
```

output
```
int((3*x^n - 2)^p/(5 - 7*x^n)^p, x)
```

Reduce [F]

$$\int (5-7x^n)^{-p}(-2+3x^n)^p\,dx=\int \frac{(3x^n-2)^p}{(-7x^n+5)^p}dx$$

input
```
int((-2+3*x^n)^p/((5-7*x^n)^p),x)
```

output
```
int((3*x**n - 2)**p/( - 7*x**n + 5)**p,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file